



Monetary Policy Response to Oil Price Shocks

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Monetary Policy Response to Oil Price Shocks

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Abstract

How should monetary authorities react to an oil price shock? The New Keynesian literature has concluded that ensuring perfect price stability is optimal. Yet, the contrast between theory and practice is striking: Inflation targeting central banks typically favor a longer run approach to price stability.

The first contribution of this paper is to show that because oil cost shares vary with oil prices, policies that perfectly stabilize prices entail large welfare costs, which explains the reluctance of policymakers to enforce them. The policy trade-off faced by monetary authorities is meaningful because oil (energy) is an input to both production and consumption.

Welfare-based optimal policies rely on unobservables, which makes them hard to implement and communicate. The second contribution of this paper is thus to analytically derive a simple interest rate rule that mimics the optimal plan in all dimensions but that only depends on observables: core inflation and the growth rates of output and oil prices.

It turns out that optimal policy is hard on core inflation but cushions the economy against the real consequences of an oil price shock by reacting strongly to output growth and negatively to oil price changes. Following a Taylor rule or perfectly stabilizing prices during an oil price shock are very costly alternatives.

Keywords: optimal monetary policy, oil shocks, divine coincidence, simple rules

JEL Class: E32, E52, E58

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1 Introduction

In the last ten years a new macroeconomic paradigm has emerged centered around the New Keynesian (NK henceforth) model, which is at the core of the more involved and detailed dynamic stochastic general equilibrium (DSGE) models used for policy analysis at many central banks. Despite its apparent simplicity, the NK model is built on solid theoretical foundations and has therefore been used to draw normative conclusions on the appropriate response of monetary policy to economic shocks. One important result from this literature is that optimal monetary policy should aim at replicating the real allocation under flexible prices and wages, or *natural* output, which features constant average markups and no inflation.

In the case of an oil price shock, the canonical NK prescription to policymakers is thus fairly simple: Central banks must perfectly stabilize inflation¹, even if that leads to large drops in output and employment. Since the latter are considered efficient, monetary policy should focus on minimizing inflation volatility. There is a *divine coincidence*,² i.e., an absence of trade-off between stabilizing inflation and stabilizing the welfare relevant output gap.

The contrast between theory and practice is striking, however. When confronted with rising commodity prices, policymakers in inflation-targeting central banks do indeed perceive a trade-off. They typically favor a long run approach to price stability by avoiding second-round effects — when wage inflation affects inflation expectations and ultimately leads to upward spiralling inflation — but by letting first-round effects on prices play out. So why the difference? Do policymakers systematically conduct irrational, suboptimal policies? Or should we reconsider some of the assumptions embedded in the NK model?

Clearly, this paper is not the first to examine the consequences of different monetary

¹As noted by Galí (2008 chapter 6), different assumptions on nominal rigidities give rise to different definitions of target inflation. Goodfriend and King (2001) and Aoki (2001) argue that monetary policy should stabilize the stickiest price. By introducing sticky wages alongside sticky prices, Erceg et al. (2000) and Bodenstein, Erceg and Guerrieri (2008) find that optimal monetary policy should *perfectly* stabilize a weighted average of core prices and (negative) wage inflation.

²The expression is from Blanchard and Galí (2007).

policy reactions to oil price shocks. In a series of empirical papers, Bernanke et al. (1997, 2004) and Hamilton and Herrera (2004) simulate counterfactual monetary policy experiments in order to evaluate the marginal impact of monetary policy on output and inflation in the aftermath of a typical oil price shock. Unfortunately, these types of exercises suffer from a Lucas' critique problem so that the discussion remains largely inconclusive. Moreover, the results do not seem to be robust across different monetary policy regimes (see Kilian and Lewis, 2010). To overcome these difficulties, Leduc and Sill (2004) and Carlstrom and Fuerst (2006), for example, conduct the same type of exercise in microfounded, calibrated general equilibrium models. Although the results largely depend on the models specifications, one general insight from this line of work is that monetary policy potentially plays an important role in explaining the transmission of an oil shock to the economy. From a normative point of view, their analysis also suggests that tough medicine - a policy consisting of perfectly stabilizing prices³ - is the best policy. Note, however, that this conclusion rests on comparing the stabilization properties of different simple monetary policy rules.

More recently, a rapidly growing literature has started to look into the design of optimal monetary policy responses to oil price shocks in calibrated or estimated NK models. Not surprisingly, the findings largely depend on the rigidities and production structures assumed. Yet, despite the differences, most studies come to the conclusion that there is indeed a trade-off between stabilizing inflation and the welfare relevant output gap.⁴ One potential reason - as argued by Blanchard and Galí (2007) (hence-

³Dhawan and Jeske (2007) introduce energy use at the household level and obtain that stabilizing core inflation instead of headline inflation is preferable.

⁴Drawing on Erceg et al.'s (2000) fundamental insight, a series of papers attribute the policy trade-off to the simultaneous presence of price and wage stickiness. Bodenstein et al. (2008) and Plante (2009) find that optimal monetary policy should stabilize a weighted average of core and nominal wage inflation. Winkler (2009) considers anticipated and unanticipated (deterministic) oil shocks and also finds that optimal policy cannot stabilize, at the same time prices, wages and the welfare relevant output gap. Nevertheless, following an oil price shock, optimal policy requires a larger output drop than under a traditional Taylor rule. This result can be contrasted with Kormilitsina (2009), who finds that optimal policy dampens output fluctuations compared to a Taylor rule. Her model is richer than Winkler's one and estimated on US data. Montoro (2007) derives the Ramsey optimal policy in a closed economy setting where oil is a non-produced input in the production function. Despite flexible wages, he still finds a monetary policy trade-off that he attributes to the fact that oil shocks affect output and labour differently, generating a wedge between the effects on the utility of consumption and the disutility of labour when oil and labor are gross complement. Drawing on the work of Barsky

forth BG07) - relates to the presence of real wage rigidity in an otherwise canonical NK model which introduces a time-varying wedge between natural and *efficient*⁵ output. In this case, stabilizing prices (targeting natural output) introduces inefficient output variations and the divine coincidence does not hold anymore.

Here I focus on an alternative explanation that does not hinge on real wage rigidities but on the characterization of technology and its interaction with the assumption of monopolistic competition. The first contribution of this paper is thus to show that increases in oil prices lead to a *quantitatively meaningful* monetary policy trade-off once it is acknowledged (*i*) that oil cannot easily be substituted by other factors in the short run, (*ii*) that there is no fiscal transfer available to policymakers to offset the steady-state distortion due to monopolistic competition, and (*iii*) that oil is an input both to production and to consumption (via the impact of the price of crude oil on the prices of gasoline, heating oil and electricity).

In a nutshell, oil price hikes temporarily lead to higher oil cost shares. The larger the market distortion due to monopolistic competition, the larger is the effect of a given increase in oil price on firms' real marginal cost and the more important is the drop in output and real wages required to stabilize prices. This explains why perfectly stabilizing prices in a non-competitive economy introduces inefficient output variations and an endogenous monetary policy trade-off. By assuming Cobb-Douglas production (and thus constant cost shares) or an efficient economy in the steady-state, the canonical NK model dismisses out of hand the mere possibility of a trade-off.⁶

and Kilian (2004) and Kilian (2008), Nakov and Pescatori (2009) expand the canonical NK model to include an optimization-based model of the oil industry featuring both monopolistic and competitive oil producers. They show that the deviation of the best targeting rule from strict inflation targeting is substantial due to inefficient endogenous price markup variations in the oil industry. Finally, opening the NK model, De Fiore et al. (2006) build a large three-country DSGE model - featuring two oil-importing countries and one oil exporting country - that they estimate on US and EU data. In contrast with the other papers mentioned above, the authors consider a whole array of shocks and search for the simple welfare maximizing rule. Their main finding in this context is that the optimal rule reacts strongly to inflation but accomodates output gap fluctuations, suggesting again a policy trade-off.

⁵The undistorted level of output that would prevail in the absence of nominal frictions in a perfectly competitive economy.

⁶Note that a central bank is usely thought to face a policy trade-off between stabilizing the output gap and stabilizing inflation. In microfounded models, this trade-off is typically modelled as arising from an exogenous markup shock (see for example Galì, 2008). In contrast, this paper provides an endogenous derivation of a cost-push shock.

While conditions (i) and (ii) are necessary to introduce a microfounded monetary policy trade-off, they are not sufficient to explain the policymakers' concern for the real consequences of oil price shocks. Hence, this paper stresses that perfectly stabilizing inflation becomes particularly costly when the impact of higher oil prices on households' consumption is also taken into account. Changes in oil prices act as a distortionary tax on labor income and amplify the monetary policy trade-off. The lower the elasticity of substitution between energy and other consumption goods, the larger is the tax effect and the more detrimental are the consequences on employment and output of a given increase in oil prices. Importantly, these findings do not hinge on particular functional forms for production or consumption. All that is needed is that oil cost shares be allowed to vary with the price of oil.

This paper also shows that central banks can improve on both the perfect price stability solution and the recommendation of a simple Taylor rule. And the welfare gains are large. One problem with welfare-based optimal policies, however, is that they rely on unobservables such as the efficient level of output or various shadow prices. This makes them difficult to communicate and to implement. The second contribution of this paper is thus to analytically derive a simple interest rate rule that mimics the optimal plan along all relevant dimensions but that relies only on observables — namely core inflation and the growth rates of output and oil prices.⁷ It turns out that optimal policy is hard on core inflation but cushions the economy against the real consequences of an oil price shock by reacting strongly to output growth and negatively to oil price changes. In other words, the optimal response to a persistent increase in oil price resembles the typical response of inflation targeting central banks: While long-term price stability is ensured by a credible commitment to stabilize inflation and inflation expectations, short-term real interest rates drop right after the shock to help dampen real output fluctuations.

The rest of the paper is structured as follows. Section 2 starts by building a two-sector NK model where oil enters as a gross complement to both production and con-

⁷See Orphanides and Williams (2003) for a thorough discussion of implementable monetary policy rules.

sumption, thus featuring both core and headline inflation like in Bodenstein et al. (2008). Section 3 shows that the oil price shock leads to a monetary policy trade-off that is increasing in the degree of monopolistic competition and is inversely related to production and consumption elasticities of substitution. In Section 4, a linear-quadratic solution to the optimal policy problem in a timeless perspective is derived to show that the optimal weight on inflation in the policymaker's loss function decreases with the oil elasticity of substitution. Section 5 derives a simple, implementable interest rate rule that replicates the optimal plan. Section 6 revisits the 1979 oil shock and computes the welfare losses associated with standard alternative policy rules in order to give a sense of the costs incurred when following suboptimal monetary policies. Finally, since oil price elasticity is very low in the short run but close to one in the long run⁸, Section 7 highlights that this paper's findings are robust to a production framework that features time-varying elasticities of substitution in the spirit of putty-clay⁹ models of energy use.

2 The model

Following Bodenstein et al. (2008) (thereafter BEG08), I assume a two-layer NK closed-economy setting¹⁰ composed of a core consumption good, which takes labor and oil as inputs, and a consumption basket consisting of the core consumption good and oil. In order to keep the notations as simple as possible, there is only one source of nominal rigidity in this economy: core goods prices¹¹ are sticky and firms set prices according to a Calvo scheme.

In contrast to BEG08, however, I relax the assumption of a unitary elasticity of substitution between oil and other goods and factors. I also explicitly consider a dis-

⁸See Pindyck and Rotemberg (1983) for an empirical analysis using cross section data.

⁹See Gilchrist and Williams (2005).

¹⁰This assumption allows one to ignore income distribution and international risk-sharing related issues.

¹¹Introducing nominal wage stickiness like in BEG08 would not change the thrust of the argument. As shown by Woodford (2003) and Galí (2008), one can usually define a composite index of wage and price inflation such that, for most reasonable calibrations, there is virtually no trade-off between stabilizing the composite index and the welfare-relevant output gap.

torted economy: There is no fiscal transfer to neutralize the monopolistic competition distortion. The model being quite standard (see BEG08) I only present the main building blocks here. A full description is relegated to Appendix I available on the journal's website.

2.1 Households

Households maximize utility out of consumption and leisure. Their consumption basket is defined as a CES aggregator C_t of the core consumption goods basket $C_{Y,t}$ and the household's demand for oil $O_{C,t}$ ¹²

$$C_t = \left((1 - \omega_{oc}) C_{Y,t}^{\frac{\chi-1}{\chi}} + \omega_{oc} O_{C,t}^{\frac{\chi-1}{\chi}} \right)^{\frac{\chi}{\chi-1}}, \quad (1)$$

where ω_{oc} is the oil quasi-share parameter and χ is the elasticity of substitution between oil and non-oil consumption goods.

Allowing for real wage rigidity (which may reflect some unmodeled imperfection in the labor market as in BG07), the labor supply condition relates the marginal rate of substitution between consumption and leisure to the geometric mean of real wages in periods t and $t - 1$.

$$\left(C_t^\sigma \nu H_t^\phi \right)^{(1-\eta)} = \frac{W_t}{P_t} \left(\frac{W_{t-1}}{P_{t-1}} \right)^{-\eta}. \quad (2)$$

In the benchmark calibration, i.e., unless stated otherwise, real wages are perfectly flexible, i.e., $\eta = 0$.

2.2 Firms

Aggregating over all firms producing core consumption goods, we get the total demand for intermediate goods $Y_t(i)$ as a function of the demand for core consumption goods $C_{Y,t}$

$$Y_t(i) = \left(\frac{P_{Y,t}(i)}{P_{Y,t}} \right)^{-\varepsilon} C_{Y,t}, \quad (3)$$

¹²The consumption basket can be regarded as produced by perfectly competitive consumption distributors whose production function mirrors the preferences of households over consumption of oil and non-oil goods.

Each intermediate goods firm produces a good $Y_t(i)$ according to a constant returns-to-scale technology represented by the CES production function

$$Y_t(i) = \left((1 - \omega_{oy}) H_t(i)^{\frac{\delta-1}{\delta}} + \omega_{oy} O_{Y,t}(i)^{\frac{\delta-1}{\delta}} \right)^{\frac{\delta}{\delta-1}}, \quad (4)$$

where $O_{Y,t}(i)$ and $H_t(i)$ are the quantities of oil and labor required to produce $Y_t(i)$ given the quasi-share parameters, ω_{oy} , and the elasticity of substitution between labor and oil, δ .

The real marginal cost in terms of core consumption goods units is given by:

$$MC_t(i) \equiv MC_t = \left((1 - \omega_{oy})^\delta \left(\frac{W_t}{P_{Y,t}} \right)^{1-\delta} + \omega_{oy}^\delta \left(\frac{P_{O,t}}{P_{Y,t}} \right)^{1-\delta} \right)^{\frac{1}{1-\delta}}. \quad (5)$$

2.3 Government

To close the model, I assume that oil is extracted with no cost by the government, which sells it to the households and the firms and transfers the proceeds in a lump sum fashion to the households. I abstract from any other role for the government and assume that it runs a balanced budget in each and every period so that its budget constraint is simply given by

$$T_t = P_{O,t} O_t,$$

for O_t the total amount of oil supplied.

2.4 Calibration

For the sake of comparability, the model calibration closely follows BEG08. The quarterly discount factor β is set at 0.993, which is consistent with an annualized real interest rate of 3 percent. The consumption utility function is chosen to be logarithmic ($\sigma = 1$) and the Frish elasticity of labor supply is set to unity ($\phi = 1$).

In the baseline calibration, I set the consumption, χ , and production, δ , oil elasticities of substitution to 0.3.¹³ Following BEG08, ω_{oc} is set such that the energy

¹³Our calibration is on the high side of estimates of short-term oil price elasticity of demand reported by Hamilton (2009) (ranging from 0.05 to 0.34) but corresponds quite closely to the median estimate reported by Kilian and Murphy (2010).

component of consumption (gasoline and fuel plus gas and electricity) equals 6 percent, which is in line with US NIPA data, and ω_{oy} is chosen such that the energy share in production is 2 percent. Prices are assumed to have a duration of four quarters, so that $\theta = 0.75$. The core goods elasticity of substitution parameters ε is set to 6, which implies a 20 percent markup of (core) prices over marginal costs. Finally, the logarithm of the real price of oil in terms of the consumption goods bundle $po_t = \log(P_{o,t})$ is supposed to follow an AR(1) process ($\rho_o = 0.95$).

3 Divine coincidence?

Because of monopolistic competition in intermediate goods markets, the economy's steady state is distorted. Production and employment are suboptimally low. Fully acknowledging this feature of the economy instead of subsidizing it away for convenience (as is usually done), entails important consequences for optimal policy when oil is difficult to substitute in the short run.

This section shows that the divine coincidence breaks down when Cobb-Douglas production, a hallmark of the canonical NK model, is replaced by CES — or any production function that implies that oil cost shares vary with changes in oil prices.¹⁴ Cobb-Douglas production functions greatly simplify the analysis and permit nice closed-form solutions, but because they assume a unitary elasticity of substitution between factors they feature constant cost shares over the cycle *regardless of the size of the monopolistic competition distortion*. Following an oil price shock, natural (distorted) output drops just as much as efficient output and perfectly stabilizing prices is then the optimal policy to follow.

Yet, the case for a unitary elasticity of substitution between oil and other factors is not particularly compelling, especially at business cycle frequency. If oil is instead considered a gross complement for other factors (at least in the short run), the response of output to an oil price shock will *depend on the size of the monopolistic competition distortion*. The larger the distortion, the larger is the dynamic impact of a given oil

¹⁴See Section 7 for an illustration with a production function featuring time-varying price elasticity of oil demand: very low elasticity in the short run and unitary elasticity in the long run.

price shock on the oil cost share — and therefore on output — in the flexible prices and wages equilibrium. Natural (distorted) output will drop more than efficient output. Strictly stabilizing inflation in the face of an oil price shock is thus no longer the optimal policy to follow; the divine coincidence breaks down.¹⁵

Perfectly stabilizing prices becomes particularly costly when the impact of higher oil prices on households' consumption is also taken into account. As stated in the introduction, increases in oil prices act as a tax on labor income; the lower the elasticity of substitution, the larger is the tax effect which amplifies the trade-off faced by monetary authorities.

Although an accurate welfare analysis requires a second order approximation of the household's utility function and the model's supply side (see Section 4 and Appendix III), some intuition on the mechanisms at stake can be gained by analyzing the properties of the log-linearized (see Appendix II for details) model economy at the FPWE.

