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The Collateral Costs of Clearing*

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Abstract

In this working paper, we study the three generic clearing arrangements in the presence of two-sided limited commitment: simple bilateral clearing, segregated collateral clearing through a third party, and – most sophisticated of all – central counterparty (CCP) clearing. Clearing secures the settlement of obligations from over-the-counter (OTC) forward contracts that smooth the income of risk-averse agents. Clearing requires collateral to guarantee settlement; this is costly, as it reduces income from investment. While welfare is greater under more sophisticated clearing arrangements, we find that these are also more demanding in terms of collateral.

Keywords: clearing, central counterparty, segregation, novation, mutualization

JEL Classification: G13, G14, G18, G2, G28, D53, D82

1. Introduction

The most recent financial regulatory overhaul in US and Europe targeted specifically over-the-counter trades by requiring that they be cleared centrally.¹ However, little or no work

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¹Regulators have long recognized that a sound financial system requires safe and efficient post trade infrastructures capable of sustaining large shocks. With the G20 commitment to centrally clear over-the-

carefully assessed the impact of such requirement on OTC markets. In this paper, we compare the collateral and welfare implications of different post-trade arrangements for OTC trades. Post-trade arrangements include the clearing and the settlement of obligations. Clearing serves to process and secure orders to transfer money and/or securities, prior to settlement.² Settlement refers to the actual discharge of obligations, i.e. the “physical” exchange itself. We analyze three generic clearing arrangements that serve to secure the settlement of trades in the presence of two-sided limited commitment; e.g. through specifying collateral requirements.

Clearing is important because most trades are usually settled after the trade was agreed upon. The lag between trade and settlement, i.e. the clearing phase, can vary from just a couple of seconds to several years. For example, in daily life, there is no meaningful distinction between clearing and settlement in spot trades, as both happen quasi-simultaneously. In financial markets, “spot” trades of bonds, say, are understood to settle within a few days after the trade. So, the clearing phase is just a few days long. However, when it comes to repos, futures or forward markets, these contracts involve promises that can span weeks to years between the agreement and the final settlement of obligations. For these financial contracts, it is crucial to have effective clearing arrangements to secure that trade obligations will be honored. Otherwise, trading would become meaningless: you can make empty promises if you know you can renege.

We analyze an environment where collateral is costly and the terms of the OTC contracts are endogenous so that they react to a change in the clearing arrangement. To model OTC trades, we consider risk-averse agents facing risk on the return to their investment (and so their consumption). Agents can insure against this risk by trading contingent contracts. Since there is no centralized market, agents have to negotiate the terms of these (forward) contracts bilaterally. With perfect commitment, forward contracts allow agents to insure themselves perfectly and at no cost. However, if agents cannot commit, they can default strategically on their future obligations when the uncertainty resolves to their advantage. For instance, consider an interest rate swap (IRS) where each counterparty agrees to pay either a fixed or floating rate to the other counterparty. At the point of initiation of the swap it is priced so that it has a net present value of zero but provides insurance to both

counter (OTC) derivatives in 2009, more trades are now required to go through such infrastructures. Notable new pieces of legislation are the Dodd-Frank Act in the US and EMIR in Europe which both require relatively standard OTC derivatives trades to be cleared via a central counterparty (CCP).

²Our definition of clearing differs from the Committee on Payment and Settlement Systems’s (CPSS) definition insofar as ours includes the securing of the settlement of the trade.

counterparties against future interest rate movements. Consequently, an interest rate change will ex post make one counterparty better and the other worse off, the latter counterparty being tempted to break the contract. Thus, to control for endogenous default, agents have no choice but to require collateral as part of the contract. Here, collateral will be part of an agent's endowment. Hence, collateral requirements are costly as they reduce investment with a corresponding loss of return.

We consider three generic clearing arrangements: (1) simple bilateral clearing, where each agent has to secure the trade by giving collateral to his counterparty, (2) segregated collateral clearing, where each agent has to secure the trade by placing collateral in a segregated account managed by a third party³, and (3) centralized clearing with segregation combined with a loss sharing rule, akin to a CCP.

The benefit of each clearing arrangement is intuitive: simple bilateral clearing allows futures trade to take place. In the absence of commitment and without any collateral requirement, only spot trade could happen. Still, the gains from futures trade are limited as defaulting agents have their hands on the collateral pledged by their counterparty. Segregated clearing improves upon bilateral clearing by eliminating this possibility. Since collateral is stored in a safe place, agents can no longer abscond with the collateral of their counterparty. In addition to segregation, a CCP offers services that are often referred to as novation and mutualization. Novation allows a CCP to enter a forward contract as the counterparty to the original seller and the original buyer. Due to novation, however, the CCP faces the risk of default. The CCP manages default risk by requiring collateral (or margins) and by introducing mutualization of its losses through a default fund. Mutualization enables a CCP to run a loss-sharing arrangement to share potential losses of an agent's default among surviving agents.

We show that all clearing arrangements trade-off insurance with the cost of pledging collateral. In our model, debt is secured by collateral and for a given level of debt, a lower collateral level increases the chance that a counterparty reneges. This counterparty risk implies that a lower level of collateral necessarily means a lower level of debt. This, in turn, limits insurance. Therefore, limited commitment, by inducing counterparty risk, implies that the optimal clearing arrangements never fully insure agents. Still, there are welfare gains (in a Pareto sense) from investing into more sophisticated clearing arrangements, if they are designed optimally.

Segregated collateral clearing is always welfare improving in comparison to bilateral clear-

³This is similar to tri-party agents used in some repo markets.

ing as it relaxes incentives constraints. However, the collateral requirements could be higher (allowing for more income smoothing) or lower (allowing for more investment) than under bilateral clearing. Also, CCP clearing always improves on segregation through a loss sharing rule (mutualization) although it always requires more collateral. Interestingly, relative to bilateral clearing, segregation may account for most of the welfare gains of a CCP. For mutualization to become more beneficial, the possibility for diversification of risk, price volatility, return and risk aversion must be at high levels. We further analyze the optimal degree of mutualization of a CCP relative to the collateral pledged by traders (the waterfall ratio). In line with current risk management practices, we find that the waterfall ratio is decreasing with price volatility.

The next section provides the basic model setup for our economy as well as the full commitment benchmark. Section 3 analyzes bilateral clearing. Segregated collateral clearing is investigated in Section 4. Section 5 is devoted to CCP clearing. Section 6 reviews this paper's findings in the light of the existing literature. Section 7 concludes.

2. The Economy

2.1. Model

We consider an economy that lasts for three periods, $t = 0, 1, 2$. There is a continuum of two types of agents that we denote by 1 and 2. There are three goods, X , Y and Z . Agents are endowed with one perfectly divisible seed. Agents 1 are endowed with a seed of good X while agents 2 are endowed with a seed of good Y .

Both types of agents have access to two technologies that transform seeds invested at $t = 1$ into goods at $t = 2$. First, agents can use a short-term technology that instantaneously gives one unit of good X (resp. Y) for each unit of seed X (resp. Y) invested. Second, agents can use a long-term technology that returns $R > 1$ units of good X (resp. Y) for each unit of seed X (resp. Y) invested. However, agent 1 (2) disposes only of the long-term technology for good X (Y). That means the other agents' seeds have to be stored.

Agents 1 and 2 consume only good Z . They both have the same preferences represented by utility function $u(\cdot)$ which is increasing, concave, satisfies the usual conditions $u'(0) = \infty$ and $u'(\infty) = 0$ and has a constant coefficient of relative risk aversion α .

Agents meet pairwise at $t = 1$. An agent 1 always meets an agent 2. Pairs of agents can be in two possible environments at $t = 1$. A measure σ of pairs of agents 1 and 2 enters a safe

environment, while a measure $1 - \sigma$ of pairs of agents 1 and 2 enters a risky environment. In the safe environment one unit of good X can buy $1/2$ units of good Z at $t = 2$ and one unit of good Y can buy $1/2$ units of good Z at $t = 2$. Therefore, there is no risk involved for agents in the safe environment and, as a consequence, there is no scope for writing an insurance contract at $t = 1$.

When the environment is risky, the price of good X or Y in terms of good Z at $t = 2$ is stochastic. There are two states, h and ℓ . In state h , good X can buy a lot of good Z while good Y cannot. Symmetrically, in state ℓ , good X cannot buy a lot of good Z while good Y can. Formally, in state h one unit of good X can buy $p \leq 1$ units of good Z , while one unit of good Y can buy $1 - p$ units of good Z . In state ℓ , it is the reverse: one unit of good X can buy $1 - p$ units of good Z , while one unit of good Y can buy p units of good Z . We assume that $1 \geq p \geq 1 - p$. This implies that there is scope for some insurance for agents 1 and 2 in the risky state at $t = 1$. We also assume that the return of the long-term technology is low enough that investing some seeds in storage may be good, or

$$p > (1 - p)R. \tag{1}$$

For an agent 2 (and similarly for an agent 1) in state h , p is the expected return of investing one unit of good X in storage while $(1 - p)R$ is the expected return of investing one unit of good Y in the long-term technology. If agent 2 could foresee that the state is h , he would invest nothing in the long-term technology and everything in storage. Similarly, if he could foresee state ℓ , he would invest everything in the long-term technology and nothing in storage. We take p as given, which is similar to considering a small open economy.

Let us discuss some of our assumptions. First, we assume that agents 1 and 2 can be in a safe environment or a risky environment at date $t = 1$. This is to get to the idea that agents may end up trading more or less risky contracts. In the safe environment at $t = 1$ agents will only trade safe contracts, i.e. contracts that do not involve any price uncertainty, and so no counterparty risk. In the risky environment agents know that their contracts involve price risk that they want to insure. Insurance comes in the form of forward contracts traded at $t = 1$. However, trading forward contracts generates counterparty risk as a counterparty to the forward contract could choose to renege on its promises if the terms of the contract are not in his favor given the realized price at $t = 2$.

Second, we assume that agents invest at date $t = 1$ rather than at date $t = 0$. We made this choice in order to obtain the notions of “default fund” and “margin” when we

analyze CCP clearing: generally, CCPs define initial margin contributions as a function of price fluctuations in some predetermined period of the past. So, we assume that the risky environment is one where agents know that price fluctuations are much higher than in the safe environment where there is no price uncertainty at all. Also, when analyzing CCP clearing, agents will decide whether to join the CCP at date $t = 0$. This allows us to introduce the “default fund” as contributions that are collected before risk realizes. Naturally, we will not consider date $t = 0$ when we analyze bilateral and segregated collateral clearing as there are only gains from trade in the risky environment.

Third, it might appear restrictive to set the price in the safe environment to $1/2$ and prices in the state h and ℓ to $p \geq 1/2$ and $1 - p$ respectively. In the Appendix, we show that this is a normalization.

Finally, one may doubt that our notion of price risk is what financial markets care about. In the Appendix we sketch a way to introduce price volatility and uncertainty over the extent of price volatility. All conclusions of the paper go through in such an environment.

2.2. Benchmarks

In this subsection, we first consider the maximum payoff that agents can obtain in autarky, i.e. when they cannot trade. We then turn to an economy where agents can engage in spot trades at $t = 1$ or $t = 2$. Finally, we consider an economy with full commitment where agents can trade contingent contracts at $t = 1$ and spot trades at $t = 1$ or $t = 2$.

