Central Bank Account for All: Efficiency and Risk Taking
(preliminary and incomplete)

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Abstract

We analyze the effects on the banking sector and the economy of an interest-bearing central bank digital currency (CBDC), that competes with bank deposits as a medium of exchange. Monopolistic banks lend to fund productive investment projects. We show that a CBDC bearing interest promotes intermediation because it forces banks to increase the remuneration on their deposits. This increases their value when deposits are used as means of payment. Increasing the rate on CBDC makes banks' deposits more valuable and investment less expensive. When considering banking sector risk, we find that high levels of CBDC interest rate induce banks to hold a higher portion of risk-free assets in their portfolio than is efficient. We show that when CBDC and reserves pay the same positive interest, increasing the remuneration on CBDC promotes investment efficiency.

Keywords: Central bank digital currency, banking sector, disintermediation, risk

JEL codes: E42, E50

1 Introduction

The issuance of a central bank digital currency (CBDC) places high on the current agenda of most central banks. Although the introduction of a CBDC may only seem a natural step in line with the ongoing digitization of the economy, issuing a retail CBDC can have dramatic implications for the banking sector and the financing of the real economy.

Retail CBDC that is widely accepted as a medium of exchange (and can be used as a store of value) could be considered a perfect substitute to insured bank deposits and better than bank deposits when the latter are not fully insured. Hence, policymakers fear that the introduction of CBDC results in withdrawals of deposits into the safe haven of CBDC thus forcing banks to increase the remuneration of deposits as a compensation for the risk of holding deposits relative to CBDC. This would affect bank's funding cost, which could increase lending rates and reduce lending volume. Hence, overall the fear is that CBDC may lead to disintermediation and increase the fragility in the banking sector.

In this paper we analyze the efficiency and stability consequences of giving households access to an interest-bearing CBDC as a means of payment. We focus on the effect CBDC has on the level of bank intermediation, captured as the volume of bank lending into productive investment projects and the composition of these investments into safe and risky projects. Furthermore, we characterize the optimal interest rate on CBDC consistent with efficient investment levels and comment on the ideal relationship between CBDC and reserves.

We introduce interest-bearing CBDC and a monopolistic banking sector in Lagos and Wright (2005). Bankers lend to entrepreneurs that have access to risky investment projects, which payoff is perfectly correlated. One project requires one unit of capital. Bankers are endowed with a monitoring technology. Monitored projects become risk-free and they pay a guaranteed return. However monitoring is costly and more so as more projects are monitored. To fund their loans to entrepreneurs, bankers issue deposits to households. Households acquire bank deposits with CBDC. In turn, bankers use the CBDC to lend to entrepreneurs. And entrepreneurs use the borrowed CBDC to buy capital from suppliers. Therefore, one unit of CBDC will buy more capital when the CBDC pays a higher interest rate. In other words, an increase in the interest rate paid on CBDC lowers the real price of a unit of capital.

Households are indifferent between using bank deposits or CBDC as means of payment as long as the risk adjusted remuneration is the same across both payment instruments, Therefore, as the interest rate paid on CBDC increases, bankers need to increase the value of their deposits to attract and retain depositors.

As a benchmark case we assume that risky projects are never successful and always yields zero. Therefore, bankers will always monitor all projects they invest in. Bankers are also able to hold central bank reserves, which are risk-free assets. Thus, we refer to this case as the risk-free benchmark.

In this benchmark, bankers set the interest rate paid on deposits equal to the interest rate paid on CBDC. Therefore, depositors enjoy a higher level of consumption when the CBDC rate increases as bank deposits can buy more goods. Furthermore, we find that introducing an interest-bearing CBDC leads to increased intermediation in the banking sector because depositors are willing to then hold greater CBDC balances. This leads to an equivalent increase in the volume of deposits. Bankers respond to the increased level of funds by increasing their lending. Once the interest rate on CBDC reaches a certain threshold, the level of lending attains the socially efficient level of investment, and any further increase in the CBDC interest rate leads banks to increase their reserve holdings. Therefore, beyond the threshold, disintermediation occurs in the sense that level of lending reduces relative to the size of the banker balance sheet. However, this disintermediation result is optimal.[1] In

[1] As in Chiu et al (2020), depositors do not need to hold CBDC for increased intermediation. The mere existence of an interest-bearing CBDC is enough. Chiu et al (2020) also find increased inter-

order to achieve the socially efficient investment level, the central bank needs to set the CBDC interest rate equal to or higher than the interest rate paid on reserves. Otherwise, if the interest rate on reserves is too high it crowds out investment, leading to under-investment for all values of CBDC interest rate.

Depositors' consumption increases as the interest on CBDC, and thus deposits, increases. Furthermore, and perhaps surprisingly, the profit of banks does not decline when the interest rate on CBDC increases. Since CBDC is used as a means of payment to acquire capital, capital becomes cheaper as the interest rate on CBDC increases. This makes investment, and hence bank lending, more profitable despite higher funding costs.[2]Therefore, CBDC increases overall welfare. It is optimal for the central bank to set the interest rate paid on CBDC equal to the rate an illiquid asset would command given inflation – the equivalent to the Friedman rule in our model – allowing for optimal investment and consumption levels.

We then consider the economy when projects have a positive net present value and give a positive payoff with some probability (in the high state) and zero otherwise (in the low state). Then bankers can hold three assets: monitored projects and central bank reserves that are risk-free, and unmonitored projects that are risky. The bankers' deposit contract then specifies payments in the high state and the low state. The larger the dispersion between payments in the high and low states is, the greater the risk to depositors is. Unlike conventional wisdom, incorporating the risk dimension does not make bank disintermediation more likely. For CBDC interest rate below a certain threshold, it is optimal for bankers to monitor a large fraction of projects, which ensures they always have enough resources to guarantee that their deposits pay the same amount in both states.

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mediation for low levels of CBDC interest rate, while if the rate becomes too high, disintermediation occurs.

[2]Our result is reminiscent of the work of Stockman (1981), among others. Stockman showed that, when capital investment is modeled as requiring monetary trade, inflation has a negative effect on the capital stock that reverts the classic Tobin effect

The investment decision of banks is efficient, as long as the interest rate on reserves is low enough. When the interest rate on reserves is too high, banks substitute some of their productive investment by reserves, which is inefficient. Therefore, as in Andolfatto (2018), disintermediation is driven by interest rate on reserves and not by the interest rate on CBDC, because the interest rate on reserves is the banks' true opportunity cost of investment. Still, CBDC introduces inefficiencies in the composition of the investments when its remuneration is too generous: As the interest rate on CBDC increases, bankers need to make deposits more valuable. To do so, bankers prefer to monitor projects to guarantee a safe return. Then they can increase the payments in both state by a little, which keeps their deposit contracts safe. Otherwise they would make their deposit contracts risky, and they would have to pay a risk premium to depositors in the high state. This however yields to over-investment in monitored projects relative to what a planner would choose. Since monitoring projects has a convex cost, banks will eventually prefer to introduce some risk into deposit contracts once the amount of deposits becomes too large (when the CBDC rate is high). When deposit contracts are risky, increasing the interest rate paid on reserves reduces the level of over-investment in the monitored assets but also reduces the level of risk on banks' balance sheet.

1.1 Literature review

The feature that makes CBDC be welfare improving in this paper is closely linked to the insight in Calomiris and Kahn (1991). They show that demandable debt, such as demand deposit, is an important incentive scheme for disciplining banks. Withdrawal of funds is a vote of no-confidence in the activity of the banker. Due to being able to withdraw, depositor have an incentive to monitor banks. Calomiris and Kahn (1991) demonstrate that the threat of withdrawal and forced liquidation due to the depositors monitoring of the bank will discipline the bank. We also show that threat of withdrawals disciplines the banking sector. However, the focus of our paper differs from Calomiris and Kahn (1991) as we show that it is the value of the depositors outside option that disciplines banks. The higher the interest on a central bank account, the more discipline the outside option will provide. Furthermore, Calomiris and Kahn (1991) do not focus on disintermediation in the banking sector.