3.1 Flexible price and wage equilibrium (FPWE)

Solving the system for $mc_t = 0$ and assuming $\sigma = 1$ and $\eta = 0$ for simplicity, we get¹⁶ :

$$h_t = - \left[\frac{\widetilde{\omega}_{oy} (1 - \delta)}{\Lambda} + \Theta \right] po_t, \quad (6)$$

$$y_t = - \left[\frac{\widetilde{\omega}_{oy} (1 + \delta\phi)}{\Lambda} + \Theta \right] po_t, \quad (7)$$

and

$$w_t = - \frac{\widetilde{\omega}_{oy} (1 - \widetilde{\omega}_{oc}) + \widetilde{\omega}_{oc}}{(1 - \widetilde{\omega}_{oc}) (1 - \widetilde{\omega}_{oy})} po_t \quad (8)$$

where $\Theta = \frac{\widetilde{\omega}_{oc} [1 - \chi s y^{-1} (1 - \widetilde{\omega}_{oc})]}{(1 - \widetilde{\omega}_{oc}) (\phi + 1)}$, $\Lambda = (1 - \widetilde{\omega}_{oy}) (1 - \widetilde{\omega}_{oc}) (\phi + 1)$ and $0 < MC (\equiv \frac{\varepsilon - 1}{\varepsilon}) \leq 1$ reflecting the degree of monopolistic competition in the economy. Note also that $\widetilde{\omega}_{oy} \equiv \omega_{oy}^\delta \left(\frac{P_o}{MC \cdot P_y} \right)^{1 - \delta}$ is the share of oil in the real marginal cost, $\widetilde{\omega}_{oc} \equiv \omega_{oc}^\chi P_o^{1 - \chi}$ is

¹⁵In line with the general theory of the second best (see Lipsey and Lancaster, 1956), monetary authorities can aim at a higher level of welfare by trading some of the costs of inefficient output fluctuations against the distortion resulting from more inflation.

¹⁶Note that lowercase letters denote the percent deviation of each variable with respect to its steady state (e.g., $c_t \equiv \log \left(\frac{C_t}{\bar{C}} \right)$).

the share of oil in the CPI, $sy \equiv (1 - \omega_{oc}) \left(\frac{Y}{C}\right)^{\frac{\chi-1}{\chi}}$ is the share of the core good in the consumption goods basket, and marginal cost is :

$$0 = mc_t = (1 - \widetilde{\omega}_{oy}) (w_t - py_t) + \widetilde{\omega}_{oy} (po_t - py_t)$$

If oil is considered a gross complement to labor in production ($\delta < 1$), the oil price elasticity of real marginal costs, $\widetilde{\omega}_{oy}$, is increasing in the degree of monopolistic competition distortion as measured by $\frac{1}{MC}$. The less competitive the economy, the larger is $\widetilde{\omega}_{oy}$ and the more sensitive are real marginal costs to increases in oil prices. As perfect price stability means constant real marginal costs, the more distorted the economy's steady-state, the bigger is the real wage drop required to compensate for higher oil prices. In equilibrium, labor and output must then fall correspondingly.

Increases in oil prices also act as a tax on labor income when $\chi < 1$; the lower the elasticity of substitution, the larger the tax effect which compounds with the effect of oil price increases on marginal costs.¹⁷ More specifically, as changes in oil prices affect headline more than core prices, increases in oil prices have a differentiated effect on real oil prices faced by consumers, po_t , and (higher) real oil prices faced by firms $po_t - py_t$.¹⁸ This discrepancy is exacerbated by the fact that changes in oil prices drive a wedge between the real wage relevant for households (the consumption real wage, w_t) and the real wage faced by firms (the production real wage, $w_t - py_t$). The lower the elasticity of substitution between energy and other consumption goods, χ , the larger is the effect of a given increase in oil prices on py_t and the larger is the required drop in real wages w_t (and in labor and output) to stabilize real marginal costs.

Indeed, equations (6), (7) and (8) show that the response of employment, output and the real wage are increasing in $\widetilde{\omega}_{oy}$ and $\widetilde{\omega}_{oc}$, which are themselves decreasing in δ and χ .¹⁹

¹⁷Note that the tax effect tends to zero when the elasticity of substitution $\chi \rightarrow \infty$ as in this case, $\widetilde{\omega}_{oc} \rightarrow 0$, and the solution of the model collapses to the one where oil is an input to production only.

¹⁸Because immediately after an increase in oil prices, the ratio of core to headline prices deteriorates ($py_t < 0$).

¹⁹Note that equation (6) shows that when $\delta = \chi = 1$, which occurs when the production functions for intermediate and final goods are Cobb-Douglas, substitution and income effects compensate one another on the labor market and employment remains constant after an oil price shock ($\Theta = h_t = 0$).

3.2 Endogenous cost-push shock

The cyclical wedge between the natural and efficient levels of output after an oil price shock — the endogenous cost-push shock — can be analyzed by comparing the log-linearized flex-price output responses in the distorted (y_t^N , natural) and undistorted (y_t^* , efficient) economies.²⁰

Starting from equation (7), the cyclical distortion can be written as :

$$y_t^N - y_t^* = -(1 + \delta\phi) \left(\widetilde{\omega}_{oy}^N / \Lambda^N - \widetilde{\omega}_{oy}^* / \Lambda^* \right) p o_t, \quad (9)$$

where I assume $MC = 1$ in $\widetilde{\omega}_{oy}^*$ and Λ^* , and $MC < 1$ in $\widetilde{\omega}_{oy}^N$ and Λ^N .

This cyclical distortion can be mapped into a cost-push shock that enters the New Keynesian Phillips curve (NKPC henceforth) :

$$\pi_{y,t} = \beta \mathbb{E}_t \pi_{y,t+1} + k_y x_t + \mu_t, \quad (10)$$

where $x_t = y_t - y_t^*$ is the percent deviation of output with respect to the welfare relevant output y_t^* and $\mu_t = k_y (y_t^* - y_t^N)$ is the cost-push shock. Note that $y_t^N = -\frac{k\mathcal{B}}{k_y} p o_t$ for \mathcal{B} a decreasing function of δ and χ , $k = \left(\frac{1-\theta}{\theta}\right) (1 - \theta\beta)$ and $k_y = k(1 - \eta)(\sigma + \phi) \left[\frac{(1 - \widetilde{\omega}_{oy})}{1 + \widetilde{\omega}_{oy}(1 - \eta)\phi\delta} \right]$ (see Appendix III and IV for detailed derivations).

First, note that the wedge is constant ($y_t^N - y_t^* = 0$) and the divine coincidence holds when a fiscal transfer is available to offset the steady state monopolistic distortion. In this case, $\widetilde{\omega}_{oy}^N = \widetilde{\omega}_{oy}^*$ and $\Lambda^N = \Lambda^*$. There is no cost-push shock and no policy trade-off. Second, when production functions are Cobb-Douglas ($\delta = \chi = 1$), $\widetilde{\omega}_{oy}^* = \widetilde{\omega}_{oy}^N = \omega_{oy}$ and $\widetilde{\omega}_{oc} = \omega_{oc}$ so that $\widetilde{\omega}_{oy}^N / \Lambda^N - \widetilde{\omega}_{oy}^* / \Lambda^* = 0$; there is again no-trade-off and $y_t^N - y_t^* = 0$. Third, allowing for gross complementarity ($\delta, \chi < 1$) in a world without fiscal transfer, y_t^N will drop more than y_t^* after an oil shock as $\widetilde{\omega}_{oy}^N > \widetilde{\omega}_{oy}^*$ when $MC^N < MC^* \equiv 1$.

Clearly, the lower δ and χ and the larger the steady-state distortion (i.e., the lower MC^N), the larger is the cyclical wedge between y_t^N and y_t^* ; the larger is the cost-push

²⁰The social planner's efficient allocation is the same as the one in the decentralized economy when prices and wages are flexible and there is no steady-state monopolistic distortion ($MC \equiv \frac{\varepsilon-1}{\varepsilon} = 1$). The natural allocation, on the other hand, corresponds to the flex-price and wage equilibrium in a distorted economy ($MC < 1$).

shock. Moreover, note that the elasticity of substitution between energy and other consumption goods, χ , plays an important role in amplifying the effect of oil prices on the gap between y_t^N and y_t^* : The lower χ , the larger is $\widetilde{\omega}_{oc}$, the lower is Λ and the larger is the difference $\widetilde{\omega}_{oy}^N/\Lambda^N - \widetilde{\omega}_{oy}^*/\Lambda^*$.

Figure 1 shows the instantaneous response of the gap between natural (YN) and efficient (Y*) output to a (one period) 1-percent increase in the real price of oil as a function of δ , the production elasticity of substitution, and for different values of the consumption elasticity of substitution, χ .²¹ The gap is exponentially decreasing in both the elasticities δ and χ . Looking at the northeastern extreme of the figure, where both elasticities are equal to one (the Cobb-Douglas case), we see that the reaction of natural and efficient outputs are the same²² (the gap is zero), so that stabilizing inflation or output at its natural level is welfare maximizing. Lowering the production elasticity only (along the curve CHI=1) gives rise to a monetary policy trade-off. Yet, the wedge becomes really large when both the consumption and production elasticities are small (like on the curve labeled CHI=0.3).

< Figure 1 >

Figure 2 performs a similar exercise, but varies the degree of net steady-state markups ($\frac{1}{MC} - 1$) for different values of the elasticities δ and χ . Again, the wedge between efficient and natural output swells for large distortions and low elasticities.

< Figure 2 >

4 Optimal monetary policy

What weight should the central bank attribute to inflation over output gap stabilization? Rotemberg and Woodford (1997) and Benigno and Woodford (2005) have shown that the central bank's loss function - defined as the weighted sum of inflation and the

²¹Note that the amplitude of the gap also depends on the Frish-elasticity of labor supply as measured by $\frac{1}{\phi}$. The smaller ϕ (the larger the elasticity), the larger are the labor demand and output drops needed to stabilize the real marginal cost, and the larger is the cyclical gap between efficient and natural output.

²²Solving equations (A3.13) and (A4.3) in Appendices III and IV for $\sigma = \phi = \delta = \chi = 1$, we get that $y_t^N = y_t^* = -\frac{\alpha}{(1-\alpha)(1-\omega_{oc})}p^o t$.

welfare relevant output gap - could be derived from (a second order approximation of) the households utility function, thereby setting a natural criterion to answer this question (see Appendix III for details). Section 4.1 shows that the parameters governing the nominal and real rigidities in the model interact with the elasticities of substitution (that are assumed smaller than one) and have important consequences on the choice of policy. For reasonable parameter values, however, the weight on inflation stabilization remains larger than the one on the output gap, a result also obtained by Woodford (2003) in a more constrained environment.

Given optimal weights in the central bank loss function, what is the optimal policy response to an oil price shock? Section 4.2 contrasts the dynamic transmission of oil price shocks under strict inflation targeting and under the optimal precommitment policy in a timeless perspective.

4.1 Lambda

Given that the central bank's objective is to minimize the loss function

$$\Upsilon \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \{ \lambda x_t^2 + \pi_{y,t}^2 \}$$

under the economy constraint, the welfare implications of alternative policies crucially depend on the value of λ .²³ Figure 3 describes the variation of λ , the relative weight assigned to output gap stabilization as a function of the elasticity of substitution δ and the degree of price stickiness θ . Stickier prices (larger θ) result in larger price dispersion and therefore larger inflation costs. In this case, monetary authorities will be less inclined to stabilize output and, for given elasticities of substitution, λ decreases when θ becomes larger.

But λ also depends crucially on the elasticities of substitution. The lower the elasticities, the flatter is the New Keynesian Phillips curve (NKPC), the larger is the sacrifice ratio²⁴, and the more concerned will be the central bank with the distortionary cost of inflation (i.e., the smaller will be λ). Assuming perfectly flexible real wages

²³Appendix III describes how λ and the loss function are derived from first principles.

²⁴When the NKPC is flat, a large change in output is required to affect inflation.

($\eta = 0$), our baseline calibration ($\theta = 0.75$, $\delta = 0.3$) leads to $\lambda = 0.028$, which implies a targeting rule that places a larger weight on inflation stabilization than on the output gap (in annual inflation terms, the ratio output gap to inflation stabilization is $\sqrt{0.022} \times 4 = 0.66$).²⁵ Note that the focus of policy is very sensitive to the degree of price stickiness. Setting $\theta = 0.5$ results in $\lambda = 0.138$ and a policy that sets a larger weight on output gap stabilization ($\sqrt{0.138} \times 4 = 1.48$).

< Figure 3 >

BG07 argue that the optimal policy choice depends crucially on the degree of real wage stickiness. Figure 4 verifies this claim by letting the degree of real wage stickiness vary between $\eta = 0$ and $\eta = 0.9$. The larger the real wage stickiness, the larger is the cost-push shock but the larger is the sacrifice ratio as a relatively larger drop in labor demand and output is necessary to engineer the required drop in real wages that stabilizes the real marginal cost and inflation. The central bank will tend to be more concerned with inflation stabilization and λ will be smaller when η is high. Assuming $\eta = 0.9$ and our baseline calibration, Figure 4 shows that $\lambda = 0.002$ ($\sqrt{0.002} \times 4 = 0.18$).

< Figure 4 >

4.2 Analyzing the trade-off

How different is the transmission of an oil price shock under the optimal precommitment policy in a timeless perspective characterized by the targeting rule (see Woodford, 2003 and Appendix IV) :

$$x_t = x_{t-1} - \frac{k_y}{\lambda} \pi_{y,t}, \quad (11)$$

which holds for all $t = 0, 1, 2, 3, \dots$, from strict inflation targeting which replicates the FPWE solution? Figure 5 shows that, assuming $\delta = \chi = 0.3$, the latter implies an increase in real interest rates (which corresponds to the expected growth of future consumption), while optimal policy recommends a temporary drop for one year following

²⁵The traditional Taylor rule that places equal weights on output stabilization and inflation stabilization would imply $\lambda = 1/16$ with quarterly inflation.

the shock. The output drop on impact is more than three times larger in the FPWE allocation, which is the price for stabilizing core inflation perfectly.

< Figure 5 >

Figure 6 shows how acute the policy trade-off is by displaying the differences in both the welfare relevant output gaps and inflation reactions to a 1-percent increase in the price of oil under optimal policy and strict inflation targeting. The "oil-in-production-only" case (dotted line) is compared with the case where energy is an input to both consumption and production (solid line). In both cases, optimal policy lets inflation increase and the welfare relevant output gap decrease. But the difference with strict inflation targeting is three times as large when oil is *both* an input to production and consumption, as could be inferred from Section 3.

< Figure 6 >

5 Simple rules

Welfare-based optimal policy plans may not be easy to communicate as they typically rely on the real-time calculation of the welfare relevant output gap, an abstract, non-observable theoretical construct. Accountability issues can be raised, which may cast doubt on the overall credibility of the precommitment assumption that underlies the whole analysis. As an alternative, some authors (see McCallum, 1999, Söderlind, 1999 or Dennis, 2004) have advocated the use of simple optimal interest rate rules. Those rules should approximate the allocation under the optimal plan but should not rely on an overstretched information set.

In what follows, a simple rule that is equivalent to the optimal plan is derived from first principles. I show that it must be based on core inflation and on current and lagged deviations of output and the real price of oil from the steady state. As the mere notion of steady-state can also be subject to uncertainty in real-time policy exercises, I also show that the optimal simple rule can be approximated by a 'speed limit' interest rate rule (see Walsh, 2003 and Orphanides and Williams, 2003) that relies only on the rate of change of the variables, i.e., on current core inflation, oil price inflation, and

the growth rate of output; this rule remains close to optimal even when real wages are sticky.

5.1 The optimal precommitment simple rule

Using the minimal state variable (MSV) approach pioneered by McCallum²⁶, one can conjecture the no-bubble solution to the dynamic system formed by i) the optimal targeting rule under the timeless perspective optimal plan and ii) the NKPC equation to get:

$$\pi_{y,t} = \alpha_{11}x_{t-1} + \alpha_{12}\mu_t, \quad (12)$$

$$x_t = \alpha_{21}x_{t-1} + \alpha_{22}\mu_t, \quad (13)$$

where α_{ij} for $i, j = 1, 2$ are functions of β , k_y , and λ .

Combining (12) and (13) with the Euler equation for consumption, one can solve for r_t , the nominal interest rate, and derive the optimal simple rule consistent with the optimal plan (see Appendix V):

$$r_t = \Phi\alpha_{11}^{-1}\pi_{y,t} + \Omega y_t - \Gamma y_{t-1} + (\Xi + \Psi\Omega)po_t - \Psi\Gamma po_{t-1}, \quad (14)$$

for $\Phi \equiv \rho_o - \sigma\alpha_{22}\alpha_{12}^{-1}(1 - \rho_o)$, $\Omega \equiv \alpha_{11} + \sigma\alpha_{21}$, $\Gamma \equiv \Phi + \sigma\alpha_{21}$ and $\Xi \equiv (\rho_o - 1)\left(\frac{\widetilde{\omega}_{oc}}{1 - \widetilde{\omega}_{oc}} - \Psi\sigma\right)$.

The optimal interest rate rule is a function of core inflation and current and lagged output and real oil price, all taken as log deviations from their respective steady states. Its parameters are functions of households preferences, technology, and nominal frictions.

For a permanent shock, $\rho_o = 1$, the rule simplifies to

$$r_t = \alpha_{11}^{-1}\pi_{y,t} + \Omega(y_t - \Gamma\Omega^{-1}y_{t-1}) + \Psi\Omega(po_t - \Gamma\Omega^{-1}po_{t-1}),$$

as $\Phi = 1$, and $\Xi = 0$. Looking at Γ and Ω shows that the closer α_{11} is to 1, the more precisely a speed limit policy (a rule based on the rate of growth of the variables) replicates optimal policy as in this case $\Gamma = \Omega$.

²⁶See McCallum (1999b) for a recent exposition of the MSV approach.

In Section 6, I show that for $\rho_o = 0.95$, a degree of persistence which corresponds closely to the 1979 oil shock, the speed limit policy approximates almost perfectly the optimal feedback rule despite a value of α_{11} clearly below 1.

5.2 Optimized simple rules

Analytical solutions to the kind of problem described in Section 5.1 rapidly become intractable when the number of shocks and lagged state variables is increased (e.g., by allowing for the possibility of real wage rigidity).

An alternative is to rely on numerical methods in order to estimate a simple rule mimicking the optimal plan's allocation along all relevant dimensions. The following distance minimization algorithm is defined over the n impulse response functions of m variables of interest to the policymakers and searches the parameter space of a simple interest rate rule that minimizes the distance criterion :

$$\arg \min_{\vartheta} (IRF_{SR}(\vartheta) - IRF_O)' (IRF_{SR}(\vartheta) - IRF_O),$$

where $IRF_{SR}(\vartheta)$ is an $mn \times 1$ vector of impulses under the postulated simple interest rate rule, and IRF_O is its counterpart under the optimal plan.²⁷ The algorithm matches the responses of eight variables (output, consumption, hours, headline inflation, core inflation, real marginal costs, and nominal and real interest rates) over a 20-quarter period using constrained versions of the following general specification of the simple interest rate rule derived from equation (14) :

$$r_t = g_\pi \pi_{y,t} + g_y y_t + g_{y1} y_{t-1} + g_{po} p_{o,t} + g_{po1} p_{o,t-1} + g_{w1} w_{t-1}, \quad (15)$$

where $\vartheta = (g_\pi, g_y, g_{y1}, g_{po}, g_{po1}, g_{w1})'$.