Autarky An agent in the safe environment invests all his resources in the long term technology. If the agent is in a risky environment at $t = 1$, he chooses his short-term investment s and his long-term investment $1 - s$ and his consumption of good z to solve

$$\begin{aligned} \max_{z, s \in [0, 1]} & 0.5u(z_\ell) + 0.5u(z_h) \\ \text{s.t.} & z_i \leq p_i R(1 - s) + p_i s \end{aligned}$$

where $p_\ell = 1 - p$ and $p_h = p$. The budget constraints read as follows: the agent invests $1 - s$ in the long-term technology, with a return $R(1 - s)$ of good $g = X$ or Y , and s in the short-term technology (savings or storage) and sells the return at price $p_i \in \{p, 1 - p\}$ for good Z depending on the price at $t = 2$. Clearly, the agent maximizes his payoff setting $s = 0$ since storage is dominated in rate of return. Hence, the expected payoff of any agent

at $t = 1$ is

$$V(p) = 0.5u(pR) + 0.5((1 - p)R),$$

and at $t = 0$, the expected payoff is

$$\sigma u(R/2) + (1 - \sigma)V(p).$$

Spot trades Consider agents can only trade in a spot market at $t = 1$ or $t = 2$. At $t = 2$ there are no gains from trading goods X and Y . However, at $t = 1$ agents may be willing to purchase some of the seeds they do not own. In the safe environment there are no gains from trade and an agent invests all his resources in the long term technology. In a risky environment at $t = 1$, he chooses his investment and his consumption of good z to maximize his expected utility. Clearly, he invests his seeds in the long-term technology and the other type of seeds in the short-term technology. We assume agents trade in a Walrasian market and we denote by q the price of a unit of seed X for a seed of good Y . Therefore, the budget constraint of agents 1 is

$$q = qx_1 + y_1$$

where x_1 denotes their demand of seed X and y_1 their demand of seed Y . The problem of agents 1 when they trade in a Walrasian market at $t = 1$ is

$$\begin{aligned} \max_{z_i^1, x_1, y_1 \in [0,1]} & 0.5u(z_l^1) + 0.5u(z_h^1) \\ \text{s.t.} & q = qx_1 + y_1 \\ & z_i^1 \leq p_i R x_1 + (1 - p_i) y_1 \end{aligned}$$

All the constraints bind and the first order condition for an interior solution is

$$(pR - (1 - p)q) u'(z_h^1) + ((1 - p)R - pq) u'(z_l^1) = 0$$

Agents 2 solve a similar problem, facing the constraints $1 = qx_2 + y_2$ and $z_i^2 \leq p_i x_2 + (1 - p_i) R y_2$ instead. The first order conditions for an interior solution is

$$(p - (1 - p)Rq) u'(z_h^2) + ((1 - p) - pRq) u'(z_l^2) = 0$$

A symmetric solution requires that $z_h^1 = z_l^2 = z_h$ and $z_l^1 = z_h^2 = z_l$. Using the budget constraints and the market clearing condition $x_1 = 1 - x_2$, it is straightforward to show

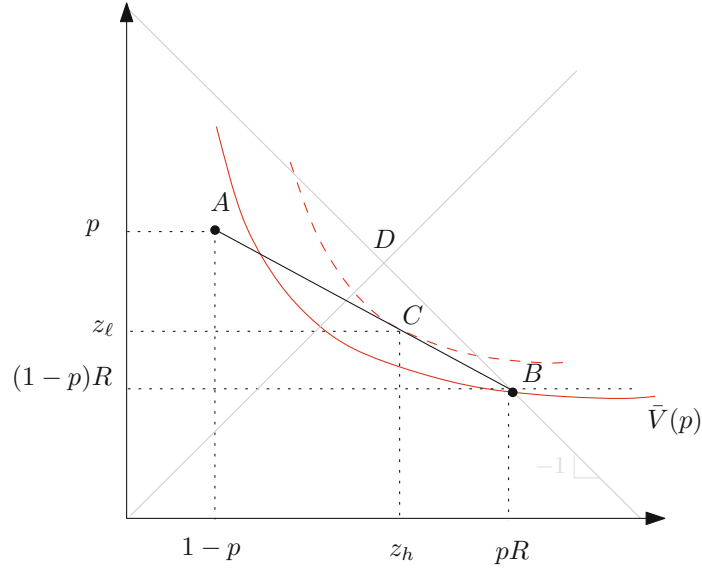


Figure 1: Spot trade

that symmetry implies $q = 1$. Therefore, the unique symmetric spot trade equilibrium is given by y the amount of seed Y purchased by agents 1, characterized by the following set of equations,

$$u'(z_h) = \frac{p - R(1 - p)}{pR - (1 - p)} u'(z_l). \quad (2)$$

with

$$z_h = pR(1 - y) + (1 - p)y \quad (3)$$

$$z_l = (1 - p)R(1 - y) + py \quad (4)$$

Notice that full insurance – i.e. $z_h = z_l$ – is feasible if and only if $R = 1$. Otherwise, if $R > 1$ equation (2) and strict concavity of the utility function imply that agents always consume more in the state where their good is valued the highest, that is $z_h > z_l$.⁴ Intuitively, purchasing the other type of seeds at $t = 1$ is beneficial, as it allows self-insurance, but it is costly, as it implies a forgone investment return. Agents fully internalize the cost of insurance and as a result do not fully insure against price risk.

Figure 1 shows the equilibrium with spot trade at $t = 1$. The two points A and B

⁴Using (3) and (4) this implies $y < R/(1 + R)$ where $R/(1 + R)$ is the amount of the other's seeds that agents have to purchase to fully insure against price risk.

correspond to the feasible consumptions in the high state (on the x-axis) and the low state (on the y-axis) when agents do not purchase any of the other's seeds ($y = 0$ gives point B) or when they swap all their seeds ($y = 1$ gives point A). Point A is below the inverted 45°-line (in grey) as agents who purchase seeds cannot benefit from the return of investing in the long-term technology. The line AB corresponds to the budget constraint of agents and, as shown, full insurance is not feasible. So, the agents prefer allocation C that does not lie on the 45°-line. However, as R tends to 1, the budget constraint AB rotates upward towards the 45°-line and C moves closer to full insurance (point D).

Full Commitment We now investigate the allocation in the environment where a pair of agents 1 and 2 writes a forward contract at $t = 1$ given they can fully commit to their promises. As we consider OTC markets, we assume that there is no centralized market for forward contracts (or that they cannot access it). A forward contract (F_h, F_ℓ) is the promise to deliver F_i units of good Z in state $i = h, \ell$ at $t = 2$. We assume that agents 1 and 2 bargain over (F_h, F_ℓ) . Before specifying the bargaining problem it is useful to consider the budget constraint of each agent. The constraint of agent 1 given contract (F_h, F_ℓ) given state $i = h, \ell$ is

$$z_i^1 \leq p_i R - F_i$$

where z_i^1 is his consumption of good Z . The constraint reads as follows: in state i , agent 1 sells his output of good X for $p_i R$ units of good Z but he has to deliver F_i units of good Z to agent 2 in state i .⁵ Similarly, the constraint of agent 2 given contract (F_h, F_ℓ) is

$$z_i^2 \leq (1 - p_i)R + F_i$$

as he – in addition to his investment in state i – receives F_i units of good Z . Adding both constraints, we notice that the resource constraint the pair faces is simply

$$z_i^1 + z_i^2 \leq R.$$

Given agents can commit, there is no need to store goods so that $s_1 = s_2 = 0$. Then, the

⁵Agents 1 receive F_i and agents 2 deliver F_i if $F_i < 0$.

forward contract is a solution to the following bargaining problem.⁶

$$\begin{aligned} \max_{z_i^1, z_i^2} & \left[\frac{1}{2}u(z_h^1) + \frac{1}{2}u(z_\ell^1) - V(p) \right] \left[\frac{1}{2}u(z_h^2) + \frac{1}{2}u(z_\ell^2) - V(p) \right] \\ & \text{s.t.} \\ & z_i^1 + z_i^2 \leq R \end{aligned}$$

The resource constraint will always bind and the first order conditions with respect to z_i^1 give us $z_i^1 = z^1$ and $z_i^2 = z^2$ for $i = h, \ell$ where $z^2 = R - z^1$ and z^1 solves

$$\frac{u'(z^1)}{u'(R - z^1)} = \frac{u(z^1) - V(p)}{u(R - z^1) - V(p)}$$

and the unique solution is $z^1 = z^2 = R/2$. As expected, the forward contract gives full insurance to both agents, irrespective of whether $R = 1$ or not. Clearly, this is also the first best solution. Naturally, a spot market at $t = 1$ (or $t = 2$) cannot achieve the first best solution as the consumption can only depend on the amount of the investment in both technologies at $t = 1$, which itself cannot depend on the realization of the state at $t = 2$. A forward contract is beneficial insofar as it allows state-dependent transfers at $t = 2$.

Figure 2 shows the solution with full commitment. Notice that, with full commitment, agents 1 and 2 invest their seeds in the long-term technology, so that the aggregate amount of resources available at date 2 is R . The forward contract allows both agents to transform one unit of resources in the high state into one unit in the low state, so that the marginal rate of substitution has to equal 1 in the equilibrium with full commitment. Finally, the figure shows the autarky level of utility $V(p)$ for both agents.

3. Bilateral Clearing

From now on, we assume agents cannot commit to make good on their promises. Then, the contract with full commitment is not compatible with agents' incentives: agents 1 will refuse to pay in state h as $pR > z^1$ and agents 2 will refuse to pay in state ℓ as $pR > z^2$. Therefore, with no commitment, agents will have to pledge collateral to prevent strategic default.

In this section, we concentrate on bilateral clearing where agents pledge collateral with each other. This means that, at date $t = 1$, agent 1 will surrender some amount of good

⁶To simplify the argument, we assume agents have the same bargaining power.

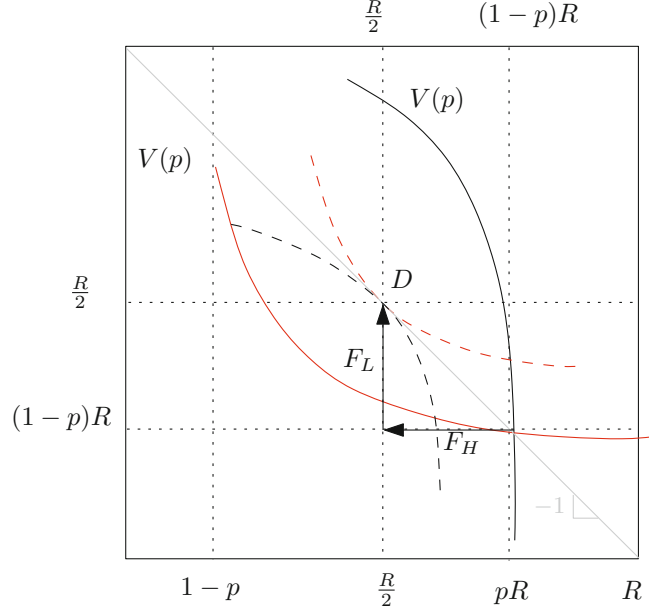


Figure 2: Full commitment forward contract.

X to agent 2 and agent 2 will surrender some amount of good Y to agent 1. Since agent 1 (respectively agent 2) does not have the technology to invest good Y (respectively good X), the goods serving as collateral will be stored, i.e. collateral will take the form of storage with a return of 1. Thus, storage is costly because it does not earn the return R . In addition, with bilateral clearing, agents will be able to abscond with the collateral pledged by their counterparty. This implies that – in addition to the delivery in each state – the forward contract must also specify some collateral constraint.

Given p_i , the collateral requirement must be such that agents are not better off defaulting. Note that since collateral is pledged ahead of date $t = 2$, collateral is state-independent. If c_1 and c_2 denote the collateral requirement from agent 1 and 2 respectively (and agents can run with the collateral posted by the other agents), the incentive constraint of agent 1 is

$$p_i [(1 - c_1)R + c_1] - F_i \geq p_i(1 - c_1)R + (1 - p_i)c_2 \quad (5)$$

and the incentive constraint of agent 2 is

$$(1 - p_i) [(1 - c_2)R + c_2] + F_i \geq (1 - p_i)(1 - c_2)R + p_i c_1 \quad (6)$$

Thus, F_i is bounded above and below by $p_i c_1 - (1 - p_i) c_2$ so that

$$F_i = p_i c_1 - (1 - p_i) c_2 \quad (7)$$

In other words, incentive compatibility requires that the forward contract is exactly equal to the difference in value of collateral pledged. The budget constraints of agents 1 and 2 are

$$\begin{aligned} z_i^1 &\leq p_i [(1 - c_1)R + c_1] - F_i \\ z_i^2 &\leq (1 - p_i) [(1 - c_2)R + c_2] + F_i \end{aligned}$$

Using expression (7), we obtain the following resources constraints

$$\begin{aligned} z_i^1 &\leq p_i(1 - c_1)R + (1 - p_i)c_2 \\ z_i^2 &\leq (1 - p_i)(1 - c_2)R + p_i c_1 \end{aligned}$$

It should be clear that these constraints will hold with equality. Therefore, the bargaining problem is

$$\begin{aligned} \max_{c_1, c_2 \in [0, 1]} & \left[\sum_{i=h, \ell} \frac{1}{2} u(p_i(1 - c_1)R + (1 - p_i)c_2) - V(\bar{p}) \right] \\ & \times \left[\sum_{i=h, \ell} \frac{1}{2} u((1 - p_i)(1 - c_2)R + p_i c_1) - V(\bar{p}) \right] \end{aligned}$$

The first order conditions give us

$$\begin{aligned} \frac{t_2}{t_1} R \left[\sum_{i=h, \ell} p_i u'(z_i^1) \right] &\geq \sum_{i=h, \ell} p_i u'(z_i^2) \quad (= \text{ if } c_1 > 0) \\ \frac{t_2}{t_1} \left[\sum_{i=h, \ell} (1 - p_i) u'(z_i^1) \right] &\leq R \sum_{i=h, \ell} (1 - p_i) u'(z_i^2) \quad (= \text{ if } c_2 > 0) \end{aligned}$$

where t_j is the trading surplus for agent $j = 1, 2$. Using these first order conditions, we obtain the following existence result for a symmetric solution $\bar{c}_j = \bar{c}$ and $\bar{z}_i^j = \bar{z}_i$ for $j = 1, 2$ for the levels of collateral and consumption with bilateral clearing (omitted proofs are found in the Appendix).