There is a growing literature on the issuance of a central-bank digital currency and its effects on the banking sector. Among the recent papers that study the introduction of CBDC, Brunnermeier and Niepelt (2019) argue that a CBDC need not affect equilibrium allocations since it involves a swap of commercial-bank liabilities and central-bank liabilities with similar liquidity properties. In their framework, a CBDC would not reduce credit or crowd out investment if combined with the appropriate monetary policy. Fernandez-Villaverde et al. (2020) build a Diamond-Dybvig model to assess how a CBDC would affect banks' maturity transformation. Their analysis shows that the introduction of central-bank accounts open to the public could result in the central bank being a monopolistic provider of deposits due to its special strength to deal with bank runs.[3] Keister and Sanches (2019) focus on the trade-off that results from the increase in the supply of safe assets: banks' funding costs increase and thereby investment decreases, but at the same time exchange becomes more efficient. Andolfatto (2018) and Chiu et al. (2019) show that CBDC can improve the efficiency of intermediation when the banking market is not competitive. However, these papers abstract from bank risk.[4] By introducing an endogenous bank monitoring decision, we add another dimension into this analysis and show how the introduction of CBDC affects depositors' consumption risk.

Currently, access to a central bank's reserve account is restricted to eligible de-

[3]Kahn et al. (2018) explore the link between the issuance of central bank digital currency in the form of accounts or tokens and competition in the banking sector, and in particular discuss the incentives of private banks to distribute central-bank tokens to their customers. Garratt and Zhu (2021) study the effects of CBDC on the banking sector when banks differ in their market share.

[4]Skeie (2020) studies the banking sector response to the issuance of a central-bank digital currency in a high inflation country.

pository institutions. With CBDC, everyone, including non-financial businesses and individuals would be able to open a deposit account at the central bank. Cecchetti and Schoenholtz (2017) argue that the offering of a CBDC can lead to financial instability. By having a risk-free outside option, agents will withdraw deposits at the first sign of instability in financial markets, which can lead to self-fulfilling bank panic as demonstrated in Diamond and Dybvig (1983). Williamson (2020) studies how the replacement of physical money by digital central-bank money affects the occurrence of bank runs. While CBDC favours financial instability, the bank run outcome is not that socially costly as it is in a physical-currency regime, as the payments systems is less severely disrupted.Our analysis of bankers' response to CBDC availability suggests another angle to address this issue: banks adapt to CBDC competition as a depositors' risk-free outside option by changing their balance sheet and highly investing in safe assets. As a result, consumption risk is higher in a CBDC regime only when a relatively high level of average consumption has been attained.[5]

2 Environment

The model builds on the Lagos and Wright (2005) alternating markets framework. Time is discrete and continues forever. The economy is populated by a continuum of five types of agents: buyers, sellers, bankers, suppliers and entrepreneurs, each with measure one. All agents discount across periods at rate $\beta \in (0, 1)$. There are two perishable consumption goods: a numeraire good and a consumption good, and two nominal assets: bank deposits and central bank money. Each period is divided into two sub-periods: decentralized market (DM) where the consumption good is traded followed by a centralized market (CM) which is the market for the numeraire.

Buyers' preferences within a period are given by $u(c)+x-h$. Buyers obtain utility

[5]By considering another aspect of bank activities, the provision of credit lines, Piazzesi and Schneider (2020) argue that CBDC can have negative welfare effects.

$u(c)$ from consuming c units of the consumption good in the DM, with $u'(c) > u(0) = 0 > u''(c)$. We assume linear (dis)utility in the CM, where x is CM consumption of the numeraire and h is CM labor supply. Buyers labor supply produces the numeraire good using a one-to-one production technology.

Sellers' preferences within a period are given by $-h_s + x_s$. Sellers produce the consumption good at linear effort cost h_s in the DM and obtain linear utility x_s from consuming the numeraire in the CM. Buyers and sellers meet bilaterally and at random in the DM. However, there are no matching frictions and every buyer will meet a seller in the DM. We assume that the buyer makes a take-it-or-leave-it offer to the seller. We assume anonymity in the DM and that agents cannot commit. Therefore, sellers require compensation in the form of a medium of exchange.

There exists a central bank that can potentially issue three types of liabilities. Physical currency, central bank digital currency (CBDC) and reserves. Physical currency is only accepted by a portion $\alpha \in (0, 1)$ of sellers and does not pay interest. In the baseline model, we assume the central bank only issues CBDC and leave the case with physical currency for an extension of the model. CBDC is an interest bearing digital currency that is accepted by all sellers (same as bank deposits).[6] It pays gross interest $(1 + i_m)$ with $i_m \leq \frac{\gamma}{\beta} - 1$. Here $\gamma \equiv \frac{\phi_t}{\phi_{t+1}}$ denotes the gross inflation rate and ϕ_t is the price of money in terms of the numeraire at time t. When $i_m = \frac{\gamma}{\beta} - 1$, the central bank runs the Friedman rule. We consider that CBDC is available to households and we analyze how our results change when bankers have access to it or not. Reserves is an interest-bearing digital currency that is only available to bankers. It pays gross interest $(1 + i_r)$. Physical currency and CBDC are two types of central bank monies.

Buyers work for central bank money in the CM at time $t - 1$ to bring as medium of exchange into DM at time t. Before meeting with a seller in the DM, the buyer

[6] The fact that it is digital is important when we interpret $1 - \alpha$ as the share of transactions that are conducted with online sellers.

can meet with a banker. Bankers can issue deposits $\phi_t d_t$ at the beginning of period t and before the DM opens. Deposits and money can be traded one-for-one. Bankers make buyers take-it-or-leave-it offers in terms of a deposit contract. If buyers accept, they will deposit their money and bring deposits as medium of exchange into the DM to trade with sellers.

We assume that bankers use the central bank money received from depositors to lend to entrepreneurs. Entrepreneurs have access to investment projects. Entrepreneurs buy capital from suppliers using central bank money, and invest one unit of capital good in each investment project. Suppliers are hand-to-mouth agents, and they use the proceeds from selling the capital to consume in the subsequent CM.[7]

Bankers are monopolistic and thus have full market power. In this model, bankers have a useful role because they increase the net present value of projects. Specifically, bankers have access to a technology to perfectly monitor entrepreneurs: monitored projects yield R with probability 1. Bankers choose how many projects q to invest in and monitor. The cost of monitoring q projects is $\kappa(q)$, with $\kappa'(q) > 0$ and $\kappa''(q) > 0$ and q a continuum of projects. Banker's return from lending to and monitoring q projects is $R - \kappa(q)$. When bankers do not monitor a project, they incur no monitoring cost, but the project only returns R with probability p and 0 otherwise, so that these projects are risky. Bankers choose the number of risky projects, n. We refer to the q projects and the n projects as "monitored" and "risky", respectively.[8] In a first benchmark, we will assume that $p = 0$ so that a bank will always monitor projects and will never invest in risky projects. Therefore, effectively the bankers' assets are all risk free and we refer to this case as the "risk-free" benchmark. In the general case, we assume risky projects have positive net present value, $pR > 1$.

[7]This timing implies that buyers money holdings are sunk. That is, when the buyer meets with banker they cannot work anymore.

[8]In a more general specification, the low-monitoring technology would potentially entail a lower (positive) cost. The cost of monitoring the q projects and the n projects would be $\kappa(q + \nu n)$ with $\nu < 1$. In this setup, $\nu = 0$.

In addition to lending to entrepreneurs, bankers can also invest in reserves, r. Reserves is central bank money that pays gross interest $(1 + i_r)$. Bankers have no endowment other than their monitoring technology and they must borrow central bank money in order to invest. Bankers borrow central bank money from buyers by issuing tradable demand deposits (IOUs) to fund their investment.[9]

3 Risk-free benchmark model

We begin by analyzing the case when $p = 0$ so bankers have to monitor each and every loan they make. Then all of their assets are effectively risk-free, i.e. their investment and reserves. This risk-free case provides the benchmark for our analysis.

3.1 Planner solution

When we consider the efficiency of the decentralized economy, it will make sense to consider the problem of a planner at the time of the DM, once buyers have made a choice of how much real balances z to bring. For such a constrained planner that maximizes welfare every period, the planner solves:

$$\max_{q,c \geq 0} u(c) + z - c + Rq - q - \kappa(q)$$

subject to

$$q \leq z$$

where z is the real value of deposits brought by buyers. The efficient solution satisfies

$$u'(c^*) = 1$$

[9]Effectively, banks in our model are pass-throughs between buyers and entrepreneurs. However, we would obtain similar results in an environment where banks could issue deposits to entrepreneurs, subject to a minimum reserve requirements that have to be fulfilled with central bank money. Banks would obtain this central bank money by issuing deposits to buyers.

and for investment, $q = z$ if $\kappa'(z) < R - 1$ and $q = q^*$ with

$$\kappa'(q^*) = R - 1$$

otherwise. In this latter case the planner holds $z - q^*$ in storage (reserves).

