Understanding FX risk premia

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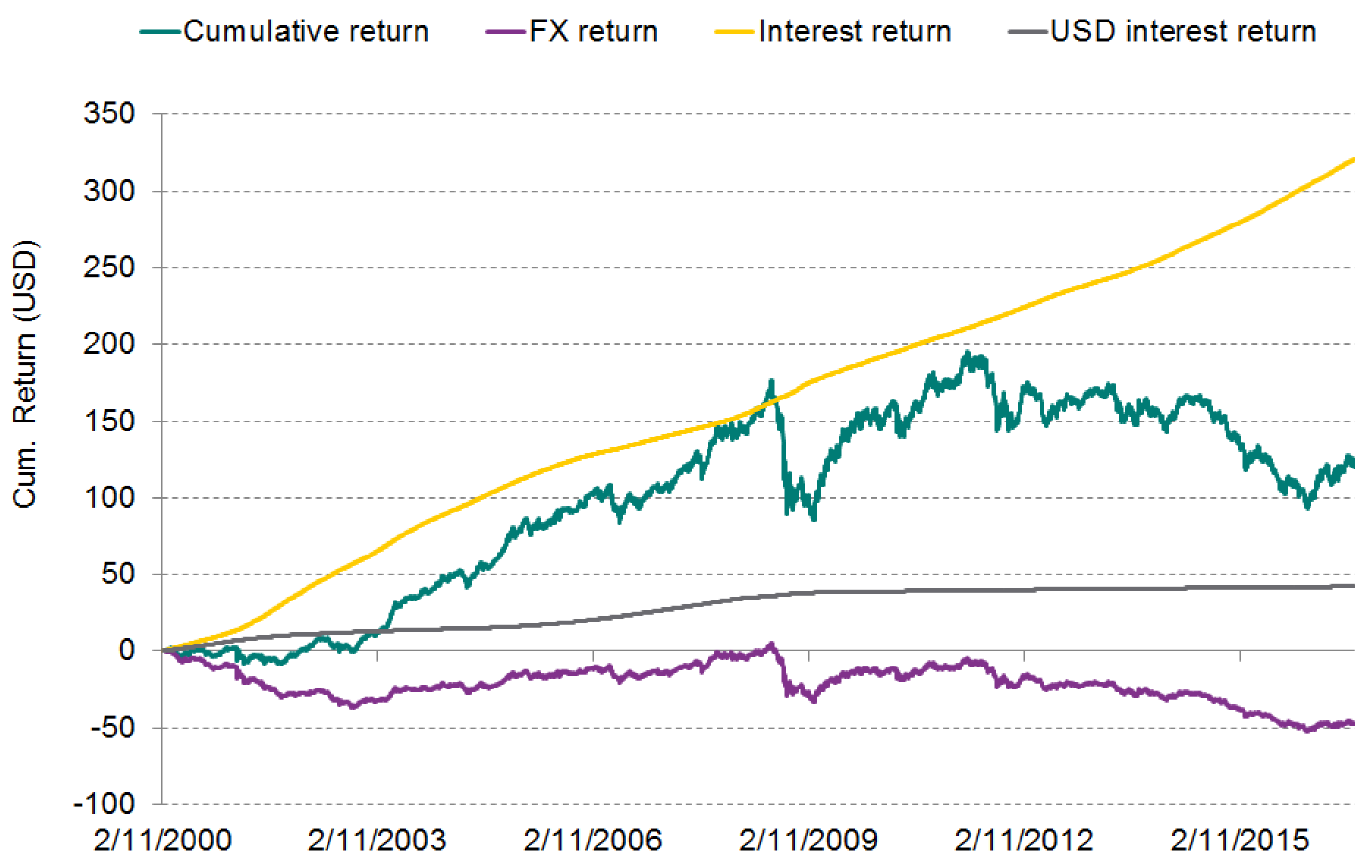
The big puzzle: why are carry trades profitable?

Example: long equally weighted basket of high vol EM currencies: PLN, TRY, ZAR, MXN, BRL funded by rolled short-term USD interbank borrowing



Large abrupt FX moves without news – aka "currency crashes"

Bottom line: carry trade returns capture a risk premium for exposure to crashes.

How we can we define and analyze that premium?

