Mortgages and Monetary Policy[*]

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Abstract

Mortgages are long-term loans with nominal payments. Consequently, under incomplete asset markets, monetary policy can affect housing investment and the economy through the cost of new mortgage borrowing and real payments on outstanding debt. These channels, distinct from traditional real rate channels, are embedded in a general equilibrium model. The transmission mechanism is found to be stronger under adjustable- than fixed-rate mortgages. Further, monetary policy shocks affecting the level of the nominal yield curve have larger real effects than transitory shocks, affecting its slope. Persistently higher inflation gradually benefits homeowners under FRMs, but hurts them immediately under ARMs.

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1 Introduction

Mortgages are long-term loans with regular *nominal* payments, consisting of interest and amortization. The payments are set up so as to guarantee that, given the mortgage interest rate, the principal is gradually repaid in full by the end of the mortgage term, typically 15 to 30 years. A fixed-rate mortgage (FRM) has a fixed nominal interest rate and constant nominal payments, set at origination, for the entire term of the loan; an adjustable-rate mortgage (ARM), in contrast, sets nominal payments on a period-by-period basis so as, given the current short-term nominal interest rate, the loan is expected to be repaid in full during its remaining term. Various mortgage loans build on these two basic contracts and in most countries, typically, one or the other type dominates.[1]

While the long-term nominal aspect of mortgages—a form of nominal 'rigidity'—has been studied in the household finance literature (e.g., Campbell and Cocco, 2003), a formal analysis of its consequences at the aggregate level has been missing (see Campbell, 2013). This is despite the fact that mortgages, in the minds of policy makers, constitute an integral part of a monetary transmission mechanism (e.g., Bernanke and Gertler, 1995; Bernanke, 2007; Mishkin, 2007).

In major economies, already the size of mortgage finance would suggest that its role in the monetary transmission mechanism must be important. Mortgage payments are equivalent to 15-22% of homeowners' pre-tax income in the U.S. (average for the past 30-40 years); 15-20% in the U.K. (Hancock and Wood, 2004); 27% in Germany (European Mortgage Federation, 2012b); 36.5% in Denmark (first-time homeowners; European Mortgage Federation, 2012c); and 30% in France (first-time homeowners; European Mortgage Federation, 2009). And the

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[1]Countries in which FRMs—with interest rates fixed for at least 10 years—have traditionally dominated the mortgage market include Belgium, Denmark, Germany, France, and the United States; in most other countries, either ARMs or FRMs with interest rates fixed for less than 5 years prevail; see Scanlon and Whitehead (2004) and European Mortgage Federation (2012a). Research is still inconclusive on the causes of the cross-country heterogeneity, but likely reasons include government policies, historical path dependence, sources of mortgage funding (capital markets vs. bank deposits), and inflation experience; see Miles (2004), Green and Wachter (2005), and Campbell (2013). Countries also differ in terms of prepayment penalties, costs of refinancing, recourse, prevalence of teaser rates, the frequency of ARM resets, and the ARM reference rate. Our analysis abstracts from these details.

mortgage debt to (annual) GDP ratio in developed economies has reached on average 70% in 2009, although there is a large cross-country variation (International Monetary Fund, 2011, Chapter 3). In some countries outstanding mortgage debt is even larger than government debt and its maturity is longer.[2]

In light of these facts, this paper provides a conceptual and quantitative analysis of the role of the long-term nominal nature of mortgages in the monetary transmission mechanism at the aggregate level. In order to accommodate the mortgage market structure of different countries, both FRM and ARM contracts are studied. First, in a simple partial equilibrium setup, the paper describes the channels through which mortgage contracts transmit nominal shocks to the real economy. These channels are then embedded in a calibrated general equilibrium model. The purpose of this exercise is to impose general equilibrium discipline on the problem, given its aggregate context, and assess the quantitative importance of the mechanism. To keep the analysis transparent, we abstract from all other nominal rigidities (sticky prices, wages), as well as other channels through which housing finance affects the macroeconomy (e.g., default or the role of home equity lines of credit in providing liquidity).

The transmission mechanism is studied in an environment in which homeowners do not trade a full set of state-contingent securities with mortgage investors. Consequently, the effective discount factors of the two agent types are not equalized state by state and risk sharing is limited. In this setup, we identify two channels through which mortgage contracts transmit nominal shocks into the real economy. Both channels are distinct from the traditional real rate channel of monetary policy transmission.[3]

One channel works through the cost of *new* mortgage loans ('price effect'). In essence, it is a dynamic version of the tilt effect, previously studied in static settings (e.g., Schwab, 1982). Expected future inflation, transmitted into nominal mortgage interest rates, redistributes real mortgage payments over the life of the loan so as to leave mortgage investors indifferent

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[2]Hilscher, Raviv, and Reis (2014) document that in the United States government debt has predominantly short-term maturity.

[3]The real rate channel, whereby the central bank directly affects the ex-ante short-term real interest rate, is described by, e.g., Bernanke and Gertler (1995). In our model, the real interest rate responds to monetary policy shocks only indirectly through general equilibrium effects.

between new mortgages and other assets. If this results in real payments increasing in periods/states in which income has high value to homeowners, the effective cost of the mortgage to homeowners increases. This effect is qualitatively the *same* under both FRM and ARM.

The other channel works through the effects of inflation on the real value of mortgage payments of *outstanding* debt ('income effect').[4] Under incomplete asset markets, the nominal nature of mortgages translates inflation shocks into shocks to real disposable income. This channel is qualitatively *different* under the two contracts. Under FRM, higher inflation reduces real mortgage payments; under ARM, higher inflation increases real mortgage payments, if it sufficiently transmits into higher nominal interest rates. Under FRM, the effects grow gradually over time through accumulated inflation; under ARM they are immediate and resemble the effects of an increase in the real interest rate, even when the real rate itself does not change.

The general equilibrium model, which embeds the two channels, consists of homeowners and capital owners/mortgage investors. There is a representative agent of each type. Such a coarse split of the population is motivated by Campbell and Cocco (2003): in the data, the 3rd and 4th quintiles of the wealth distribution represent typical homeowners; the 5th quintile represents capital owners.[5] As an approximation to the characteristics in the data, homeowners in the model derive income from labor and invest in housing capital, financing a given fraction of housing investment with mortgages. Capital owners do not work and invest in capital used in production, one-period nominal bonds, and mortgages, pricing the assets competitively by arbitrage. Homeowners can also access the one-period bond market, but at a cost. The production side is standard and monetary policy is characterized by an interest rate rule.

We study the transmission mechanism under (i) FRM and ARM in isolation, (ii) the

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[4]Bernanke and Gertler (1995) and Mishkin (2007) refer to this effect also as a 'cash flow' or 'household balance sheet' effect.

[5]The 1st and 2nd quintiles are essentially renters with no assets and little liabilities and are not included in the model.

co-existence of the two contracts and the endogenous choice between them, and (iii) refinancing. In equilibrium, the stochastic properties of short- and long-term nominal interest rates depend on the parameters of exogenous shock processes. Using the model, the persistence and standard deviations of two shocks—to the monetary policy rule (a nominal shock) and the one-period bond market (a real shock)—are estimated by matching the volatility and persistence of the long-term nominal interest rate and the long-short spread. Similarly to Atkeson and Kehoe (2009), the nominal shock turns out to be highly persistent, shifting the level of the nominal yield curve and inflation, the other shock fairly temporary, affecting the slope of the yield curve and the ex-ante real interest rate. Given these estimates, the model is used as a laboratory to study the effects of the nominal shock, which—like the level factor in the data, e.g., Piazzesi (2006)—is the more important of the two shocks for nominal interest rates (in contrast, existing money-macro literature mainly studies shocks affecting the real rate). The main focus is on housing investment, where the shock should matter the most, but responses of other variables are also studied.[6]

The findings can be summarized as follows. First, the real effects of the nominal shock turn out to be stronger under ARM than FRM: a one percentage point (annualized) downward shift of the nominal yield curve and inflation generates, on impact, a 1.8% increase in housing investment under ARM, whereas a 0.7% decline under FRM (0.15% decline with refinancing).[7] The magnitude of the response under ARM is similar to those reported in the literature as occurring due to real rate shocks, making the transmission mechanism under ARM as potent as traditional real rate channels. The finding that the effects are stronger under ARM than FRM may seem surprising as, a-priori, one may expect that a contract fixing nominal payments for the entire term will generate larger real effects. The failure to do so is because under FRM the increase in real mortgage payments due to lower inflation

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[6]There are two additional shocks: to total factor productivity and the marginal rate of transformation between housing and nonhousing use of output. Together, the four shocks produce standard deviations of endogenous variables, and their correlations with output, consistent with the cyclical moments of U.S. data. In addition, we cross-validate the model against existing empirical studies with respect to the responses of housing investment, under FRM and ARM, to shocks to the real interest rate and the size of the marginal propensity to consume out of homeowners' income. The model lines up well with both.

[7]The responses are symmetric for an upward shift of the yield curve, except in the case of refinancing.

is only gradual and is partially offset by the price effect (less expensive new loans). Under ARM, the decline in real mortgage payments is immediate and is further strengthened by the price effect. Refinancing, while in principle making FRM look like ARM when interest rates decline, does not overturn the result. This is because, once calibrated to the data, refinancing accounts for only a small fraction of the outstanding debt (2% per quarter on average for total refinancing and less than 1% when cash-out refinancing, not necessarily related to interest rates, is excluded).[8] When FRMs and ARMs coexist, the real effects lie in-between the two separate cases. Under U.S. calibration, the results are still closer to the FRM case than the ARM case.

Second, under both contracts, the size of the real effects declines with the persistence of the nominal shock (i.e., when the shock starts to affect mainly the slope of the yield curve). This is because what matters with long-term loans is the expected path of inflation, and nominal interest rates, over the entire term of the loan.[9] And third, persistently higher inflation redistributes income from capital owners to homeowners under FRM, but (at least initially) from homeowners to capital owners under ARM. The redistribution is gradual under FRM, but immediate under ARM.

The paper proceeds as follows. Section 2 connects the paper with the literature. Section 3 explains the main ideas. Section 4 describes the general equilibrium model. Section 5 maps it into data. Section 6 cross-validates the model. Section 7 reports findings from the main computational experiments. Section 8 concludes and offers suggestions for future research. Supplemental material contains (i) equilibrium conditions, (ii) computation, (iii) data counterparts to the variables, and (iv) estimates of mortgage debt servicing costs for the United States.

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[8]Even though the refinancing fraction can as much as double in response to realistic drops in interest rates, it is still insufficient to overturn the findings. Of course, given the large size of the stock, the small variation in the fraction of the stock shows up as a large variation in the share of refi loans in new loans, reported periodically by Fredie Mac.

[9]The model abstracts from the possibility that monetary policy may tilt the slope of the yield curve by affecting term premia. All the effects on the slope work in the model through the expectations hypothesis. We also abstract from potential effects of monetary policy on mortgage markups and the primary-secondary spread.

2 Related studies

The paper is related to different strands of a growing literature on monetary policy, housing, and debt.[10] First, following Iacoviello (2005), a number of studies focus on the interaction between sticky prices and the collateral value of housing, whereby housing facilitates borrowing for general consumption purposes, similar to home equity lines of credit (Iacoviello, 2010, contains a brief summary of this line of research). To this end, loans in these models are one-period loans.[11] We abstract from this channel, focusing instead on the role of mortgages as loans for house purchase (i.e., first mortgages) and stressing their long-term nominal aspect in transmitting nominal shocks to the real economy.[12]

Second, following Doepke and Schneider (2006), there is work on the redistributive effects of inflation under incomplete markets and nominal debt (Meh, Rios-Rull, and Terajima, 2010; Sheedy, 2013; Doepke, Schneider, and Selezneva, 2015). This research focuses exclusively on debt contracts with fixed nominal payments, similar to FRM loans. However, we show that the redistributive consequences of inflation are quite opposite under ARM contracts. Policy recommendations based on this literature, such as nominal GDP targeting (Sheedy, 2013), are thus inappropriate for countries with a large fraction of ARMs.[13]

Third, a line of research investigates, in various contexts, the effects of inflation on housing: the tax code (Piazzesi and Schneider, 2012), money illusion (Piazzesi and Schneider, 2007; Brunnermeier and Julliard, 2008), and substitution between market and home pro-

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[10]The role of housing finance, both first mortgages and home equity lines of credit, has been also studied in the context of home ownership, consumption smoothing, optimal mortgage choice, refinancing, default, the recent financial crisis, and the cyclical dynamics of residential investment (see Davis and Van Nieuwerburgh, 2015, for an overiew of this broader literature). Many of these studies model housing market decisions in great detail, but the models are either partial equilibrium or steady-state models. Our model is general equilibrium model with aggregate shocks, but compromises on the details of the housing market.

[11]Rubio (2011) extends the Iacoviello (2005) framework by considering one-period loans with interest rates evolving in a sluggish manner, as a weighted average of past interest rates, interpreting such loans as FRMs. Calza, Monacelli, and Stracca (2013) distinguish between ARMs, modeled as one-period loans, and FRMs, modeled as two-period loans.

[12]Ghent (2012) considers long-term FRM loans denominated in real, rather than nominal, terms. The monetary transmission mechanism in her model works through the long-term real interest rate, which responds only little to monetary policy shocks.

[13]Auclert (2014) studies redistributive effects of monetary policy working through a real rate channel. Redistributive effects of monetary policy are also at the heart of the transmission mechanism proposed by Sterk and Tenreyro (2013).

duction (Aruoba, Davis, and Wright, 2012).[14] In our model, inflation transmits to housing investment through mortgage contracts.

Finally, in recent years, a few studies investigated empirically connections between monetary policy and mortgage contracts. Villar Burke (2015) compares interest rates on outstanding mortgage debt and new mortgage loans in a sample of Eurozone countries in a period around the cut in the European Central Bank's policy rate in 2008/2009. In countries in which FRMs dominate (Germany and France) there was almost no change in the interest rate on the pool of outstanding mortgages, despite relatively stable house prices. In contrast, in countries in which ARMs dominate (Spain and, to a smaller extent, Italy), the interest rate on the pool declined almost in parallel with the policy rate. The responses of interest rates on new loans also differed across countries, declining more moderately in FRM countries than in ARM countries. Consistency with these empirical findings is at the core of the mechanism in our model.

Calza et al. (2013), using a VAR model based on the usual identification strategy, find stronger negative responses of housing investment to positive monetary policy shocks in ARM then FRM countries.[15] The shock they identify is closer to the real interest rate shock in our model, than the nominal shock we focus on, in the sense that it increases the real interest rate. Our model is consistent with their finding, producing stronger negative responses to the real interest rate shock under ARM than FRM. Even quantitatively, the responses are comparable.

Di Maggio, Kermani, and Ramcharan (2014) and Keys, Piskorski, Seru, and Yao (2014) exploit the variation across U.S. counties in the use of FRM and ARM contracts to investigate the responses of consumption to the 2008 cut in the Fed funds rate using household-level data. Both studies find that counties with a larger share of ARMs in the existing pool of loans experienced a much larger boost in homeowners' consumption than counties with

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[14]Earlier studies include Lessard and Modigliani (1975), Kearl (1979), Schwab (1982), Alm and Follain (1984), and Poterba (1984).

[15]Earlier VAR studies of housing investment include Bernanke and Gertler (1995) and Iacoviello and Minetti (2008).

a larger share of FRMs. While the authors are careful to control for other channels that could potentially lead to such an outcome, identification can always be an issue in reduced-form analysis. Our model provides a theoretical general-equilibrium underpinning for such empirical findings, generating larger responses under ARM than FRM for both types of interest rate shocks, nominal and real. Furthermore, the model replicates the empirical marginal propensities of consumption, out of the extra income saved on mortgage payments, for a slightly lower than the benchmark persistence of the interest rate shocks.[16]

3   A simple partial equilibrium model

The main ideas can be conveyed within a simple partial equilibrium model. We describe the two channels of transmission (price and income effects) under FRM and ARM, explain why the combined effect is stronger under ARM, and discuss under what conditions monetary policy is neutral and to what extent the results also apply to other forms of nominal debt. Refinancing and the choice between FRM and ARM are delayed until the full model, although we touch on these issues in Section 3.7, which provides numerical illustrations of the price and income effects.

Throughout this section, the real interest rate and real labor income are held constant, the one-period nominal interest rate is varied exogenously, and the *current* inflation rate is also exogenous. All these variables are endogenized in the general equilibrium model. The details of FRM and ARM loans differ across countries. We abstract from the details and focus on the key common features.[17]

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[16]Cloyne, Ferreira, and Surico (2015) also use household-level data. They attempt to identify changes in nominal interest rates in the United States and the United Kingdom due to monetary policy surprises. While they find substantial differences in the responses between outright homeowners and mortgagors, they do not find large differences between the responses of mortgagors in the two countries.

[17]For instance, in the United States, the ARM rate may not change at the same frequency as the monetary policy rate, as in our model. Typically, the ARM rate changes only once a year. In addition, many ARMs have an initial period for which the rate is fixed. In contrast, tracker mortgages, common for instance in Ireland, change the mortgage rate whenever the policy rate of the European Central Bank changes.

3.1 A mortgage-financed house purchase

There are three periods, with time denoted by $t = 1, 2, 3$. Each period a household is endowed with constant real income w and in $t = 1$ has no outstanding mortgage debt (outstanding debt is considered later). In $t = 1$, the household makes a once-and-for-all house purchase, financing a fraction θ of the purchase with a loan and a fraction $1 - \theta$ with income. The loan can be used only for house purchase and the house lasts for $t = 2, 3$, then it fully depreciates. The life-time utility function of the household is $V = \sum_{t=1}^{3} \beta^{t-1} u(c_t) + \sum_{t=2}^{3} \beta^{t-1} g(h)$, where β is a discount factor, c_t is period-t nonhousing consumption, h is housing, and $u(.)$ and $g(.)$ have standard properties. The household maximizes utility with respect to c_1, c_2, c_3, and h, subject to three per-period budget constraints: $c_1 + h = w + l/p_1$, $c_2 = w - m_2/p_2$, and $c_3 = w - m_3/p_3$, where $l = \theta p_1 h$ is the nominal value of the loan, m_2 and m_3 are nominal mortgage payments, and p_t is the aggregate price level (i.e., the price of goods in terms of an abstract unit of account; this section abstracts from house prices).

Assume there is a financial market that prices assets by the no-arbitrage principle but in which the household does not participate due to, for instance, high entry costs. This exclusion is reflected in the sequence of the budget constraints above, which do not allow for financial instruments other than the mortgage (in the full model this assumption is partially relaxed, but maintaining it at this stage brings out the key aspects of the mechanism more clearly). Assume that monetary policy controls the one-period nominal interest rate i_t. No-arbitrage pricing implies the Fisher effect: $1 + \pi_{t+1} = (1 + i_t)/(1 + r)$, where $1 + r$ is a gross real interest rate, given by some exogenous effective discount factor (intertemporal rate of substitution) of investors, $\mu^* = (1 + r)^{-1}$, and $\pi_{t+1} \equiv p_{t+1}/p_t - 1$ is the inflation rate between periods t and $t + 1$.

Mortgage payments have a general form, $m_2 \equiv (i_2^M + \gamma)l$ and $m_3 \equiv (i_3^M + 1)(1 - \gamma)l$. Here, i_t^M denotes the mortgage interest rate, henceforth referred to as the 'mortgage rate'. Under FRM, $i_2^M = i_3^M = i^F$; under ARM, i_2^M and i_3^M may be different. Further, γ is the amortization rate in the first period of the life of the mortgage, when the outstanding nominal debt is l.