3.2 Buyers

In the CM, buyers consume the numeraire good, supply labor and readjust their portfolio. In particular, they redeem deposits and CBDC holdings and decide on the real balances to bring forward into the subsequent period. Let $W_t^b(z_t, \delta_t)$ denote the value function of a buyer who holds an amount $z_t = \phi_t m_t$ of CBDC paying interest rate $i_m \leq \frac{\gamma}{\beta} - 1$ and a real value $\delta_t = \phi_t d_t$ of bank deposits paying interest rate i_d, at the beginning of the CM.

$$W_t^b(z_t, \delta_t) = x - h + \beta V_{t+1}^b(z_{t+1})$$

subject to

$$x + \gamma z_{t+1} = h + (1 + i_m)z_t + (1 + i_d)\delta_t + T$$

The function V_{t+1}^b represents the DM value function for buyers.

Simplifying:

$$W_t^b(z_t, \delta_t) = (1 + i_m)z_t + (1 + i_d)\delta_t + T + \max_{z_{t+1}} \left\{-\gamma z_{t+1} + \beta V_{t+1}^b(z_{t+1})\right\} \quad (1)$$

In the DM the buyer has all the bargaining power and thus extracts the full surplus from trade, by making a real offer to the seller that just covers the cost of producing c_t. Thus, using CBDC the DM expected payoff for buyers is:

$$V_t^b(z_t, \delta_t) = u(c) - c + W_t^b(0) + (1 + i_m)z_t + (1 + i_d)\delta_t \quad (2)$$

subject to

$$c_t \leq (1+i_m)z_t + (1+i_d)\delta_t \tag{3}$$

Therefore,

$$c_t(z_t,\delta_t) = \begin{cases} c^* & \text{if } c^* \leq (1+i_m)z_t + (1+i_d)\delta_t \\ (1+i_m)z_t + (1+i_d)\delta_t & \text{otherwise} \end{cases} \tag{4}$$

3.2.1 Choice of real balances

Since the buyer can acquire δ_{t+1} deposits with its real CBDC holding from the bank, the buyer's choice of CBDC real balances in the CM is:

$$\max_{z_{t+1},\delta_{t+1}} \{-\gamma z_{t+1} + \beta\left[u(c_{t+1}(z_{t+1}-\delta_{t+1},\delta_{t+1})) + (1+i_m)\left(z_{t+1}-\delta_{t+1}\right) + (1+i_d)\delta_{t+1} - c_{t+1}(z_{t+1},\delta_{t+1})\right]\}$$

s.t.

$$\begin{aligned} c_{t+1} &\leq (1+i_m)z_{t+1} + (1+i_d)\delta_{t+1} \\ \delta_{t+1} &\leq z_{t+1} \end{aligned}$$

We assume $\frac{\gamma}{\beta} > (1+i_m)$ and hence CBDC is costly to hold. Also, since the bank is a monopolist, we guess and verify later that the bank will set $i_d = i_m$, so that buyers are just indifferent between holding CBDC or bank deposits. Thus $c_{t+1} = (1+i_m)z_{t+1}$ and buyer's choice of CBDC balances reduces to:

$$\max_z \{\beta\left[u((1+i_m)z)\right] - \gamma z\}$$

with z solving:

$$u'\left[(1+i_m)z\right] = \frac{\gamma}{\beta(1+i_m)} \tag{5}$$

and $\frac{d((1+i_m)z}{di_m} > 0$. When $1+i_m = \frac{\gamma}{\beta}$, buyers choose the efficient amount of CBDC z^* such that $u'\left(\frac{\gamma}{\beta}z^*\right) = 1$.

3.3 Suppliers

Suppliers produce capital and sell it in a competitive market for nominal price ρ to entrepreneurs. They accept CBDC as payment for capital. Therefore, suppliers receive the interest rate paid on CBDC and thus i_m affects the real return of capital.

Suppliers maximize profits:

$$\max_q \; -q + \phi\rho(1+i_m)q$$

From the first order conditions, $\phi\rho(1+i_m) = 1$. Hence, the price of capital is a decreasing function of the interest rate on CBDC. In other words, the higher i_m is the cheaper it becomes to invest in projects.

3.4 Bankers

Before entering the DM, buyer meets with a banker. The monopolistic banker makes a take-it-or-leave-it deposit contract to a buyer who holds an amount of real balances z. If the buyer accepts the contract, he will transfer his real CBDC holdings z to the banker in exchange for $\phi d \equiv \delta$ units of real deposits, with $z \geq \delta$. The banker is bound by the CBDC holdings buyers bring into the DM given by (5).[10] Therefore, the banker cannot freely choose the level of bank deposits.[11] Rather, the banker can choose whether to issue deposits for all or a portion of the CBDC holdings buyers bring. Given δ, the deposit contract stipulates the interest rate paid on deposits i_d. If $z = \delta$, then the buyer can use bank deposits equivalent to the real value $c = (1+i_d)\delta$ with sellers in the DM. If $z > \delta$, then the buyer uses both

[10] Banks are bound by market conditions when issuing deposits. Hence, there is a limit to the amount of deposits bankers can issue. We model this limitation by assuming banker needs to acquire funds from buyers to issue deposits and extend loans. Therefore, banker is bound by buyer's choice of real balances.

[11] In Appendix A we solve the model assuming the banker can choose the level of deposits. If a monopolistic banker can freely issue deposit, the banker will always choose the efficient investment level, regardless of the interest rate on CBDC.

deposits and CBDC as means of payment in the DM, with real consumption value $c = (1+i_d)\delta + (1+i_m)(z-\delta)$.

We assume the banker funds and monitor a measure q of projects. The unit price of capital is $\phi\rho = \frac{1}{(1+i_m)}$. Each project pays a return R and the monopolistic banker obtains the full return, leaving entrepreneurs with zero profits. When the banker obtains an interest rate i_r on reserves and CBDC pays i_m, the problem of the banker is:

$$\max_{q,r,\delta,i_d} \; Rq + r(1+i_r) - (1+i_d)\delta - \kappa(q)$$

subject to

$$\begin{aligned}
\beta u\left((1+i_d)\delta + (1+i_m)(z-\delta)\right) &\geq \beta u\left((1+i_m)z\right) \quad (\mu) &(6)\\
\frac{q}{(1+i_m)} + r &\leq \delta \quad (\lambda) &(7)\\
\delta &\leq z \quad (\eta) &(8)\\
q, r &\geq 0 &(9)
\end{aligned}$$

where μ, λ, and η are the Lagrange multipliers on the (6), (7) and (8) respectively.

Here (6) captures the participation constraint of a buyer who holds z units of CBDC. If the buyer uses CBDC as means of payment to acquire the DM consumption good he will be able to consume $(1+i_m)z$ units. If the buyer deposits an amount δ of his CBDC holdings with the bank and uses deposits to acquire the DM good he will be able to consume $c = (1+i_d)\delta+(1+i_m)(z-\delta)$ units. From the participation constraint, the banker has to ensure that the buyer obtains at least as high a payoff from using deposits as he would using CBDC. Bankers' resource constraint is captured by (7). When the constraint binds, the banker uses all the deposits it issues to invest and/or hold reserves. (8) captures the deposit constraint of bankers.

A monopolist banker does not leave surplus to depositors so (6) binds and $i_d = i_m$.

In other words, the banker offers a deposit contract that makes the buyer indifferent between holding bank deposits and CBDC. Therefore, $c = (1 + i_m)z$, and the banker ensures the return on deposits is the same as the return on CBDC. Here $u'(c) \geq 1$, with efficiency, $u'(c^*) = 1$ at the Friedman rule.

Analyzing the first order conditions on δ, q and r and using $\mu = \frac{1}{\beta u'[(1+i_d)\delta + (1+i_m)(z-\delta)]}$

$$\delta \ : \ (1 + i_m) + \eta = \lambda \tag{10}$$

$$q \ : \ (1 + i_m)\left[R - \kappa'(q)\right] \leq \lambda \tag{11}$$

$$r \ : \ (1 + i_r) \leq \lambda \tag{12}$$

With $\lambda > 0$, the resource constraint (7) binds and the bank invests all the deposits it issues. The deposit constraint binds if $\lambda > (1 + i_m)$, while $\delta < z$ and $\eta = 0$ if $\lambda = (1 + i_m)$.

The return on monitored projects relative to the return on reserves will determine the banker's portfolio choice. If $(1 + i_m)\left[R - \kappa'(q)\right] > (1 + i_r)$, the return on monitored projects is higher than on reserves, the banker invests all deposits into monitored projects and $q > 0$, $r = 0$. On the other hand, if $\left[R - \kappa'(q)\right] = \frac{(1+i_r)}{(1+i_m)}$, the gross return on monitored projects and reserves is the same, the banker will invest in both and $q, r > 0$.