FX returns risk premium: the difference between *ex ante* probability of large and abrupt currency moves and *ex post* frequency of such moves.

But many questions remain:

- How do we assess the probability of a crash?
- Is crash probability observable?
- How do we distinguish ex ante from ex post?
- What drives the premium in FX returns?
- Is such a premium tradeble?

Options traders are in the business of quantifying probability:

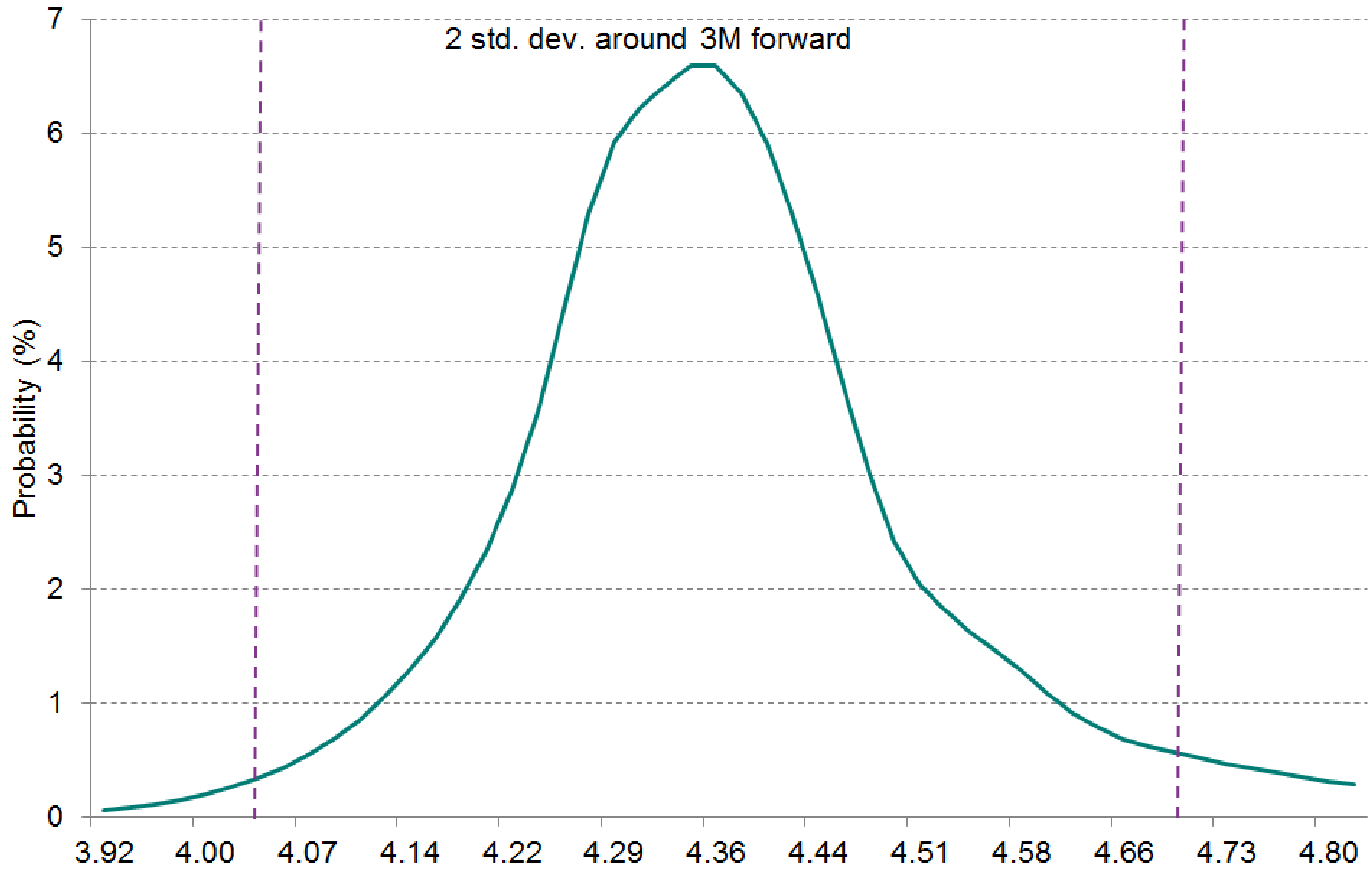
$$C(K,T) = e^{-rT}\mathbb{E}\{\max(S_T - K)\} = e^{-rT}\int_K^{\infty}(S_T - K)\varphi(S)dS$$