Proposition 1. *There is a unique symmetric solution where $\bar{z}_h > \bar{z}_\ell$ and $\bar{c} > 0$ is uniquely*

given by

$$u'(\bar{z}_h) = \frac{p - R(1 - p)}{pR - (1 - p)} u'(\bar{z}_\ell). \quad (8)$$

with

$$\bar{z}_h = pR(1 - \bar{c}) + (1 - p)\bar{c} \quad (9)$$

$$\bar{z}_\ell = (1 - p)R(1 - \bar{c}) + p\bar{c} \quad (10)$$

Otherwise, $\bar{c} = 0$.

It should be clear that \bar{c} is a function of p and R . Also, \bar{z}_h is decreasing in \bar{c} while \bar{z}_ℓ is increasing in \bar{c} such that increasing collateral increases insurance (respectively the level of income smoothing trade). In the economy with bilateral clearing, the total collateral requirement is $(1 - \sigma)\bar{c}$. Figure 3 shows the equilibrium relative to bilateral trading with full commitment. By pledging collateral c , agents agree to a lower aggregate amount of resources in any state at date 2, as depicted by the grey area in the figure. Therefore, while the value of staying in autarky is still given by point B , now the indifference curves of agent 2 when $c > 0$ have to be taken relative to the new origin O_2 . With bilateral clearing, the transformation rate of a unit of consumption in state h into a unit in state ℓ is given by the line AB with slope $-\frac{p-R(1-p)}{pR-(1-p)} > -1$. Feasible allocations are given by those consumption levels that are feasible and satisfy the incentive compatibility constraints for both agents (light green area for agents 1 and light blue area for agents 2). Notice that with no collateral, the green and blue area would meet at B , so that autarky would be the only feasible allocation. By increasing collateral, agents are reducing the aggregate amount of resources, but agents 1 are able to move “up” the line AB , thus, getting closer to full insurance. Still, as aggregate resources are lowered, point D is never attainable. At the optimal solution, agents vary c so that the only feasible allocation is point C , as depicted. This point is where the marginal rate of substitution is equal for both agents.⁷

It may be surprising that there is trade even though there is no commitment at all. In particular, agents could keep the collateral once the state is realized at $t = 2$. After all, if the price is high for agent 1 then he has no incentive to keep his promises unless the

⁷Note that it would be misleading to show the indifference curve (IC) of agent 2 at point C : it would have been placed beyond the IC passing through point D , while agents 2 do not obtain a higher utility than at point D (the full insurance allocation). The reason is that point D has its origin at O , while the ICs of agents 2 when $c > 0$ have their origin at O_2 . Therefore, the IC of agents 2 passing through C would correspond to a level of utility that would be translated north-east by the translation $O_2 \rightarrow O$.

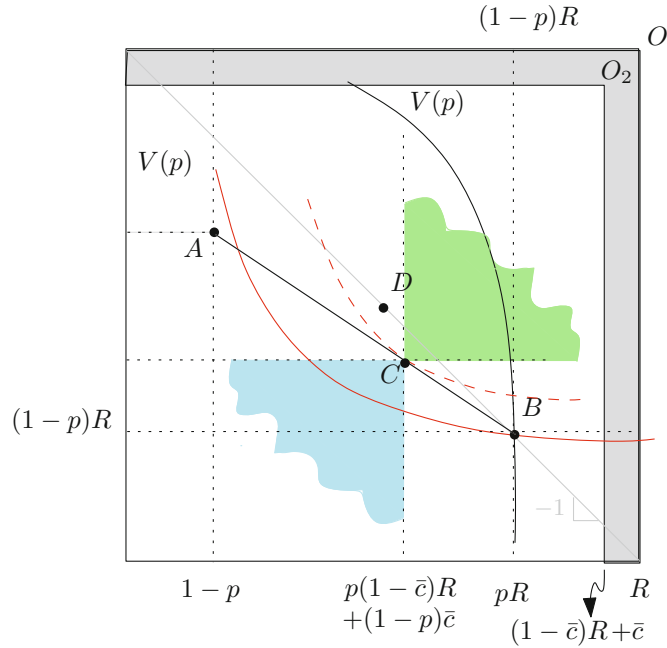


Figure 3: Bilateral Clearing

collateral he posted is worth less than what he owes agent 2. But then, agent 2 has no incentive to surrender the collateral back to agent 1 unless the value of agent 1's promise is higher than the value of the collateral itself. Hence, the value of collateral must equal the value of the promise. However, this implies that clearing does not add anything to what agents could achieve by merely trading collateral on the spot market in $t = 1$. Indeed, comparing the spot-trade solution (2)-(4) and the solution with bilateral clearing (8)-(10), one realizes that there are no gains from using forward contracts if agents cannot commit and they use a simple bilateral clearing arrangement. To understand the mechanics consider the consumption levels \bar{z}_i : agents can obtain the same consumption levels if they trade \bar{c} seeds at date $t = 1$. They are willing to do so to insure some consumption of good Z . However, agents are not willing to exchange half of their endowment as they only have the technology to get the return on their own seed. Therefore, with bilateral clearing and no commitment, the forward contract achieves no better than agents swapping and storing \bar{c} units of their endowments at $t = 1$, with no further interaction later. We summarize this result as

Proposition 2. *The allocation when agents trade forward contracts and clear bilaterally is the same as when agents trade in a spot market, therefore bilateral clearing of forward contracts is not essential.*

While theoretically bilateral clearing is not essential as swapping and storing endowments is equivalent, the economic rationale behind the exchange of collateral is to preserve settlement incentives.⁸ Bilateral clearing or swapping endowments, however, increases welfare relative to autarky only if the return is not too high. As the return represents the opportunity cost of collateral, there is a trade-off between forgone investments and income smoothing. This trade-off is relaxed the lower price risk is.

Corollary 1. *With bilateral clearing, there is never full insurance unless $R = 1$. There is $\bar{R} > 1$ such that $\bar{c}(p) = 0$ whenever $R \geq \bar{R}$. For all $R \in (1, \bar{R})$, the collateral is positive, $\bar{c}(p) > 0$.*

In state \bar{p} at $t = 1$, the expected welfare under bilateral clearing is then

$$V^b(p) = \sum_{i=h,\ell} \frac{1}{2} u(p_i(1 - \bar{c}(p))R + (1 - p_i)\bar{c}(p))$$

where $p_h = p$ and $p_\ell = 1 - p$, so that total expected welfare (as of $t = 0$) under bilateral clearing is

$$W^b = \sigma u\left(\frac{R}{2}\right) + (1 - \sigma)V^b(p)$$

4. Segregated Collateral Clearing

In this section, we study the effect on the optimal contract and welfare of introducing a technology that allows agents to pledge collateral at a third party. We refer to this technology as “segregation”. This technology prevents a defaulter from accessing the collateral while, at the same time, allowing the non-defaulter to sell the defaulting agent’s collateral for his own consumption. One could imagine a third-party clearer that allows agents to store their collateral in its vault. In contrast to agents, the third-party clearer can commit to his collateral policy e.g. to give all collateral to the non-defaulting agent.

Segregation changes the incentive constraints of the two agents. For agent 1 incentive compatibility now requires

⁸This is similar to a property of clearing that is meant to preserve settlement incentives, namely the exchange of variation margins. With a certain frequency, additional collateral (variation margins) are exchanged to adjust the value of collateral of each counterparty to the changing market value of its obligations. In our model setup, this emerges endogenously insofar as the changing market value of seeds pledged as collateral reflects variation margins (remember, seeds change their market value from $p = \frac{1}{2}$ at $t = 1$ to $p = \frac{1}{2}, p_L, p_H$ at $t = 2$).

$$p_i [(1 - c_1)R + c_1] - F_i \geq p_i(1 - c_1)R \quad (11)$$

and for agent 2 incentive compatibility requires

$$(1 - p_i) [(1 - c_2)R + c_2] + F_i \geq (1 - p_i)(1 - c_2)R \quad (12)$$

Indeed, if agents 1 or 2 were to default, they would not pay F_i but they would only be left with the value of their investment. Therefore, it should be clear that incentive constraints no longer depend on the other agent's collateral. As a consequence, incentive constraints are relaxed by the fact that the agent who defaults can not run away with the other agent's collateral. Therefore, from expressions (11) and (12), the forward contract is restricted by the following inequalities,

$$\begin{aligned} pc_1 &\geq F_h \\ pc_2 &\geq -F_\ell \end{aligned}$$

Since collateral is costly, these constraints hold with equality. Therefore, we can simplify the budget constraints for agents 1 as

$$\begin{aligned} z_h^1 &= p(1 - c_1)R \\ z_\ell^1 &= (1 - p) [(1 - c_1)R + c_1] + pc_2 \end{aligned}$$

and similarly for agents 2,

$$\begin{aligned} z_h^2 &= (1 - p) [(1 - c_2)R + c_2] + pc_1 \\ z_\ell^2 &= p(1 - c_2)R \end{aligned}$$

Hence, the bargaining problem can be written as

$$\max_{c_1, c_2} \left[\sum_{i=h, \ell} \frac{1}{2} u(z_i^1) - V(p) \right] \left[\sum_{i=h, \ell} \frac{1}{2} u(z_i^2) - V(p) \right]$$

subject to $c_1 \in [0, 1]$ and $c_2 \in [0, 1]$. The first order conditions for c_1 and c_2 are

$$\begin{aligned} \frac{t_2}{t_1} [pRu'(z_h^1) + (1-p)(R-1)u'(z_\ell^1)] &\geq pu'(z_h^2) \quad (= \text{ if } c_1 > 0) \\ \frac{t_1}{t_2} [pRu'(z_\ell^2) + (1-p)(R-1)u'(z_h^2)] &\geq pu'(z_\ell^1) \quad (= \text{ if } c_2 > 0) \end{aligned}$$

Then, using these first order conditions and denoting by $\hat{c}(p)$ and \hat{z}_i^j the solutions for the collateral and consumption levels with segregated collateral, we obtain

Proposition 3. *There is a unique symmetric solution with $\hat{z}_h > \hat{z}_\ell$ where $\hat{c} > 0$ is uniquely given by*

$$u'(\hat{z}_h) = \frac{1 - (1-p)R}{pR} u'(\hat{z}_\ell). \quad (13)$$

with

$$\begin{aligned} \hat{z}_h &= p(1 - \hat{c})R \\ \hat{z}_\ell &= (1-p)(1 - \hat{c})R + \hat{c} \end{aligned}$$

Otherwise $\hat{c} = 0$.

Again, it should be clear that \hat{c} is a function of p and R . Figure 4 summarizes the equilibrium allocation with segregation. First, notice that segregation expands the set of incentive feasible allocations relative to bilateral clearing, as there is now more consumption feasible for each collateral level c . While bilateral clearing implied a unique incentive feasible allocation for each collateral level, the yellow area (intersection of the blue and green areas) shows the set of incentive feasible allocations with segregation. Second, segregation allows the rate of transformation from consumption in the high state to consumption in the low state to increase relative to bilateral clearing: indeed, it takes pR units of resources in the high state to obtain $1 - (1-p)R$ units of resources in the low state.⁹ Therefore, the rate of transformation is now the line $\hat{A}B$ (instead of AB with bilateral clearing). Finally, the pair of agents choose the best allocation that is in the yellow area and on the line $\hat{A}B$. This is clearly point \hat{C} , where only the incentive constraint in the high state binds. Since \hat{C} lies on the line $\hat{A}B$ which is an upward rotation of the line AB , agents 1 achieve a higher utility at \hat{C} . Again, we do not show the indifference curve of agents 2 as this may lead to confusion due to the change in origin.

⁹More precisely, $-\frac{1-(1-p)R}{pR} < -\frac{p-R(1-p)}{pR-(1-p)}$.