3.5 Interest rate on CBCD and bankers' decisions

Define i_m^* as the interest rate at which investment efficiency $q^* = (1 + i_m)(z - r)$ is reached, which as demonstrated by the planner solution occurs when $R - \kappa'(q^*) = 1$ is achieved.

Case 1 $i_m = i_r$:

1. For $i_m < i_m^*$: then $R - \kappa'(q) > 1$ and bankers choose to invest all their deposits into monitored projects and do not hold reserves, $q > 0$ and $r = 0$.

Furthermore, $\eta > 0$ and $z = \delta$ and bankers accept and issue deposits for all the CBDC real balances buyers bring.

2. For $i_m \geq i_m^*$: then $R - \kappa'(q^*) = 1$ and the gross return on monitored projects and reserves is equivalent. Thus bankers invest in projects and hold reserves, $q, r > 0$. Furthermore, $\eta = 0$ and $z \geq \delta$.

For low levels of the CBDC interest rate (and therefore for low levels of interest rate on deposits since $i_d = i_m$), buyers do not bring high enough levels of real balances for bankers to be able to achieve the unconstrained efficient investment level. Bankers accept and issue deposits for all the real balances buyers bring and invest all of it in monitored projects. Hence, from the resource constraint, $q = (1 + i_m)z$. As the interest rate paid on CBDC increases, $(1 + i_m)z = q$ increases as well. Therefore, when $i_m < i_m^*$, the level of investment increases with i_m. Thus $\frac{dq}{di_m} > 0$ and an increase in the interest rate paid on CBDC leads to higher levels of intermediation. Furthermore, due to $i_d = i_m$ and the binding participation constraint, bankers also respond to higher interest rate on CBDC by increasing the remuneration on bank deposits, allowing depositors higher real consumption levels in the DM.

Due to the cost of monitoring projects and since $\frac{dq}{di_m} > 0$, the return on investment $R - \kappa'(q^*)$ decreases with i_m. When the interest rate on CBDC reaches i_m^*, then $R - \kappa'(q^*) = 1$ and the return on investment, reserves and CBDC is equivalent. From this point, bankers will not adjust their investment levels in response to a change in i_m. Thus, for $i_m \geq i_m^*$, $\frac{dq}{di_m} = 0$. Bankers become indifferent between accepting further deposits and holding it as reserves, or allowing buyers to hold their extra balances as CBDC. If the central bank decides to pay higher interest on CBDC than on reserves and allows bankers access to CBDC, the investment level would also stay fixed at the efficient level q^*. However, the banker would be indifferent between accepting the extra deposits and holding as CBDC or allowing buyers to hold it as CBDC.

Case 2 $i_r > i_m$**:** Define $\overrightarrow{i}_m$ as the interest rate on CBDC when bankers start holding reserves in an economy where $i_r > i_m$ always holds

1. For $i_m < \overrightarrow{i}_m$: then $R - \kappa'(q) > \frac{(1+i_r)}{(1+i_m)} > 1$ and bankers choose to invest all their deposits into monitored projects and do not hold reserves, $q > 0$ and $r = 0$. Furthermore, $\eta > 0$ and $z = \delta$ and bankers accept and issue deposits for all the CBDC real balances buyers bring.

2. For $i_m \geq \overrightarrow{i}_m$: then $R - \kappa'(q) = \frac{(1+i_r)}{(1+i_m)} > 1$ and the gross return on monitored projects and reserves is equivalent. Bankers choose to both invest in monitored projects and to hold reserves, $q, r > 0$. Furthermore, $\eta > 0$ and $z = \delta$. Thus, the deposit constraint binds.

If the interest rate paid on reserves is higher than the interest rate paid on CBDC, the efficient investment level cannot be achieved. The interest rate paid on reserves crowds out investment and we observe under-investment for all levels of $i_m \leq \frac{\gamma}{\beta} - 1$. Therefore, central bank seeking to promote efficient investment levels should not offer a higher interest rate on reserves than it offers on CBDC.

As in case 1 when CBDC and reserves are essentially the same asset, we observe increased intermediation for low levels of CBDC. As the interest rate on CBDC increases, buyers bring more real balances to the banker. The banker issues deposits for all those balances and uses all of it to invest in monitored projects. Thus with $(1 + i_m)z = q$, when $i_m < \overrightarrow{i}_m$ then $\frac{dq}{di_m} > 0$. The return on investment $R - \kappa'(q^*)$ decreases with i_m until $i_m = \overrightarrow{i}_m$ is reached. At this point, $R - \kappa'(q) = \frac{(1+i_r)}{(1+i_m)}$ and the effect of a change in i_m on investment and the level of reserves depends on the relationship between i_m and i_r. If the ratio $\frac{(1+i_r)}{(1+i_m)}$ stays fixed, then $\frac{dq}{di_m} = 0$. Hence, if there is an equivalent change in interest rates on CBDC and reserves it will not affect the risk-free investment level. It will increase the level of reserves banker holds, $\frac{dr}{di_m} > 0$. If the increase in the interest rate paid on reserves is greater than the change in the interest rate paid on CBDC, and the ratio $\frac{(1+i_r)}{(1+i_m)}$ increases, it will make reserves relatively more attractive. Hence, banker will respond by increasing its reserve holding and reduce investment. Therefore, leading to disintermediation, $\frac{dq}{di_m} < 0$. Lastly, if the ratio $\frac{(1+i_r)}{(1+i_m)}$ decreases, then reserves are becoming less attractive relative to investment[12]. Hence, leading to increased investment level, $\frac{dq}{di_m} > 0$.

For all i_m bankers respond to an increase in the interest rate on CBDC by increasing the remuneration on bank deposits.

Case 3 $i_m > i_r$: Here we assume that bankers cannot hold CBDC. More precisely, bankers can accept CBDC from buyers but cannot hold CBDC accounts and have to convert CBDC into other assets.

1. For $i_m < i_m^*$: then $R - \kappa'(q) > 1 > \frac{(1+i_r)}{(1+i_m)}$ and bankers choose to invest all their deposits in monitored projects and do not hold reserves, $q > 0$ and $r = 0$. Furthermore, $\eta > 0$ and $z = \delta$ and bankers accept and issue deposits for all the CBDC real balances buyers bring.

2. For $i_m \geq i_m^*$: then $R - \kappa'(q^*) = 1 > \frac{(1+i_r)}{(1+i_m)}$. The unconstrained efficient investment level is achieved. However, the return on reserves is too low and bankers invest all their deposits in monitored projects and do not hold reserves, $q > 0$ and $r = 0$. Furthermore, $\eta = 0$ and $z > \delta$. Therefore, the banker will not issue any further deposits.

As in the other two cases, for low levels of the interest rate on CBDC, bankers choose to accept all balances buyers bring, they only invest in monitored projects and as i_m increases, the level of investment increases in line with the higher level of deposits. However, when $i_m > i_r$, and bankers cannot hold CBDC, as the efficient investment level is achieved, bankers do not accept any further deposits. Hence, the investment level stays fixed, with $\frac{dq}{di_m} = 0$ and buyers bring both deposits and CBDC into the

[12]the interest rate paid on CBDC affects the price of capital

DM as means of payment. The remuneration on deposits increases in line with the increased remuneration on CBDC.

3.5.1 Optimal interest rate and overall welfare

Central bank wanting to promote efficient investment levels should set $i_m = i_r$ if bankers do not have access to CBDC or $i_m \geq i_r$ if bankers have access to CBDC.[13] If $i_r > i_m$, then the high return on reserves crowds out investment leading to lower levels of intermediation than when $i_m \geq i_r$.

Bankers make their investment and remunerations decisions in order to maximize profits. Therefore, as demonstrated above, bankers choose the level of deposits to issue, remuneration on deposits, as well as how much of its funds to invest and how much to hold as reserves in order to maximize $Rq + r(1+i_r) - (1+i_d)\delta - \kappa(q)$, subject to (6), (7) and (8).

Claim 1 *Assuming $i_m \geq i_r$: i) for $i_m < i_m^*$ an increase in i_m increases bank profits; ii) for $i_m \geq i_m^*$ an increase in i_m keeps bank profits constant*

When $i_m < i_m^*$, a higher interest rate on CBDC reduces the profit margin. However, the deposit base increases with i_m, increasing investment level and raising overall bank profits. When $i_m \geq i_m^*$, investment into monitored projects stays fixed and the banker puts the extra resources received due to the growing deposit base into reserves.[14]. Even though investment level stays fixed and the remuneration on deposits is increasing, banker profits stay constant. The cost of acquiring the capital used as inputs into investment goes down as the interest rate on CBDC increases, since CBDC is used as the means of payment to acquire capital.

