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## The Safety Premium of Safe Assets

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#### Abstract

Safe assets usually trade at a premium due to their high credit quality and deep liquidity. To understand the role of credit quality for such premia, we focus on Swiss Confederation bonds, which are extremely safe but not particularly liquid. We therefore refer to their premia as *safety premia* and quantify them using an arbitrage-free term structure model that accounts for time-varying premia in individual bond prices. The estimation results show that Swiss safety premia are large and exhibit long-lasting trends. Furthermore, our regression analysis suggests that they shifted upwards persistently following the launch of the euro but have been depressed in recent years by the asset purchases of the European Central Bank.

JEL Classification: C32, E43, E52, F34, G12

*Keywords:* affine arbitrage-free term structure model, bond-specific risk premia, euro launch, negative interest rates

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In recent years, a growing body of research has emphasized the importance of safe assets for a number of global macroeconomic trends. Due to global investment and savings imbalances between developed and emerging market economies,<sup>1</sup> as argued by Caballero et al. (2017), a shortage of safe assets partially explains the low level of interest rates, including the natural real rate, in many countries.<sup>2</sup> This situation may in turn have implications for financial stability if it makes investors reach for higher yields, as discussed in Gorton (2017). Furthermore, since nominal yields are constrained by a lower bound near zero, a lack of safe assets may also be one of the factors that has contributed to constraining monetary policy around the world since the global financial crisis.<sup>3</sup> Hence, the properties of safe assets and their pricing merit further examination.

To better understand the market pricing of safe assets, it is useful to first discuss what makes a safe asset. In the existing literature, there are many overlapping definitions. Gorton (2017) describes a safe asset as one that is (almost always) valued at face value without expensive and prolonged analysis, i.e., there is little to no value of private information and hence no risk of selection bias or other strategic motives for trading. Gourinchas and Jeanne (2012) define it as a liquid debt claim with negligible default risk that provides a secure store of value. Caballero et al. (2017) emphasize that it is a debt instrument that is expected to preserve its value during adverse systemic events. Hence, common threads across these definitions are that a safe asset should have the properties of providing security, such that it pays close to par with near certainty in the future, and providing liquidity, such that it is similar to money in its availability and acceptability.

As a consequence of these characteristics, safe assets serve as a useful benchmark for several important tasks. They play a transaction role by serving as collateral in financial transactions and regulatory capital in meeting liquidity requirements. They serve as an accessible store of value by providing a reliable return. In addition, they serve an accounting role as a benchmark for the pricing of other assets (see IMF 2012). In light of these useful attributes, we should expect the value of these services to be reflected in the prices of safe assets.

In terms of this question, previous research has focused mainly on U.S. Treasury securities and documented the existence of a premium in their prices arising from a combination of high credit quality and deep liquidity.<sup>4</sup> Such premia are typically referred to as convenience yields and defined as the difference between the observed yield and the fundamental yield that the assets would pay without any value attached to their special attributes.

The convenience yield can be broken down further into a liquidity premium and a safety

<sup>&</sup>lt;sup>1</sup>Bernanke (2005) is frequently referenced regarding this topic.

 $<sup>^{2}</sup>$ Glick (2019) provides an excellent overview of the recent literature on this topic with a global perspective. <sup>3</sup>See Reifschneider and Williams (2000) for a discussion of constraints on monetary policy in low-interestrate environments.

<sup>&</sup>lt;sup>4</sup>Early examples of papers that estimate the liquidity convenience yields of U.S. Treasuries include Amihud and Mendelson (1991) and Longstaff (2004), among many others.

premium. The liquidity premium represents the value investors are willing to forgo in exchange for the ability to easily buy and sell the asset. This premium is a function of the outstanding amount of the considered class of assets and the structure and size of the market in which they are traded, and it is clearly not limited to safe assets. The safety premium, on the other hand, is the value investors attach to the safety of the asset. This part of the convenience yield is ultimately related to extreme events at the tail of the return distribution and for which other assets may not be likely to pay off in full, as stressed by Caballero et al. (2017). Private companies or financial institutions, such as Lehman Brothers, can go bankrupt, and governments may be unable to keep servicing their public debt; Argentina and Greece would be notable examples. Assets that are perceived not to be susceptible to such risks are valuable and may pay a lower yield in exchange for maintaining their value under such stressed financial market conditions.

While it may be possible to estimate the convenience yield of a safe asset, it is generally challenging to separate it into the underlying liquidity and safety premia. To achieve a better understanding of each of these components, Krishnamurthy and Vissing-Jorgensen (2012) scrutinize the yield spreads of assets with differences in liquidity but similar safety and those of assets with different levels of safety but similar liquidity. They find that U.S. Treasury yields averaged 73 basis points lower than they otherwise would have during the 1926-2008 period due to their safety and liquidity. Using the yield spreads of AAA- and BBB-rated corporate bonds, their analysis further suggests that up to 46 basis points of this convenience yield reflect the liquidity advantage of U.S. Treasuries, which leaves at least 27 basis points to be explained by their safety premium.

Nagel (2016) focuses on matching the three-month Treasury bill and repo rates. Given that the latter is equivalent to a collateralized loan, these are both claims that are essentially free of credit risk. Hence, the Treasury bills' convenience yield averaging 24 basis points in the 1991-2011 period should exclusively represent liquidity premia arising from their extreme liquidity. Combined with the results from Krishnamurthy and Vissing-Jorgensen (2012), this would leave a security premium of about 50 basis points in U.S. Treasury yields.

In this paper, we aim to follow an alternative approach and analyze assets that are extremely safe but relatively illiquid. By implication, any convenience yield in their pricing must mainly reflect a safety premium and is unlikely to represent any liquidity premium. This should allow us to shed light on the role of credit quality and safety for the convenience yields of safe assets.

To achieve this empirically, we focus on the Swiss Confederation bond market, which is widely viewed as one of the safest government bond markets in the world but lacks the high liquidity of the U.S. Treasuries. Clearly, if credit quality is indeed a factor in the convenience yields of safe assets described in the literature, such a convenience premium should exist and matter for the pricing of Swiss Confederation bonds. To quantify the convenience premia in this market, we use the approach of Andreasen et al. (2020, henceforth ACR), who augment a standard term structure model with a risk factor to accurately measure bond-specific risk premia. The identification of the risk factor in the ACR approach comes from its unique loading, which mimics the idea that, over time, an increasing fraction of the outstanding notional amount of a given security tends to become locked up in the buy-and-hold investors' portfolios. This raises its sensitivity to a variation in the marketwide systemic component of the bond-specific premium captured by the added risk factor. By observing a cross section of securities over time, the marketwide bond-specific risk factor can be separately identified and distinguished from the conventional fundamental risk factors in the model.

Given the much smaller size and lower liquidity of the Swiss Confederation bond market compared to that of the other major sovereign bond markets, these bonds are unlikely to command a liquidity premium. As a consequence, we refer to the estimated convenience premia as *safety premia*. Furthermore, if Swiss Confederation bonds are considered to be illiquid and carry an outright liquidity discount, our estimated safety premia should be viewed as lower bound estimates of the underlying true safety premium. Thus, we consider our analysis conservative from this perspective.

Our focus on Swiss Confederation bond data is motivated by a few additional observations. First, few papers have estimated either liquidity or safety premia for standard government bond markets outside of the U.S. Treasuries market. Thus, little is known about the magnitudes of such premia in regular sovereign bond markets.<sup>5</sup>

Second, given that the Swiss National Bank (SNB) was one of the first central banks to introduce negative policy rates and has gone further in that direction than any of its peers, Switzerland offers a unique case for studying the behavior of safety premia in various regimes: (1) the normal period before the financial crisis; (2) the 2008-2014 period in which the Swiss monetary policy rate was cut decisively but remained in positive territory; and (3) the 2015-2019 period in which Swiss interest rates moved deeply into negative territory after the SNB discontinued its Swiss franc to euro minimum exchange rate. Specifically, this latter episode allows us to analyze whether safety premia are affected when yields turn negative. The answers to these questions have become pertinent given the growing number of central banks that either have introduced negative rates already or are at risk of doing so in coming years.

Last, our data also allow us to study whether asset purchases by the European Central Bank (ECB) have affected the Swiss safety premium. Given that such purchases globally reduce the available stock of safe assets, they could increase the safety premia in many countries, including Switzerland. Alternatively, in a narrow European context, such purchases reduce the stock of euro-area safe assets. This reduction makes Swiss safe assets less exclu-

<sup>&</sup>lt;sup>5</sup>Christensen et al. (2019) apply an approach similar to ours to estimate illiquidity premia in the sovereign bond market of Mexico, a major emerging economy.

sive relative to those of its European neighbors, potentially lowering the safety premium that Swiss safe assets can require.<sup>6</sup> Ultimately, which of these effects dominates in the pricing of Swiss safe assets is an empirical question.

Our results can be summarized as follows. First, we find that the augmented model improves model fit and delivers robust estimates of the conventional risk factors that drive the variation in the frictionless part of the Swiss Confederation bond yield curve. Furthermore, the fit is not affected by either the 2008-2014 period near the zero lower bound (ZLB) or the negative rate environment prevailing since December 2014. Actually, the fit is better in these periods than during the first 15 years of our sample, with interest rates far above zero and more variable.

Second, our estimation results show that the safety premium has been positive on average, with a mean of 68 basis points and a standard deviation of 20 basis points. This likely reflects the flight-to-safety advantages Swiss government-backed securities offer investors.<sup>7</sup> Their sizable mean and variability also underscore that safety premia are important components in the pricing of Swiss Confederation bonds, in particular with Swiss interest rates near historical lows recently.

Most interestingly, the introduction of the euro in January 1999 appears to have given rise to an upward shift in the Swiss safety premium. A regression analysis with a variety of control variables suggests that the premium experienced a long-lasting spike of between 35 and 40 basis points around this event. We conjecture that this shift might be caused by the perceived implicit risk-sharing across eurozone countries that could have reduced the safety of government bonds in core eurozone countries such as Germany or France relative to that of Swiss government bonds.

Furthermore and relatedly, our regression results indicate that the ECB's purchases of government bonds since January 2015 have exerted a persistent downward pressure on the Swiss safety premium with a cumulative effect of about 20 basis points.<sup>8</sup> In contrast, we find that the introduction of negative interest rates by the SNB cannot explain the gradual decline of the average safety premium since late 2014, once we control for all confounding factors. Hence, our results suggest that the scarcity of euro-area safe assets created by the ECB's asset purchases might have reduced the relative exclusiveness of Swiss safe assets and thereby depressed their safety premium.

Finally, as an additional contribution, we exploit our yield curve model to study the term structure of safety premia. The results show that there was an upward slope in the

 $<sup>^{6}</sup>$ Krishnamurthy and Vissing-Jorgensen (2012) find that a lower supply of U.S. Treasury debt widens the U.S. Treasury yield spread relative to AAA-rated corporate bonds. We therefore hypothesize that a similar effect could be present in the relative pricing between the assets in the euro-area and the Swiss safe assets.

<sup>&</sup>lt;sup>7</sup>Consistent with our results, Jäggi et al. (2016) describe flight-to-safety effects in the Swiss franc.

<sup>&</sup>lt;sup>8</sup>Note that this effect on convenience yields of safe assets is different from the international spillover effects of central bank asset purchases, such as the signaling or portfolio balance effects traditionally discussed in the literature, which affect the general expectations and risk premium components of bond yields; see Bauer and Neely (2014) for an example.

premia in the 1990s, followed by a relatively flat term structure in the period from the launch of the euro until the global financial crisis. Since 2009, however, the term structure has turned negative, driven by a much sharper decline in the long-term safety premia than in the medium-term safety premia. We speculate that this may be tied to the very high duration risk of long-term bonds in low-interest-rate environments and could be an indication that even the most desirable safe assets can lose some of their attractiveness when interest rates become sufficiently low.

Before drawing any conclusions based on the apparent connection between the lower trend in the safety premium and the coincidental decline in longer-term Swiss interest rates, note that these changes happened in the context of relatively low financial market volatility, at least compared to that in both the global financial crisis of 2008-2009 and the subsequent European sovereign debt crisis in 2011-2012. Thus, to what extent flight-to-safety effects, which tend to spike when economic uncertainty is elevated, are truly smaller in low-interestrate environments is difficult to determine based on our data.

As for the underlying cause for the existence of the safety premia we identify in Swiss Confederation bond prices, these premia may be related to the "safe haven" status of the Swiss franc, which is typically taken to mean that safe Swiss franc-denominated assets offer a hedging value against global risk, both on average and specifically in crisis episodes; see Grisse and Nitschka (2015) for evidence and a discussion.

In a final exercise, we repeat our analysis using Danish government bonds. Similar to Swiss Confederation bonds, these bonds are widely viewed as being among the safest government bonds in the world but lack the liquidity superiority of U.S. Treasuries. Therefore, we consider any bond-specific premia in their prices to represent safety premia. The regression analysis using Danish bonds produces results very similar to those derived from the Swiss Confederation bonds. First and foremost, the launch of the euro appears to have boosted Danish safety premia by 20-30 basis points. Second, Danish safety premia were not affected by the introduction of negative rates by the Danish National Bank (DNB) in July 2012. Finally, the ECB's purchases of safe government bonds from the euro area have also put significant downward pressure on the Danish safety premia.

The remainder of the paper is organized as follows. Section 2 describes the Swiss Confederation bond data, while Section 3 details the no-arbitrage term structure model we use and presents the empirical results. Section 4 analyzes the estimated Confederation bond safety premium and its determinants, while Section 5 is dedicated to a similar analysis of the estimated Danish safety premia. Section 6 concludes. An online appendix contains details of the data and a description of our control variables.



(a) Maturity distribution of Confederation bonds

(b) Number of Confederation bonds

#### Figure 1: The Swiss Confederation Bond Market

Panel (a) shows the maturity distribution of the Swiss Confederation bonds considered. The solid grey rectangle indicates the subsample used throughout the paper. Panel (b) reports the number of Swiss Confederation bonds at each date.

## 2 Swiss Confederation Bond Data

Our Swiss Confederation bond price data are collected daily by staff at the SNB and are available back to the 1980s. However, an inspection of the data reveals that the early part of the sample is characterized by many stale or erratic prices. For these reasons we choose to start the data sample in January 1993, a date at which the data appear to be systematically reliable across all available bonds. For the purposes of factor identification, the model needs at least four bonds to be traded on each day in the sample. As a consequence, we track a few select bonds that were issued before January 1993 and appear to have reliable prices. These considerations lead us to focus on the universe of Swiss Confederation bonds listed in online Appendix A. Importantly, the sample contains every Swiss Confederation bond issued since 1993. Thus, our analysis is complete and comprehensive for the period since then. While the bond data are available at a daily frequency, the analysis was performed on monthly data to facilitate empirical implementation.

Figure 1(a) shows the maturity distribution of the universe of Swiss Confederation bonds across time since 1993. The vertical solid grey lines indicate the start and end dates for our sample, while the horizontal solid grey lines indicate the top and bottom of the maturity range considered. The top of the range equals 50 years and is determined by the longest bond maturity issued by the Swiss Confederation, while the bottom of the range is fixed at three months to avoid erratic price patterns for bonds as they approach maturity, see Gürkaynak et al. (2007).



#### Figure 2: Yield to Maturity of Swiss Confederation Bonds

Illustration of the yield to maturity of the Swiss Confederation bonds considered in this paper, which are subject to two sample choices: (1) sample limited to the period from January 29, 1993, to March 29, 2019; (2) censoring of a bond's price when it has less than three months to maturity.

Figure 1(b) shows the number of bonds in our sample at each point in time. We note that the total number of outstanding bonds have been fairly stable since the mid-1990s. At the end of our sample period, a total of 22 bonds remained outstanding.