So if we know prices of all options for all strikes we can determine the distribution of the underlying implied by these prices (aka implied distribution):

$$\varphi(S) = e^{rT}\frac{\partial^2 C}{\partial K^2}$$

Bottom line: If we know the ex ante implied distribution and its moments, we can compare it to their respective ex post realizations to arrive at an estimate of risk premium on currency exposures.

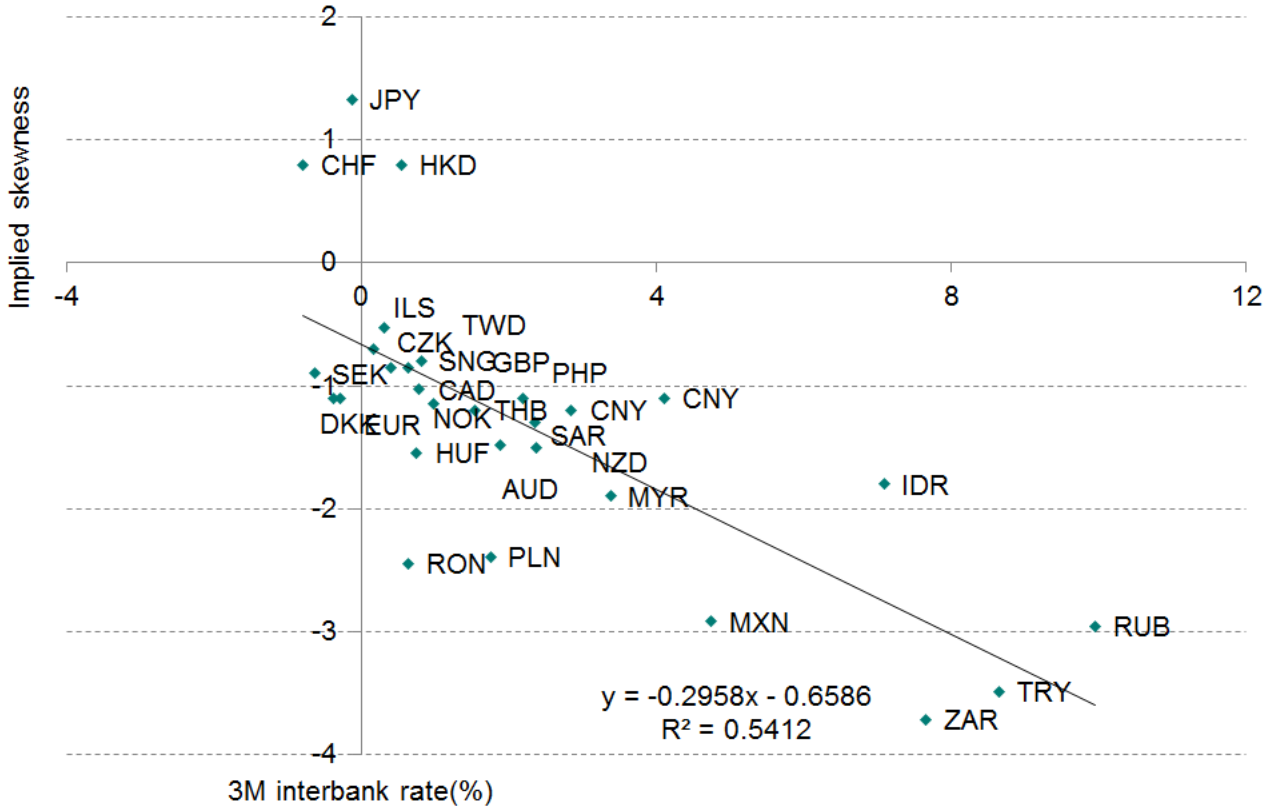
Here's an example for a 3M EURPLN option



Note: it's asymmetric & probability mass in the appreciation tail is 4×lower than in the depreciation tail

- Distributions of EM currencies typically exhibit:
  - skewness which measures distribution asymmetry
  - kurtosis which measures tail thickness relative to normal distribution
- Skewness and kurtosis are proxies for crash risk.
- The more investors fear a crash, the more they demand in crash protection, and the higher should be the compensation for assuming crash risk directly or indirectly.