I start with a version of the model that assumes perfect real wage flexibility and run the minimum distance algorithm on an unconstrained version of equation (15) and on a speed limit version where $g_y + g_{y1} = 0$, $g_{po} + g_{po1} = 0$, $g_{w1} = 0$ and $g_y, g_{po} \geq 0$. Figure 7

²⁷Another possibility is to search within a predetermined space of simple interest rate rules for the one that minimizes the central bank loss function (see e.g., Söderlind, 1999 and Dennis, 2004). However as different combinations of output gaps and inflation variability could in principle produce the same welfare loss, I rely on IRFs instead.

shows the response to a 1 percent shock to oil prices under the optimal precommitment policy (solid line), the optimized simple rule (OR, dotted line) and the speed limit rule (SLR, dashed line). The responses under the OR stand exactly on top of the ones under the optimal policy, which is not surprising given that a closed-form solution to the problem can be derived (see previous sub-section). More remarkable, however, is how well the SLR (dashed line) is able to match the optimal precommitment policy (solid line). For most variables they are almost indistinguishable.

< Figure 7 >

The coefficients of the different rules are reported in Table 1. They are quite large compared to the coefficients typically found for Taylor-type interest rate rules. Both the OR and the SLR react strongly to demand shocks that push inflation and the output gap in the same direction, but adopt a more balanced response to oil cost-push shocks by accommodating decreases in output and by dropping real rates when oil price increases.

How robust are these findings to the introduction of real wage stickiness? Assuming $\eta = 0.9$ Figure 8 shows that, again, the OR (dotted line) is almost on top of the optimal plan benchmark (solid line).

< Figure 8 >

Table 1 (OR_W) shows that the monetary authorities react strongly to both inflation and the output gap (defined as deviation from steady state), but also to changes in oil prices. This means that the rules tends toward perfect prices stability in the case of a demand shock, but acknowledges the trade-off in the case of an oil price shock.

< Table 1 >

Although our analysis assumes that oil prices are exogenous supply-side events, the OR suggests that policy will be swiftly tightened in the case of a demand-driven increase in oil prices as growth would accelerate and core inflation increase beyond its steady-state level.²⁸

²⁸See Kilian, 2009 for a discussion of the importance of disentangling demand and supply shocks in the oil market.

6 Oil price shocks and US monetary policy

All US recessions since the end of World War II — and the latest vintage is no exception — have been preceded by a sharp increase in oil prices and an increase in interest rates.²⁹ But are US recessions really caused by oil shocks, or should the monetary policy responses to the shocks be blamed for this outcome? Empirical evidence seems to suggest a role for monetary policy (Bernanke et al. 2004), but its importance remains difficult to assess. One major stumbling block is the role of expectations. To evaluate the effect of different monetary policies in the event of an oil price shock one has to take into account the effect of those policies on the agents' expectations, which is typically not feasible using reduced-form time series models whose estimated parameters are not invariant to policy (see Lucas, 1976, and Bernanke et al., 2004, for a discussion in the context of an oil shock).

The alternative approach is to rely on a structural, microfounded model to simulate counterfactual policy experiments. I start by describing the dynamic behavior of the economy under different monetary policies during the 1979 oil price shock. I have chosen to focus on this episode for the oil shock was clearly exogenous to economic activity (Iranian revolution) and as such corresponds to the model definition of an oil price shock. I then compute the welfare loss associated with suboptimal policies. Finally, I compare the optimal rule to standard alternatives in the 2006-2008 oil price rallye.

6.1 1979 oil price shock

Figure 9 shows that the pattern of real oil prices between 1979 and 1986 can be well replicated by an AR(1) process $po_t = \rho_o po_{t-1} + \varepsilon_{o,t}$ for $\rho_o = 0.95$. I will thus rely on this shock process to perform all simulations and compute welfare losses.

< Figure 9 >

Figure 10 compares the IRFs under optimal policy (OR henceforth, solid line), the traditional Taylor rule (HTR, dashed line) and a Taylor rule based on core inflation

²⁹See Hamilton (2009) for a recent analysis.

(CTR henceforth, dotted line).

Under optimal policy, the central bank credibly commits to a state-contingent path for future interest rates that involves holding real interest rates positive in the next five years despite negative headline inflation and close to zero core inflation. In doing so it is able to dampen inflation expectations without having to resort to large movements in real interest rates and it attains superior stabilization outcome in the short to medium run. At the peak, output falls twice as much and core inflation is five times larger under HTR than under OR. Because inflation never really takes off under OR, nominal interest rates remain practically constant over the whole period. This suggests that if monetary policy had been conducted according to OR during the oil shock of 1979, the recession would not have been averted but it would have been much milder with almost no increase of core inflation beyond steady-state inflation. CTR leads to less output gap fluctuations than OR but at the cost of much higher core inflation.

< Figure 10 >

Some authors (Bernanke et al., 1999) have argued that monetary policy should be framed with respect to a *forecast* of inflation rather than *realized* inflation. And, indeed, many inflation-targeting central banks communicate their policy by referring to an explicit goal for their forecast of inflation to revert to some target within a specified period. Like BEG08, I define a forecast-based rule as a Taylor-type rule where realized inflation has been replaced by a one-quarter-ahead forecast of core or headline inflation; the parameters remain the same with $g_\pi = 1.5$ and $g_y = 0.5$. Figure 11 shows that forecast-based rules fulfill their goal of stabilizing both headline and core inflation in the long run but appear too accommodative³⁰ in the short run.

< Figure 11 >

But how costly is it to follow suboptimal rules? Table 2 summarizes the main results. The first column shows the cumulative welfare loss from following alternative policies between 1979 Q1 and 1983 Q4 expressed as a percent of one year steady-state

³⁰Under a temporary oil price shock, oil price inflation is negative next period, pushing down headline inflation. A Taylor rule based on a forecast of headline inflation completely eliminates the output consequence of the 1979 oil shock, as can be seen in the upper left panel, but with dire effects on inflation in the short to medium run.

consumption. The second and third columns report the λ -weighted decomposition of the loss arising from volatility in the output gap or in core inflation.

The numbers seem to be unusually large. They are about 100 times larger than the ones reported by Lucas (1987), for example. One has to keep in mind, however, that our calculation refers to the cumulative welfare loss associated with *one* particularly painful episode and not the *average* cost from garden variety oil price shocks. Indeed, Galí et al. (2007) report that the welfare costs of recessions can be quite large. Their typical estimate for the cumulative cost of a 1980-type recession is in the range of 2 to 8 percent of one year steady-state consumption, depending on the elasticities of labor supply and intertemporal substitution.³¹

< Table 2 >

Table 2 shows that despite very good performances in terms of stabilizing the welfare relevant output gap, forecast-based HTR ranks last among the rules considered because of much higher core inflation. Taylor rules based on contemporaneous headline inflation are also quite costly if there is no inertia in interest rate decisions. The results also suggest that having followed a policy close to the benchmark Taylor rule (HTR)³² during the 1979 oil shock instead of the optimal policy may have cost the equivalent of 2.1 percent of one year steady-state consumption to the representative household (or about 200 billions of 2008 dollars). The overall cost would have been 40 percent smaller if monetary policy had been based on an inertial interest rate rule such as CTR or HTR with $\rho = 0.8$.

As mentioned above, our utility-based welfare metric tends to weigh heavily inflation deviations as a source of welfare costs.³³ This notwithstanding, the results suggest that welfare losses under the perfect price stability policy remain three times as large as under optimal policy and amount to 1.8 percent of one year steady-state consumption

³¹They also acknowledge that their estimates are probably a lower bound as they ignore the costs of price and wage inflation resulting from nominal rigidities.

³²Admittedly, HTR is only a rough approximation of the actual Federal Reserve behavior, but it seems sufficiently accurate to describe how US monetary policy has been conducted during the oil shock of 1979. See Orphanides (2000) for a detailed analysis using real-time data.

³³Assuming $\theta = 0.75$ and $\eta = 0.9$ amounts to setting λ to 0.02, which means that the central bank attributes about twice as much importance to inflation stabilization as to output gap stabilization when inflation is expressed in annual terms.

because of disproportionately large fluctuations in the welfare relevant output gap.

6.2 US monetary policy 2006-2008

How does the optimal rule (SLR) compare to actual policy in the US and to usual benchmarks during the last run up in oil prices, from 2006 to 2008 ? This episode is of great interest as some recent empirical evidence tends to show that the policy rule followed by the Federal Reserve was different in the post-Volcker period from the one followed during the 1979 oil price shock (see Kilian and Lewis, 2010). Figure 12 shows that in the period 2000-2005 the SLR is not very different from a classical Taylor rule based on headline (HTL) or core inflation (CTR) : All rules tend to suggest higher interest rates than the actual 3 months market rates.

Things become more interesting during the 2006-2008 oil price rallye (shaded area). In this period, the Federal reserve accommodated the oil price increase by dropping interest rates in the second half of 2007. This is also what would have been recommended by CTR , whereas HTL, reacting to increases in headline inflation would have supported further interest rate hikes until mid-2008. SLR, on the contrary, because it takes into account the detrimental impact of higher oil prices on consumption, would have suggested to start dropping interest rates a year and a half before the Fed did. Following the SLR, the Fed would also have started to tighten in 2009 already in an effort to keep inflation and inflation expectations in check.

< Figure 12 >

7 Time-varying elasticities of substitution

It is a well-know empirical fact that the demand for energy is almost unrelated to changes in its relative price in the short run. In the long run, however, persistent changes in prices have a significant bearing on the demand for energy.³⁴

How are the result of the precedent sections affected by the possibility of time-varying elasticities of substitution? Is the short run monetary trade-off after an oil

³⁴Pindyck and Rotemberg (1983), for example, report a cross-section long-run price elasticity of oil demand close to one.

price shock the mere reflection of some CES-related specificity, or is it a more general argument related to low short-term substitutability in a distorted economy ?

To allow for time-varying elasticities of substitution, I transform the production processes of Section 2 by introducing a convex adjustment cost of changing the input mix in production, as in Bodenstein et al. (2007) (see Appendix VI). Figure 13 shows impulse responses to a 1 percent shock to the price of oil and compares the flex-price equilibrium allocation with the optimal precommitment policy³⁵ when a fiscal transfer is available to neutralize the steady-state inefficiency due to monopolistic competition. Because of the adjustment costs — which add two state variables to the problem — the IRFs are not exactly similar to the ones obtained under CES production (see Figure 6). However, the message remains the same: price stability is the optimal policy when the economy’s steady-state is efficient.

Figure 14 performs the same exercise but allows for the same degree of monopolistic competition distortion in steady-state as in previous sections (a 20 percent steady-state markup of core prices over marginal costs). It shows that allowing for time-varying elasticities of substitution does not affect the paper’s main finding: In a distorted equilibrium, an oil price shock introduces a significant monetary policy trade-off if the oil cost share is allowed to vary in the short-run.

< Figure 13 >

< Figure 14 >

8 Conclusion

Most inflation targeting central banks understand their mandate to be ensuring long-term price stability. Following an oil price shock, however, none of them would be ready to expose the economy to the type of output and employment drops recommended by standard New Keynesian theory for the sake of stabilizing prices in the short term.

³⁵Optimal monetary policy is here also derived under the timeless perspective assumption. Since we are only interested in the dynamic response of variables to an oil price shock under optimal policy, we do not compute the LQ solution. We directly solve the non-linear model for the Ramsey policy that would maximize utility under the constraint of our model using Andy Levin’s Matlab code (see Levin 2004, 2005).

This paper argues that policies which *perfectly* stabilize prices entail significant welfare costs, explaining the reluctance of policymakers to enforce them.

Interestingly, I find that the optimal monetary policy response to a persistent increase in oil price indeed resembles the typical response of inflation targeting central banks: While long-term price stability is ensured by a credible commitment to keep inflation and inflation expectations in check, short-term real rates drop right after the shock to help dampen real output fluctuations. By managing expectations efficiently, central banks can improve on both the flexible price equilibrium solution and the recommendation of simple Taylor rules. Using standard welfare criteria, I reckon that following a standard Taylor rule in the aftermath of the 1979 oil price shock may have cost the US household about 2% of annual consumption.

These findings are based on the assumptions that monetary policy is perfectly credible and transparent and that agents and the central banks have the right (and the same) model of the economy. Further work should explore how sensitive the policy conclusions are to the incorporation of imperfect information and learning into the analysis. Moreover, further research should establish the robustness of the simple rule to the incorporation of more shocks into a larger DSGE model. Another potential limitation of the analysis is that oil price shocks are treated here as exogenous events. The optimal monetary policy response could vary if the oil price increase was due to an increase in world aggregate demand instead of an oil supply disruption (see Kilian, 2009, Kilian and Murphy, 2010 and Nakov and Pescatori, 2007 for a first attempt at modelling the oil market explicitly). A related issue concerns the treatment of the open economy aspect. Bodenstein et al. (2007), for example, have shown that the effect of an oil price shock on the terms of trade, the trade balance and consumption depends on the assumption made on the structure of financial market risk-sharing. These important considerations are left for future research.

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Appendix I: The model

AI.1 Households

There exists a unit mass continuum of infinitely lived households indexed by $j \in [0, 1]$, which maximize the discounted sum of present and expected future utilities defined as follows

$$\mathbb{E}_t \sum_{t=s}^{\infty} \beta^{t-s} \left\{ \frac{C_t(j)^{1-\sigma}}{1-\sigma} - \nu \frac{H_t(j)^{1+\phi}}{1+\phi} \right\}, \quad (\text{A1.1})$$

where $C_t(j)$ is the consumption goods bundle, $H_t(j)$ is the (normalized) quantity of hours supplied by household of type j , the constant discount factor β satisfies $0 < \beta < 1$ and ν is a parameter calibrated to ensure that the typical household works eight hours a day in steady state.

In each period, the representative household j faces a standard flow budget constraint

$$P_t B_t(j) + P_t C_t(j) = R_{t-1} B_{t-1}(j) + W_t H_t(j) + \tilde{\Pi}_t(j) + T_t(j), \quad (\text{A1.2})$$

where $B_t(j)$ is a non-state-contingent one period bond, R_t is the nominal gross interest rate, P_t is the CPI, $\tilde{\Pi}_t(j)$ is the household j share of the firms' dividends and $T_t(j)$ is a lump sum fiscal transfer to the household of the profits from sovereign oil extraction activities.

Because the labor market is perfectly competitive, I drop the index j such that $H_t \equiv H_t(j) = \int_0^1 H_t(j) dj$, and I write the consumption goods bundle³⁶ C_t as a CES aggregator of the core consumption goods basket $C_{Y,t}$ and the household's demand for oil $O_{C,t}$

$$C_t = \left((1 - \omega_{oc}) C_{Y,t}^{\frac{\chi-1}{\chi}} + \omega_{oc} O_{C,t}^{\frac{\chi-1}{\chi}} \right)^{\frac{\chi}{\chi-1}}, \quad (\text{A1.3})$$

where ω_{oc} is the oil quasi-share parameter and χ is the elasticity of substitution between oil and non-oil consumption goods.

Households determine their consumption, savings, and labor supply decisions by maximizing (A1.1) subject to (A1.2). This gives rise to the traditional Euler equation

$$1 = \beta \mathbb{E}_t \left\{ \left(\frac{C_t}{C_{t+1}} \right)^\sigma \frac{R_t}{\Pi_{t+1}} \right\}, \quad (\text{A1.4})$$

which characterizes the optimal intertemporal allocation of consumption and where Π_t represents headline inflation.

Allowing for real wage rigidity (which may reflect some unmodeled imperfection in the labor market as in BG07), the labor supply condition relates the marginal rate of

³⁶The consumption basket can be regarded as produced by perfectly competitive consumption distributors whose production function mirrors the preferences of households over consumption of oil and non-oil goods.

substitution between consumption and leisure to the geometric mean of real wages in periods t and $t - 1$.

$$\left(C_t^\sigma \nu H_t^\phi\right)^{(1-\eta)} = \frac{W_t}{P_t} \left(\frac{W_{t-1}}{P_{t-1}}\right)^{-\eta}. \quad (\text{A1.5})$$

In the benchmark calibration, i.e., unless stated otherwise, $\eta = 0$; real wages are perfectly flexible and equal to the marginal rate of substitution between labor and consumption in all periods.

Finally, households optimally divide their consumption expenditures between core and oil consumption according to the following demand equations:

$$C_{Y,t} = P_{y,t}^{-\chi} (1 - \omega_{oc})^\chi C_t, \quad (\text{A1.6})$$

$$O_{C,t} = P_{o,t}^{-\chi} \omega_{oc}^\chi C_t, \quad (\text{A1.7})$$

where $P_{y,t} \equiv \frac{P_{Y,t}}{P_t}$ is the relative price of the core consumption good and $P_{o,t} \equiv \frac{P_{O,t}}{P_t}$ is the relative price of oil in terms of the consumption good bundle and where

$$P_t = \left((1 - \omega_{oc})^\chi P_{Y,t}^{1-\chi} + \omega_{oc}^\chi P_{O,t}^{1-\chi}\right)^{\frac{1}{1-\chi}} \quad (\text{A1.8})$$

represents the overall consumer price index (CPI).

AI.2 Firms

Core goods producers

I assume that the core consumption good is produced by a continuum of perfectly competitive producers indexed by $c \in [0, 1]$ that use a set of imperfectly substitutable intermediate goods indexed by $i \in [0, 1]$. In other words, core goods are produced via a Dixit-Stiglitz aggregator

$$Y_t(c) = \left(\int_0^1 Y_t(i, c)^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}, \quad (\text{A1.9})$$

where ε is the elasticity of substitution between intermediate goods. Given the individual intermediate goods prices, $P_{Y,t}(i)$, cost minimization by core goods producers gives rise to the following demand equations for individual intermediate inputs:

$$Y_t(i, c) = \left(\frac{P_{Y,t}(i)}{P_{Y,t}}\right)^{-\varepsilon} Y_t(c), \quad (\text{A1.10})$$

where $P_{Y,t} = \left(\int_0^1 P_{Y,t}(i)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}$ is the core price index.

Aggregating (A1.10) over all core goods firms, the total demand for intermediate goods $Y_t(i)$ is derived as a function of the demand for core consumption goods Y_t

$$Y_t(i) = \left(\frac{P_{Y,t}(i)}{P_{Y,t}}\right)^{-\varepsilon} Y_t, \quad (\text{A1.11})$$

using the fact that perfect competition in the market for core goods implies $Y_t(c) \equiv Y_t = \int_0^1 Y_t(c) dc$.