Figure 2 shows the Swiss Confederation bond prices converted into yield to maturity. Note the following regarding these yield series. First, yield levels have generally trended lower during this 25-year period, ranging from close to 5 percent in the early 1990s to zero by the end of our sample. Second, business cycle variation in the shape of the yield curve is pronounced around the lower trend. The yield curve tends to flatten ahead of recessions and steepen during the initial phase of economic recoveries. It is these characteristics that are the practical motivation behind our choice of using a three-factor model for the frictionless part of the Swiss yield curve, adapting an approach similar to what is standard for U.S. and U.K. data; see Christensen and Rudebusch (2012).

To support that choice more formally, we note that researchers have typically found that three factors are sufficient to model the time-variation in the cross section of U.S. Treasury yields (e.g., Litterman and Scheinkman 1991). To perform a similar analysis based on our sample of Swiss Confederation bond prices, we construct synthetic zero-coupon bond yields. For the period from January 1993 to December 1997 with a limited range of available bond maturities, we follow De Pooter et al. (2014) and use the Nelson and Siegel (1987) yield curve function for this construction, while we switch to the more flexible Svensson (1995)

Maturity	First	Second	Third
in months	P.C.	P.C.	P.C.
24	0.42	0.84	-0.24
36	0.39	0.14	0.45
60	0.38	-0.15	0.46
84	0.37	-0.21	0.24
120	0.36	-0.24	-0.06
144	0.36	-0.25	-0.22
240	0.36	-0.28	-0.65
% explained	93.21	5.83	0.79

Table 1: Factor Loadings of Swiss Confederation Bond Yields

The top rows show the eigenvectors corresponding to the first three principal components. Put differently, they show how bond yields at various maturities load on the first three principal components. In the final row the proportion of all bond yield variability explained by each principal component is shown. The data are daily Swiss Confederation zero-coupon bond yields from January 4, 1993, to March 29, 2019, a total of 6,578 observations for each yield series.

yield curve for the period since January 1998 due to the greater dispersion in the available set of bond maturities.<sup>9</sup> To have a yield panel representative of the underlying bonds in our sample, we include yields for seven constant maturities: 2, 3, 5, 7, 10, 12, and 20 years. The data series are daily series, covering the period from January 4, 1993, to March 29, 2019.

The result of a principal component analysis of the yield panel is reported in Table 1. The top panel reports the eigenvectors that correspond to the first three principal components. The first principal component accounts for 93.2% of the variation in the bond yields, and its loading across maturities is uniformly positive. Thus, similarly to a level factor, a shock to this component changes all yields in the same direction irrespective of maturity. The second principal component accounts for 5.8% of the variation in these data and has sizable positive loadings for the shorter maturities and sizable negative loadings for the long maturities. Thus, similarly to a slope factor, a shock to this component steepens or flattens the yield curve. Finally, the third component, which accounts for 0.8% of the variation, has a hump shaped factor loading as a function of maturity, which is naturally interpreted as a curvature factor. Combined these three factors account for 99.83% of the total variation. This motivates our choice to focus on the Nelson and Siegel (1987) model with its level, slope, and curvature factors for modeling this sample of Swiss bond yields. However, for theoretical consistency, we use the arbitrage-free version of this class of models derived in Christensen et al. (2011). Furthermore, to explain the remaining variation in the bond yield data not accounted for by the level, slope, and curvature factors, we augment the model with a liquidity risk factor to allow for bond-specific risk premia using the approach described in ACR and detailed in Section 3. Importantly, we stress that the estimated state variables in our model are not identical to the principal component factors discussed here, but estimated through Kalman

<sup>&</sup>lt;sup>9</sup>Technically, for both periods, we proceed as described in Andreasen et al. (2019).

filtering.<sup>10</sup>

Finally, we note that Swiss yields had come close to the ZLB already in 2003-2004. Thus, Swiss fixed-income markets have a long history of being at or near the ZLB, only matched by their Japanese equivalents. However, unlike Japanese yields that mostly respected the ZLB until 2015 (see Christensen and Spiegel 2019), Swiss yields have been less constrained by the ZLB. Considering the data available into 2019 when several yields were close to negative one percent, it is not clear what the lower bound for Swiss yields is, if there is any. As a consequence, we choose to focus on models with Gaussian dynamics, which can easily handle negative interest rates.<sup>11</sup>

## 2.1 The Bond-Specific Risk of Confederation Bonds

In this paper, we apply the ACR model approach to the universe of Swiss Confederation bonds. To support the use of the ACR approach as a way of identifying bond-specific risk premia in Confederation bond prices, we point to the structure of bonds' bid-ask spreads. Figure 3 shows two series of bid-ask spreads for thirty-year Confederation bonds, one represents the bid-ask spread of the first thirty-year bond in our sample (4 percent Eidgenosse maturing 4/8/2028), the other tracks the bid-ask spread of the most recently issued thirty-year bond in our sample (1.5 percent Eidgenosse maturing 4/30/2042) issued in April 2012. Both series are smoothed four-week averages and are measured in basis points. Similar to the findings of ACR for U.S. Treasury inflation-protected securities (TIPS), the pattern in the bid-ask spreads of Swiss Confederation bonds is such that the more seasoned securities are less liquid than the recently issued securities. Rational, forward-looking investors are aware of these dynamics and the fact that future market liquidity of a given security is likely to be below its current market liquidity. These patterns give rise to security-specific premia in the bond prices.<sup>12</sup> The ACR approach detailed below is designed to capture such premia in individual bond prices. Also, these series underscore the relatively low liquidity of seasoned Confederation bonds, which dominate our sample.

## **3** Model Estimation and Results

In this section, we detail the term structure model that serves as the benchmark in our analysis and describe the restrictions imposed to achieve econometric identification of the model. We then compare its estimates to those from the model without any adjustment for bond-specific risk premia. Finally, we provide a more detailed analysis of the fit of the model relative to a

 $<sup>^{10}</sup>$ A number of recent papers use principal components as state variables. Joslin et al. (2011) is an early example.

<sup>&</sup>lt;sup>11</sup>This choice is also supported by the analysis of Grisse and Schumacher (2018), who find that, with the exception of a brief period, long-term Swiss Confederation bond yields have responded symmetrically to changes in the short rate since 2000, a pattern well captured by Gaussian models.

<sup>&</sup>lt;sup>12</sup>Fontaine and Garcia (2012) document pricing differences of this nature in the U.S. Treasury market.



Figure 3: **Bid-Ask Spreads of Thirty-Year Swiss Confederation Bonds** Illustration of the four-week moving average of bid-ask spreads of two thirty-year Swiss Confederation bonds. The series are daily covering the period from June 12, 2012, to March 29, 2019.

set of relevant benchmark models.

## 3.1 The AFNS-R Model

The fundamental frictionless Swiss yields that would prevail in a world without any frictions to trading or any excess demand attributable to safety concerns regarding other assets are modeled by using a standard Gaussian model, namely the arbitrage-free Nelson-Siegel (AFNS) model introduced in Christensen et al. (2011). We augment this model with a risk factor structured as in the ACR approach and refer to it as the AFNS-R model.

To begin the model description, let  $X_t = (L_t, S_t, C_t, X_t^R)$  denote the state vector of the four-factor AFNS-R model. Here,  $L_t$  denotes a level factor, while  $S_t$  and  $C_t$  represent slope and curvature factors. Finally,  $X_t^R$  is the added marketwide bond-specific risk factor.

The instantaneous risk-free rate is defined as follows:

$$r_t = L_t + S_t. \tag{1}$$

The risk-neutral dynamics of the state variables used for pricing are given by the following:

$$\begin{pmatrix} dL_t \\ dS_t \\ dC_t \\ dX_t^R \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \lambda & -\lambda & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \kappa_R^{\mathbb{Q}} \end{pmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ \theta_R^{\mathbb{Q}} \end{pmatrix} - \begin{pmatrix} L_t \\ S_t \\ C_t \\ X_t^R \end{pmatrix} \end{bmatrix} dt + \Sigma \begin{pmatrix} dW_t^{L,\mathbb{Q}} \\ dW_t^{S,\mathbb{Q}} \\ dW_t^{C,\mathbb{Q}} \\ dW_t^{R,\mathbb{Q}} \end{pmatrix},$$

where  $\Sigma$  is a lower-triangular matrix.

Based on the Q-dynamics above, the standard frictionless zero-coupon bond yields preserve a Nelson and Siegel (1987) factor loading structure:

$$y_t(\tau) = L_t + \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau}\right)S_t + \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right)C_t - \frac{A(\tau)}{\tau},\tag{2}$$

where  $\frac{A(\tau)}{\tau}$  is a convexity term that adjusts the functional form in Nelson and Siegel (1987) to ensure absence of arbitrage (see Christensen et al. (2011)).

Importantly, due to bond-specific premia in the Confederation bond market, individual bond prices are sensitive to the variation in the bond-specific risk factor  $X_t^R$ . As a consequence, the pricing of Confederation bonds is not performed with the standard discount function, but rather with a discount function that accounts for the bond-specific risk:

$$\overline{r}_{t}^{i} = r_{t} + \beta^{i} (1 - e^{-\lambda^{R,i} (t - t_{0}^{i})}) X_{t}^{R},$$
(3)

where  $t_0^i$  denotes the date of issuance of the specific security and  $\beta^i$  is its sensitivity to the variation in the market-wide bond-specific risk factor. Furthermore, the decay parameter  $\lambda^{R,i}$  is assumed to vary across securities as well.

As shown in Christensen and Rudebusch (2019), the net present value of one Swiss franc paid by Confederation bond i at time  $t + \tau$  has the following exponential-affine form:

$$P_t^i(t_0^i,\tau) = E^{\mathbb{Q}} \left[ e^{-\int_t^{t+\tau} \overline{r}^i(s,t_0^i)ds} \right]$$
  
=  $\exp\left( B_1^i(\tau)L_t + B_2^i(\tau)S_t + B_3^i(\tau)C_t + B_4^i(t_0^i,t,\tau)X_t^R + A^i(t_0^i,t,\tau) \right).$ 

This implies that the model belongs to the class of Gaussian affine term structure models. Note also that, by fixing  $\beta^i = 0$  for all *i*, we recover the AFNS model.

Consider the whole value of the Swiss Confederation bond issued at time  $t_0^i$  with maturity at  $t + \tau$  that pays a coupon C annually. Its price is given by the following:<sup>13</sup>

$$P_t^i(t_0^i,\tau) = C(t_1-t)E^{\mathbb{Q}}\left[e^{-\int_t^{t_1} \overline{\tau}^i(s,t_0^i)ds}\right] + \sum_{j=2}^N CE^{\mathbb{Q}}\left[e^{-\int_t^{t_j} \overline{\tau}^i(s,t_0^i)ds}\right] + E^{\mathbb{Q}}\left[e^{-\int_t^{t+\tau} \overline{\tau}^i(s,t_0^i)ds}\right].$$
 (4)

<sup>&</sup>lt;sup>13</sup>This is the clean price that does not account for any accrued interest and maps to our observed bond prices.

At present, the description of the AFNS-R model has relied solely on the dynamics of the state variables under the Q-measure used for pricing. However, to complete the description of the model and to implement it empirically, we will need to specify the risk premia that connect these factor dynamics under the Q-measure to the dynamics under the real-world (or physical) P-measure. Note that there are no restrictions on the dynamic drift components under the empirical P-measure beyond the requirement of constant volatility. To facilitate empirical implementation, we use the essentially affine risk premium specification introduced in Duffee (2002). In the Gaussian framework, this specification implies that the risk premia  $\Gamma_t$  depend on the state variables; that is,

$$\Gamma_t = \gamma^0 + \gamma^1 X_t,$$

where  $\gamma^0 \in \mathbf{R}^4$  and  $\gamma^1 \in \mathbf{R}^{4 \times 4}$  contain unrestricted parameters.

Thus, the resulting unrestricted four-factor AFNS-R model has  $\mathbb{P}$ -dynamics given by the following:

$$\begin{pmatrix} dL_t \\ dS_t \\ dC_t \\ dX_t^R \end{pmatrix} = \begin{pmatrix} \kappa_{11}^{\mathbb{P}} & \kappa_{12}^{\mathbb{P}} & \kappa_{13}^{\mathbb{P}} & \kappa_{14}^{\mathbb{P}} \\ \kappa_{21}^{\mathbb{P}} & \kappa_{22}^{\mathbb{P}} & \kappa_{23}^{\mathbb{P}} & \kappa_{24}^{\mathbb{P}} \\ \kappa_{31}^{\mathbb{P}} & \kappa_{32}^{\mathbb{P}} & \kappa_{33}^{\mathbb{P}} & \kappa_{34}^{\mathbb{P}} \\ \kappa_{41}^{\mathbb{P}} & \kappa_{42}^{\mathbb{P}} & \kappa_{43}^{\mathbb{P}} & \kappa_{44}^{\mathbb{P}} \end{pmatrix} \begin{pmatrix} \theta_1^{\mathbb{P}} \\ \theta_2^{\mathbb{P}} \\ \theta_3^{\mathbb{P}} \\ \theta_4^{\mathbb{P}} \end{pmatrix} - \begin{pmatrix} L_t \\ S_t \\ C_t \\ X_t^R \end{pmatrix} \end{pmatrix} dt + \Sigma \begin{pmatrix} dW_t^{L,\mathbb{P}} \\ dW_t^{S,\mathbb{P}} \\ dW_t^{C,\mathbb{P}} \\ dW_t^{R,\mathbb{P}} \end{pmatrix}.$$

This is the transition equation in the extended Kalman filter estimation.

## 3.2 Identification of Bond-Specific Risk Premia

As described above, our model discounts coupon and principal payments from individual Confederation bonds by using bond-specific short rates, which differ in their loadings on the common bond-specific risk factor. This is a very direct way of identifying bond-specific risk premia and only requires a panel of bond market prices, which is readily available. A key advantage of our approach is that we do not need any data on bond-specific trading dynamics such as bid-ask spreads, trading volumes, quote sizes, or trade sizes to be able to identify the unique premia embedded in individual bond prices. Furthermore, as emphasized by ACR, such measures reflect *current* market conditions whereas our concept of bond-specific premia is centered around the investors' expectations about *future* market conditions and how those expectations should affect *current* bond prices.

Our model structure stipulates that bond-specific premia will be a function of time since issuance and remaining time to maturity. The model structure allows this time interval to affect the expectations investors use when pricing bonds, which makes the model arbitrage free and theoretically consistent. Intuitively, any buyer today is clearly not in the market to sell, but buyers will be mindful of what market conditions they are likely to face in the future if they were to sell the bond back to the market. As a consequence, their expectations about *future* market conditions affect *current* bond prices, and the model structure we use is a flexible, yet very simple and robust method to identify exactly that component in the cross section of bond prices.

A theoretical implication of our model is that bond-specific premia are identified from the implied price differential of otherwise identical principal and coupon payments. The proposed identification scheme is thus related to the analysis of Fontaine and Garcia (2012). However, unlike that study, our approach does not require the existence of nearly matching pairs of securities for identification and hence can be applied to any bond market. Furthermore, our model and its application differ along other dimensions from the analysis in Fontaine and Garcia (2012). In particular, we impose the absence of arbitrage and hence allow the bond-specific risk premium adjustment to factor into the expectations formed by investors.

## 3.3 Model Estimation and Econometric Identification

Due to the nonlinear relationship between state variables and bond prices in equation (4), the model cannot be estimated with the standard Kalman filter. Instead, we use the extended Kalman filter as in Kim and Singleton (2012), see Christensen and Rudebusch (2019) for details. Furthermore, to make the fitted errors comparable across bonds of various maturities, we scale each bond price by its duration. Thus, the measurement equation for the bond prices takes the following form:

$$\frac{P_t^i(t_0^i,\tau^i)}{D_t^i(t_0^i,\tau^i)} = \frac{\widehat{P}_t^i(t_0^i,\tau^i)}{D_t^i(t_0^i,\tau^i)} + \varepsilon_t^i.$$

Here,  $\widehat{P}_t^i(t_0^i, \tau^i)$  is the model-implied price of bond *i*,  $D_t^i(t_0^i, \tau^i)$  is its duration, which is calculated before estimation, and  $\varepsilon_t^i$  represents independent and Gaussian distributed measurement errors with mean zero and a common standard deviation  $\sigma_{\varepsilon}$ . See Andreasen et al. (2019) for evidence supporting this formulation of the measurement equation.