There is some tentative confirmation for our reasoning...



Skewness and kurtosis will be central to our definition of crash risk premium

Skewness and kurtosis can be extracted from FX option strategies: butterfly (BF) and risk reversal (RR)

- Risk reversal: long a call and short a put with the same maturity, but different strike (symmetric around the forward rate)

- Butterfly: long a call and a put spread symmetrically around the forward, and short a call struck at the forward rate

These strategies are so popular they've become quoting convention in OTC markets:

EURCHF | EURCZK | EURGBP | EURHUF | EURNOK | EURPLN

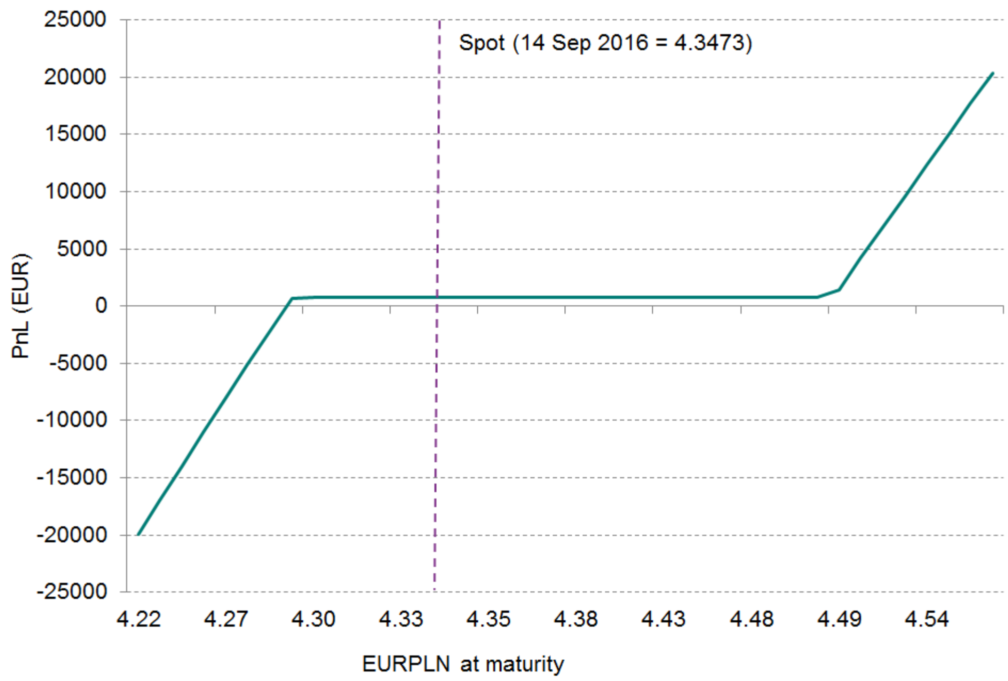
NEX Data EURPLN 66) MSG Contributor 09:41:27