In the second period, the outstanding nominal debt is $(1 - \gamma)l$ and the amortization rate is equal to one (i.e., the mortgage is repaid in full). For a standard mortgage, γ is calculated so as to ensure $m_2 = m_3$, conditional on i_2^M.[18]

In the FRM case, no-arbitrage pricing by investors means that i^F satisfies

$$1 = Q_1^{(1)}(i^F + \gamma) + Q_1^{(2)}(i^F + 1)(1 - \gamma), \tag{1}$$

where $Q_1^{(1)} = (1+i_1)^{-1} = (1+\pi_2)^{-1}\mu^*$ and $Q_1^{(2)} = [(1+i_1)(1+i_2)]^{-1} = [(1+\pi_2)(1+\pi_3)]^{-1}(\mu^*)^2$ are the period-1 prices of one- and two-period zero-coupon bonds, determined according to the expectations hypothesis. The no-arbitrage condition (1) states that the present value of payments from a mortgage of size one dollar is equal to one dollar. In the ARM case, $i_2^M = i_1$ and $i_3^M = i_2$ satisfies no-arbitrage pricing, as

$$Q_1^{(1)}(i_2^M + \gamma) + Q_1^{(2)}(i_3^M + 1)(1 - \gamma) = \frac{i_1 + \gamma}{1 + i_1} + \frac{(i_2 + 1)}{(1 + i_1)}\frac{(1 - \gamma)}{(1 + i_2)} = 1. \tag{2}$$

3.2   Price effect

The price effect refers to the effect of monetary policy on the cost of new mortgages.[19] By 'monetary policy' we mean a sequence of short-term nominal interest rates i_1 and i_2. The sequence is known to the household (and the investors). Generally speaking, different sequences of nominal interest rates lead to different sequences of real mortgage payments over the life of the loan, so as to ensure that the present value of the one-dollar loan is equal to one dollar, as dictated by the no-arbitrage pricing by investors. This, however, affects the value of the loan from the household's perspective, as the household does not have the same

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[18]It is easy to verify that a higher i_2^M implies higher mortgage payments, despite the fact that γ changes to equalize the payments between the two periods. Observe that to equalize the payments, $\gamma = 1/(2 + i_2^M)$. Therefore, $d\gamma/di_2^M = -1/(2+i_2^M)^2 \in (-0.25, 0)$. That is, when the mortgage rate increases, the amortization rate needs to decline, but the decline is small. Overall, mortgage payments therefore increase. As a first pass, we can therefore abstract from changes in the amortization rate when studying the effects of interest rates on mortgage payments.

[19]While the price effect is discussed in a deterministic setup, which is the simplest setup in which it can be demonstrated, it is straightforward to extend it to stochastic settings.

valuation of the real mortgage payments as the investors. This is due to the assumed market incompleteness, which prevents an equalization of the effective discount factors across the two agent types.

Specifically, after substituting the budget constraints in the utility function, the first-order condition for the utility maximization problem of the household with respect to h gives

$$u'(c_1)(1 + \tau_H) = \beta(1 + \beta)g'(h),$$

where

$$\tau_H = -\theta \left\{ 1 - \left[\mu_{12} \frac{i_2^M + \gamma}{1 + \pi_2} + \mu_{12}\mu_{23} \frac{(i_3^M + 1)(1 - \gamma)}{(1 + \pi_2)(1 + \pi_3)} \right] \right\} \tag{3}$$

is a wedge between the marginal utility of period-1 nonhousing consumption and the marginal lifetime utility of housing. Further, $\mu_{t,t+1} \equiv \beta u'(c_{t+1})/u'(c_t)$ is the household's effective discount factor. The wedge—working like a relative price of housing—captures the cost of mortgage finance from the household's perspective. To see this, observe that the expression in the square brackets is the present value of real mortgage payments, discounted with the household's effective discount factor, $\mu_{t,t+1}$, rather than the effective discount factor of the investor, μ^*. In the case of FRM

$$\tau_H^{FRM} = -\theta \left\{ 1 - \left[\mu_{12} \frac{i^F + \gamma}{1 + \pi_2} + \mu_{12}\mu_{23} \frac{(i^F + 1)(1 - \gamma)}{(1 + \pi_2)(1 + \pi_3)} \right] \right\},$$

in the case of ARM

$$\tau_H^{ARM} = -\theta \left\{ 1 - \left[\mu_{12} \frac{i_1 + \gamma}{1 + \pi_2} + \mu_{12}\mu_{23}(\mu^*)^{-1} \frac{1 - \gamma}{1 + \pi_2} \right] \right\},$$

where, in the second term, we have used the Fisher effect $(1 + i_2)/(1 + \pi_3) = (\mu^*)^{-1}$.

To explain the effect of monetary policy on the wedge, it is easier to start with the ARM case. A decline in i_1, for instance, reduces the real payments in the first period of the life of the loan, $(i_1 + \gamma)/(1 + \pi_2)$. To see this, observe that through the Fisher effect, π_2 declines

one-for-one with i_1 but—as γ is less than one—the effect of i_1 on the numerator is larger than the effect of π_2 on the denominator. The decline in π_2, however, increases the real payments in the second period of the life of the loan, $(\mu^*)^{-1}(1-\gamma)/(1+\pi_2)$. If the household's effective discount factor assigns a sufficiently large weight on payments in the first period of the life of the loan, mortgage finance becomes cheaper for the household and the wedge declines, inducing more housing investment. In the FRM case, the effect is similar, as long as i_F sufficiently declines in response to the decline in the short rate. This will be the case if the decline in the short rate is persistent; i.e., both i_1 and i_2 decline. The lower inflation rates, π_2 and π_3, increase real payments in the second period of the life of the FRM loan, but again, the wedge declines if the those payments are sufficiently discounted.[20]

3.3 Income effect

The income effect refers to the effect of monetary policy on the real value of payments on outstanding mortgage debt. It concerns ex-post changes in inflation and, in the case of ARM, also nominal interest rates. Effectively, the nominal aspect of mortgages translates nominal shocks to inflation and nominal interest rates into shocks to real mortgage payments and disposable income.

To make the discussion concrete, consider a household in period $t = 1$, that took out a mortgage in period $t = 0$. The mortgage matures in $t = 3$ (to show the income effect under ARM, the minimum length of the loan has to be three periods). By 'monetary policy' we mean a sequence π_1, i_1, and i_2, which becomes known to the household in period $t = 1$; i.e., one period after the household took out the loan (in this sense it is a monetary policy

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[20]Holding the nominal payments to income ratio constant across loans of different maturities (i.e., adjusting the size of the loan accordingly), the front- and back-end effects become stronger the longer is the term of the loan. At the front end, for a sufficiently small inflation rate, $\frac{i_{t+1}^M+\gamma_{t+1}}{1+\pi_{t+1}} \approx i_{t+1}^M + \gamma_{t+1} \approx i_{t+1}^M$, which holds since $\gamma_{t+1} \to 0$, as the term of the loan increases. Changes in real mortgage payments at the front end are thus approximately equal to changes in the nominal interest rate. At the back end, for a small enough nominal interest rate, $\frac{i_{t+\iota}^M+\gamma_{t+\iota}}{(1+\pi_{t+\iota})+...+(1+\pi_{t+1})} \approx \frac{\gamma_{t+\iota}}{(1+\pi_{t+\iota})+...+(1+\pi_{t+1})}$, which holds since $\gamma_{t+\iota} \to 1$ as the end of the term approaches. The denominator becomes larger with the term of the loan, increasing the accumulated inflation effect at the back end. In contrast, when the loan is a one-period loan ($\gamma_{t+1} = 1$), $(i_{t+1}^M + 1)/(1 + \pi_{t+1}) = 1 + r$, as $i_{t+1}^M = i_t$, and there is no price effect.

'surprise'). Specifically, suppose that monetary policy reduces π_1, i_1, and i_2.[21] Through the Fisher effect, i_1 and i_2 again pin down π_2 and π_3, respectively.

The real mortgage payments that the homeowner has to make on the outstanding loan are

$$\frac{m_1}{p_1} = \frac{i_1^M + \gamma_1}{1 + \pi_1}\widetilde{l}_0, \qquad\qquad\qquad\qquad\text{in } t = 1,$$

$$\frac{m_2}{p_2} = \frac{i_2^M + \gamma_2}{(1 + \pi_1)(1 + \pi_2)}(1 - \gamma_1)\widetilde{l}_0, \qquad\qquad\text{in } t = 2,$$

$$\frac{m_3}{p_3} = \frac{i_3^M + 1}{(1 + \pi_1)(1 + \pi_2)(1 + \pi_3)}(1 - \gamma_2)(1 - \gamma_1)\widetilde{l}_0, \qquad\text{in } t = 3,$$

where γ_1 and γ_2 are the amortization rates in the first and second periods of the life of the loan and $\widetilde{l}_0 \equiv l_0/p_0$ is the real size of the loan in period $t = 0$. Under FRM, $i_1^M = i_2^M = i_3^M = i_0^F$. Under ARM, $i_1^M = i_0$, $i_2^M = i_1$, and $i_3^M = i_2$.

Unlike the case of the price effect, the income effect works in opposite directions under FRM and ARM. Broadly speaking, in the FRM case, accumulated inflation affects the real value of mortgage payments over the life of the loan. In the ARM case, at least in the near term, changes in the nominal interest rate have the dominating effect on the real payments, as in the case of the price effect. The following paragraphs provide the details.

Starting with the payments in $t = 1$, the mortgage rate i_1^M is predetermined; it is equal to some i_0^F under FRM and to i_0, the period-0 short rate, under ARM, both determined in period $t = 0$. Clearly, a decline in π_1 generates a negative income effect for the household in $t = 1$.

Regarding payments in $t = 2$, there is still the lingering effect of the decline in π_1. More importantly, however, the payments are affected by i_1. In the FRM case, $i_2^M = i_0^F$ and a decline in i_1 increases the real payments further due to the resulting decline in π_2. In the ARM case, the effects are different. Here, $i_2^M = i_1$ and a decline in i_1 reduces the real mortgage payments, even though π_2 declines one-for-one with i_1 (again, as $\gamma_2 \in (0, 1)$, the

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[21]In the general equilibrium model, such a joint decline in nominal interest rates and current inflation is an equilibrium outcome.

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effect of i_1 on the numerator is larger than the effect of π_2 on the denominator). The decline in the nominal interest rate thus works like a decline in the real interest rate, reducing real interest payments on the loan.

Finally, the payments in $t = 3$ increase under both contracts, due to the accumulated effect of persistently lower inflation.

3.4 Both effects

What happens when both effects are taken into account? Under FRM, a decline in the nominal interest rate reduces the cost of new housing investment, but increases real payments on outstanding debt. Under ARM, both the cost of new housing investment and real payments on outstanding debt decline (the latter at least in the near term). Furthermore, the increase in real mortgage payments on outstanding debt under FRM is gradual, whereas the decline under ARM is immediate. Potential general equilibrium adjustments aside, one would therefore expect monetary policy to be more potent under ARM than FRM loans.

3.5 Monetary policy neutrality

Generally speaking, if asset markets were complete, the Modigliani-Miller theorem would apply in our setting and debt finance (including its form, FRM or ARM) would be irrelevant. As a result, monetary policy would be neutral.

Specifically, start by observing that the mortgage pricing conditions (1) and (2) can be re-written as

$$1 = \mu^* \frac{i_2^M + \gamma}{1 + \pi_2} + \mu^* \mu^* \frac{(i_3^M + 1)(1 - \gamma)}{(1 + \pi_2)(1 + \pi_3)}.$$

Thus, if asset markets were complete—implying $\mu_{t,t+1}$ equal to μ^*—the expression in the square brackets in equation (3) would be equal to one and the wedge τ_H would be equal to zero. Intuitively, the household's valuation of mortgage payments in every period (and more generally in every state) would be the same as that of the investor. And because the investor's no-arbitrage pricing implies that the present value of mortgage payments is

equal to one, the household's valuation would also be equal to one. This holds regardless of what monetary policy does. Furthermore, under complete markets, the agents would be mutually insured against redistributive income shocks, making their income immune to monetary policy surprises. The income effect would therefore be eliminated.

Monetary policy would also be neutral if asset markets were generally incomplete, but complete with respect to inflation. That is, if the nominal rigidity in mortgages could be effectively removed from the economy. Suppose $\mu_{t,t+1} \neq \mu^*$, but consider, for instance, mortgages that are index-linked, adjusting the principal for changes in the price level. The nominal payments of such mortgages are $m_2 = (i_2^M + \gamma)(1 + \pi_2)l$ and $m_3 = (i_3^M + 1)(1 - \gamma)(1 + \pi_2)(1 + \pi_3)l$ and no-arbitrage pricing implies $i_2^M = i_3^M = r$. Converting the nominal payments to real, $m_2/p_2 = (r + \gamma)\widetilde{l}$ and $m_3/p_3 = (r + 1)(1 - \gamma)\widetilde{l}$, shows that the real payments do not depend on nominal variables. The income effect is thus absent. The wedge also does not depend on nominal variables, but is generally nonzero: $\tau_H = -\theta \{1 - [\mu_{12}(\gamma + r) + \mu_{12}\mu_{23}(r + 1)(1 - \gamma)]\}$.

3.6 Discussion: other types of nominal debt

The focus of the paper is on mortgages, as opposed to corporate debt, as long-term corporate assets are less debt-dependent than housing and presumably corporations operate closer to complete asset markets than do households.[22] Nevertheless, the two channels of transmission in principle apply also to the corporate sector. Long-term corporate debt usually takes the form of coupon bonds, which result in our setup when $\gamma_1 = \gamma_2 = 0$. With such amortization structure, the nominal payments are concentrated in the final period of the loan and the real effects of monetary policy work predominantly by affecting the real value of those payments through accumulated inflation.[23]

Within the household sector, our analysis applies also to auto loans, which have a similar payment structure as FRMs. We abstract from auto loans as mortgage debt has a longer

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[22]Long-term corporate assets are typically more than 75% financed through retained earnings and other forms of equity (Rajan and Zingales, 1995).

[23]Gomes, Jermann, and Schmid (2013) study nominal corporate debt in a model in which monetary policy affects its default value.

term (the usual term of auto loans is only five years) and makes up a much larger fraction of household debt than auto loans.

3.7 Numerical illustrations

The three-period model has limitations to illustrate how the price and income effects vary with inflation persistence and what is the realistic size of the effects for a typical household. For this purpose, we rely on numerical examples, based on a standard 30-year mortgage.

Figure 1 plots debt-servicing costs—the ratio of real mortgage payments (m_t/p_t) to real income (w)—over the term of the loan (120 quarters) under two alternative paths of i_t; a constant 'steady-state' $i_t = 4\%$ and a mean-reverting decline of i_t to 1% in period 1, which we refer to as 'monetary policy easing'. The persistence of the decline is 0.95, which is the average quarterly autocorrelation of the short rate in the data. All the assumptions of the simple model—constant r and w and no-arbitrage pricing, with equation (1) extended to 120 quarters—are maintained here. The parameterization is $r = 1\%$ per annum and $\widetilde{l} = 16w$, i.e., four times annual income. In the full model of the next section, the household chooses $\widetilde{l}$ optimally (by choosing h). The point here is simply to illustrate the size of these effects for one particular loan size.[24]

At the steady-state interest rate, debt-servicing costs are front-loaded and decline monotonically over the life of the mortgage from 29% to 6.5%. This is the standard 'tilt effect' (e.g., Schwab, 1982), occurring due to a positive inflation rate (in this case 3%). This path is a baseline against which to compare the debt-servicing costs under the monetary policy easing.

Starting with the case of a new loan, under both FRM and ARM, monetary policy easing reduces debt-servicing costs at the front end, where they are the highest, and somewhat

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[24]The parameterization of the loan size is based on the average ratio, 1975-2010, of the median price of a new home (assuming a loan-to-value ratio of 76%) to the median household net income (assuming an income tax rate of 23.5%). The data on house prices and gross incomes are from the U.S. Census Bureau. The loan-to-value ratio is the average ratio for single family newly-built home mortgages (Federal Housing Finance Agency, Monthly Interest Rate Survey, Table 10); the tax rate is a NIPA-based estimate. A historical 2% markup is added to the interest rate.

increases them at the back end, where they are the smallest. The decline under FRM is smaller than under ARM because the FRM interest rate, due to the mean-reverting nature of the short rate in this example, declines by less than the short rate itself. The flattening of the path of debt-servicing costs results in smoother consumption and thus a decline in τ_H under a concave utility function (and/or sufficiently small β). Using a log utility function and $\beta = 0.9883$, a parameterization of the model of the next section, τ_H declines by 1.66 percentage points in the case of FRM and by 3.83 percentage points in the case of ARM. Recall that $(1 + \tau_H)$ can be interpreted as the effective price of new housing investment and, thus, these numbers represent quarterly percentage declines in this price.[25]

For the case of an existing loan, we consider a loan with 119 periods remaining (the magnitudes of the income effect decline as the remaining term of the loan gets shorter for reasons analogous to those in footnote 20). In the case of ARM, as the loan is only one period into its life, the expected path of debt-servicing costs is essentially the same as that for the new loan. Under FRM, however, the persistently low inflation leads to a gradual increase in debt-servicing costs for the remainder of the term of the loan. The income effect under the two contracts thus goes in opposite directions.[26]

Figure 2 plots the results of the same experiment, but for two alternative degrees of persistence: 0.99 and 0.5. In the 0.99 case, the magnitudes are much larger than in the 0.95 case. Furthermore, for new loans, the results under FRM and ARM are more similar to each other than in the 0.95 case, as the long rate drops almost as much as the short rate. For existing loans, however, the effects under FRM and ARM diverge further apart. When the persistence is 0.5, the effects on both new and existing loans are small, in fact hardly noticeable in the FRM case.

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[25] As the decline in the wedge is larger under ARM than FRM, given the choice, the household would prefer the ARM loan over the FRM loan. (Through out the paper we assume constant risk and therefore the household's responses are driven only by conditional means of inflation and interest rates.)

[26] With refinancing, in the aggregate, a fraction of the outstanding FRM debt would get refinanced (the whole stock would get refinanced if refinancing was costless). This would not be the case in the opposite experiment of monetary policy tightening.

4 General equilibrium model

The general equilibrium model extends the model of the previous section to infinite horizon, formally introduces shocks, and endogenizes the variables that were either held constant (real labor income and the real interest rate) or were treated as exogenous (the short-term nominal interest rate and current inflation).

4.1 Environment

The economy's population is split into two groups, 'homeowners' and 'capital owners', with measures Ψ and $(1 - \Psi)$, respectively. Within each group, agents are identical. An aggregate production function combines capital and labor to produce a single good, which can be used for consumption, capital investment, or as new housing structures; new homes consist of new structures and new land. Capital owners own the economy's capital stock, homeowners supply labor. Capital owners play the role of mortgage investors, kept outside of the simple model.[27] Where applicable, the notation is the same as in Section 3. Only new variables and functions are therefore defined. When a variable's notation is the same for both agent types, an asterisk ($*$) denotes the variable pertaining to capital owners. Through out the model, fiscal variables—taxes, transfers, and government spending—are included only to ensure sensible calibration, explained in Section 5.

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[27]The split of the population is motivated by Campbell and Cocco (2003). In the data, homeowners (corresponding to the 3rd and 4th quintiles of wealth distribution) have one major asset, a house, and one major liability, a mortgage. Their main source of income is labor income. In contrast, capital owners (the 5th quintile) hold almost the entire corporate equity in the economy and housing is a less important component of their asset composition; labor income is also a less important source of their income. The 1st and 2nd quintiles are essentially renters with no assets and little liabilities and are not included in the model.

4.1.1 Homeowners

The representative homeowner's problem is an extended version of the problem in Section 3. The homeowner maximizes expected life-time utility

$$E_t \sum_{t=0}^{\infty} \beta^t \left\{ v(c_t, h_t) - \chi_t \right\}, \quad \beta \in (0, 1), \tag{4}$$

where $v(., .)$ has standard properties and χ_t is a utility cost related to refinancing and mortgage choice, described below. The maximization is subject to a sequence of constraints

$$c_t + p_{Ht} x_{Ht} + \frac{b_{t+1}}{p_t} = (1 - \tau_N)(w_t n - \tau) + \frac{l_t}{p_t} - \frac{m_t}{p_t} + (1 + i_{t-1} + \Upsilon_{t-1})\frac{b_t}{p_t} - \psi_t, \tag{5}$$

$$\frac{l_t}{p_t} = \theta p_{Ht} x_{Ht}, \tag{6}$$

$$h_{t+1} = (1 - \delta_H)h_t + x_{Ht}. \tag{7}$$

Here, x_{Ht} is newly purchased homes, p_{Ht} is their relative price, b_{t+1} is the homeowner's holdings of a one-period nominal bond between periods t and $t+1$, w_t is a real wage rate, n is labor, which the household supplies inelastically, τ_N is a labor income tax rate, τ is a transfer to capital owners, and $\delta_H \in (0, 1)$ is a depreciation rate of the housing stock.[28] Υ_{t-1} is a bond market participation cost, governed by a function $\Upsilon(-\widetilde{b}_t)$, where $\widetilde{b}_t \equiv b_t/p_{t-1}$ is the homeowner's real holdings of the bond. The function $\Upsilon(.)$ is assumed to be increasing and convex and it controls the extent to which homeowners can smooth out consumption in the presence of income fluctuations.[29] In order to avoid the participation cost affecting

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[28] As in the three-period model, θ is a parameter. Chambers, Garriga, and Schlagenhauf (2009a) make a similar assumption and empirical evidence supports this assumption at the aggregate level: over the period 1973-2006, there has been very little variation in the cross-sectional average of the loan-to-value ratio for single family newly-built home first mortgages, despite large changes in interest rates and other macroeconomic conditions (Federal Housing Finance Agency, Monthly Interest Rate Survey, Table 10).