Furthermore, since the marketwide bond-specific risk factor is a latent factor that we do not observe, its level is not identified without additional restrictions. We therefore let the first thirty-year, 4 percent coupon Confederation bond, which was issued on April 8, 1998 and matures on April 8, 2028, have a unit loading on this factor. That is,  $\beta^i = 1$  for this security. This choice implies that the  $\beta^i$  sensitivity parameters measure the sensitivity to this factor relative to that of the thirty-year 2028 Confederation bond.

Finally, we note that the  $\lambda^{R,i}$ -parameters can be hard to identify if their values are too large or too small. Consequently, we impose the restriction that they fall within the range from 0.0001 to 10, which is a restriction that has no practical consequences. In addition, for numerical stability during model optimization, we impose the restriction that the  $\beta^{i}$ parameters fall within the range from 0 to 250.



Figure 4: Fitted Errors of Swiss Confederation Bond Yields

Illustration of the fitted errors of Swiss Confederation bond yields-to-maturity implied by the AFNS-R model estimated with all bonds. The data are monthly and cover the period from January 29, 1993, to March 29, 2019.

## 3.4 Estimation Results

This section presents our benchmark estimation results. In the interest of simplicity, we devote the paper's focus to a version of the AFNS-R model in which  $K^{\mathbb{P}}$  and  $\Sigma$  are diagonal matrices. As shown in ACR, these restrictions have hardly any effects on the estimated bond-specific risk premia, because they are identified from the model's Q-dynamics, which are independent of  $K^{\mathbb{P}}$  and only display a weak link to  $\Sigma$  through the small convexity adjustment in the yields. Furthermore, we confirm this later on in our robustness analysis in Section 4.2 when we consider more flexible specifications of the AFNS-R model's P-dynamics.

The impact of accounting for the bond-specific risk premia is apparent in our results. To examine the model fit and to make pricing errors comparable accross securities, we compute the pricing errors based on the implied yield on each coupon bond. For the price on the *i*th coupon bond  $P_t^i(\tau, C^i)$ , we find the value of  $y_t^{i,c}$  that solves the following:

$$P_t^i(\tau^i, C^i) = C^i(t_1 - t) \exp\left\{-y_t^{i,c}(t_1 - t)\right\} + \sum_{j=2}^N C^i \exp\left\{-y_t^{i,c}(t_j - t)\right\} + \exp\left\{-y_t^{i,c}(t_N - t)\right\}.$$
 (5)

For the model-implied estimate of this bond price, denoted  $\hat{P}_t^i(\tau, C^i)$ , we find the corresponding implied yield  $\hat{y}_t^{i,c}$  and report the pricing error as  $y_t^{i,c} - \hat{y}_t^{i,c}$ . The bond pricing errors produced by the AFNS model indicate a reasonable fit with an overall root mean-squared error (RMSE) of 9.08 basis points. However, we see a substantial improvement in the pricing

Parameter	AFNS		AFNS-R		
1 arameter	Est.	SE	Est.	SE	
$\kappa_{11}^{\mathbb{P}}$	0.0103	0.0286	0.0097	0.0373	
$\kappa_{22}^{\mathbb{P}}$	0.2065	0.1234	0.1924	0.1352	
$\kappa_{33}^{\mathbb{P}}$	0.8432	0.2537	0.5012	0.2880	
$\kappa_{44}^{\mathbb{P}}$	-	-	0.0072	0.0345	
$\sigma_{11}$	0.0028	0.0000	0.0033	0.0001	
$\sigma_{22}$	0.0077	0.0002	0.0088	0.0006	
$\sigma_{33}$	0.0162	0.0008	0.0191	0.0014	
$\sigma_{44}$	-	-	0.0114	0.0010	
$ heta_1^\mathbb{P}$	0.0345	0.0061	0.0423	0.0161	
$ heta_2^{ar{\mathbb{P}}}$	-0.0242	0.0063	-0.0263	0.0105	
$ heta_3^{\mathbb{P}}$	-0.0137	0.0039	0.0120	0.0122	
$ heta_4^{\mathbb{P}}$	-	-	0.1232	0.2795	
$\lambda^{-}$	0.2746	0.0026	0.1881	0.0032	
$\kappa^{\mathbb{Q}}_R$	-	-	2.9361	0.1404	
$ heta_R^{\mathbb{Q}}$	-	-	-0.0057	0.0004	
$\sigma_{arepsilon}$	$0.0010  3.38 \times 10^{-6}$		0.0007	$3.89 \times 10^{-6}$	
Max $\mathcal{L}^{EKF}$	35,445.24		$37,\!527.98$		

### Table 2: Estimated Dynamic Parameters

The table shows the estimated dynamic parameters for the AFNS and AFNS-R models estimated with a diagonal specification of  $K^{\mathbb{P}}$  and  $\Sigma$ .

errors when correcting for the bond-specific risk premia, as the AFNS-R model has a much lower overall RMSE of just 5.78 basis points.<sup>14</sup>

The time series of the individual fitted yield error series from the AFNS-R model are shown in Figure 4. Since 2010 the model has been able to deliver a very accurate fit to the entire cross section of bond yields, effectively keeping all bonds within a 10-basis-point error band. Thus, neither the 2009-2014 period with shorter-term interest rates constrained by the ZLB nor the negative rates prevailing since December 2014 appear to cause the model any problems.

Table 2 reports the estimated dynamic parameters. With the exception of the curvature factor, which is notably more persistent and has a higher mean in the AFNS-R model, the dynamics of the first three factors are qualitatively very similar across the two estimations. Furthermore,  $\lambda$  is significantly smaller in the AFNS-R model. This implies that the yield loadings of the slope factor decay toward zero at a slower pace as maturity increases. At the same time, the peak of the curvature yield loadings is located at a later maturity. Thus, in the AFNS-R model, slope and curvature matter more for longer-term yields. Since long-term yields tend to be more persistent than short-term yields, this helps explain the greater persistence of the curvature factor in the AFNS-R model.

The estimated paths of the level, slope, and curvature factors from the two models are

<sup>&</sup>lt;sup>14</sup>The full details of the model fit comparison are provided in online Appendix B.



Figure 5: **Estimated State Variables** Illustration of the estimated state variables from the AFNS and AFNS-R models.

shown in Figure 5. The two models' level and slope factors are fairly close to each other during the entire sample, while there are notable level differences between the two estimated curvature factors throughout most of the sample period. Accordingly, the main impact of accounting for the bond-specific risk premia is on the curvature of the frictionless yield curve. The marketwide factor driving the variation in the bond-specific risk premia unique to the AFNS-R model is very persistent with moderate volatility. Its estimated path exhibits a persistent decline until the late 1990s and has fluctuated close to zero since then.

Maturity	No.	AF	FNS	D	NS	Sver	nsson	AF	GNS	AFI	NS-R
bucket	obs.	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE
0-2	784	-0.39	15.61	-0.45	14.78	-0.58	13.67	-0.03	10.34	0.08	9.56
2-4	924	0.15	9.77	0.80	9.46	0.83	7.92	0.03	7.97	0.21	6.06
4-6	935	-0.46	8.12	-0.81	7.89	-0.39	7.41	-0.68	7.28	-0.89	5.62
6-8	838	0.34	6.08	-0.41	6.16	-0.28	5.75	0.11	5.96	0.73	4.48
8-10	773	-0.36	6.09	-1.03	6.71	-0.87	4.82	-0.14	4.78	0.67	4.20
10-12	583	-0.30	6.74	-0.67	7.01	-0.64	5.61	0.05	5.38	0.45	4.03
12-14	334	-0.53	6.63	-0.12	6.64	0.47	4.46	0.58	4.37	0.03	3.72
14-16	212	1.24	7.62	2.30	7.68	2.08	5.41	2.00	5.98	0.23	4.96
16-18	181	1.56	6.75	3.07	6.40	2.05	4.36	1.31	4.51	0.00	4.96
18-20	214	2.72	7.90	4.50	8.17	2.75	6.76	2.29	6.95	1.05	5.55
20-22	120	-0.65	6.08	1.96	5.84	1.51	5.52	-0.11	5.59	0.74	5.08
22-24	132	-3.09	7.24	-0.57	7.20	-1.60	4.30	-3.38	5.36	-1.44	4.54
24-26	108	-3.70	8.51	-1.80	8.28	-1.50	6.46	-3.30	7.22	-0.13	5.80
26 - 28	118	-7.55	9.64	-4.68	8.41	-5.09	6.98	-7.38	8.91	-1.96	6.28
28 - 30	90	-5.33	10.98	-3.75	12.07	-6.80	8.20	-7.60	9.03	-2.24	5.56
30 <	344	1.65	10.14	-1.01	10.77	-0.64	6.40	0.51	6.83	0.06	5.58
All bonds	6,690	-0.22	9.08	-0.18	8.95	-0.21	7.56	-0.22	7.03	0.09	5.78

Table 3: Summary Statistics of Fitted Errors of Swiss Confederation Bond Yields This table reports the mean pricing errors (Mean) and the root mean-squared pricing errors (RMSE) of the Swiss bond prices for various models estimated from the sample of Swiss Confederation bond prices. The pricing errors are reported in basis points and computed as the difference between the implied yield on the coupon bond and the model-implied yield on this bond. The data are monthly and cover the period from January 29, 1993, to March 29, 2019.

## 3.5 Analysis of Model Fit

In this section, we aim to provide some perspective on how well the AFNS-R model fits the Swiss Confederation bond price data. Therefore, we compare its fit to that of a set of well-established yield curve models. Specifically, we compare it to the AFNS model already considered, the dynamic Nelson-Siegel (DNS) model of Diebold and Li (2006), a similar dynamic version of the Svensson (1995) model, and the arbitrage-free generalized Nelson-Siegel (AFGNS) model described in Christensen et al. (2009).<sup>15</sup>

Table 3 evaluates the ability of all these models to match the market prices of the coupon bonds. As before, the pricing errors are computed based on the implied yield on each coupon bond to make these errors comparable across securities.

Table 3 reports the summary statistics for the fit to all bonds in the sample from the five model estimations broken down into maturity buckets. The AFNS-R model provides the closest fit to the data across the entire term structure. Compared to the AFNS and DNS models, which have only three factors, this result is not surprising. However, this result may be a little bit surprising compared to the flexible five-factor AFGNS model. Importantly, these results underscore a couple of takeaways. First, they show that it is crucial to account

<sup>&</sup>lt;sup>15</sup>In online Appendix C, we also use these models to assess the quality of our Swiss data by comparing the model-implied yield curves to those produced by SNB staff and made publicly available on the SNB website.

for bond-specific risk premia when it comes to modeling Swiss Confederation bond prices. Second, the really competitive fit of the AFNS-R model suggests that this model structure is well specified and strikes an appropriate balance between the number of frictionless factors and the bond-specific risk premium components. This view is supported further by our robustness analysis in Section 4.2.

## 4 The Confederation Bond Safety Premium

In this section, we analyze the Confederation bond safety premia implied by the estimated AFNS-R model described in the previous section. First, we formally define the bond safety premium and study its historical evolution. We then use a regression analysis to assess whether the safety premium experienced a structural break following the launch of the euro in January 1999 and the introduction of negative rates in Switzerland in December 2014, and whether it has been affected by recent ECB safe-asset purchases. We end the section with an analysis of the term structure of safety premia.

## 4.1 The Estimated Confederation Bond Safety Premium

We now use the estimated AFNS-R model to extract the safety premium in the Confederation bond market. To compute this premium, we first use the estimated parameters and the filtered states  $\{X_{t|t}\}_{t=1}^{T}$  to calculate the fitted Confederation bond prices  $\{\hat{P}_{t}^{i}\}_{t=1}^{T}$  for all outstanding securities in our sample. These bond prices are then converted into yields to maturity  $\{\hat{y}_{t}^{c,i}\}_{t=1}^{T}$  by solving the following fixed-point problem:

$$\hat{P}_{t=1}^{i} = C(t_{1}-t) \exp\left\{-(t_{1}-t)\hat{y}_{t}^{c,i}\right\} + \sum_{k=2}^{n} C \exp\left\{-(t_{k}-t)\hat{y}_{t}^{c,i}\right\} + \exp\left\{-(T-t)\hat{y}_{t}^{c,i}\right\},$$
(6)

for i = 1, 2, ..., n, which denotes that  $\left\{\hat{y}_{t}^{c,i}\right\}_{t=1}^{T}$  is approximately the rate of return on the *i*th Confederation bond if held until maturity (see Sack and Elsasser 2004). To obtain the corresponding yields without correcting for the safety premium, a new set of model-implied bond prices are computed from the estimated AFNS-R model but using only its frictionless part, i.e., with the constraints that  $X_{t|t}^{R} = 0$  for all t and  $\sigma_{44} = 0$ . These prices are denoted  $\left\{\tilde{P}_{t}^{i}\right\}_{t=1}^{T}$  and converted into yields to maturity  $\tilde{y}_{t}^{c,i}$  by using (6). They represent estimates of the prices that would prevail in a world without any financial frictions or convenience yields. The safety premium for the *i*th Confederation bond is then defined as follows:

$$\Psi_t^i \equiv \tilde{y}_t^{c,i} - \hat{y}_t^{c,i}.$$
(7)



## Figure 6: Average Estimated Swiss Confederation Bond Safety Premium

Illustration of the average estimated Swiss Confederation bond safety premium for each observation date, as implied by the AFNS-R model. The Confederation bond safety premiums are measured as the estimated yield difference between the frictionless yield to maturity of individual Confederation bonds with the market risk factor turned off and the corresponding fitted yield to maturity. The data cover the period from January 29, 1993, to March 31, 2019. The solid red lines indicate the average of the safety premium series from January 31, 1993, to December 31, 1998, and from January 31, 1999, to November 30, 2014.

Figure 6 shows the average Swiss Confederation bond safety premium  $\Psi_t$  across the outstanding Confederation bonds at each point in time. The average estimated safety premium clearly varies notably over time with a maximum of around 100 basis points achieved in the summer of 2007 at the onset of the financial crisis and again following the bankruptcy of Lehman Brothers in September 2008.

Interestingly, however, there seems to be a persistent upward shift in the safety premium happening somewhere in the period between late 1997 and mid-1999. Such a dramatic shift could be linked to the introduction of the euro, which may have reduced the safe-haven attractiveness of euro-area assets, by possibly changing perceptions about implicit risk-sharing between core and noncore eurozone countries. Consequently, the introduction of the euro might have increased the appeal of Swiss assets, including Swiss Confederation bonds, as safe-haven assets.

In addition, note that the average safety premium appears to have trended lower ever since the SNB introduced negative interest rates. In an effort to curb the appreciation of the Swiss franc by making investments in Swiss assets less attractive, the SNB lowered the interest rate on sight deposits, first to -25 basis points in December 2014, and then to -75 basis points in January 2015.<sup>16</sup> In response, the entire yield curve of Swiss confederation bonds shifted downwards, as documented in Christensen (2019), which increased the opportunity costs of holding these bonds and might have reduced their safety premium.

Since safe asset purchases by foreign central banks tend to reduce the available stock of foreign safe assets, the government bond purchases made by the ECB since January 2015 through its public sector purchase program (PSPP) could have made Swiss Confederation bonds relatively less attractive and reduce the Swiss safety premium. Alternatively, these asset purchases could have reduced the available stock of safe assets globally and hence could have raised the premium of all safe assets. Which of these two effects is more important is an empirical question we aim to explore as well.