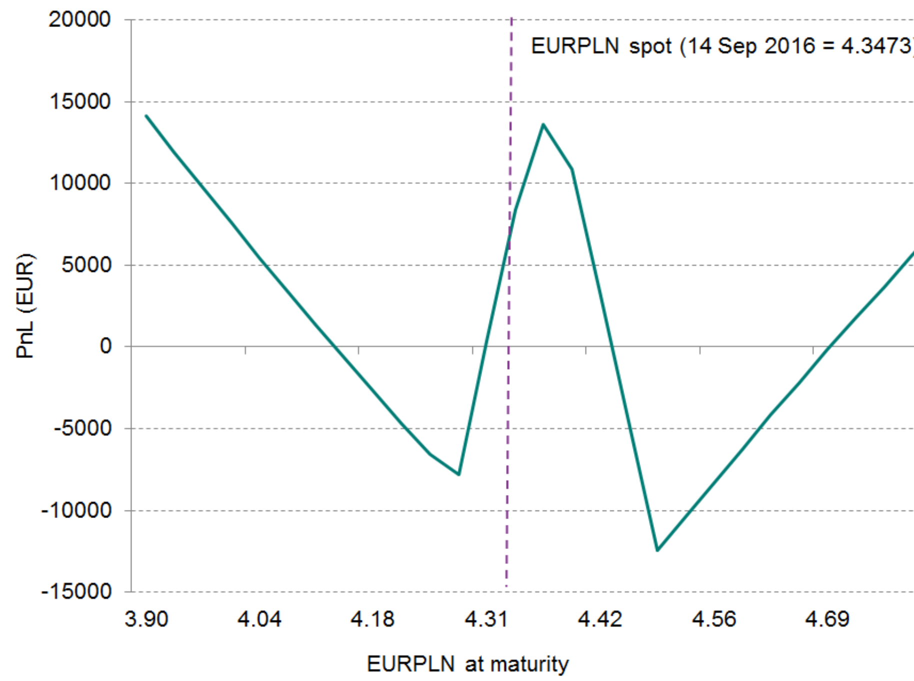
Zoom 100%

NEX Global menu -> FX -> TFS FX Options -> EMEA -> EURPLN (GDCO 2495 20)

	ATM		25D RR		25D BF		10D RR		10D BF		
Term	Bid	Ask	Bid	Ask	Bid	Ask	Bid	Ask	Bid	Ask	Time
1) 1 W	5.15	7.15	-0.40	2.10			0.05	3.05			
2) 1 M	5.60	6.40	0.65	2.00	0.05	0.55	1.85	2.85	0.52	1.27	06:02
3) 2 M	5.60	6.40	0.70	2.05	0.10	0.60	2.00	3.00	0.63	1.38	09:25
4) 3 M	5.65	6.35	0.80	2.10	0.13	0.63	2.15	3.15	0.75	1.50	06:02
5) 6 M	5.85	6.45	0.90	2.15	0.17	0.67	2.35	3.35	0.88	1.63	06:02
6) 1 Y	6.65	7.15	1.15	2.35	0.25	0.75	2.85	3.85	1.13	1.88	06:02

By design, butterflies and risk reversals offer protection against currency crashes





Using a little math, we can show that RR and BF are linked to the moments of the implied distribution.

Let:

- $r_t = \ln(S_t/S_{t-1})$ be log return on currency S at t
- $\mu = \mathbb{E}(r_t)$ be population mean
- $\sigma^2 = \mathbb{E}(r_t - \mu)^2$ be variance of returns
- $s = \frac{1}{\sigma^3}\mathbb{E}(r_t - \mu)^3$ be return skewness
- $k = \frac{1}{\sigma^4}\mathbb{E}(r_t - \mu)^4 - 3$ be return kurtosis

Backus et al. (1997) show that:

$$IV(d) \approx \sigma\left(1 - \frac{1}{6}sd - \frac{1}{24}k(1-d^2)\right), \ \ d = \frac{\ln\left(\frac{F}{K}\right) + \frac{1}{2}IV^2T}{IV\sqrt{T}}$$

