Risk and Balance Sheet Size
in a Model of Fractional Reserve Banking
(preliminary and incomplete)

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Abstract

We analyze the optimal risk-return trade-off when banks create deposits (or inside money). Optimally the quantity of deposits is restricted by some reserve or liquid asset requirements. Increasing these requirements or inflation makes loans to the private sector more expensive. This induces borrowers to take more risk. They also invest less when loans are more expensive. As a result leverage declines. This induces borrowers to take safer decision. The optimal combination of reserve or liquid asset requirements and inflation trades-off investment and risk. The Friedman rule or zero reserve requirement is not necessarily optimal, as it would induce too much leverage. In spite of being the safest system, fully backed deposits, or 100% reserve requirement, is likely not optimal as it reduces leverage too much. A floor system is optimal only if the level of reserves is sufficiently large. Deposit insurance unambiguously reduces welfare while (anticipated) bailout may increase it.

1 Introduction

A fundamental question in monetary economics and banking is whether an economy that allows banks to freely create deposits is more or less stable than the same economy relying solely on deposits fully backed by reserves.\(^1\) The famous Chicago plan called for 100% reserve requirements at a time when the Great Depression gave ammunitions to those arguing for limiting the use of inside money. The Great Recession revived the academic and policy debate: Movements to reform the monetary system towards 100% reserve

\(^1\)We will use deposits and deposits to mean “inside money”. We will use cash, currency, reserves or money to mean “money”.
requirements exist in more than twenty countries.\footnote{See http://internationalmoneyreform.org.} Switzerland will vote in 2018 on a binding national referendum initiated to drastically limit banks’ ability to create unbacked deposits (a.k.a. the VollGeld initiative).

The theoretical and empirical literature is large and below we only review its most recent development, but let us mention two recurrent themes. A system relying on the free creation of deposits is arguably more efficient because banks are more flexible to respond to loan demand (Williamson, 1999). But this system is inherently unstable because it allows multiple equilibria, which opens the door to exotic dynamics, cycles, and crashes (Sanches, 2015). One puzzling aspect of the literature is that risk is missing from the analysis: Banks and their borrowers do not engage in risk taking activities and in equilibrium banks do not take any risk. It is the source of fundings that is fragile. In this paper, we analyze the stability properties of an economy with deposit creation by putting risk-taking at the center of our analysis.

More precisely, we introduce the moral hazard of risk taking in an otherwise standard monetary model with banks. We model risk as a continuous choice variable. As a result, there is an optimal amount of risk that borrowers should take. So even at the optimum projects will fail and so may banks that financed those projects. However, moral hazard and limited liability implies that borrowers take too much risk in equilibrium and the more indebted they are the more risk they take. When borrowers face a low loan rate, they will tend to borrow more, thus increasing their indebtedness and taking more risk. In this context, we study if and how a fractional reserve banking system can help achieve the (constrained) optimal level of debt and risk taking.

We show that reserve requirements combined with inflation exploit a trade-off between risk-taking and the level of investment. In an inflationary environment, reserve requirements are costly for banks when there is inflation and they recoup this cost by adjusting the loan rate they charge borrowers. So borrowers will tend to reduce the amount they borrow and their leverage. Since they have more at stake, they increase the quality of their project and take less risk. In general, we show that when the real loan rate is low, the entrepreneur takes on a higher leverage and chooses riskier investment. This trade-off is reminiscent of the two themes we mentioned earlier.

However, another effect of imposing higher reserve requirements when there is no deposit insurance is to increase the return on deposits when borrowers fail. As a result, deposits backed by more reserves can buy more: the borrower faces a better effective loan rate. Again, this can improve stability by lowering the terms of loans but this may also hamper it by inducing more borrowing.

The optimal combination of reserve requirement and inflation trades-off these effects. The Friedman rule or zero reserve requirement is not necessarily optimal, as it would induce too much leverage. In spite of being the safest system, fully backed deposit may not be optimal as it can reduce leverage too much. Also, we
show the central bank should implement the Friedman rule only if the amount of real balances in the system is sufficiently large. When reserves are too low, the central bank should deviate from the Friedman rule, as again borrowers will otherwise take too much risk.

The rest of the paper is organized as follows. Section 2 presents some preliminary considerations that help understand the working of the model. We present the full-fledged model in Section 3 and we derive the equilibrium in Section 4. We consider the case where the central bank’s balance sheet is the policy variable in Section 5. Section 6 contains several extensions such as the effect of bail-out policies. The last section summarizes the findings and concludes.

2 Preliminaries

We present the backbone of the model before delving into its details. Consider an economy with a supplier, an entrepreneur, and a banker. All three agents are risk neutral. The entrepreneur has no equity but operates a production function $f(k,q)$ given by

$$f(k,q) = \begin{cases} zF(k) & \text{with probability } q, \\ 0 & \text{otherwise.} \end{cases}$$

(1)

where $F(k)$ is a neoclassical production function that transforms capital $k$ into some numeraire goods. We assume $F(k)$ is homogeneous of degree $\sigma$. The entrepreneur chooses $q \in [0,1]$ by suffering a cost $\gamma q^2 F(k)/2$. So a higher $q$ makes the positive outcome more likely but also lowers the surplus from production. We think of $q$ as the quality of the project but also as inversely related to the level of risk taking. In this section, we set $z = \gamma = 1$. The supplier can produce any level of $k$ at cost $Ck$. The supplier will not lend to the entrepreneur because he does not trust him to repay his loan. But the supplier trusts the banker. As a result, production takes place only when the banker intermediates the trade between the entrepreneur and the supplier.

The entrepreneur offers the banker to pay $Rk$ if the project succeeds. Out of this payment, the banker will pay the supplier. The supplier expects the entrepreneur to choose quality $Q$. The supplier has no bargaining power and the banker promises $Ck/Q$ to the supplier if the project succeeds. The banker’s outside option is $\bar{R}k$.\footnote{For example, the banker borrows $k$ and invests it in a safe project that returns $\rho k$ and the payoff of the banker is $(\rho - C)k \equiv \bar{R}k.$} The choice of $q$ is contractible by the banker (but not by suppliers) so that there is no moral hazard between the entrepreneur and the banker in this backbone version of our model. We assume the banker has
no bargaining power, so a contract is a tuple \((k, R, q)\) that maximizes the entrepreneur’s payoff,

\[
\max_{q,R,k} q[F(k) - Rk] - \frac{1}{2} q^2 F(k),
\]

subject to the bank’s participation constraint

\[
q \left( R - \frac{C}{Q} \right) k \geq Rk.
\]

This constraint is binding so we can simplify the problem as

\[
\max_{q,k} q \left( 1 - \frac{q}{2} \right) F(k) - \frac{q}{Q} Ck - Rk
\]

The first order condition with respect to \(q\) is

\[
q = 1 - \frac{Ck}{Q F(k)}, \tag{2}
\]

so that quality is an inverse function of the leverage of the entrepreneur-banker pair \(\mathcal{L} = Ck / (Q F(k))\). Then more leverage leads to more risk taking. Also, how \(k\) affects leverage is directly related to \(\sigma\), a feature which will be important in the rest of this paper. In particular the elasticity of leverage with respect to investment is

\[
\frac{\partial \mathcal{L}}{\partial k} \frac{k}{\mathcal{L}} = \frac{C F(k) - F'(k)k k}{Q} \frac{F'(k)F(k)}{F(k)^2} \mathcal{L} = 1 - \sigma
\]

As \(\sigma\) increases, leverage is less sensitive to investment and so is quality. Notice that (2) defines the best response \(q\) to the expectation of the supplier, \(Q\). In particular, everything else held constant, a more optimistic supplier (higher \(Q\)) leads to a higher \(q\). The reason is that a more optimistic supplier will lower the bank’s funding costs, inducing the entrepreneur to choose a better quality.

We now turn to the first order condition with respect to \(k\):

\[
q \left( 1 - \frac{q}{2} \right) F'(k) = \frac{q}{Q} C + \bar{R} \tag{3}
\]

which says that the (expected) marginal benefit of investment must equal the marginal cost. The two curves (2) and (3) can be solved jointly to find \(q\) and \(k\) as a function of \(Q, C,\) and \(R\). In equilibrium \(q = Q\), so that combining (3) together with (2) gives

\[
q F'(k) = (2 - \sigma) C + 2 \bar{R} \tag{4}
\]

and (2) then gives

\[
q = 1 - \frac{C \sigma}{(2 - \sigma) C + 2 \bar{R}}
\]
Consider now the problem of a planner seeking to maximize welfare, given by

$$\max_{q,k} q \left( 1 - \frac{q}{2} \right) F(k) - Ck$$

The first order conditions are

$$q^* = 1$$

and (replacing $q^* = 1$):

$$\frac{1}{2} F'(k^*) = C$$

Clearly, the solution to (2) is less than $q^*$, so that the contract involves too much risk. Since $q < q^*$, we know the solution $k$ to (3) is lower than $k^*$ so that the equilibrium contract displays under-investment relative to the optimum.\(^4\)

In the sequel, we endogenize $C$ and $\bar{R}$ as functions of the mix of fundings between inside and money, as well as policy variables such as monetary policy and reserve requirements. Then we study the optimal policy choice. It is easy to see that there can be a trade-off between risk and output: $k$ is decreasing in $\bar{R}$ if is large while $q$ is always increasing in $\bar{R}$. While these are rather general considerations, we need to enrich the model to endogenize $C$ and $\bar{R}$.

To be clear, the model in the next section differs from this simple model along several dimensions. First, we introduce a moral hazard problem between the bank and the entrepreneur so that $q$ will no longer be contractible. As the example shows, this is not central to our analysis, but it gives some interesting results. Second, banks can fund their operation using inside money (deposits or banknotes) or money. They can borrow money in an interbank market and their deposits (if any) are traded in a Walrasian market. So interbank market participants and buyers of deposits forms expectation about the quality of the projects the banks fund. This endogenizes $C$. Third, and finally, the central bank pays interest rate on reserves, which together with the interbank market endogenizes the banks’ outside option $\bar{R}$.

3 Environment

The model is a version of Rocheteau, Wright, and Zhang (2016). Time $t = 1, 2, \ldots$ is discrete and infinite and each period is divided in two subperiods. There are two goods, capital and the numeraire that are not storable. There are three types of risk neutral agents, each with measure one, entrepreneurs (e), suppliers (s), and bankers (b). Suppliers are infinitely lived, but entrepreneurs and banks live finite life. In each

\(^4\)There is under-investment whenever

$$F'(k) = \frac{q}{q(1 - \frac{q}{2})} > F'(k^*) = 2C$$

In equilibrium $q = q^* < 1$ so that $q(1 - q/2) < 1/2$. Hence $(C + \bar{R})/(q(1 - q/2)) > 2C$. \(5\)
period, a measure one of entrepreneurs is born at the beginning of the first stage and die at the end of the second stage. Also, a measure one of bankers is born in the second stage of each period and die in the second stage of the next period. Therefore, we have an overlapping generation structure of bankers. As will be clear, this implies that entrepreneurs cannot build equity so that they have to borrow, and bankers will be short-sighted but will be able to build equity when young.

Suppliers are endowed with two linear technologies: In the first subperiod, they can transform hours worked one-for-one into capital, while in the second subperiod they can transform hours worked one-for-one into the numeraire good. Young bankers are also endowed with two technologies: they can transform hours worked one-for-one into the numeraire good, they can also create deposits (banknotes) that can be traded but cannot be counterfeited. A deposit is a promise to 1 unit of the numeraire good in the next subperiod. Old bankers cannot work, but they are endowed with a debt enforcement technology. A fraction \( \lambda \in [0, 1] \) of entrepreneurs are endowed with the technology \( f(k, q) \) given by (1) that transforms capital into the numeraire good. All in all, suppliers produce the capital good \( k \) in the first subperiod when it is invested by the entrepreneurs with a return in the second subperiod. The numeraire is only produced and/or consumed in the second subperiod.

Preferences of suppliers and bankers are represented by the utility function \( U(c, h) = c - h \) where \( c \geq 0 \) is the consumption of the numeraire and \( h \geq 0 \) is hours worked. Entrepreneurs’ preferences for consumption of the numeraire is just \( u(c) = c \). Banks and suppliers discount the future at a rate \( \beta \in (0, 1) \).

In the first subperiod, there is an OTC market for banking services with search and bargaining, followed by an interbank market and a Walrasian market for capital. As not all entrepreneurs are productive, some banks are unmatched and will become lenders on the interbank market. In the second subperiod, there is a frictionless centralized market where agents produce and/or consume the numeraire and settle debts.

Entrepreneurs cannot commit to repay suppliers and they have no equity. So entrepreneurs have to borrow assets from a bank and use them to purchase capital. There are two types of assets. We already mentioned deposits. There is also money (currency, or reserves) which stock evolves according to \( M_{t+1} = (1 + \pi)M_t \). The price of money in terms of the numeraire is \( v_t \). In stationary equilibrium \( v_{t+1}M_{t+1} = v_tM_t \), so \( v_t = (1 + \pi)v_{t+1} \). We let the nominal rate be \( i = (1 + \pi)/\beta - 1 \).