[29] It is further assumed that $\Upsilon(.) = 0$ when $\widetilde{b}_t = 0$, $\Upsilon(.) > 0$ when $\widetilde{b}_t < 0$ (the homeowner is borrowing), and $\Upsilon(.) < 0$ when $\widetilde{b}_t > 0$ (the homeowner is saving). We think of $\Upsilon(.) > 0$ as capturing a premium for unsecured consumer credit, which is increasing in the amount borrowed. $\Upsilon(.) < 0$ can be interpreted as higher intermediation costs for homeowners than capital owners, which reduces the homeowers' returns on savings below those of capital owners. In steady state, $\widetilde{b} = 0$. A technical role of the cost function is that, as in two-country business cycle models with incomplete markets, it prevents the one-period debt from

the definition of aggregate output, it is rebated to the homeowner in a lump-sum way as ψ_t, which the homeowner takes as given. The description of mortgage payments follows after describing the capital owner.

4.1.2 Capital owners

A representative capital owner maximizes expected life-time utility

$$E_t \sum_{t=0}^{\infty} \beta^t u(c_t^*),$$

where $u(.)$ has standard properties, subject to a sequence of constraints

$$c_t^* + x_{Kt} + \frac{b_{t+1}^*}{p_t} + \frac{l_t^*}{p_t} = [(1 - \tau_K)r_t + \tau_K \delta_K]\, k_t + (1 - \tau_{bt})(1 + i_{t-1})\frac{b_t^*}{p_t} + \frac{m_t^*}{p_t} + \tau_t^* + \frac{p_{Lt}}{1 - \Psi}, \quad (8)$$

$$k_{t+1} = (1 - \delta_K)k_t + x_{Kt}. \quad (9)$$

Here, x_{Kt} is investment in capital, r_t is the real rate of return on capital, τ_K is a capital income tax rate, $\delta_K \in (0, 1)$ is a capital depreciation rate (tax deductible), k_t is capital, τ_t^* is a lump-sum transfer, $1/(1 - \Psi)$ is new residential land, which the capital owner receives each period as an endowment, and p_{Lt} is its relative price. In addition, τ_{bt} is a stochastic tax rate on income from the one-period bond market. The tax rate follows a stationary AR(1) process $\tau_{b,t+1} = \rho_b \tau_{bt} + \epsilon_{b,t+1}$, where $\epsilon_{bt} \sim iidN(0, \sigma_b)$. The tax proceeds from this tax are rebated back to the capital owner in a lump-sum way as a part of τ_t^*. As discussed below, τ_{bt} shows up as a wedge in the capital owner's Euler equation for bonds and affects the ex-ante real interest rate. In a reduced form way, this wedge captures various shocks and frictions in financial markets. Šustek (2011), for example, shows how monetary policy transmission through liquidity frictions (liquidity effects) in bond markets can be mapped into movements in a

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becoming a random walk in a log-linear solution of the model. In other words, it keeps the log-linearized model stationary.

wedge like τ_{bt}.[30] Under no-arbitrage pricing, the capital owner is indifferent across investing in mortgages, bonds, and capital. His composition of period-t investment is therefore pinned down by homeowners' demand for new mortgages and the one-period bond.

4.1.3 Mortgages

In the interest of clarity, we start by describing FRMs and ARMs in their simple forms; i.e., no refinancing or choice between the contracts. The modeling of mortgages is based on Kydland, Rupert, and Šustek (forthcoming), who consider infinitely-lived loans, which nevertheless contain the key characteristics of standard (finitely-lived) mortgages. Denoting by d_t the period-t stock of outstanding nominal mortgage debt of the homeowner, the nominal mortgage payments the homeowner has to make in period t are

$$m_t = (R_t + \gamma_t)d_t, \tag{10}$$

where R_t and γ_t are, respectively, the interest and amortization rates of the outstanding debt. The variables determining m_t are state variables evolving as

$$d_{t+1} = (1 - \gamma_t)d_t + l_t, \tag{11}$$

$$\gamma_{t+1} = (1 - \phi_t)(\gamma_t)^{\alpha} + \phi_t \kappa, \tag{12}$$

$$R_{t+1} = \begin{cases} (1 - \phi_t)R_t + \phi_t i_t^F, & \text{if FRM,} \\ i_t, & \text{if ARM,} \end{cases} \tag{13}$$

with $\phi_t \equiv l_t/d_{t+1}$ denoting the fraction of new loans in the outstanding debt next period. The amortization rate γ_{t+1} and the interest rate R_{t+1} in the FRM case thus evolve as weighted averages of the amortization and interest rates, respectively, on the existing stock and new

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[30]By subjecting only the capital owner to this wedge, it is assumed that the wedge captures frictions related only to the 'wholesale' financial market, in which mortgage investors participate, as opposed to the 'retail' financial market, in which homeowners participate. As discussed in Section 4.3, this assumption, however, is fairly innocuous for our results.

loans. $\kappa, \alpha \in (0,1)$ are parameters. Specifically, κ is the initial amortization rate of a new loan and α controls the evolution of the amortization rate over time. Notice that setting $\alpha = 0$ and $\kappa = 1$ implies $\gamma_t = 1 \; \forall t$. That is, l_t becomes a one-period loan. Setting $\alpha = 1$ results in a constant amortization rate $\gamma_t = \kappa$ and thus declining nominal mortgage payments over the life of the loan. Recall from Section 3 that in order to keep mortgage payments constant over the life of a loan, the amortization rate has to be increasing. When $\kappa, \alpha \in (0,1)$, the amortization rate increases, converging to one. Kydland et al. (forthcoming) show that κ and α can be chosen so as to approximate the payments of a standard 30-year mortgage.[31]

Mortgage payments received by the capital owner are specified analogously

$$m_t^* = (R_t^* + \gamma_t^*)d_t^*, \tag{14}$$

$$d_{t+1}^* = (1 - \gamma_t^*)d_t^* + l_t^*, \tag{15}$$

$$\gamma_{t+1}^* = (1 - \phi_t^*)(\gamma_t^*)^{\alpha} + \phi_t^* \kappa, \tag{16}$$

$$R_{t+1}^* = \begin{cases} (1 - \phi_t^*)R_t^* + \phi_t^* i_t^F, & \text{if FRM}, \\ i_t, & \text{if ARM}, \end{cases} \tag{17}$$

where $\phi_t^* \equiv l_t^*/d_{t+1}^*$. Under FRM, the first-order condition for l_t^* pins down i_t^F such that the capital owner is indifferent between new mortgages and rolling over the one-period bond from period t on. The first-order condition is an infinite-horizon counterpart to equation (1); see Appendix A.

4.1.4 Refi loans

Next, we turn to refinancing. Chen, Michaux, and Roussanov (2013) document that, on average, 60% of refi loans in the United States between 1985 and 2013 were unrelated to

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[31]Under appropriate choice of κ and α, even though the loan has an infinite life, it gets essentially repaid within 30 years and the nominal payments are approximately constant for most of these 30 years, conditional on a constant mortgage rate. Such modeling of mortgages is convenient, as both the agents and loans have an infinite life, thus allowing a simple recursive representation of the model with only a few state variables.

interest rate changes. Instead, they served the purpose of liquidity withdrawals, similar to home equity lines of credit.[32] Given our interest, we abstract from this type of refinancing and focus on the remaining 40%. At the aggregate level, as Chen et al. (2013) demonstrate, this fraction is negatively related to a difference between the current FRM interest rate and an average of its past values. This is the relationship we intend to capture.

With refinancing, newly originated loans consist of mortgages used for new house purchase (first mortgages) and refi loans

$$l_t = \theta(p_t p_{Ht} x_{Ht}) + \varrho_t (1 - \gamma_t) d_t. \tag{18}$$

Here, ϱ_t is the fraction of the outstanding debt that is being refinanced in period t (refinancing takes place after the regular period-t amortization payment is made). Mortgage payments are still given by $m_t = (R_t + \gamma_t) d_t$, but now with laws of motion

$$d_{t+1} = (1 - \varrho_t)(1 - \gamma_t) d_t + l_t, \tag{19}$$

$$\gamma_{t+1} = (1 - \phi_t) (\gamma_t)^\alpha + \phi_t \kappa, \tag{20}$$

$$R_{t+1} = (1 - \Theta_t) R_t + \Theta_t i_t^F, \tag{21}$$

where $\phi_t \equiv \theta(p_t p_{Ht} x_{Ht})/d_{t+1}$ is the fraction of first mortgages in the stock of debt next period and $\Theta_t \equiv l_t/d_{t+1}$ is the fraction of all newly originated loans in the stock of debt next period. Observe that combining equations (18) and (19) gives back the original law of motion for debt (11); i.e., prepayments and refi loans cancel out and only first mortgages are added to the stock of debt. An implicit assumption in the law of motion (20) is that a new refi loan starts with an amortization rate of the loan that it replaces (this captures that, e.g., a loan that is refinanced 10 years before maturity gets replaced with a 10-year loan). This leads to a sharp characterization of refinancing. Namely, in the homeowner's constraints, refinancing

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[32]Hurst and Stafford (2004) also focus on the liquidity role of refinancing

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shows up only in the law of motion (21), as a change in the weights on the old effective interest rate R_t and the current market interest rate i_t^F, without tieing this change to new house purchases. When $\varrho_t = 1$, i.e., the whole existing stock is refinanced, $\Theta_t = 1$ and the evolution of R_{t+1} becomes the same as under ARM. When $\varrho_t = 0$, $\Theta_t = \phi_t$ and the evolution of R_{t+1} becomes the same as in the case of the basic FRM loan, equation (13).

The capital owner's laws of motion are analogous,

$$d_{t+1}^* = (1 - \varrho_t)(1 - \gamma_t^*)d_t^* + l_t^*$$

$$\gamma_{t+1}^* = (1 - \phi_t^*)(\gamma_t^*)^\alpha + \phi_t^* \kappa,$$

$$R_{t+1}^* = (1 - \Theta_t^*)R_t^* + \Theta_t^* i_t^F,$$

where $\phi_t^* = [l_t^* - \varrho_t(1 - \gamma_t^*)d_t^*]/d_{t+1}^*$ and $\Theta_t^* \equiv l_t^*/d_{t+1}^*$, with ϱ_t taken as given. The capital owner's first-order condition for l_t^* (gross lending) again determines i_t^F. Observe that, as the evolution of d_{t+1}^* depends on ϱ_t, the FRM rate i_t^F depends on the refinancing behavior of homeowners (i.e., ρ_t shows up in a first-order condition for l_t^*). The current FRM rate i_t^F thus prices in the expected refinancing behavior of homeowners.[33]

The homeowner's decision to refinance is modeled as follows

$$\varrho_t = \begin{cases} \Delta & \text{and} & \chi_t = 0 & \text{if } i_t^F \geq R_t, \\ \widetilde{\varrho}_t & \text{and} & \chi_t = \Gamma(\widetilde{\varrho}_t) & \text{if } i_t^F < R_t. \end{cases}$$

Here, Δ is a parameter, representing a refi fraction occurring due to exogenous reasons (e.g., moving house), $\widetilde{\varrho}_t$ is chosen optimally, and $\Gamma(\widetilde{\varrho}_t)$ is a function with $\Gamma(\widetilde{\varrho}_t)'' > 0$ and a minimum $\Gamma(\widetilde{\varrho}_t) = 0$ at $\widetilde{\varrho}_t = \Delta$. The utility cost χ_t can be interpreted, for instance, as a time loss.[34]

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[33]With refinancing, the left-hand side of the capital owner's budget constraint (8) needs to be modified: l_t^* gets replaced with $l_t^* - \varrho_t(1 - \gamma_t^*)d_t^*$ (i.e., it is first mortgages in both cases).

[34]Even though, at an individual level, refinancing is a discrete choice involving a fixed cost, at the aggregate level of the representative homeowner, assuming a cost function like $\Gamma(.)$ is a more appropriate modeling choice. Such approach is common in aggregate macro models (e.g., Kydland and Prescott, 1991, in the context of discrete choices, subject to fixed costs, of moving in and out of the labor market).

When we abstract from refinancing, we set $\varrho_t \equiv 0$ and $\chi_t \equiv 0$. Characterization of $\widetilde{\varrho}_t$ is provided in Section 7, together with the quantitative findings.

4.1.5   Mortgage choice

Allowing for endogenous choice between new FRM and ARM loans requires keeping track of the two types of debt separately. Therefore, mortgage payments of the homeowner are $m_t = m_{1t} + m_{2t}$, where m_{1t} are payments on outstanding FRM debt and m_{2t} are payments on outstanding ARM debt. These two variables are determined by separate laws of motion (11)-(13), as applicable, each with its own new loans, l_{1t} for FRM and l_{2t} for ARM. The mortgage payments received by the capital owner are specified analogously. The financing constraint of the homeowner becomes

$$l_{1t} + l_{2t} = l_t, \quad \text{with} \quad l_t = \theta p_t p_{Ht} x_{Ht}. \tag{22}$$

The homeowner faces a cost function $\Phi(l_{2t}/l_t)$, assumed to satisfy $\Phi(l_{2t}/l_t)'' > 0$ with a minimum $\Phi(l_{2t}/l_t) = 0$ at $l_{2t}/l_t = \Lambda$, where $\Lambda > 0$ is a parameter. As in the case of refinancing, it is assumed that the cost takes the form of the utility loss χ_t. A way to think about the parameter Λ is as representing a 'normal' share of ARM loans in the economy, determined by various institutional and historical factors, noted in footnote 1 and not modeled here, and the cost function represents the economy's costs of deviating away from this norm. This is a reasonable modeling strategy, given our focus on temporary fluctuations, rather than the long run (assuming costs of deviating away from an institutional norm is consistent with Badarinza, Campbell, and Ramadorai, 2014, who suggest, for instance, costs of changing marketing strategies by banks or costs to households of familiarizing themselves with new mortgage products).

The capital owner takes the mortgage choice by the homeowner as given but the first-order condition for l_{1t}^* again determines the mortgage rate i_t^F, so as to leave the capital owner indifferent between the two types of new loans (and other assets). The characterization of

optimal mortgage choice is provided in Section 7.3, together with the quantitative findings.

4.1.6 Technology

An aggregate production function, operated by perfectly competitive producers, is given by $Y_t = A_t f(K_t, N_t)$, where K_t is the aggregate capital stock, N_t is aggregate labor, and $f(.,.)$ has the standard neoclassical properties. Total factor productivity (TFP) follows a stationary AR(1) process $\log A_{t+1} = (1 - \rho_A) \log A + \rho_A \log A_t + \epsilon_{A,t+1}$, where A is the unconditional mean and $\epsilon_{At} \sim iidN(0, \sigma_A)$. The real rate of return on capital, r_t, and the real wage rate, w_t, are determined by the marginal products of capital and labor, respectively. The resource constraint of the economy is $C_t + X_{Kt} + q_t X_{St} + G = Y_t$, where C_t is aggregate consumption, X_{Kt} is aggregate investment in capital, X_{St} is new housing structures, and G is (constant) government expenditures. Here, q_t is the marginal rate of transformation between new housing structures and the other uses of output, and hence the relative price of new housing structures. It is given as $q_t = \eta_t q(X_{St})$, where $q(X_{St})$ is a convex function that makes the economy's production possibilities frontier (PPF) concave in the space of $(C_t + X_{Kt} + G)$ and (X_{St})—a specification akin to that of Huffman and Wynne (1999), a stand-in for the costs of moving factors of production across different sectors of the economy. The purpose of the function $q(.)$ is to ensure realistic volatility of new housing structures in response to shocks; if the production possibilities frontier was linear, the volatility would be too high. Further, η_t is a shock following a stationary AR(1) process $\eta_{t+1} = (1 - \rho_q) + \rho_q \eta_t + \epsilon_{q,t+1}$, with unconditional mean equal to one and $\epsilon_{qt} \sim iidN(0, \sigma_q)$. This shock can be thought of as a housing construction shock. The shocks A_t and η_t are relevant only for cross-validating the model against business cycle moments of the data.

As in Davis and Heathcote (2005), new homes consist of new housing structures and land and are produced by perfectly competitive homebuilders according to an aggregate production function $X_{Ht} = g(X_{St}, X_{Lt})$. Here, X_{Ht} is the aggregate number of new homes constructed in period t, X_{Lt} is the aggregate new residential land, and g has the standard

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neoclassical properties.

4.1.7 Monetary policy

Monetary policy is characterized by a Taylor-type rule with a stochastic inflation target (e.g.,
Ireland, 2007)

$$i_t = (i + \overline{\pi}_t - \overline{\pi}) + \nu_\pi(\pi_t - \overline{\pi}_t), \quad \nu_\pi > 1, \tag{23}$$

where i is the steady-state short-term nominal interest rate, and $\overline{\pi}_t$ is the inflation target.
The inflation target follows a stationary AR(1) process $\overline{\pi}_{t+1} = (1 - \rho_\pi)\overline{\pi} + \rho_\pi\overline{\pi}_t + \epsilon_{\pi,t+1}$, where
$\overline{\pi}$ is a steady-state inflation rate, and $\epsilon_{\pi,t+1} \sim iidN(0, \sigma_\pi)$. Notice that the interest rate rule
can be rewritten in a more typical form as $i_t = i + \nu_\pi(\pi_t - \overline{\pi}) + \xi_t$, where $\xi_t \equiv -(\nu_\pi - 1)(\overline{\pi}_t - \overline{\pi})$.

As shown below, when ρ_π is close to one, $\overline{\pi}_t$ works in equilibrium like a level factor, moving
short and long rates equally, and allows the model to reproduce the observed volatility and
persistence of the 30-year mortgage rate.