Intermediate goods firms

Each intermediate goods firm produces a good $Y_t(i)$ according to a constant returns-to-scale technology represented by the CES production function

$$Y_t(i) = \left((1 - \omega_{oy}) (\mathcal{H}_t H_t(i))^{\frac{\delta-1}{\delta}} + \omega_{oy} (O_{Y,t}(i))^{\frac{\delta-1}{\delta}} \right)^{\frac{\delta}{\delta-1}}, \quad (\text{A1.12})$$

where \mathcal{H}_t is the exogenous Harrod-neutral technological progress whose value is normalized to one. $O_{Y,t}(i)$ and $H_t(i)$ are the quantities of oil and labor required to produce $Y_t(i)$ given the quasi-share parameters, ω_{oy} , and the elasticity of substitution between labor and oil, δ .

Each firm i operates under perfect competition in the factor markets and determines its production plan so as to minimize its total cost

$$TC_t(i) = \frac{W_t}{P_{Y,t}} H_t(i) + \frac{P_{O,t}}{P_{Y,t}} O_{Y,t}(i), \quad (\text{A1.13})$$

subject to the production function (A1.12) for given W_t , $P_{Y,t}$, and $P_{O,t}$. Their demands for inputs are given by

$$H_t(i) = \left(\frac{W_t}{MC_t(i) P_{Y,t}} \right)^{-\delta} (1 - \omega_{oy})^\delta Y_t(i) \quad (\text{A1.14})$$

$$O_{Y,t}(i) = \left(\frac{P_{O,t}}{MC_t(i) P_{Y,t}} \right)^{-\delta} \omega_{oy}^\delta Y_t(i), \quad (\text{A1.15})$$

where the real marginal cost in terms of core consumption goods units is given by

$$MC_t(i) \equiv MC_t = \left((1 - \omega_{oy})^\delta \left(\frac{W_t}{P_{Y,t}} \right)^{1-\delta} + \omega_{oy}^\delta \left(\frac{P_{O,t}}{P_{Y,t}} \right)^{1-\delta} \right)^{\frac{1}{1-\delta}}. \quad (\text{A1.16})$$

Price setting

Final goods producers operate under perfect competition and therefore take the price level $P_{Y,t}$ as given. In contrast, intermediate goods producers operate under monopolistic competition and face a downward-sloping demand curve for their products, whose price elasticity is positively related to the degree of competition in the market. They set prices so as to maximize profits following a sticky price setting scheme à la Calvo. Each firm contemplates a fixed probability θ of not being able to change its price next period and therefore sets its profit-maximizing price $\overline{P_{Y,t}}(i)$ to solve

$$\arg \max_{P_{Y,t}(i)} \left\{ \mathbb{E}_t \sum_{n=0}^{\infty} \theta^n \mathcal{D}_{t,t+n} \tilde{\Pi}_{t,t+n}(i) \right\},$$

where $\mathcal{D}_{t,t+n}$ is the stochastic discount factor defined by $\mathcal{D}_{t,t+n} = \beta^n \left(\frac{C_{t+n}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+n}}$ and profits are

$$\tilde{\Pi}_{t,t+n}(i) = \overline{P_{Y,t}}(i) Y_{t+n}(i) - MC_{t+n} P_{t+n}^Y Y_{t+n}(i).$$

The solution to this intertemporal maximization problem yields

$$\frac{\overline{P_{Y,t}}(i)}{P_{Y,t}} = \frac{K_t}{F_t}, \quad (\text{A1.17})$$

where

$$K_t \equiv \left(\frac{\varepsilon}{\varepsilon - 1} \right) \mathbb{E}_t \sum_{n=0}^{\infty} (\beta\theta)^n (X_{t+n}^Y)^{-\varepsilon} \left(\frac{Y_{t+n}}{C_{t+n}^\sigma} \right) \left(\frac{P_{t+n}^Y}{P_{t+n}} \right) MC_{t+n},$$

and

$$F_t \equiv \mathbb{E}_t \sum_{n=0}^{\infty} (\beta\theta)^n (X_{t+n}^Y)^{1-\varepsilon} \left(\frac{Y_{t+n}}{C_{t+n}^\sigma} \right) \left(\frac{P_{t+n}^Y}{P_{t+n}} \right).$$

Since only a fraction $(1 - \theta)$ of the intermediate goods firms are allowed to reset their prices every period while the remaining firms update them according to the steady-state inflation rate, it can be shown that the overall core price index dynamics is given by the following equation

$$(P_{Y,t})^{1-\varepsilon} = \theta (P_{Y,t-1})^{1-\varepsilon} + (1 - \theta) (\overline{P_{Y,t}}(i))^{1-\varepsilon} \quad (\text{A1.18})$$

Following Benigno and Woodford (2005), I rewrite equation (A1.18) in terms of the core inflation rate $\Pi_{Y,t}$

$$\theta (\Pi_{Y,t})^{\varepsilon-1} = 1 - (1 - \theta) \left(\frac{K_t}{F_t} \right)^{1-\varepsilon}, \quad (\text{A1.19})$$

for

$$K_t = \frac{\varepsilon}{\varepsilon - 1} \left(\frac{Y_t P_{Y,t}}{C_t^\sigma P_t} \right) MC_t + \beta\theta \mathbb{E}_t \{ (\Pi_{Y,t+1})^\varepsilon K_{t+1} \},$$

and

$$F_t = \frac{Y_t P_{Y,t}}{C_t^\sigma P_t} + \beta\theta \mathbb{E}_t \{ (\Pi_{Y,t+1})^{\varepsilon-1} F_{t+1} \}.$$

AI.3 Government

To close the model, I assume that oil is extracted with no cost by the government, which sells it to the households and the firms and transfers the proceeds in a lump sum fashion to the households. I abstract from any other role for the government and assume that it runs a balanced budget in each and every period so that its budget constraint is simply given by

$$T_t = P_{O,t} O_t,$$

for O_t the total amount of oil supplied.

A1.4 Market clearing and aggregation

In equilibrium, goods, oil, and labor markets clear. In particular, given the assumption of a representative household and competitive labor markets, the labor market clearing condition is

$$H_t^D \equiv \int_0^1 H_t(i) di = \int_0^1 H_t(j) dj \equiv H_t.$$

Because I assume that the real price of oil $P_{o,t}$ is exogenous in the model, the government supplies all demanded quantities at the posted price. The oil market clearing condition is then given by

$$\int_0^1 O_{C,t}(j) dj + \int_0^1 O_{Y,t}(i) di = O_t,$$

for O_t the total amount of oil supplied.

As there is no net aggregate debt in equilibrium,

$$\int_0^1 B_t(j) dj = B_t = 0,$$

we can consolidate the government's and the household's budget constraints to get the overall resource constraint

$$C_{Y,t} = Y_t.$$

Finally, Calvo price setting implies that in a sticky price equilibrium there is no simple relationship between aggregate inputs and aggregate output, i.e., there is no aggregate production function. Namely, defining the efficiency distortion related to price stickiness $P_t^* \equiv \frac{P_t^{disp}}{P_t}$ for $P_t^{disp} \equiv \left(\int_0^1 (P_{Y,t}(i))^{-\varepsilon} di \right)^{-\frac{1}{\varepsilon}}$, I follow Yun (1996) and write the aggregate production relationship

$$Y_t = \left((1 - \omega_{oy}) H_t^{\frac{\delta-1}{\delta}} + \omega_{oy} O_{Y,t}^{\frac{\delta-1}{\delta}} \right)^{\frac{\delta}{\delta-1}} P_t^*, \quad (\text{A1.20})$$

where price dispersion leads to an inefficient allocation of resources given that

$$P_t^* : \begin{cases} \leq 1 \\ = 1 \end{cases} \quad P_{Y,t}(r) = P_{Y,t}(s), \quad \text{all } r = s.$$

The inefficiency distortion P_t^* is related to the rate of core inflation $\Pi_{Y,t}$ by making use of the definition

$$P_t^* = \left(\theta \left(P_{t-1}^{disp} \right)^{-\varepsilon} + (1 - \theta) \left(\overline{P_{Y,t}}(i) \right)^{-\varepsilon} \right)^{-\frac{1}{\varepsilon}},$$

and equations (A1.19) and (A1.17) to get

$$P_t^* = \left((1 - \theta) \left(\frac{K_t}{F_t} \right)^{-\varepsilon} + \frac{\theta (\Pi_{Y,t})^\varepsilon}{P_{t-1}^*} \right)^{-1}.$$

Appendix II: log-linearized economy

The allocation in the decentralized economy can be summarized by the following five equations. Log-linearizing the labor supply equation (A1.5) (and setting $\eta = 0$ for flexible real wages), the labor demand equation (A1.14), and the real marginal cost (A1.16), gives equations (A2.1), (A2.2), and (A2.3). Substituting out oil consumption (A1.7) in (A1.3) and making use of the overall resource constraint gives (A2.4). Finally, equation (A2.5) is the log-linear version of (A1.8) and describes the evolution of the ratio of core to headline price indices as a function of the real price of oil in consumption units. Lowercase letters denote the percent deviation of each variable with respect to their steady states (e.g., $c_t \equiv \log\left(\frac{C_t}{C}\right)$):

$$w_t = \phi h_t + \sigma c_t \quad (\text{A2.1})$$

$$h_t = y_t - \delta (w_t - mc_t - py_t) + \Delta_t \quad (\text{A2.2})$$

$$mc_t = (1 - \widetilde{\omega}_{oy}) (w_t - py_t) + \widetilde{\omega}_{oy} (po_t - py_t) \quad (\text{A2.3})$$

$$c_t = -\chi \frac{\widetilde{\omega}_{oc}}{sy} po_t + y_t \quad (\text{A2.4})$$

$$py_t = -\frac{\widetilde{\omega}_{oc}}{1 - \widetilde{\omega}_{oc}} po_t \quad (\text{A2.5})$$

where $w_t = \log\left(\frac{W_t P}{P_t W}\right)$ is the consumption real wage, $po_t = \log\left(\frac{P_{o,t}}{P_o}\right)$ is the real oil price in consumption units, $py_t = \log\left(\frac{P_{y,t}}{P_y}\right)$ is the relative price of the core goods in terms of consumption goods, $\widetilde{\omega}_{oy} \equiv \omega_{oy}^\delta \left(\frac{P_o}{MC \cdot P_y}\right)^{1-\delta}$ is the share of oil in the real marginal cost, $\widetilde{\omega}_{oc} \equiv \omega_{oc}^\chi P_o^{1-\chi}$ is the share of oil in the CPI, and $sy \equiv (1 - \omega_{oc}) \left(\frac{Y}{C}\right)^{\frac{\chi-1}{\chi}}$ is the share of the core good in the consumption goods basket.

Also, the real marginal cost is equal to the inverse of the desired gross markup in the steady state, itself determined by the degree of monopolistic competition as measured by the elasticity of substitution between goods ε . So $MC = \frac{\varepsilon-1}{\varepsilon}$ in the steady state and $MC \rightarrow 1$ when $\varepsilon \rightarrow \infty$ in the perfect competition limit.

Appendix III : Deriving a quadratic loss function

The policy problem originally defined as maximizing households utility can be rewritten in terms of a quadratic loss function defined over the welfare relevant output gap $y_t - y_t^*$ and core inflation $\pi_{y,t}$ as shown in Benigno and Woodford (2005).

AIII.1 Second-order approximation of the model supply side

Starting with the labor market, labor demand can be rewritten as:

$$H_t = \left(\frac{W_t}{MC_t p_{y,t}} \right)^{-\delta} (1 - \omega_{oy})^\delta \frac{Y_t}{P_t^*},$$

and the labor supply as:

$$C_t^{\sigma(1-\eta)} \nu H_t^{\phi(1-\eta)} = \frac{W_t}{P_t} \left(\frac{W_{t-1}}{P_{t-1}} \right)^{-\eta}$$

Rewriting them in log-deviations from steady state:

$$h_t = y_t - \delta (w_t - mc_t - py_t) + \Delta_t$$

$$w_t = \eta w_{t-1} + (1 - \eta) (\phi h_t + \sigma c_t)$$

where Δ_t is the log deviation of the price dispersion measure $\frac{1}{P_t^*}$ from its steady state and measures the distortion due to inflation. Note that these two log-linear equations are exact transformations of the nonlinear equations.

Combining the labor demand and supply with a second-order approximation of the real marginal cost

$$mc_t = (1 - \widetilde{\omega}_{oy}) [w_t - py_t] + \widetilde{\omega}_{oy} [po_t - py_t] + \frac{1}{2} \widetilde{\omega}_{oy} (1 - \widetilde{\omega}_{oy}) (1 - \delta) [w_t - po_t]^2 + O(\|\xi\|^3)$$

and a first-order approximation of the demand for consumption (where the demand for energy consumption has been substituted out)

$$c_t = -\chi \frac{\widetilde{\omega}_{oc}}{sy} p_{o,t} + y_t + O(\|\xi\|^2), \quad (\text{A3.1})$$

we obtain a second-order accurate equilibrium relation linking total hours to output and the real price of oil

$$\begin{aligned} h_t &= (1 - \mathcal{D}(\sigma + \phi)) y_t + \frac{\mathcal{D}}{(1 - \eta)} \mathcal{M} p_{o,t} + \frac{\mathcal{W}}{(1 - \widetilde{\omega}_{oy})} \Delta_t \\ &+ \frac{1}{2} \frac{\mathcal{D}}{(1 - \eta)} \mathcal{W}^2 \frac{(1 - \delta)}{(1 - \widetilde{\omega}_{oy})} [\mathcal{J} y_t + \mathcal{L} p_{o,t}]^2 + O(\|\xi\|^3) + t.i.p \end{aligned} \quad (\text{A3.2})$$

where

$$\begin{aligned} \mathcal{M} &\equiv \frac{[\widetilde{\omega}_{oc}(1-\eta) \frac{\sigma\chi}{sy} - \frac{\widetilde{\omega}_{oc}}{1-\omega_{oc}} + \mathcal{B}]}{1-\mathcal{W}} \\ \mathcal{B} &\equiv \frac{1-\mathcal{W} + \mathcal{W}(1+(1-\eta)\delta\phi)[\widetilde{\omega}_{oc}(1+(1-\eta)\mathcal{A}) - (1-\eta)\mathcal{A}]}{1-\widetilde{\omega}_{oc}}, \\ \mathcal{A} &\equiv \frac{\phi\delta}{(1+(1-\eta)\phi\delta)} \left(\frac{\widetilde{\omega}_{oc}}{1-\widetilde{\omega}_{oc}} \right) + \frac{\chi\sigma}{(1+(1-\eta)\phi\delta)} \left(\frac{\widetilde{\omega}_{oc}}{sy} \right), \end{aligned}$$

$$\begin{aligned}
\mathcal{W} &\equiv \frac{(1-\widetilde{\omega}_{oy})}{1+\widetilde{\omega}_{oy}(1-\eta)\phi\delta}, \\
1-\mathcal{W} &\equiv \frac{\widetilde{\omega}_{oy}(1+(1-\eta)\phi\delta)}{1+\widetilde{\omega}_{oy}(1-\eta)\phi\delta}, \\
\mathcal{J} &\equiv (1-\eta)(\sigma+\phi), \\
\mathcal{L} &\equiv \frac{(1-\eta)\phi\delta}{(1+(1-\eta)\delta\phi\mathcal{W})}\mathcal{B} - (1+\widetilde{\omega}_{oy}(1-\eta)\phi\delta)(1+(1-\eta)\mathcal{A}), \\
\mathcal{D} &= (1-\eta)\delta\mathcal{W}\frac{\widetilde{\omega}_{oy}}{(1-\widetilde{\omega}_{oy})}.
\end{aligned}$$

As the price dispersion measure can be written as

$$P_t^* = \left((1-\theta) \left(\frac{K_t}{F_t} \right)^{-\varepsilon} + \frac{\theta (\Pi_{Y,t})^\varepsilon}{P_{t-1}^*} \right)^{-1},$$

Benigno and Woodford (2004) demonstrate that Δ_t — the log deviation of the price dispersion measure — has a second-order approximation that depends only on second-order inflation terms and lagged dispersion

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} \Delta_t = f(\Delta_{t_0-1}) + \frac{1}{2} \frac{\varepsilon}{k} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{\pi_{y,t}^2}{2} + O(\|\xi\|^3). \quad (\text{A3.3})$$

AIII.2 Second-order approximation to NKPC

In this section I derive a second-order approximation to the NKPC, which can be used to substitute out the term linear in y_t in the second-order approximation to utility when the steady state is distorted.