Bank profits are not declining in i_m and investment stays at the efficient level even when $i_m > i_m^*$. Furthermore, buyer consumption levels, and thus utility are increasing

[13]If bankers have access to CBDC and $i_m > i_r$, bankers will never choose to hold reserves. Hence, for co-existence of CBDC and reserves, central banks should set $i_m \leq i_r$.

[14]Into CBDC if $i_m > i_r$

in i_m. Therefore, overall welfare is increasing in i_m and the introduction of CBDC increases welfare. It would be optimal for central banks to set $i_m = \frac{\gamma}{\beta} - 1$, which is consistent with the Friedman rule. Running the Friedman rule allows optimal investment levels and optimal consumption levels at $u'(c) = 1$.

4 Model with risk

In this section we extend the benchmark model by assuming $p > 0$ and $pR > 1$ so that projects have positive net present value even without bank's monitoring. So bankers can either extend risky loans when they do not monitor projects, or they can extend safe loans when they monitor projects. In addition bankers can hold reserves. With this modification we can analyze the effect CBDC has on the banker's balance sheet composition as well as the level of risk in the banking sector. Also, we assume the payoff of risky projects is perfectly correlated, so that they all succeed or fail at the same time. Finally, we assume the payoff of risky project becomes publicly known at the start of the DM.

4.1 Planner solution

A constrained planner that wants to maximize welfare solves:

$$\max_{q,n,c\geq 0} u\left(c\right) + z - c + Rq - q - \kappa\left(q\right) + pRn - n$$

subject to

$$q + n \leq z$$

where z is the real value of deposits, q is the number of monitored projects and n is the number of risky projects. The efficient solution satisfies

$$u'(c^*) = 1$$

and

$$
\begin{aligned}
R - 1 - \kappa'(q) &= \lambda_z - \lambda_q \\
pR - 1 &= \lambda_z - \lambda_n
\end{aligned}
$$

where λ_z is the multiplier on the resource constraint and λ_x is the multiplier on $x \geq 0$. We assume $\kappa'(0) = 0$ so that the planner would always prefer investing in the safe projects. Hence $\lambda_q = 0$. The planner solution can be summarized in the following claim:

Claim 2 *Suppose $pR > 1$. Define $\widehat{z}$ as the solution to $(1-p)R = \kappa'(\widehat{z})$. For all $z < \widehat{z}$ the planner invests $q = z$ in risk-free projects only. For all $z > \widehat{z}$, the planner invests $q = \widehat{z}$ in risk-free projects and $n = z - \widehat{z}$ in risky projects. The planner never invests in storage (reserves)*

4.2 Timing

Since the banker's deposits are possibly state contingent assets, it is important to understand the timing. We assume the following sequence of events;
CM time t:

1. Buyers work in the CM
2. Buyers choose their CBDC holdings

DM time $t+1$:

1. Buyers purchase a deposit contract from the banker with CBDC
2. Banker lends CBDC to entrepreneurs
3. Entrepreneurs use CBDC to buy capital from suppliers, and invest the capital good

4. True signal on the return of risky project realized

5. Buyers trade deposits with sellers (real value depends on signal)

CM time $t+1$:

1. Sellers (deposit holders) redeem deposits in the CM.

4.3 Suppliers, sellers, and buyers

The supplier's problem remains the same, so that the price of capital is still $\phi\rho = \frac{1}{1+i_m}$. Regarding the buyer's problem, while the banker's deposits can now be risky, we guess and later verify that the banker leaves depositors indifferent between holding CBDC or bank deposits. Therefore, the buyer's choice of CBDC balances, z, solves $u'\left[(1+i_m)z\right] = \frac{\gamma}{\beta(1+i_m)}$. Finally, sellers will just produce enough to be exactly compensated by the value of the bank's deposits they accept from buyers.

4.4 Banker

Recall that the bank invests in a number q of monitored projects and in a number n of projects that the banker will not monitor. Banker incurs a monitoring cost $\kappa(q)$. Monitored projects deliver output R, while non-monitored projects deliver output R with probability p and zero output with probability $(1-p)$. The probability p is exogenous and banker cannot affect it. Hence, the gross return of investing in a measure q of monitored projects is $R - \kappa(q)$ and investing in a measure n of non-monitored projects yields an expected return of pR. We assume that the return of non-monitored projects is perfectly correlated.

Buyers meet with a banker before entering the DM. The banker offers the buyer δ units of a deposit contract that the buyer can purchase using CBDC. A deposit contract is a promise to pay $(1+i_h)$ to the depositor in the high state, and $(1+i_l)$

in the low state, with $i_h \geq i_l$.[15] We will focus on the case where reserves and CBDC pay the same interest rate with $i_r = i_m$. Since $i_r = i_m \geq 0$ we can assume that the banker will offer $\delta = z$ and the banker can always use reserves in case she has too many resources. To further solve for the deposit contract, let us define for any $w \geq 0$,

$$v(w) = \max_{c \leq w} u(c) + w - c$$

This function is well defined, always positive, increasing and (weakly) concave, with

$$v'(w) = \begin{cases} u'(w) & \text{if } w \leq c^* \\ 1 & \text{if } w > c^* \end{cases}$$

Finally, we use $w_h = (1 + i_h)z$ and $w_l = (1 + i_l)z$ as the payment promised by the banker. Then the banker's offer a deposit contract that solves

$$\max_{q,n,r,w_h,w_l} qR + npR + r(1 + i_m) - pw_h - (1 - p)w_l - \kappa(q)$$

subject to

$$\begin{aligned} pv(w_h) + (1 - p)v(w_l) &\geq v((1 + i_m)z) \qquad (\mu) && (13) \\ \frac{q + n}{1 + i_m} + r &\leq z \qquad (\lambda) && (14) \\ w_l &\leq r(1 + i_m) + qR \qquad (1 - p)\nu && (15) \\ w_h &\leq r(1 + i_m) + qR + nR && (16) \\ q, n, r &\geq 0 && (17) \end{aligned}$$

Here (16) is the resource constraint for the high state. Banker can use the return from the monitored and risky projects as well as reserves to pay depositors in the

[15] The high state occurs with probability p and is the case when the risky asset pays output, whereas the low state occurs with prob. $(1 - p)$ and is the state when the risky project pays no output.

high state. We assume this constraint is not binding, since a monopolistic banker obtains profits in the high state. (15) is the resource constraint in the low state. In the low state, the risky project does not produce output and banker can only use the return from the risk-free project and reserves to compensate depositors in the low state.

Furthermore, (13) captures the participation constraint of depositors: The banker has to ensure that buyers are at least as well off depositing their CBDC as they are carrying CBDC into the DM. From the first order conditions, $\mu = 1/\beta u'(c_h)$. Therefore, $\mu > 0$ and the participation constraint binds. Hence, banker remunerates deposits such that depositors are equally well off depositing with the bank as they are using CBDC in the DM.

We measure the level of risk in the deposit contract by the ratio of marginal utility of buyers in both states, $v'(c_l)/v'(c_h)$. This ratio will be important for the banker's decisions to invest in monitored or non-monitored projects. A higher ratio indicates lower insurance due to higher dispersion between revenue in the high and low states. Therefore, under risk, the banker has two potential margins of adjustments to make deposits more attractive. Banker can increase the total payment to depositors as well as providing extra insurance through adjusting the dispersion of consumption across low and high states.

The first order condition on w_l yield:

$$\nu = \frac{v'(w_l)}{v'(w_h)} - 1 \tag{18}$$

Therefore, in the case with full insurance, meaning depositors get the same remuneration in the high and low states, $w_h = w_l$, the low state constraint is not binding. Hence, when $w_h = w_l = w$, then $\nu = 0$ and $w_l < r(1 + i_m) + qR$. If the banker does not provide full insurance, such that there is a spread between consumption in the high and low states, $w_h > w_l$, then the low state constraint binds, $\nu > 0$.

The first order conditions for banker investment choices yield:

$$
\begin{aligned}
q &: \quad (1+i_m)\left[(R-\kappa'(q)) + R(1-p)\left(\frac{v'(w_l)}{v'(w_h)}-1\right)\right] \leq \lambda \\
n &: \quad (1+i_m)pR \leq \lambda \\
r &: \quad (1+i_m)\left[1+(1-p)\left(\frac{v'(w_l)}{v'(w_h)}-1\right)\right] \leq \lambda
\end{aligned}
$$

Hence, $\lambda > 0$ and the resource constraint binds, such that the banker takes all the resources it receives from depositors and invests it. The choice of investment depends on the relative returns from monitoring projects and reserves. The return on monitored projects and reserves take into account that holding those assets can relax the resource constraint in the low state.