4.2 Equilibrium

To economize on space and notation, the equilibrium is defined only for the case of separate
FRM and ARM loans in their basic form.[35] Let $z_t \equiv [\log A_t, \overline{\pi}_t, \tau_{bt}, \eta_t, p_{t-1}]$ be the vector
of exogenous state variables and a lagged value of the price level, $s_t^* \equiv [k_t, b_t^*, d_t^*, \gamma_t^*, R_t^*]$ the
vector of the capital owner's state variables, $s_t \equiv [h_t, b_t, d_t, \gamma_t, R_t]$ the vector of the home-
owner's state variables, and $S_t \equiv [K_t, H_t, B_t, D_t, \Gamma_t, \Re_t]$ the vector of aggregate endogenous
state variables, where the elements are, respectively, aggregate capital, housing stock, bonds,
outstanding mortgage debt, and its amortization and interest rates. Next, write the capital

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[35]Accommodating refinancing and mortgage choice simply involves expanding the vector of decision vari-
ables of the homeowner by including the additional choice variable, either ϱ_t or l_{2t}. In the case of mortgage
choice, the state space is larger than in the case of the separate contracts, as there are two sets of the
mortgage state variables, one for each type of debt.

owner's optimization problem as

$$U(z, S, s^*) = \max_{[x_K, (b^*)', l^*]} \left\{ u(c^*) + \beta E[U(z', S', (s^*)')|z] \right\}, \tag{24}$$

where a prime denotes a value next period and the constraints (8), (9), and (14)-(17) are thought to have been substituted in the utility and value functions. Similarly, write the homeowner's problem as

$$V(z, S, s) = \max_{[x_H, b']} \left\{ v(c, h) - \chi + \beta E[V(z', S', s')|z] \right\}, \tag{25}$$

where the constraints (5)-(7) and (10)-(13) are thought to have been substituted in the utility and value functions. Let $W_t \equiv [X_{Kt}, p_t, i_t^M, X_{Ht}, B_{t+1}]$ be the vector of aggregate decision variables and prices, where $i_t^M = i_t^F$ under FRM and $i_t^M = i_t$ under ARM. Define a function $W_t = W(z_t, S_t)$.[36]

A recursive competitive equilibrium consists of the functions U, V, and W such that: (i) U and V solve (24) and (25), respectively; (ii) r_t and w_t are given by the respective marginal products of capital and labor, p_{Ht} and p_{Lt} are given by the respective marginal products of structures and land, and $q_t = \eta_t q(X_{St})$; (iii) i_t is given by the monetary policy rule (23); (iv) the bond, mortgage, housing, and land markets clear: $(1 - \Psi)b_{t+1}^* + \Psi b_{t+1} = 0$, $(1 - \Psi)(l_t^*/p_t) = \Psi\theta p_{Ht}x_{Ht}$, $\Psi x_{Ht} = g(X_{St}, X_{Lt})$, and $X_{Lt} = 1$; (v) aggregate consistency is ensured: $K_t = (1 - \Psi)k_t$, $X_{Kt} = (1 - \Psi)x_{Kt}$, $X_{Ht} = \Psi x_{Ht}$, $B_t = \Psi b_t$, $H_t = \Psi h_t$, $(1 - \Psi)m_t^* = \Psi m_t$, $(1 - \Psi)d_t^* = \Psi d_t = D_t$, $\gamma_t^* = \gamma_t = \Gamma_t$, $R_t^* = R_t = \Re_t$, and $(1 - \Psi)\tau_t^* = \tau_K(r_t - \delta_K)K_t + \tau_N(w_tN - \Psi\tau) + \Psi\tau + \tau_{bt}(1 - i_{t-1})B_t/p_t - G$; (vi) the exogenous state

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[36]With refinancing, the capital owner's laws of motion (15)-(17) are replaced with their refi counterparts. Similarly, the homeowner's laws of motion (11)-(13) are replaced with their refi counterparts. In addition, the homeowner's maximization is also with respect to ϱ_t and an aggregate counterpart to ϱ_t is an element of W_t. Further, the mortgage market clearing condition noted below is replaced with $(1 - \Psi)(l_t^*/p_t) = \Psi[\theta(p_{Ht}x_{Ht}) + \varrho_t(1 - \gamma_t)d_t/p_t]$. With mortgage choice, the homeowner's maximization is also with respect to l_{1t} and l_{2t}, subject to $l_{1t} = \theta(p_t p_{Ht}x_{Ht}) - l_{2t}$, and the aggregate counterpart to l_{2t} is a part of W_t. In the capital owner's problem, maximization with respect to l_t^* is replaced with l_{1t}^*. Mortgage market clearing requires $(1 - \Psi)(l_{1t}^* + l_{2t}^*) = \Psi\theta(p_t p_{Ht}x_{Ht})$ and $(1 - \Psi)l_{2t}^* = \Psi l_{2t}$. The choice of x_{Ht} and l_{2t} by the homeowner thus pins down l_{1t}^* and l_{2t}^*.

variables follow their respective stochastic processes and the endogenous aggregate state variables evolve according to aggregate counterparts to the laws of motion for the respective individual state variables; and (vii) the individual optimal decision rules of the capital owner (for x_K, $(b^*)'$, and l^*) and the homeowner (for x_H and b') are consistent with $W(z, S)$.[37]

The computational procedure is described in Appendix B. Before computing the equilibrium, the model is made stationary by dividing all nominal variables by either p_t or p_{t-1}. Given a relatively large number of state variables, the model is solved by log-linear approximation, specifically, adopting a version of the method of Hansen and Prescott (1995). Thus, in the quantitative results presented, the linear equilibrium decision rules and pricing functions are characterized by certainty equivalence, whereby only current realizations and the conditional first moments of future shocks (not higher moments) affect agents' decisions and market clearing prices. The FRM rate in the log-linear model is thus based on expectations hypothesis.[38] In the case of refinancing, the computational method is modified by utilizing the ideas developed by Guerrieri and Iacoviello (2015).

4.3 Equilibrium nominal interest rates and inflation

The capital owner's first-order conditions for b^*_{t+1} and x_{Kt} yield the Fisher equation. In a linearized form

$$i_t - E_t \pi_{t+1} \approx E_t r_{t+1} + E_t \tau_{b,t+1}, \tag{26}$$

where (abusing notation) the variables are in percentage point deviations from steady state. Notice that, for a given expected rate of return on capital $E_t r_{r+1}$, determined by the expected marginal product of capital, an increase in $E_t \tau_{b,t+1}$ increases the ex-ante real interest rate,

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[37]In the case of ARM, $i_t^M = i_t$ makes the capital owner indifferent between new mortgages and bonds and the first-order condition for l_t^* can be dropped from the description of the equilibrium. In the case of FRM, the first-order condition is needed to determine i_t^F.

[38]We have explored the effects of (constant) second moments of the shocks on the equilibrium decision rules and pricing functions, using a special case of Epstein-Zin preferences (i.e., with unitary intertemporal elasticity of substitution) that is easy to handle by the computational method. But like Tallarini (2000) and Backus, Ferriere, and Zin (2015), we have found little effect of the second moments on the model dynamics for a wide range of the risk aversion parameter.

$i_t - E_t \pi_{t+1}$. In this sense, τ_{bt} is a *real rate shock*.

Given stochastic processes for r_t and τ_{bt}, the Fisher equation and the monetary policy rule (23) determine, by forward substitutions, i_t and π_t. For ρ_π close to one, excluding explosive paths for inflation (a common assumption), the resulting expression for i_t is

$$i_t \approx \sum_{j=0}^{\infty} \left(\frac{1}{\nu_\pi} \right)^j E_t r_{t+1+j} + \frac{\nu_\pi \rho_b}{\nu_\pi - \rho_b} \tau_{bt} + \overline{\pi}_t. \tag{27}$$

Substituting i_t from equation (27) back into the policy rule (23) gives the equilibrium inflation rate

$$\pi_t \approx \frac{1}{\nu_\pi} \sum_{j=0}^{\infty} \left(\frac{1}{\nu_\pi} \right)^j E_t r_{t+1+j} + \frac{\rho_b}{\nu_\pi - \rho_b} \tau_{bt} + \overline{\pi}_t. \tag{28}$$

Thus, under the assumption that ρ_π is close to one, the equilibrium short-term nominal interest rate and inflation move, subject to general equilibrium adjustments in the expected path of r_t, one for one with the highly persistent $\overline{\pi}_t$ shock. In this sense, in contrast to the τ_{bt} shock, the $\overline{\pi}_t$ shock is a purely *nominal shock*.

Because movements in i_t occurring due to the $\overline{\pi}_t$ shock are highly persistent, the long rate i_t^F moves almost as much as the short rate in response to this shock. In this sense, the $\overline{\pi}_t$ shock works like a level factor shock. In contrast, the τ_{bt} shock, if less persistent, has only a temporary effect on the short rate. It thus has only a small effect on the long rate and moves predominantly the long-short spread. We exploit these properties in calibration.

Recall from Section 3 that the price effect (in the case of both FRM and ARM), as well as the income effect (in the case of ARM), are in the short run driven mainly by the nominal interest rate, regardless of whether the increase in the nominal interest rate comes from an increase in the real rate or inflation. Thus, by affecting the nominal interest rate, both shocks $\overline{\pi}_t$ and τ_{bt} show up in the price effect (FRM and ARM) and the income effect (ARM). Given that the price and income effects work in the same direction under ARM, we would expect both shocks to have a stronger effect on homeowners under ARM than FRM.[39]

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[39]In the present case, where τ_{bt} affects only the capital owner's Euler equation, the ex-ante real rate is given by equation (26) and, in a log-linearized form, $E_t c_{t+1}^* - c_t^* + E_t \tau_{b,t+1} \approx E_t c_{t+1} - c_t - \Upsilon_t$ (again

5 Calibration

As most of the required historical data are readily available for the United States, the calibration is based on U.S. data, even though the mechanism applies more generally. One period in the model corresponds to one quarter.

5.1 Functional forms

The following standard functional forms are used: $u(c^*) = \log c^*$, $v(c, h) = \xi \log c + (1 - \xi) \log h$, $f(K, N) = K^\varsigma N^{1-\varsigma}$, $g(X_S, X_L) = X_S^{1-\varphi} X_L^\varphi$. Further, as in Kydland et al. (forthcoming), $q(X_{St}) = \exp(\zeta(X_{St} - X_S))$, where $\zeta > 0$ and X_S is the steady-state ratio of new housing structures to output (Y is normalized to be equal to one in steady state). A similar functional form is used also for the bond market participation cost: $\Upsilon(-\widetilde{b}_t) = \exp(-\vartheta \widetilde{b}_t) - 1$, where $\vartheta > 0$ and $\widetilde{b}_t = 0$ in steady state. It is straightforward to check that this function satisfies the properties set out in Section 4.

5.2 Debt-servicing costs

A particular challenge arises due to the need to match debt-servicing costs—mortgage payments to income ratio—of homeowners. This requires the model to be consistent with the cross-sectional distribution of income, in addition to the standard aggregate targets: $X_K/Y = 0.156$, $X_S/Y = 0.054$, $K/Y = 7.06$, $H/Y = 5.28$, and $rK/Y = 0.283$, all averages for 1958-2006. Official data for mortgage debt servicing costs are not published for the United States. Estimates, however, can be obtained from different sources (see Appendix D), resulting in long-run averages in the ballpark of 18.5% of homeowners' pre-tax income. The model's steady-state counterpart to this ratio is $\widetilde{M}/(wN - \Psi\tau)$, where $\widetilde{M} = (R + \gamma)\widetilde{D}/(1 + \pi)$,

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abusing notation). If τ_{bt} affects both the capital owner's and homeowner's Euler equations, then equation (26) still holds and $E_t c^*_{t+1} - c^*_t \approx E_t c_{t+1} - c_t - \Upsilon_t$. If τ_{bt} affects only homeowners, then we can write $i_t - E_t \pi_{t+1} \approx E_t c_{t+1} - c_t - \Upsilon_t + E_t \tau_{b,t+1}$ and $E_t c^*_{t+1} - c^*_t \approx E_t c_{t+1} - c_t - \Upsilon_t + E_t \tau_{b,t+1}$. In all three cases, τ_{bt} (in so far it is autocorrelated) affects the ex-ante real rate. The only difference is how the real rate relates to the consumption paths of the two agents. Experimenting with the three cases, the quantitative differences on our findings are small.

with $\widetilde{D}$ being the debt-to-output ratio.

Consistency with the observed cross-sectional distribution of income is achieved through τ. Recall that homeowners in the model are an abstraction for the 3rd and 4th quintiles of the wealth distribution, while capital owners are an abstraction for the 5th quintile. In the data, while the 5th quintile get substantial income from capital, they also receive income from transfers and labor. Thus, if the only source of income of capital owners in the model was capital, and given that the model is required to match the observed average capital share of output ($rK/Y = 0.283$), capital owners would account for too small fraction of aggregate income (28.3% in the model v.s. 48% in the data; SCF 1998), while homeowners' share would be too large. As a result, the steady-state debt-servicing costs would be too low (or the required debt-to-output ratio would have to be too high, making it inconsistent with the observed X_S/Y ratio, the land share in new housing, the loan-to-value ratio, and the amortization schedule). The parameter τ adjusts for this discrepancy, transferring some of the homeowners' labor income to capital owners.

5.3   Parameter values

The parameter values are listed in Table 1, where they are organized into eight categories: Ψ (population); β, ξ (preferences); δ_K, δ_H, ς, ζ, φ (technology); τ_K, τ_N, G, τ (fiscal); θ, α, κ (mortgage contracts); ϑ (bond market); $\overline{\pi}$, ν_π (monetary policy); and ρ_A, σ_A, ρ_π, σ_π, ρ_b, σ_b, ρ_q, σ_q (stochastic processes).[40] Wherever possible, calibration targets are for the period 1958-2006. The calibration is mainly based on steady-state relations and unconditional first moments of the data. Most parameters can be assigned values without solving a system of steady-state equations, three parameters (ξ, τ_K, τ) have to be obtained jointly, and

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[40]Parameters specific to refinancing and mortgage choice are dealt with in Sections 7.2 and 7.3, respectively. Because in steady state interest rates are constant and the short rate is equal to the FRM rate, the values of the parameters related to refinancing and mortgage choice do not affect the values of the parameters calibrated from steady state relations described in this section. (As shown below, second moments used to calibrate some parameters are found to be broadly invariant to FRM vs. ARM, and thus should not be affected by mortgage choice or refi either.) The additional parameters can therefore be chosen separately and the model does not need to be recalibrated once these additional parameters are introduced.

two parameters (ζ, ϑ) are assigned values together with the parameters of the stochastic processes by matching unconditional second moments of the data. Calibration of the three sets of parameters is described in turn.

First, the share of homeowners Ψ is set equal to 2/3. The capital share ς is set equal to 0.283, an estimate from National Income and Product Accounts (NIPA) for aggregate output close to our measure of output (see Appendix C). The share of land in new housing φ is set equal to 0.1, an estimate reported by Davis and Heathcote (2005). The depreciation rates δ_K and δ_H are set equal to 0.02225 and 0.01021, respectively, to be consistent with the average flow-stock ratios for capital and housing, respectively. The labor income tax rate is derived from NIPA using a procedure of Mendoza, Razin, and Tesar (1994), yielding $\tau_N = 23.5\%$. The role of this parameter is to map the observed debt-servicing costs, which are based on gross income, into net income. The parameter G is measured to be equal to 0.138 (see Appendix C).[41] We focus on conventional mortgages and therefore set θ equal to 0.6. This is based on the observation that conventional loans make up, on average, 80% of single family newly-built home mortgages (Construction Survey, 1973-2006) and the average cross-sectional mean of their loan-to-value ratio is 0.76 (Freddie Mac's Monthly Interest Rate Survey, 1973-2006). As in Kydland et al. (forthcoming), the amortization parameters are $\kappa = 0.00162$ and $\alpha = 0.9946$, approximating the amortization schedule of a 30-year mortgage. The weight on inflation in the monetary policy rule ν_π is set equal to a fairly standard value of 1.5. The steady-state inflation rate $\overline{\pi}$ is set equal to 0.0113, the average (1972-2006) quarterly inflation rate. In steady state, the first-order condition for l_t^* constrains i^F to equal to i. The first-order condition for b_t^* then relates i and $\overline{\pi}$ to β. The above value of $\overline{\pi}$ and $i^F = 9.31\%$ per annum (the 1972-2006 average for 30-year FRM rate) imply $\beta = 0.9883$.

Given the above parameter values, the second set of parameters (ξ, τ_K, τ) is calibrated by matching the observed average K/Y ratio, H/Y ratio, and debt-servicing costs. The relationship between the three parameters and the targets is given by the steady-state versions of

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[41]Having government expenditures in the model ensures a realistic expense of the tax revenues, which otherwise would have to go into transfers, thus affecting the distribution of income.

the first-order conditions for x_{Kt} and x_{Ht} and the expression for steady-state debt-servicing costs noted above (see Appendix A for the first-order conditions). These restrictions yield $\xi = 0.5003$, $\tau_K = 0.3362$, and $\tau = 0.4693$.

Finally, given the values of the first and second set of parameters, ζ and ϑ are calibrated, together with the parameters of the stochastic processes, by minimizing an equally-weighted distance between the following second moments of the data and their simulated model counterparts[42]: (i) the standard deviations and autocorrelations of the 10-year government bond yield (a longer series used as a proxy for the 30-year mortgage rate) and of the long-short spread (10-year minus 3-month), (ii) the standard deviation and autocorrelation of output, (iii) the standard deviation and autocorrelation of house prices, and (iv) the standard deviations of housing and capital investment. These are 10 targets for 10 parameters. The resulting parameter values are reported in Table 1. Here, instead of going over the values, we briefly discuss how the targets map into the parameters. Targets (i) help pin down the parameters of the inflation target and bond market shocks, as explained in Section 4; targets (ii) help pin down the parameters of the TFP shock; targets (iii) help pin down the parameters of the shock to the marginal rate of transformation; and targets (iv) help pin down the parameter controlling the curvature of PPF and the parameter of the cost function for homeowners' participation in the bond market.[43] As reported and discussed in the next section, the cyclical properties of the model are similar under FRM and ARM, in line with cross-country data. The parameter values, therefore, are not particularly sensitive to which contract is used to simulate the model to match the moments (the reported values are based on FRM).

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[42]Both the U.S. and simulated data are expressed as percentage (percentage point, for interest rates) deviations from HP-filter trend.

[43]Specifically, the bond market cost function controls the extent to which homeowners can indirectly invest in capital, thus affecting the volatility of capital investment. The curvature of PPF controls the volatility of housing investment.

6 Model cross-validation

Before proceeding to quantitatively assess the transmission mechanism under investigation, we use three different sets of available empirical observations to cross-validate the model: (i) business cycle moments, (ii) responses to a real interest rate shock, and (iii) the marginal propensity to consume (MPC) of ARM homeowners. As the empirical evidence is conditioned on FRM vs. ARM, or based on ARM alone, the model in this section is also solved under only either FRM or ARM, without refinancing.

6.1 Business cycle moments

First, the business cycle moments—standard deviations and correlations with output—are reported in Panel A of Table 2.[44] Six standard deviations in the table are part of the set of moments used to calibrate the model under FRM. Nonetheless, the remaining standard deviations—of consumption, the short rate, and inflation—also line up well with the data. The model also correctly predicts positive correlations with output of consumption and the two types of investment, with the correlation of consumption being the highest and the correlation of housing investment being the lowest of the three. However, it overpredicts the strength of these relations. This also applies to house prices. Such finding should be expected, as the number of shocks in the model is limited. The short and long interest rates, the long-short spread, and the inflation rate have correlations with output similar to those in the data.

The findings in Table 2 show that the cyclical properties of the model are not particularly sensitive to whether FRM or ARM contract is used to simulate the model. This may seem surprising as one would expect that, especially, the moments of housing investment should be sensitive to the type of the loan. Housing investment is indeed less positively correlated with output under ARM than FRM, and its volatility is lower under ARM than FRM, but

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[44]The data are quarterly. Both the actual and simulated data are expressed as percentage (or percentage point, for interest rates and the inflation rate) deviations from HP-filter trend.

the differences are small. Is this true in the data? Panel B of the table shows that the differences in the model are similar to those between a set of FRM countries and a set of ARM countries.[45] The reason for the similarity is that the TFP shock is the dominating shock in the model and housing investment responds similarly to such a shock under both contracts. However, as in the data, it responds a little less under ARM than FRM because the responses get dampened by procyclical movements of interest rates, to which housing investment is more sensitive under ARM than FRM, as shown below.

6.2 Responses to a real rate shock

Second, we scrutinize the model responses to the τ_{bt} shock, which directly affects the ex-ante real interest rate. Recall that this shock can be thought of as capturing the real rate channel of monetary policy transmission. The identification of such shocks in the data is well understood, unlike the identification of the shock we focus on. Calza et al. (2013) employ a typical identification strategy to study the responses of housing investment in a sample of FRM and ARM countries. They find that, in response to a one-percentage point (annualized) increase in the short-term nominal interest rate, increasing the real rate, housing investment declines more in ARM than FRM countries. Figure 3 shows that the model is consistent with this finding. Panel A shows responses under the calibrated persistence of the τ_{bt} shock, panel B shows responses under calibration that reproduces the persistence of the nominal interest rate in Calza et al. (2013).[46] In the latter case, the quantitative response under FRM falls within the error bands reported in their paper. Under ARM, the initial decline somewhat exceeds their error bands (-1.95% in the model vs. -1.4% in their paper). This is likely due to the fact that the interest rate of the ARM contract in the model adjusts immediately, whereas in their study the classification of countries as ARM countries is based on interest

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[45]The countries used are the only countries for which quarterly housing investment data are available going back to at least 1980. FRM counties: BEL, FRA, US; ARM countries: AUS, CAN, UK. The moments are taken from Kydland et al. (forthcoming).

[46]The calibrated persistence is based on the long-short spread, which may be affected by other factors, such as time-varying risk premia, than just the channels identified by Calza et al. (2013).

rates fixed for up to five years.

6.3 MPC of homeowners

Finally, in Table 3, we compare the MPC of ARM homeowners in the model to the empirical MPC from the study by Di Maggio et al. (2014) described briefly in Section 2 (the study does not contain MPC of FRM homeowners). In both the data and the model, the MPC is calculated as an increase in consumption of ARM homeowners divided by the extra income brought about by a decline in real mortgage payments occurring due to a drop in the nominal interest rate. In the model, the nominal interest rate can decline due to both the τ_{bt} and $\overline{\pi}_t$ shocks (i.e., it can decline due to a decline in the real rate or in line with inflation), whereas the empirical study is silent on the sources of the decline. We therefore consider both scenarios. As the empirical study is based on reduced form analysis, it does not address the question of the agents' expectations of the persistence of the interest rate decline. When we use the calibrated persistence of the shocks, the model implies MPC of around 0.28. However, a modest reduction in the persistence of the $\overline{\pi}_t$ shock, from 0.994 to 0.95, replicates the empirical MPC of 0.17 (the MPC value of 0.28, however, is in the ballpark of estimates from the fiscal stimulus literature, e.g., Johnson, Parker, and Souleles, 2006; Parker, Souleles, Johnson, and McClelland, 2013). As demonstrated below, reducing the persistence of the shock to 0.95 does not change the main message of the paper. In the case of the τ_{bt} shock, the persistence needs to be brought down to 0.7 (interestingly, a value in-between our calibrated value of 0.9 and the value implied by Calza et al., 2013, of 0.5).