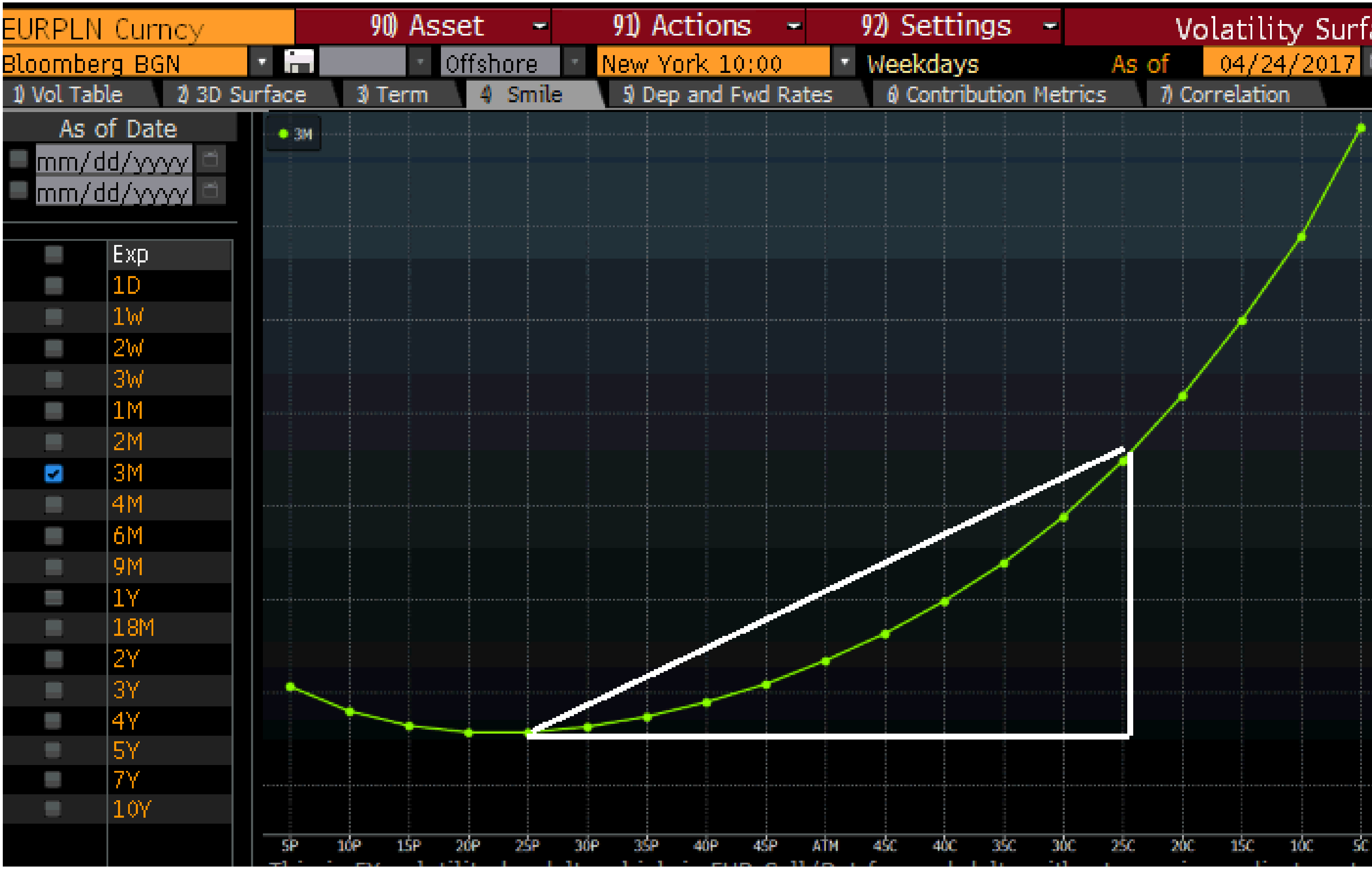
A little math reveals that

$$IV(0) \approx \sigma\left(1 - \frac{1}{24}k\right) \approx \sigma$$

and

$$\frac{\partial IV(d)}{\partial d}|_{d=0} = -\frac{1}{6}s\sigma, \ \ \frac{\partial^2 IV(d)}{\partial d^2}|_{d=0} = \frac{1}{12}k\sigma$$

Slope and curvature of implied volatility smile is directly related to moments of implied distribution...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91) Actions
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1) Vol Table
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3) Term
4) Smile
5) Dep and Fwd Rates
6) Contribution Metrics
7) Correlation
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...which implies that prices of butterflies and risk reversals are related to moments of implied distribution. Indeed