The timing is as follows. In each period, the OTC market for banking services open first. There, entrepreneur-bank pairs are formed. One entrepreneurs is matched randomly with one bank with probability \( \alpha \) where \( \alpha \leq 1 \) is the measure of operating banks in the economy. As a result, a measure \( \alpha \lambda \) of entrepreneurs are productive and can possibly obtain a loan from a bank. With probability \( (1 - \alpha)\lambda \) an entrepreneur is productive but does not meet a bank. Once matched, the bank and the productive entrepreneur bargain over the terms of the loan in a way we describe below. Concurrently banks with too little reserves can borrow

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\(^5\)Conveniently, this implies that we can also interpret capital as labour.

\(^6\)We take \( \alpha \) as given for most of the paper, and we endogenize it in one of our extensions.
from banks with too much reserves in the interbank market. Then, entrepreneurs who managed to obtain a bank loan use it to purchase capital from suppliers in the Walrasian market for capital. Then they invest capital and choose the quality of their project.

In the second subperiod, successful entrepreneurs repay their (now old) banker by transferring some of their output. The bank redeems its deposits using reserves and some of the output from the entrepreneur. Unsuccessful entrepreneurs cannot pay back their banker who then only has reserves to redeem their deposits. The banker fails when reserves are not enough to pay the par-value of the deposits. In this case, the holders of deposits are paid pro-rata. Because old bankers cannot produce the numeraire goods, deposits can be risky if the banker does not hold enough reserves. We assume the failed bank is replaced by a new bank. One of our extensions modifies this assumption so that there is a real cost of failure.

Finally, suppliers and successful banks may hold reserves or currency but have little use for it. Then they can sell them to a young banker. The young banker has the ability to produce the numeraire and so can build equity in the form of reserves (this is sweat equity) for the upcoming loan market.

We now describe each market in more details and in chronological order.

### 3.1 Markets for bank loans and reserves

A productive entrepreneur needs a bank loan because he has no equity and he is not trustworthy to obtain credit directly from suppliers. A bank can grant a loan using a mix of currency and deposits. Let $p^o$ and $p^n$ be the real price of currency and deposits in the market for capital, respectively. Then entrepreneurs need to borrow $p^o k^o$ in money and an amount $p^n k^n$ of deposits for them to buy $k^o + k^n$ units of capital. The bank charges a fee $\phi$ for this loan. So a bank loan is a list $(p^n k^n, p^o k^o, \phi)$. With this loan, an entrepreneur buys $k = k^o + k^n$ from suppliers and pay them the real amount $\psi = p^n k^n + p^o k^o$ with a mix of deposits and money. To simplify notation, we will say that a bank loan is a list $(k^n, k^o, \phi)$.

Limited liability implies that the expected value of a loan $(k^n, k^o, \phi)$ for an entrepreneur is

$$\max_{q \leq 1} q \left[ \left( z - \frac{\gamma}{2} q \right) F(k^n + k^o) - \psi - \phi \right]$$

since entrepreneurs only repay their loans when their project succeeds. Also, notice that the entrepreneur chooses $q$ after contracting with the bank. The level of quality that solves this problem is

$$q = \frac{z}{\gamma} - \frac{p^n k^n + p^o k^o + \phi}{\gamma F(k^n + k^o)} \quad (5)$$

The entrepreneur takes more risk as the principal $\psi$ or the interest $\phi$ increase. The reason is the usual moral-

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7These prices may differ because we do not assume deposit insurance (however, see an extension). So banks may default on their deposits but the CB does not default on money.
The hazard problem inherent to bank loans: the entrepreneur is less diligent as part of the outcome accrues to the bank, and he is less so as his debt is higher.

We now define the value of a loan for a bank. First, banks face reserve requirements. We assume that a young bank who issued deposits worth $k^a$ has to set aside enough reserves (in the form of money) to pay at least $\tilde{\tau} k^a$ when old to its note holders, where $\tilde{\tau} \in [0, 1]$ is a policy variable. So reserve requirements differ from capital requirements insofar as reserve requirements are not invested and can be used to pay holders of deposits if investments turn bad. The regulator pays an interest $r \in \mathbb{R}$ (positive or negative) on required and excess reserves. So banks who extend a contract $(k^a, k^o, \phi)$ must hold reserves $R$ so as to satisfy the constraint (in real terms)

$$\tilde{\tau} k^a \leq (1 + r)R.$$  \hspace{1cm} (6)

As we will show below, (6) binds whenever the rate of inflation is higher than $r$, as banks try to economize on the amount of reserves they hold. Also, for simplicity we use the normalization $\tilde{\tau} \equiv (1 + r)\tau$.  \hspace{1cm} (8)

Banks can change their holdings of reserves in the interbank market. This market is organized as a Walrasian market and $i_m$ is the market clearing rate. Banks can increase their current holdings $m^b$ by borrowing $b$ reserves on the interbank market, or they can lend $\ell$ reserves when they have excess reserves. We assume that interbank loans are unsecured and so junior to any other loans in case of bankruptcy. Hence, if the entrepreneur fails, its suppliers get paid first by the bank from whatever amount of reserves it has. Since we consider fractional reserves ($\tilde{\tau} k^a \leq p^a k^a$), a failing bank with a binding reserve requirements can only (partially) reimburse holders of deposits (suppliers) and not its junior interbank loans. In this case, banks lending on the interbank market expect their loans to fail with probability $Q$ and to garnish $R(\bar{\ell})$ in case of default. \hspace{1cm} (9)

So the reserve management problem of a bank with contract $(k^a, k^o, \phi)$ is

$$U(k^a, k^o, m^b) = \max_{\ell, \ell, b} \{ (1 + r)R + Q(1 + i_m)\ell + (1 - Q)R(\ell) - (1 + i_m)b \}$$  \hspace{1cm} (7)

subject to

$$R = m^b - k^o - \ell + b$$  \hspace{1cm} (8)

$$\tilde{\tau} k^a \leq (1 + r)R$$  \hspace{1cm} (9)

$$0 \leq \ell, b$$  \hspace{1cm} (10)

Notice that there is no reserve requirement when the bank lends money instead of just creating deposits. This is inconsequential as in equilibrium with inflation above $r$, banks are economizing real balances such that $k^o = 0$. Also, we could have imposed the constraint $(1 + r)R \geq \tau p^a k^o$, but it simplifies the analysis to work with the constraint $(1 + r)R \geq \tau k^a$ instead.

\hspace{1cm} (16)

We assume that banks perfectly diversify their interbank lending, so that they know a fraction $Q$ of banks are matched with a successful entrepreneur and so can reimburse their interbank loans. The other banks can only pay once they redeem their deposits. We show later that banks borrowing on the interbank market do not lend there. Hence, those banks have wealth $(1 + r)\bar{R} - p^a \bar{k}^a$ once they redeem their deposits. When those banks borrowed $\bar{b} > 0$ then

$$\mathcal{R}(\ell) = \min \left\{ (1 + i_m); \max \left\{ (1 + r)\bar{R} - p^a \bar{k}^a; 0 \right\} \frac{1}{\bar{b}} \right\} \ell$$
The first constraint (8) is the definition of reserves. The second constraint (9) is the reserves requirement constraint. A bank holding reserves \( m_b \) who does not meet an entrepreneur has value \( U(0,0,m_b) \). We will denote by \( b^1, \ell^1 \) (resp. \( b^0, \ell^0 \)) the level of borrowing and lending on the interbank market by banks who are matched (resp. not matched) with a productive entrepreneur. We can now define the bank’s surplus from contract \((k^n, k^o, \phi)\), as

\[
q [\psi + \phi - p^n k^n + U(k^n, k^o, m)] + (1 - q) \max \{-p^n k^n + U(k^n, k^o, m); 0\} \geq U(0,0,m)  
\] (11)

If the entrepreneur fails, the bank’s only resource from which it has to pay its deposits is the net income from managing its reserves \( U(k^n, k^o, m) \). The bank goes bankrupt and naturally gets zero payoff if it does not have enough reserves to pay its liabilities. If the entrepreneur succeeds, the bank can redeem its deposits using \( U(k^n, k^o, m) \) as well as the principal plus interest \( \psi + \phi \).

To solve for the equilibrium bank loan contract, we assume the entrepreneur takes the entire surplus,

\[
S(k^n, k^o, m_b) = q \left[ (z - \frac{\gamma}{2}q)F(k^n + k^o) + U(k^n, k^o, m_b) - p^n k^n \right] 
+ (1 - q) \max \left[ U(k^n, k^o, m_b) - p^n k^n; 0 \right] - U(0,0,m)  
\] (12)

Therefore, the contract \((k^n, k^o, \phi)\) will maximize the expected total surplus (12) subject to \( k^n, k^o \geq 0, q \) solving (5) and the bank getting zero surplus, or

\[
\mathcal{P}(m_b) \equiv \max_{k^n \geq 0, k^o \geq 0, \phi} S(k^n, k^o, m_b)  
\]

subject to (5) and (11) which says that the interest rate on the loan \( \phi \) has to make the bank at least indifferent between lending and not. Our assumption that loan contracts give the entire surplus to entrepreneurs implies that the moral hazard problem originating from lending is minimized. Any other surplus sharing rule would make the moral hazard problem even more acute thus reinforcing our result.

Once banks loans are agreed upon and required reserves are set aside, the market for capital opens.

### 3.2 Capital market

The demand for capital is given by the bank loan contract. To determine the supply of capital, we turn to the problem of suppliers in the capital market. Obviously, suppliers are aware of the moral hazard problem and they expect each entrepreneur (and their bank) to fail with probability \( 1 - Q \). In addition, when the bank fails, they expect that each of its deposit returns \( R^n \leq 1 \). The capital market being Walrasian, suppliers are able to perfectly diversify by selling capital to every productive entrepreneurs.\(^{10}\) Hence, the problem of

\(^{10}\)The ability to diversify plays no role in this model where suppliers are risk neutral.
a supplier entering the capital market with $m^s$ real units of money is

\[ V^s(m^s) = \max_{k^m,k^o \geq 0} \{ m^s - k^m - k^o + (1 - Q) R^m p^n k^m + Q p^n k^m + p^o k^o + W^s(0) \} \]  

(13)

where $k^m$ is the capital sold for deposits at (real) price $p^n$ and $k^o$ is the capital sold for money at (real) price $p^o$, and we already anticipated that the value of net worth in the second subperiod $W^s(\omega)$ is linear. The first order conditions give us

\[ p^o = 1. \]  

(14)

As well as

\[ (1 - Q) R^m p^n + Q p^n = 1 \]

When a bank holding reserves $R$ defaults, its resources $(1 + r)R$ are split evenly among the holders of its deposits. Since there are $p^n k^m$ deposits in circulation, each deposit receives

\[ R^m = \min\{ (1 + r) \frac{R}{p^n k^m}, 1 \} \]

when the bank defaults. So we can write the first order condition for $k^m$ as

\[ (1 - Q) \min\{ (1 + r) \frac{R}{p^n k^m}, 1 \} p^n + Q p^n = 1. \]  

(15)

If banks have enough reserves to redeem their deposits then they are safe and $p^n = 1$. However, if $(1 + r)R < k^m$, then deposits carry a risk-premium.

### 3.3 Centralized market (CM)

In the last centralized market, successful entrepreneurs settle their debt and their bank redeem their deposits. They consume whatever is left. Suppliers consume their real net worth $\omega$ and solve the following savings problem

\[ W^s(\omega) = \omega + T + \max_{m^s \geq 0} \{ -(1 + \pi) m^s + \beta V^s(m^s) \}, \]  

(16)

where $T$ is a real transfer (possibly negative) and $m^s$ is the real amount of money the suppliers chooses to carry over to the next period. So using the expression for $V^s(m)$ in (13) which is linear in $m$, the problem of suppliers becomes

\[ \max_{m^s} \{ -(1 + \pi) + \beta m^s \} \]

so that $m^s = 0$ iff $(1 + \pi) / \beta = 1 + i > 1$, it is indeterminate when $i = 0$ and it is infinite whenever $i < 0$. So the only equilibrium is one where the nominal interest rate is positive, $i \geq 0$.\footnote{Equilibrium with $1 + \pi < \beta$ are possible whenever the regulator remunerates the reserves of suppliers at $r < 0$.} Given this result, we will naturally assume that suppliers hold no real balances since they have no liquidity needs.