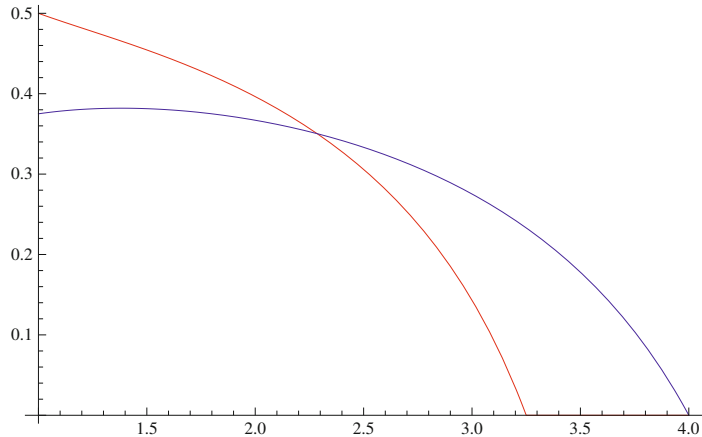


Figure 6: Average collateral with bilateral clearing (red) and segregated collateral clearing (blue) as a function of $R \leq p/(1-p)$ for $p = 0.8$, and $\alpha = 2$.

bilateral clearing, while it is higher under segregated clearing. This is the trade-off between consumption smoothing and the return on the productive technology.

Another way to see this result is to consider two polar cases. First, with $R = 1$, the opportunity cost of collateral is nil and agents swap half of their endowment with bilateral clearing. With $R = 1$, agents naturally exchange less collateral to achieve perfect insurance with segregated collateral clearing as incentive constraints are relaxed. While the level of collateral is different under both clearing arrangements, the agents' utility is the same as $R = 1$. Second, consider the return level \bar{R} where there is no trade taking place with bilateral clearing (in Figure 5, this is the case when the line $\bar{A}B$ is tangent to the autarky IC at B). When we consider collateral segregation at this level of return, the incentive constraints are relaxed and it pays off to increase income smoothing by posting collateral even though the opportunity cost of collateral is high. Optimal collateral policies are non-linear. Figure 6 shows this for $p = 0.8$ and $\alpha = 2$.

Furthermore, it is easy to see that segregated collateral clearing always dominates bilateral clearing.¹⁰

Proposition 4. *Welfare under segregated collateral clearing is higher than under bilateral*

¹⁰We refrain from claiming that segregation is essential for the following reason. While segregation is essential in this simple environment, in the sense that it expands the set of feasible allocations beyond what is attainable with spot trades or bilateral clearing, this may not be the case in a more general environment. For instance, if we consider a dynamic setting, dynamic contracts without collateral could do better. For instance, Kocherlakota (1996) considers an dynamic environment with double-sided lack of commitment and endowment risk, and finds that trade is feasible although agents do not pledge collateral. Here, our static framework is akin to imposing a lack of public record keeping so that dynamic contracts are somehow infeasible.

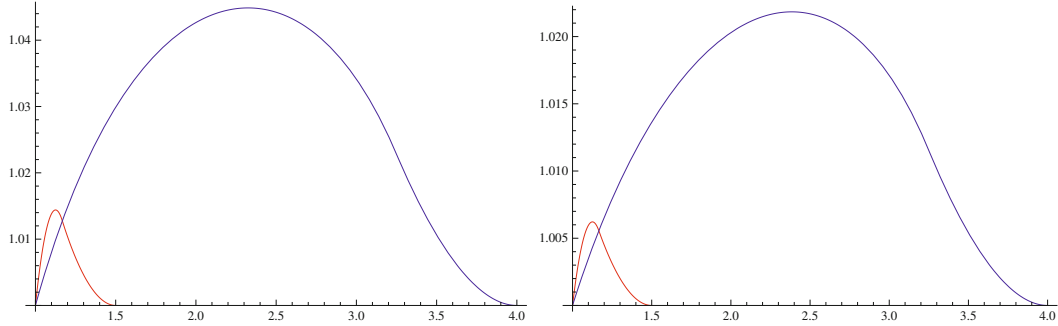


Figure 7: γ as a function of R . Left: $\sigma = 0.3$. Right: $\sigma = 0.7$. Red: $p = 0.6$. Blue: $p = 0.8$.

clearing.

The constraint set of the bilateral clearing contract is given by expressions (5) and (6) for $i = h, \ell$ and the constraint set of segregated collateral clearing is given by expressions (11) and (12). Clearly, any symmetric contract (c, F_i) that satisfies the bilateral clearing constraint set also satisfies the segregated collateral clearing constraint set. Since the objective function under both clearing arrangement is the same, it must be that the solution with segregation achieves a higher payoff for agents.

Finally, we compute the consumption equivalent γ of moving from bilateral to segregated collateral clearing as of $t = 0$. We compute γ as the value that solves

$$\sigma u\left(\gamma \frac{R}{2}\right) + (1 - \sigma) \sum_{i=h,\ell} \frac{1}{2} u(\gamma \bar{z}_i(p)) = \sigma u\left(\frac{R}{2}\right) + (1 - \sigma) \sum_{i=h,\ell} \frac{1}{2} u(\hat{z}_i(p))$$

Therefore, an agent is willing to give up a fraction $\gamma - 1$ of his consumption to move from an economy with bilateral clearing to an economy with segregated collateral clearing.

Figure 7 shows γ (y-axis) as a function of the cost of pledging collateral $R \leq 0.8/0.2 = 4$ for different levels of risk as measured by p and σ , with $\alpha = 2$. The value for p is either 0.6 (red curve) or 0.8 (blue curve), while the value for σ is either 0.3 (left panel) or 0.7 (right panel). As the reader can notice, increasing σ reduces the welfare gains from segregation, as there is less scope for insurance as of $t = 0$.

To choose an example, we compute the welfare equivalent for a relatively low value for the cost of pledging collateral. Suppose $R = 1.04$, then agents are willing to give up around 0.3% of their consumption in order to shift from bilateral to segregated clearing when $p = 0.6$, while they are only willing to give up 0.15% when $p = 0.8$. Surprisingly, agents are more willing to switch to segregated clearing when the risk is low and collateral is cheap. The

reason is that agents pledge more collateral under segregated clearing than under bilateral clearing when the risk is higher (i.e. for low R , $\bar{c}(p) - \hat{c}(p)$ is decreasing in p). So, the gains from switching from one to the other form of clearing naturally decreases. Finally, the gains are hump-shaped in R . The gains amount to a maximum of 4.5% of consumption when σ is small. The bottom line is that there seems to be non-negligible welfare gains from segregation, specially when it is relatively more costly to pledge collateral.

5. Central Counterparty Clearing

In this section, we consider central counterparty clearing. In order to solve for the CCP's problem, we adopt an approach similar to a constrained planner's problem: the CCP seeks to maximize the date $t = 0$ welfare of agents by choosing their collateral and consumption levels, subject to numerous constraints. In this sense, we obtain the highest date $t = 0$ level of welfare that agents could ever hope to achieve by clearing through a CCP.

Formally, the CCP promises payments $z(p)$ at $t = 2$ to agents in the safe environment and payments $(z_h(p), z_\ell(p))$ to agents in the risky environment. Figure (8) shows the collateral requirements as well as the consumption level that the CCP promises to agents in each state.

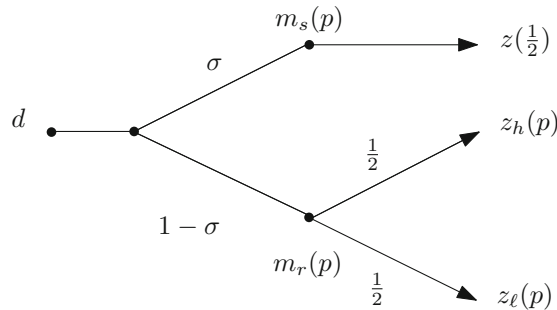


Figure 8: CCP Collateral requirements and consumption levels

Practitioners might argue that this does not mirror the *raison d'être* of a CCP, as a CCP only insures that a trade goes through by requiring a default fund contribution to cover potential losses that go beyond margin contributions of the one or two members that generate the largest loss potential to the CCP. While the CCP's financial resources are meant to guarantee the economic viability of the CCP to withstand its losses, at the same time, they prevent a default from agents in the high state. By allowing the CCP to choose how to collect financial resources, we focus on the optimal composition of these financial resources to determine the biggest welfare gains from mutualization via a default fund. This is a key

ingredient to explain the clearing market, on the one hand, and to think about policy issues, on the other hand.

As before, we consider a symmetric solution to the CCP. Therefore, the CCP solves the following problem:

$$\max_{d, m(p), z_i(p)} \sigma u(z(p)) + \frac{(1-\sigma)}{2} [u(z_h(p)) + u(z_\ell(p))]$$

where the expectation is over the price p prevailing at date 2 as of date 0. The CCP has to satisfy the following constraints. First, the allocation has to be feasible for each p which requires,

$$\begin{aligned} \sigma z(p) + \frac{(1-\sigma)}{2} (z_h(p) + z_\ell(p)) &= \sigma(1-d-m_s(p)) \frac{R}{2} + (1-\sigma)(1-d-m_r(p)) \frac{R}{2} \\ &\quad + \sigma \frac{1}{2} (d+m_s(p)) + (1-\sigma) \frac{1}{2} (d+m_r(p)) \end{aligned}$$

To understand the feasibility constraint, at date 1 the CCP receives collateral $m_s(p)$ from a measure σ of safe agents. This collateral has value $1/2$. Also, the CCP receives collateral $m_r(p)$ from a measure $1-\sigma$ of agents facing the risky state. Half of this collateral has value p while the other half has value $1-p$. Therefore, the average value of the collateral posted by risky agents is $1/2$. Hence, the RHS of the feasibility constraint represents the resources available to the CCP which are independent of volatility. The LHS mirrors the CCP's expenditure as a function of the possible realization of the price p . The CCP has to finance the consumption of a measure σ of agents in the safe state and a measure $1-\sigma$ agents in the risky state.

Second, the CCP faces a participation constraint at $t=0$, or

$$\sigma u(z(p)) + \frac{(1-\sigma)}{2} [u(z_h(p)) + u(z_\ell(p))] \geq \sigma u\left(\frac{R}{2}\right) + (1-\sigma)V(p)$$

where the RHS is the value of trading only in the spot market at $t=2$ with no consumption smoothing through forward trading. This constraint imposes that the overall collateral requirement is not too high.

Third, the CCP has to satisfy interim participation constraints at $t=1$ for agents in the safe environment as well as for agents in the risky environment p . These constraints are, for

agents in the safe and risky environment respectively,

$$u(z(p)) \geq u\left((1-d)\frac{R}{2}\right)$$

$$u(z_h(p)) + u(z_\ell(p)) \geq u(p(1-d)R) + u((1-p)(1-d)R)$$

These constraints put limits on the margins that agents are willing to pledge, given they already contributed to the default fund.

Fourth, the CCP incentive compatibility constraints for each agent in the risky environment:

$$z_h(p) \geq pR(1-d-m_r(p))$$

$$z_\ell(p) \geq (1-p)R(1-d-m_r(p))$$

These constraints make sure that agents prefer the CCP's allocation to default. Finally, the CCP also faces a feasibility constraint, $1-d-m_i(p) \geq 0$ for $i = s, r$.

A quick look at the constraint set suffices to note that $m_s(p)$ has no impact on incentive constraints, but reduces the resources available to the CCP. Therefore, the CCP optimally sets $m_s(p) = 0$. To simplify notation, we will now set $m_r(p) = m(p)$ as the margin requirement for agents in the risky environment.

Proposition 5. *With CCP clearing there is never full insurance. Whenever $d^* \in (0, 1)$ and $m^*(p) \in (0, 1-d^*)$, the unique solution is given by*

$$z_h^*(p) = pR(1-d^*-m^*(p)). \quad (14)$$

$$z_\ell^*(p) = (1-p)R(1-d^*-m^*(p)) + \frac{1}{1-\sigma}d^* + m^*(p). \quad (15)$$

$$z^*(p) = (1-d^*)\frac{R}{2} \quad (16)$$

and

$$u'(z_h^*(p)) = \frac{1-(1-p)R}{pR}u'(z_\ell^*(p)) \quad (17)$$

$$u'(z_\ell^*(p)) = Ru'(z^*(p)) \quad (18)$$

Several aspects of the CCP's solution are worth emphasizing. First, expressions (14) and

(15) show that agents in state h are indifferent between defaulting or not, while agents in state ℓ get all the available collateral. Again, this is not very surprising, as the CCP wants to insure agents as much as possible and, thus, transfers the maximum amount of resources feasible to the poorest agents, those in state ℓ .