## 4.2 Robustness Analysis

This section examines the robustness of the previously reported average safety premium to some of the main assumptions imposed so far. Throughout this section, the AFNS-R model with diagonal  $K^{\mathbb{P}}$  and  $\Sigma$  matrices serves as the benchmark.

First, we assess whether the specification of the dynamics within the AFNS-R model matters for the estimated Swiss Confederation bond safety premium. To do so, we estimate the AFNS-R model with unrestricted  $K^{\mathbb{P}}$  and diagonal  $\Sigma$  matrix and the AFNS-R model with unconstrained dynamics, i.e., the AFNS-R model with unrestricted  $K^{\mathbb{P}}$  and lower triangular  $\Sigma$  matrix.

Figure 7 shows the estimated Swiss safety premium series from these two estimations and compares them to the series produced by our benchmark model. Note that the three series are barely distinguishable. Thus, we conclude that the specification of the dynamics within the AFNS-R model play only a very modest role for the estimated bond-specific risk premia, which is consistent with the findings of ACR in the context of U.S. TIPS. This result also provides support for our choice to focus only on the most parsimonious specification of the AFNS-R model in our empirical analysis.

Second, we assess whether the data frequency plays any role for our results. To do so, we estimate the AFNS-R model using daily, weekly, and monthly data. Based on the results above, we focus on the most parsimonious AFNS-R model with diagonal  $K^{\mathbb{P}}$  and  $\Sigma$  matrices.

Figure 8 shows the estimated Swiss safety premium series from all three estimations. Note that they are hardly distinguishable. Thus, we conclude that data frequency matters little for our results. Clearly, at the high daily and weekly frequency, there are a few isolated spikes that are absent in the monthly series, but they are too few to have an impact on the estimation results.

Finally, we explore whether allowing for stochastic volatility in one or more of the frictionless factors within the AFNS-R model affects the estimated Swiss Confederation bond safety

<sup>&</sup>lt;sup>16</sup>See SNB's press releases from December 18, 2014, and January 15, 2015, respectively.



# Figure 7: Average Estimated Swiss Confederation Bond Safety Premium: Alternative $\mathbb{P}$ Dynamics

Illustration of the average estimated Swiss Confederation bond safety premium for each observation date, as implied by the AFNS-R model when estimated with three specifications of its dynamics as detailed in the text. In all cases the Confederation bond safety premiums are measured as the estimated yield difference between the fitted yield to maturity of individual Confederation bonds and the corresponding frictionless yield to maturity with the liquidity risk factor turned off. The data cover the period from January 29, 1993, to March 29, 2019.

premium. Specifically, we consider the four admissible combinations of allowing for spanned stochastic volatility generated by one or two factors in the model following Christensen et al. (2014). In light of the results above, we focus on the most parsimonious specification of each model with diagonal  $K^{\mathbb{P}}$  and  $\Sigma$  matrices.

We refer to these models as AFNS models because they share the key properties of the AFNS model. First, the three frictionless state variables have joint dynamics under the risk-neutral probability measure used for pricing and closely matching the arbitrage-free Nelson-Siegel models described in Christensen et al. (2011). Furthermore, the frictionless short rate remains defined as  $r_t = L_t + S_t$ . Therefore, to keep the notation simple, we use AFNS(i) to denote a model, as defined above. *i* refers to the number of factors generating stochastic volatility, while the letters—L, S, and C—are used to indicate the source(s) of stochastic volatility in the model.

Figure 9 shows the estimated Swiss safety premium series from these estimations. Note that they are barely distinguishable. Thus, we conclude that allowing for stochastic volatility within the AFNS-R model only plays a very modest role for our results. Hence, this provides support for our choice to only focus on the Gaussian AFNS-R model with constant volatility.



# Figure 8: Average Estimated Swiss Confederation Bond Safety Premium: Data Frequency

Illustration of the average estimated Swiss Confederation bond safety premium for each observation date implied by the AFNS-R model when estimated using daily, weekly, and monthly data. In all cases the Confederation bond safety premiums are measured as the estimated yield difference between the fitted yield to maturity of individual Confederation bonds and the corresponding frictionless yield to maturity with the liquidity risk factor turned off. The data cover the period from January 4, 1993, to March 29, 2019.

To summarize, we find the estimated safety premium series to be robust to both alternative model dynamics and dynamic assumptions. As a consequence, we are comfortable using the parsimonious AFNS-R model with diagonal  $K^{\mathbb{P}}$  and  $\Sigma$  matrices as our benchmark. Furthermore, given the persistence of the monetary policy changes we are interested in, we are also comfortable focusing on the results from monthly data, which greatly reduces the computational burden and simplifies the regression analysis detailed in the following section.

## 4.3 Regression Analysis

To test the hypotheses laid out in Section 4.1, we first look for structural breaks in the time series of the average Swiss safety premium. We run the CUSUM test from Page (1954) to measure the stability of the intercept coefficient in a regression of a vector of ones on the safety premium  $\bar{\Psi}_t$ . The test shows that there is one significant break in the premium series starting from March 1999 and lasting until the end of the sample. Therefore, the launch of the euro in January 1999 appears to have given rise to a break in the time series of the Swiss safety premium, while this is not the case for either the introduction of negative interest rates by the SNB or the ECB's asset purchases.



# Figure 9: Average Estimated Swiss Confederation Bond Safety Premium: Allowing for Stochastic Volatility

Illustration of the average estimated Swiss Confederation bond safety premium for each observation date as implied by the AFNS-R model when estimated with and without allowing for stochastic volatility as detailed in the text. In all cases the Confederation bond safety premiums are measured as the estimated yield difference between the fitted yield to maturity of individual Confederation bonds and the corresponding frictionless yield to maturity with the liquidity risk factor turned off. The data cover the period from January 29, 1993, to March 29, 2019.

To explore this result further, we measure the average treatment effect of the launch of the euro, the introduction of negative interest rates, and the ECB's operation of the PSPP on the Swiss safety premium within a regression framework. In particular, we run the following regression:

$$\bar{\Psi}_t = \alpha + \delta_{euro} d_t^{euro} + \delta_{nr} d_t^{nr} + \delta_{pspp} d_t^{pspp} + \delta_c D_t + \sum_{l=0}^L \delta_l X_{t-l} + \varepsilon_t, \tag{8}$$

where  $d_t^{euro}$  and  $d_t^{nr}$  are dummy variables that take the value of one onwards from January 1999 and from December 2014, respectively, and take the value of zero before then, while  $d_t^{pspp}$  is the stock of bonds acquired by the ECB through the PSPP, expressed as a percent of nominal GDP in the euro area;<sup>17,18</sup>  $D_t$  and  $X_t$  are vectors of a control dummy and of continuous

 $<sup>^{17}</sup>$ We use nominal GDP for each calendar year except for 2019, for which we use a 4-quarter rolling sum.

<sup>&</sup>lt;sup>18</sup>In principle, this measure should only include the truly safe assets acquired by the ECB and should exclude bonds issued by high-risk countries in the periphery of the euro area. However, in the absence of that granular data, we use the entire stock of purchased government-backed securities as a proxy for the amount of absorbed safe assets. Provided the ratio of truly safe assets in the ECB's portfolio is close to being constant, which is a reasonable assumption given the fixed distribution key for the purchases, there should be little bias in the estimated parameter.

variables, respectively. L is the number of lags included; and  $\varepsilon_t$  is a random residual. We consider the estimates of  $\delta_{euro}$  and  $\delta_{nr}$  to be the average treatment effects of the euro launch and the introduction of negative rates on the safety premium under the assumption that the confounding variables in the vector  $X_t$  are exogenous and that  $E[\varepsilon_t|X_t] = 0$ , while  $\delta_{pspp}$ measures the effect on the safety premium of a one percentage point change in the stock of bonds held by the ECB.

We control for a host of confounding factors. In a core set of controls, we consider the CBOE Volatility Index (VIX), the spread between the Italian and the German 10-year government bond yields, the TED spread, and the on-the-run premium in U.S. Treasuries to proxy for investors' risk aversion, financial market uncertainty, and related demand for safe-haven assets;<sup>19</sup> the spread between the German and Swiss 10-year government bond yields proxies for the opportunity cost of holding Swiss confederation bonds and the debtto-GDP ratio is used to control for effects tied to the supply of Confederation bonds;<sup>20</sup> the three-month CHF LIBOR is considered to proxy for the opportunity cost of holding money and the associated liquidity premiums of Confederation bonds, as explained in Nagel (2016). Furthermore, we include the average Confederation bond age and the one-month realized volatility of the ten-year Confederation bond yield as additional proxies for bond liquidity following the work of Houweling et al. (2005). Inspired by the analysis of Hu et al. (2013), we also include a noise measure of Swiss Confederation bond prices to control for the variation in the amount of arbitrage capital available in this market. Finally, we also add a dummy  $D_t$ that takes the value of one in the period of minimum exchange rate from September 2011 to January 2015 and zero otherwise, to control for any indirect effects of that particular SNB policy on the safety premium.

In addition to the set of core control variables, we consider several other confounding factors in the regressions. We add the overnight federal funds rate to proxy for the U.S. safe-asset liquidity premium as in Nagel (2016), and we use reported earnings per share of companies in the S&P 500 to account for opportunity costs in the equity market. We also consider the MOVE volatility index to proxy for risk aversion in the bond market. Finally, we include the total sight deposits at the SNB to control for any possible reserve-induced effects of the SNB's FX interventions; see Christensen and Krogstrup (2019).

Table 4 reports the results in which column (1) contains the outcomes without any controls, while columns (2) and (3) report the regressions conducted by using the core and extended group of controls, respectively.

The main finding is that the introduction of the euro had a significant *positive* effect on the Swiss safety premium. The estimated effect ranges from 25 to 44 basis points depending on the control variables included in the regression. On the other hand, the introduction of negative interest rates by the SNB had a significant *negative* effect on the premium, but this result is

<sup>&</sup>lt;sup>19</sup>See Grisse and Nitschka (2015).

<sup>&</sup>lt;sup>20</sup>See Nagel (2016) and Krishnamurthy and Vissing-Jorgensen (2012), respectively.

	1	2	3
$\alpha$	36.2	113.9	279.6
	(0.00)	(0.00)	(0.00)
$\delta_{euro}$	44.2	24.6	26.6
	(0.00)	(0.00)	(0.00)
$\delta_{nr}$	-6.4	-17.6	-5.7
	(0.00)	(0.00)	(0.38)
$\delta_{pspp}$	-1.1	-1.2	-1.6
	(0.00)	(0.00)	(0.00)
controls	no	core	all
Adj. $R^2$	0.82	0.89	0.93
DW	0.39	0.74	1.11

 Table 4: Average Treatment Effects of Euro Introduction and SNB's Negative

 Rates

The table reports the coefficient estimates from regression (8) together with their respective *p*-values (in brackets) obtained by using Newey-West standard errors. The first column reports the regression without controls, the second column reports the estimates with the core set of control variables, and the third column contains the results including all the controls. The last two rows report the adjusted  $R^2$  and the Durbin-Watson statistic. The number of lags *L* in regressions (2) and (3) is set to 12. The sample starts in January 1993 and ends in March 2019.

not robust to including the extended set of control variables, most notably the sight deposits at the SNB, which grew substantially with the sizable amount of SNB interventions in the foreign exchange markets following the financial crisis. Finally, we do observe systematically significant *negative* effects from including the ECB's safe asset purchases under the PSPP in the regressions. The results suggest that an increase of one percentage point in these holdings lowers the Swiss safety premium by slightly more than one basis point. Hence, the cumulative effect between January 2015 and March 2019 lowered the Swiss safety premium by about 20 basis points.

We note that adding controls and their lags increases the adjusted  $R^2$ . Relatively high  $R^2$ s indicate that most confounding factors are controlled for and that our estimates of  $\delta_{euro}$  could be largely considered as causal. However, if there are omitted variables correlated with the euro dummy, our estimate of the treatment effect would be upward biased. We therefore try to sharpen the inference regarding our three specific questions in the following sections.

### 4.3.1 Effects of the Introduction of the Euro in Isolation

To focus narrowly on the impact of the launch of the euro in isolation and avoid potentially polluting effects from the financial crisis, the Swiss introduction of negative rates, and the

Sample	No controls	Core controls	All controls	
+/-1 year	26.6	6.7	-53.1	
	(0.00)	(0.65)	(0.11)	
+/-2 years	33.6	24.6	22.2	
	(0.00)	(0.01)	(0.03)	
+/-3 years	36.3	30.6	36.4	
	(0.00)	(0.00)	(0.00)	
+/-4 years	40.6	36.6	42.0	
	(0.00)	(0.00)	(0.00)	
+/-5 years	40.3	33.7	41.3	
	(0.00)	(0.00)	(0.00)	
+/-6 years	41.4	34.0	36.1	
	(0.00)	(0.00)	(0.00)	

Table 5: Average Treatment Effects of the Euro Introduction in Isolation

The table reports the estimated coefficients of the euro dummy variable in regression (9) together with their respective *p*-values (in brackets) obtained by using Newey-West standard errors. The first column reports the regression without controls, the second column reports the estimates with the core set of control variables, and the third column contains the results including all the controls. The six samples start in January 1998, January 1997, January 1996, January 1995, January 1994, and January 1993, and end in December 1999, December 2000, December 2001, December 2002, December 2003, and December 2004, respectively.

ECB asset purchases, we run simplified regressions of the following form:<sup>21</sup>

$$\bar{\Psi}_t = \alpha + \delta_{euro} d_t^{euro} + \delta_l X_t + \varepsilon_t, \tag{9}$$

where we first use a sample that contains one year of data before the January 1, 1999 euro launch and one year of data after this date. We then expand this sample in a symmetric fashion by adding one more year of data before and after the euro launch and run the regression. We repeat this exercise until the sample contains six years of data before and after the euro launch, which is the maximum period given the January 1993 start date for our data. Overall, this allows us to compare the residual variation in the immediate pre-euro period to that during the immediate post-launch period.

The results are reported in Table 5. For the smallest (+/-1 year) sample, the results are erratic, which may be due to low statistical power. However, for the most informative samples defined as those containing at least three years of data before and after the euro launch, the estimated dummy coefficients are relatively stable, systematically highly statistically significant, and not sensitive to the set of controls used. Therefore, we see clear evidence of a statistically significant *positive* effect of roughly 35-40 basis points on the Swiss safety premium following the introduction of the euro.

 $<sup>^{21}</sup>$ In these regressions, we have to drop the lags of the explanatory variables due to the limited number of observations and high number of control variables.

One might view this result as surprising given that investors knew the launch date well in advance and therefore could have rebalanced their portfolios towards Swiss assets earlier. However, there was lingering uncertainty about the rollout and the general implementation of the euro project even relatively close to the launch date. Furthermore, this happened in the context of general financial market uncertainty surrounding the Asian financial crisis of 1997 and the collapse of Long-Term Capital Management (LTCM) during the Russian sovereign debt crisis in the fall of 1998, both of which provided temporary boosts to the Swiss safety premium. Thus, it is not clear at what point and how fast we should expect to see a sustained fundamental change in the safety premium. To see no change would be an extreme example of irrational investor behavior that is not supported by our safety premium series as it had clearly started its upward trend before January 1, 1999. On the other hand, to see the full difference priced in ahead of the euro launch would require a level of foresight and rationality among market participants rarely documented in the empirical finance literature. Our results suggest that the effect starts to be reflected in the safety premium a couple of years before the launch and is fully priced in a couple of years after it. This seems reasonable given that the involved portfolio flows likely were informed by the general experience with the euro and perceptions about the operation of the new common monetary policy in the euro area.

### 4.3.2 Effects of the Introduction of Negative Rates in Isolation

To assess the impact of the effect in isolation of the introduction of negative rates, we follow a similar approach and run simplified regressions of the form:

$$\bar{\Psi}_t = \alpha + \delta_{nr} d_t^{nr} + \delta_l X_t + \varepsilon_t, \tag{10}$$

where we first use a sample that contains one year of data before the December 18, 2014, introduction of negative rates and one year of data after this date. As before, the regressions are repeated with symmetrically expanded samples that each time contain one more year of data before and after the December 18, 2014 event date. Given that our sample ends in March 2019, we can add up to four years of data around both sides of the event date in this exercise.