Let us now turn to the problem of young banks in the CM. Given the form of the contract $(k^m, k^o, \phi)$ determined in the previous section as a solution to $P(m^b)$ – and functions of $m^b$ – banks will choose real
balances \( m^b \) to maximize their lifetime net worth:

\[
V^b(m^b) = \max_{m^b} -(1 + \pi)m^b + \beta(1 - \lambda)U(0,0,m^b) \\
+ \beta \lambda \left\{ q \begin{array}{l}
\psi + \phi - p^n k^n + U(k^n,k^o,m^b) \\
\text{received by E paid to S}
\end{array} \right\} + (1 - q) \max \begin{array}{l}
0; -p^n k^n + U(k^n,k^o,m^b) \\
\text{paid to S}
\end{array} (17)
\]

\[
= \max_{m^b} -(1 + \pi)m^b + \beta U(0,0,m^b) (18)
\]

where the second equality follows from our assumption that bank loans gives all the surplus to the entrepreneurs. The reader should notice the existence of a hold-up problem when there is reserve requirement and \( i > r \): While banks incur the cost of bringing real balances in the loan market, they do not obtain any surplus from it. Therefore, if the interbank market was absent, there would not be any equilibrium with lending. However, the interbank market gives them a viable (outside) option with payoff \( U(0,0,m^b) \).

The entrepreneur has to promise the bank at least what it would get on the interbank market, and this is sufficient for an equilibrium with lending to exist. Still, in equilibrium, the bank is indifferent as to the amount of real balances it brings (it would obtain the same payoff by bringing none).

### 3.4 Equilibrium

We can now define a symmetric steady state equilibrium.

**Definition 1.** A symmetric steady state equilibrium is a list consisting of reserve management choices \( \{(R^i, \ell^i, b^i)\}_{i=0,1} \), loan contracts \( (k^n,k^o,\phi) \), project quality \( q \) and \( Q \), prices \( p^n, p^o, i^m, r \), choice of real balances \( m^b, m^s \), inflation \( \pi \), and a fraction of active bank \( \alpha \) such that: given prices \( \pi, r, i^m, p^n \) and \( p^o \), the amount of capital \( k^n \) and \( k^o \) solve \( P(m^b) \), \( q \) is given by (5), \( \phi \) is given by (11), \( m^b \) solves (17), \( m^s \) solves (16), \( (R^1, \ell^1, b^1) \) is the solution to (7) for banks who met an active entrepreneur, \( (R^0, \ell^0, b^0) \) is the solution to (7) for banks who met an inactive entrepreneur, \( p^n \) is given by (15), \( p^o = 1 \), \( i^m \) clears the interbank market, the market for balances clear \( m^b + m^s = m \), aggregate quality is consistent with individual choices \( Q = q \).

Since we consider a symmetric equilibrium, the interbank market clearing condition is

\[
(1 - \lambda)(\ell^0 - b^0) + \lambda(\ell^1 - b^1) = 0.
\]

There are two types of equilibrium to consider. First, banks could obtain a positive surplus even when their entrepreneur fails. In this case the bank has enough reserves to pay all of its liabilities and we can write its surplus as

\[
q \left[ (p^n k^n + k^o) + \phi \right] - p^n k^n + U(k^n,k^o,m) \geq U(0,0,m^b)
\]
while the total surplus (12) is

\[ S(k^n, k^o, m^b) = q(z - \frac{1}{2}q)F(k^n + k^o) + U(k^n, k^o, m^b) - p^n k^n - U(0, 0, m^b) \]  

(19)

In a second case, banks only obtain a positive surplus when their entrepreneur succeeds, and they default on some or of all their liabilities otherwise. In this case the bank’s participation constraint is

\[ q \left( p^n k^n + k^o \right) + \phi - p^n k^n + U(k^n, k^o, m) \geq U(0, 0, m^b) \]

\[ q \left( p^n k^n + k^o \right) + \phi \geq U(0, 0, m^b) + p^n k^n - U(k^n, k^o, m) \]

while the total surplus is

\[ S(k^n, k^o, m^b) = q \left[ (z - \frac{1}{2}q)F(k^n + k^o) + U(k^n, k^o, m^b) - p^n k^n \right] - U(0, 0, m^b) \]  

(20)

To solve for an equilibrium, of either type, we proceed as follows.

1. Suppose banks default or not when the entrepreneur fails.

2. Solve problem \( P(m^b) \) using either (19) or (20) in several steps. Use the bank’s participation constraint to find the expression for \( \phi \). Replace this expression for \( \phi \) in \( q \) and solve for \( \partial q / \partial k^i \) for \( i = n, o \) and \( \partial q / \partial b^1 \). Take first order conditions with respect to \( b^1, k^o, \) and \( k^n \). Solve for the different possible cases, \( b^1 \geq 0, k^o \geq 0, k^n \geq 0 \) and the reserve requirement constraint binding or not.

3. In case \( b^1 > 0 \) so that the interbank market is active, use the market clearing condition to find \( b^1 \) or \( m^b \). If the interbank market is active, it means that banks must get a higher payoff by lending on the interbank market than keeping reserves. In this case, the relevant outside option is to lend on the interbank market and to obtain \( (1 + i_m)\ell \) or whatever is paid out in case banks default.

4. Given the solution for the contract \((k^n, k^o, \phi)\) solve for \( q \) given in equilibrium \( q = Q \).

5. Find conditions on parameters such that banks default or not (as guessed in step 1 above), such that \( k^o > 0, k^n > 0 \). etc. and for banks to accumulate cash.

The equilibrium when \( i > r \) refers to the case when real balances are costly to carry. In this case, the equilibrium will be of the second type: a bank will not fail only if the entrepreneur’s project succeeds. Still, banks could decide to finance entrepreneur either with inside or money only depending on \( r \) and \( \bar{r} \). We separate the analysis between the two policy instruments of the regulator: Reserve requirements when \( i > r \) and the size of the central bank’s sheet when \( i = r \).
4 Reserve requirements $\tau$ ($i > r$)

In this section, we analyze the case where the regulator remunerates reserves but not enough to compensate banks for the cost of holding them. Because reserves are costly to hold, we guess that banks will finance their operation in the interbank market, whether they lend inside or money. If they lend deposits, the reserve requirement will bind so that $(1 + r)R = \tilde{\tau}k^n$. If they lend only money, then they will not keep any reserves. As a consequence, we guess that banks will default on their interbank loans whenever the entrepreneur fails (and partially on their deposits). So in this case, interbank lenders cannot recover any of their loans when the borrower fails, that is $R(\ell) = 0$. Then the bank’s participation constraint is

$$q \left[ (p^n k^n + k^o) + \phi - p^n k^n + U(k^n, k^o, m^b) \right] \geq U(0, 0, m^b) = U(m^b)$$

while the total surplus is

$$S(k^n, k^o, m^b) = q \left[ (z - \frac{\gamma}{2} q) F(k^n + k^o) + U(k^n, k^o, m^b) - p^n k^n \right] - U(0, 0, m^b) \tag{21}$$

Using (21), the problem $\mathcal{P}(m^b)$ becomes

$$\mathcal{P}(m^b) \equiv \max_{k^n, k^o, b^1} q \left[ (z - \frac{\gamma}{2} q) F(k^n + k^o) + U(k^n, k^o, m^b) - p^n k^n \right]$$

subject to $k^n, k^o, b^1 \geq 0$, and

$$\begin{align*}
q &= \frac{U(m^b)}{q} - U(k^n, k^o, m^b) + p^n k^n \\
\frac{\gamma F(k^n + k^o)}{\gamma F(k^n + k^o)} &\leq m^b + b^1 - k^o \tag{22}
\end{align*}$$

where

$$U(k^n, k^o, m^b) = (1 + r) \left( m^b + b^1 - k^o \right) - (1 + i_m) b^1$$

and we used $\tau = \tilde{\tau}/(1 + r)$.

In the Appendix, we show that the interbank market is active, $b^1 > 0$, whenever $i > r$ and the reserve requirement binds because the interbank market rate is greater than the interest rate on reserves, $i_m > r$. Then after simple but tedious algebra we obtain the first conditions for $k^o$ and $k^n$, respectively,

$$\begin{align*}
k^o : \quad & F'(k^n + k^o) \left( z - \frac{\gamma}{2} q \right) - \frac{1}{2} \sigma \frac{U(m^b)}{q(k^n + k^o)} = (1 + i_m) - \tilde{\lambda}_{k^o} \tag{23} \\
k^n : \quad & F'(k^n + k^o) \left( z - \frac{\gamma}{2} q \right) - \frac{1}{2} \sigma \frac{U(m^b)}{q(k^n + k^o)} = p^n + \tau (i_m - r) - \tilde{\lambda}_{k^n} \tag{24}
\end{align*}$$

where $\tilde{\lambda}_{k^o}$ and $\tilde{\lambda}_{k^n}$ are the appropriately scaled Lagrange multipliers on the positivity constraints $k^o, k^n \geq 0$. 
The second term on the left-hand side of these conditions \( \frac{1}{2} \frac{F'(k^n+k^o)}{qF(k^n+k^o)} U(m^b) \) captures the moral hazard affecting the quality choice, which increases the cost of investment. The entrepreneur’s choice of quality is a direct function of what the bank gets paid, relative to total production. The implicit payment to the banks is \( U(m^b) \) because it receives no surplus from trade. The parameter \( \sigma \) will play an important role in the effect on moral hazard on the equilibrium investment because it magnifies the effect on the banks’ compensation.

Clearly, we obtain from (23) and (24),

\[
\begin{align*}
k^n > 0 \text{ and } k^a = 0 & \quad \text{iff} \quad 1 + i_m < p^n + \tau(i_m - r) \\
k^n = 0 \text{ and } k^a > 0 & \quad \text{iff} \quad 1 + i_m \geq p^n + \tau(i_m - r)
\end{align*}
\]

The condition for lending inside or money has a straightforward interpretation: in an equilibrium where banks borrow on the interbank market, the marginal cost of lending an additional unit of money is the interbank rate \( 1 + i_m \). The marginal cost of lending deposits is the cost to redeem them \( p^n \), as well as the cost of holding reserves \( \tau \) against them: \( \tau(i_m - r) \). This is the cost of borrowing reserves on the interbank market when reserves are remunerated at rate \( r \).

The two conditions above show that a bank either lends its entire stock of money to the entrepreneur (including what it borrows on the interbank market) or it issues deposits and keeps just enough reserves to satisfy the reserve requirement. In either case, the reserve requirement binds and \( R = \tau k^n \). Then, using (15), the price of deposits (if any) is,

\[
p^n = \frac{1}{Q} - \frac{1}{Q} (1 - Q)(1 + r)\tau
\]

(25)

Since a bank cannot pay all of its deposits when it fails, it cannot be any of its interbank liability because they are junior to deposits in bankruptcy. Therefore, bank’s expected payoff from lending one unit of reserve on the interbank market is just \( Q(1 + i_m) \). As a result, the bank’s outside option in bargaining is

\[
U(m^b) = Q(1 + i_m)m^b.
\]

(26)

Also problem (17) implies that the interbank market rate is the nominal rate adjusted for risk,

\[
1 + i_m = (1 + i)/Q.
\]

(27)

We now analyze the two types of equilibrium below, starting with the case where \( k^n > k^a = 0 \).
money  We now analyze the case where $k^o > 0$ and $k^n = 0$. As banks do not issue deposits, they do not keep reserves. Hence $m^b + b^1 = k^o$ and market clearing implies

$$m^b = \lambda k^o$$

Then $k^o$ solves

$$F'(k^o) \left( z - \frac{\gamma}{2} q^o \right) - \frac{1}{2} \sigma \frac{U(\lambda k^o)}{q^o} = 1 + i_m$$

and the quality level with money $q^o$ is given by

$$q^o = \frac{z}{\gamma} - \frac{U(m^b)}{q^o} - \frac{U(0, k^o, m^b)}{\gamma F(k^o)}$$

$$= \frac{z}{\gamma} - \frac{U(\lambda k^o) + q^o(1 + i_m)(1 - \lambda)k^o}{\gamma q^o F(k^o)}$$

Using (29) to replace $\gamma q^o$ in (28), we obtain the marginal benefit of investment

$$z F'(k^o) = [2 - (1 - \lambda)\sigma] \frac{(1 + i)}{Q}.$$  

Notice that the level of investment is an increasing function of the expected choice of quality and it does not depend on the entrepreneur’s own choice of quality $q^o$. Then using (29), (26), (27), as well as $\sigma F(k) = F'(k)k$, give us the best response of the entrepreneur’s choice of quality $q^o$ as a function of the expected choice $Q$, (and $k^o$)

$$q^o = \frac{z}{\gamma} - \left( \frac{\lambda}{q^o} + \frac{1 - \lambda}{Q} \right) \frac{1 + i}{\gamma F'(k^o)}$$

Figure 1a shows the above curve. There are two solutions to (31). Since the entrepreneur is maximizing his payoff taking everything else as given, he will always choose the highest $q$ as his best response, and so the best response is the upper part of the curve (31) shown in red in the figure. So everything else constant, individual quality choice is increasing in expected quality (individual and expected quality are strategic complement) whenever $\lambda$ is low enough so that there is a lot of interbank market borrowing as measured by $(1 - \lambda)m^b/\lambda$. This is intuitive: increasing expected quality makes interbank loans cheaper. So the entrepreneur’s financing cost will decline when the banks funds a large share of its lending activities by borrowing on the interbank market. As a consequence, the entrepreneur chooses a higher level of quality. In contrast, when inflation increases, the bank pays more when it borrows on the interbank market but also when it builds reserves when young. As a consequence, the borrowing cost increases and the entrepreneur may choose a lower quality even if $Q$ increases. Of course, these partial equilibrium arguments neglects the fact that the entrepreneur may wish to invest more or less. Substituting $k^o$ using (30), we obtain the “total”
(a) “Partial” best-response $q^o(Q, k^o)$ in (31)

(b) Best response $q^o(Q)$ in (32)

Figure 1: Best responses

The best response function $q^o(Q)$,

$$q^o = \frac{z}{\gamma} - \left( \frac{\lambda}{q^o} + \frac{1 - \lambda}{Q} \right) \frac{Q\sigma}{\gamma [2 - (1 - \lambda)\sigma]}$$

as shown in the Figure 1b. Again, there are two solutions to (32) and the entrepreneur will always choose the highest $q$ as his best response, as shown by the red curve. Maybe surprisingly, the “total” best response is a decreasing function. So the direct “partial” effect of $Q$ on $q$ which plays through the market price of deposits is reversed when we consider the effect of $Q$ on investment. The intuition should be clear: Lower interbank rates transmit into cheaper bank loans. On the one hand, this induces the borrower to take less risk. On the other hand, this induces the borrower to invest more. But as her loan size is larger, a borrower takes more and not less risk. Finally, notice that the best response function (32) does not depend on inflation, as the effect of inflation is completely reflected in the marginal cost of investment.