Second, the margin requirements $m(p)$ only equate the marginal utility for those agents in the risky state. Thus, as can be seen in expression (17), the rate of transformation between consumption in the high and low state is the same with a CCP as with segregation only. Thus, the added value from CCP clearing over segregated collateral clearing comes from the use of the default fund. The default fund allows for an insurance device at $t = 0$ that allows to reduce the collateral requirement of agents in a risky match. This can be seen by summing (14) and (15) and denoting $k(p) = d^* + m^*(p)$ as the collateral pledged by agents in a risky match, we obtain

$$z_h^*(p) + z_\ell^*(p) > R(1 - k(p)) + k(p)$$

so that agents in a risky match, and for the same level of posted collateral, have more resources when they clear through a CCP than using segregation only. These additional resources are provided, of course, by those agents that end up in the safe match.

Third, the CCP sets the default fund contribution so as to equate the marginal utility for agents in the safe environment to the expected marginal utility of agents in a risky environment at $t = 1$. In so doing, the CCP insures that $z_h^*(p) > z^*(p) > z_\ell^*(p)$.

Finally, without the use of the default fund, the equilibrium allocation with the CCP is the same as with segregation. This can be illustrated with two polar cases. First, if $\sigma = 1$, all agents end up in the safe environment and, thus, will not trade. More interesting, if $\sigma = 0$, or in other words, when there is no safe state, a default fund cannot add financial resources to those agents in the risky environment.

Figure 9 illustrates the CCP solution relative to segregated clearing. The figure clearly shows the benefit of CCP clearing for those agents in the risky state. Indeed, while they have contributed d to the default funds, they obtain $pR(1 - d)$ in the high state when they do not pledge margins, but they obtain $(1 - p)(1 - d)R + \frac{d}{1 - \sigma}$ in the low state, as they obtain the payment from the default fund. Therefore, the new “autarky” origin is point E, which is on a higher line to the rate of transformation $\hat{A}B$. Since the rate of transformation is the same as under segregation, the chosen allocation C^* has to be on a higher indifference curve than the allocation under segregation, \hat{C} . The bottom line is that agents in the risky state are better off at date 1, as they benefit from the extra resources in the default funds.

We also obtain results on the collateral structure of a CCP.

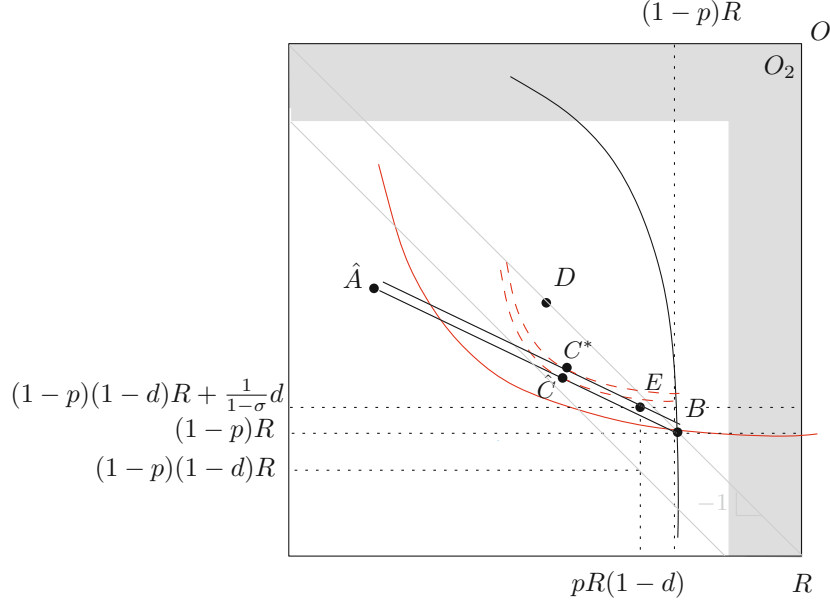


Figure 9: CCP

Corollary 4. *There are two thresholds $R^* > R_m^* > 1$ such that $m^*(p, R) = d^*(R) = 0$ whenever $R \geq R^*$, and $d^*(R) > m^*(p, R) = 0$ whenever $R \in (R_m^*, R^*)$. For all $R \in (1, R_m^*)$ the margins and the default funds are positive. In addition, $R^* > \hat{R} > R_m^*$.*

This result shows that the CCP prefers to use default fund contributions when collateral is expensive. This is akin to the results in models with exogenous default risk such as Haene and Sturm (2009) and Nahai-Williamson (2012). The reason is to spread the cost of pledging collateral over more agents, thus, achieving a higher consumption smoothing. Also, the presence of the default fund allows the CCP to reduce margins to zero for a lower cost of collateral than under segregated clearing. In this sense, the CCP saves on collateral. However, taking into account the aggregate amount of posted collateral, the CCP turns out to be more costly, as we show below.

Proposition 6. *CCP clearing requires more collateral than segregated clearing, i.e.*

$$d^* + (1 - \sigma)m^* > (1 - \sigma)\hat{c}.$$

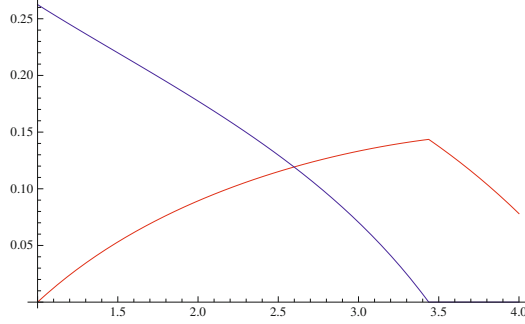


Figure 10: d^* (red) and $(1 - \sigma)m^*$ (blue)

In the figures, we illustrate how the aggregate amount of collateral posted differs among different clearing arrangements for different levels of R and risk p given an economy where $\sigma = 0.3$, $\alpha = 2$. Precisely, Figure 10 shows the decomposition of the aggregate collateral requirement of the CCP, while Figure 11 shows the average amount of collateral under bilateral clearing $(1 - \sigma)\bar{c}(p)$ (red curve), the average amount of collateral under segregated clearing, $(1 - \sigma)\hat{c}(p)$ (green curve), and the average amount of collateral under centralized clearing, $d^* + (1 - \sigma)m^*(p)$ (blue curve).

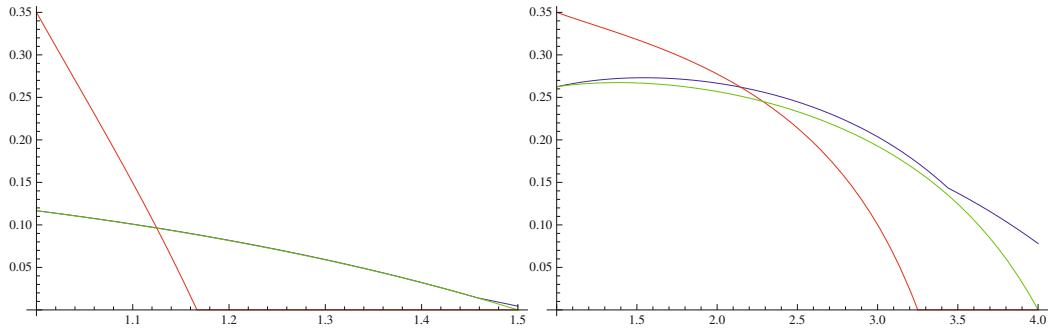


Figure 11: Average collateral. Left: $p = 0.6$, Right: $p = 0.8$. Red: bilateral clearing. Green: segregated collateral clearing. Blue: CCP clearing.

Although the mark-up in the left figure is almost inexistent, centralized clearing is always more greedy in terms of collateral than segregated clearing. The kink on the right panel of Figure 11 occurs as $m(p)$ is zero as collateral becomes very costly to pledge. We can further evaluate the extent of mutualization with the following corollary.

Corollary 5. Suppose $m^*(p) > 0$ then given d^* , $m^*(p)$ is increasing in p .

A higher p implies a more severe default as the CCP has more at stake the larger p is. Therefore, given d , the CCP will require additional collateral to each trader in the risky

environment. Clearly, this implies that the optimal waterfall ratio, defined as $d^*/m^*(p)$, is decreasing with increasing p . Thus, the optimal CCP waterfall rests on more defaulter-pays resources as the price risk increases.

Interestingly, models with exogenous counterparty risk – such as in Haene and Sturm (2009) and Nahai-Williamson et al. (2013) – draw the opposite conclusion. However, these models pin down margin and default fund contributions such as to let the CCP survive the losses of an exogenous default in case of a plausible but extreme price scenario. In contrast, our model takes $m^*(p)$ as a function of the actually observed price volatility. If we extend our model to consider a range of price volatilities (as suggest in the Appendix), we could make d a function of the maximum price volatility considered, while $m^*(p)$ remains a function of actually observed price volatility. Then, the waterfall ratio increases with maximum price volatility and, at the same time, decreases with actually observed price volatility. Considering differing maximum price volatilities could be taken as different financial contracts being cleared by separate CCPs. Thus, the CCP clearing the more volatile contracts would use a higher default fund and, hence, would show a higher waterfall ratio.

Taking into account current CCP practices, Heller and Vause (2012) estimate margin and default fund contributions to global CCPs that either clear credit default swaps (CDS) or IRS. They do so with artificial trade data generated on the basis of G14 dealers' balance sheet data. Considering market data between 2004 and 2010, clearly, CDS contracts show a higher level of price volatility than IRS contracts. Combining stylized facts, we find that for a single CCP the waterfall ratio decreases with increasing price volatility and that a CCP clearing products with a higher maximum level of price volatility shows a higher waterfall ratio. While our model could be calibrated to real data, accessibility of data prevents us from doing so.¹¹ We now turn to welfare.

Proposition 7. *Welfare under CCP-clearing is higher than under segregated collateral clearing.*

This result is intuitive and we skip a formal proof: there is nothing that segregated clearing does that CCP clearing cannot do. In particular, setting $d = 0$, the constraints of the CCP are the same as under segregated clearing. Therefore, the CCP can never do worse than segregated collateral clearing.

We can compute the welfare gains of moving from bilateral clearing to CCP clearing and from segregated collateral clearing to CCP clearing. In this way, we can decompose the gains

¹¹We calibrated a version of our model using data from Heller and Vause (2012). The results are available from the authors.

from CCP clearing that originate from the CCP offering collateral segregation and the CCP offering mutualization. We compute the consumption equivalent η_b as follows:

$$\sigma u\left(\eta_b \frac{R}{2}\right) + (1 - \sigma) \sum_{i=h,\ell} \frac{1}{2} u(\eta_b \bar{z}_i(p)) = \sigma u\left(\frac{R}{2}\right) + (1 - \sigma) \sum_{i=h,\ell} \frac{1}{2} u(\eta_b z_i^*(p))$$

so that an agent is willing to give up a fraction $\eta_b - 1$ of its consumption to move from an economy with bilateral clearing to an economy with CCP clearing. Again, using $u(z) = z^{1-\alpha}/(1-\alpha)$ with $\alpha = 2$ and $\sigma = 0.3$, Figure 12 shows η_b (y-axis) as a function of R (x-axis) for $p = 0.6$ (blue curve) and $p = 0.8$ (red curve).

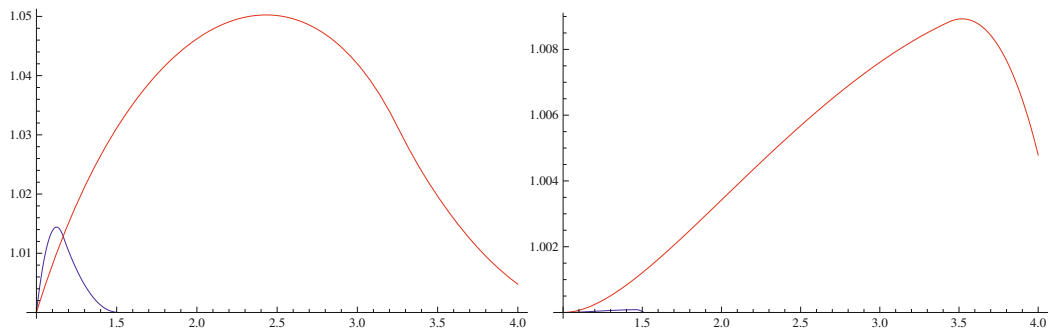


Figure 12: η_b (left) and η_s (right) as a function of R with $\alpha = 2$. Blue: $p = 0.6$. Red: $p = 0.8$.