I start by writing a second-order approximation to the model inflation/marginal cost nexus. For convenience, I rewrite from the main text

$$\begin{aligned}
K_t &= \left(\frac{\varepsilon}{\varepsilon-1} \right) \left(\frac{Y_t}{C_t^\sigma} \right) P_{y,t} MC_t + \beta \theta E_t (\Pi_{Y,t+1})^\varepsilon K_{t+1} \\
F_t &= \left(\frac{Y_t}{C_t^\sigma} \right) P_{y,t} + \beta \theta E_t (\Pi_{Y,t+1})^{\varepsilon-1} F_{t+1} \\
\frac{K_t}{F_t} &= \left[\frac{1 - \theta (\Pi_{Y,t})^{\varepsilon-1}}{1 - \theta} \right]^{\frac{1}{1-\varepsilon}}.
\end{aligned}$$

Taking a second-order approximation of the three preceding equations, I follow Benigno and Woodford (2004) and Castillo et al. (2006) and express the NKPC as

$$V_t = kmc_t + \frac{1}{2} kmc_t [2(y_t - \sigma c_t + p_{y,t}) + mc_t] + \frac{1}{2} \varepsilon \pi_t^2 + \beta E_t V_{t+1} + O(\|\xi\|^3) \quad (\text{A3.4})$$

where I define the auxiliary variable V_t

$$V_t = \Pi_{Y,t} + \frac{1}{2} \left(\frac{\varepsilon-1}{1-\theta} + \varepsilon \right) (\Pi_{Y,t})^2 + \frac{1}{2} (1-\theta\beta) \Pi_{Y,t} z_t$$

and the linear expansion of z_t

$$z_t = 2(y_t - \sigma c_t + p_{y,t}) + mc_t + \theta\beta E_t \left(\frac{2\varepsilon - 1}{1 - \theta\beta} \Pi_{Y,t+1} + z_{t+1} \right).$$

Using the first-order³⁷ approximation of c_t and $p_{y,t}$ and a second-order approximation of mc_t , I write

$$\begin{aligned} mc_t &= \mathcal{W}\eta w_{t-1} + (1 - \eta)(\sigma + \phi)\mathcal{W}y_t \\ &\quad + (1 - \eta)\phi\mathcal{W}\Delta_t + \mathcal{B}p_{o,t} \\ &\quad + \frac{1}{2} \frac{(1 - \delta)}{(1 - \widetilde{\omega}_{oy})} \mathcal{W}^2 (1 - \mathcal{W}) [(1 - \eta)(\sigma + \phi)y_t + \mathcal{L}p_{o,t}]^2 \\ &\quad + O(\|\xi\|^3) + t.i.p. \end{aligned}$$

which I substitute in (A3.4) to get

$$\begin{aligned} V_t &= k_y y_t + k_p p_{o,t} + k\mathcal{W}\phi\Delta_t \\ &\quad + \frac{1}{2}k(c_{yy}y_t^2 + 2c_{yp}y_t p_{o,t} + c_{pp}p_{o,t}^2) \\ &\quad + \frac{1}{2}\varepsilon\pi_{y,t}^2 + \beta E_t V_{t+1} + O(\|\xi\|^3) \end{aligned} \tag{A3.5}$$

for

$$k_y \equiv k(1 - \eta)(\sigma + \phi)\mathcal{W}$$

$$k_p \equiv k\mathcal{B}$$

$$c_{yy} \equiv F(1 - \eta)^2(\sigma + \phi)^2 + 2(1 - \eta)(\sigma + \phi)(1 - \sigma)\mathcal{W} + (1 - \eta)^2(\sigma + \phi)^2\mathcal{W}^2$$

$$c_{yp} \equiv (1 - \eta)(\sigma + \phi)\mathcal{W}(\Sigma + \mathcal{B}) - F(\sigma + \phi)(1 - \eta)\mathcal{L} + \mathcal{B}(1 - \sigma)$$

$$c_{pp} \equiv F\mathcal{L}^2 + 2\Sigma\mathcal{B} + \mathcal{B}^2$$

$$F \equiv \frac{1 - \delta}{1 - \widetilde{\omega}_{oy}} \mathcal{W}^2 (1 - \mathcal{W})$$

$$\Sigma \equiv \sigma\chi \frac{\widetilde{\omega}_{oc}}{s_y} - \frac{\widetilde{\omega}_{oc}}{1 - \widetilde{\omega}_{oc}}.$$

Note that the natural level of output can be found from the preceding equation by rewriting it as

$$\begin{aligned} V_t &= k_y \left\{ y_t + \overbrace{k_y^{-1}k_p p_{o,t}}^{-Y_t^N} + k_y^{-1}k\mathcal{W}\phi\Delta_t + \frac{1}{2}k_y^{-1}k(c_{yy}y_t^2 + 2c_{yp}y_t p_{o,t} + c_{pp}p_{o,t}^2) + \frac{1}{2}k_y^{-1}\varepsilon\pi_t^2 \right\} \\ &\quad + \beta E_t V_{t+1} + O(\|\xi\|^3) \end{aligned}$$

and ignoring all second-order terms.

³⁷A second-order approximation is not necessary here as these two variables enter multiplicatively with mc_t .

Using the law of iterated expectation and (A3.3), equation (A3.5) can be rewritten as an infinite discounted sum

$$\begin{aligned} \sum_{t=t_0}^{\infty} \beta^{t-t_0} y_t &= \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{k_y} (V_{t_0} - f(\Delta_{t_0-1})) \\ &\quad - \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ k_y^{-1} k_p p_{o,t} + \frac{1}{2} k_y^{-1} k (c_{yy} y_t^2 + 2c_{yp} y_t p_{o,t} + c_{pp} p_{o,t}^2) \right. \\ &\quad \left. + \frac{1}{2} k_y^{-1} \varepsilon (1 + \phi \mathcal{W}) \pi_{y,t}^2 \right\} \\ &\quad + O(\|\xi\|^3). \end{aligned} \tag{A3.6}$$

AIII.3 Second-order approximation to utility

I take a second-order approximation of the representative household utility function in t_o

$$U_{t_o} = E_{t_o} \sum_{t=t_o}^{\infty} \beta^{t-t_o} \{u(C_t) - v(H_t)\}. \tag{A3.7}$$

The second-order approximation of the first term is given by

$$u(C_t) = C u_c \left\{ c_t + \frac{1}{2} (1 - \sigma) c_t^2 \right\} + O(\|\xi\|^3) + t.i.p. \tag{A3.8}$$

Substituting (A3.1) and its square into (A3.8), I get

$$u(C_t) = C u_c \left\{ u_y y_t + \frac{1}{2} u_{yy} y_t^2 + u_{yp} y_t p_{o,t} \right\} + O(\|\xi\|^3) + t.i.p. \tag{A3.9}$$

for $u_y \equiv 1$, $u_{yy} \equiv 1 - \sigma$, $u_{yp} \equiv -\chi \frac{\omega_{oc}}{s_y} (1 - \sigma)$ and *t.i.p.* stands for terms independent of policy.

The second term in household utility is approximated by

$$v(H_t) = H v_h \left\{ h_t + \frac{1}{2} (1 + \phi) h_t^2 \right\} + O(\|\xi\|^3) + t.i.p. \tag{A3.10}$$

Substituting (A3.2) and its square in (A3.10) and getting rid of variables independent of policy, I obtain

$$v(H_t) = H v_h \left\{ v_y y_t + v_{\Delta} \Delta_t + \frac{1}{2} v_{yy} y_t^2 + v_{yp} p_{o,t} y_t \right\} + O(\|\xi\|^3) + t.i.p. \tag{A3.11}$$

for

$$\begin{aligned}
v_y &\equiv 1 - \mathcal{D}(\phi + \sigma), \\
v_\Delta &\equiv \frac{\mathcal{W}}{1 - \widetilde{\omega}_{oy}}, \\
v_{yy} &\equiv \frac{\mathcal{D}}{(1-\eta)} \mathcal{W}^2 \frac{(1-\delta)}{(1-\widetilde{\omega}_{oy})} \mathcal{J}^2 + (1 + \phi) (1 - \mathcal{D}(\phi + \sigma))^2 \\
v_{yp} &\equiv \frac{\mathcal{D}}{(1-\eta)} \left[(1 + \phi) (1 - \mathcal{D}(\phi + \sigma)) \mathcal{M} + \mathcal{W}^2 \frac{(1-\delta)}{(1-\widetilde{\omega}_{oy})} \mathcal{J} \mathcal{L} \right].
\end{aligned}$$

Now, since the technology is constant returns to scale, the share of labor in total cost is equivalent to its share in marginal cost and the following equilibrium relationship at the steady state

$$Y \mathbf{u}_c MC (1 - \widetilde{\omega}_{oy}) = H v_h,$$

which can be used to rewrite total utility U_{t_0} by substituting (A3.11) and (A3.9) into (A3.7), to get

$$U_{t_0} = (Y \mathbf{u}_c) E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ u_y y_t + \frac{1}{2} u_{yy} y_t^2 + u_{yp} p_{o,t} y_t + \frac{1}{2} u_\pi \pi_t^2 \right\} + O(\|\xi\|^3) + t.i.p., \quad (\text{A3.12})$$

where

$$\begin{aligned}
u_y &= \frac{C}{Y} - MC (1 - \widetilde{\omega}_{oy}) v_y, \\
u_{yy} &= \frac{C}{Y} (1 - \sigma) - MC (1 - \widetilde{\omega}_{oy}) v_{yy}, \\
u_{yp} &= -\frac{C}{Y} \left(\frac{\widetilde{\omega}_{oc}}{sy} \right) \chi (1 - \sigma) - MC (1 - \widetilde{\omega}_{oy}) v_{yp} \\
u_\Delta &= -MC (1 - \widetilde{\omega}_{oy}) v_\Delta = -MC \mathcal{W} \\
u_\pi &= \frac{\varepsilon}{k} u_\Delta = \frac{\varepsilon}{k} MC \mathcal{W}.
\end{aligned}$$

For the last step, substituting the expression (A3.6) for $\sum_{t=t_0}^{\infty} \beta^{t-t_0} y_t$ in (A3.12) obtains

$$\begin{aligned}
U_{t_0} &= -Y \mathbf{u}_c E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2} (u_y k_y^{-1} k c_{yy} - u_{yy}) y_t^2 \\
&\quad + \frac{1}{2} (2 u_y k_y^{-1} k c_{yp} - 2 u_{yp}) y_t p_{o,t} \\
&\quad + \frac{1}{2} (u_y k_y^{-1} \varepsilon (1 + \phi \mathcal{W}) - u_\pi) \pi_t^2 + O(\|\xi\|^3) + t.i.p.,
\end{aligned}$$

that can be rewritten equivalently as

$$\begin{aligned}
U_{t_0} &= -Y \mathbf{u}_c E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2} (u_y k_y^{-1} k c_{yy} - u_{yy}) [y_t - y_t^*]^2 \\
&\quad + \frac{1}{2} (u_y k_y^{-1} \varepsilon (1 + \phi \mathcal{W}) - u_\pi) \pi_t^2 + O(\|\xi\|^3) + t.i.p.
\end{aligned}$$

which ends up as the central banks's loss function to minimize

$$U_{t_0} = \Upsilon \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \{ \lambda x_t^2 + \pi_{y,t}^2 \} + O(\|\xi\|^3) + t.i.p.$$

for $\Upsilon \equiv \frac{1}{2} \lambda_{\pi_y} Y \mathbf{u}_c$, $\Psi \equiv \frac{k u_y k_y^{-1} c_{yp} - u_{yp}}{k u_y k_y^{-1} c_{yy} - u_{yy}}$, and where $\lambda \equiv \frac{\lambda_y}{\lambda_{\pi_y}}$ for $\lambda_y = u_y k_y^{-1} k c_{yy} - u_{yy}$ and $\lambda_{\pi_y} = u_y k_y^{-1} \varepsilon (1 + \phi \mathcal{W}) - u_{\pi}$. The output gap $x_t = y_t - y_t^*$ is now the percent deviation of output with respect to the welfare relevant output y_t^* itself defined as

$$y_t^* \equiv - \frac{k u_y k_y^{-1} c_{yp} - u_{yp}}{k u_y k_y^{-1} c_{yy} - u_{yy}} p_{o,t} = -\Psi p_{o,t}. \quad (\text{A3.13})$$

The values of λ_y and λ_{π_y} are functions of the model parameters and describe the weights assigned by the central bank to stabilize the welfare relevant output gap and core inflation. In what follows I summarize this information with $\lambda \equiv \frac{\lambda_y}{\lambda_{\pi_y}}$, which determines how concerned about the output gap a central bank should be after an oil price shock. Typically, λ decreases with the sacrifice ratio and the degree of price stickiness.

Appendix IV: Characterizing optimal policy

As shown in Section 3, acknowledging the low level of short-term substitutability between oil and other factors gives rise to a cyclical distortion coming from the interaction between the steady-state efficiency distortion and the oil price shock. In terms of the model equations, this cyclical distortion is translated into a cost-push shock that enters the New Keynesian Phillips curve (NKPC henceforth). Taking a log-linear approximation of equation (A1.19) around the zero inflation steady-state yields the standard result that (core) inflation is a function of next period inflation and this period real marginal cost: the NKPC

$$\pi_{y,t} = \beta \mathbb{E}_t \pi_{y,t+1} + k m c_t, \quad (\text{A4.1})$$

where $m c_t$ is the log-deviation of real marginal cost from its (distorted) steady state and $k = \left(\frac{1-\theta}{\theta}\right) (1 - \theta\beta)$ is the elasticity of inflation to the real marginal cost.

Substituting the labor market clearing level of the real wage into the real marginal cost equation (A1.16), we can rewrite (A4.1) as

$$\pi_{y,t} = \beta \mathbb{E}_t \pi_{y,t+1} + k_y g a p_t, \quad (\text{A4.2})$$

where the output gap $g a p_t = y_t - y_t^N$ measures the deviation between current output and the natural level of output, and where

$$y_t^N = - \frac{k \mathcal{B}}{k_y} p_{o,t}, \quad (\text{A4.3})$$

for \mathcal{B} a decreasing function of δ and χ , the oil production and consumption elasticities of substitution (see Appendix III).

But (A4.2) can be equivalently rewritten as

$$\pi_{y,t} = \beta \mathbb{E}_t \pi_{y,t+1} + k_y x_t + \mu_t, \quad (\text{A4.4})$$

for $\mu_t = k_y (y_t^* - y_t^N)$ the cost-push shock that arises as a direct function of the cyclical wedge between the natural and the welfare maximizing level of output. and $x_t = y_t - y_t^*$ as defined in Appendix III.

Obviously, the divine coincidence obtains when $y_t^* = y_t^N$, which is the case for $\chi = \delta = 1$, as shown in Section 3.

Following Benigno and Woodford's (2005) linear-quadratic approach, I circumvent the usual time consistency issues associated with fully optimal monetary policies by assuming that the central bank is able to commit with full credibility to an optimal policy plan which specifies a full set of state-contingent sequences $\{x_t, \pi_{y,t}\}_{t=0}^{\infty}$ that minimize

$$\Upsilon \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \{ \lambda x_t^2 + \pi_{y,t}^2 \}$$

subject to the following sequence of constraints

$$\pi_{y,t} = \beta \mathbb{E}_t \pi_{y,t+1} + k_y x_t + \mu_t \quad (\text{A4.5})$$

and a constraint on the initial inflation rate

$$\pi_{y,t_0} = \bar{\pi}_{y,t_0}, \quad (\text{A4.6})$$

where $\bar{\pi}_{y,t_0}$ is defined as the inflation rate in time t_0 that is consistent with optimal policy in a "timeless perspective" or, in other words, the inflation rate that would have been chosen a long time ago and which is consistent with the optimal precommitment plan.

Solving this problem under the timeless perspective gives rise to the following set of first-order conditions

$$x_t = x_{t-1} - \frac{k_y}{\lambda} \pi_{y,t}, \quad (\text{A4.7})$$

which are supposed to hold for all $t = 0, 1, 2, 3, \dots$ and which characterize the central bank's optimal policy response.

Appendix V : Deriving an optimal simple rule

Following McCallum (1999b) and McCallum and Nelson (2004), the no bubble MSV solution to the system formed by equations (A4.4) and (A4.7) can be written as:

$$\pi_{y,t} = \alpha_{11} x_{t-1} + \alpha_{12} \mu_t \quad (\text{A5.1})$$

$$x_t = \alpha_{21} x_{t-1} + \alpha_{22} \mu_t, \quad (\text{A5.2})$$

where α_{ij} for $i, j = 1, 2$ are functions of β , k_y and λ .

Because the supply of oil is supposed perfectly elastic at a given exogenous real price, one can write the following definitions:

$$c_t - c_t^* = y_t - y_t^*$$

$$\pi_{c,t} = \pi_{y,t} + \frac{\widetilde{\omega}_{oc}}{1 - \widetilde{\omega}_{oc}} (po_t - po_{t-1})$$

that are used to rewrite the consumption Euler equation in deviation from efficient consumption as follows:

$$x_t = -\frac{1}{\sigma} \left(r_t - \mathbb{E}_t \pi_{y,t+1} - \frac{\widetilde{\omega}_{oc}}{1 - \widetilde{\omega}_{oc}} \mathbb{E}_t (po_{t+1} - po_t) - rr_t^* \right) + \mathbb{E}_t x_{t+1} \quad (\text{A5.3})$$

for $rr_t^* = \sigma \mathbb{E}_t \{ \Delta y_{t+1}^* \} = -\Psi \sigma (1 - \rho_o) po_t$.

Combining (A5.1), (A5.2), and (A5.3) leads to

$$\alpha_{21} x_{t-1} + \alpha_{22} \mu_t = -\frac{1}{\sigma} \left(\begin{array}{c} r_t - \alpha_{11} x_t - \alpha_{12} \rho_o \mu_t \\ -\frac{\widetilde{\omega}_{oc}}{1 - \widetilde{\omega}_{oc}} (\rho_o - 1) po_t - \Psi \sigma (1 - \rho_o) po_t \end{array} \right) + \alpha_{21} x_t + \alpha_{22} \rho_o \mu_t,$$

which can be solved for r_t :

$$\begin{aligned} r_t &= (\alpha_{11} + \sigma \alpha_{21}) x_t - \sigma \alpha_{21} x_{t-1} - (\sigma \alpha_{22} - \alpha_{12} \rho_o - \sigma \alpha_{22} \rho_o) \mu_t \\ &\quad + \underbrace{(\rho_o - 1) \left(\frac{\widetilde{\omega}_{oc}}{1 - \widetilde{\omega}_{oc}} - \Psi \sigma \right)}_{\Xi} po_t. \end{aligned}$$

From (A5.1), $\mu_t = \frac{\pi_{y,t} - \alpha_{11} x_{t-1}}{\alpha_{12}}$, so that the above equation can be rewritten as:

$$r_t = \frac{\Phi}{\alpha_{11}} \pi_{y,t} + (\alpha_{11} + \sigma \alpha_{21}) x_t - (\Phi + \sigma \alpha_{21}) x_{t-1} + \Xi po_t,$$

for $\Phi = (\rho_o - \sigma \alpha_{22} \alpha_{12}^{-1} (1 - \rho_o)) \alpha_{11}$.

Finally, substituting $y_t^* = -\Psi po_t$ in the above, I get:

$$\begin{aligned} r_t &= \frac{\Phi}{\alpha_{11}} \pi_{y,t} + (\alpha_{11} + \sigma \alpha_{21}) (y_t + \Psi po_t) \\ &\quad - (\Phi + \sigma \alpha_{21}) (y_{t-1} + \Psi po_{t-1}) + \Xi po_t \end{aligned}$$

so

$$r_t = \Phi \alpha_{11}^{-1} \pi_{y,t} + \Omega y_t - \Gamma y_{t-1} + (\Xi + \Psi \Omega) po_t - \Psi \Gamma po_{t-1}.$$

Appendix VI : Time varying elasticities

To allow for time-varying elasticities of substitution, I transform the production processes of Section 2 by introducing a convex adjustment cost of changing the input mix in production, as in Bodenstein et al. (2007) and redefine equations (A1.12) and (A1.3), such that

$$Y_t = \left((1 - \omega_{oy}) H_t^{\frac{\delta-1}{\delta}} + \omega_{oy} [\varphi_{OY,t} O_{Y,t}]^{\frac{\delta-1}{\delta}} \right)^{\frac{\delta}{\delta-1}}, \quad (\text{A6.1})$$

and

$$C_t = \left((1 - \omega_{oc}) C_{Y,t}^{\frac{\chi-1}{\chi}} + \omega_{oc} [\varphi_{OC,t} O_{C,t}]^{\frac{\chi-1}{\chi}} \right)^{\frac{\chi}{\chi-1}}. \quad (\text{A6.2})$$

The variables $\varphi_{OY,t}$ and $\varphi_{OC,t}$ represent the costs of changing the oil intensity in the production of the core good and the consumption basket, and are supposed to take the following quadratic form

$$\varphi_{OY,t} = \left[1 - \frac{\varphi_{OY}}{2} \left(\frac{O_{Y,t}/H_t}{O_{Y,t-1}/H_{t-1}} - 1 \right)^2 \right] \quad (\text{A6.3})$$

$$\varphi_{OC,t} = \left[1 - \frac{\varphi_{OC}}{2} \left(\frac{O_{C,t}/C_{Y,t}}{O_{C,t-1}/C_{Y,t-1}} - 1 \right)^2 \right]. \quad (\text{A6.4})$$

This specification allows for oil demand to respond quickly to changes in output and consumption, while responding slowly to relative price changes. In the long-run, the elasticity of substitution is determined by the value of δ and χ . Although somewhat ad hoc, this form of adjustment costs introduces a time-varying elasticity of substitution for oil, an important characteristic of putty-clay models such as in Atkeson and Kehoe (1999) or Gilchrist and Williams (2005).³⁸ The presence of adjustment costs transforms the static cost-minimization problem of the representative intermediate firms and final consumption goods distributors into forward-looking dynamic ones. They can be regarded as choosing contingency plans for $O_{Y,t}$, H_t , $O_{C,t}$, and $C_{Y,t}$ that minimize their discounted expected cost of producing Y_t and C_t subject to the constraints represented by equations (A6.1) to (A6.4).