The results are reported in Table 6, in which we note a small and mostly insignificant effect on the Swiss safety premium following the introduction of negative rates in December 2014. Thus, any negative effects on the safety premium in recent years appear to be due to other factors.

## 4.3.3 Effects of the ECB's PSPP in Isolation

One particular factor we are interested in exploring further is the effect on the Swiss safety premium from the ECB's government bond purchases, which were announced in January

Sample	No controls	Core controls	All controls
+/-1 year	0.5	-0.6	-4.6
	(0.83)	(0.89)	(0.29)
+/-2 years	-2.7	1.2	2.4
	(0.13)	(0.76)	(0.43)
+/-3 years	-9.6	-2.0	2.6
	(0.00)	(0.71)	(0.38)
+/-4 years	-14.7	-4.7	1.3
	(0.00)	(0.41)	(0.70)

### Table 6: Average Treatment Effects of SNB's Negative Rates in Isolation

The table reports the estimated coefficients of the negative rates dummy variable in regression (10) together with their respective *p*-values (in brackets) obtained by using Newey-West standard errors. The first column reports the regression without controls, the second column reports the estimates with the core set of control variables, and the third column contains the results including all the controls. The four samples start in December 2013, December 2012, December 2011, and December 2010, and end in November 2015, November 2016, November 2017, and November 2018, respectively.

2015. Given that such purchases reduce the available stock of government-backed safe assets in the euro area, they could also reduce the relative safety and exclusivity premium of Swiss safe assets.<sup>22</sup> Alternatively, this could have also raised the premia of all safe assets globally, although we stress that any broad-based effects from asset purchases that lower the general interest rate level in the euro area should affect the frictionless yields within our model and not the bond-specific safety premia.

To test this hypothesis, we include ECB government bond holdings divided by the nominal GDP of the euro area in regressions that take a form similar to the two previous exercises:

$$\bar{\Psi}_t = \alpha + \delta_{nr} d_t^{pspp} + \delta_l X_t + \varepsilon_t, \tag{11}$$

where we first use a sample that contains one year of data before the January 22, 2015, launch of the PSPP and one year of data after this date. As before, the regressions are repeated with symmetrically expanded samples that each time contain one more year of data before and after the event date. Given that our sample ends in March 2019, we can add up to four years of data around both sides of the event date, as in the previous section.

The results are reported in Table 7. Overall, they point to a significant *negative* effect on the Swiss safety premium from the ECB's asset purchases, and the effect becomes more significant as we expand the data window around the event date. The magnitude of the effect varies with the length of the sample and the inclusion of controls, but with at least six years of data included the estimate ranges from a -0.4 basis point to a -1.2 basis point per 1 percentage

 $<sup>^{22}</sup>$ Koijen et al. (2017) find that foreigners exhibited the strongest reaction to the ECB asset purchases under this program in terms of rebalancing their portfolios toward more attractive investment opportunities elsewhere. That process could also have led them to reassess their perceptions about the relative safety of Swiss safe assets.

Sample	No controls	Core controls	All controls
+/-1 year	-1.3	-1.4	1.7
	(0.00)	(0.13)	(0.13)
+/-2 years	-0.8	-1.1	0.1
	(0.00)	(0.02)	(0.91)
+/-3 years	-1.1	-1.1	-0.4
	(0.00)	(0.00)	(0.30)
+/-4 years	-1.2	-1.1	-0.6
	(0.00)	(0.00)	(0.00)

### Table 7: Average Treatment Effects of ECB's PSPP in Isolation

The table reports the estimated coefficients of the PSPP variable in regression (11) together with their respective *p*-values (in brackets) obtained by using Newey-West standard errors. The first column reports the regression without controls, the second column reports the estimates with the core set of control variables, and the third column contains the results including all the controls. The four samples start in January 2014, January 2013, January 2012, and January 2011, and end in December 2015, December 2016, December 2017, and December 2018, respectively.

point of GDP in asset purchases. Given the increase in the ECB's asset holdings through the end of our sample, a point estimate of approximately a -1 basis point implies a cumulative effect of close to 20 basis points between spring 2015 and spring 2019. Thus, the declines in the available stock of safe assets in the euro area has materially negatively affected the safety premium commanded by Swiss safe assets. Note also that these results and conclusions are fully consistent with the results from the full sample regressions reported in Table 4.

In terms of tangible evidence from the euro area supportive of our results, we point to Arrata et al. (2019), who find that the asset purchases under the PSPP constrained the free flow of bonds in the highly rated government bond markets in the euro area and caused bonds from those markets to trade at a special premium in repo markets. Furthermore, Roh (2019) documents that this drove up the prices of this class of bonds. These findings suggest the outstanding amount of euro-area safe assets available for trading was significantly reduced, which could reduce the relative exclusiveness of Swiss safe assets and depress their safety premium.

## 4.4 The Term Structure of Safety Premia

Krishnamurthy and Vissing-Jorgensen (2012) explicitly assume that safety attributes may differ across short- and long-term assets and hence lead to differences in their respective convenience yields. Therefore, after having studied the average safety premium and its determinants, we now turn to the behavior of the term structure of safety premia.

To study this in greater detail, we generate a generic standardized measure of safety premia across maturities by taking the fitted frictionless yield of a given maturity from the AFNS-R model and subtract the fitted yield of the same maturity derived from the flexible



Figure 10: **Term Structure of Swiss Confederation Bond Safety Premia** Illustration of the estimated term structure of generic safety premia calculated as described in the main text and compared to the average estimated Swiss Confederation bond safety premium implied

by the AFNS-R model. The data cover the period from January 29, 1993 to March 31, 2019.

AFGNS model considered in Section 3.5. Since the latter yield just represents a very flexible fit to raw bond price data, this construction essentially provides us with a smoothed measure of the safety premium at a given maturity. This has both advantages and disadvantages. On the positive side, it smooths out some of the variation in the bond-specific premia that tends to be reflected in the average safety premium series and provides us with a consistent measure for fixed maturities. On the negative side, it neglects the compositional changes in the underlying bond universe, such as changes in the number of bonds outstanding, their coupon rates, and remaining time to maturity that may play a role in some of the changes we see in the average safety premium series. Importantly, however, the generic measures are highly positively correlated with the average safety premium analyzed thus far.

In Figure 10, we plot these generic smoothed estimates of the Swiss safety premium at the five-, seven-, and ten-year maturities and compare them to the average estimated Swiss safety premium considered so far. As shown in online Appendix D, the average time to maturity started around nine years in 1993, gradually declined to a low close to five years in 1999, and rapidly increased to around ten years, where it remained until 2012; then started rising to close to fifteen years by the end of our sample. Based on the variation in the average time to maturity for our bond sample, it is therefore not surprising that the average estimated safety premium series started out close to the generic seven-year safety premium, then approached the five-year safety premium by late 1995 and tracked it until 1999. All safety premium series stayed fairly close to each other for the next ten years. However, since 2009 we have started to see a development of negative term structure in which long-term safety premia have declined

more than shorter-term safety premia. Furthermore, since the average time to maturity of our bonds is slightly above ten years during most of this period, it is reasonable to see the average safety premium series track the generic ten-year safety premium during this part of the sample.

The negative slope of the term structure of safety premia over the past decade suggests that the very low interest rates in Switzerland since the financial crisis are exerting greater downward pressure on the safety premia of long-term bonds compared to the safety premia of shorter-term bonds. This seems reasonable since the duration risk of very long-term bonds in low and negative interest rate environments is really high. Hence, the variation in the term structure of safety premia would seem to lend support to the view that, provided interest rates become sufficiently low, even the most desirable safe assets can lose some of their attractiveness, in part due to the sizable duration risk their exposure entails.

## 5 Danish Government Bond Safety Premium Analysis

In this section, we seek to establish further support for our findings for Swiss safety premia by repeating the analysis for a monthly sample of Danish government bond prices during the period from January 31, 1995 to March 29, 2019. Similar to Swiss Confederation bonds, Danish government bonds are widely viewed as being among the safest government bonds in the world, but lack the liquidity superiority of U.S. Treasuries. Hence, as in our Swiss analysis, we consider any bond-specific premia in their bond prices to reflect safety premia. To estimate these bond-specific premia, we continue to use our parsimonious benchmark AFNS-R model with diagonal  $K^{\mathbb{P}}$  and  $\Sigma$  matrices.

Figure 11 shows the average Danish government bond safety premium  $\bar{\Psi}_t$  across the outstanding bonds at a given point in time. The average estimated liquidity premium clearly varies notably over time, reaching a minimum of -62 basis points in late 1995 and achieving a maximum of 69 basis points in January 2015 shortly after the SNB had discontinued its minimum exchange rate to the euro and the ECB had announced its first open-ended large-scale purchases of euro area government bonds. For the entire period, the series has an average of 12.14 basis points with a standard deviation of 26.12 basis points.

One notable difference between the Swiss and Danish safety premia is observed during the financial crisis. Events like the onset of the crisis in the summer of 2007 and the bankruptcy of Lehman Brothers in September 2008 coincide with spikes in the Swiss safety premium, while they tend to be associated with a persistent decline in the Danish safety premium. We speculate that Denmark with its peg to the euro was viewed by global investors as less of a safe haven during this period than Switzerland.

Interestingly and more importantly for our analysis, there seems to be a persistent upward shift in the safety premium happening somewhere in the period between late 1997 and mid-1999. Therefore, the introduction of the euro might have increased the appeal of Danish



Figure 11: Average Estimated Danish Government Bond Safety Premium Illustration of the average estimated Danish government bond safety premium for each observation date, as implied by the AFNS-R model. The bond safety premiums are measured as the estimated yield difference between the frictionless yield to maturity of individual government bonds with the market risk factor turned off and the corresponding fitted yield to maturity. The data cover the period from January 31, 1995, to January 31, 2019.

government bonds as safe-haven assets in much the same way as it did for Swiss safe assets. In terms of magnitudes, the Danish safety premium averaged -25.5 basis points from 1995 through December 1998, while its average from January 1999 through June 2012 was 15.6 basis points. Thus, there is a visual upward level shift in the Danish safety premium of more than 40 basis points between these two periods. However, as before, a comprehensive time series analysis is required to establish any connection between the introduction of the euro and the Danish safety premia.

Furthermore, note that the average Danish safety premium appears to have trended higher in the years following the introduction of negative interest rates by the DNB. In an effort to curb the appreciation pressure of the Danish kroner relative to the euro, the DNB lowered its key policy interest rate to -20 basis points in July 2012. In response, the entire yield curve of Danish government bonds shifted downwards as documented in Christensen (2019), and part of this yield decline appears to be driven by upticks in the Danish safety premium. However, to link the introduction of negative interest rates to these trends obviously requires a careful econometric analysis that controls for the effects of confounding factors that may have influenced the safety premium during this period.

Finally, since safe asset purchases by foreign central banks reduce the available stock of foreign safe assets, the government bond purchases made by the ECB since January 2015 through its public sector purchase program (PSPP) could make Danish government bonds relatively less attractive and reduce the Danish safety premium. Alternatively, they could be

viewed as reducing the available stock of safe assets globally and hence raise the premium of all safe assets. Which of these two effects is more important is an empirical question we aim to explore as well.

## 5.1 Regression Analysis

To test the stated hypotheses above, we again first look for structural breaks in the time series of the average estimated Danish safety premium. We run the CUSUM test from Page (1954) to measure the stability of the intercept coefficient in a regression of a vector of ones on the safety premium  $\bar{\Psi}_t$ . The test shows that there is one significant break in the premium series starting from September 1998 and lasting until the end of the sample. Therefore, the launch of the euro in January 1999 also appears to have given rise to a break in the time series of the Danish safety premium, while this is once more not the case for either the introduction of negative interest rates by the DNB or the ECB's asset purchases.

Similar to the Swiss analysis, we control for a host of confounding factors. In a core set of controls, we consider the VIX, the spread between the Italian and the German 10-year government bond yields, the TED spread, and the on-the-run premium in U.S. Treasuries to proxy for investors' risk aversion, financial market uncertainty, and related demand for safe-haven assets. We consider the spread between German and Danish 10-year government bond yields to proxy for the opportunity cost of holding Danish government bonds and we use the debt-to-GDP ratio to control for effects tied to the supply of Danish government bonds. The overnight deposit rate should proxy for the opportunity cost of holding money and the associated liquidity premia of Danish government bonds. Furthermore, we include the average Danish government bond age and the one-month realized volatility of the ten-year Danish government bond yield as additional proxies for bond liquidity following the work of Houweling et al. (2005). Inspired by the analysis of Hu et al. (2013), we also include a noise measure of Danish government bond prices to control for the variation in the amount of arbitrage capital available in this market.

In addition to the set of core control variables, we again consider several other confounding factors in the regressions. We add the overnight Fed funds rate to proxy for the U.S. safe-asset liquidity premium as in Nagel (2016), and use reported earnings per share of companies in the S&P 500 to account for opportunity costs in the equity market. We also consider the MOVE volatility index to proxy for risk aversion in the bond market.

### 5.1.1 Effects of the Introduction of the Euro in Isolation

To focus narrowly on the impact of the launch of the euro in isolation and avoid potentially polluting effects from the financial crisis, the introduction of negative rates, and the ECB

Sample	No controls	Core controls	All controls
+/-1 year	6.5	2.5	-0.1
	(0.10)	(0.68)	(0.99)
+/-2 years	17.6	7.6	2.6
	(0.00)	(0.24)	(0.71)
+/-3 years	19.8	-1.9	-0.6
	(0.03)	(0.77)	(0.93)
+/-4 years	29.6	21.4	31.0
	(0.00)	(0.00)	(0.00)

Table 8: Average Treatment Effects of Euro Introduction in Isolation

The table reports the estimated coefficients of the euro dummy variable in regression (12) together with their respective *p*-values (in brackets) obtained by using Newey-West standard errors. The first column reports the regression without controls, the second column reports the estimates with the core set of control variables, and the third column contains the results including all the controls. The four samples start in January 1998, January 1997, January 1996, January 1995, and end in December 1999, December 2000, December 2001, and December 2002, respectively.

asset purchases, we repeat our simplified regressions of the form:<sup>23</sup>

$$\bar{\Psi}_t = \alpha + \delta_{euro} d_t^{euro} + \delta_l X_t + \varepsilon_t, \tag{12}$$

where we first use a sample that contains one year of data before the January 1, 1999 euro launch and one year of data after this date. We then expand this sample in a symmetric fashion by adding one more year of data before and after the euro launch and re-run the regression. This exercise is repeated until the sample contains four years of data before and after the euro launch, which is the maximum given the January 1995 start date for our Danish bond price data. Overall, this allows us to compare the residual variation in the immediate pre-euro period to that during the immediate post-launch period.

The results are reported in Table 8. For the shorter samples with up to three years of data before and after the euro launch, the results are erratic and insignificant, which may be due to low statistical power. However, for the most informative sample with four years of data before and after the euro launch, the estimated dummy coefficients are relatively stable, systematically highly statistically significant, and not very sensitive to the set of controls used. Therefore, we see clear evidence of a statistically significant *positive* effect on the Danish safety premium following the introduction of the euro of roughly 20-30 basis points.

One might view this result as surprising given that investors knew the launch date well in advance and therefore could have rebalanced their portfolios towards Danish assets earlier. Our results suggest that the effect starts to be reflected in the safety premium a couple of years before the launch and is fully priced in a couple of years after it. As in the Swiss

 $<sup>^{23}</sup>$ In these regressions, we again have to drop the lags of the explanatory variables due to the limited number of observations and high number of control variables.