Similarly, we compute η_s as

$$\sigma u\left(\eta_s \frac{R}{2}\right) + (1 - \sigma) \sum_{i=h,\ell} u(\eta_s \hat{z}_i(p)) = \sigma u\left(\frac{R}{2}\right) + (1 - \sigma) \sum_{i=h,\ell} u(z_i^*(p))$$

so that an agent is willing to give up a fraction $\eta_s - 1$ of its consumption to move from an economy with segregated collateral clearing to an economy with CCP clearing. For the same parameters as before, Figure 12 shows η_s (y-axis) as a function of R (x-axis) for different values of p . Comparing Figure 7 with the left panel of Figure 12, under the parametrization of our example, agents who are clearing bilaterally would mostly benefit from moving towards segregation, while their additional welfare increase from moving to centralized clearing is much lower, especially when there is little price risk (i.e. when p is close to $1/2$). Hence, using this parametrization, there does not seem to be much gains from mutualization. However, this is not generally the case. Indeed, the benefit of introducing

a CCP is very sensitive to the project return R but also to the coefficient of relative risk aversion α . To show this, Figure 13 plots η_b (left panel) and η_s (right panel) as functions of R for $\alpha = 4$ (again for $p = 0.6$ in blue and $p = 0.8$ in red).¹²

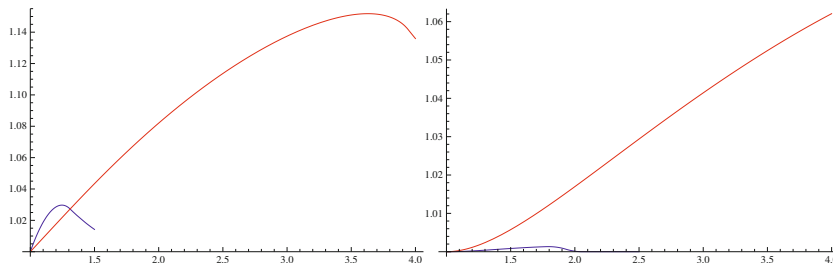


Figure 13: η_b (left) - η_s (right) as a function of R with $\alpha = 4$. Blue: $p = 0.6$. Red: $p = 0.8$.

As σ increases, mutualization becomes more costly as more agents will not benefit from centralized clearing although they have to contribute to the default fund. This effect decreases the gains from centralized clearing over just segregated clearing. However, central clearing is more desirable as agents become more risk averse.

As in Figure 7, Figure 13 shows that also for relatively high risk aversion, and for low enough R the benefit of segregation outweighs the benefits of mutualization by large, so that adding a CCP to segregated clearing would not add much to agents welfare, even if agents are very averse to risk. However, as R and p increase, mutualization clearly becomes more valuable and more so as risk aversion increases.

6. Literature and Discussion

Our main friction is that agents cannot commit to their promises. We believe limited commitment is relevant in clearing for three reasons. First, limited commitment is understood as a crucial friction that motivates diverse institutional settings in financial intermediation more generally.¹³ For instance, segregation in the form of escrow accounts is understood as an essential feature of payment systems as highlighted in Kahn, McAndrews and Roberds (2003) and Kahn and Roberds (2007). As pointed out by McAndrews and Roberds (2003) and Gu et al. (2013) limited commitment fosters the understanding of banking. Second,

¹²The blue curve in the left figure stops at $R = 1.5$ as expression (1) is violated after this.

¹³For an informal discussion, see Nosal and Steigerwald (2010) and Nosal (2011) on the role of limited commitment in clearing.

limited commitment remains a relevant friction even in economies with a functioning legal system and a developed regulatory framework. While Madoff and AIG may serve as illustrations at one extreme of the spectrum, limited commitment is relevant more generally. For instance, strategic default is often referred to as technical default that is acknowledged as a real world phenomenon. For repo markets this is well illustrated by Flemming and Garbade (2002 and 2005). Third, a CCP's risk management essentially reflects limited commitment. Entering between two counterparties, a CCP promises to replace financial contracts if one of the counterparties defaults. As a consequence, the CCP assumes credit risk. In so doing, the CCP's financial resources are designed to absorb potential losses fully, i.e. the CCP is neither designed to cover credit risk up to a limit nor to cope with a notion of expected losses as it would have to do if we were to consider limited liability instead.

To our knowledge this is the first paper analyzing clearing from the perspective of two-sided limited commitment, i.e. both the seller and the buyer to a forward contract can strategically default. While Duffie and Zhou (2011), Maegerle and Nellen (2011) and Duffie et al. (2014) acknowledge two-sided counterparty risk, they focus on multilateral netting, analyzing netting efficiency of different clearing structures. Haene and Sturm (2009) and Nahai-Williamson et al. (2013) consider exogenous default risk to derive the optimal waterfall ratio. Koepl and Monnet (2009), Koepl and Monnet (2010), Koepl (2012), Acharya and Bisin (2013), Biais, Heider and Hoerova (2012a and b) and Carappella and Mills (2012) assume counterparty risk to be one-sided. In contrast to these papers, we model two-sided limited commitment and allow the strategically defaulting counterparty to be endogenously determined by the realizing state of nature.

Also, to our knowledge this is the first paper to discuss the three generic clearing arrangements in a unified framework. In particular, we add segregated collateral clearing to the set of clearing frameworks analyzed. While the literature almost exclusively focuses on the analysis of CCP clearing or on the comparison between bilateral and CCP clearing, we believe that our paper is a first step towards a more comprehensive analysis of clearing. This is important as relevant markets are bilaterally cleared or rely on a third-party clearing agent. For instance, repo markets are often cleared by a so-called tri-party agent, i.e. via segregated collateral clearing. In terms of segregated collateral clearing, a notable exception is Biais, Heider and Hoerova (2012a). They consider bilateral clearing with escrow accounts that ring-fence collateral from moral hazard in an environment with one-sided default risk and exogenous collateralization.

Koepl and Monnet (2010) analyze the benefits of novation and mutualization and discuss

the feasibility to clear non-fungible OTC financial contracts through a CCP. We complement their work by analyzing the welfare gains of clearing. The model enables us to decompose the welfare gains from CCP clearing into (1) the gains a third-party clearer can achieve with segregation, and (2) with novation and mutualization. For instance, the optimal clearing structure might depend on market characteristics, in particular on return (respectively the opportunity cost of collateral), price volatility and the potential for risk diversification (or mutualization). Our model allows to identify conditions when mutualization adds little to welfare. This could explain why certain OTC markets are not cleared by CCPs but by third-party agents that allow for collateral segregation such as tri-party agents in repo markets.

Biais, Heider and Hoerova (2012b) also motivate CCP clearing by the introduction of mutualization. They find that mutualization fully insures against idiosyncratic risk but cannot provide insurance against aggregate risk. Moral hazard and aggregate risk result in a second best where protection buyers also bear risk when there is CCP clearing. In a similar vein, we find full insurance is not achievable even in the absence of aggregate risk and moral hazard. In contrast to them, CCP clearing in our model does not add much through mutualization in the presence of aggregate risk. However, clearing remains beneficial by means of collateral segregation via a third-party.

Haene and Sturm (2009) and Nahai-Williamson et al. (2013) analyze the optimal division of a CCP's waterfall (loss-absorbing financial resources) into defaulter-pays resources (usually associated with margins) and survivors-pay resources (default fund). With exogenous counterparty risk and risk neutral agents, Haene and Sturm (2009) find that it is always optimal to establish a default fund and, in some cases, a sufficiently large default fund is even all it takes. Similarly, Nahai-Williamson et al. (2013) find that default fund contributions are preferable to margins if the probability of default is low and the opportunity cost of collateral is high. We complement their work endogenizing counterparty risk. We go beyond their findings conditioning margins on actually observed price volatility. Then, the optimal CCP arrangement is characterized by a decreasing waterfall ratio with increasing price volatility. Furthermore, the waterfall ratio increases with the opportunity cost of collateral. If the cost of collateral is high, a level can be reached where only mutualization remains active and, if collateral is even more costly, trading stops all together.

Carapella and Mills (2012) investigate an information problem in bilaterally and CCP cleared OTC markets, namely asymmetric information. They emphasize its role in the massive dry up seen in OTC markets during the financial crisis starting 2007, arguing that many OTC products are information sensitive. CCP clearing is understood to make securities

information insensitive and, as a consequence, to prevent market dry up. In contrast, we highlight the fact that markets may simply vanish if the pay-off structure of financial products changes adversely such that it is too costly to put up collateral to enforce trades. Rather, agents prefer to face their income risk in full instead of smoothing it by means of forward trading. Asymmetric information has certainly been a key source of the financial disruptions seen, however, we show that it is not a necessary condition for a market breakdown. We go beyond this as we are able to relate potential effects on trading to the prevalent clearing framework. With third-party clearers, a complete market breakdown can be withstood for more adverse market conditions. Furthermore, given a diversified clearing portfolio, CCP clearing can prevent a market breakdown for higher levels of return, price volatility and risk aversion than segregated collateral clearing. However, third-party clearers too are not able to prevent a market breakdown if conditions become too adverse.

Amini et al. (2013) use a network representation of OTC markets to analyze contagion effects with and without central counterparty clearing. Their aim is to find the optimal form of clearing to control the value of the financial network or its systemic risk by maximizing stakeholder value while accounting for liquidation costs. They find that a CCP not always reduces systemic risk and provide sufficient conditions for this to hold. Acharya and Bisin (2013) understand CCP clearing as a way to eliminate counterparty risk externalities. Because OTC markets are opaque, market participants with limited liability are unable to find efficient prices. In particular, default risk cannot be priced as it depends on unknown trade positions. This lack of transparency allows agents to build up excessive short positions which provokes inefficient risk sharing and systemic risk.

These papers consider a centralized clearing mechanism or CCP clearing to eliminate or relax externalities as information on trade positions are collected and made transparent or externalities are priced by the collection of collateral. We ignore externalities to focus on the interdependency of trade and clearing. We show that CCP clearing is always welfare increasing absent any set-up costs, although it increases aggregate collateral requirements. Thus, whether mandatory clearing increases welfare crucially depends on the systemic risk effects that origin from counterparty risk externalities. A promising line for future research could, thus, be to analyze the extent of network and counterparty risk externalities in an optimal clearing framework. In addition, one might want to think about information dissemination that allows to internalize remaining externalities.

7. Conclusion

This paper nests the three generic clearing arrangements – namely bilateral clearing, segregated collateral clearing and central counterparty clearing – in a common framework of two-sided limited commitment. We find clearing to be essential as it allows trade to happen. Moreover, investing in more elaborated forms of clearing allows to increase feasible allocations in a Pareto sense, although it requires more collateral. Also, we are able to capture the different instruments of loss-absorbing financial resources applied in actual clearing arrangement to secure the settlement of trades – namely variation margins, initial margins and default fund contributions.

This paper takes a first step in quantifying the welfare gains from different clearing arrangements. While CCP clearing is second-best, we find that most gains may stem from segregating collateral rather than from mutualization. Thus, our findings help to explain the absence of CCP clearing in many markets, a stylized fact a model of clearing should be able to account for. We go beyond this by characterizing the optimal waterfall ratio, the composition of defaulter-pays (initial margins) and survivors-pay instruments (total default fund contributions). We do so in dependence of agent’s risk aversion and the relevant characteristics of the markets cleared, namely the potential for the diversification of idiosyncratic risk, return and price volatility.

To focus on the essence of clearing – namely securing the settlement of obligations resulting from trade – we abstract from many important issues. A CCP might be able to offer other benefits that increase welfare, multilateral netting being one of them. Also, a CCP is often understood as a requirement to allow for anonymous trade on exchanges. Furthermore, we do not consider counterparty risk externalities which could potentially give rise to systemic risk.

There may be other frictions that we have not taken into account and that may affect welfare. For example, one might have to model counterparties more seriously, adding capital frictions for instance, in order to explain prevailing (although suboptimal) clearing frameworks. Finally, we have left issues related to bargaining aside. Market power is a real issue in OTC markets. Further integrating the analysis of trading/bargaining and clearing might enhance our understanding of the functioning of OTC markets. All these important issues are beyond the focus of our paper and are left for future research.