Imposing the equilibrium condition $q''(Q) = Q$, we then obtain the solution for the choice of quality

$$Q^o = \frac{z}{\gamma} \left[ 1 - \frac{(1 + i)k^o}{Q^o z F(k^o)} \right]$$

where $(1 + i)k^o$ is the (real) debt of the entrepreneur towards the bank when the bank makes no surplus, while $Qz F(k)$ is aggregate output. Therefore, we can write $Q^o$ as

$$Q^o = \frac{z}{\gamma} \left( 1 - \frac{\text{debt}}{\text{GDP}} \right)$$
or

\[ Q^o = \frac{z}{1 + \mathcal{L} z} \]

where \( \mathcal{L} = \text{debt}/\text{GDP} \) is leverage. Hence overall quality increases iff leverage drops. It would seem that restricting debt would be good in this economy. However, this view overlooks the effect of debt on investment and the potential increase in GDP that it may bring about. To solve for the equilibrium, we can arrange (32) with \( q^o = Q^o \) to find the equilibrium level of quality,

\[ Q^o = \frac{z}{\gamma} \left[ \frac{2 - (2 - \lambda)\sigma}{2 - (1 - \lambda)\sigma} \right], \]

and use this result in (30) to find \( F'(k^o) \) and the equilibrium level of investment \( k^o \),

\[ F'(k^o) = \frac{\gamma}{z^2} \left[ \frac{2 - (1 - \lambda)\sigma}{2 - (2 - \lambda)\sigma} \right] (1 + i). \]

The last two equations are giving us the equilibrium when banks do not create deposits. We now turn to the equilibrium with inside money.

**Deposits** Using similar steps, we now analyze the equilibrium when \( 1 + i - \left( \frac{i - r}{1 + r} \right) \hat{\tau} \geq 1 \). Then the bank only lends deposits, \( k^n > 0 \) and \( k^o = 0 \). The best response function is now

\[ q^n = \frac{z}{\gamma} - \left\{ \left( \frac{\lambda}{q^n} + \frac{(1 - \lambda)}{Q} \right) (1 + i)\tau + \frac{1 - (1 + r)\tau}{Q} \right\} \frac{\sigma}{\gamma F'(k^n)} \]  

(33)

The main differences with the money equilibrium is the presence of the reserve requirement parameter \( \tau \) – explained by the fact that the bank holds reserves (money) in proportion to \( \tau \) – and the fact that banks earn interest rate \( r \) on its reserves which lowers the risk of deposits holders and contributes to diminish the funding cost for the entrepreneurs. Then after some algebra, we find the marginal benefit of investment as

\[ zF'(k^n) = \{(2 - \sigma) [1 + \tau (i - r)] + \sigma (1 + i) \lambda \tau \} \frac{1}{Q} \]  

(34)

Everything else given, investment will decrease with inflation and reserve requirements, but increase with the interest rate paid on reserves. Replacing (34) in (33), we find the best response function \( q^n(Q) \) in implicit form,

\[ q^n = \frac{z}{\gamma} - \frac{\sigma z}{\gamma} \left\{ \left( \frac{Q}{Q'} - 1 \right) (1 + i)\lambda \tau + (i - r)\tau + 1 \right\} \left\{ (2 - \sigma) [1 + \tau (i - r)] + \sigma (1 + i) \lambda \tau \right\} \]  

(35)

Compared with the best response function in the equilibrium with money, the entrepreneur now reacts to all the policy variables: inflation, reserve requirement, and interest rate on reserves. Figure 2 shows how (35) reacts to an increase in inflation (again the relevant portion of the decreasing curve): the best response function shifts upward from the blue to the red curve, so that for all \( Q \), the entrepreneur chooses a higher
$q^n$. While we have seen that the direct effect of inflation on $q^n$ could be positive (see (33)), inflation always reduces investment. As a result leverage falls, and the entrepreneur ends up choosing a higher quality for any $Q$.

Now, in equilibrium $q^n = Q$ and we obtain the equilibrium $Q^n$,

$$Q^n = \frac{z}{\gamma} \left[ 1 - \frac{(1 + i \tau) k^n}{Q^n z F(k^n)} \right],$$

(36)

where $(1 + i \tau) k^n$ is now the entrepreneur’s (real) debt towards the bank in equilibrium. Notice that, given $i$, it is smaller than when the bank finances the entrepreneur with money because it only requires $\tau$ units of money. Also given investment $k^n$, an increase in $\tau$ implies that the entrepreneur must compensate the bank for holding more reserves (and the compensation is higher as inflation is higher) making the moral hazard problem more acute. Using (36), as well as the market clearing condition on the interbank market $m^b = \lambda \tau k$, we can find the two equations determining the equilibrium with deposits: the risk equation,

$$Q^n \left( \frac{z}{\gamma} - Q^n \right) = \frac{z}{\gamma} \left( \frac{1 + \frac{\tau}{1 + \tau}}{z F(k^n)} \right) k^n, \quad \text{(risk)}$$

and the investment equation,

$$Q^n \left( \frac{z}{\gamma} - \frac{1}{2} Q^n \right) = \frac{1}{\gamma F(k^n)} \left( 1 + \frac{\tau (i - r)}{(1 + r)} + \frac{1}{2} \frac{(1 + \epsilon)}{(1 + r)} \lambda \tau \right), \quad \text{(investment)}$$

Below we plot these two equations in the $(Q^n, k^n)$-dimension. It is easy to see that the curves depicting both equations are bell-shaped. The LHS of the risk-curve achieves a maximum at $Q^n = z/2 \gamma$, while the LHS of
the investment curve achieves a maximum at \( Q^n = z/\gamma \) (=1 in the figure). The equilibrium is given by the intersection between the two curves. Figure (3a) shows an example where there is no inflation \((i = 0)\) and the equilibrium is at point \(A\).

**Summary and discussion** The following proposition summarizes the characteristics of the equilibrium when \(i > r\).

**Proposition 1.** When \(i > r\), a unique equilibrium exists where the reserve requirement always binds and the interbank market is active. If \(i < \left( \frac{1 - \lambda}{1 + r} \right) \) the solution is given by

\[
Q^n = \frac{z}{\gamma} \left[ \frac{2 - (2 - \lambda) \sigma}{2 - (1 - \lambda) \sigma} \right]
\]

and \(k^n = 0\) while \(k^o\) solves

\[
F'(k^o) = \frac{\gamma}{z^2} \left[ \frac{2 - (1 - \lambda) \sigma}{2 - (2 - \lambda) \sigma} \right] (1 + i)
\]

and the bank defaults on its liabilities (in this case, the interbank loans).

If \(i \geq \left( \frac{1 - \lambda}{1 + r} \right) \) the solution is given by

\[
Q^n(i, r, \tilde{\tau}) = \frac{z}{\gamma} \frac{2 (1 - \sigma) \left[ 1 + \tilde{\tau} \left( \frac{i}{1 + \tilde{\tau}} \right) \right] + \sigma \lambda \tilde{\tau} \left( \frac{1 + i}{1 + \tilde{\tau}} \right)}{(2 - \sigma) \left[ 1 + \tilde{\tau} \left( \frac{i}{1 + \tilde{\tau}} \right) \right] + \sigma \lambda \tilde{\tau} \left( \frac{1 + i}{1 + \tilde{\tau}} \right)}
\]

(37)

and \(k^n = 0\) while \(k^o\) solves

\[
F'(k^o) = \frac{\gamma}{z^2} \left[ \frac{(2 - \sigma) \left[ 1 + \tilde{\tau} \left( \frac{i}{1 + \tilde{\tau}} \right) \right] + \lambda \sigma \tilde{\tau} \left( \frac{1 + i}{1 + \tilde{\tau}} \right)}{(2 - \sigma) \left[ 1 + \tilde{\tau} \left( \frac{i}{1 + \tilde{\tau}} \right) \right] + \sigma \lambda \tilde{\tau} \left( \frac{1 + i}{1 + \tilde{\tau}} \right)} \right]^2
\]

(38)

and the bank defaults on its liabilities (deposits and interbank loans).

We illustrate how the equilibrium with deposits changes with an increase in inflation in Figure (3a). The equilibrium with no inflation is at point \(A\). As inflation rises from, the marginal cost of capital increases, everything else constant. Hence, given \(Q^n\), investment will decline, and the investment-curve shifts down. This induces a decrease in capital, which, if nothing would change would induce a move down on the risk-curve from \(A\) to \(B\): As leverage declines, risk drops (quality increases). However, inflation induces the bank to charge a higher interest rate to the entrepreneur (the risk-curve shift down), so that the entrepreneur chooses a lower quality. So the equilibrium moves from \(B\) to \(C\). In the new equilibrium, there is lower investment, but it is not clear if the reduction in investment is sufficient to undo the increase in cost of funds, so that leverage still declines. Our result implies that the investment effect is always stronger than the direct effect so that \(Q^n\) always increases with inflation. This may be due to the presence of a positive
feedback loop: Because quality increases, deposits are now safer, so the risk premium in $p^n$ declines. This contributes to a further reduction in leverage and higher average quality.

In addition, there are several remarks worth making on Proposition 1.

- A bank loan consists of only deposits or only money and it is never a mix of both payment instruments. The condition that determines the form of the loans (deposits or money) implies that the bank lends money only when $r < 0$. Intuitively, the bank seeks to economize on the (required and excess) reserve it holds because they are taxed.

- When $r < 0$, policy variables do not affect risk-taking and only inflation (negatively) affects investment. The reason is that both debt and output moves one for one with inflation so that the entrepreneur’s leverage (and so quality) is independent of inflation. Finally, it should be obvious that when he is financed with money, the entrepreneur chooses $Q^n$ independently of the level of reserve requirement or the interest rate on reserves. So when $r < 0$, policy variables do not affect risk in the economy and only inflation affects the size of investment.

- When $i > r \geq 0$, reserve requirement, inflation, and interest rate on reserves are complement in affecting risk, and not substitute. Indeed, when $r \geq 0$ the bank only lends deposits to the entrepreneur and keep just enough reserves to satisfy the reserve requirements. Setting $\tau = 0$, it is straightforward that $Q^n$ is independent of inflation or the interest rate on reserves. Hence, for policies to affect risk-taking it must be that $\tau > 0$. However, $\tau$ should not be too large: setting $\tau = 1$ (100% required reserves and no money creation by banks), the solution for $Q$ is identical to the one with only money loans, which is again independent of $i$ and $r$.\(^{12}\)

\(^{12}\)There is a discontinuity in $Q$ as the bank switches from lending deposits to lending money. The origin of this discontinuity lies in a discontinuous reserve requirement between the two forms of fundings: it is effectively 100% when the bank lends money.
When $\hat{\tau} > 0$, the comparative statics of $Q^n$ with respect to its arguments are

$$\frac{\partial Q^n}{\partial \tau} > 0, \quad \frac{\partial Q^n}{\partial \hat{\tau}} > 0, \quad \text{and} \quad \frac{\partial Q^n}{\partial r} < 0.$$ 

So more inflation and higher reserve requirements reduce risk-taking, but higher interest rate on reserves increases risk taking. The intuition for the last result is simple: When the interest rate on reserves is higher, the bank has more to lose by lending to the entrepreneur (e.g. in case the entrepreneur fails, the bank loses the interest on the reserves it holds) and so requires a higher payment. This reduces the entrepreneur’s incentives to exert an effort and $Q^n$ drops.