7 Findings from computational experiments

The purpose of the computational experiments is to impose general equilibrium discipline on the transmission mechanism laid out in Section 3 and to explore its quantitative importance. In order to demonstrate the general equilibrium effects of FRM and ARM contracts in their starkest form, we start by studying the responses of the model economy under the

two contracts separately, abstracting from the choice between them and refinancing. This exercise, while not strictly applicable to the United States, is relevant for other countries, in addition to its purpose of isolating the effects of each contract. In many countries, ARMs (of various types) are the only contract available; see footnote 1. And in some FRM countries, refinancing is less common than in the United States. For instance, as noted in Section 2, Villar Burke (2015) shows that the interest rate on the pool of existing mortgages stayed almost intact in Germany and France, following the European Central Bank's interest rate cut to near zero in 2008/2009, despite declines of interest rates on new loans and relatively stable house prices. After this initial exercise, we scrutinize the mechanism under refinancing and mortgage choice.

7.1 The transmission mechanism under FRM and ARM

7.1.1 Baseline responses

The quantitative assessment of the transmission mechanism under the two separate contracts is contained in Figure 4. The figure plots the responses of selected variables to a positive (1 percentage point, annualized) $\overline{\pi}_t$ shock in period 1. The first two charts demonstrate the nominal and level factor nature of the shock: the short-term nominal interest rate and the inflation rate, and in the case of the FRM economy also the FRM rate, all increase more or less in parallel by approximately one percentage point (annualized).[47] The next chart plots the responses of real mortgage payments (m_t/p_t). The chart confirms the effects discussed in Section 3. Due to higher inflation, real mortgage payments decline on impact (period 1) under both contracts by the same amount. But this decline is dwarfed by the magnitudes in subsequent periods. Under FRM, real mortgage payments display a persistent gradual decline, while under ARM the payments increase sharply (and persistently) one period after the shock. Under FRM, both homeowners' consumption (C_t) and housing investment (X_{St})

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[47]The model is stationary. Thus, even though convergence back to steady state may not be apparent from the plots, eventually all variables converge back to steady state. This, however, takes longer than the 40 periods displayed in the charts.

increase in response to growing real disposable income.[48] In contrast, under ARM, both variables decline in response to the drop in real disposable income. Further, the responses under ARM are stronger than under FRM. For instance, the decline in housing investment is 1.8% on impact under ARM, compared with a 0.7% increase under FRM.[49] House prices (p_{Ht}), not plotted due to space constraints, increase on impact by 0.2% under FRM, and decline by 0.5% under ARM. The last chart shows that capital owners' consumption (C_t^*) responses are opposite to those of homeowners, but are smaller and smoother. This reflects the fact that mortgage payments are a quantitatively smaller fraction of capital owners' than homeowners' income (6.4% vs 24.2% of after-tax income) and that capital owners can better smooth out fluctuations in income.

7.1.2 Comparison with a real rate channel

It was argued in Section 3 that, under ARM, the effect of a one-for-one increase in the nominal interest rate and inflation works like an increase in the real interest rate. How does, then, the transmission mechanism under ARM quantitatively compare with a real interest rate channel? To answer this question, we rely again on the τ_{bt} shock that affects the ex-ante real interest rate. Using again the calibration based on the VAR study of Calza et al. (2013), ρ_b is set equal to 0.5. Figure 5 contains the results of this comparison. It plots the responses of selected variables ($i_t - E_t\pi_{t+1}$, m_t/p_t, X_S, and C_t) to a one percentage point (annualized) increase in the short term nominal interest rate, occurring due to either the τ_{bt} or the $\overline{\pi}_t$ shock. Observe that under the τ_{bt} shock the ex-ante real interest rate ($i_t - E_t\pi_{t+1}$) increases, whereas under the $\overline{\pi}_t$ shock it stays almost unchanged (it gradually declines due to small general equilibrium effects working through capital accumulation). The immediate effect on real mortgage payments, however, is the same under the two shocks and the subsequent effect is even more persistent under the $\overline{\pi}_t$ shock than under the τ_{bt} shock. Housing investment

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[48]The figure plots responses of X_{St} but the responses of X_{Ht} are similar.

[49]The responses of consumption and housing investment do not exactly copy the responses of real mortgage payments as homeowners have some access to the one-period bond market to smooth out the impact of the changes in disposable income.

under both shocks declines by roughly 2% on impact, but the decline is more persistent under the $\overline{\pi}_t$ shock. The same applies to consumption. The transmission mechanism studied in this paper is thus at least as potent, under ARM, as the traditional real rate channel.

7.1.3 The role of persistence

Recall that the calibration of ρ_π is based on replicating the persistence of the 10-year nominal government bond yield, used as a proxy for the 30-year mortgage rate. How do the quantitative findings change when the persistence of the shock is reduced? Figure 6 provides the answer. Focusing on the ARM case, in which the real effects are generally larger, the initial response of housing investment is reduced from 1.8% to 1.1% as the persistence is reduced from the calibrated value of 0.994 to 0.95 (recall that the latter value is required to match the MPC of homeowners in Di Maggio et al., 2014). When the persistence is further reduced to 0.5, the real effects become negligible. At this degree of persistence, the real effects working through the traditional real rate channel are much stronger (refer back to the bottom panel of Figure 3). Notice that as the persistence is reduced, the shock starts to affect the long-short spread, rather than the level of the yield curve. The transmission mechanism proposed in this paper is thus quantitatively more relevant for monetary policy shocks that primarily affect the level of the yield curve, rather than its slope.

7.2 Optimal refinancing

The previous subsection abstracted from refinancing. This is a reasonable approximation for countries like Germany and France, as argued above, but not for the United States. Does optimal refinancing make the responses of the economy under FRM look more like under ARM when interest rates decline? As described in Section 4.1.4., when $i_t^F \geq R_t$ the homeowner sets $\varrho_t = \Delta > 0$, which is a parameter capturing exogenous reasons to refinance, such as moving home. However, when $i_t^F < R_t$, the homeowner sets $\varrho_t = \widetilde{\varrho}_t$, where $\widetilde{\varrho}_t$ is chosen optimally, weighting the benefits and costs of refinancing. Here we characterize this

decision.

In Section 4.1.4, we described the refi cost function as $\Gamma(\widetilde{\varrho}_t)'' > 0$ with a minimum $\Gamma(\widetilde{\varrho}_t) = 0$ at $\widetilde{\varrho}_t = \Delta$. Recall that the cost takes the form of a utility loss χ_t in the homeowner's utility function (4). To make the cost function operative, assume a quadratic parametric form $\upsilon(\widetilde{\varrho}_t - \Delta)^2$, with $\upsilon > 0$. Under this assumption, the optimal refi problem yields a simple solution with an intuitive interpretation. Observe that $\widetilde{\varrho}_t$ appears in the homeowner's problem only in two places: it enters linearly the law of motion for R_{t+1}, equation (21), and it appears in the quadratic utility loss χ_t.[50] It is convenient to work with the recursive formulation of the homeowner's problem, summarized by equation (25). Substituting the law of motion (21) for R_{t+1} in the value function V and taking the first derivative of the objective function with respect to $\widetilde{\varrho}_t$ and setting it equal to zero yields

$$\widetilde{\varrho}_t - \Delta = \frac{(1-\gamma_t)}{(1+\pi_t)} \frac{\widetilde{d}_t}{\widetilde{d}_{t+1}} \left(\frac{-\beta E_t V_{R,t+1}}{2\upsilon} \right) (R_t - i_t^F). \tag{29}$$

In equation (29), $\widetilde{d}_t \equiv d_t/p_{t-1}$ and $V_{Rt} < 0$ is the derivative of the homeowner's value function with respect to R_t, representing the marginal life-time gain of reducing the effective interest rate on outstanding debt by one unit.[51] By refinancing, the homeowner trades off the current marginal refi cost against the expected marginal life-time benefit. The further is i_t^F below R_t, the larger is the gain from refinancing and thus the larger is the fraction of debt that gets refinanced.[52]

Observe from equation (29) that as R_t converges to i_t^F, $\widetilde{\varrho}_t$ converges to Δ, when the ratios in equation (29) are bounded. The numerical solution of the model described in Appendix B

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[50]Recall that the law of motion for debt (19) boils down to the law of motion under no refi (11) once we substitute in for l_t from equation (18). $\widetilde{\varrho}_t$ thus does not affect d_{t+1}.

[51]The derivative of the value function is given recursively by a Benveniste-Scheinkman condition as $V_{R,t+1} = -\upsilon_{c,t+1} \frac{\widetilde{d}_{t+1}}{1+\pi_{t+1}} + \beta \frac{(1-\varrho_{t+1})(1-\gamma_{t+1})}{(1+\pi_{t+1})} \frac{\widetilde{d}_{t+1}}{\widetilde{d}_{t+2}} E_{t+1} V_{R,t+2}$.

[52]The second derivative of the objective function with respect to $\widetilde{\varrho}_t$ is equal to $-2\upsilon + \left[\frac{(1-\gamma_t)}{(1+\pi_t)} \frac{\widetilde{d}_t}{\widetilde{d}_{t+1}} (i_t^F - R_t) \right]^2 E_t V_{RR,t+1} < 0$, where the inequality follows from $V_{RR,t+1} < 0$, which can be established by taking the derivative of $V_{R,t+1}$, given in footnote 51, with respect to R_{t+1}: $V_{RR,t+1} = \upsilon_{cc,t+1} \left(\frac{\widetilde{d}_{t+1}}{1+\pi_{t+1}} \right)^2 + \beta \left[\frac{(1-\varrho_{t+1})(1-\gamma_{t+1})}{(1+\pi_{t+1})} \frac{\widetilde{d}_{t+1}}{\widetilde{d}_{t+2}} \right]^2 E_t V_{RR,t+2}$. By recursion, $V_{RR,t+1} < 0$ as $\upsilon_{cc,t+1} < 0$ $\forall t$, a standard assumption. Condition (29) thus characterizes a unique maximum with respect to $\widetilde{\varrho}_t$.

yields a stationary state space representation of the equilibrium (i.e., the highest eigenvalue of the state space system is less than one in absolute value). Thus, after a shock that moves i_t^F below R_t, and absent any further shocks, the ratios in equation (29) converge to a constant and the two interest rates converge back to their steady-state value equal to the parameter i (see Section 5). As a result, $\widetilde{\varrho}_t$ converges to Δ. When i_t^F is equal to R_t, there are no gains from refinancing and thus only the costless exogenous refinancing activity, $\varrho_t = \Delta$, takes place.

The two parameters related to refinancing, Δ and υ, are calibrated as follows. The parameter Δ is set equal to 0.02, which is the long-run average fraction of outstanding debt that is refinanced per quarter. This fraction is reported by Chen et al. (2013) and corresponds to a long-run share of refi loans in newly originated loans equal to 0.4, which is consistent with observations from the Freddie Mac's Primary Mortgage Market Survey (1987-2007). The parameter υ is set equal to 12, so as to match, in equilibrium, the local elasticity of the share of refi loans in *new* loans with respect to the FRM rate. Specifically, we identify periods of declines in the FRM rate (from a local peak to a local trough) accompanied with increases in refi activity and calculate the average percentage point increase in the refi share per one percentage point decline in the FRM rate during these periods. This calculation gives approximately a 32 percentage point increase in the refi share of new loans for a one percentage point decline in the 30-year FRM rate.[53]

Figure 7 shows the responses of selected variables to a 25 basis point quarterly (one percentage point, annualized) decline in the FRM rate, occurring due to the $\overline{\pi}_t$ shock, with the calibrated persistence of $\rho_\pi = 0.994$. For comparison, the figure also plots responses under FRM without refinancing and under ARM. As expected, the responses under refinancing lie in-between the other two cases. Quantitatively, they are closer to the FRM responses. This is because the increase in the quarterly refi share of outstanding debt, $\widetilde{\varrho}_t$, is fairly small: an

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[53]The average refi share in the subsample used for this calculation is close to the average refi share in the whole sample. The elasticity will necessarily be smaller in periods in which the share of refi loans in new loans approaches the upper bound of 100%. The elasticity we estimate is thus "local" in the sense that it holds around the average level of the refi share.

increase from 0.02 to 0.026 on impact. Notice, however, that (given the size of the stock) such a modest increase in ϱ_t shows up as a large increase in the refi share of *new* loans: an increase from 0.4 to 0.48 on impact (equivalent to an increase to 0.72 for a one percentage point decline in the FRM rate). The increase in refinancing is persistent, resulting in a sustained, though modest, decline in real mortgage payments for several periods after the shock. As R_t and i_t^F converge back to steady state, $\widetilde{\varrho}_t$ converges to Δ (the figure plots $\widetilde{\varrho}_t - \Delta$). Over time, however, the effect of persistently low inflation gets stronger and real mortgage payments start to increase, converging to the basic FRM case. Due to the attenuated response of real mortgage payments, the negative responses of consumption and housing investment observed under the basic FRM contract are now weaker, close to zero, in fact, for several periods.

7.3   Optimal mortgage choice

In many countries, the only types of mortgage contract available are various forms of ARM loans (see footnote 1). However, in the United States, and a few other countries, homeowners have a choice between FRMs and ARMs. On average (1982-2007), ARMs in the United States accounted for 30 percent of new loan originations but this share has not been stable over time. Various studies document that the ARM share moves positively with the long-short spread, or its various proxies (Koijen, Van Hemert, and Van Nieuwerburgh, 2007; Moench, Vickery, and Aragon, 2010; Badarinza et al., 2014).[54] Is our model consistent with this behavior once we allow for mortgage choice? And does mortgage choice affect the responses of the model economy to the $\overline{\pi}_t$ shock? Campbell and Cocco (2003) provide a thorough theoretical analysis of mortgage choice at the household level. They show that mortgage choice is sensitive to short-run movements in interest rates (and thus in the long-short spread) when households have limited ability to smooth out fluctuations in real mortgage payments and are thus affected by their timing. Such a friction is present in our

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[54]Koijen et al. (2007) go further and argue that it is the term premium component of the long-short spread that is the key driver of the movements in the ARM share. Botsch and Malmendier (2015), in contrast, argue that people's mortgage choice is affected by personal inflation experience.

model and the timing of real mortgage payments matters, as shown in Section 3.

As in the case of refinancing, the mortgage choice cost function is made operative by assuming a quadratic form, $\Phi(.) = \omega(l_{2t}/l_t - \Lambda)^2$, where (as noted in Section 4) l_{2t}/l_t is the fraction of ARM loans in newly originated loans and Λ is a parameter capturing the institutional norm of the ARM share in the economy. To characterize the optimal mortgage choice, it is again convenient to work with the recursive formulation of the homeowner's problem (25). Observe that l_{1t} and l_{2t} appear only in the constraint (22) and the respective laws of motion (11)-(13). New ARM loans, l_{2t}, appear also in the quadratic cost function, where $l_t = \theta p_t p_{Ht} x_{Ht}$.

Let λ_t denote the Lagrange multiplier associated with the constraint (22). Substituting the laws of motion into the value function and taking the first derivatives with respect to l_{1t} and l_{2t} and setting them equal to zero yields

$$-\lambda_t + E_t \left[\beta \widetilde{V}_{d1,t+1} + \zeta_{D1t}(\kappa - \gamma_{1t}^\alpha)\beta V_{\gamma 1,t+1} + \zeta_{D1t}(i_t^F - R_{1t})\beta V_{R1,t+1} \right] = 0,$$

$$-\frac{\partial}{\partial l_{2t}} \Phi(l_{2t}/l_t) - \lambda_t + E_t \left[\beta \widetilde{V}_{d2,t+1} + \zeta_{D2t}(\kappa - \gamma_{2t}^\alpha)\beta V_{\gamma 2,t+1} + \zeta_{D2t}(i_t^F - R_{2t})\beta V_{R2,t+1} \right] = 0,$$

where $\zeta_{Djt} \equiv \left(\frac{1-\gamma_{jt}}{1+\pi_t} \widetilde{d}_{jt} \right) / \left(\frac{1-\gamma_{jt}}{1+\pi_t} \widetilde{d}_{jt} + \widetilde{l}_{jt} \right)^2 \in (0,1)$, for $j = 1, 2$. Eliminating λ_t, using the functional form for $\Phi'(l_{2t}/l_t)$, and carrying out some algebra yields

$$l_{2t}/l_t - \Lambda = \frac{v_{ct} p_{Ht} x_{Ht}}{2\omega} \left(\tau_{H1t} - \tau_{H2t} \right), \tag{30}$$

where τ_{H1t} and τ_{H2t} are the wedges capturing the price effect, as discussed in Section 3 (for the full model, the wedges are derived in Appendix A, but they are straightforward generalizations of the wedges in the simple model of Section 3). Recall, that the wedges capture the costs of the respective mortgages (FRM vs. ARM) to the homeowner. The optimality condition (30) thus has an intuitive interpretation. It states that the ARM share in new loans l_{2t}/l_t increases when the homeowner perceives ARM loans as cheaper than FRM

loans; i.e., $\tau_{H1t} > \tau_{H2t}$.[55]

Observe that if the ratio $(v_{ct}p_{Ht}x_{Ht})/(2\omega)$ in equation (30) is bounded, l_{2t}/l_t converges to Λ as τ_{H1t} and τ_{H2t} converge to each other. In steady state, $\tau_{H1t} = \tau_{H2t} = 0$ and thus there are no benefits of deviating away from the institutional norm Λ.[56] The numerical solution, described in Appendix B, produces a stationary state space representation of the equilibrium, with all eigenvalues less than one in absolute value. Thus, after a shock, the above ratio converges to a constant, the wedges converge back to their steady state, and the ARM share l_{2t}/l_t therefore converges to Λ.

The parameter Λ is set equal to 0.3, the long-run average of ARM share in the U.S. economy, and the cost parameter ω is set so as to replicate the empirical elasticity of the ARM share to the long-short spread, equal to 7.0 (i.e., a seven percentage point increase in the ARM share in response to a one percentage point increase in the long-short spread). This elasticity implies $\omega = 0.0055$.[57] To assess if the model is consistent with the empirical observations on the movements in the ARM share described in the literature, we study responses to the τ_{bt} shock, which is the main driver of the long-short spread in the model. Panel A of Figure 8 shows that the model is consistent with the basic feature of the data: the ARM share of new loans declines when the long-short spread declines. Similarly to the case of refinancing, while the response of the share in new loans is substantial, it translates to only a small change in the share of outstanding debt. The response of housing investment is mainly affected by the parameter Λ and lies in-between the responses under the separate FRM and ARM contracts reported in panel A of Figure 3.

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[55]The second-order condition is difficult to establish analytically. A simple condition such as the one in the refi case is not available. We therefore instead check whether the second-order condition holds for the approximate economy computed numerically. In the approximate economy, the value function has a quadratic form and the associated matrix is negative definite. This establishes that the first-order condition characterizes a unique maximum.

[56]As discussed in the paper, because in the steady state homeowners have the same discount factor as capital owners, β, both agents price future mortgage payments as equal to one unit of current consumption in present value terms. Thus, $\tau_{H1t} = \tau_{H2t} = 0$ in the steady state.

[57]This value is much smaller than that for the cost parameter in the case of refinancing. This is because the potential gain from refinancing is much larger than the gain from mortgage choice as refinancing applies to the stock, whereas mortgage choice applies only to new loans. The fact that in the data only a small fraction of the stock is refinanced in response to a percentage fall in the mortgage rate implies a relatively large refi cost in the model.

Given that the model is consistent with the basic observations on mortgage choice in the data, we ask if mortgage choice plays an important role in the transmission mechanism with respect to the $\overline{\pi}_t$ shock. Panel B of Figure 8 provides a negative answer. As the $\overline{\pi}_t$ shock is highly persistent, it has almost no effect on the long-short spread and thus no effect on mortgage choice. The response of housing investment is only affected by the parameter Λ and lies in-between the responses under FRM and ARM, reported in Figure 4.

8 Concluding remarks

Mortgage payments constitute a substantial part of homeowners' mandatory expenses. In combination with the fact that mortgages are long-term loans set in nominal terms, it is natural to ask: what role do mortgage contracts play in the transmission of monetary policy? This paper attempts to establish these connections. Like goods market imperfections provide a breeding ground for nominal price rigidities to play a role in the transmission of monetary policy in New-Keynesian models, financial market imperfections (incomplete asset markets) facilitate transmission of monetary policy through mortgage contracts in our framework.