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9. Appendix

9.1. Normalization

Our set-up is a normalized version of the following environment. The return to the productive technology is \tilde{R} . In the safe environment, the price of good Z in terms of good X or Y is $q > 0$, so that agents 1 and 2 are able to consume $q\tilde{R}$ for each unit of investment. In the risky environment, however, the price of good X is $q + \tilde{x}$ in state h and $q - \tilde{x}$ in state ℓ (and inversely for the price of good Y). Here, we could set q to any positive number while \tilde{x} is restricted to be below q . Then we get our original set-up by setting $R = 2q\tilde{R}$ and $p = \frac{1}{2}(1 + \tilde{x}/q)$. Notice that this bears consequences for some of the statistics of the model. In particular, the variance of prices in the original model is $Var(q) = (1 - \sigma)\tilde{x}^2$ while it is $Var(p) = (1 - \sigma)(p - 1/2)^2$ in the normalized model. Therefore, the normalized variance is at most $Var(p) = (1 - \sigma)/4$ which seems at first very small, but the original variance can be as high as $Var(q) = (1 - \sigma)q^2$ which can be made large with q . Finally, notice that R could be quite high. In particular, it would be misleading to think of R as the risk free rate. For example, q could be very large and correspondingly increase R . For instance, if X is gold,

Y is platinum, and Z is bread and \tilde{R} is the risk free rate, then R could be much higher than 2. Therefore, in what follows, we will not restrict R to be necessarily arbitrarily close to a risk free rate. All this to stress that our assumption on prices is only a normalization and, thus, is without any loss of generality.

9.2. Price Volatility

We could introduce price volatility in two ways. First, at $t = 1$ agents know that $p \in [\frac{1}{2}, \bar{p}]$ is distributed according to some distribution function. For simplicity, we assume that agents know that $p \in \{\frac{1}{2}, \bar{p}\}$. Furthermore, the probability that $p = 1/2$ is $1 - \pi$ while the probability that $p = \bar{p}$ is π . To be clear, agents in a risky environment at $t = 1$ will be able to sell their goods at price $1/2$ with probability $1 - \pi$, at price \bar{p} with probability $\pi/2$ and at price $1 - \bar{p}$ with probability $\pi/2$. Then, we can measure price volatility as the standard deviation in market price, which is increasing with \bar{p} . Finally, we assume that there are different volatility regimes (i.e. agents can be in several risky environments at $t = 1$ that differ in their amount of risk). \bar{p} is distributed according to the distribution $\Lambda(\bar{p})$, so that at $t = 0$, agents do not know which \bar{p} they will face at $t = 1$.

Second, instead of just one price together with a distribution across the possible realization, we could simply assume different levels of prices at $t = 2$, i.e. $\tilde{p} \in [p_1, \dots, p_n]$ with $p_i < p_{i+1}$.

Note that the conclusions of the paper are not going to change. What matters is the binding incentive constraint. Looking at the latter way to model price volatility, the binding incentive constraint would be the one for those agents in state h when the price is p_n . Consumption for those agents would then be given by $z_h^*(p_n)$.

9.3. Proofs

Proof of Proposition 1:

If we restrict the analysis to a symmetric contract, then $c_1 = c_2 = c(p) \geq 0$. In this case, notice that $z_h^1 = z_\ell^2 = \bar{z}_h = p(1 - c)R + (1 - p)c$ while $z_\ell^1 = z_h^2 = \bar{z}_\ell = (1 - p)(1 - c)R + pc$. Clearly, $\bar{z} > \underline{z}$ and $s_1 = s_2$. Since $c(p) > 0$, both first order conditions must hold at equality, so that

$$\begin{aligned} R[pu'(\bar{z}_h) + (1 - p)u'(\bar{z}_\ell)] &= pu'(\bar{z}_\ell) + (1 - p)u'(\bar{z}_h) \\ (1 - p)u'(\bar{z}_h) + pu'(\bar{z}_\ell) &= R[(1 - p)u'(\bar{z}_\ell) + pu'(\bar{z}_h)] \end{aligned}$$

Thus, we obtain

$$u'(\bar{z}_h) = \frac{p - R(1 - p)}{pR - (1 - p)} u'(\bar{z}_\ell)$$

This gives us one equation in one unknown which is uniquely solved for c . Therefore, with symmetric collateralization, c is feasible if and only if $p > R(1 - p)$ and π is large enough. Also, notice that in this case $u'(\bar{z}_h) < u'(\bar{z}_\ell)$ as expected. ■

Proof of Corollary 1:

Full insurance requires that $\bar{z}_h = \bar{z}_\ell$ which can only be the case if $R = 1$. To see the second claim, suppose the FOCs are holding with strict inequality at $c_1 = c_2 = 0$, then the solution is no trade. In this case, the surplus from trade is equal, i.e. $s_2 = s_1$. Hence, this is the case when

$$\begin{aligned} R[pu'(pR) + (1 - p)u'((1 - p)R)] &> \\ pu'((1 - p)R) + (1 - p)u'(pR) & \end{aligned}$$

or

$$R > \frac{pu'((1 - p)R) + (1 - p)u'(pR)}{(1 - p)u'((1 - p)R) + pu'(pR)} \quad (19)$$

Notice that as $p > 1 - p$ then $u'((1 - p)R) > u'(pR)$, and the RHS of expression (19) is greater than 1. The derivative of the RHS with respect to R is

$$\frac{(2p - 1)}{(\text{denominator})^2} \frac{u'(pR)u'((1 - p)R)}{R} \left[\frac{(1 - p)Ru''((1 - p)R)}{u'((1 - p)R)} - \frac{pRu''(pR)}{u'(pR)} \right] = 0$$

as we assumed that the utility is isoelastic, with a constant coefficient of relative risk aversion. Therefore there is \bar{R}_1 such that for all $R > \bar{R}$, then $c = 0$. Since the RHS is always greater than 1, if \bar{R} exists, then \bar{R} satisfies expression (19) at equality. ■

Proof of Corollary 2:

First, we can show that full insurance cannot be achieved with $R > 1$. Indeed, suppose that $\hat{z}_\ell = \hat{z}_h$. This requires that $\hat{c} > 0$, so that the first order conditions hold with equality, and, from expression (13), we necessarily have $R = 1$. Second, we can show that given the (symmetric) solution $\bar{c}(p)$ of the bilateral clearing case, there is more insurance built in the third-party clearing case. To show this, it suffices to compare the level of consumption,

$(\bar{z}_h, \bar{z}_\ell)$ in the bilateral and $(\hat{z}_h, \hat{z}_\ell)$ in the tri-party clearing case for the same amount of collateral, i.e.

$$\begin{aligned} p(1 - \bar{c})R + (1 - p)\bar{c} &= \bar{z}_h > \hat{z}_h = p(1 - \bar{c})R \\ (1 - p)(1 - \bar{c})R + p\bar{c} &= \bar{z}_\ell < \hat{z}_\ell = (1 - p)[(1 - \bar{c})R + \bar{c}] + p\bar{c} \end{aligned}$$

Third, we can show that the threshold \hat{R} such that there is no trade with tri-party clearing is higher than the threshold \bar{R} under bilateral clearing. Indeed, suppose the FOCs are holding with strict inequality in tri-party clearing at $c_1 = c_2 = 0$, then the solution is no trade. Hence, this is the case when

$$R > \frac{u'((1 - p)R)}{pu'(pR) + (1 - p)u'((1 - p)R)} \quad (20)$$

Using the same argument as for the bilateral clearing case, we obtain \hat{R} solving for expression (20) with equality. Now we want to compare \bar{R} with \hat{R} . Concavity implies that $pu'((1 - p)R) + (1 - p)u'(pR) < u'((1 - p)R)$. Therefore, the RHS of expression (19) is lower than the RHS of expression (20) when evaluated at the same R . As a consequence, evaluated at \hat{R} , we obtain that expression (19) holds with strict inequality. Therefore, it must be that $\bar{R} < \hat{R}$. ■

Proof of Corollary 3:

We know that under bilateral clearing there is no trade for $\hat{R} > R \geq \bar{R}$, while there is trade with segregated collateral clearing, i.e. $\hat{c}(p, R) > \bar{c}(p, R) = 0$ for $R \in [\bar{R}, \hat{R})$. Now consider optimal collateralization for $R = 1$. We know that both with bilateral and with segregated collateral clearing, full insurance can be achieved and is optimal. We further know that full insurance with bilateral clearing results in $\bar{c} = \frac{R}{R+1} = \frac{1}{2}$. With segregated collateral clearing perfect insurance and $R = 1$ imply $\hat{c} = \frac{2p-1}{2p}$. Thus, $\hat{c}(R) < \bar{c}(R)$ for $R = 1$. Because the problem is well-behaved, by continuity, we can deduce that there is a $\tilde{R} \in [1, \bar{R})$ such that for $R \geq \tilde{R}$ then $\hat{c}(R) \geq \bar{c}(R)$, while for $R < \tilde{R}$ then $\hat{c}(R) < \bar{c}(R)$ and $\hat{c}(\tilde{R}) = \bar{c}(\tilde{R})$. ■

Proof of Proposition 5:

To understand the impact of d and m notice how they affect the CCP's feasibility constraint. If we denote the right-hand side of the feasibility constraint by F (these are the resources available to the CCP), we have $\partial F / \partial d = \frac{1}{2}(1 - R)$. Therefore, increasing the default fund reduces the CCP's resources by the forgone return $1 - R$ on investment in the safe and risky projects. Similarly, $\partial F / \partial m(\bar{p}) = \frac{(1-\sigma)}{2}(1 - R)$. Thus, by increasing margins,

the CCP gives up the return on the risky investment only. Hence, for the sake of saving resources, margins are preferred to default funds. However, notice that the CCP has an insurance motive, as it wants to have z , z_h and z_ℓ as close as possible to each other. Looking at the interim PC for the safe agents, the CCP may want to increase the default fund in order to achieve more insurance across agents. So, we guess that the CCP will first increase m leaving $d = 0$ and then increase d .

Now, suppose that the participation constraint of those agents in the risky state is not binding. Then the interim participation constraints for agents in the safe state at date $t = 1$ has to bind. The reason is simple. Suppose this constraint does not bind. Then, the CCP can decrease d by a little and increase $m(p)$ by the same amount to keep $d + m(p)$ constant for all p . Then no incentive constraints are modified and the CCP can free up resource, as $\frac{\partial F}{\partial m(\bar{p})} - \frac{\partial F}{\partial d} = \frac{1}{2}\sigma(R - 1) > 0$. Therefore, the interim participation constraint of the safe agents has to bind so that $z(p) = R(1 - d)/2$.

Therefore, given d the consumption levels for agents h and ℓ are solving the following sub-problem:

$$\begin{aligned}
& \max_{m(p), z_i(p)} \frac{(1 - \sigma)}{2} [u(z_h(p)) + u(z_\ell(p))] \\
& \text{subject to} \\
& (1 - \sigma)(z_h(p) + z_\ell(p)) = \text{Feasibility} \\
& (1 - \sigma)(1 - d - m(p))R + \sigma d + (1 - \sigma)(m(p) + d) \\
& u(z_h(p)) + u(z_\ell(p)) \geq u(p(1 - d)R) + u((1 - p)(1 - d)R) \quad \text{Interim PC} \\
& z_h(p) \geq pR(1 - d - m(p)) \text{ for all } p \quad \text{IC} \\
& 1 - d - m(p) \geq 0 \text{ for all } p \quad \text{for } i = s, r
\end{aligned}$$

where we have already eliminated the IC of agents in state ℓ as it is never binding with $p \geq 1/2$. Looking at this problem, it should be clear that incentive compatibility constraint in the high state binds. If it does not, then the CCP could reduce $m(p)$ to increase resources without changing any incentive or feasibility constraints. Hence,

$$z_h(p) = pR(1 - d - m(p)). \quad (21)$$

Therefore, in state p , the consumption of agent ℓ is given by the feasibility constraint,

$$z_\ell(p) = (1-p)R(1-d-m(p)) + \frac{1}{1-\sigma}d + m(p). \quad (22)$$

Therefore, given d , the CCP solves¹⁴

$$\begin{aligned} & \max_{m(p)} u(z_h(p)) + u(z_\ell(p)) \\ & \text{subject to} \\ & u(z_h(p)) + u(z_\ell(p)) \geq u(p(1-d)R) + u((1-p)(1-d)R) \quad \text{Interim PC} \\ & 1-d-m(p) \geq 0 \text{ for all } p \quad \text{for } i = s, r \end{aligned}$$

where $z_h(p)$ and $z_\ell(p)$ satisfy (21) and (22) respectively. Since the CCP will provide some settlement insurance, we can work under the assumption that Interim PC does not bind. Hence, the first order condition for $m(p) \in [0, 1-d]$ gives us

$$-pRu'(z_h(p)) + (1-(1-p)R)u'(z_\ell(p)) \stackrel{\leq}{=} 0 \quad (24)$$

(with equality if $m(p) \in (0, 1-d)$, with $>$ if $m(p) = 1-d$ and with $<$ if $m(p) = 0$). Therefore, as long as $R > 1$ the CCP does not fully insure agents in any state \bar{p} . If $m(p) \in (0, 1)$ then

$$u'(z_h(p)) = \frac{1-(1-p)R}{pR}u'(z_\ell(p)).$$

That is the transformation rate is the same under the CCP as under segregation.