The reaction of investment $k^n$ to the policy variable depends on $\sigma$. While Figure 3a shows that $k^n$ drops with inflation, the decline in investment is not ineluctable. In particular, the investment curve is directly susceptible to $\sigma$. Increasing $\sigma$ shifts the investment curve down, but also makes it steeper, so that the investment-curve will cross the risk-curve when it is increasing, as illustrated in Figure 3b. Then, as we show below, increasing inflation might increase equilibrium investment.

Suppose $\sigma$ is high enough (as specified in the proof). When $i \simeq 0$, investment $k^n$ increases in $\hat{\tau}$. When $\hat{\tau} \approx 0$ and $i \approx r$ (Friedman rule), investment $k^n$ increases in $i$ but decreases in $r$. In words, if money is relatively cheap for banks to hold ($i = r = 0$), then investment increases with reserve requirement. This is intuitive: Since there is little inflation, the cost to raise reserve requirement is small for the bank. But higher reserves, implies that deposits are safer. As a result, entrepreneurs can now invest more for a given loan size. Similarly, investment increases when there is little reserve requirement and inflation is raised from the FR. This seems counterintuitive at first because we concluded above that $Q$ increases with inflation thanks to lower leverage. But a general equilibrium effect plays through the decline in the risk premium on inside money. As it is cheaper to fund entrepreneurs, and as $\hat{\tau}$ is small, banks do not suffer much from the rising inflation and the overall cost of funds for the entrepreneur can decline. Our result shows that this decline in the cost of funds, while enticing the entrepreneur to invest more, can be large enough to reduce leverage.

and $< 100\%$ otherwise. When $\hat{\tau} = 1$ there is no such discontinuity.
• We can compare how risk, investment, and leverage differ across the inside and money regimes. For any policy parameters \((i, r, \tilde{\tau})\), we have \(Q^o > Q^n(i, r, \tilde{\tau})\): Abstracting from considerations related to equilibrium existence, the entrepreneur chooses a higher quality when he is financed with a money loan. Therefore leverage is always higher with inside money loans. As a result, the monetary authority can select an equilibrium with a higher quality of bank’s assets and lower corporate leverage by decreasing \(r\) below the threshold for which banks finance entrepreneurs with money loans, i.e. \(i < \left(\frac{i - \lambda}{1 + \tau}\right) \tilde{\tau}\). However, this comes at a cost because it may be that \(k^n(i, r, \tilde{\tau}) > k^o\).

For example if \(\tilde{\tau} = 1\) we end up with (when \(r > 0\)):

\[
F'(k^n) = \frac{\gamma [2 - (1 - \lambda)\sigma]^2}{z^2 [2 - (2 - \lambda)\sigma]} \left(1 + i\right) 
\leq F'(k^o) = \frac{\gamma [2 - (1 - \lambda)\sigma]^2}{z^2 (2 - (2 - \lambda)\sigma)} (1 + i)
\]

So higher quality and lower leverage can imply lower investment. Again, there may be a trade-off between safety and production. In the next Section we analyze this trade-off in more details, by looking at welfare.

While we studied steady state, we may be able to extrapolate to conclude that a system relying on deposits will be characterized by higher growth, but also a higher risk of failure for its entrepreneurs.

### 4.1 Welfare when \(i > r\)

In this section we study the welfare consequences of the risk-investment trade-off. First, we compute the number of operative banks in steady state. As all agents are risk neutral, welfare is given by aggregate output net of the cost of producing the investment good and the entrepreneur’s cost of effort,

\[
W = \alpha \lambda \left[Q(z - \frac{\gamma}{2} Q) F(k) - k\right]
\]

where \(k = k^n + k^o\).
A planner seeking to maximize welfare will choose investment \( k^\ast \) and quality \( Q^\ast \) to solve

\[
Q^\ast \left( z - \frac{\gamma}{2} Q^\ast \right) F'(k^\ast) = 1
\]

and

\[
Q^\ast = \frac{z}{\gamma}
\]

So that \( F'(k^\ast) = 2\gamma/z^2 \). Finally, using (39) we can express welfare as

\[
W^\ast = \frac{\alpha \lambda}{\sigma} k^\ast (1 - \sigma)
\]

We now determine an expression for welfare in equilibrium. Notice we can rewrite (23) and (24), as

\[
k^o : \quad \sigma Q \left( z - \frac{\gamma}{2} Q \right) \frac{F(k^n + k^o)}{k^n + k^o} = \frac{1}{2} \sigma \frac{U(m^k)}{(k^n + k^o)} + Q(1 + i_m) - Q \lambda k^o
\]

\[
k^n : \quad \sigma Q \left( z - \frac{\gamma}{2} Q \right) \frac{F(k^n + k^o)}{k^n + k^o} = \frac{1}{2} \sigma \frac{U(m^k)}{(k^n + k^o)} + Q \left[ p^n + \tau(i_m - r) \right] - Q \lambda k^n
\]

Therefore, we can use (40) to express welfare in the money equilibrium, when \( i < \left( \frac{\tau}{1 + \tau} \right) \tilde{T} \), as

\[
W^o = \frac{\alpha \lambda}{\sigma} k^o \left[ (1 + i) \left( \frac{1}{2} \lambda \sigma + 1 \right) - \sigma \right]
\]

Using the equilibrium expression for \( k^o \) we obtain

\[
W^o = \frac{\alpha \lambda}{\sigma} (1 + i)^{\frac{\tau}{1 + \tau}} \left[ \frac{1}{2} \frac{\gamma}{\sigma z^2} \frac{2 - (1 - \lambda) \sigma^2}{2 - (2 - \lambda) \sigma} \right] \left[ (1 + i) \left( \frac{1}{2} \lambda \sigma + 1 \right) - \sigma \right]
\]

This function is always decreasing in \( i \geq 0 \). Hence the optimal (feasible) inflation is \( i = 0 \). The monetary authority can implement this optimal level of inflation iff \( r < 0 \). Notice that the monetary authority would like to bring \( i < 0 \) but cannot because suppliers would then demand infinite real balances.

Now, we can use (41) to express welfare in the equilibrium with deposits, if \( i \geq \left( \frac{i - r}{1 + r} \right) \tilde{T} \), as

\[
W^a = \frac{\lambda}{\sigma} k^n \left[ \frac{1}{2} \frac{\gamma}{1 + r} \left( 1 + i \right) + \gamma \tilde{T} \left( \frac{i - r}{1 + r} \right) + 1 - \sigma \right]
\]

and using the equilibrium expression for \( k^n \), we obtain a rather simple expression for welfare (where we used \( \tau = \tilde{T} / (1 + r) \) to simplify exposition,

\[
W^a = \frac{\alpha \lambda}{\sigma} \left[ \frac{1}{2} \frac{\gamma}{\sigma z^2} \frac{(2 - \sigma) [1 + \tau (i - r)] + \lambda \sigma \tau (1 + i)^2}{[1 + \tau (i - r)] + \sigma \lambda \tau (1 + i)} \right] \left[ \frac{1}{2} \frac{\gamma}{\sigma z^2} (1 + i) \lambda \tau + \gamma (i - r) + 1 - \sigma \right]
\]
From this expression, it is feasible to compute the values for \( i > r \) and \( \tau \) that will maximize welfare, at least numerically. As anticipated, when \( i = r \) and \( \tau = 1 \) then \( \mathcal{W}^o = \mathcal{W}^n \).

Below we plot \( \mathcal{W}^n \) as a function of inflation (or the nominal interest rate) when \( r = 0 \) and where \( i = 0 \) is the Friedman rule. We use the following parametrization, \( F(k) = k^\lambda \), \( \lambda = 0.95 \), \( \gamma = 10 \) and \( z = 9.9 \). In the first two plots, we show welfare for \( \sigma = 0.6 \) for different values of \( \tau \) and nominal rate \( i \). In a second series of plot, we lower \( \sigma \) to 0.2.

![Graphs of Welfare](image)

**Figure 5: Welfare**

The result that welfare is increasing with inflation is however susceptible to the form of the production function. For the same set of parameters as above, figures (5c) and (5d) shows welfare as a function of \( \tau \) and \( i \) respectively when \( \sigma = 0.2 \) for different configurations of \( \tau \) and \( i \). Then welfare can be decreasing if inflation is too high.

Finally, we compare welfare in the two types of equilibrium. Figure (6) shows welfare as a function of the interest rate on reserves \( r \) on the x-axis for some arbitrary parameters \( (i = 0.1) \). When \( r < r^* \), the equilibrium with money prevails. When \( r > r^* \), and in particular when \( r \) is sufficiently close to \( i = 0.1 \), the equilibrium with deposits prevails instead. When \( r \) is sufficiently close to \( i \), money is relatively inexpensive
to hold so that the money economy can achieve a higher welfare. This can be achieved by raising the reserve requirement. However, as $r$ declines, welfare with deposits becomes higher and is maximized at $r^*$. However, dropping the rate on reserves further will induce a sharp reduction in welfare due to a shift in the equilibrium. We are not making a general statement here: there are parameters for which deposits are always better (or always worse).

![Figure 6: Welfare comparison](image)

5 Large balance sheet or the floor system ($i = r$)

We now analyze the case where the central bank sets $i = r$. This policy is akin to a floor system (or large balance sheet) whereby the bank is setting the nominal rate to the rate it pays on reserves. Recall that when $i > r$ it is costly to accumulate real balances, and banks are accumulating reserves only because the (active) interbank market provides them with a relatively attractive outside option.

When $i = r$ however, there may still be an equilibrium even if the interbank market is inactive, as the bank’s outside option just compensates its cost of accumulating reserves. Precisely, there are three types of equilibrium when $i = r$: (1) The amount of real balances allows banks to redeem their deposits independently of the entrepreneur’s success. (2) The amount of real balances allows banks to satisfy their reserves requirement without borrowing on the interbank market, but not enough to redeem their deposits if the entrepreneur fails. In this equilibrium, the interbank market is inactive. (3) The amount of real balances is insufficient for banks to satisfy their reserve requirement and the interbank market is active. In each of these cases, banks are indifferent as to the amount of real balances they bring, so that the amount of real balances is a choice of the monetary authority. We go through each cases in turn.
5.1 No default

When there is enough real balances in the economy, banks will not need to borrow to satisfy their reserve requirement, so \( b^1 = 0 \) and the interbank market rate is the interest rate on reserves \( 1 + i_m = 1 + r \). When banks do not default on their deposits, their price do not carry any risk premium so that \( p^n = 1 \), which occurs whenever
\[
(1 + r)(m^b - k^n) > p^n k^n = k^n.
\]
In this case, the reserve requirement constraint is always satisfied and we can just ignore it when solving the contracting problem. Then this problem becomes,
\[
\mathcal{P}(m^b) \equiv \max_{k^n, k^o} q(z - \frac{\gamma}{2} q) F(k^n + k^o) + (1 + r) (m^b - k^o) - p^n k^n
\]
subject to \( k^n, k^o \geq 0 \), as well as
\[
q = \frac{z}{\gamma} - \frac{p^n k^n + k^o + \phi}{\gamma F(k^n + k^o)},
\]
and the participation constraint of the bank,
\[
q (p^n k^n + k^o + \phi) + (1 + r) (m^b - k^o) - p^n k^n \geq (1 + r) m^b.
\]
The first order conditions are
\[
k^o : \quad - \frac{(z - \gamma q)}{(2 \gamma q - z)} [1 + r - q(z - \gamma q) F'(k^n + k^o)] + q(z - \frac{\gamma}{2} q) F'(k^n + k^o) - (1 + r) + \lambda_{k^o} = 0
\]
\[
k^n : \quad - \frac{(z - \gamma q)}{(2 \gamma q - z)} [p^n - q(z - \gamma q) F'(k^n + k^o)] + q(z - \frac{\gamma}{2} q) F'(k^n + k^o) - p^n + \lambda_{k^n} = 0
\]
As \( p^n = 1 \), the bank does not use money \( k^n = 0 \) whenever \( r > 0 \). When \( r = 0 \) it does not matter if the bank finances the entrepreneur through inside or money, so we consider the case where \( k^n > k^o = 0 \). When there is no interbank borrowing or risk premium on the price of deposits, the contract solution only depends on the individual choice of quality \( q \) and is no longer a function of aggregate quality \( Q \). In other words, there is no market externality. After some algebra, we can find the solution as
\[
q = Q = \frac{z (2 - \sigma)}{\gamma}.
\]
and
\[
F'(k^n) = \frac{4 \gamma}{z^2 (2 - \sigma)}.
\]
It is useful to notice that the solution does not depend on the policy variables, \( i, r \) or \( \tau \). It does not depend
on \( \tau \) because the reserve requirement is not binding. It does not depend on \( r = i \) because, with no default, the bank’s is indifferent between lending one additional unit or keeping it in reserves (so \( \phi \) does not depend on \( r \)). Also, it does not depend on \( r = i \) because there is no interbank market activity. Still, the solution is not the optimal solution with \( q = \frac{z}{\gamma} \) because of the moral hazard problem between the bank and the entrepreneur.