$$RR = IV(d_{25}^c) - IV(d_{25}^p)$$

$$BF = \frac{IV(d_{25}^c) + IV(d_{25}^p)}{2} - IV(0)$$

Hence

$$s \approx 4.447 \times RR/IV(0)$$

$$k \approx 52.75 \times BF/IV(0)$$

Implied ex ante moments can be compared to their ex post realized values

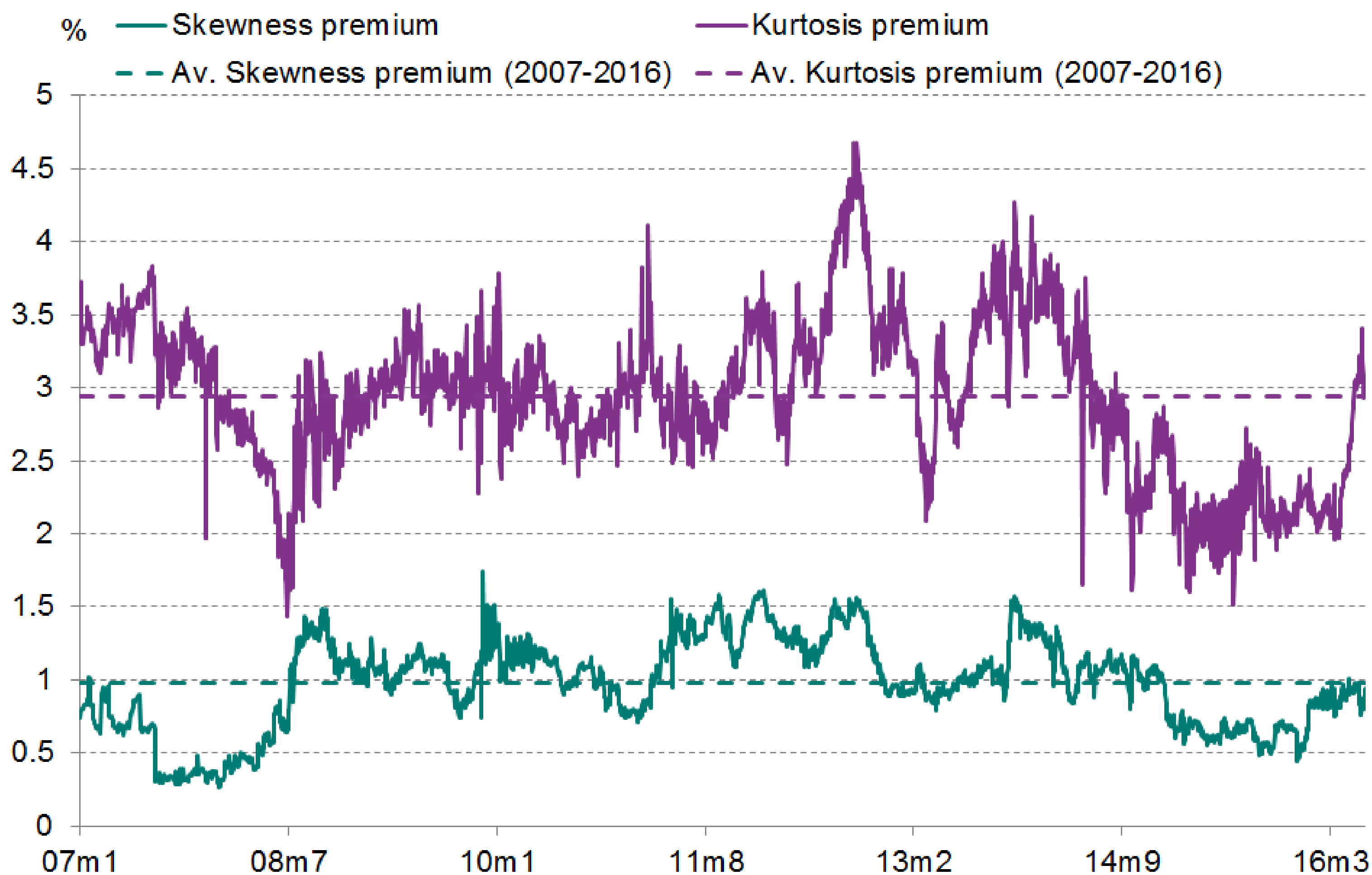
$$\widehat{\sigma} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(r_t)^2}$$

$$s = \frac{1}{N}\frac{\sum_{i=1}^{N}(r_t)^3}{\widehat{\sigma}^3}$$

$$\widehat{k} = \frac{1}{N}\frac{\sum_{i=1}^{N}(r_t)^4}{\widehat{\sigma}^4} - 3$$