Given expressions (21)-(24), the CCP chooses d in order to maximize the ex-ante utility

¹⁴The CCP could set $m(p)$, so that $z_h(p) = z_\ell(p)$. In this case the CCP would achieve full insurance as of period 1. In particular $m(p)$ would be given by

$$m(p) = \frac{(2p-1)R(1-d) - \frac{1}{1-\sigma}d}{1-R(2p-1)}. \quad (23)$$

However, this may turn out to be too expensive for the CCP, so that we cannot assume that the CCP would set such a high margin.

of agents (using λ_i as the probability to end up in state i):

$$\begin{aligned} \max_d \sigma u(z) + \frac{(1-\sigma)}{2} [u(z_h(p, d)) + u(z_\ell(p, d))] \\ \text{subject to} \\ u(z_h(\bar{p}, p)) + u(z_\ell(\bar{p}, p)) \geq u(p(1-d)R) + u((1-p)(1-d)R) \quad \text{Interim PC} \\ d \in [0, 1] \end{aligned}$$

as well as the ex-ante participation constraint. Assuming that no constraints are binding and using expressions (21)-(24) as well as

$$z = (1-d)\frac{R}{2},$$

we obtain the following first order condition for d being an interior solution:

$$-\sigma \frac{R}{2} u'((1-d)\frac{R}{2}) + \frac{(1-\sigma)}{2} \left[-p R u'(z_h(p)) - \left((1-p)R - \frac{1}{1-\sigma} \right) u'(z_\ell(p)) \right] = 0$$

Since we will be interested in the case where $d > 0$ and $m(\bar{p}) > 0$ for all \bar{p} , we can use (24) at equality to simplify the FOC for d as

$$u'(z_\ell(p)) = R u'((1-d)\frac{R}{2}).$$

■

Proof of Corollary 4:

Using the first order conditions of the CCP problem with respect to m , we obtain that $m(R) = 0$ whenever

$$R > \frac{u'((1-p)R(1-d) + \frac{1}{1-\sigma}d)}{p u'(pR(1-d)) + (1-p)u'((1-p)R(1-d) + \frac{1}{1-\sigma}d)} \quad (25)$$

Notice that the RHS is equal to the RHS of (20) whenever $d = 0$, while it is decreasing with d . Therefore, when $d \geq 0$, and $R \geq \hat{R} > RHS$ then $m^*(d) = 0$. Also, the RHS of (25) is constant in R (given d) so (25) defines $R_m^*(d) < \hat{R}$. To show that $R^* > \hat{R}$, notice that the

RHS of (25) equals \hat{R} when $d = 0$. However, $d = 0$ whenever

$$-\sigma \frac{R}{2} u' \left(\frac{R}{2} \right) + \frac{1-\sigma}{2} \left[\frac{1}{1-\sigma} u'((1-p)R) - pRu'(pR) - (1-p)Ru'((1-p)R) \right] \leq 0$$

Evaluating the LHS at \hat{R} we obtain using (20),

$$-\sigma \frac{\hat{R}}{2} u' \left(\frac{\hat{R}}{2} \right) + \frac{1-\sigma}{2} \left[\frac{1}{1-\sigma} u'((1-p)\hat{R}) - u'((1-p)\hat{R}) \right]$$

and arranging,

$$-\sigma \frac{\hat{R}}{2} u' \left(\frac{\hat{R}}{2} \right) + \frac{1}{2} \sigma u'((1-p)\hat{R})$$

But this is strictly positive as $u'(\frac{\hat{R}}{2}) < pu'(p\hat{R}) + (1-p)\hat{R}u'((1-p)\hat{R}) = u'((1-p)\hat{R})$. ■

Proof of Corollary 5:

Suppose that $m(p) \in (0, 1-d)$. We want to show that $m'(p) > 0$. As (24) holds with equality, we can use the implicit function theorem to obtain $m'(p)$. To save on notation, we use $z_i = z_i(p)$ for $i = \ell$ and h . Then

$$pRu'(z_h) - (1 - (1-p)R)u'(z_\ell) = 0.$$

and taking the derivative with respect to $m(p)$ and p we obtain

$$\begin{aligned} R[u'(z_h) - u'(z_\ell)] + pRu''(z_h) \frac{dz_h}{dp} - (1 - (1-p)R)u''(z_\ell) \frac{dz_\ell}{dp} &= 0 \\ R[u'(z_h) - u'(z_\ell)] + pRu''(z_h) \left(\frac{\partial \bar{z}_h}{\partial \bar{p}} - pRm'(p) \right) \\ - (1 - (1-p)R)u''(z_\ell) \left(\frac{\partial \bar{z}_\ell}{\partial \bar{p}} + (1 - (1-p)R)m'(p) \right) &= 0 \end{aligned}$$

so that

$$\begin{aligned} R[u'(z_h) - u'(z_\ell)] + pRu''(z_h) \frac{\partial \bar{z}_h}{\partial \bar{p}} - (1 - (1-p)R)u''(z_\ell) \frac{\partial \bar{z}_\ell}{\partial \bar{p}} \\ = [(pR)^2 u''(z_h) pR + (1 - (1-p)R)^2 u''(z_\ell)] m'(p) \end{aligned}$$

Since $u'(\bar{z}_\ell) > u'(\bar{z}_h)$, $\frac{\partial \bar{z}_\ell}{\partial \bar{p}} < 0$ and $\frac{\partial \bar{z}_h}{\partial \bar{p}} > 0$ the LHS of the inequality is negative. Also, the RHS of the equality is only negative if $m'(\bar{p}) > 0$. ■

Proof of Proposition 6:

To show this result, define the function $m(d)$ such that given d , $m(d)$ is defined implicitly by (14)-(15) and (17). Then we know that $m(0) = \bar{c}$. Therefore, we want to know how $d + (1 - \sigma)m(d)$ is changing with d . And if $1 + (1 - \sigma)m'(d) > 0$ for all d then we are done as $d \geq 0$. We can compute $m'(d)$ implicitly using (14)-(15) and (17) to find

$$\begin{aligned} m'(d) &= -\frac{\frac{u'(z_h)}{u'(z_\ell)} \left[\frac{1}{1-\sigma} - (1-p)R \right] + \frac{u''(z_h)}{u''(z_\ell)} pR}{\frac{u'(z_h)}{u'(z_\ell)} \left[1 - (1-p)R \right] + \frac{u''(z_h)}{u''(z_\ell)} pR} \\ &= \frac{1}{1-\sigma} \frac{\frac{u'(z_h)}{u'(z_\ell)} \left[1 - (1-p)(1-\sigma)R \right] + \frac{u''(z_h)}{u''(z_\ell)} pR(1-\sigma)}{\frac{u'(z_h)}{u'(z_\ell)} \left[1 - (1-p)R \right] + \frac{u''(z_h)}{u''(z_\ell)} pR} \end{aligned}$$

Clearly, $m'(d) < -1$ as long as $\sigma < 1$. Plugging this expression back in $1 + (1 - \sigma)m'(d)$ and arranging, we obtain

$$\begin{aligned} 1 + (1 - \sigma)m'(d) &= 1 - \frac{\frac{u'(z_h)}{u'(z_\ell)} \left[1 - (1-p)(1-\sigma)R \right] + \frac{u''(z_h)}{u''(z_\ell)} pR(1-\sigma)}{\frac{u'(z_h)}{u'(z_\ell)} \left[1 - (1-p)R \right] + \frac{u''(z_h)}{u''(z_\ell)} pR} \\ &= \frac{\sigma \left[\frac{u''(z_h)}{u''(z_\ell)} pR - \frac{u'(z_h)}{u'(z_\ell)} (1-p)R \right]}{\frac{u'(z_h)}{u'(z_\ell)} \left[1 - (1-p)R \right] + \frac{u''(z_h)}{u''(z_\ell)} pR} \end{aligned}$$

Then we can use the fact that risk aversion is constant, to simplify further, as

$$\begin{aligned} 1 + (1 - \sigma)m'(d) &= \frac{u'(z_h)}{u'(z_\ell)} \frac{\sigma \left[\frac{u''(z_h)}{u''(z_\ell)} \frac{u'(z_\ell)}{u'(z_h)} pR - (1-p)R \right]}{\frac{u'(z_h)}{u'(z_\ell)} \left[1 - (1-p)R \right] + \frac{u''(z_h)}{u''(z_\ell)} pR} \\ &= \frac{u'(z_h)}{u'(z_\ell)} \frac{\sigma \left[\frac{u''(z_h)}{u''(z_\ell)} \frac{u'(z_\ell)}{u'(z_h)} \frac{z_h}{z_\ell} \frac{z_\ell}{z_h} pR - (1-p)R \right]}{\frac{u'(z_h)}{u'(z_\ell)} \left[1 - (1-p)R \right] + \frac{u''(z_h)}{u''(z_\ell)} pR} \\ &= \frac{u'(z_h)}{u'(z_\ell)} \frac{\sigma \left[\frac{z_\ell}{z_h} pR - (1-p)R \right]}{\frac{u'(z_h)}{u'(z_\ell)} \left[1 - (1-p)R \right] + \frac{u''(z_h)}{u''(z_\ell)} pR} \end{aligned}$$

Hence, $1 + (1 - \sigma)m'(d) > 0$ iff $\frac{z_\ell}{z_h} pR - (1-p)R > 0$. Now at $d = 0$ we obtain

$$\begin{aligned} \frac{z_\ell(0)}{z_h(0)} pR - (1-p)R &= \frac{(1-p)R(1-m) + m}{pR(1-m)} pR - (1-p)R \\ &= \frac{m}{(1-m)} > 0 \end{aligned}$$

so that $1 + (1 - \sigma)m'(d) > 0$ at $d = 0$. Therefore, increasing the default fund slightly from zero will increase the overall collateral of the CCP beyond what is required from segregated clearing, while increasing welfare. This proves the claim for d close to zero. Now, recall that preferences represented by a constant relative risk aversion utility function are homogeneous. Therefore (17) implies that any CCP solution lies along the same ray from the origin than $(\bar{z}_h, \bar{z}_\ell)$. Therefore, $z_\ell(0)/z_h(0) = z_h^*/z_\ell^*$ so that $1 + (1 - \sigma)m'(d) > 0$ for all d (note that we have used $d = 0$ only in the last step of the derivation above). This concludes the proof. ■

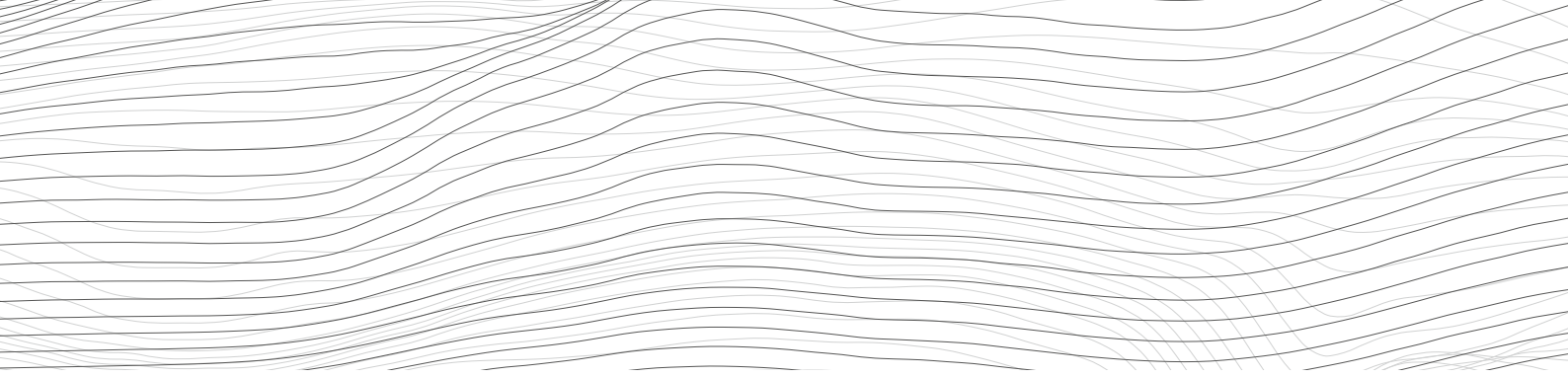
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