This equilibrium requires that \((1 + r)m^b > k^n\). That is

\[
m^b > \frac{1}{1 + r} \left[ \frac{4 \gamma}{\sigma z^2 (2 - \sigma)} \right]^{\frac{1}{2 - \gamma}}.
\]

In this case welfare is

\[
W^*(k^n) = \alpha \lambda \left[ Q \left( z - \frac{\gamma}{2} Q \right) F(k^n) - k^n \right]
\]

Therefore, with no default,

\[
W^* = \frac{\lambda}{1 + \lambda \left( 1 - \frac{z}{\gamma} \right)^2} \left( \frac{4 \gamma}{\sigma z^2 (2 - \sigma)} \right)^{\frac{1}{2 - \gamma}} \left( \frac{2 - \sigma}{2\sigma} \right)
\]

This is the highest achievable welfare level because the bank internalizes most of the entrepreneur’s decision as limited liability is not biting in this case and there is no market externality as the price of deposits do not carry a risk premium.

### 5.2 Default but no interbank market

In this case, the reserve requirement constraint is not binding, there is no activity on the interbank market, but the bank still does not have enough reserves to redeem its entire stock of deposits. The bank only lends deposits whenever the cost \( p^n \) of doing so is lower than the opportunity cost of lending money, or

\[
p^n \leq 1 + r.
\]

We can use (22) to obtain the expression for the choice of quality

\[
q = \frac{z}{\gamma} - \left\{ \left( \frac{Q}{q} - 1 \right) \left( 1 + r \right) \frac{m^b}{k^n} + 1 \right\} \frac{k^n}{\gamma Q F(k^n)}
\]

And the first order conditions with respect to \( k^n \) is

\[
F'(k^n) \left( z - \frac{\gamma}{2} q \right) - \frac{1}{2} \sigma \frac{U(m^b)}{qk^n} = p^n.
\]
In this equilibrium, the price of deposits is

\[ p^n = \frac{1}{Q} - \frac{1}{Q} (1 - Q)(1 + r) \frac{m^b}{k^n} \]

so that using the FOC for \( k^n \),

\[ F'(k^n) \left( z - \frac{\gamma}{2} q \right) - \frac{1}{2} \sigma \frac{U(m^b)}{qk^n} = p^n \]

as well as (42) we obtain the marginal benefit of investment,

\[ zQF'(k^n) = 2 - \sigma - (2(1 - Q) - \sigma)(1 + r) \frac{m^b}{k^n} \]

Again, as in the model with \( i > r \), this expression is independent of the entrepreneur’s choice of quality but it is a function of the aggregate quality level. Plugging it back in (42), we obtain the best response function \( q(Q) \) of the entrepreneur,\(^{13}\)

\[ q = \frac{z}{\gamma} - \frac{z}{\gamma} \frac{\sigma \left\{ \left( \frac{Q}{\gamma} - 1 \right) (1 + r) \frac{m^b}{k^n} + 1 \right\}}{2 - \sigma - (2(1 - Q) - \sigma)(1 + r) \frac{m^b}{k^n}} \]

In equilibrium \( q(Q) = Q \), so that the equilibrium level of quality is

\[ Q = \frac{z}{\gamma} \left[ \frac{2(1 - \sigma) + [\sigma - 2(1 - Q)] (1 + r) \frac{m^b}{k^n}}{2 - \sigma + [\sigma - 2(1 - Q)] (1 + r) \frac{m^b}{k^n}} \right] \]

We can parametrize this equilibrium by using \( m^b = \mu k^n \). Then

\[ Q = \frac{z}{\gamma} \left[ \frac{2(1 - \sigma) + [\sigma - 2(1 - Q)] (1 + r) \mu}{2 - \sigma + [\sigma - 2(1 - Q)] (1 + r) \mu} \right] \]

and

\[ zQF'(k^n) = 2 - \sigma - (2(1 - Q) - \sigma)(1 + r) \mu. \]

\(^{13}\)Relative to the other case with \( i > r \), here there is no interbank market, and the \( Q \) in the denominator – which is absent from the case with interbank market – comes from deposits.

28
So given $\mu$ we obtain $Q(\mu)$, and $k^n(\mu)$. Then we obtain the amount of real balances as $m^b = \mu k^n(\mu)$. This is an equilibrium whenever $1/(1 + r) \geq \mu \geq \tilde{\tau}/(1 + r)$. Welfare is parametrized by $\mu$

$$W(\mu) = \alpha \lambda \left[ Q(\mu) \left( z - \frac{\gamma}{2} Q(\mu) \right) - \frac{\gamma}{2} Q(\mu) \right]$$

and it is decreasing in $\mu \in [\tilde{\tau}/(1 + r), 1]$. The reason is intuitive: as $\mu$ increases, the banks has more real balances to fund the entrepreneur who can invest more. As usual, this induces him to take too much risk relative to the planner’s allocation because he benefits from limited liability. (To be sure, the objective function of the entrepreneur is increasing in $\mu$.) Therefore, maximum welfare in the region $\mu \in [\tilde{\tau}/(1 + r), 1]$ is attained at $\mu = \tilde{\tau}/(1 + r)$. For later references, we denote this level of welfare by $W(\tilde{\tau}/(1 + r)) < W^*$. At this level of real balances, the equilibrium features over-investment.

### 5.3 Default with interbank market

The third solution is for $(1 + r)m^b < \tilde{\tau}k^n$: Banks do not bring enough real balances to satisfy the reserve requirements at the desired level of investment and they borrow on the interbank market. Then there is default and the price of deposits

$$p^n = 1 - \frac{1}{Q} \left( 1 - Q \right) (1 + r) \tau$$

As banks lend in the interbank market, it must be that the interbank rate compensate for the risk of failure, that is $1 + i_m = (1 + r)/Q$. Then it is easy to show that $k^o = 0$ for all $r > 0$. The first order condition for $k^n$ can be simplified to

$$QF'(k^n) \left( z - \frac{\gamma}{2} Q \right) = 1 + \frac{1}{2} \sigma \left( 1 + r \right) \frac{m^b}{k^n}$$

(43)

The interbank market clearing condition together with the condition that $\ell \leq m^b$, and the binding reserve requirement $m^b + b^1 = \tau k^n$, give

$$m^b \geq \lambda \frac{\tilde{\tau}}{1 + r} k^n.$$

---

14 This equilibrium exists iff

$$p^n = 1 - \frac{1}{Q} \left( 1 - Q \right) (1 + r) \frac{m^b}{k^n} \leq 1 + r$$

$$\frac{1 - (1 + r) Q}{1 - Q} \leq (1 + r) \frac{m^b}{k^n}$$

also, as $p^n \geq 1$ it must be that

$$(1 + r) \frac{m^b}{k^n} \leq 1$$

Therefore, we need $r$ to be sufficiently high for this equilibrium to exist. Under some uninteresting conditions on $r$, $\sigma$, and $\tau$ we obtain that at the solution

$$\frac{1 - (1 + r) Q}{1 - Q} < \tau$$

also, when $\sigma < 1/2$ the unique (real) solution for $Q$ is the “positive”-root.
Also using the expression for the entrepreneur’s choice of quality, and

\[ q = \frac{z}{\gamma} - \frac{k^n}{\gamma QF(k^n)} \]

Once again, we can parametrize the equilibrium using \( \mu = \frac{m^b}{k^n} \), so that the equilibrium quality and investment levels are

\[ Q = \frac{z}{\gamma} \left[ \frac{2(1 - \sigma) + \sigma(1 + r)\mu}{2 - \sigma + \sigma(1 + r)\mu} \right] \]

with

\[ F'(k^n) = \frac{\gamma}{\sigma^2} \frac{[2 - \sigma + \sigma(1 + r)\mu]^2}{2(1 - \sigma) + \sigma(1 + r)\mu} \]

This is an equilibrium whenever

\[ \frac{\tilde{\tau}}{1 + r} \geq \mu \geq \frac{\tilde{\tau}}{1 + r}. \]

In this case, welfare is

\[ \tilde{W}(\mu) = \alpha \lambda k^n \left[ Q(z - \frac{\gamma}{2} Q) (k^n)^{\sigma - 1} - 1 \right] \]

and using (43),

\[ \tilde{W}(\mu) = \alpha \lambda \left[ \frac{\gamma}{\sigma^2} \frac{(2 - \sigma + \sigma(1 + r)\mu)^2}{2(1 - \sigma) + \sigma(1 + r)\mu} \right] ^\frac{1}{\sigma - 1} \left( \frac{1}{\sigma} + \frac{1}{2}(1 + r)\mu - 1 \right) \]

which is increasing in \( \mu \). Therefore the highest welfare for \( \mu \in [\lambda \frac{\tilde{\tau}}{1 + r}, \frac{\tilde{\tau}}{1 + r}] \) is

\[ \tilde{W} \left( \frac{\tilde{\tau}}{1 + r} \right) = \alpha \lambda \left[ \frac{\gamma}{\sigma^2} \frac{(2 - \sigma + \sigma\tilde{\tau})^2}{2(1 - \sigma) + \sigma\tilde{\tau}} \right] ^\frac{1}{\sigma - 1} \left( \frac{1}{\sigma} + \frac{1}{2}\tilde{\tau} - 1 \right) \]

### 5.4 Welfare comparison

The following table summarizes the equilibrium in the floor system.

<table>
<thead>
<tr>
<th>( \frac{m^b}{k^n} = \mu )</th>
<th>( \tilde{W}(\mu) )</th>
<th>( F'(k^n) )</th>
<th>( Q )</th>
<th>Default/b ≥ 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \in \left[ \frac{\tilde{\tau}}{1 + r}, \frac{\tilde{\tau}}{1 + r} \right] )</td>
<td>( \tilde{W}(\mu) )</td>
<td>( \frac{\gamma}{\sigma^2} \frac{(2 - \sigma + \sigma(1 + r)\mu)^2}{2(1 - \sigma) + \sigma(1 + r)\mu} )</td>
<td>( \frac{z}{\gamma} \frac{2(1 - \sigma) + \sigma(1 + r)\mu}{2 - \sigma + \sigma(1 + r)\mu} )</td>
<td>Default/b ≥ 0</td>
</tr>
<tr>
<td>( \in \left[ \frac{\tilde{\tau}}{1 + r}, \frac{\tilde{\tau}}{1 + r} \right] )</td>
<td>( \tilde{W}(\mu) )</td>
<td>( \frac{2 - \sigma - (2(1 - Q) - \sigma(1 + r))\mu}{zQ} )</td>
<td>( \frac{z}{\gamma} \frac{2(1 - \sigma) + \sigma(1 + r)\mu}{2 - \sigma + \sigma(1 + r)\mu} )</td>
<td>Default/b &gt; 0</td>
</tr>
<tr>
<td>( \geq \frac{1}{1 + r} )</td>
<td>( \tilde{W}^*(\mu) )</td>
<td>( \frac{4\gamma}{\sigma^2(2 - \sigma)} )</td>
<td>( \frac{z}{\gamma} \frac{(2 - \sigma)^2}{2} )</td>
<td>No default/b = 0</td>
</tr>
</tbody>
</table>

Table 1: Summary of equilibrium in the floor system

In this section, we compare the different welfare levels for different levels of real balances \( m^b \), but also with
welfare levels away from the Friedman rule, that is for \( i > r \). Maybe surprisingly, we have

\[
W^* > \tilde{W} \left( \frac{\tilde{\tau}}{1 + r} \right) > W \left( \frac{i}{1 + r} \right)
\]

While the planner would prefer the unconstrained equilibrium with high levels of real balances, the planner would prefer to have few levels of real balances than intermediate levels. The intuition is again, that with relatively few reserves, banks have to borrow on the interbank market at a premium interest rate, thus reducing the level of investment but increasing its quality.

Now, suppose banks cannot accumulate real balances (because their sweat equity is limited) then it may be better to deviate from the FR. Recall that welfare with deposits and \( i > r \) is

\[
\mathcal{W}(i, r) = \alpha \left( \frac{1}{\sigma^2} \left( \frac{1}{1 + \tilde{\tau}} \right) + \lambda \frac{\tilde{\tau}(1 + \tilde{\tau})}{1 + \tilde{\tau}} \right) \left( \frac{1}{2} \lambda \tilde{\tau} \left( \frac{1}{1 + r} \right) + \frac{1}{\sigma} \tilde{\tau} \left( \frac{i - r}{1 + r} \right) + \frac{1}{\sigma} - 1 \right)
\]

Notice that

\[
\mathcal{W}(i, i) = \tilde{W} \left( \frac{i}{1 + r} \right) < \bar{W} \left( \frac{i}{1 + r} \right)
\]

However, as Figure 7 below shows, for some parameter configurations, increasing \( i \) above \( r \) (while leaving \( r \) the same) can increase \( \mathcal{W}(i, r) \) above \( \tilde{W} \left( \frac{i}{1 + r} \right) \) or even \( \bar{W} \left( \frac{i}{1 + r} \right) \) (in red), although never above \( W^* \).\(^{15}\) So while a deviation from the FR can never attain the constrained first best welfare level \( W^* \) it can do better whenever \( \mu < 1 \). The reason is that increasing \( i \) makes accumulating real balances more expensive. As a consequence, banks reduce their holdings of real balances, which limits the entrepreneur’s “over”-leverage problem in the region where there is default and no interbank market.