Three key properties of the mortgage transmission mechanism emerge. First, the transmission mechanism is found to be stronger under adjustable- than fixed-rate mortgages. Second, monetary policy shocks affecting the level of the nominal yield curve have larger real effects than transitory shocks, affecting its slope. And third, persistently higher inflation gradually benefits homeowners under FRMs, but hurts them immediately under ARMs. In terms of quantities, on impact, housing investment increases by 1.8% under ARM and declines by 0.7% (0.15% with refinancing) under FRM, in response to a one percentage point (annualized) downward shift of the nominal yield curve (and inflation). Under ARM, the strength of the transmission mechanism is comparable to a traditional real rate channel. This is despite the fact that, in equilibrium, the real interest rate in our model hardly moves in response to monetary policy shocks.

In the interest of transparency, we have abstracted from the usual nominal frictions and

other channels through which housing finance affects the macroeconomy. The number of shocks was also limited. A natural extension is therefore to incorporate these additional features and study their interaction with the mechanism proposed here.

Another interesting avenue is to study the price and income effects in an overlapping generations setup with realistic life-cycle dynamics. It is likely that the agents who face the price effect are different from those who face the income effect, and the importance of the income effect further varies with the homeownership life-cycle. This can lead to an additional dimension of monetary policy redistribution, redistribution across different homeowners.

Finally, an important normative question regards optimal monetary policy. Such a policy is likely to depend on the prevalent mortgage type, FRM or ARM, in the economy. Different countries may thus follow different policies depending on their mortgage markets. This question may be quite complex for the Eurozone, which includes countries with very different mortgage markets. Extending the model to allow for default and banking may also generate interesting interactions between monetary and macroprudential policies. The framework developed in this paper provides a groundwork for addressing these additional questions.

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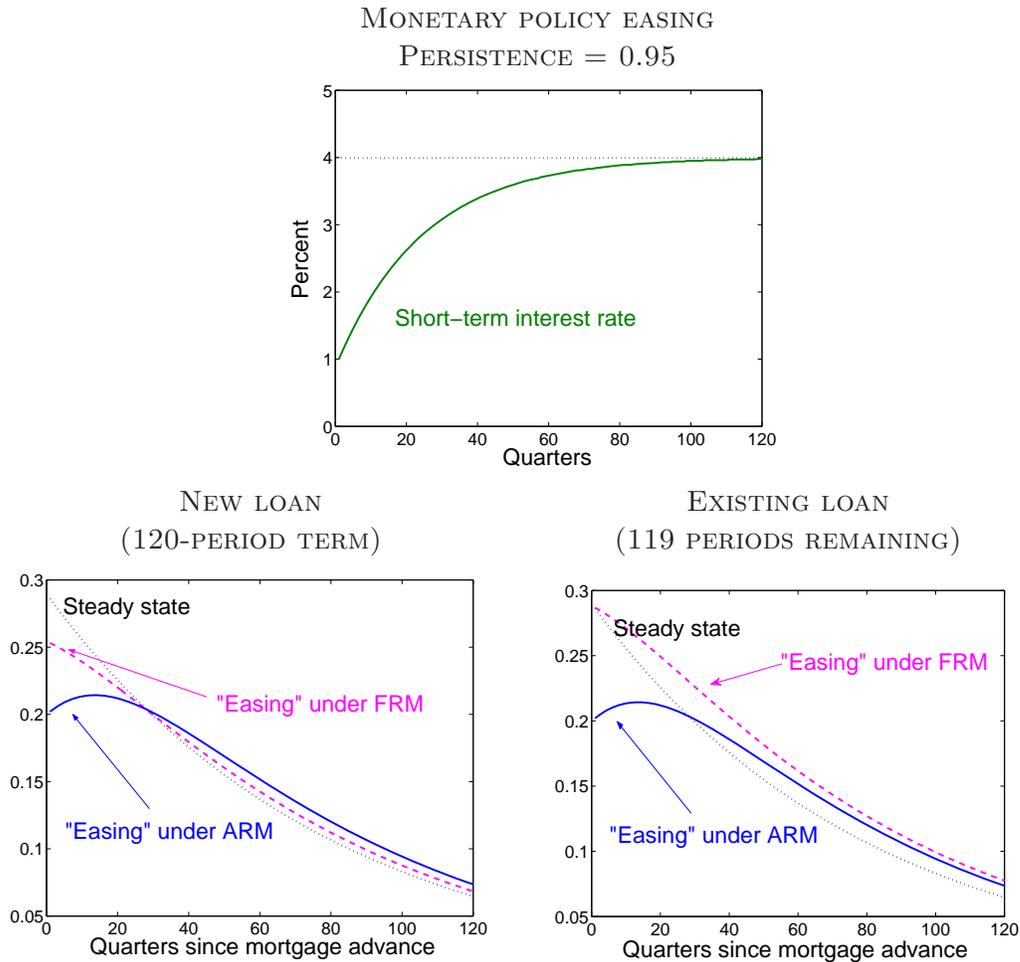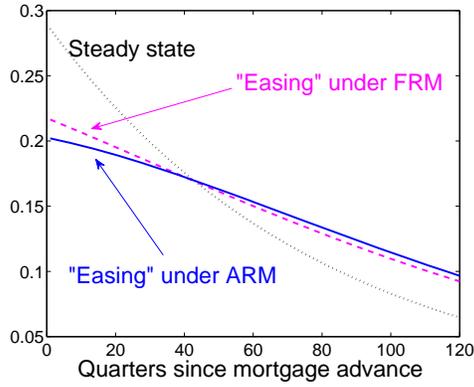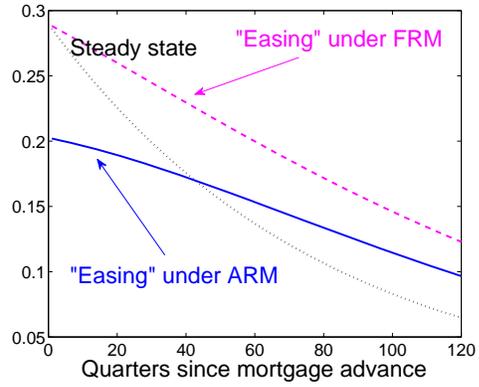
Figure 1: Illustration of price and income effects. Debt-servicing costs over a term of a new and an existing 30-year mortgage under alternative paths of the short-term nominal interest rate. The label 'steady-state' refers to the case when the short rate is at its steady-state level of 4%. The mortgage is equal to four times the household's annual income; the real interest rate is held constant at 1% per annum.

A. High persistence (0.99) of the short rate decline
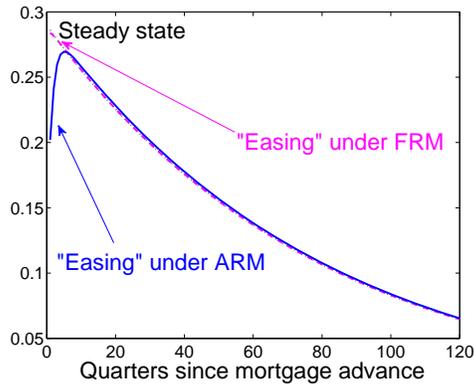
New loan
(120-period term)

Existing loan
(119 periods remaining)

B. Low persistence (0.5) of the short rate decline

New loan
(120-period term)
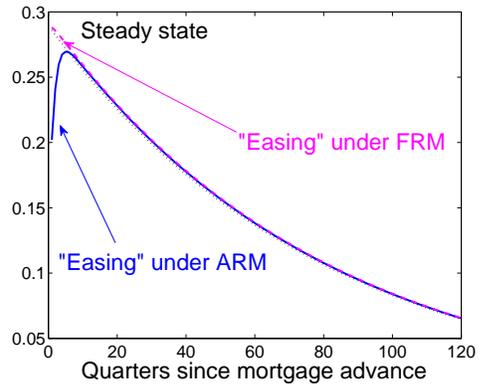
Existing loan
(119 periods remaining)

Figure 2: Illustration of price and income effects for high and low persistence of the mean-reverting short rate decline by 3 percentage points.

Table 1: Calibration

Symbol	Value	Description
Population		
Ψ	2/3	Share of homeowners
Preferences		
β	0.9883	Discount factor
ξ	0.5003	Weight on nonhousing consumption
Technology		
ς	0.283	Capital share of output
δ_K	0.02225	Depreciation rate of capital
δ_H	0.01021	Depreciation rate of housing
ζ	3.0	Curvature of PPF
φ	0.1	Land share of new housing
Fiscal		
G	0.138	Government expenditures
τ_N	0.235	Labor income tax rate
τ_K	0.3362	Capital income tax rate
τ	0.4693	Transfer
Mortgage contract		
θ	0.6	Loan-to-value ratio
κ	0.00162	Initial amortization rate
α	0.9946	Amortization adjustment factor
Bond market		
ϑ	0.0345	Participation cost function
Monetary policy rule		
ν_π	1.5	Weight on inflation
$\overline{\pi}$	0.0113	Steady-state inflation rate
Exogenous processes		
ρ_A	0.9641	Persistence of TFP shock
σ_A	0.0148	Std. of TFP innovations
ρ_π	0.994	Persistence of nominal shock
σ_π	0.0012	Std. of nominal innovations
ρ_b	0.903	Persistence of real rate shock
σ_b	0.00075	Std. of real rate innovations
ρ_q	0.798	Persistence of PPF shock
σ_q	0.012	Std. of PPF innovations
Refinancing		
Δ	0.02	Long-run fraction of refi in outst. debt
υ	12.0	Cost function parameter
Mortgage choice		
Λ	0.3	Long-run share of ARMs
ω	0.0055	Cost function parameter

Table 2: Model cross-validation I: business cycle moments

A.	U.S. data	Model			U.S. data	Model	
		FRM	ARM ‖			FRM	ARM
Std				**Corr with Y**			
Y	1.92	1.90	1.91	Y	1.00	1.00	1.00
C	0.85	0.78	0.78	C	0.79	0.95	0.96
X_S	13.32	12.98	12.77	X_S	0.60	0.82	0.77
X_K	4.70	5.09	5.58	X_K	0.73	0.89	0.86
i	0.28	0.29	0.29	i	0.36	0.42	0.38
i^F	0.17	0.18	N/A	i^F	0.01	0.16	N/A
$i^F - i$	0.20	0.20	N/A	$i^F - i$	-0.49	-0.43	N/A
π	0.34	0.23	0.22	π	0.36	0.36	0.33
p_H	3.01	2.90	2.87	p_H	0.55	0.96	0.91
B.	**Other countries**			**Model**			
		FRM	ARM ‖			FRM	ARM
$\mathrm{std}(X_S)/\mathrm{std}(Y)$		5.80	5.50	$\mathrm{std}(X_S)/\mathrm{std}(Y)$		6.83	6.68
$\mathrm{corr}(X_S, Y)$		0.65	0.55	$\mathrm{corr}(X_S, Y)$		0.84	0.77

Notes A: All moments are for HP-filtered series, quarterly data. U.S. data: 1958-2006. 10-year government bond yield is used as a proxy for i_t^F due to longer time availability; CPI ex-energy inflation rate is used for π_t; 3-month T-bill yield is used for i_t; the ratio of the average price of new homes sold (Census Bureau) and the GDP deflator is used for p_{Ht} (1975-2006). The model moments are averages of moments for 150 runs; the artificial data of each run are of the same length as the U.S. data and are also HP filtered. Details of U.S. data are in Appendix C.

Notes B: FRM countries are BEL, FRA, US; ARM countries are AUS, CAN, UK. The moments, based on HP-filtered data, are taken from Kydland et al. (forthcoming). The classification is based on Footnote 1.
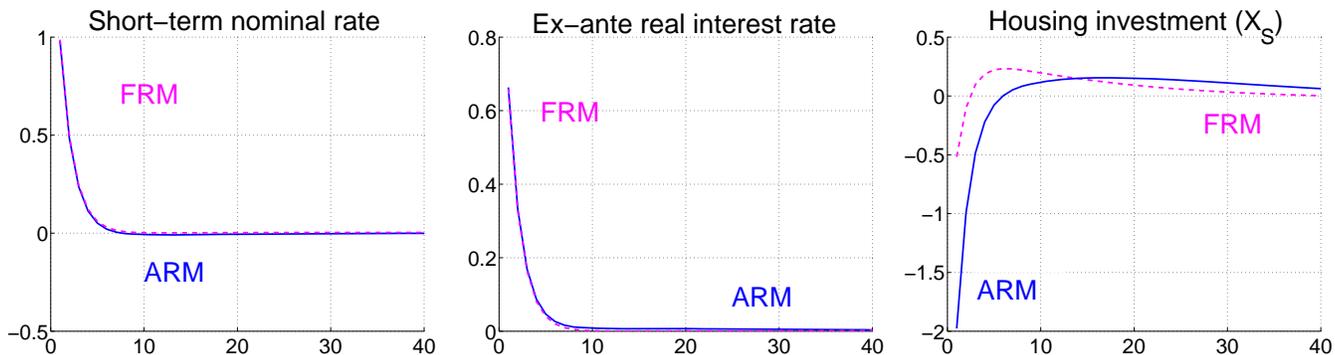
Figure 3: Model cross-validation II: a real rate shock. The effect of an increase of the ex-ante real interest rate (τ_b shock). Panel A: calibrated shock persistence ($\rho_b = 0.9$) based on the persistence of the long-short spread. Panel B: shock persistence ($\rho_b = 0.5$) chosen to replicate the persistence of the response of the interest rate in Calza et al. (2013). Interest rates are in percentage point deviations (annualized), housing investment is in percentage deviations. One period = one quarter.

Table 3: Model cross-validation III: marginal propensity to consume (MPC) of ARM home-owners

	Di Maggio et al. (2014)	Calibrated		Matched	
		Nom. shock	Real rate shock	Nom. shock	Real rate shock
Shock persistence	N/A	0.994	0.90	0.95	0.70
MPC	0.17*	0.29	0.27	0.17	0.17

Note: Real rate shock = direct effect on the ex-ante real interest rate (τ_b shock); nominal shock = no direct effect on the ex-ante real interest rate ($\overline{\pi}_t$ shock). Calibrated = persistence as in Table 1; matched = persistence so as to reproduce MPC in Di Maggio et al. (2014).
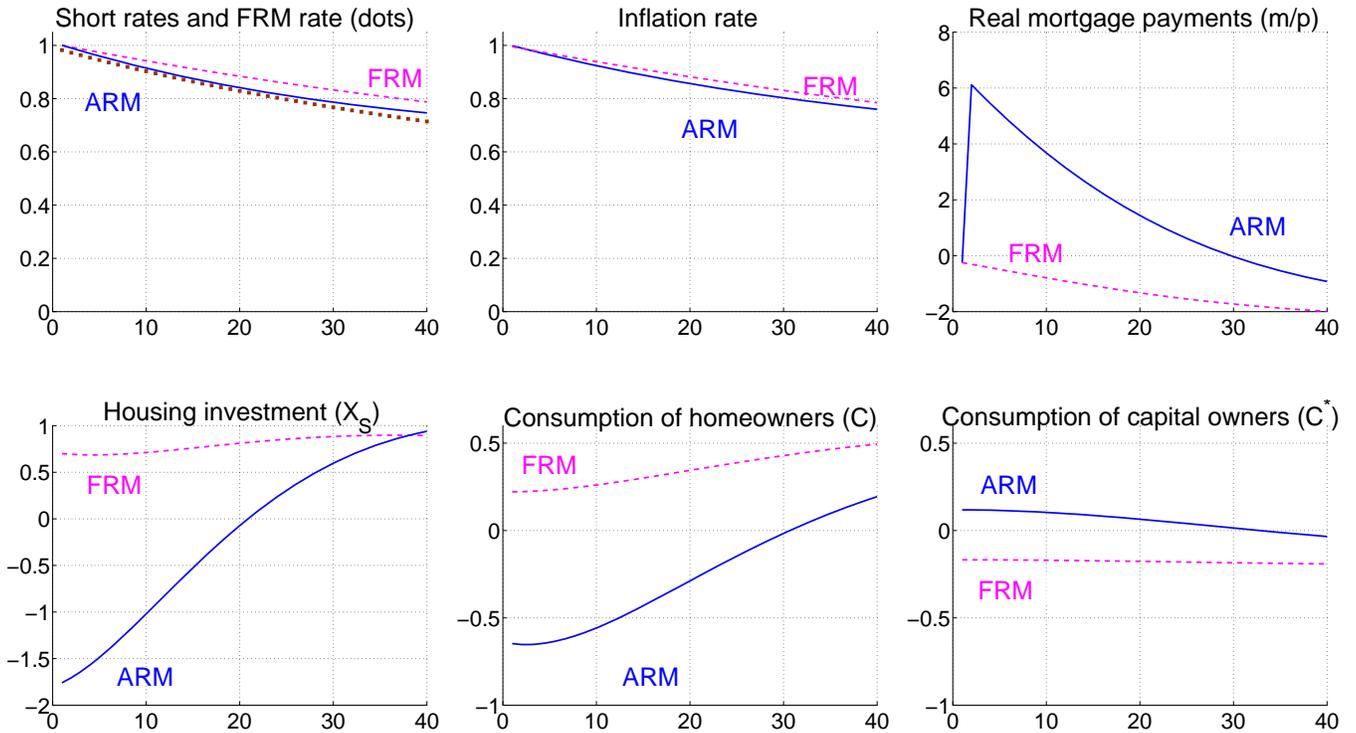* Based on their reported $150 increase in consumption in response to $900 increase in disposable income.