I calibrate φ_{OY} and φ_{OC} such that the instantaneous price elasticities of demand for oil correspond to the baseline calibration chosen in the CES setting of the previous sections. The *de facto* short-term elasticities are then set to 0.3. In the long run, I assume a unitary elasticity of substitution ($\delta = \chi = 1$) such that (A6.1) and (A6.2) are *de facto* Cobb-Douglas functions when $t \rightarrow \infty$.

³⁸In putty-clay models of energy use, a large variety of types of capital goods are combined with energy in different fixed proportions, making the short-term elasticity of substitution low. In the longer run, the elasticity goes up as firms invest in capital goods with different fixed energy intensities.

Tables

Table 1: Optimized simple rule (OR) and speed limit rules (SLR)

Simple rule	g_π	g_y	g_{y1}	g_{po}	g_{po1}	g_{w1}
OR	5.123	4.742	-4.731	0.007	-0.014	-
SLR	5.101	4.742	-4.742	0.008	-0.008	-
OR_w ($\eta = 0.9$)	5.134	8.708	-7.884	0.276	-0.240	0.088
SLR_w ($\eta = 0.9$)	2.054	3.404	-3.404	0.096	-0.096	-

note: all coefficients are consistent with annualized interest rates and inflation

Table 2: Welfare costs under alternative policies (percent of annual consumption)

Policy		Total Loss	y Loss	π_y Loss
optimal		0.6	0.4	0.2
core	Strict inflation target	1.8	1.8	0
	CTR	1.9	0.2	1.7
	CTR inertia ($\rho = 0.8$)	1.7	0.2	1.5
	Forecasting CTR	4.4	0.2	4.2
headline	HTR	2.7	0.4	2.3
	HTR inertia ($\rho = 0.8$)	1.7	0.3	1.4
	Forecasting HTR	9.4	0.1	9.3

Figures

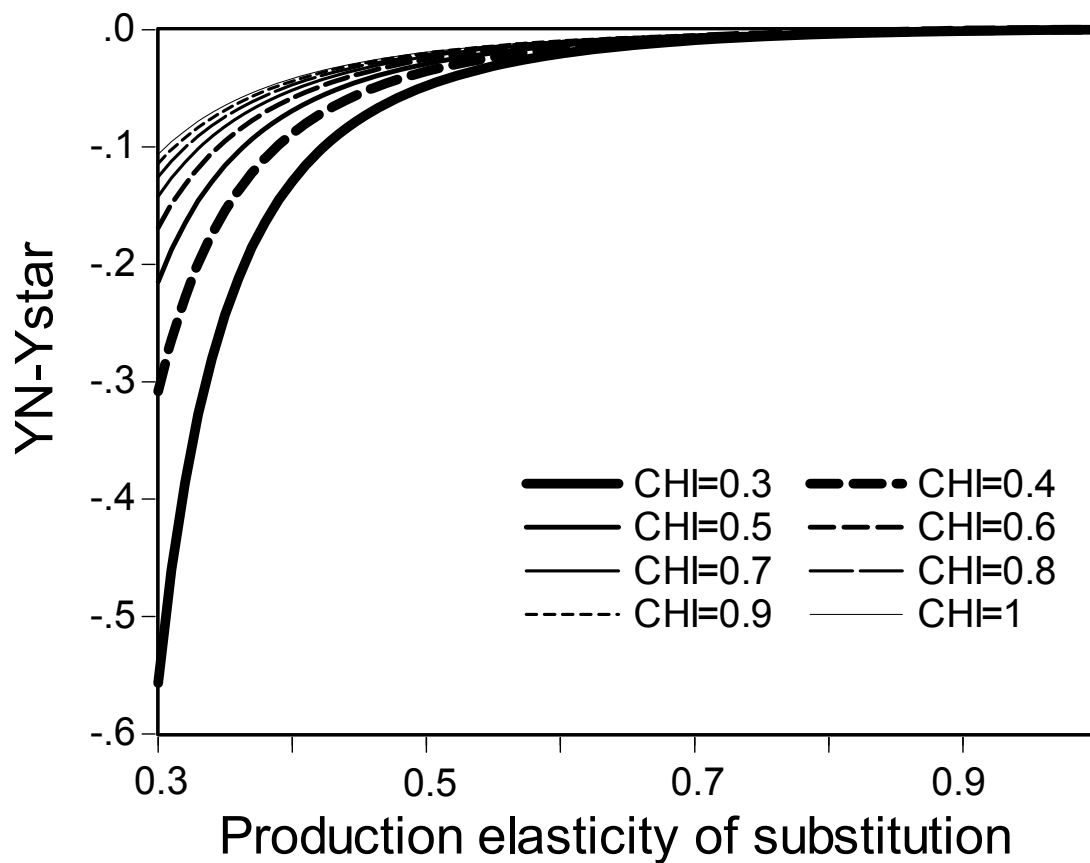


Figure 1: (Y_N) and efficient (Y_{star}) output to a 1-percent increase in oil price as a function of the production and the consumption elasticity of substitution (χ); baseline calibration with 2-percent oil share of output and 6-percent energy component of consumption.

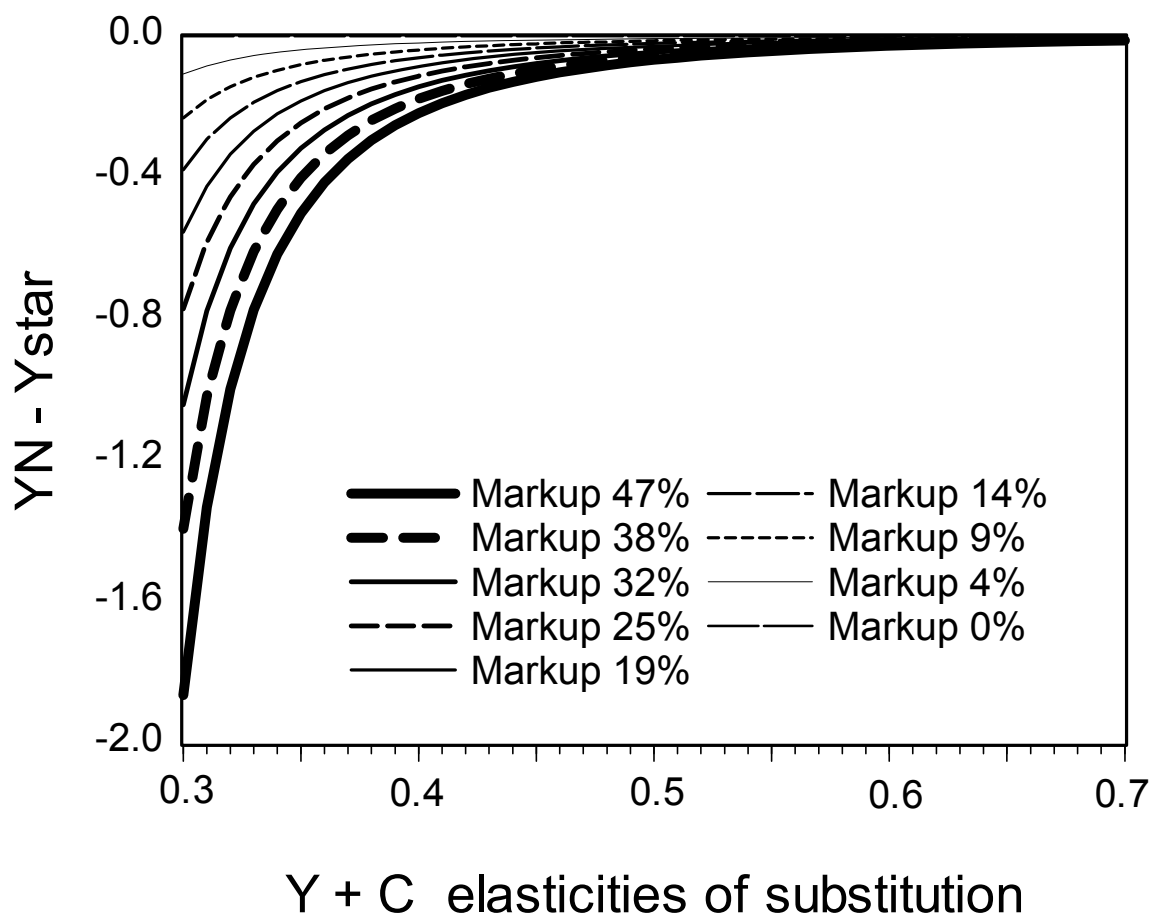


Figure 2: Response of the gap between natural (Y_N) and efficient (Y_{star}) output to a 1-percent increase in oil price as a function of the degree of monopolistic competition; baseline calibration with 2-percent oil share of output and 6-percent energy component of consumption.

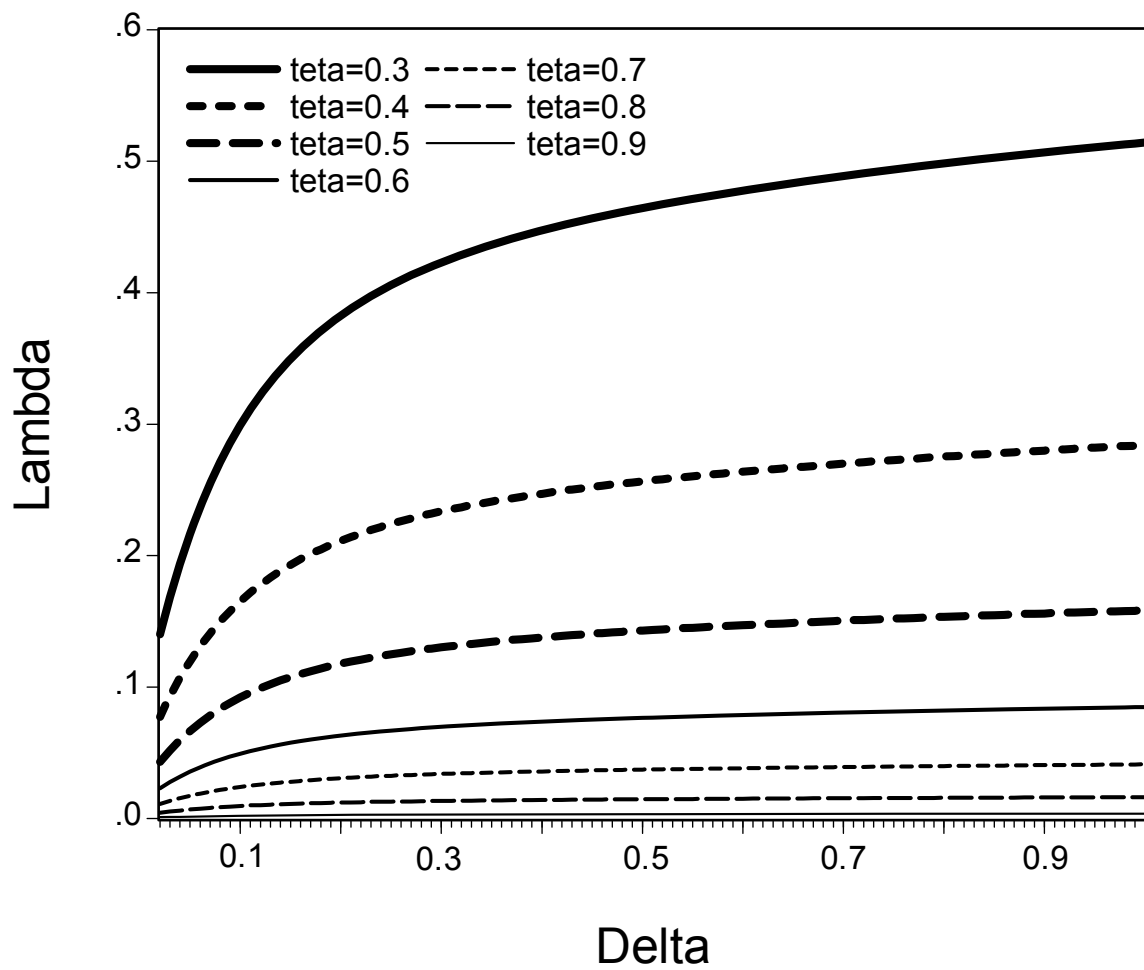


Figure 3: Change in the weight (Lambda) assigned to output gap stabilization as a function of the elasticities of substitution (Delta) and the degree of price stickiness (teta)

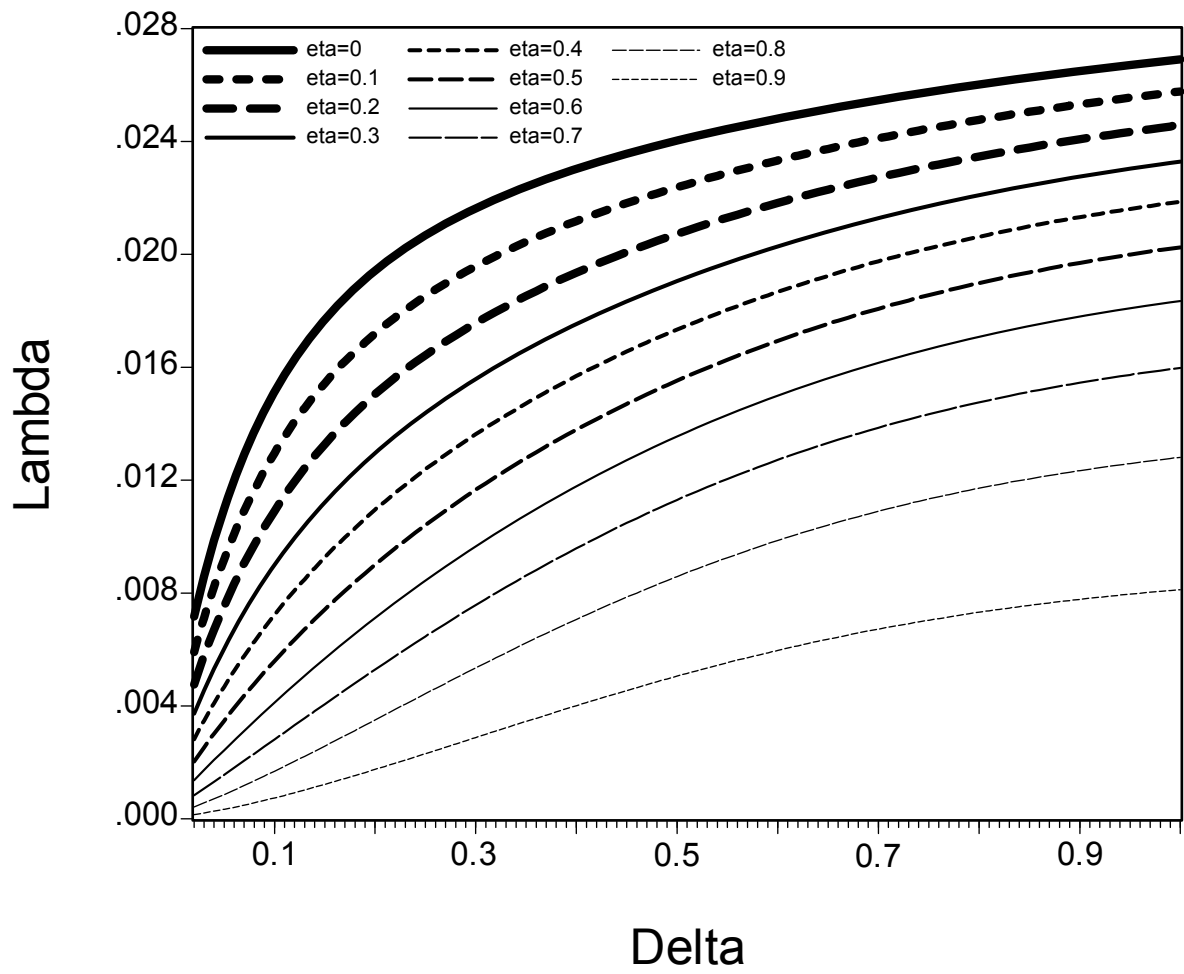


Figure 4: Change in the weight (Lambda) assigned to output gap stabilization as a function of the elasticities of substitution (Delta) and the degree of real wage rigidity (eta)

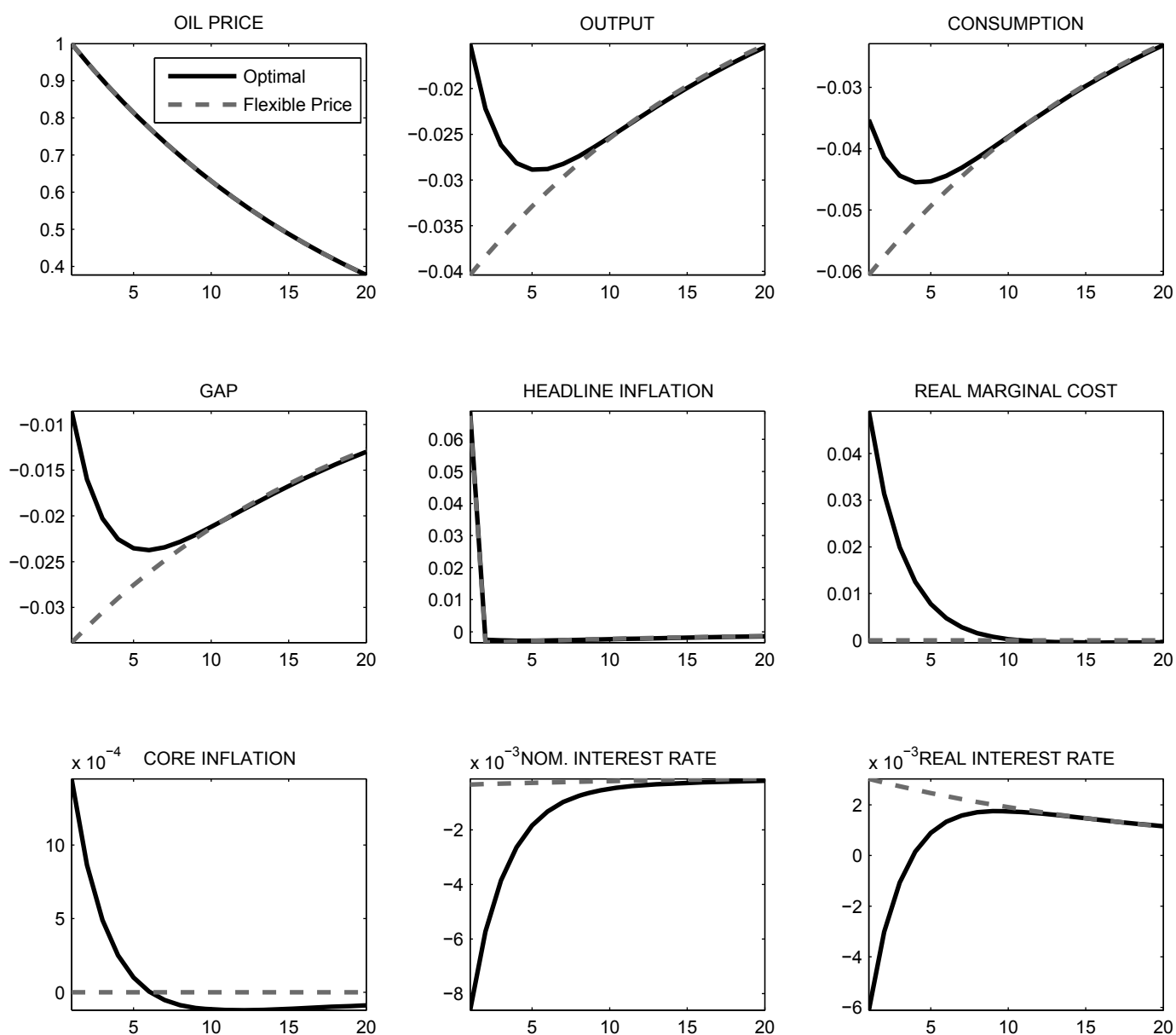


Figure 5: Impulse response functions to a 1-percent increase in oil price; comparison of optimal precommitment monetary policy with flexible price equilibrium; CES technology with low elasticity ($\psi = \chi = 0.3$); baseline calibration.