6 Extensions

6.1 Operating banks \( \alpha \)

We endogenize \( \alpha \) in this section. We assume that if a bank fails, it is replaced by a new bank, but with a one period lag (the time necessary to unwind the bank). As a result, bankers will not fully internalize the cost of their default. Most, if not all, regulators will agree that it is socially costly for any banks to fail. One such cost is the disruption of the payment system, and some argue that there is a loss of expertise. Our qualitative results that reserve requirements can be welfare improving do not depend on this assumption, although it may affect the quantitative predictions of the model.

We now compute the number of operating banks in period \( t \). Since banks that financed a failing entrepreneur

\(^{15}\)The parameters used to construct the figure are \( z = \gamma = 1; \sigma = 0.3; \tau = 0.05; \lambda = 0.95; r = 0 \).
lose their license for one period, the number of operating banks in period \( t \) is \( \alpha_t \):

\[
\alpha_t = Q \underbrace{\lambda \alpha_{t-1}}_{\#b \text{ with a match}} + (1 - \lambda) \alpha_{t-1} + \underbrace{(1 - \alpha_{t-1})}_{\#b \text{ not operating last period}} + \underbrace{\#e \text{ financed}}_{\text{financed}}.
\]

So in steady state \( \alpha \equiv \alpha_t = \alpha_{t-1} \),

\[
\alpha = \frac{1}{1 + \lambda(1 - Q)}.
\]

If operating banks always meet productive entrepreneurs (i.e. \( \lambda = 1 \)) then \( \alpha = 1/(2 - Q) \). Naturally, if \( Q = 1 \) then all banks are operating, while less banks are operating otherwise.

Then all of our equilibrium analysis go through. Welfare is given by aggregate output net of the cost of producing the investment good and the entrepreneur’s cost of effort,

\[
W = \alpha \lambda \left[ Q(z - \frac{\gamma}{2}Q)F(k) - k \right]
\]

where \( \alpha \) is now given by (44). So optimal investment \( k^* \) and quality \( Q^* \) solve

\[
Q^* \left( z - \frac{\gamma}{2}Q^* \right) F'(k^*) = 1
\]
and either \(Q^* = 1\) or it is given by\(^{16}\)

\[
[1 + \lambda(1 - Q^*)](z - \gamma Q^*) F(k^*) = \lambda k^* \left(1 - \frac{1}{\sigma}\right). 
\]

Since \(\sigma \leq 1\), the optimal quality level is \(Q^* \geq z / \gamma\), so that \(F'(k^*) \geq \frac{2\gamma}{z^2}\). As we have seen, absent any externality, these inequality holds with equality, \(Q^* = z / \gamma\) and \(F'(k^*) = 2\gamma / z^2\). The planner requires a higher quality because he internalizes the cost of bank failure.

6.2 (Anticipated) bail-outs

In this section, we consider the case where the government decides to bail-out failing banks when it does not remunerate reserves, \(r = 0\). This means that the government will pay off all of the failing bank’s liabilities by taxing suppliers. When banks are bailed out, all of their liabilities are always repaid so none of their liabilities carry a risk premium. More precisely, the real price of deposits becomes \(p^n = 1\) while the interbank market rate is

\[1 + i_m = \frac{1 + \pi}{\beta}.
\]

The rest of the model is as before. In particular, \(q\) still solves (5), \(\psi = p^n k^n + k^n = k\) and the bank surplus gives \(\phi\) as

\[
\phi = \frac{(1 + i_m)m^b}{q} + (1 + i_m) (k^o - m^b) + i_m \tau k^n - k^n 
\]

Notice that bailing-out the interbank market loans improves the outside option of the bank of lending on the interbank market. As a result, (everything else kept constant) the expectation of a bail-out increases \(\phi\).

Using \(k^n = 0\) and \(k^n = k\), \(q\) is the solution to

\[
q \gamma F(k) = z F(k) - \left(\frac{(1 + i_m)m^b}{q} - (1 + i_m)m^b + (1 + \tau i_m)k^o\right) 
\]

and

\[
\frac{dq}{dk} = \frac{q \left[(z - \gamma q) F'(k) - \frac{\partial(\psi + \phi)}{\partial k}\right]}{- (z - 2q\gamma) F(k) + \frac{\partial(\psi + \phi)}{\partial k} k - (1 + i_m)m^b} 
\]

where \(\frac{\partial(\psi + \phi)}{\partial k} = 1 + \tau i_m < \frac{\partial(\psi + \phi)}{\partial k}\big|_{p_h^k > 1}\). So quality is more sensitive to investment when agents expect a bail-out. Then the expression for \(F'(k)\) is

\[
F'(k) = \frac{2F(k)(1 + \tau i_m)}{z F(k) + (1 + \tau i_m)k - (1 + i_m)m^b} 
\]

\(^{16}\)The first order condition is

\[
\lambda \left[1 + \lambda(1 - Q)\right]^2 \left[Q(z - \frac{\gamma Q}{2})F(k) - k\right] + \frac{1}{1 + \lambda(1 - Q)} (z - \gamma Q) F(k) = 0 
\]

which simplifies in the expression above.
An equilibrium with bail-out is a list \((p^k_n, i_m, k, q, Q, m^b)\) such that given the bail-out policy, and policies \(\tau, \pi\), prices \(p^k_n\) and \(i_m\), and aggregate risk \(1 - Q\), banks optimally choose \(m^b\), entrepreneurs choose \(k\) and \(q\) to maximize their surplus, \(p^k_n = 1\), \(i_m\) clears the interbank market and \(q = Q\). Market clearing condition still requires \(\lambda \tau k = m^b\) and from the banks’ demand for money,

\[
1 + i_m = \frac{1 + \pi}{\beta}.
\]

To solve for the equilibrium risk-return trade-off we use \(k^n = k\) as well as (46) and the expression for \(i_m\) to obtain

\[
q = \frac{z}{\gamma} \left[ 1 - \frac{(1 - \tau + \frac{1 + \pi}{\beta} \tau + \frac{1}{q} (1 - 1)) \frac{1 + \pi}{\beta} \lambda \tau}{z F(k)} k \right] \tag{48}
\]

The term \(\left(\frac{1}{q} - 1\right) \frac{1 + \pi}{\beta} \lambda \tau\) refers to the risk premium the bank charges the entrepreneur, as it would incur no risk from lending on the interbank market, but will suffer a loss if the entrepreneur fails. Notice that there are two quality levels solving (48). The entrepreneur will choose the level of quality that maximizes its expected payoff. Before finding this level it is useful to solve for \(k\) by re-arranging (47) and using the expression for the interbank rate, as well as \(\lambda \tau k = m^b\),

\[
\frac{F(k)}{k} = \frac{1}{\sigma z} \left( (2 - \sigma)(1 + i \tau) + \sigma(1 + i) \lambda \tau \right) \tag{49}
\]

So with the expectation of a full bail-out, risk is not affecting the level of investment. The RHS is increasing with \(\tau\) and \(i\) and the LHS is decreasing with \(k\). Therefore, increasing \(\tau\) and/or \(i\) will decrease investment.

Now we can find the expected payoff of the entrepreneur by using the equilibrium expression for \(q\),

\[
q \left[ z F(k) - (1 + \rho) k \right] - \frac{q^2}{2} \gamma F(k) = \frac{q^2}{2} \gamma F(k)
\]

Since \(F(k)\) is not a function of \(q\) the entrepreneur will always choose the highest \(q\), as this maximizes his payoff. Given the equilibrium level of investment, the aggregate quality under bail-out \(Q^b\) is given by (48), or replacing the expression for \(F(k)/k\):

\[
Q^b = \frac{z}{\gamma} \left[ \frac{2 (1 - \sigma)(1 + i \tau) + \sigma \left(2 - \frac{1}{Q^b}\right)(1 + i) \lambda \tau}{(2 - \sigma)(1 + i \tau) + \sigma(1 + i) \lambda \tau} \right] \tag{50}
\]

where we have used \(i\) to denote the gross nominal rate of interest \((1 + \pi)/\beta\), and the solution to \(Q^b\) is the largest root to this equation (or 1 whichever is smaller).

**Proposition 2.** In an equilibrium with bail-out, the investment level is \(k\) given by (49) and the quality of
projects is $Q^b$ given by the largest root to equation (50), $Q^+$. $k$ is independent of $Q^b$ and always declines with inflation or reserve requirements. $Q^+$ is decreasing in $i$ and $\tau$.

Proof. See Appendix.

We now compare welfare under bail-out and no bail-out, say when $r = 0$. First compare the levels of investment as given by (38) and (49) with $r = 0$ and $\hat{\tau} = \tau$. For any $Q < 1$, $F(k)/k$ is higher and so $k$ is lower whenever it is given by (38) and there is no bail-out. The level of investment with bail-out corresponds to the one with no bail-out when there is no moral hazard problem. Turning to quality, the RHS of (37) is higher than the RHS of (50), except for $Q = 1$ when they are equal, whenever $\tau > 0$. Therefore quality under bail-out will be smaller than with no bail-out.\(^{17}\) The welfare consequences of bail-out then depend on whether the benefits of higher investment dominates the detrimental effects of lower quality. For some parametrization, we found that the investment effect dominates so that welfare under bail-out is higher.

### 6.3 Deposit “insurance”

In this section, we analyze whether an insurance scheme for deposits can do better than reserve requirements, or bail-out policies of the previous section. Banks would have to work in the CM when they are born and pledge resources to the deposit insurance fund, as a fraction $\alpha$ of their loans/deposits $k$. In this sense, the insurance scheme is like a lending tax. However, it is more than a tax, as the deposit insurance would then tap into the funds to guarantee deposits (but not the interbank market loans). Sustainability of the deposit insurance mechanism requires that it has enough resources to cover the shortfall, i.e. $\alpha k = (1 - Q)p^nk^n$. Since in an equilibrium with deposit insurance deposits are as safe as money, $p^n = 1$ and since inflation implies $k = k^n$, we obtain $\alpha = 1 - Q$. Notice that banks always lose their contributions to the deposit insurance fund.

With such an insurance scheme in place, the bank’s problem becomes

$$V^b(m^b) = \max_{m^b} \left\{ -(1 + \pi)m^b + (1 - \lambda)\beta Q(1 + i_m)m^b + \beta \lambda \max_k -\alpha k + q \left( \psi + \phi - \frac{(1 - \tau) + Q(1 + i_m)\tau k}{Q} (1 + i_m) m^b \right) \right\}$$

As before, given $k$ the banks’ zero surplus condition gives $\phi$

$$-\alpha k + q \left( \psi + \phi + \tau k - p^nk^n + (1 + i_m) (m^b - \tau k) \right) \geq Q(1 + i_m)m^b$$

and using $\psi = p^nk^n + k^o = k$ and $k^o = 0$ we obtain

$$\phi = \frac{\alpha k + Q(1 + i_m)m^b}{q} - (1 + i_m)m^b + i_m \tau k$$

\(^{17}\)The 45\(^o\)-line crosses the RHS of (50) before the RHS of (37) so the two solutions of (50) have to be lower than the solution for (37).
where \( q \) is the solution to
\[
q \gamma F(k) = zF(k) - \left( \frac{\alpha k + Q(1 + \imath_m)m^b}{q} - (1 + \imath_m)m^b + (1 + \imath_m)k \right) \tag{51}
\]
and
\[
\frac{dq}{dk} = \frac{q \left( (z - \gamma q) F'(k) - \frac{\partial(\psi + \phi)}{\partial k} \right)}{-(z - 2q \gamma) F(k) + \frac{\partial(\psi + \phi)}{\partial k} k - (1 + \imath_m)m^b}
\]

where
\[
\frac{\partial(\psi + \phi)}{\partial k} = \frac{\alpha}{q} + 1 + \tau \imath_m = 1 + \frac{1 - Q}{q} + \tau \imath_m \\
\geq \left| \frac{\partial(\psi + \phi)}{\partial k} \right|_{\text{no DI}} = p^n + \tau \imath_m = \frac{1 - (1 - Q)}{Q} + \tau \imath_m
\]

where the inequality holds given \( Q \) and when evaluated in equilibrium with \( q = Q \) and \( \tau > 0 \). So, everything else constant, quality is less sensitive to investment when there is deposit insurance. An important change relative to our previous analysis is that \( q \) now affects the marginal cost of bank’s intermediation. Then the expression for \( F'(k) \) is
\[
F'(k) = \frac{2F(k)\left( \frac{z}{q} + 1 + \tau \imath_m \right)}{zF(k) + \left( \frac{\alpha}{q} + 1 + \tau \imath_m \right) k - (1 + \imath_m)m^b} \tag{52}
\]

An equilibrium with deposit insurance is a list \((p^k_n, \imath_m, k, q, Q, m^b)\) such that given the deposit insurance policy \( \alpha \), and policies \( \pi, \tau \), prices \( p^k_n \) and \( \imath_m \), and aggregate risk \( 1 - Q \), banks optimally choose \( m^b \), entrepreneurs choose \( k \) and \( q \) to maximize their surplus, \( p^k_n = 1, \imath_m \) clears the interbank market and \( q = Q \).