4.5 Interest rate on CBDC and investment choice under risk

In this section, we characterize the equilibrium when the banker can decide to invest in unmonitored projects. The details are left to the Appendix.

1. For $i_m < \bar{i}_m$: the banker invests all funds in projects and monitor all of them so that $q = (1+i_m)z > 0$, $n = r = 0$, and the deposit contract features full insurance with $w_l = w_h = w = (1+i_m)z$. Furthermore, $dq/di_m > 0$ and $dw/i_m > 0$. This is equivalent to the efficient solution when $z < \widehat{z}$. The level of interest rate $\bar{i}_m$ is defined as $(1-p)R = \kappa'((1+\bar{i}_m)z)$.

2. For $\bar{i}_m \leq i_m < \widetilde{i}_m$: the banker invests all funds in projects some of which will be monitored, $q, n > 0$, and $r = 0$, with $q = (1+i_m)z - n$ and the deposit contract features full insurance with $w_l = w_h = w = (1+i_m)z$. Furthermore, $dq/di_m = 0$ and $dn/di_m > 0$ and $dw/di_m > 0$. This is equivalent to the efficient solution when $z = \widehat{z}$. In this equilibrium, $(1-p)R = \kappa'(q)$ and $\widetilde{i}_m$ is defined as the level of interest rate such that $qR = (1+\widetilde{i}_m)z$ so that above $(1+\widetilde{i}_m)$ full insurance is no longer feasible.

3. For $\widetilde{i}_m \leq i_m < \widehat{i}_m$: the banker invests all funds in projects some of which will be monitored, $q, n > 0$ and $r = 0$. The deposit contract features some risk with $w_h > w_l$. At this level of i_m full insurance is not optimal for the banker as it would have to monitor too many projects and incur too high a monitoring cost. Still, in this region, $dq/di_m > 0$. Investment in risky projects can increase or decrease, $dn/di_m > 0$ if $dq/di_m < 1$ whereas $dn/di_m < 0$ otherwise. Furthermore,and as i_m increases, the banker increases remuneration both in the low and high states while reducing the level of insurance it provides to depositors. This region is inefficient because there is over-investment in monitored projects. In this region, $1 \leq \frac{v'(w_l)}{v'(w_h)} \leq \frac{p}{1-p}(R-1)$ and the level of interest rate $\widehat{i}_m$ is the one for which $\frac{v'(w_l)}{v'(w_h)} = \frac{p}{1-p}(R-1)$. [over-investment in monitored projects: because the bank finds it too expensive to attract risk averse depositors with risky deposit contracts and prefers to provide more insurance to depositors by investing more in monitored projects.]

4. For $\widehat{i}_m \leq i_m$: the banker invests some funds into monitored projects, some funds into unmonitored projects, and the rest in reserves, q, n and $r > 0$. The deposit contract features some risk with $w_h > w_l$. Furthermore, $dq/di_m = d\left(\frac{v'(w_l)}{v'(w_h)}\right)/di_m = 0$ and $dw_l/di_m, dw_h/di_m > 0$. Banker responds to an increase in i_m by increasing remuneration in both the high and low states however keeping overall insurance, $v'(w_l)/v'(w_h)$, constant. Holding reserves and investing in unmonitored project provides the same linear return for the banker. However, increasing n increases the wedge between w_h and w_l. Thus, n and r adjust with i_m such that $v'(w_l)/v'(w_h)$ stays constant. This region is inefficient because there is over-investment in monitored projects.

Raising the interest rate on CBDC does not lead to disintermediation, as $q + n$ does not decrease in i_m.

Efficiency is achieved for $i_m < \widetilde{i}_m$. However, in order to increase investment levels

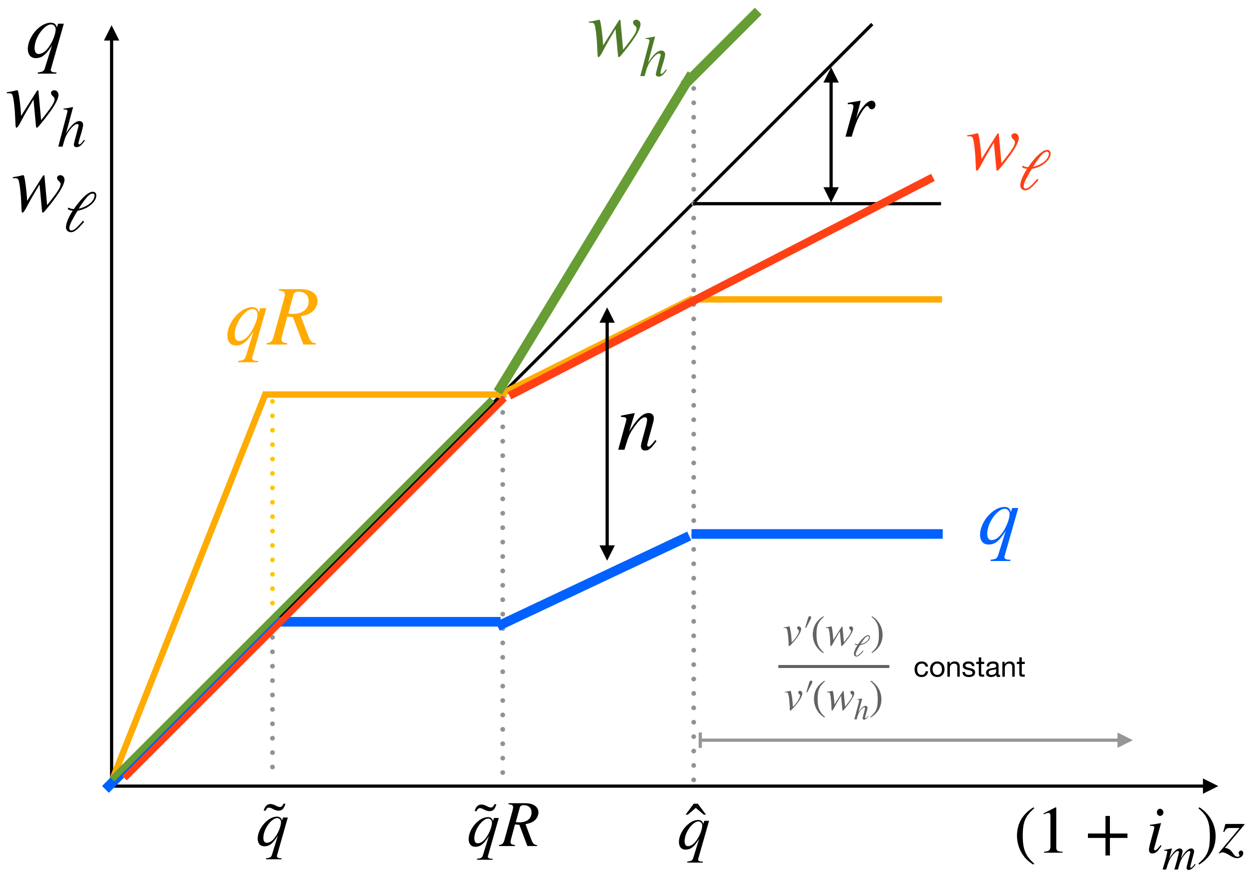

Figure 1: Equilibrium investment levels

and achieve unconstrained investment levels in the risk-free asset, the central bank should choose i_m in the interval: $\bar{i}_m \leq i_m < \widetilde{i}_m$. Figure **??** shows the equilibrium investment levels and deposit value in the high and the low state as a function of the value of CBDC holdings.

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4.6 Banker problem if $i_r > i_m$

Thus far, we have analyzed the model assuming that $i_m = i_r$. In this section we analyze whether the choice of investment between monitored and unmonitored projects is affected if reserves always pay more than CBDC, i.e. $i_r > i_m$ always holds. The banker's problem and first order conditions remain the same as in section 4.4, except the interest rate paid on reserves is now $i_r > i_m$.

4.6.1 Deposit contract featuring full insurance

If $pR > \frac{(1+i_r)}{(1+i_m)}$, then for all $i_m < \widetilde{i}_m$ the banker choice is identical to the case when $i_m = i_r$. The banker provides deposit contract featuring full insurance, with investment only in monitored projects for $i_m < \bar{i}_m$ and in monitored and unmonitored projects for $\bar{i}_m \leq i_m < \widetilde{i}_m$.