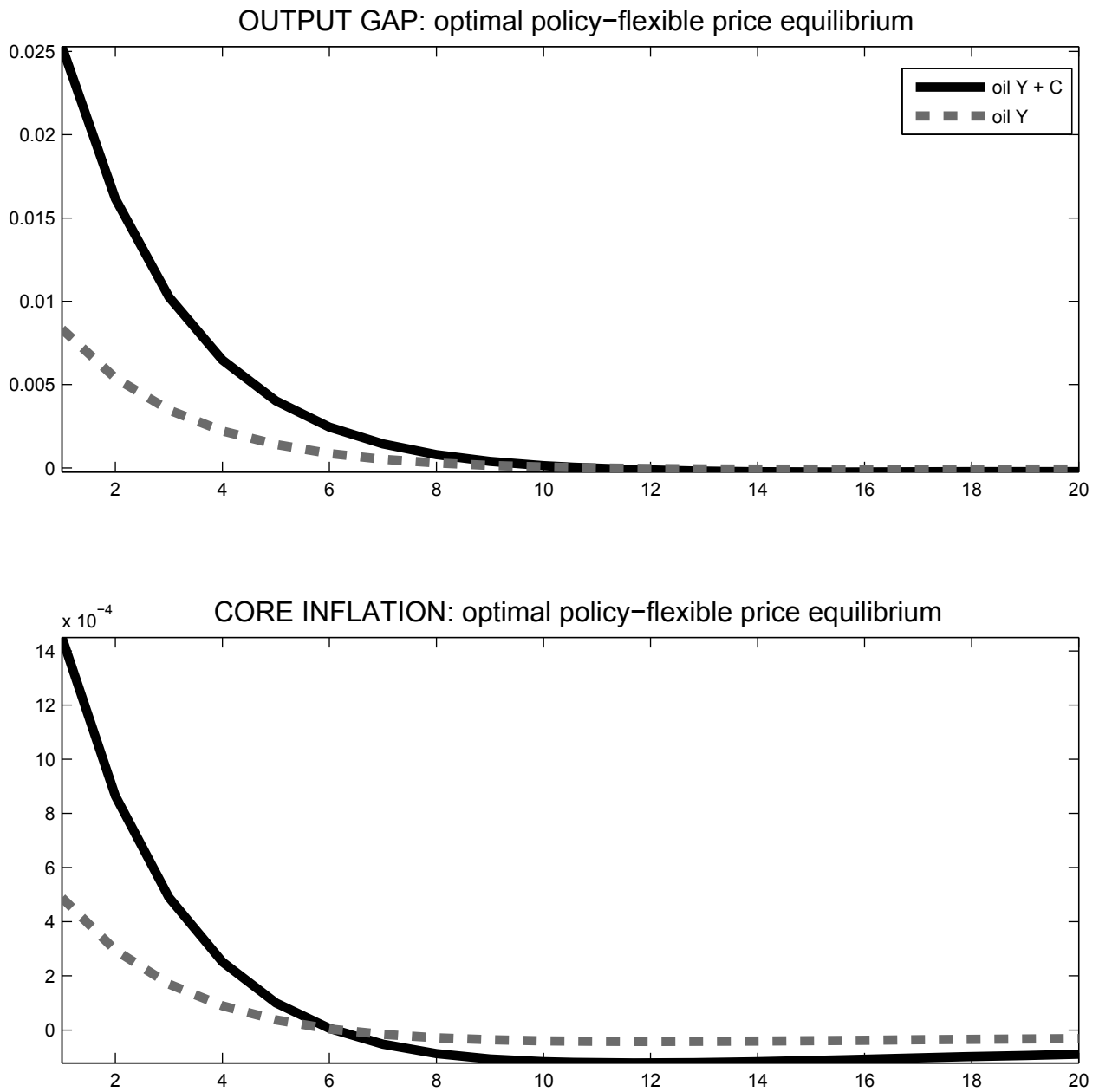


Figure 6: Tradeoff magnification effect; difference between optimal policy and FPWE when oil is an input to production only (dashed line) and when oil is an input to both production and consumption (solid line); baseline calibration.

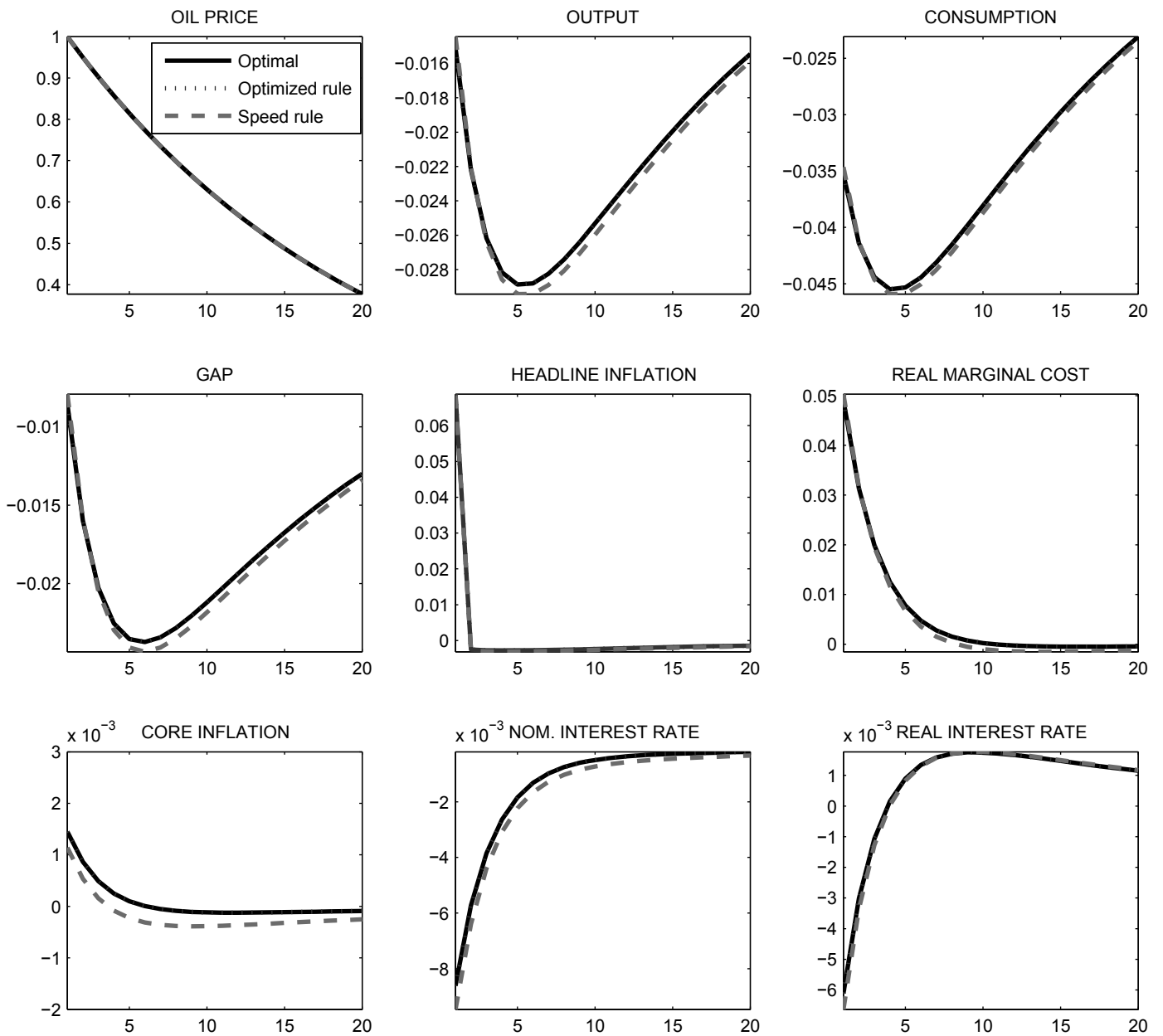


Figure 7: Impulse response functions to a 1-percent increase in oil price; comparison of optimal precommitment monetary policy with optimized simple rule and speed limit policy; CES technology with low elasticity ($\psi = \chi = 0.3$); baseline calibration.

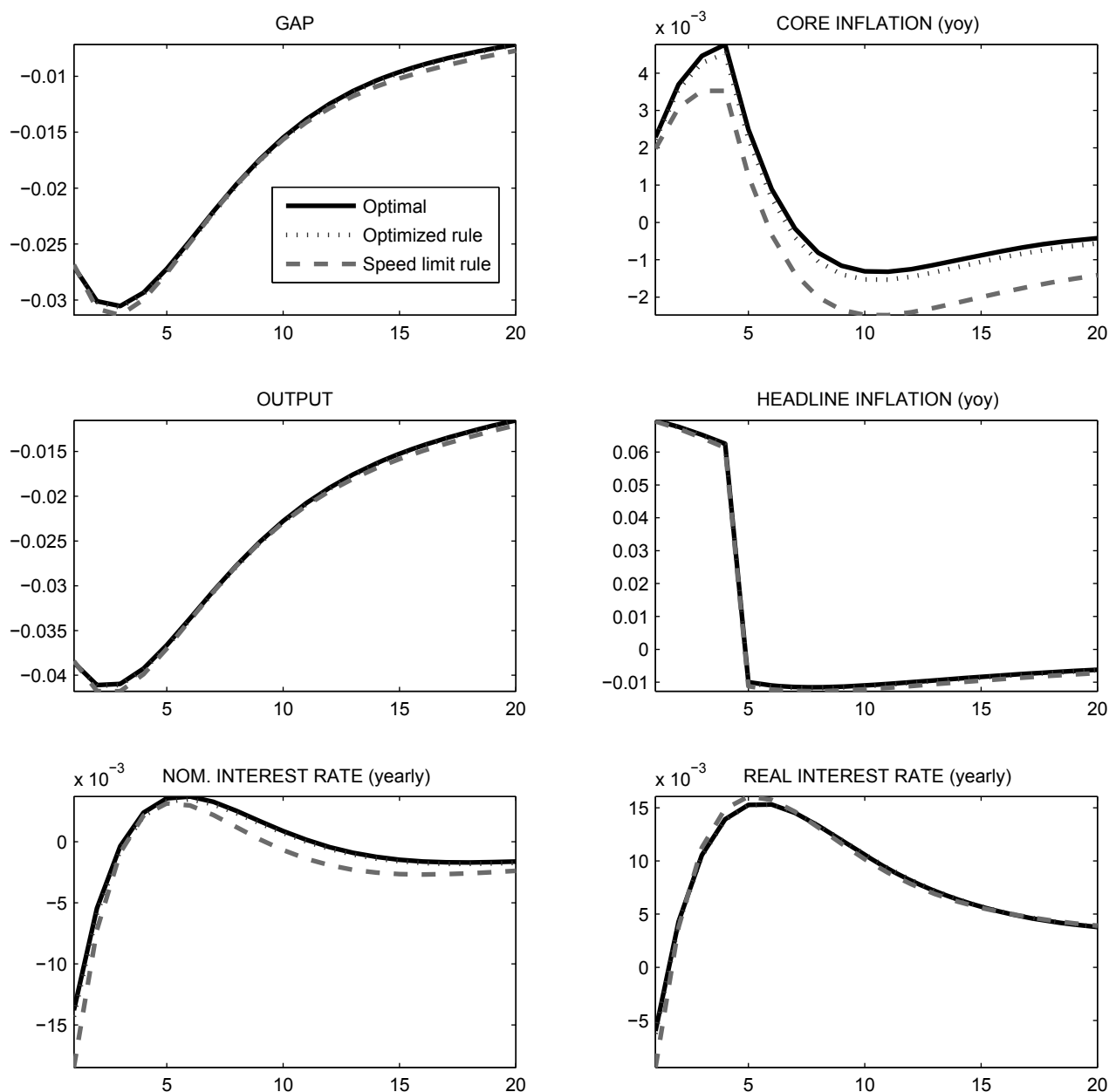


Figure 8: Impulse response functions to a 1-percent increase in oil price; comparison of optimal precommitment monetary policy with optimized simple rule and speed limit policy based on four quarters moving average of core inflation; CES technology with low elasticity ($\psi = \chi = 0.3$); real wage rigidity ($\eta = 0.9$).

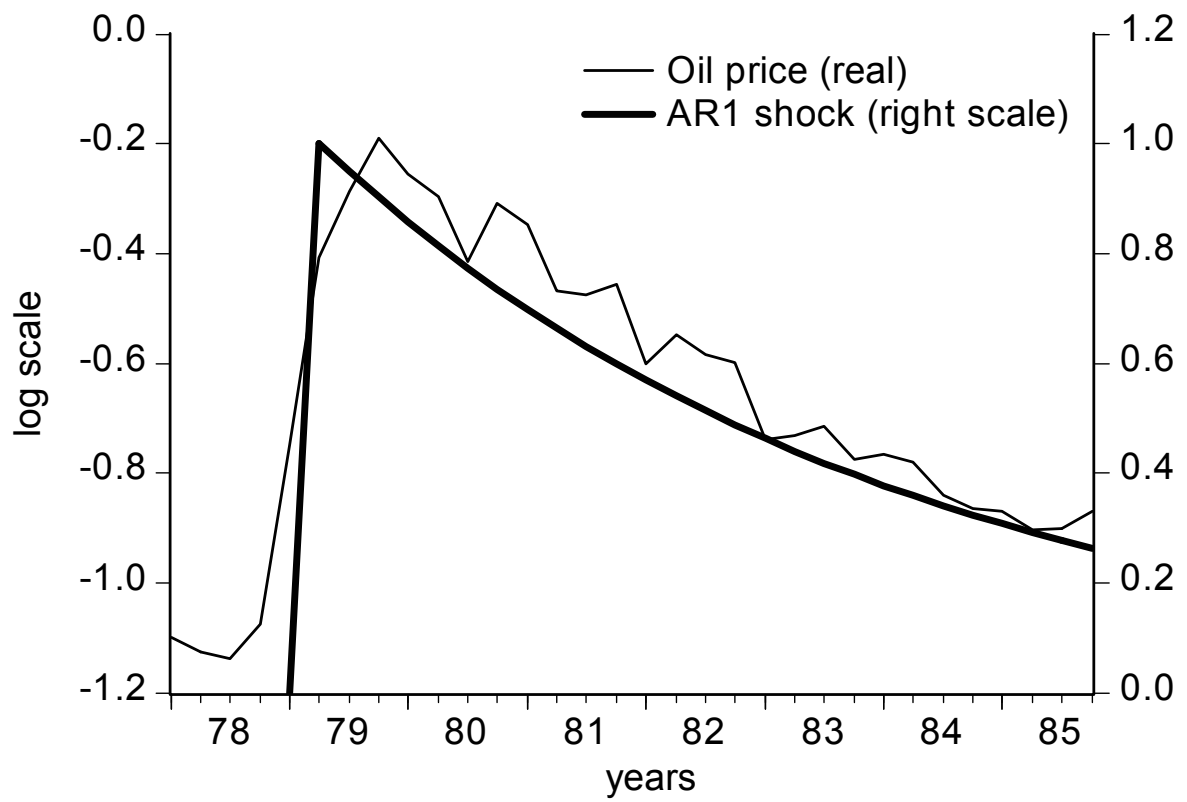


Figure 9: 1979 oil price shock and comparable AR(1) exogenous process for the real price of oil; log scales