Sample	No controls	Core controls	All controls
+/-1 year	16.3	3.5	8.6
	(0.00)	(0.54)	(0.29)
+/-2 years	16.8	10.1	16.5
	(0.00)	(0.03)	(0.00)
+/-3 years	24.4	2.8	2.0
	(0.00)	(0.54)	(0.71)
+/-4 years	25.5	6.1	10.0
· · ·	(0.00)	(0.17)	(0.05)

Table 9: Average Treatment Effects of the DNB's Negative Rates in Isolation The table reports the estimated coefficients of the negative rates dummy variable in regression (13) together with their respective *p*-values (in brackets) obtained by using Newey-West standard errors. The first column reports the regression without controls, the second column reports the estimates with the core set of control variables, and the third column contains the results including all the controls. The four samples start in July 2011, July 2010, July 2009, and July 2008, and end in June 2013, June 2014, June 2015, and June 2016, respectively.

case, this seems reasonable given that the involved portfolio flows likely were informed by the general experience with the euro and the new common monetary policy in the euro area.

### 5.1.2 Effects of the Introduction of Negative Rates in Isolation

To assess the impact of the effect in isolation of the introduction of negative rates, we follow a similar approach and run simplified regressions of the following form:

$$\bar{\Psi}_t = \alpha + \delta_{nr} d_t^{nr} + \delta_l X_t + \varepsilon_t, \tag{13}$$

where we first use a sample that contains one year of data before the July 5, 2012, introduction of negative rates and one year of data after this date. As before, the regressions are repeated with symmetrically expanded samples that each time contain one more year of data before and after the event date. For consistency with the previous exercise, we add up to four years of data around both sides of the event date.

The results are reported in Table 9, in which we note a significant and large positive effect on the Danish safety premium from the introduction of negative interest rates when we exclude controls. Once they are included, the magnitude of the estimated coefficients is significantly diminished, and their significance mostly vanishes. Similar to what we found for Switzerland, any effects on the Danish safety premium during this period appear to be therefore due to other factors.

Sample	No controls	Core controls	All controls
+/-1 year	-3.4	-11.2	-16.6
	(0.01)	(0.02)	(0.02)
+/-2 years	-0.9	-8.0	-7.0
	(0.10)	(0.00)	(0.00)
+/-3 years	-0.6	-5.8	-6.1
	(0.14)	(0.00)	(0.00)
+/-4 years	-0.4	-3.5	-4.6
	(0.21)	(0.00)	(0.00)

Table 10: Average Treatment Effects of ECB's PSPP in Isolation

The table reports the estimated coefficients of the PSPP variable in regression (14) together with their respective *p*-values (in brackets) obtained by using Newey-West standard errors. The first column reports the regression without controls, the second column reports the estimates with the core set of control variables, and the third column contains the results including all the controls. The four samples start in January 2014, January 2013, January 2012, and January 2011, and end in December 2015, December 2016, December 2017, and December 2018, respectively.

## 5.1.3 Effects of the ECB's PSPP in Isolation

One particular factor we are interested in exploring further is the effect of the ECB's government bond purchases on the Danish safety premium. Given that such purchases reduce the available stock of government-backed safe assets in the euro area, they could potentially reduce the relative safety and exclusivity premium of Danish safe assets similar to what we observe for the Swiss safety premia.

To test the stated hypotheses, we include the ECB government bond holdings divided by the nominal GDP of the euro area in regressions that take a form similar to the two previous exercises:

$$\bar{\Psi}_t = \alpha + \delta_{nr} d_t^{pspp} + \delta_l X_t + \varepsilon_t, \tag{14}$$

where we first use a sample that contains one year of data before the January 22, 2015, launch of the PSPP and one year of data after this date. As before, the regressions are repeated with symmetrically expanded samples that each time contain one more year of data before and after the event date. Given that our sample ends in March 2019, we can add up to four years of data around both sides of the event date as in the exercises in the previous sections.

The results are reported in Table 10. Overall, they point to a significant and sizable *negative* effect on the Danish safety premium from the ECB's asset purchases. The effect becomes more significant as we expand the data window around the January 22, 2015, event date. The magnitude of the effect varies with the length of the sample and the inclusion of controls, but with at least six years of data and controls included the estimate ranges from -3.5 basis points to -6.1 basis points per percentage point of GDP in asset purchases. Given the increase in the ECB's asset holdings through the end of our sample, a point estimate of



Figure 12: Dynamic Conditional Correlations of Swiss and Danish Safety Premia Illustration of the average estimated Danish government bond safety premium for each observation date, as implied by the AFNS-R model. The bond safety premiums are measured as the estimated yield difference between the frictionless yield to maturity of individual government bonds with the market risk factor turned off and the corresponding fitted yield to maturity. The data cover the period from January 31, 1995, to January 31, 2019.

around -4 basis points implies a cumulative effect of close to 80 basis points between spring 2015 and spring 2019. Thus, the declines in the available stock of safe assets in the euro area has materially negatively affected the safety premium commanded by Danish safe assets.

The estimated effects are qualitatively similar to those found for Swiss safety premia. However, the magnitude is an order of magnitude larger. The difference in sensitivity is likely tied to the fact that Denmark is a member of the European Union and that the Danish kroner is pegged to the euro. Thus, the Danish financial markets are extremely closely tied to those in the euro area. In contrast, Switzerland is not a member of the European Union, and the Swiss franc has been freely floating against the euro since January 2015, creating some room for the exchange rate to adjust in response to adjustments in Swiss bond premia.

## 5.2 Dynamic Correlations between Swiss and Danish Safety Premia

As we have shown in the previous sections, both Swiss and Danish safety premia reacted significantly to the introduction of the euro and to the ECB's asset purchases. In this section, we therefore examine their co-movement and analyze whether these events might have affected their dynamic relationship.

To begin, we look at the sample correlation of first differences in the safety premia in three sub-samples: the period before the introduction of euro (January 1995-December 1998), the period after the euro introduction but before the start of ECB's asset purchase program (January 1999-December 2014), and the period after the ECB's asset purchases started (January 2015-March 2019). We find that the correlations in the three periods are -0.05, 0.15, and -0.08, respectively. The difference between the correlations in the first and the second period, and between those in the second and third period are statistically significant, with Fisher's z-score of -2.49 and 2.95, respectively.

We also estimate a time-varying correlation coefficient between the first differences in the Swiss and Danish safety premia by using a version of the General Autoregressive Conditional Heteroskedasticity (GARCH) model that allows for Dynamic Conditional Correlation (DCC) as proposed by Engle (2002). Figure 12 plots the time-varying correlations estimated using a variety of DCC GARCH (m,n) models, in which m = 1,2,3 and n = 1,2,3 (gray-shaded area) and the DCC GARCH (1,1) model with the lowest BIC (solid black line).

The correlation between the safety premia turned positive around the time of the introduction of the euro and reached its maximum around the time of the peak of the European sovereign debt crisis in the summer of 2012. After the ECB started to purchase government bonds in the euro area in January 2015, the correlation trended lower but recovered somewhat toward the end of the sample. This is a pattern that would be consistent with a gradual decline in the importance of safety premia in Swiss and Danish government bond yields since 2015, as also indicated by our regression analysis.

To summarize, the analysis of the Danish safety premium produces results that are very similar to our findings for the Swiss safety premium. First and most importantly, the launch of the euro appears to have boosted Danish safety premia by 20-30 basis points. Second, the Danish safety premia were not affected by the introduction of negative rates. Last, the ECB's purchases of safe government bonds from the euro area have put significant downward pressure on Danish safety premia. Therefore, we take the presented evidence to provide full support for the conclusions we draw from the analysis of the Swiss safety premium.

## 6 Conclusion

In this paper, we are among the first to estimate bond-specific premia, frequently labeled convenience yields, in a conventional government bond market, specifically the market for Swiss Confederation bonds. Since these bonds are much less liquid than U.S. Treasuries, we attribute their premia to the high safety and creditworthiness of the Swiss Confederation and therefore refer to them as *safety premia*. To support this interpretation in our empirical analysis, we control for a large number of factors, including investor risk aversion as well as liquidity and debt supply effects.

We find a sizable safety premium of Swiss safe assets averaging 68 basis points and exhibiting significant variation over time. One important implication of this result is that the yields of Swiss Confederation bonds are below those we would observe in a world without excess demand tied to safety concerns. As a consequence, these yields represent a downward biased measure of the risk-free rate in Switzerland—a point also made by Krishnamurthy and Vissing-Jorgensen (2012) in the context of the U.S. Treasury market.

In addition to describing the variation of the Swiss safety premium, we also explore its relationship with three key events over the past 25 years: the launch of the euro in January 1999, the introduction of negative interest rates in Switzerland in December 2014, and the ECB's launch of its main asset purchase program, the PSPP, in January 2015.

Based on a range of tests of a break in the Swiss safety premium in January 1999, we conclude that the launch of the euro led to a long-lasting upward shift in the Swiss safety premium of between 35 and 40 basis points. We conjecture that this effect came about because, once the euro was created, even German and French bonds could be exposed to some extreme tail risk such as a bailout of one or more euro member states, which would leave them seriously indebted. All else being equal, such scenarios would increase the attractiveness of safe assets in a safe-haven country such as Switzerland.

More recently, a combination of negative interest rates and ECB asset purchases appear to have given rise to a persistent negative pressure on the Swiss safety premium. Negative effects from these two developments could be expected. A move to negative rates should make government debt less attractive in general. Furthermore, safe asset purchases by foreign central banks reduce the supply of foreign safe assets and, in turn, reduce the relative uniqueness of Swiss Confederation bonds within the class of highly safe European assets. These conclusions are supported by matching evidence from Danish government bond prices, which represent another class of extremely safe assets. Our results hence point to a potentially important international spillover channel of central bank asset purchases that operates through its impact on the relative scarcity of safe assets. These findings also raise the prospect that even the safest assets can become unattractive when interest rates are sufficiently low. To shed light on that question, it may be relevant to look at other countries with very safe government debt and negative interest rates. However, we leave it for future research to explore those avenues.

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# **Online Appendix**

# The Safety Premium of Safe Assets

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The views in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Federal Reserve Bank of San Francisco, the Federal Reserve System, or the Swiss National Bank.

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## A Swiss Confederation Bond Data

This appendix provides the details of the Swiss Confederation bonds considered in the analysis.

Confederation bond	No.	Issuance		Number of	Total
Confederation bond	obs.	Date	Amount	auctions	amount
$(1) \ 6.75\% \ 1/22/1999$	69	1/22/1991	324	2	824
(2) 6.25% 3/15/1999	71	3/15/1991	206	1	206
(3) 6.25% 7/15/1999	74	5/17/1991	497	1	497
(4) 6.25% 7/15/2000	87	7/15/1991	302	1	302
(5) 6.5% 2/5/2002	106	2/5/1992	600	2	1,400
(6) 6.5% 4/10/2004	132	4/10/1992	704	1	704
(7) 6.75% $6/11/2003$	122	6/11/1992	1,205	1	1,205
(8) 7% 7/9/2001	99	7/9/1992	500	2	1,000
(9) 7% 9/10/2003	124	9/10/1992	700	1	700
(10) 6.25% 11/5/2004	139	11/5/1992	700	1	700
(11) 6.25% $1/7/2003$	117	1/7/1993	998	2	1,609
(12) 5.25% $2/11/1998$	57	2/11/1993	800	2	1,154
(13) 5% 3/11/2000	81	3/11/1993	500	5	1,948
(14) 4.5% 4/8/2006	153	4/8/1993	477	10	4,291
(15) 4.5% $6/10/2000$	81	6/10/1993	550	5	2.351
(16) 4.5% 7/8/2002	105	7/8/1993	231	9	2.858
(17) 4.5% $10/7/2004$	129	10/7/1993	862	6	3.816
(18) 4.25% 1/6/2014	237	1/6/1994	676	8	3,208
(19) 4% $3/10/1999$	56	3/10/1994	353	3	907
(20) 5.5% $10/7/2001$	81	10/7/1994	685	2	1.140
(21) 5% $11/10/1996$	21	11/10/1994	555	1	555
(22) 5.5% 1/6/2005	117	1/6/1995	726	3	1.769
(23) 4.25% 1/8/2008	141	1/8/1996	1.000	7	5,366
(24) 4.5% $6/10/2007$	129	6/10/1996	302	7	4,468
(25) 4.25% 6/5/2017	237	6/5/1997	228	10	3,260
(26) 3.5% $8/7/2010$	153	8/7/1997	561	10	8,902
(27) 3.25% 2/11/2009	118	2/11/1998	938	8	8,523
(28) 4% 2/11/2023	243	2/11/1998	231	9	2.858
(29) 4% 4/8/2028	243	$\frac{4}{8}/1998$	467	10	3.012
(30) 2.75% $6/10/2012$	149	6/10/1999	569	13	6,360
(31) 4% 1/6/2049	234	1/6/1999	189	9	1,595
(32) 4% 2/11/2013	151	2/11/2000	586	9	5,380
(33) 4% 6/10/2011	128	6/13/2000	1,332	7	6,182
(34) 3.75% 6/10/2015	164	6/11/2001	524	7	3,139
(35) 3% 1/8/2018	174	1/8/2003	1,139	11	6,136
(36) 2.5% $3/12/2016$	152	3/12/2003	715	11	5,704
(37) 3.5% 4/8/2033	192	4/8/2003	486	7	2,433
(38) 3% 5/12/2019	177	5/12/2004	309	10	5,399
(39) 1.75% 11/5/2009	57	11/5/2004	628	3	1,756
(40) 2.25% 7/6/2020	165	7/6/2005	423	11	4,101
(41) 2% 10/12/2016	128	10/12/2005	483	6	2,667
(42) 2% 11/9/2014	105	11/9/2005	547	3	1,606
(43) 2.5% 3/8/2036	157	3/8/2006	222	9	2,303
(44) 3.25% $6/27/2027$	142	6/27/2007	101	8	1,909
(45) 2% 4/28/2021	108	4/28/2010	675	11	3,958
(46) 2% 5/25/2022	95	5/25/2011	531	8	3,233
(47) 2.25% 6/22/2031	94	6/22/2011	478	7	1,920
(48) 1.5% $4/30/2042$	83	4/30/2012	872	8	3,196
(49) 1.25% $6/11/2024$	81	6/11/2012	216	11	2,743
(50) 1.25% 6/27/2037	81	6/27/2012	267	11	3,049
(51) 1.5% 7/24/2025	68	7/24/2013	540	8	2,467
(52) 1.25% $5/28/2026$	58	5/28/2014	301	13	2,062
(53) 2% 6/25/2064	57	6/25/2014	565	5	1,487
(54) 0.5% 5/27/2030	46	5/27/2015	318	10	1,798
(55) 0.5% 5/30/2058	34	5/30/2016	169	8	1,208
(56) 0% 6/22/2029	33	6/22/2016	308	11	2,213
(57) 0.5% 5/24/2055	22	5/24/2017	80	7	1,102
(58) 0.5% 6/28/2045	21	6/28/2017	255	7	999
(59) 0.5% 6/27/2032	9	6/27/2018	206	5	664

## Table 1: The Universe of Swiss Confederation Bonds

The table reports the characteristics, issuance dates, initial issuance amounts, total number of auctions, and total issuance amounts in millions of Swiss francs for each Confederation bond. Also reported are the number of monthly observation dates for each bond during the sample period from January 29, 1993, to March 29, 2019.

## **B** Model Fit and Bond-Specific Risk Parameter Estimates

The fit of the AFNS and AFNS-R models along with the estimated bond-specific risk sensitivity parameters in the latter model for each Swiss Confederation bond are given in Table 2.

The first two columns in Table 2 show that the bond pricing errors produced by the AFNS model indicate a reasonable fit, with an overall RMSE of 9.08 basis points. The following two columns reveal a substantial improvement in the pricing errors when correcting for the bond-specific risk premia, because the AFNS-R model has a much lower overall RMSE of just 5.78 basis points. Hence, accounting for these risk premia leads to a notable improvement in the ability of our model to explain the confederation bond prices. Without exception there is uniform improvement in model fit from augmenting the AFNS model.