Figure 4: Findings I: the transmission mechanism under separate contracts. Responses to the nominal ($\overline{\pi}_t$) shock under FRM and ARM; no refinancing, no mortgage choice. Interest rates and the inflation rate are measured as percentage point (annualized) deviations from steady state, quantities are in percentage deviations. One period = one quarter.
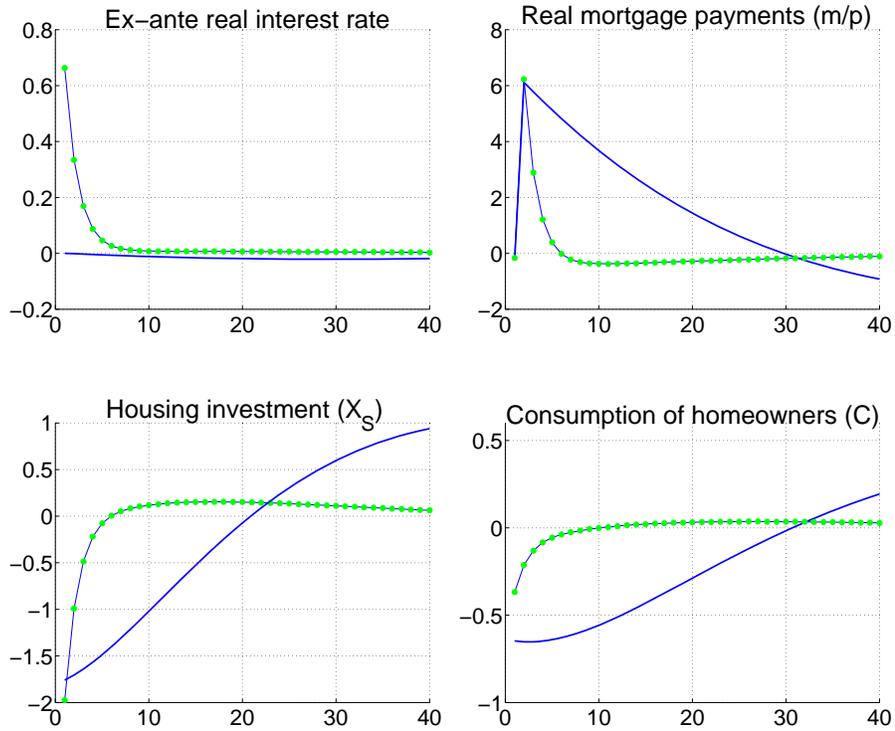
Figure 5: Findings II: comparison of two transmission mechanisms under ARM. Responses to the real rate (τ_{bt}) shock and to the nominal ($\overline{\pi}_t$) shock under ARM. Real rate shock = circles. Nominal shock = solid line. Interest rates are measured as percentage point (annualized) deviations from steady state, quantities are in percentage deviations. The persistence of the real rate shock is 0.5, based on the VAR estimate of Calza et al. (2013). The persistence of the nominal shock is the calibrated value of 0.994, based on matching the autocorrelation of the long rate. In both cases, the initial response of the short-term nominal interest rate (not plotted) is one percentage point (annualized). One period = one quarter.
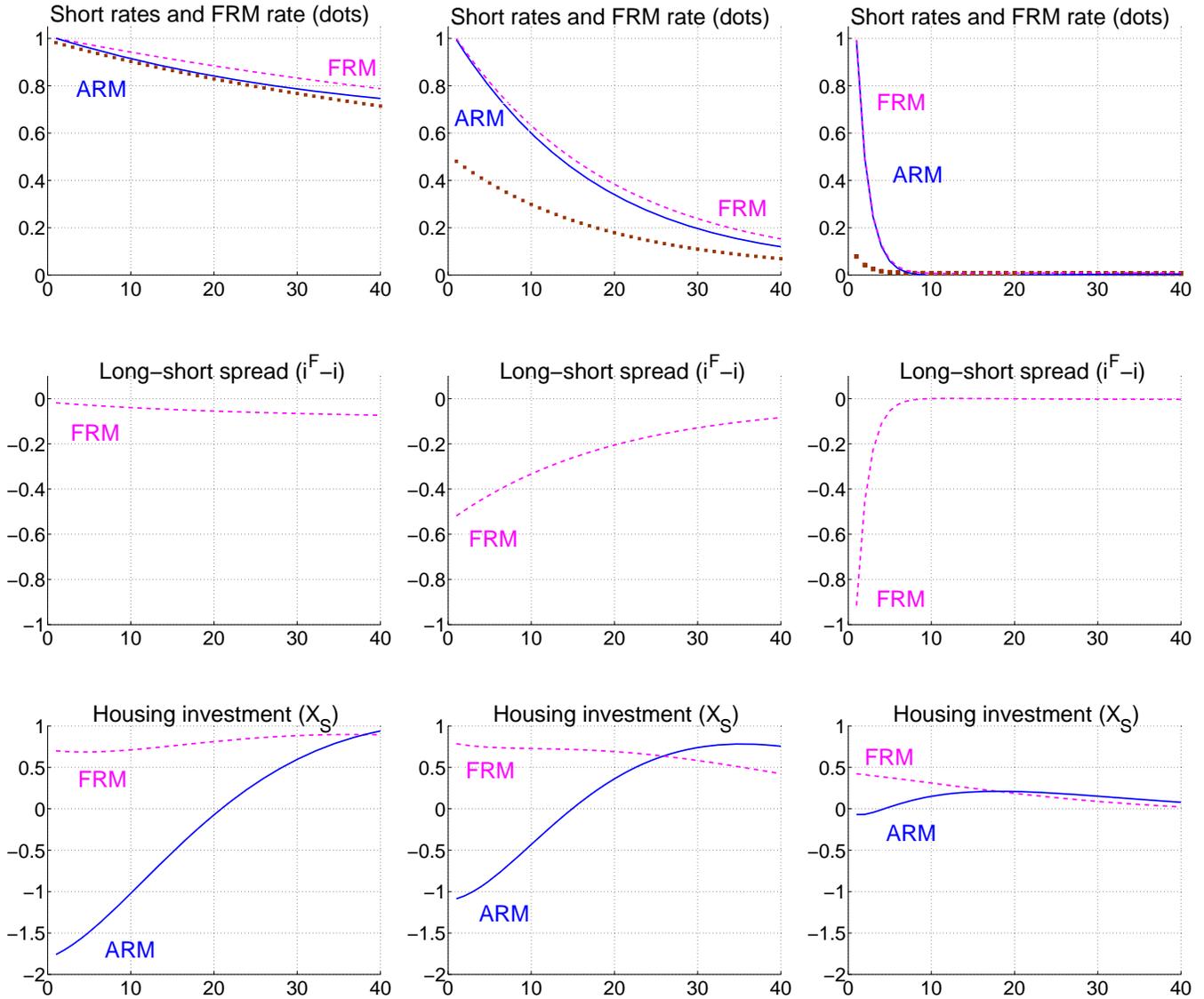
Figure 6: Findings III: the effect of the persistence of the nominal $(\overline{\pi}_t)$ shock under FRM and ARM; no refinancing, no mortgage choice. Interest rates and the inflation rate are measured as percentage point (annualized) deviations from steady state, quantities are in percentage deviations. One period = one quarter.
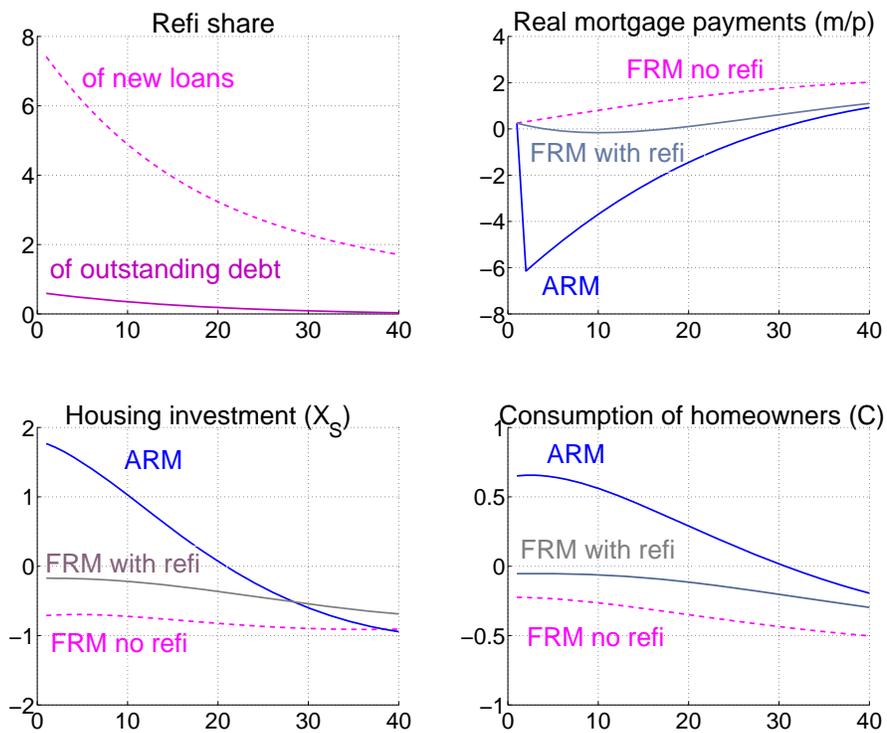
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Figure 7: Findings IV: the transmission mechanism under refinancing. Responses to a one percentage point (annualized) decline in the FRM rate due to the nominal $(\overline{\pi}_t)$ shock, with the calibrated persistence $\rho_\pi = 0.994$. Refi shares are measured as percentage point deviations from steady state, quantities are in percentage deviations. One period = one quarter.

A. REAL RATE SHOCK, $\rho_b = 0.9$
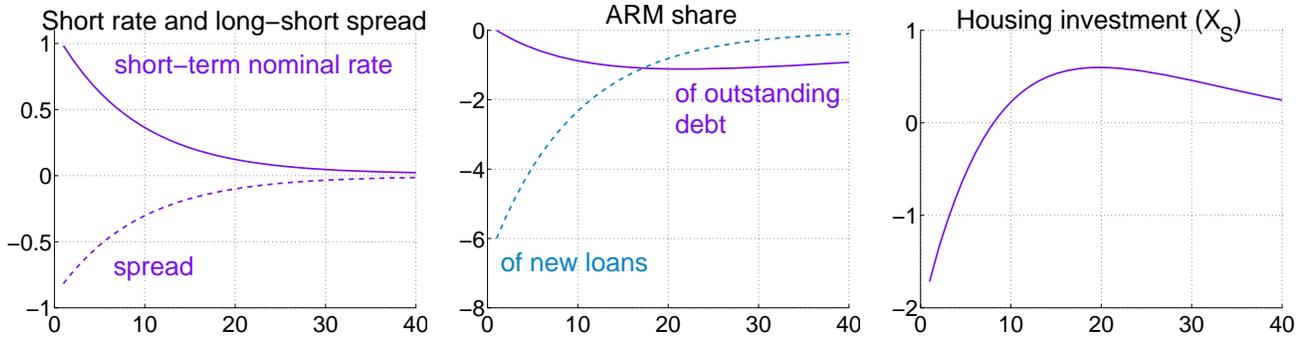
B. NOMINAL SHOCK, $\rho_\pi = 0.994$
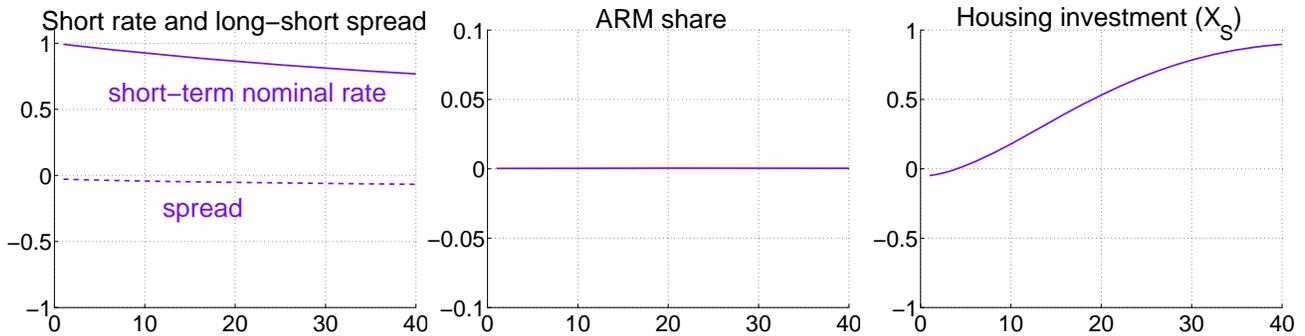

Figure 8: Findings V: the transmission mechanism under mortgage choice. Both shocks are scaled so that, on impact, the short rate increases by one percentage point (annualized). ARM share is measured as percentage point deviations from steady state, housing investment is in percentage deviations. The steady-state share of ARMs is 30 percent (1982-2007 average). One period = one quarter.

Supplemental material

The supplemental material covers (A) equilibrium conditions, (B) computation, (C) data used in calibration, and (D) estimates of mortgage debt servicing costs for the United States.

Appendix A: Equilibrium conditions

This appendix lists the conditions characterizing the equilibrium defined in Section 4 for the basic FRM and ARM cases. We use these conditions to solve for the steady state (a situation in which all shocks are equal to their unconditional means and all real variables, interest rates, and the inflation rate are constant). The steady state is used to calibrate the model and serves as the point of a local approximation of the economy in our solution method described in Appendix B. Interested readers may also find these conditions useful if they wish to compute the equilibrium by log-linearization of the equilibrium conditions. Our computational method does not rely on approximating these conditions. Instead, it takes a linear-quadratic approximation of the Bellman equations within the recursive competitive equilibrium problem described in Section 4 and then solves the LQ problem. Details are provided in Appendix B. As is well known, the two methods yield the same approximate decision rules and pricing functions. To solve for the case of refinancing, the standard method needs to be modified, as explained in Appendix B. The steady state, however, is the same for the basic FRM case, the ARM case, refinancing, and the mortgage choice problem. This is because, in the steady state, $i_t = i_t^F = R_t$. (As a result, steady-state refinancing and ARM share are parameterized, as noted in the text.) The conditions below can thus be used to solve for the steady state under all contract types considered in this paper.

Throughout, the notation is that, for instance, u_{ct} denotes the first derivative of the function u with respect to c, evaluated in period t. Alternatively, v_{2t}, for instance, denotes the first derivative of the function v with respect to the second argument, evaluated in period t.

Capital owner's optimality

First-order conditions:

$$1 = E_t \left\{ \beta \frac{u_{c,t+1}}{u_{ct}} \left[1 + (1 - \tau_K)(r_{t+1} - \delta_K) \right] \right\}, \tag{A1}$$

$$1 = E_t \left\{ \beta \frac{u_{c,t+1}}{u_{ct}} \left[\frac{(1 - \tau_{b,t+1})(1 + i_t)}{1 + \pi_{t+1}} \right] \right\}, \tag{A2}$$

$$1 = E_t \left\{ \beta \frac{\widetilde{U}_{d,t+1}}{u_{ct}} + \beta \frac{U_{\gamma,t+1}}{u_{ct}} \zeta_{Dt}^* \left[\kappa - (\gamma_t^*)^\alpha \right] + \beta \frac{U_{R,t+1}}{u_{ct}} \zeta_{Dt}^* (i_t^F - R_t^*) \right\}. \tag{A3}$$

Note: the last equation (for l_t^*) applies only in the FRM case, as explained in the text.

Benveniste-Scheinkman conditions:

$$\widetilde{U}_{dt} = u_{ct} \frac{R_t^* + \gamma_t^*}{1 + \pi_t} + \beta \frac{1 - \gamma_t^*}{1 + \pi_t} E_t \left\{ \widetilde{U}_{d,t+1} + \zeta_{lt}^* \left[(\gamma_t^*)^\alpha - \kappa \right] U_{\gamma,t+1} + \zeta_{lt}^* (R_t^* - i_t^F) U_{R,t+1} \right\}, \tag{A4}$$

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$$U_{\gamma t} = u_{ct}\left(\frac{\widetilde{d}_t^*}{1+\pi_t}\right) - \beta\left(\frac{\widetilde{d}_t^*}{1+\pi_t}\right)E_t\widetilde{U}_{d,t+1} \tag{A5}$$

$$+\beta\left(\frac{\widetilde{d}_t^*}{1+\pi_t}\right)\left\{\zeta_{lt}^*[\kappa - (\gamma_t^*)^\alpha] + \frac{(1-\gamma_t^*)\alpha(\gamma_t^*)^{\alpha-1}}{\frac{1-\gamma_t^*}{1+\pi_t}\widetilde{d}_t^* + \widetilde{l}_t^*}\right\}E_t U_{\gamma,t+1}$$

$$+\beta\left(\frac{\widetilde{d}_t^*}{1+\pi_t}\right)\zeta_{lt}^*(i_t^F - R_t^*)E_t U_{R,t+1},$$

$$U_{Rt} = u_{ct}\left(\frac{\widetilde{d}_t^*}{1+\pi_t}\right) + \beta\left(\frac{\frac{1-\gamma_t^*}{1+\pi_t}\widetilde{d}_t^*}{\frac{1-\gamma_t^*}{1+\pi_t}\widetilde{d}_t^* + \widetilde{l}_t^*}\right)E_t U_{R,t+1}. \tag{A6}$$

Constraints:

$$c_t^* + k_{t+1} + \widetilde{b}_{t+1}^* + \widetilde{l}_t^* = [1 + (1-\tau_K)(r_t - \delta_K)]\,k_t + (1-\tau_{bt})(1+i_{t-1})\frac{\widetilde{b}_t^*}{1+\pi_t} + \widetilde{m}_t^* + \tau_t^* + \frac{p_{Lt}}{1-\Psi}, \tag{A7}$$

$$\widetilde{m}_t^* = (R_t^* + \gamma_t^*)\frac{\widetilde{d}_t^*}{1+\pi_t}, \tag{A8}$$

$$\widetilde{d}_{t+1}^* = \frac{1-\gamma_t^*}{1+\pi_t}\widetilde{d}_t^* + \widetilde{l}_t^*, \tag{A9}$$

$$\gamma_{t+1}^* = (1-\phi_t^*)(\gamma_t^*)^\alpha + \phi_t^*\kappa, \tag{A10}$$

$$R_{t+1}^* = \begin{cases} (1-\phi_t^*)R_t^* + \phi_t^* i_t^F, & \text{if FRM,} \\ i_t, & \text{if ARM,} \end{cases} \tag{A11}$$

$$x_{Kt} = k_{t+1} - (1-\delta_k)k_t. \tag{A12}$$

Homeowner's optimality

First-order conditions:

$$v_{ct}(1-\theta)p_{Ht} = \beta E_t\left\{V_{h,t+1} + p_{Ht}\theta\left[\widetilde{V}_{d,t+1} + \zeta_{Dt}(\kappa - \gamma_t^\alpha)V_{\gamma,t+1} + \zeta_{Dt}(i_{t+1}^M - R_t)V_{R,t+1}\right]\right\}, \tag{A13}$$

$$1 = E_t\left[\beta\frac{v_{c,t+1}}{v_{ct}}\left(\frac{1+i_t+\Upsilon_t}{1+\pi_{t+1}}\right)\right]. \tag{A14}$$

Note: $i_{t+1}^M = i_t^F$ (FRM case), $i_{t+1}^M = i_t$ (ARM case).

Benveniste-Scheinkman conditions:

$$\widetilde{V}_{dt} = -v_{ct}\frac{R_t + \gamma_t}{1+\pi_t} + \beta\frac{1-\gamma_t}{1+\pi_t}E_t\left[\widetilde{V}_{d,t+1} + \zeta_{lt}(\gamma_t^\alpha - \kappa)V_{\gamma,t+1} + \zeta_{lt}(R_t - i_{t+1}^M)V_{R,t+1}\right], \tag{A15}$$

$$V_{\gamma t} = -v_{ct}\left(\frac{\widetilde{d}_t}{1+\pi_t}\right) - \beta\left(\frac{\widetilde{d}_t}{1+\pi_t}\right)E_t\widetilde{V}_{d,t+1} \tag{A16}$$

$$+\beta\left(\frac{\widetilde{d}_t}{1+\pi_t}\right)\left[\zeta_{lt}(\kappa-\gamma_t^\alpha) + \frac{(1-\gamma_t)\alpha\gamma_t^{\alpha-1}}{\frac{1-\gamma_t}{1+\pi_t}\widetilde{d}_t + \widetilde{l}_t}\right]E_tV_{\gamma,t+1}$$

$$+\beta\left(\frac{\widetilde{d}_t}{1+\pi_t}\right)\zeta_{lt}(i_{t+1}^M - R_t)E_tV_{R,t+1},$$

$$V_{Rt} = -v_{ct}\left(\frac{\widetilde{d}_t}{1+\pi_t}\right) + \beta\left(\frac{\frac{1-\gamma_t}{1+\pi_t}\widetilde{d}_t}{\frac{1-\gamma_t}{1+\pi_t}\widetilde{d}_t + \widetilde{l}_t}\right)E_tV_{R,t+1}, \tag{A17}$$

$$V_{ht} = v_{ht} + \beta(1-\delta_H)E_tV_{h,t+1}. \tag{A18}$$

Constraints

$$\widetilde{l}_t = \theta p_{Ht}x_{Ht}, \tag{A19}$$

$$x_{Ht} = h_{t+1} - (1-\delta_H)h_t. \tag{A20}$$

Production

First-order conditions:
$$r_t = A_tf_1\left((1-\Psi)k_t, \Psi n\right), \tag{A21}$$
$$w_t = A_tf_2\left((1-\Psi)k_t, \Psi n\right), \tag{A22}$$
$$Y_t = A_tf\left((1-\Psi)k_t, \Psi n\right). \tag{A23}$$

PPF curvature:
$$q_t = \eta_t q(\Psi x_{St}). \tag{A24}$$

Homebuilding

First-order conditions (after imposing land market clearing $X_{Lt} = 1$):

$$x_{St} = \frac{1}{\Psi}(\Psi x_{Ht})^{\frac{1}{1-\varphi}}, \tag{A25}$$

$$p_{Ht} = q_t\frac{(\Psi x_{St})^\varphi}{1-\varphi}, \tag{A26}$$

$$p_{Lt} = p_{Ht}\varphi(\Psi x_{St})^{1-\varphi}. \tag{A27}$$

Monetary policy

$$i_t = (i - \overline{\pi} + \overline{\pi}_t) + \nu_\pi(\pi_t - \overline{\pi}_t). \tag{A28}$$

Government budget constraint

$$G + (1 - \Psi)\tau_t^* = \tau_K(r_t - \delta_K)(1 - \Psi)k_t + \tau_N(w_t\Psi n - \Psi\tau) + \Psi\tau + \Xi_t. \tag{A29}$$

Market clearing

$$(1 - \Psi)c_t^* + \Psi c_t + (1 - \Psi)x_{Kt} + q_t\Psi x_{St} + G = Y_t, \tag{A30}$$

$$(1 - \Psi)\widetilde{b}_t^* + \Psi\widetilde{b}_t = 0, \tag{A31}$$

$$(1 - \Psi)\widetilde{l}_t^* = \Psi\widetilde{l}_t. \tag{A32}$$

Aggregate consistency

$$(1 - \Psi)\widetilde{d}_t^* = \Psi\widetilde{d}_t, \tag{A33}$$

$$\gamma_t^* = \gamma_t, \tag{A34}$$

$$R_t^* = R_t. \tag{A35}$$

Equation count: 35 equations (FRM), 34 equations (ARM). Note: the homeowner's budget constraint holds by Walras' law.