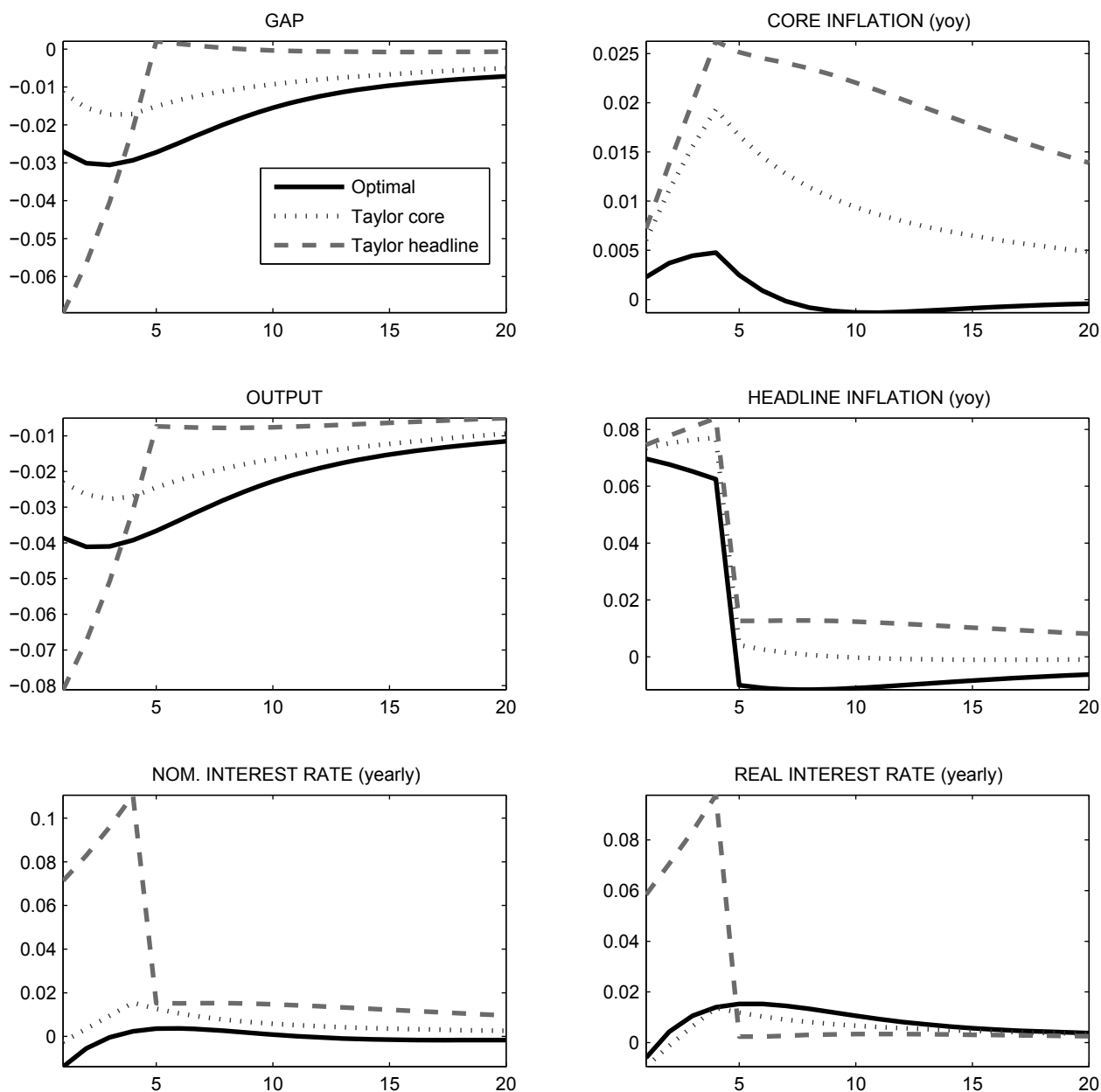


Figure 10: Impulse response functions to a 1979-like 100% log-increase in oil price; comparison of optimal precommitment monetary policy with simple Taylor rules based on four quarters moving average values of core or headline inflation; baseline calibration; real wage stickiness ($\eta = 0.9$).

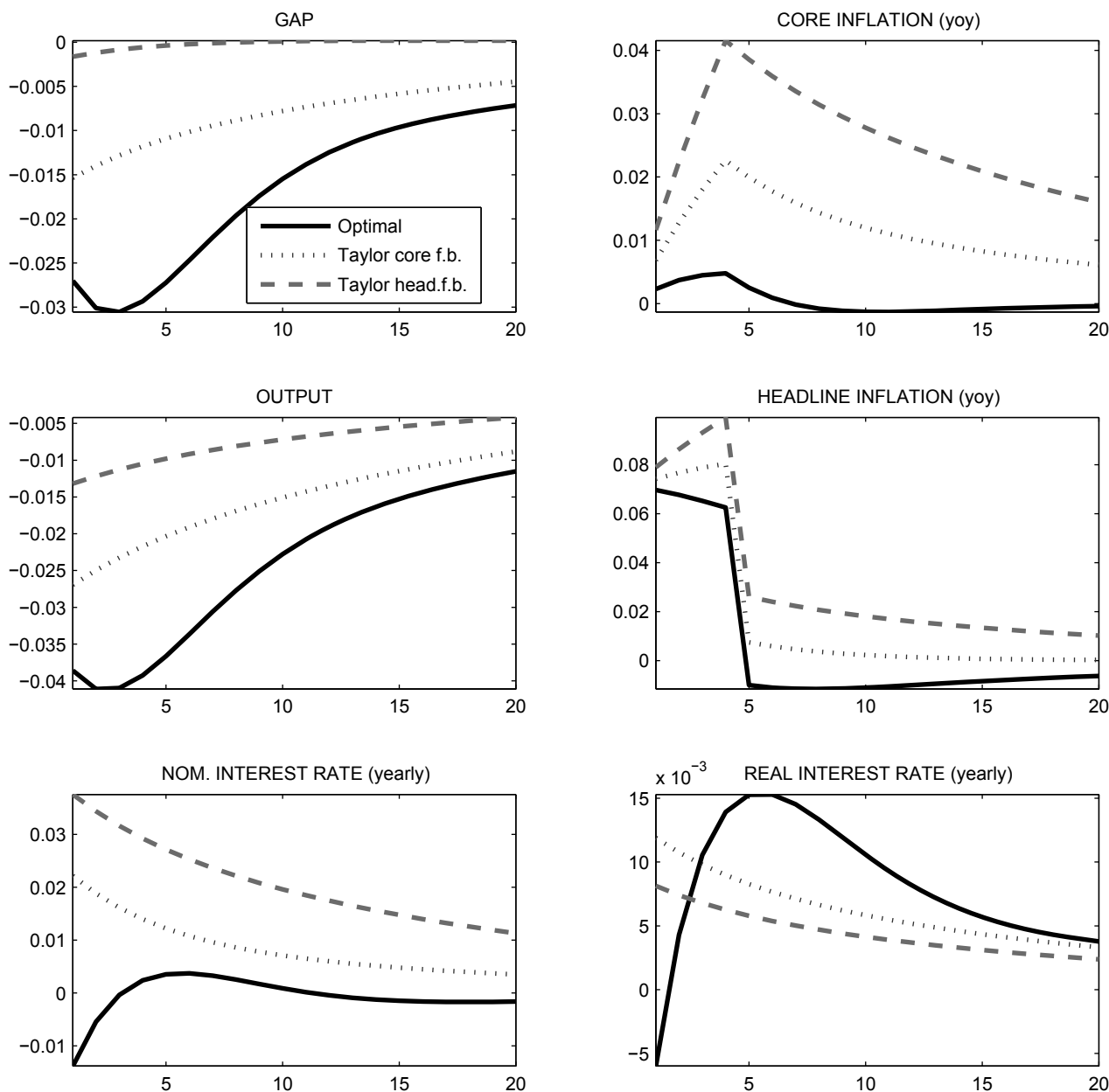


Figure 11: Impulse response functions to a 1979-like 100% log-increase in oil price; comparison of optimal precommitment monetary policy with simple forecast based Taylor rules; baseline calibration; real wage stickiness ($\eta = 0.9$).

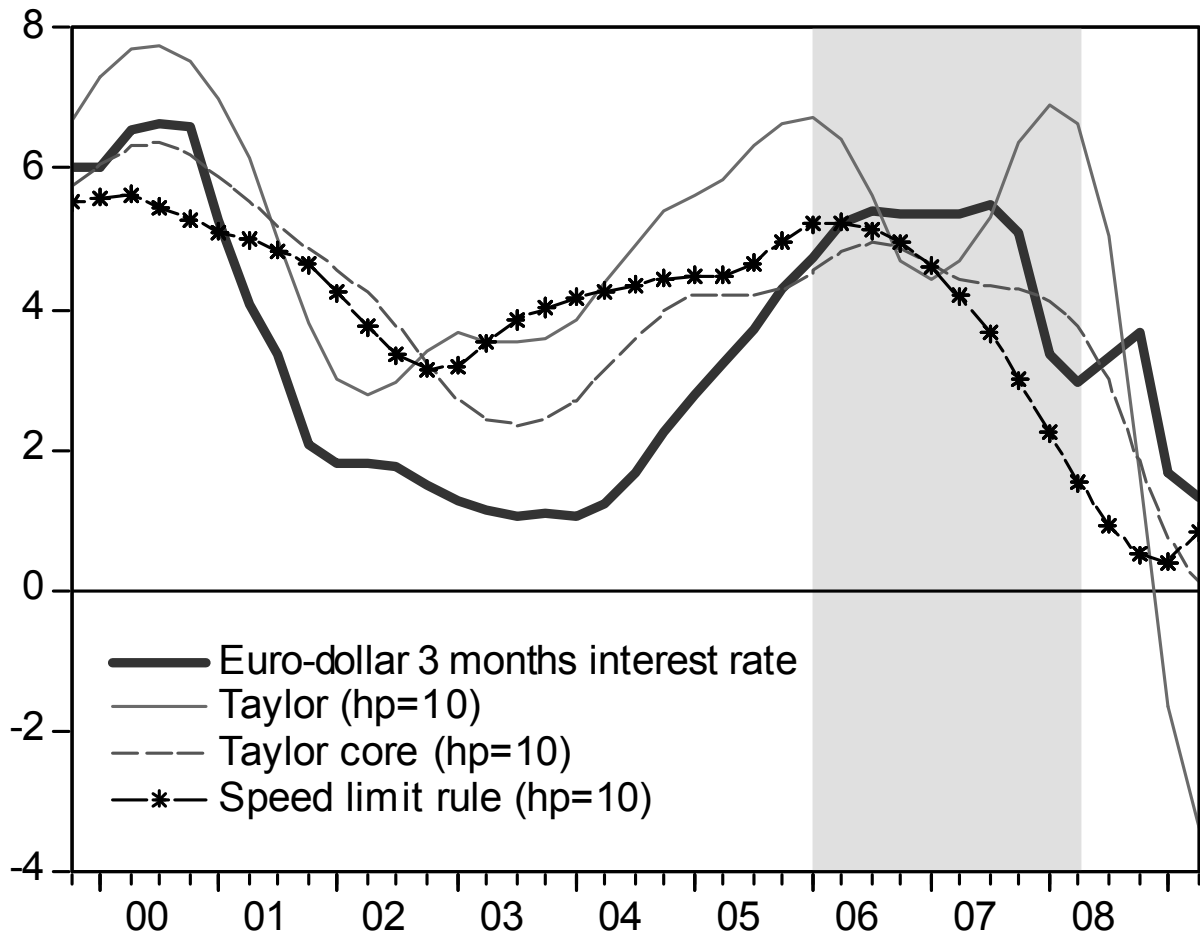


Figure 12: Comparing simple monetary policy rules in the US during the 2006-2008 oil price rallye (shaded area); all rules hp-filtered ($\lambda=10$).

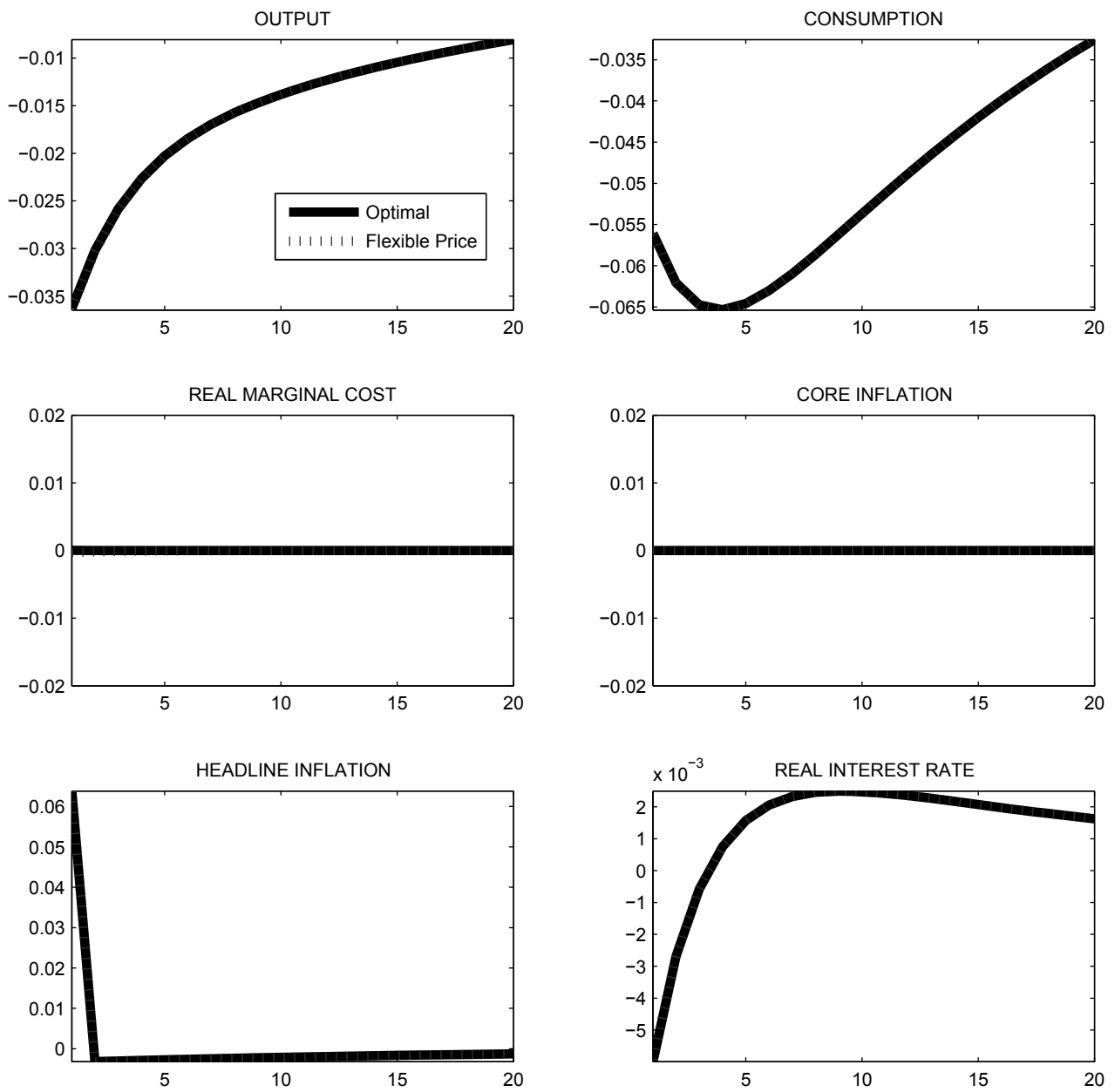


Figure 13: Impulse response functions to a 1-percent log-increase in oil price; time-varying elasticities; comparison of optimal precommitment monetary policy with flexible price equilibrium; undistorted equilibrium.

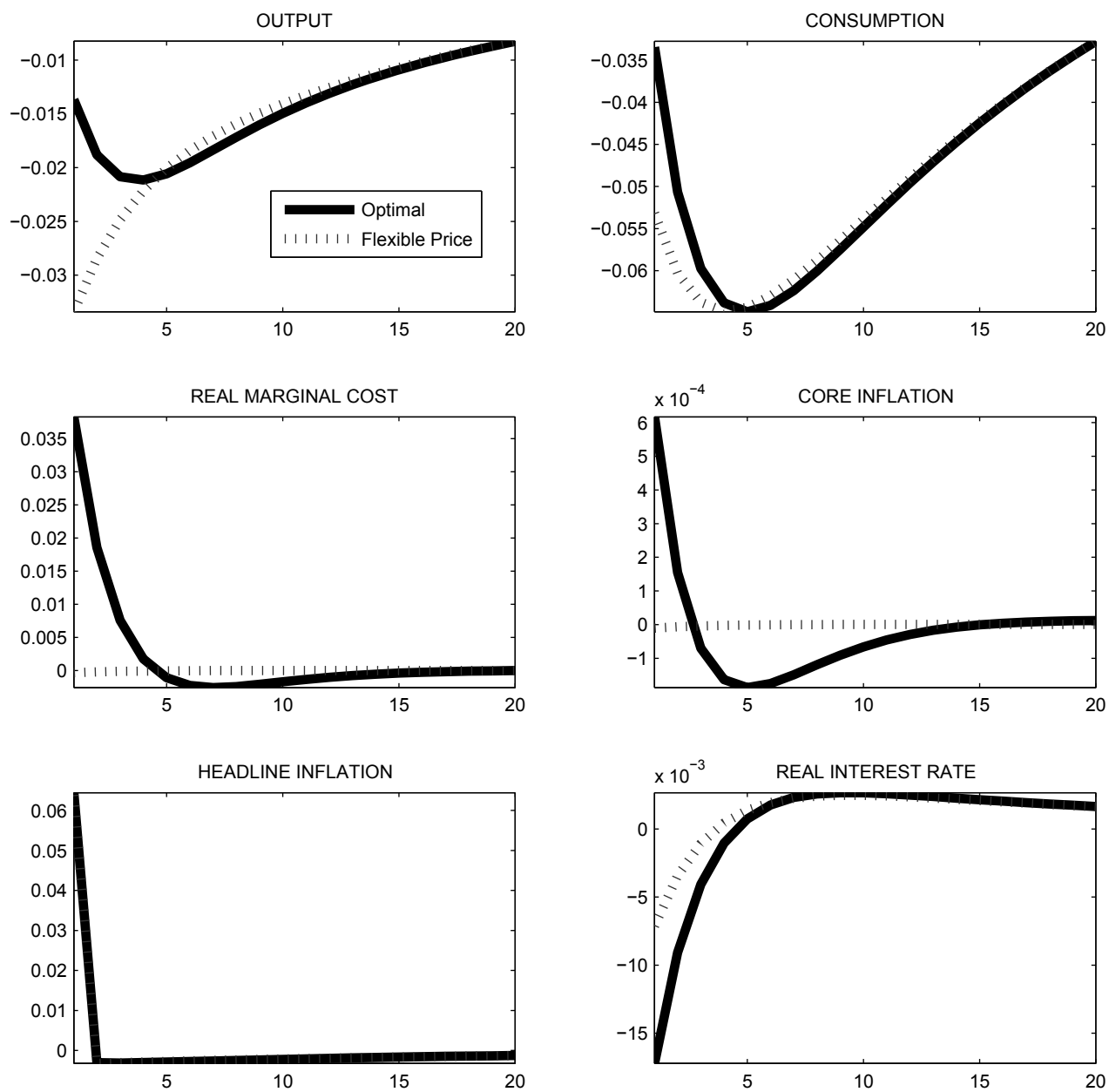


Figure 14: Impulse response functions to a 1-percent log-increase in oil price; Time varying elasticities; comparison of optimal precommitment monetary policy with flexible price equilibrium; distorted equilibrium (markup 20%).

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