The final four columns of Table 2 report the estimates of the bond-specific parameters associated with each Confederation bond. Except for a handful of bonds, including one fifty-year bond (number 53), most bonds in our sample are exposed to bond-specific risk as their  $\beta^i$  parameters are large and statistically significant.

		Pricing	errors		Estimated parameters					
Confederation bond	AFNS		AFNS-R		AFNS-R					
	Mean	RMSE	Mean	RMSE	$\beta^i$	SE	$\lambda^{R,i}$	SE		
$(1) \ 6.75\% \ 1/22/1999$	10.76	16.84	-1.33	11.18	2.3133	0.1703	9.9982	1.6093		
$(2) \ 6.25\% \ 3/15/1999$	5.32	17.22	-0.06	6.33	3.9044	0.2841	9.9934	1.0741		
$(3) \ 6.25\% \ 7/15/1999$	-6.55	18.40	0.49	10.21	2.4130	0.1626	1.2653	0.2087		
$(4) \ 6.25\% \ 7/15/2000$	2.41	17.83	2.81	14.76	1.2629	0.0903	1.0618	0.2000		
$(5) \ 6.5\% \ 2/5/2002$	4.08	8.92	0.40	4.97	1.3750	0.3164	0.2498	0.2159		
(6) $6.5\% \ 4/10/2004$	1.73	8.33	0.80	5.69	2.3906	0.5216	0.0946	0.0354		
$(7) \ 6.75\% \ 6/11/2003$	2.39	9.24	0.56	6.02	39.2904	0.8659	0.0039	0.0003		
(8) 7% 7/9/2001	1.79	7.72	0.47	5.62	1.2626	0.0916	0.6938	0.1995		
(9) 7% 9/10/2003	3.23	12.35	-0.09	10.29	72.5920	0.8295	0.0020	0.0001		
$(10) \ 6.25\% \ 11/5/2004$	4.80	9.66	0.43	6.74	1.5001	0.1448	0.2605	0.0863		
$(11) \ 6.25\% \ 1/7/2003$	1.90	8.73	0.67	5.67	23.4727	0.8298	0.0070	0.0005		
(12) 5.25% 2/11/1998	-3.66	13.70	0.43	8.14	24.0735	0.8213	0.0057	0.0005		
$(13)\ 5\%\ 3/11/2000$	-4.93	11.38	-0.24	8.09	38.5223	0.8627	0.0044	0.0004		
(14) 4.5% 4/8/2006	-1.52	6.64	-0.29	4.50	1.6162	0.1273	0.2928	0.0922		
(15) 4.5% 6/10/2000	-5.83	10.76	-1.14	7.41	24.6930	0.8188	0.0074	0.0006		
(16) 4.5% 7/8/2002	-3.16	9.28	-0.86	5.33	3.5347	0.7954	0.0638	0.0200		
(17) 4.5% 10/7/2004	-1.50	5.90	-0.27	4.48	1.6574	0.1694	0.2737	0.0967		
(18) 4.25% 1/6/2014	1.93	7.60	0.27	4.43	76.7614	0.8264	0.0014	0.0001		
(19) 4% 3/10/1999	-9.20	13.70	-1.50	8.43	140.4379	0.8460	0.0018	0.0002		
(20) 5.5% 10/7/2001	0.47	10.36	0.02	8.08	1.5722	0.5510	0.2594	0.2234		
(21) 5% 11/10/1996	-3.12	16.11	0.60	12.08	22.438	0.8986	0.0078	0.0008		
(22) 5.5% 1/6/2005	0.01	6.18	0.27	4.89	1.6485	0.1538	0.3039	0.0828		
(23) 4.25% 1/8/2008	-1.89	7.62	0.24	4.04	1.6608	0.1141	0.4256	0.1389		
(24) 4.5% 6/10/2007	-2.10	7.23	0.05	4.17	1.5642	0.1005	2.1419	0.8660		
(25) 4.25% 6/5/2017 (26) 2.5% 8/7/2010	0.07	5.88	0.03	4.01	1.3577	0.8925	0.0178	0.0027		
(26) 3.5% 8/7/2010	0.17	9.70	0.33	6.34 C.45	1.8668	0.1264	0.2807	0.0552		
(27) 3.25% 2/11/2009 (28) 4% 2/11/2022	3.38	13.30	1.11	0.45	1.0901	0.1119	0.0472	0.2007		
(26) 470 2/11/2023 (20) 407 4/8/2028	3.45	0.90	0.09	4.04	1.7695	0.2057	0.0652	0.0271		
(29) 470 4/8/2028 (20) 2.75% 6/10/2012	4.02 2.70	0.00	0.51	0.01 5.07	1 2 2 2 5 6	n.a.	0.4977	0.9000		
(30) 2.75% 0/10/2012 (31) 4% 1/6/2040	-2.10	0.01	-0.00	0.07 6.39	2.3350 0.3337	0.2128	0.1007	1.0561		
(31) 470 1/0/2049 (32) $4\% 2/11/2013$	0.83	6 66	0.00	4.03	2 2104	0.0000	9.9075	0.0400		
(32) 4% 2/11/2013 (33) 4% 6/10/2011	1.67	0.00 7 32	-0.02	4.05	1 9441	0.2141 0.1462	0.1305 0.3257	0.0490		
(34) 3 75% 6/10/2015	-1.32	7.32 5.07	-0.02	4.92	2.6417	0.1402	0.3257 0.1257	0.0890		
(35) 3% 1/8/2018	-0.88	5.56	0.21	3.96	2.0417	0.2640	0.1201 0.1401	0.0305		
(36) 2.5% 3/12/2016	-3.38	$5.00 \\ 5.72$	0.04	4 21	2.3993	0.2040 0.2047	0.1401	0.0307		
(37) 3 5% $4/8/2033$	-1.36	7.25	-0.82	6.03	0.8221	0.2041	9 9921	1.4182		
(38) 3% 5/12/2019	-1.61	5.95	0.00	4 19	2.2270	0.1962	0.0021 0.1668	0.0328		
(39) 1.75% 11/5/2009	-12.72	27.22	-0.28	14.05	164.9121	1.7678	0.0038	0.0002		
(40) 2.25% 7/6/2020	-1.02	4.86	0.18	4.00	1.9344	0.1404	0.2366	0.0545		
(41) 2% 10/12/2016	-4.52	6.99	0.19	4.44	2.2048	0.1348	0.3705	0.0890		
(42) 2% 11/9/2014	-8.22	13.51	0.19	7.68	2.3374	0.1194	0.4899	0.0764		
(43) 2.5% 3/8/2036	-4.49	8.08	-0.65	5.04	0.7164	0.0372	9.9993	1.4149		
(44) 3.25% $6/27/2027$	3.34	6.67	-0.06	4.49	1.1105	0.0250	9.9867	1.6540		
(45) 2% 4/28/2021	-0.01	3.97	0.09	2.96	1.7559	0.1293	0.6022	0.3185		
(46) 2% 5/25/2022	1.07	3.71	0.09	2.69	1.6147	0.1378	0.7530	0.6484		
(47) 2.25% $6/22/2031$	0.42	3.34	0.01	2.71	0.8866	0.0646	9.9845	6.3796		
(48) 1.5% 4/30/2042	-7.28	8.89	-0.13	2.86	0.5482	0.0740	9.9939	7.5394		
(49) 1.25% 6/11/2024	0.66	3.53	0.07	2.07	1.4239	0.1470	0.7144	0.8508		
(50) 1.25% 6/27/2037	-5.27	6.55	-0.06	2.75	0.6711	0.0787	9.9980	7.7019		
(51) 1.5% 7/24/2025	1.32	3.69	0.04	2.01	1.3203	0.1782	0.6871	1.1949		
(52) 1.25% 5/28/2026	-0.25	2.38	0.05	2.25	1.2600	0.1840	0.7983	1.5809		
(53) 2% $6/25/2064$	3.52	6.35	0.07	4.28	0.0000	0.1051	9.9897	13.1894		
(54) 0.5% 5/27/2030	-2.92	3.43	-0.04	1.98	0.9754	0.1195	8.9381	14.7901		
$(55) \ 0.5\% \ 5/30/2058$	-6.79	7.32	0.14	2.70	1.3390	23.4571	0.0087	0.1722		
$(56) \ 0\% \ 6/22/2029$	-4.13	4.83	-0.01	1.28	1.3346	4.0993	0.2696	2.5954		
$(57) \ 0.5\% \ 5/24/2055$	-8.51	8.84	0.23	2.17	0.7118	28.9378	0.0339	2.0984		
$(58) \ 0.5\% \ 6/28/2045$	-7.86	7.94	0.11	1.10	0.5003	0.2811	9.9921	30.7706		
$(59) \ 0.5\% \ 6/27/2032$	-2.52	2.58	0.05	0.88	0.8578	0.3493	9.0653	41.7957		
All yields	-0.22	9.08	0.09	5.78	-	_	-	_		

## Table 2: Pricing Errors and Estimated Bond-Specific Risk Parameters

This table reports the mean pricing errors (Mean) and the root mean-squared pricing errors (RMSE) of Swiss Confederation bonds in the AFNS and AFNS-R models estimated with a diagonal specification of  $K^{\mathbb{P}}$  and  $\Sigma$ . All errors are reported in basis points. Also reported are the estimates of the bondspecific parameters within the AFNS-R model and their associated standard errors (SE). Note that standard errors are not available (n.a.) for the normalized value of  $\beta^{29}$ .

## C Accuracy of the Swiss Confederation Bond Price Data

In this appendix, we assess the quality of our Swiss Confederation bond data. First, we describe how we construct our own synthetic zero-coupon yields based on the bond price data before we go on to compare them to the zero-coupon yields constructed by SNB staff and made publicly available on the SNB website. Finally, to identify any major differences, we compare the fit of both of these synthetic yield curves to the fit of various dynamic yield curve models estimated using our data.

## C.1 Construction of Synthetic Zero-Coupon Yields

We construct synthetic zero-coupon yields using the Svensson (1995) discount function combined with the panel of Swiss Confederation coupon bond prices described in the main text. The Svensson (1995) yield curve has a flexible functional form given by

$$y_t(\tau) = \beta_0(t) + \frac{1 - e^{-\lambda_1 \tau}}{\lambda_1 \tau} \beta_1(t) + \left(\frac{1 - e^{-\lambda_1 \tau}}{\lambda_1 \tau} - e^{-\lambda_1 \tau}\right) \beta_2(t) + \left(\frac{1 - e^{-\lambda_2 \tau}}{\lambda_2 \tau} - e^{-\lambda_2 \tau}\right) \beta_3(t), \quad (1)$$

where we impose the restrictions that  $\lambda_1 > \lambda_2 > 0$ . This function contains the level, slope, and curvature components known from Nelson and Siegel (1987) and augments them with an additional curvature factor to provide a better fit to the long end of the yield curve. The corresponding discount function is easily obtained as  $P_t^{\rm zc}(\tau) = e^{-y_t(\tau)\tau}$ . Now, consider the value at time t of a coupon bond with maturity at  $t + \tau$  that pays an annual coupon C. Its clean price, denoted  $P_t(\tau, C)$ , is simply the sum of its remaining cash flow payments weighted by the zero-coupon bond price function  $P_t^{\rm zc}(\tau)$ :

$$P_t(\tau, C) = C(t_1 - t)P_t^{\rm zc}(t_1) + \sum_{j=2}^N \frac{C}{2}P_t^{\rm zc}(t_j) + P_t^{\rm zc}(\tau), \quad t < t_1 < \ldots < t_N = \tau.$$
(2)

We estimate the parameters in the Svensson (1995) curve,  $\psi = (\beta_0, \beta_1, \beta_2, \beta_3, \lambda_1, \lambda_2)$ , for each observation date by optimizing the following objective function:

$$\min_{\psi} \sum_{i=1}^{n_t} \frac{1}{D_t^{Data,i}} (P_t^{Data,i} - \widehat{P_t^i}(\psi))^2, \tag{3}$$

where  $n_t$  is the number of coupon bond prices observed on day t,  $P_t^{Data,i}$  is the observed price for bond number i,  $\widehat{P_t^i}$  is its price implied by the Svensson (1995) discount function, and  $D_t^{Data,i}$  is its duration, which is model-free and calculated before the estimation based on

the Macaulay formula. The stated objective is to minimize the weighted sum of the squared deviations between the actual bond prices and the predicted prices, where the weights are the inverse of the durations of each individual security. This is identical to the objective function used by Gürkaynak et al. (2007, 2010). The optimization for each observation date is started at the same parameter vector:

$$\begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} 0.05010075 \\ -0.02569675 \\ 0.02326028 \\ -0.01846854 \\ 0.8378759 \\ 0.09652915 \end{pmatrix}$$

The purpose of these synthetic yields is twofold. First, they serve as a key input into the validation of the fit of the dynamic term structure models we consider in the paper. Second, they can be used to evaluate the quality of our Swiss Confederation bond data by comparing our synthetic yields to those produced and made publicly available by the SNB.<sup>1</sup>

## C.2 Synthetic Zero-Coupon Yield Comparison

In this section, we focus on the synthetic yields constructed with the Svensson (1995) yield model as described in Appendix C.1. Since this is the same yield model used by SNB staff, we can perform a clean apples-to-apples comparison of the synthetic yields from these two samples. Provided the yields are very close to each other, it would imply that our bond price data are qualitatively very similar to those used by SNB staff.

Table 3 reports summary statistics for the differences between the two data sets at various maturities. The mean absolute differences for yields in the three- to thirty-year maturity range are within four basis points and hence small. Larger deviations emerge at the shortest maturities, with mean absolute differences at the one-year maturity of 14 basis points. Large maximum outlier differences are also evident at both ends of the yield curve. Non-negligible discrepancies are also evident in the correlations between the two data sets, shown in the last two columns in Table 3. The correlations are clearly less than one at short and long maturities. On the other hand, it is clear from Table 3 that the constructed yield curves are almost indistinguishable in the range from three to twenty years remaining to maturity.

<sup>&</sup>lt;sup>1</sup>The parameters from the Svensson (1995) yield curve fitted daily by SNB staff are available at the link:

Maturity	Mean	Mean	Max.	Correlation		
in years	diff.	abs. diff.	abs. diff.	Levels	Diff.	
1	8.07	13.57	179.27	0.988	0.626	
2	5.44	7.46	37.49	0.999	0.927	
3	1.27	3.67	18.77	1.000	0.965	
4	-0.84	2.47	14.16	1.000	0.978	
5	-1.54	2.45	8.68	1.000	0.981	
6	-1.56	2.32	9.23	1.000	0.983	
7	-1.35	2.09	10.34	1.000	0.984	
8	-1.12	1.87	12.50	1.000	0.985	
9	-0.92	1.74	15.24	1.000	0.986	
10	-0.79	1.74	15.51	1.000	0.986	
11	-0.72	1.88	13.55	1.000	0.985	
12	-0.69	2.03	14.35	1.000	0.984	
13	-0.70	2.16	16.67	1.000	0.981	
14	-0.72	2.29	18.31	1.000	0.978	
15	-0.76	2.42	20.64	1.000	0.972	
16	-0.80	2.52	22.45	1.000	0.964	
17	-0.84	2.57	26.16	1.000	0.954	
18	-0.88	2.58	34.78	0.999	0.941	
19	-0.91	2.56	43.46	0.999	0.926	
20	-0.93	2.54	52.11	0.999	0.908	
21	-0.94	2.50	60.62	0.999	0.888	
22	-0.94	2.45	68.95	0.999	0.866	
23	-0.93	2.41	77.05	0.999	0.841	
24	-0.90	2.41	84.88	0.999	0.815	
25	-0.87	2.46	92.44	0.999	0.787	
26	-0.83	2.64	99.70	0.999	0.757	
27	-0.77	2.88	106.66	0.998	0.727	
28	-0.71	3.16	113.32	0.998	0.695	
29	-0.64	3.48	119.69	0.998	0.662	
30	-0.56	3.83	125.78	0.998	0.629	

## Table 3: Comparing Two Data Sets of Synthetic Zero-Coupon Yields

The table reports the summary statistics for the mean differences, the mean absolute differences, and the maximum absolute differences between synthetic Swiss zero-coupon yields from the Swiss National Bank and our implementation of the Svensson (1995) discount function. These differences are reported in annual basis points. The last two columns report the correlations between the two yield series for each maturity in levels and first differences, respectively. The data series are monthly, covering the period from January 30, 1998, to March 29, 2019.