On the other hand, if $\frac{(1+i_r)}{(1+i_m)} > pR > 1$, then the banker will never invest in unmonitored projects. Thus, the banker provides a deposit contract featuring full insurance with $w_l = w_h = w = (1 + i_m)z$, for all $i_m \leq \frac{\gamma}{\beta} - 1$.

1. For $i_m < \bar{i}_m^r$: the banker invests all funds in monitored projects so that $q = (1 + i_m)z > 0$, $n = r = 0$. Furthermore, $dq/di_m > 0$ and $dw/i_m > 0$. The level of interest rate $\bar{i}_m^r$ is defined as $R - \frac{(1+i_r)}{(1+\bar{i}_m^r)} = \kappa'((1 + \bar{i}_m^r)z)$. Note that $\bar{i}_m^r < \bar{i}_m$, thus the interest rate threshold at which banker starts using its funds for other assets than the monitored projects is lower when $i_r > i_m$ than when $i_r = i_m$.

2. For $\bar{i}_m^r \leq i_m$: the banker invests all funds in monitored projects and reserves so that $q, r > 0$, $n = 0$, where $q = (1 + i_m)(z - r)$. Furthermore, $dw/i_m > 0$ and the effect of an increase in i_m on the level of monitored investment, q, depends on the relative changes in i_r and i_m.

Therefore, if $i_r > i_m$ and the spread between the interest rates paid on reserves and CBDC is large enough that $\frac{(1+i_r)}{(1+i_m)} > pR > 1$, the risk to depositors is minimized and bankers only offer deposit contracts featuring full insurance. More specifically, since bankers are better off holding reserves than investing in risky projects bankers do not take risk and always monitor projects. This case is identical to the risk-free case with $i_r > i_m$ and $\bar{i}^r_m \equiv \overrightarrow{i}_m$. The remuneration on bank deposits increases in line with the interest rate on CBDC. Despite the risk to depositors being minimized, the case with $i_r > i_m$ does not promote efficient investment levels. We observe constrained investment efficiency for $i_m < \bar{i}^r_m$, but underinvestment for $i_m \geq \bar{i}^r_m$.

4.6.2 Risky deposit contract

If $pR > \frac{(1+i_r)}{(1+i_m)}$, then for $\widetilde{i}_m \leq i_m$ the deposit contract features some risk with $w_h > w_l$.

1. For $\widetilde{i}_m \leq i_m < \widehat{i}^r_m$: the banker invests all funds in projects, some of which are monitored and some which are not, $q, n > 0$ and $r = 0$. As in the case when $i_m = i_r$, at this level of i_m it is too costly for the bank to invest enough funds into monitored projects to be able to pay $(1 + i_m)z$ to depositors in the low state. Therefore, $w_h > w_l$. Nevertheless, $\frac{dq}{di_m} > 0$ and $\frac{dw_h}{di_m}, \frac{dw_l}{di_m} > 0$. The overall level of insurance, as measured by $v'(w_l)/v'(w_h)$, is declining in this region, with $d\left(\frac{v'(w_l)}{v'(w_h)}\right)/di_m > 0$. The level of interest rate $\widehat{i}^r_m$ is where $\frac{v'(w_l)}{v'(w_h)} = \frac{p}{1-p}(R-1)\left[\frac{1+i_m}{1+i_r}\right] > 1$ is reached.

2. For $\widehat{i}^r_m \leq i_m$: the bankers invests both in monitored and unmonitored projects and holds reserves, q, n and $r > 0$. The deposit contract features some risk with $w_h > w_l$. An increase in the interest rate on CBDC increase both w_h and w_l. But the risk faced by depositors, as measured by $\frac{v'(w_l)}{v'(w_h)}$, depends on the interest rate on reserves. If the ratio $\frac{1+i_m}{1+i_r}$ stays constant, the level of risk also stays constant. If the ratio increases, such that $di_m > di_r$, the level of risk increases. However, if the ratio $\frac{1+i_m}{1+i_r}$ decreases, meaning that the central bank increases i_r to a greater extent than it increases i_m it will reduce the wedge between w_h and w_l and lower the level of risk to depositors.

Note that $\widehat{i_m^r} < \widehat{i}_m$ and the level of depositor risk $v'(w_l)/v'(w_h)$ is lower when $i_r > i_m$, than when $i_r = i_m$.

4.7 Banker's choice and level of risk

When the interest rate on CBDC is very low relative to inflation, real balances are costly to hold and bankers can only obtain a low amount of deposits. Bankers invest all funds into monitored projects because the cost of monitoring remains low. As i_m increases, the amount of deposits also increases and the banker finds it too costly to invest all funds monitored projects. If the return to unmonitored projects is higher than the return on reserves, bankers will invest some of its funds into projects that it will not monitor. However, if i_m is not too high, despite investing insome unmonitored projects, the banker invests enough into monitored projects to be able to pay $(1 + i_m)z$ in the low state. Therefore, if the central bank sets $i_m = \widetilde{i}_m$, and ensures that the interest rate paid on reserves does not crowd out investment in unmonitored projects, it can promote efficient investment levels and ensure that there is no risk to depositors.

However, there is underinvestment if the interest rate on reserves is set too high: banks start holding reserves rather than investing in projects (monitored or not).

Assuming that the interest rate on reserves does not crowd out investment in the case where banker can offer safe deposit contracts, as i_m continues increasing, the amount bankers invest in monitored projects is not enough to pay $(1 + i_m)z$ in the low state. Therefore, the high cost of monitoring implies a lack of safe assetson the bank's balance sheet which itself creates a wedge between the remuneration on deposits in the high and low states. Hence, deposits become risky.

When deposits are risky, increasing the interest rate paid on reserves can mitigate the level of risk. Reserves allow bankers access to an alternative risk-free asset with a lower cost than the one of monitored projects. Thus, for high levels of interest rate on CBDC, if the central bank implements $i_r > i_m$ it results in a lower wedge between pay-offs in the high state and low and reduces the level of depositor risk compared to the case when $i_r = i_m$. Therefore, when the interest rate paid on CBDC is high, the interest rate paid on reserves can lower the level of risk in the banking sector, but at the cost of lower intermediation.

5 Risk-free case without CBDC

In this section we analyze an extension to the model, where we assume that non-interest bearing cash is the central bank money. Hence, it is equivalent to assuming $i_m = 0$ for every scenario. The banker problem is then:

$$\max_{q,r,c} \; Rq + r(1+i_r) - c - \kappa(q) \tag{19}$$

subject to:

$$\begin{aligned} \beta u(c) &\geq \beta u(z) && (20) \\ q + r &\leq z && (21) \\ q, r &\geq 0 && (22) \end{aligned}$$

where z^* solves $u'(z) = \frac{\gamma}{\beta}$. When cash is non-interest bearing, it does not impact the price of capital $\rho = 1$ and it is not changing real money demand (as in the case with CBDC). Hence, z does not change unless γ changes.

From the first order conditions:

$$\begin{aligned} q &: \; R - \kappa'(q) \leq \lambda && (23) \\ r &: \; (1+i_r) \leq \lambda && (24) \end{aligned}$$

If $R - \kappa'(q) > (1+i_r)$, then $q > 0$ and $r = 0$ and all deposits are invested in monitored projects. If $R - \kappa'(q) = (1 + i_r)$, then $q > 0$ and $r > 0$. Hence, there is overall less investment than in the case when $R - \kappa'(q) > (1 + i_r)$. If i_r increases then the quantity invested in monitored projects decline resulting in disintermediation. (The same amount of real deposits, but the split between investment and reserves changes). Hence, reserves can crowd-out good investment.

Currently, central bankers are offering near zero or negative interest rates on their deposit facility (reserves) in order to promote investment. As the current section shows, this is needed when non-interest bearing cash is the widely-accepted central bank money. However, if central banks were to implement an interest-bearing CBDC, they can promote overall investment while raising overall interest rate levels, with efficiency achieved at $i_r > 0$. As previously stated, it would be optimal for central bankers to always set the same interest rate on CBDC and reserves.