Market clearing condition still requires \( \lambda \tau k = m^b \) and from the banks’ demand for money,
\[
1 + \imath_m = \frac{1 + \pi}{Q \beta}.
\]

To solve for the equilibrium risk-return trade-off \( Q \), we use \( q = Q, k^n = k, \alpha = 1 - Q \), as well as (51) and the expression for \( \imath_m \) to obtain
\[
Q = \frac{z}{\gamma} \left[ 1 - \left( \frac{1}{Q} - \tau + \frac{1+i\tau}{Q} \right) \frac{k}{zF(k)} \right] \tag{53}
\]

The fact that interbank market exposures are not covered by the deposit insurance scheme is reflected by the risk premium \( \frac{1+i\tau}{Q} \), while the contribution to the deposit insurance – increasing in the aggregate risk – is captured by the term \( 1/Q \). We can now find \( k \) from re-arranging (52) and using the expression for the interbank rate, as well as market clearing \( \lambda \tau k = m^b \),
\[
\frac{F(k)}{k} = \frac{1}{\sigma z} \left\{ (2 - \sigma) \left( \frac{1}{Q} - \tau + \frac{1+i\tau}{Q} \right) + \sigma \frac{1+i\tau}{Q} \lambda \right\} \tag{54}
\]
The deposit insurance scheme is now affecting the level of investment: The higher the risk (the lower $Q$), the higher $F(k)/k$ and so the lower the investment level $k$. Now, given the equilibrium level of investment, the aggregate quality is given by (53), or replacing the expression for $F(k)/k$:

$$Q = \frac{z}{\gamma} \left\{ 1 - \frac{\sigma(1 - Q\tau + \tau(1 + i))}{(2 - \sigma)(1 - Q\tau + \tau(1 + i)) + \sigma\lambda\tau(1 + i)} \right\}$$  \hspace{1cm} (55)

where we have used $i$ to denote the nominal rate of interest $1 + i = (1 + \pi)/\beta$. In the Appendix, we show that the unique equilibrium is one with the negative root, as the other root is always greater than unity.\(^{18}\)

We can now compare the project’s quality without insurance (37) with the one with insurance (55). For convenience, we restate (37) here for the case where $r = 0$:

$$Q = \frac{z}{\gamma} \left\{ 1 - \frac{\sigma(1 + i\tau)}{(2 - \sigma)(1 + i\tau) + \sigma(1 + i)\lambda\tau} \right\}$$

Notice that the RHS of (55) is strictly increasing in $Q$ whenever $\tau > 0$ and $i > 1$. Also when $Q = 1$, the RHS of (37) equals the RHS of (55). Therefore, the RHS of (55) is always strictly smaller than the RHS of (37). Therefore, the RHS of (37) crosses the $45^\circ$-line at a higher point than the RHS of (55). This implies that, for given $\tau$ and $i$, the average quality of projects is lower – and investment is lower – when there is deposits insurance. We summarize this discussion in the following proposition.

**Proposition 3.** There is a unique equilibrium with deposit insurance, the investment level is $k$ given by (54) and the quality of projects is $Q$ given by (55). $k$ is increasing in $Q$, and always declines with inflation or reserve requirements. The average quality, the level of investment, and welfare are lower with deposit insurance than without insurance.

7 Literature Review (incomplete)

Williamson (1999) argue that the creation of tradable deposit allows productive intermediation and is thus desirable. Using a similar angle of attack, Chari and Phelan (2016) argue that the creation of deposits has the (private) benefits of insuring against liquidity shocks, while at the same time imposing a pecuniary externality by raising the price level. This implies that the social benefits of deposit creation can even be negative. As a result 100% reserve requirement can be desirable. Our mechanism also plays through a pecuniary externality, but while Chari and Phelan study the effect of consumption loans, we study the effect of corporate credit lines on the production process. Then we can show that deposits possibly increase leverage beyond its optimal level and increasing risk (in addition to the price level).

Monnet and Sanches (2015) show that 100% reserve requirements may be undesirable because bankers cannot

\(^{18}\)We are looking for a fixed point, with the understanding that given $Q$ the entrepreneur chooses the highest $q$ that is consistent with (51).
commit to repay deposits. Instead, our results are driven by limited liability and a pecuniary externality.

Jakab and Kumhof (2015) remark that, with a few exceptions, the academic literature has focused on the wrong model of banks, namely the “intermediation of loanable funds” model. In this model banks intermediate the funds from savers to borrowers. A prime example of such a model is Berentsen, Camera, and Waller (2007). Instead, Jakab and Kumhof argue that banks’ main activity is to finance firms through the creation of money (or deposits). Among many other results, they show that the financing model of banking explains why leverage is pro-cyclical. Our model belongs to the financing view of banking and we concentrate on risk taking and the optimal reserve requirement policy.

Sanches (2015) argues that a purely private monetary regime is inconsistent with macroeconomic stability. The result hinges on endogenously determined limits on private money creation and the presence of self-fulfilling equilibrium characterized by monetary collapse.

Our paper is also related to Williamson (2016) that features the moral hazard problem of creating low quality collateral when the interest rate is low.

8 Conclusion

T.b.a.

9 Bibliography


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10 Appendix

10.1 Derivation of first order conditions (23) and (24)

The first order conditions with respect to $b^1$, $k^o$, and $k^1$ are, respectively,

$$
(b^1) : \frac{\partial S(k^o, m^b)}{\partial q} \frac{\partial q}{\partial b^1} + q(r - i_m) + \lambda_b + \lambda \tau = 0
$$

$$
(k^o) : \frac{\partial S(k^o, m^b)}{\partial q} \frac{\partial q}{\partial k^o} + q \left( z - \frac{\gamma}{2} \right) F'(k^n + k^o) - (1 + r) + \lambda_{k^o} - \lambda \tau = 0
$$

$$
(k^n) : \frac{\partial S(k^n, m^b)}{\partial q} \frac{\partial q}{\partial k^n} + q \left( z - \frac{\gamma}{2} \right) F'(k^n + k^o) - p^n + \lambda_{k^n} - \tau \lambda \tau = 0
$$

The first term in these first order conditions captures the direct effects of the choice variables on the moral hazard problem and its indirect effect on the total surplus. The second terms captures the direct effects of these choice variables on the total surplus. When banks borrow a unit of reserves more on the interbank market, $(b^1)$ says that they earn a margin $r - i_m$ only when their entrepreneur succeeds, as they fail otherwise. When banks increase their lending of money to the entrepreneur, $(k^o)$ says that conditional on success, the total surplus increases by the marginal product of capital $z F'(k^n + k^o)$ net of the marginal cost of effort for the entrepreneur, $\frac{\gamma}{2} q F'(k^n + k^o)$ and net of the opportunity cost of forgiving the interest on reserves. Finally, when banks increase their lending of deposits, $(k^n)$ says that the total surplus increases by the marginal product of capital net of the marginal effort cost for the entrepreneur and the banker’s cost of redeeming deposits $p^n$ for each unit of capital financed with deposits. The last terms are the Lagrange multipliers on the positivity constraints and the reserve requirement constraint ($\lambda \tau$).

Then using (21) and the implicit function theorem on the expression for $q$, and with

$$
\xi = \frac{(z - \gamma q) + \frac{U(k^n, k^o, m) - p^n k^n}{F(k^n + k^o)}}{2 \gamma q - z + \frac{U(k^n, k^o, m) - p^n k^n}{F(k^n + k^o)}}
$$
we obtain an expression for the effects on moral hazard,

\[
\frac{\partial S(k^n, k^o, m^b)}{\partial q} \frac{\partial q}{\partial \bar{b}^i} = \xi q(r - i_m)
\]

\[
\frac{\partial S(k^n, k^o, m^b)}{\partial q} \frac{\partial q}{\partial k^n} = -\xi q \left[p^n - \frac{k^o + p^n k^n + \phi}{F(k^n + k^o)} F'(k^n + k^o)\right]
\]

\[
\frac{\partial S(k^n, k^o, m^b)}{\partial q} \frac{\partial q}{\partial k^o} = -\xi q \left[1 + r - \frac{k^o + p^n k^n + \phi}{F(k^n + k^o)} F'(k^n + k^o)\right]
\]

Naturally, we would expect these expressions to be negative, as increasing borrowing increases the principal and the fee that the entrepreneur has to pay back to banks. This makes the moral hazard problem more severe, thus decreasing the marginal benefit of additional investment. After some tedious algebra and using our assumption that the production function is isoelastic we can simplify the expressions for the first order conditions as follows:

\[
b^i : \left[\frac{\gamma q^2}{\gamma q - \frac{U(m^b)}{q F(k^n + k^o)}}\right] (i_m - r) = \lambda_b + \lambda_r
\]

\[
k^o : \quad F'(k^n + k^o) \left(z - \frac{\gamma}{2} q\right) - (1 + r) - \frac{1}{2} \sigma \frac{U(m^b)}{q(k^n + k^o)} = (\lambda_r - \lambda_{k^o}) \left[\frac{\gamma q^2}{\gamma q - \frac{U(m^b)}{q F(k^n + k^o)}}\right]^{-1}
\]

\[
k^n : \quad F'(k^n + k^o) \left(z - \frac{\gamma}{2} q\right) - p^n - \frac{1}{2} \sigma \frac{U(m^b)}{q(k^n + k^o)} = (\tau \lambda_r - \lambda_{k^n}) \left[\frac{\gamma q^2}{\gamma q - \frac{U(m^b)}{q F(k^n + k^o)}}\right]^{-1}
\]

Comparing this set of first order conditions with the previous one, notice that the moral hazard problem is simply captured by the term (using isoelasticity again):

\[
\frac{1}{2} \sigma \frac{U(m^b)}{k^n + k^o} = \frac{1}{2} \frac{U(m^b)}{F(k^n + k^o)} F'(k^n + k^o)
\]

The entrepreneur’s choice of quality is a direct function of what the bank gets paid, relative to total production. The implicit payment to the banks is \(U(m^b)\) as the bank receives no surplus from trade. The expression above shows how the payment to the bank relative to total production changes with investment. It is rather clear that moral hazard decreases the marginal benefit of investment. Also clear, \(\sigma\) will play an important role in the effect on moral hazard on the equilibrium investment because it magnifies the effect on the banks’ compensation.

We can already notice that inflation will increase the marginal cost of capital via the moral hazard term: Indeed, looking back at the bank’s problem for cash holding, it must be that in equilibrium \(1 + i = U'(m^b)\). Therefore, higher inflation will increase the moral hazard component. This is intuitive: banks have to be compensated for the cost of holding real balances, and this cost is increasing with inflation, thus reducing the entrepreneur’s incentive to choose a high quality project. Finally, as \(U(0) = 0\) moral hazard has no effect.
on the investment decision when \( m_b = 0 \). Again, this is intuitive: when the bank does not bring anything to the table, it will get a zero payoff. Then the entrepreneur fully internalizes the benefit of making effort and choose the efficient \( q^* \).

Whenever there is no interbank market activities, the outside option of the bank with value \( U(m_b) \) is to keep its reserves to earn \( (1 + r)m_b \). Looking back at problem (17), this implies \( i = r \). So given \( i > r \) we can rule out that the interbank market is inactive. Hence, \( b^1 > 0 \) and \( \lambda_b = 0 \). Now let us consider whether the reserve requirement constraint binds.

Suppose \( i_m = r \), so that the reserve requirement does not bind. In this case, the bank outside option is \( U(m_b) = (1 + r)m_b \) and there is only activities on the interbank market if it is default free (if it wasn’t, banks would rather keep their reserves and earn \( 1 + r \)). Looking back at problem (17), this implies \( i = r = i_m \). So given \( i > r \) we can rule out this case here. So it must be that \( i_m > r \) and the reserve requirement binds. Then and as \( \lambda_b = 0 \) we can use the first order condition for \( b^1 \) to find an expression for \( \lambda_r \) and replace it in the first order conditions for \( k^p \) and \( k^n \) to obtain (23) and (24).