These observations raise the question: which yield curve is the better representation? And is there a better way to construct synthetic yield curves?

To address these questions, Andreasen et al. (2019) recommend using one-step estimations and focusing on the fitted yield curves from such exercises. Therefore, we follow their

https://data.snb.ch/en/topics/ziredev#!/cube/rendopar



(b) Two-year yield

Figure 1: Comparison of Synthetic Shorter-Term Zero-Coupon Yields

advice and estimate the four dynamic term structure models described in Section 3.4 of the paper with the one-step method. We include the fitted yields from these four models in the comparison.

Figure 1 provides comparisons of the one- and two-year Swiss zero-coupon yields constructed in these six different ways, while Figure 2 shows the corresponding comparisons for the ten- and thirty-year Swiss zero-coupon yields.

We note that short-term yields are sensitive to the yield curve construction method used. We also note that the short-term zero-coupon yields constructed by SNB staff are different for extended periods from the other five yield sets. This suggests that the SNB yields may be



(b) Thirty-year yield

Figure 2: Comparison of Synthetic Longer-Term Zero-Coupon Yields

constructed using only a subset of our sample of bond prices. Alternatively, it may include some additional short-term interest rate information, which anchors the short end of these curves at a different level than the other curves. Finally, it is notable that the static Svensson model produces yield curves with significantly more volatile short-term interest rates. We ascribe this to the relatively low representation of short-term bonds in our sample for much of our sample period.

On the other hand, interestingly, any such differences are much less pronounced when it comes to the constructed long-term yields as evidenced in Figure 2. Indeed, for the benchmark ten-year yields, there are practically no material differences across any of the six series shown.

Maturity	No			One-step estimation					Static Svensson model				
bucket	oba	AFNS		AFGNS		Svensson		Gen. Svensson		Our data		SNB data	
in years	obs.	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE	Mean	RMSE
0-2	704	-0.39	15.71	-0.09	10.33	-0.59	13.84	-0.30	10.35	-2.26	16.53	-6.69	24.43
2-4	737	0.79	8.44	0.47	5.94	1.40	5.85	1.12	5.98	2.20	5.89	1.08	6.42
4-6	655	-0.84	6.48	-1.00	5.06	-0.74	5.41	-0.99	5.05	-0.92	5.20	0.53	5.46
6-8	570	-0.36	5.37	-0.49	5.40	-0.95	4.96	-1.04	5.40	-1.58	4.85	-0.37	4.98
8-10	546	-0.57	5.85	-0.24	4.08	-1.27	4.14	-0.89	3.98	-1.67	3.78	-0.86	3.71
10-12	461	-0.21	5.65	0.34	3.68	-0.59	3.94	-0.21	3.64	-0.95	3.68	-0.10	3.35
12-14	317	-0.28	6.50	0.81	4.06	0.67	4.19	0.62	3.95	0.62	3.78	0.92	4.59
14-16	212	1.24	7.62	2.00	5.98	2.08	5.41	2.25	5.95	1.87	4.64	2.60	6.05
16-18	157	1.19	5.88	0.78	3.77	1.75	3.74	1.30	3.69	2.23	3.91	2.36	4.75
18-20	183	2.45	7.45	2.06	6.71	2.82	6.83	2.67	6.77	1.93	5.53	4.25	7.57
20-22	120	-0.65	6.08	-0.11	5.59	1.51	5.52	1.19	5.52	1.34	4.71	1.65	5.63
22-24	132	-3.09	7.24	-3.38	5.36	-1.60	4.30	-2.16	4.38	-0.86	3.61	-0.73	3.87
24-26	108	-3.70	8.51	-3.30	7.22	-1.50	6.46	-1.99	6.66	-1.91	5.76	-1.07	6.28
26-28	118	-7.55	9.64	-7.38	8.91	-5.09	6.98	-5.44	7.29	-4.64	6.01	-4.55	6.47
28-30	90	-5.33	10.98	-7.60	9.03	-6.80	8.20	-6.95	8.37	-5.68	7.28	-5.70	7.36
30 <	344	1.65	10.14	0.51	6.83	-0.64	6.40	-0.20	6.39	0.09	4.21	-0.09	7.15
All bonds	$5,\!454$	-0.31	8.79	-0.30	6.34	-0.28	7.02	-0.28	6.21	-0.50	7.46	-0.62	10.16

Table 4: Summary Statistics of Fitted Errors of Swiss Confederation Bond Yields This table reports the mean pricing errors (Mean) and the root mean-squared pricing errors (RMSE) of the Swiss bond prices for various models estimated on the sample of Swiss Confederation bond prices. The pricing errors are reported in basis points and computed as the difference between the implied yield on the coupon bond and the model-implied yield on this bond. The data are monthly and cover the period from January 30, 1998, to March 29, 2019.

Thus, in the core five- to twenty-year maturity segment, the construction of zero-coupon yields is very robust and barely sensitive to the yield curve construction method used.

## C.3 Analysis of Model Fit

To begin, we compare the fit we obtain with our yield curve models to that produced by the synthetic yield curve constructed daily by SNB staff and made publicly available.<sup>2</sup> Presumably, SNB staff would only include high quality bond prices in their yield curve construction. Hence, if those yield curves are able to provide a close fit to our data, the logical implication must be that our data are of high quality. Furthermore, if our models deliver a fit that is on par or better than that obtained with the synthetic yield curves produced by SNB staff, it would be a comforting sign suggesting that our models are flexible and able to deliver what must be characterized as an accurate fit to the prices trading in the Swiss Confederation bond market. As a consequence, their fitted yield curves would be used for both academic research and monetary policy analysis.

 $<sup>^{2}</sup>$ Please note that this exercise covers data only back to 1998, when the SNB yield data become available.

In Table 4, we compare the fit of four yield curve models that we implement with a onestep estimation as recommended by Andreasen et al. (2019). Furthermore, we include the fit from our synthetic zero-coupon yield curves as well as those provided by the SNB. First, the yield curves produced by SNB staff deliver a satisfying fit with an overall RMSE of 10.16 basis points. Furthermore, its fit to bond prices with maturities greater than two years is very tight. Second, our own Svensson (1995)-based synthetic yield curves deliver an even better fit, which is not surprising given that it was fitted to this exact data sample. These two observations combined lead us to conclude that our Swiss Confederation bond data are indeed of high quality.

Finally, as for the four models considered in the one-step estimations, we note that there is only very marginal improvement from increasing flexibility by switching from the three-factor AFNS model to the very flexible AFGNS model or the generalized Svensson model, which both have five factors. We take this a further evidence that our augmented AFNS-R model is well-specified regarding its structure for the frictionless level, slope, and curvature factors.

## D Description of Regression Variables

In this appendix, we provide a detailed description of the variables used in the regression analysis in the paper, starting with our core set of control variables, followed by details about the additional set of control variables used in the analysis.

## D.1 Core Control Variables



Figure 3: VIX and U.S. Treasury On-the-Run Premium

Figure 3 shows the CBOE Volatility Index (VIX) along with the U.S. Treasury ten-year on-the-run premium derived from the difference between par-coupon yields of seasoned tenyear Treasury bonds (as per Gürkaynak et al. (2007)) and the yields on newly issued tenyear Treasury bonds (as reported in the Federal Reserve's H.15 series). We note the high positive correlation (65%) between these two measures of financial market uncertainty and risk aversion.

Figure 4 shows the three-month CHF LIBOR, which has been the leading policy rate of the SNB since 2000, when its current monetary policy framework was adopted. Furthermore, in private conversations, SNB staff confirmed that this rate is also a representative measure of the stance of Swiss monetary policy under the previous monetary policy framework. As a consequence, we use this series as a measure of the opportunity costs of holding money, which has been shown by Nagel (2016) to be a good proxy for the liquidity premium in U.S. Treasury bills, and we want to control for similar effects in the pricing of Swiss Confederation bonds.

The figure also shows the ten-year Swiss zero-coupon yield implied by the AFGNS model



Figure 4: Swiss Interest Rates

Illustration of the fitted ten-year Swiss Confederation bond yield implied by the AFGNS model along with the three-month CHF LIBOR rate downloaded from Bloomberg.

considered in Section 3.4 of the paper and estimated with a one-step approach as recommended by Andreasen et al. (2019). This series is used to construct the yield spread between German and Swiss ten-year yields and, as demonstrated in Appendix C.2, it is very robustly estimated.

Figure 5 shows the German and Italian ten-year yields downloaded from Bloomberg. In our regression analysis, we include the yield spread between the two to control for (1) effects tied to the compression of sovereign yield spreads in the run-up to the launch of the euro in January 1999; and (2) flight-to-safety effects during various phases of the European sovereign debt crisis. Furthermore, we use the German ten-year yield to calculate the yield spread relative to the matching Swiss ten-year yield shown in Figure 4. Note also that the convergence between the German and Italian yields starts well ahead of the euro launch, but is not complete until relatively close to January 1999, which is a time series pattern that is largely consistent with the time profile of the level shift we observe in the Swiss safety premium around the time of the launch of the euro.

The next variable is the U.S. TED spread, which is calculated as the difference between the three-month U.S. LIBOR and the three-month U.S. T-bill interest rate and shown in Figure 6. This spread represents a measure of the perceived general credit risk in global financial markets that could affect the pricing and trading of Swiss Confederation bonds.

Figure 7 shows the Swiss public debt-to-GDP ratio, which serves as an important control for changes in the supply of Swiss Confederation bonds and as proxy for the true, but unob-



Figure 5: German and Italian Ten-Year Government Yields Illustration of ten-year German and Italian government bond yields.



Figure 6: Three-Month U.S. TED Spread



#### Figure 7: Swiss Public Debt-to-GDP Ratio

Illustration of the Swiss public debt-to-GDP ratio. The data series is quarterly and therefore linearly interpolated to produce the monthly series used in the regression analysis.

served variation in the stock of Swiss safe assets following the analysis of Krishnamurthy and Vissing-Jorgensen (2012). Note that this series is quarterly. Hence, we use linear interpolation to convert it into a monthly series that can be used in the regression analysis.

Following the work of Houweling et al. (2005), we include the average Confederation bond age as a proxy for bond liquidity. This series is shown in Figure 8 along with the average remaining time to maturity. We note that both the age and the duration of the available universe of Swiss Confederation bonds have trended up during the sample period and are near all-time highs at the end of our sample with a value of 8.68 years and 15.55 years, respectively.

Houweling et al. (2005) also recommend using yield volatility as a proxy for bond liquidity. To that end, we use a standard measure of realized volatility based on daily data. First, we estimate the AFGNS model using the daily sample of Swiss Confederation bond prices considered in Appendix ??. This gives us daily fitted Swiss Confederation zero-coupon yields at all relevant maturities. We then generate the realized standard deviation of daily changes in interest rates for the past 31-day period on a rolling basis. The realized variance measure is used by Andersen and Benzoni (2010), Collin-Dufresne et al. (2009), and Jacobs and Karoui (2009) in their assessments of stochastic volatility models. This measure is fully nonparametric and has been shown to converge to the underlying realization of the conditional variance as the sampling frequency increases; see Andersen et al. (2003) for details. The square root of



Figure 8: Average Confederation Bond Age and Time to Maturity Illustration of the average age and time to maturity of the Swiss Confederation bonds in

Illustration of the average age and time to maturity of the Swiss Confederation bonds included in the sample, which covers the period from January 29, 1993 to March 29, 2019 and censors each bond's price when it has less than three months to maturity.

this measure retains these properties. For each observation date t we determine the number of trading days N during the past 31-day time window (where N is most often 21 or 22).<sup>3</sup> We then generate the realized standard deviation as

$$RV_{t,\tau}^{STD} = \sqrt{\sum_{n=1}^{N} \Delta y_{t+n/N}^2(\tau)},$$

where  $\Delta y_{t+n/N}(\tau)$  is the change in yield  $y(\tau)$  from trading day (n-1) to trading day  $n^4$ .

Figure 9 shows that the realized yield volatility series used in the regression analysis is not sensitive to the choice of the yield maturity or the model used to construct the fitted zero-coupon yields. To achieve the greatest accuracy in the constructed yields, we follow Andreasen et al. (2019) and focus on the fitted yield implied by the five-factor AFGNS model. Furthermore, given that the average time to maturity of the available Confederation bonds is close to ten years for much of our sample period as demonstrated in Figure 8, we choose to use the one-month realized volatility of the ten-year yield in our regressions, but we stress that our results are clearly robust to alternative choices, both in terms of the model used and

 $<sup>^{3}</sup>$ As a consequence, the realized volatility measure can be calculated for the period from February 4, 1993, to March 29, 2019.

<sup>&</sup>lt;sup>4</sup>Note that other measures of realized volatility have been used in the literature, such as the realized mean absolute deviation measure as well as fitted GARCH estimates. Collin-Dufresne et al. (2009) also use option-implied volatility as a measure of realized volatility.



(a) Comparison across maturities



(b) Comparison across models

## Figure 9: Realized Yield Volatility Comparisons

The top panel shows the one-month realized volatility of yields with three different maturities, all constructed from the AFGNS model estimated with our daily sample of Swiss Confederation bond prices. The bottom panel shows the one-month realized volatility of Swiss ten-year yields constructed from three different yield curve models, all estimated with our daily sample of Swiss Confederation bond prices.

the maturity considered.

Inspired by the analysis of Hu et al. (2013), we also include a noise measure of Swiss Confederation bond prices to control for variation in the amount of arbitrage capital available in this market. In principle, this could be constructed using any yield curve model. However, to be consistent with the rest of the analysis and limit focus to models that have been used in the existing literature, we choose to focus on the Svensson (1995) model and the AFGNS



Figure 10: Noise Measures

Illustration of the Swiss noise measures constructed based on the fitted errors implied by the estimated Svensson and AFGNS models using the Swiss Confederation bonds covering the period from January 4, 1993 to March 29, 2019 with a comparison to the U.S. noise measure constructed by Hu et al. (2013).

model already used elsewhere, both estimated with the one-step approach recommended by Andreasen et al. (2019). The two resulting noise measures, defined as the average absolute fitted errors of the yield to maturity across all available bonds at each observation date, are shown in Figure 10 with a comparison to the U.S. noise measure constructed by Hu et al. (2013) and accessed from Jun Pan's personal website with data through the end of 2018.<sup>5</sup> The correlation between the noise measure constructed based on the estimated Svensson model and that based on the estimated AFGNS model is 94.8 percent at daily frequency. Thus, this measure is not very sensitive to the specific yield curve model used, as also emphasized by Hu et al. (2013).

## D.2 Additional Control Variables

Besides the set of core control variables, we consider several additional confounding factors in the regressions. We add the overnight federal funds rate shown in Figure 11 to proxy for the U.S. safe-asset liquidity premium as in Nagel (2016), and reported earnings per share of companies in the S&P 500 to account for opportunity costs in the equity market. We also consider the MOVE volatility index to proxy for risk aversion in the bond market. Finally, we include the total sight deposits at the SNB to control for any possible reserve-induced effects of the SNB's FX interventions; see Christensen and Krogstrup (2019).

<sup>&</sup>lt;sup>5</sup>See the link: http://en.saif.sjtu.edu.cn/junpan/



Figure 11: U.S. Overnight Federal Funds Rate

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