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6 Appendix A - Proofs for risk-free benchmark

To be written into document

7 Appendix B - Proofs for risky case

Proof of planner claim

1. If $\lambda_z > 0$ the planner is not using storage (reserves).

   (a) If $\lambda_n > 0$, the planner is only investing in monitored projects. This is the case when

   $$\begin{aligned} R-1 &> \kappa'(z) \\ pR-1 &< R-1-\kappa'(z) \end{aligned}$$

   that is

   $$\begin{aligned} R &> 1+\kappa'(z) \\ \kappa'(z) &< (1-p)R \end{aligned}$$

   (b) If $\lambda_n = 0$, the planner is investing in both monitored and unmonitored projects. In this case q and n solve

   $$\begin{aligned} R-1-\kappa'(q) &= pR-1, \\ q+n &= z. \end{aligned}$$

   This is the solution when

   $$(1-p)R = \kappa'(q) < \kappa'(z).$$

   (c) Hence there is $\hat{z}$ such that the planner invests in unmonitored projects whenever $z > \hat{z}$ and $\hat{z}$ solves

   $$(1-p)R = \kappa'(\hat{z})$$

2. If $\lambda_z = 0$ the planner is using storage (reserves).

    (a) If $\lambda_n > 0$, the planner only invests in monitored projects. In this case, $pR < 1$ and q is given by

    $$R = 1 + \kappa'(q) < 1 + \kappa'(z).$$

    (b) If $\lambda_n = 0$, the planner invests in both monitored and unmonitored projects. In this case, $pR = 1$ and q is given by

    $$R = 1 + \kappa'(q) < 1 + \kappa'(z).$$

7.0.1 Safe deposit contracts: $w_h = w_l$

When deposit contracts are safe, $w_l = w_h \equiv c$ and $\nu = 0$. Therefore, the first order conditions for the banker investment choices reduce to

$$R - \kappa'(q) \leq \lambda \ (q) \tag{25}$$

$$pR \leq \lambda \ (n) \tag{26}$$

$$1 \leq \lambda \ (r) \tag{27}$$

and

$$w_l = w_h = w = (1 + i_m)z \tag{28}$$

Notice that if $pR < 1$, banker never invests in unmonitored projects as it pays a lower return than holding reserves. If $pR > 1$, then banker will not choose to hold reserves in the case deposit contracts are safe. On the other hand if $pR = 1$ the banker is indifferent between investing in unmonitored projects or holding reserves.

1. If

$$R - \kappa'(q) > pR \geq 1$$

then $q > 0$, $n = r = 0$ and $q = (1 + i_m)z$. Furthermore, $c = (1 + i_m)z$ and $qR > c$.

This is an equilibrium whenever

$$R(1 - p) > \kappa'((1 + i_m)z)$$

and

$$pR \geq 1.$$

With $q = (1 + i_m)z$, which we have shown is increasing in i_m, therefore $\frac{dq}{di_m} > 0$. Hence, as i_m increases, the banker invests more in monitored projects. As $q = (1 + i_m)z$ increases, the return on monitored projects, $R - \kappa'(q)$, decline. When i_m and therefore $\kappa'(q) = \kappa'((1 + i_m)z)$ has reached such a level that $R(1 - p) = \kappa'(q)$, then the banker starts investing in unmonitored projects in addition to monitored projects.

2. If

$$R - \kappa'(q) = pR > 1$$

then $q > 0$, $n > 0$ and $r = 0$ and $q + n = (1 + i_m)z$. This is an equilibrium whenever

$$\begin{aligned} R(1 - p) &= \kappa'((1 + i_m)z - n), \\ pR &> 1 \end{aligned}$$

and

$$\underbrace{R}_{\text{gain}} < \underbrace{pR + \kappa'((1 + i_m)z)}_{\text{"opportunity" cost of investing the last bit in safe}}$$

Furthermore,

$$c = (1 + i_m)z < qR$$

Notice that in this region, q does not depend on i_m since $R(1 - p) = \kappa'(q)$. Therefore, $\frac{dq}{di_m} = 0$. In this equilibrium, increasing i_m increases the investment in unmonitored projects up to the point where $qR < (1+i_m)z$, in which case offering a safe deposit contract is no longer possible.

3. If

$$R - \kappa'(q) = pR = 1$$

then $q > 0$, $n > 0$ and $r > 0$. Since the bank is indifferent between investing in unmonitored projects or holding reserves, we assume it invests in unmonitored projects. Then the equilibrium conditions are the same as under scenario 2.

7.0.2 Risky deposit contracts: $w_h > w_l$

In the case with risky deposit contracts, $qR < (1+i_m)z$. Therefore, the case with risky deposit contracts requires that the bank invests in unmonitored projects and $n > 0$.

1. If

$$(R - \kappa'(q)) + R(1-p)\left(\frac{v'(w_l)}{v'(w_h)} - 1\right) = pR \tag{29}$$

and

$$pR > 1 + (1-p)\left(\frac{v'(w_l)}{v'(w_h)} - 1\right) \tag{30}$$

then q, $n > 0$ and $r = 0$, with $q + n = (1+i_m)z$ and $c_l = qR$ and

$$pv(w_h) + (1-p)v\left(qR\right) = v((1+i_m)z) \tag{31}$$

Simplifying (29) and (30), this is an equilibrium whenever:

$$(1-p)R\frac{v'(qR)}{v'(w_h)} = \kappa'(q) \tag{32}$$

and

$$\frac{p}{1-p}\left(R-1\right) > \frac{v'(qR)}{v'(w_h)}. \tag{33}$$

Notice that q is higher than in the case with a safe deposit contract as $\frac{v'(qR)}{v'(w_h)} > 1$.

Using that $w_l = qR$ and taking the differential of the system (32) and (31) we find that $\frac{dw_h}{dq} > 0$ and $\frac{dq}{di_m} > 0$.[16] Since $\frac{dq}{di_m} > 0$, then $\frac{dw_h}{di_m}, \frac{dw_l}{di_m} > 0$. Therefore, when the banker offers a risky deposit contract, as the interest rate paid on CBDC increases, the banker increases his investment in monitored projects. The banker also pays out more to depositors, both in the high state and the low state. The investment in un-monitored projects n can increase or decrease, depending on $\frac{dq}{di_m} \lesseqgtr 1$, where

$$\left[p\frac{dw_h}{dq} + \kappa'(q)\right] v'(w_h)\frac{dq}{di_m} = v'((1+i_m)z)$$

Definingthe co-efficient of relative risk aversion $\sigma(q) = -qR\frac{u''(qR)}{u'(qR)}$ and using that the differential of (32) yields

$$(1-p)\frac{\frac{v''(qR)}{v'(qR)}R\kappa'(q) - \kappa''(q)}{\kappa'(q)^2}Rv'(qR)dq = v''(w_h)dc_h \tag{34}$$

$$v'(qR) = \frac{\kappa'(q)v'(w_h)}{(1-p)R}$$

Simplifying, we find that

$$\left[\frac{v''(qR)}{v'(qR)}Rq - \frac{\kappa''(q)}{\kappa'(q)}q\right]\frac{dq}{q} = \frac{v''(w_h)}{v'(w_h)}w_h\frac{dw_h}{w_h}$$

Therefore, either $w_h \geq c^*$ and $dq = 0$, or $w_h < c^*$ and assuming a constant coefficient of relative risk aversion for buyers, in this region $(dw_h/w_h)/(dq/q) > 1$. Hence w_h always (weakly) increases by more than q.

As i_m increases, then $\frac{v'(w_l)}{v'(w_h)}$ increases, until it reaches the point where $\frac{p}{1-p}(R-1) = \frac{v'(w_l)}{v'(w_h)}$

[16]We find that $(1-p)\frac{\frac{u''(qR)}{u'(qR)}R\kappa'(q)-\kappa''(q)}{\kappa'^2}Ru'(qR)dq = u''(c_h)dc_h$. Thus, $\frac{dc_h}{dq} > 0$. Furthermore, $\left[p\frac{dc_h}{dq} + \kappa'(q)\right]u'(c_h)\frac{dq}{di_m} = u'((1+i_m)z)$. Thus $\frac{dq}{di_m} > 0$ and $\frac{dc_h}{di_m} > 0$

2. If

$$(1-p)R\frac{v'(w_l)}{v'(w_h)} = \kappa'(q)$$

and

$$\frac{p}{1-p}(R-1) = \frac{v'(w_l)}{v'(w_h)} \tag{35}$$

then q, n and $r > 0$, with $q + n = (1+i_m)(z-r)$ and $w_l = qR + (1+i_m)r$ and

$$pu(w_h) + (1-p)u(qR) = u((1+i_m)z).$$

Notice, that the ratio $\frac{v'(w_l)}{v'(w_h)}$ is constant, as per (35). Therefore, $\kappa'(q)$ has to stay constant. Hence, q and i_m are not a function of i_m and $\frac{dq}{di_m} = d\left(\frac{v'(w_l)}{v'(w_h)}\right)/di_m = 0$.