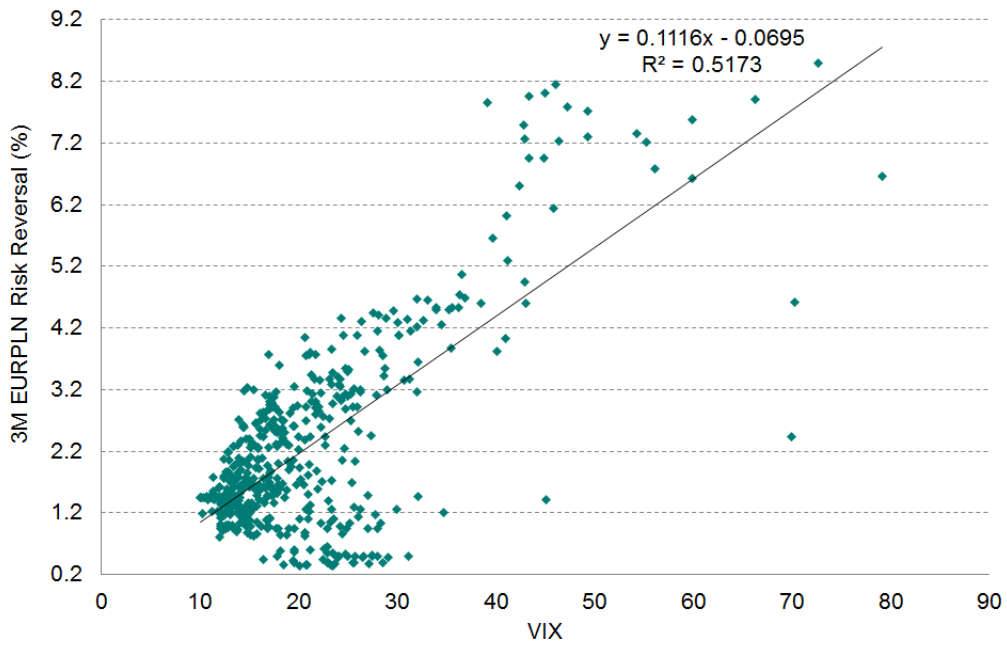
The average differences $s - \widehat{s}$ and $k - \widehat{k}$ will approximate risk premia on the respective exposures.

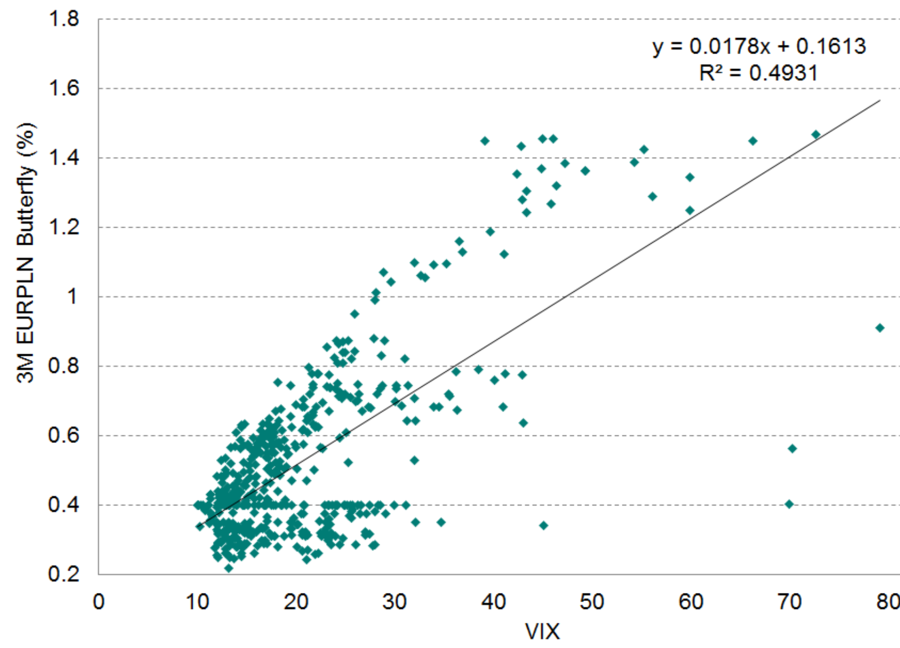
Here's an example for EURPLN



Skewness and kurtosis premia are positive & correlated

Risk premia are driven strongly by global uncertainty





Risk premia can be monetized and explain attractiveness of FX carry trades