Endogenous variables

35 variables (FRM), 34 variables (ARM):

Allocations: c_t^*, x_{Kt}, k_{t+1}, c_t, x_{Ht}, h_{t+1}, x_{St}, Y_t.	8 variables
Prices: π_t, i_t, i_t^F (FRM only), r_t, w_t, q_t, p_{Lt}, p_{Ht}.	8 vars (FRM), 7 vars (ARM)
Mortgages: $\widetilde{l}_t^*$, m_t^*, $\widetilde{d}_{t+1}^*$, γ_{t+1}^*, R_{t+1}^*, $\widetilde{l}_t$, $\widetilde{d}_{t+1}$, γ_{t+1}, R_{t+1}.	9 variables
Bonds: $\widetilde{b}_{t+1}^*$, $\widetilde{b}_{t+1}$.	2 variables
Transfer: τ_t^*.	1 variable
Value function: $\widetilde{U}_{dt}$, $U_{\gamma t}$, U_{Rt}, $\widetilde{V}_{dt}$, $V_{\gamma t}$, V_{Rt}, V_{ht}.	7 variables

Shocks

$$\log A_{t+1} = (1 - \rho_A)\log A + \rho_A \log A_t + \epsilon_{A,t+1}, \quad \epsilon_{A,t+1} \sim iidN(0, \sigma_A).$$

$$\overline{\pi}_{t+1} = (1 - \rho_\pi)\overline{\pi} + \rho_\pi\overline{\pi}_t + \epsilon_{\pi,t+1}, \qquad \epsilon_{\pi,t+1} \sim iidN(0, \sigma_\pi).$$

$$\tau_{b,t+1} = \rho_b\tau_{bt} + \epsilon_{b,t+1}, \qquad\qquad \epsilon_{b,t+1} \sim iidN(0, \sigma_b).$$

$$\eta_{t+1} = (1 - \rho_q) + \rho_q\eta_t + \epsilon_{q,t+1}, \qquad\quad \epsilon_{q,t+1} \sim iidN(0, \sigma_q).$$

Transformation of variables used in (A1)-(A35)

To ensure stationarity:

$$\widetilde{U}_{dt} \equiv p_{t-1} U_{dt},$$

$$\widetilde{m}_t^* \equiv m_t^*/p_t,$$

$$\widetilde{d}_t^* \equiv d_t^*/p_{t-1},$$

$$\widetilde{l}_t^* \equiv l_t^*/p_t,$$

$$\widetilde{b}_t^* \equiv b_t^*/p_{t-1},$$

$$\widetilde{V}_{dt} \equiv p_{t-1} V_{dt},$$

$$\widetilde{d}_t \equiv d_t/p_{t-1},$$

$$\widetilde{l}_t \equiv l_t/p_t,$$

$$\widetilde{b}_t \equiv b_t/p_{t-1}.$$

Auxiliary notation:

$$\zeta_{lt}^* \equiv \frac{\widetilde{l}_t^*}{\left(\frac{1-\gamma_t^*}{1+\pi_t}\widetilde{d}_t^* + \widetilde{l}_t^*\right)^2} \in (0,1),$$

$$\zeta_{Dt}^* \equiv \frac{\frac{1-\gamma_t^*}{1+\pi_t}\widetilde{d}_t^*}{\left(\frac{1-\gamma_t^*}{1+\pi_t}\widetilde{d}_t^* + \widetilde{l}_t^*\right)^2} \in (0,1),$$

$$\zeta_{Dt} \equiv \frac{\frac{1-\gamma_t}{1+\pi_t}\widetilde{d}_t}{\left(\frac{1-\gamma_t}{1+\pi_t}\widetilde{d}_t + \widetilde{l}_t\right)^2} \in (0,1),$$

$$\zeta_{lt} \equiv \frac{\widetilde{l}_t}{\left(\frac{1-\gamma_t}{1+\pi_t}\widetilde{d}_t + \widetilde{l}_t\right)^2} \in (0,1),$$

$$\phi_t^* \equiv \widetilde{l}_t^*/\widetilde{d}_{t+1}^* \in (0,1),$$

$$\Xi_t \equiv (1-\Psi)\tau_{Bt}(1-i_{t-1})\widetilde{b}_t^*/(1+\pi_t),$$

$$\Upsilon_t = \Upsilon\left(-\widetilde{b}_{t+1}\right).$$

Residual variable

$$\widetilde{m}_t = (R_t + \gamma_t)\frac{\widetilde{d}_t}{1+\pi_t} \quad \text{or} \quad \widetilde{m}_t = \frac{1-\Psi}{\Psi}\widetilde{m}_t^*.$$

Connecting the GE model with the simple PE model

Mortgage pricing

Notice that for a once-and-for-all mortgage loan ($l_t^* = l^*$ in period t and $l_t^* = 0$ thereafter) and no outstanding mortgage debt ($d_t^* = 0$ in period t), we have $\zeta_{Dt}^* = 0$ and $\zeta_{l,t+j}^* = 0$, for $j = 1, 2, ...$ In this case, the first-order condition for l_t^* (A3) and the BS condition for $\widetilde{U}_{dt}$ (A4) simplify, as the terms related to $U_{\gamma t}$ and U_{Rt} drop out. Once combined, the two optimality conditions result in an equation that is a straightforward infinite-horizon extension of the mortgage-pricing equation (1) in the two-period mortgage example of Section 3:

$$1 = E_t \left[Q_{1t}^* \left(i_t^F + \gamma_{t+1}^* \right) + Q_{2t}^* \left(i_t^F + \gamma_{t+2}^* \right) \left(1 - \gamma_{t+1}^* \right) + ... \right],$$

where

$$Q_{Jt}^* \equiv \prod_{j=1}^{J} \beta \frac{u_{c,t+j}}{u_{c,t+j-1}} \frac{1}{1 + \pi_{t+j}} \quad J = 1, 2, ...$$

The terms related to $U_{\gamma t}$ and U_{Rt} in the general form of the optimality conditions arise because the mortgage payment m_t^* entering the budget constraint of the capital owner pertains to payments on the entire outstanding mortgage debt, not just the new loan. In this case, the terms related to $U_{\gamma t}$ and U_{Rt} capture the marginal effect of l_t^* on the average interest and amortization rates of the outstanding debt, and thus the marginal effect of l_t^* on the mortgage payments on the outstanding debt.

The wedge

Rearranging the first-order condition for x_{Ht} (A13) yields

$$v_{ct} p_{Ht} (1 + \tau_{Ht}) = \beta E_t V_{h,t+1},$$

where the wedge τ_{Ht} is given by

$$\tau_{Ht} \equiv -\theta E_t \left[1 + \beta \frac{\widetilde{V}_{d,t+1}}{v_{ct}} + \zeta_{Dt}(\kappa - \gamma_t^\alpha) \beta \frac{V_{\gamma,t+1}}{v_{ct}} + \zeta_{Dt}(i_{t+1}^M - R_t) \beta \frac{V_{R,t+1}}{v_{ct}} \right].$$

For the same reasons as above, the wedge is more complicated than in the case of the two-period mortgage. It becomes a straightforward infinite-horizon extension of equation (3) in Section 3 if the housing investment decision is once-and-for-all and there is no outstanding mortgage debt (i.e., $\zeta_{Dt} = 0$ and $\zeta_{l,t+j} = 0$, for $j = 1, 2, ...$):

$$\tau_{Ht} \equiv -\theta E_t \left\{ 1 - \left[Q_{1t} \left(i_{t+1}^M + \gamma_{t+1} \right) + Q_{2t} \left(i_{t+2}^M + \gamma_{t+2} \right) \left(1 - \gamma_{t+1} \right) + ... \right] \right\},$$

where

$$Q_{Jt} \equiv \prod_{j=1}^{J} \beta \frac{v_{c,t+j}}{v_{c,t+j-1}} \frac{1}{1 + \pi_{t+j}}.$$

Appendix B: Computation

Overview

The equilibrium is computed using a linear-quadratic (LQ) approximation method for distorted economies with exogenously heterogenous agents (see Hansen and Prescott, 1995), adjusted along the lines of Benigno and Woodford (2006). Further modification, described below, is made to handle refinancing.

In a nutshell, the method approximates the recursive competitive equilibrium problem of the original economy, specified in Section 4, with a corresponding linear-quadratic problem. The method involves iteration on quadratic Bellman equations (one for each agent type) subject to linear individual, aggregate, and market clearing constraints. The fixed point of the iteration is a pair of (quadratic) value functions for the homeowner and capital owner, $\widehat{P}_H(z, S)$ and $\widehat{P}_C(z, S)$, and a set of (linear) aggregate decisions rules and pricing functions, $\widehat{W}(z, S)$, where z is a vector of exogenous state variables and S is a vector of endogenous aggregate state variables, as specified in Section 4. This function is accompanied by a resulting set of linear laws of motion for the endogenous state variables, $S' = \widehat{\Omega}(S, z)$.

The centering point of the LQ approximation is the steady state, satisfying the equilibrium conditions listed in Appendix A, and the LQ approximation of the original Bellman equations is computed using numerical derivatives. All variables in the approximation are either in percentage deviations or percentage point deviations (for interest, inflation, and amortization rates) from the steady state. Before computing the equilibrium, the model is made stationary by expressing all nominal variables in real terms and replacing ratios of price levels with the inflation rate, as in Appendix A (as a result, p_t is replaced with π_t in the vector W_t, specified in Section 4, and p_{t-1} drops out of the vector z_t).

Return functions in Bellman equations

The Hansen-Prescott method needs to be modified along the lines of Benigno and Woodford (2006) because the laws of motion for the mortgage variables are nonlinear. This means that these equations cannot be substituted out into the per-period utility functions, as is normally done in LQ approximation procedures. The modification involves forming a Lagrangian for each agent, consisting of the per-period utility function and the respective laws of motion for the mortgage variables. The Lagrangian is then used as the return function in the Bellman equation being approximated. This adjustment is necessary to ensure that second-order cross-derivatives of the utility function and the constraints are taken into account in the LQ approximation. This modification, as applied to the homeowner, is described in detail by Kydland et al. (forthcoming). The specification for the capital owner is analogous. We therefore refer the reader to that paper for details.

An alternative procedure would be to log-linearize the equilibrium conditions in Appendix A and use a version of the Blanchard-Kahn method, or a method of undetermined coefficients, to arrive at the equilibrium decision rules and pricing functions. As is well known, these methods yields the same linear equilibrium decision rules and pricing functions as the modified LQ approximation method.

Certainty equivalence

The linear equilibrium decision rules and pricing functions computed as described above are characterized by certainty equivalence. That is, the coefficients of the linear functions $\widehat{W}(z, S)$ loading onto z and S depend only on the conditional means of future values of the exogenous state variables, implied by their AR(1) processes. Not on higher moments. One implication of the certainty equivalence is that the FRM rate in the approximated model conforms with the expectations hypothesis.

Computing business cycle moments and impulse-responses

The business cycle moments used to cross-validate the model are based on artificial data from 150 runs of the model. In a single run, a sequence of innovations $[\epsilon_{At}, \epsilon_{\pi t}, \epsilon_{bt}, \epsilon_{qt}]$, for $t = 1, ..., 150$, is drawn from their respective *iid* normal distributions. The AR(1) processes for the exogenous state variables are then used to generate a sequence of the vector z (for $t = 1, ..., 150$), which is then fed into the endogenous functions $\widehat{W}(z, S)$ and $S' = \widehat{\Omega}(S, z)$ to recursively generate a sequence of the endogenous variables of interest.

When computing impulse-responses, either to cross-validate the model or in the main computational experiments, all exogenous state variables except the one of our interest are held constant at their unconditional means. A path for the exogenous state variable of our interest is generated by setting its innovation equal to a particular value in period $t = 1$ and to zero in all subsequent periods. The path of the state variable is then generated using the variable's AR(1) process and this path is fed into the endogenous functions $\widehat{W}(z, S)$ and $S' = \widehat{\Omega}(S, z)$ to recursively generate a sequence of the endogenous variables. Because the model is stationary, after the initial impulse in $t = 1$ that moves the economy away from the steady state, all variables converge back to the steady state.

Handling refinancing

The computational method needs to be modified in the case of refinancing. The adjustments we introduce are inspired by the method proposed by Guerrieri and Iacoviello (2015). Their ideas are inset here in the general computation procedure described so far. Essentially, with refinancing, the economy can operate under two regimes: one in which $i_t^F \geq R_t$ and $\varrho_t = \varrho$ and another in which $i_t^F < R_t$ and $\varrho_t = \widetilde{\varrho}_t$. We therefore compute two sets of approximate equilibrium decision rules and pricing functions, $\widehat{W}_1(z, S)$ for the first regime and $\widehat{W}_2(z, S)$ for the second regime (and two sets of the corresponding laws of motion for S). The point of approximation is the steady state, which (as explained in Appendix A) is common to both regimes. Given that the economy can operate under two regimes, the endogenous linear functions $\widehat{W}_1(z, S)$ and $\widehat{W}_2(z, S)$ have to take into account the fact that the economy can switch between them. This is relatively straightforward to implement in the case of the experiments of Section 7. There are two reasons for this. First, after the initial decline, in response to the negative $\overline{\pi}_t$ shock, i_t^F converges monotonically back to the steady state. Second, from the law of motion for the interest rate on outstanding debt follows that i_t^F is a marginal rate of R_{t+1}. Therefore, if i_t^F falls below the steady state and converges back monotonically, i_t^F and R_t cross paths only once. That is, the economy switches from the regime $i_t^F < R_t$ (which occurs on impact of the shock) to the regime $i_t^F \geq R_t$ only once along

the convergence path. Therefore, once the economy is in the latter regime, it stays in it until it converges to the steady state. Further, the point in which it switches is deterministic. It occurs some N periods after the initial impact of the shock, where N is endogenous.

To compute the equilibrium decision rules and pricing functions for the impulse-responses under refinancing, we proceed in two steps. First, we compute the approximation $\widehat{W}_1(z, S)$ for the case $i_t^F \geq R_t$. This involves the usual procedure described above, as the economy stays in this regime once it is in it. Denote the pair of the fixed-point quadratic value functions (one for each agent type) that result from this step as $\widehat{P}_1(z, S)$. To compute the equilibrium decision rules and pricing functions for the case of $i_t^F < R_t$, we modify the usual procedure in two ways. First, we use as a staring point of the iterative procedure on the Bellman equations the pair of value functions $\widehat{P}_1(z, S)$. Second, we do not iterate until a fixed point, but only N times. After obtaining $\widehat{W}_1(z, S)$ and $\widehat{W}_2(z, S)$ this way, we generate impulse-response functions and check whether the N we have used in computing $\widehat{W}_2(z, S)$ is consistent with the resulting impulse-responses; i.e., whether i_t^F and R_t cross paths N periods after the shock. If not, we change N and repeat the steps until consistency is obtained. For the calibrated persistence of the shock, $N = 60$. It turns out that for N this far in the future, the responses in the first 40 periods we are looking at obtained for the two-regime economy are almost identical to the responses obtained under 'naive' decision rules and pricing functions that do not take into account the fact that the economy will switch to the second regime 60 periods after the shock (the naive decision rules and pricing functions are obtained as a fixed point of the usual iterative procedure assuming that the economy is always in the $i_t^F < R_t$ regime).

Appendix C: Data counterparts to variables

This appendix describes the data used to calculate the aggregate ratios employed in calibrating the model. Adjustments to official data are made to ensure that the data correspond conceptually more closely to the variables in the model. To start, for reasons discussed by Gomme and Rupert (2007), the following expenditure categories are taken out of GDP: gross housing value added, compensation of general government employees, and net exports. In addition, we also exclude expenditures on consumer durable goods (as our 'home capital' includes only housing) and multifamily structures (which are owned by corporate entities and rented out to households mainly in the 1st and 2nd quintiles of the wealth distribution). With these adjustments, the data counterparts to the expenditure components of output in the model are constructed from BEA's NIPA tables as follows: consumption (C) = the sum of expenditures on nondurable goods and services less gross housing value added; capital investment (X_K) = the sum of nonresidential structures, equipment & software, and the change in private inventories; housing structures (X_S) = residential gross fixed private investment less multifamily structures; and government expenditures (G) = the sum of government consumption expenditures and gross investment less compensation of general government employees. Our measure of output $(Y = C + X_K + X_S + G)$ accounts, on average (1958-2006), for 74% of GDP. BEA's Fixed Assets Tables and Census Bureau's M3 data provide stock counterparts to capital and housing investment: capital stock (K) = the sum of private nonresidential fixed assets and business inventories; housing stock (H) = residential assets

less 5+ unit properties.[58]

Appendix D: Estimation of mortgage debt servicing costs

A key measurement for calibrating the model concerns the mortgage debt servicing costs of homeowners. Unfortunately, such information for the United States is not readily available. Four different procedures are therefore used to arrive at an estimate. The four procedures exploit the notion that the homeowners in the model correspond to the 3rd and 4th quintiles of the U.S. wealth distribution. Some of these estimates arguably overestimate the debt servicing costs, while others underestimate it. Nevertheless, all four procedures yield estimates in the ballpark of 18.5% of pre-tax income, the value used to calibrate the model.

1. Estimate based on income from the Survey of Consumer Finances

The first procedure, for FRM (1972-2006) and ARM (1984-2006), combines data on income from the Survey of Consumer Finances (SCF) and the model's expression for debt servicing costs. Suppose that all mortgage debt is FRM.[59] The model's expression for steady-state debt-servicing costs, $(R+\gamma)D/(pwN - p\tau\Psi)$, can then be used to compute the average debt-servicing costs of homeowners. The various elements of this expression are mapped into data in the following way: $D/(pwN - p\tau\Psi)$ corresponds to the average ratio of mortgage debt (for 1-4 unit structures) to the combined personal income (annual, pre-tax) of the 3rd and 4th quintiles, equal to 1.56; R corresponds to the average FRM annual interest rate for a conventional 30-year mortgage, equal to 9.31%; and γ corresponds to the average amortization rate over the life of the mortgage, equal to 4.7% per annum. This yields debt-servicing costs of 22%. This estimate is likely an upper bound as some of the outstanding mortgage debt in the data is owed by the 5th quintile (the 1st and 2nd quintiles are essentially renters) and the effective interest rate on the stock in the data is likely lower than the average FRM rate due to refinancing. When all mortgage debt is assumed to be ARM, this procedure yields 17.5% (based on the average Treasury-indexed 1-year ARM rate for a conventional 30-year mortgage).

2. Estimate based on Financial Obligation Ratios

The second estimate is based on Federal Reserve's Financial Obligation Ratios (FOR) for mortgages (1980-2006). FOR report all payments on mortgage debt (mortgage payments, homeowner's insurance, and property taxes) as a fraction of NIPA's share of disposable income attributed to homeowners. For our purposes, the problem with these data is that members of the 5th quintile of the wealth distribution are also counted as homeowners in the data (as long as they own a home), even though they do not represent the typical

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[58]Separate stock data on 2-4 unit properties are not available, but based on completions data from the Census Bureau's Construction Survey, 2-4 unit properties make up only a tiny fraction of the multifamily housing stock.

[59]Federal Reserve's Flow of Funds Accounts provide data on mortgages and we equalize mortgage debt in the model (D) with the stock of home mortgages for 1-4 family properties. The Flow of Funds data, however, include mortgage debt issued for purchases of existing homes, second mortgages, and home equity loans. In contrast, the model speaks only to first mortgages on new housing. The data thus provide an upper bound for D in the model.

homeowner in the sense of Campbell and Cocco (2003). To correct for this, we apply the share of the aggregate SCF personal income, attributed to the union of the 3rd, 4th, and 5th quintiles of the wealth distribution, to aggregate disposable income from NIPA. This gives us an estimate of NIPA disposable income attributed to these three quintiles. This aggregate is then multiplied by the financial obligation ratio to arrive at a time series for total mortgage payments. Assuming again that all mortgage payments are made by the 3rd and 4th quintiles, the total mortgage payments are divided by NIPA personal (pre-tax) income attributed to just these two quintiles (calculated by applying the SCF shares). This procedure yields average debt-servicing costs of 20%.

3. Estimate based on wealth quintiles from the Survey of Consumer Finances

Third, we use the ratio of all debt payments to pre-tax family income for the 50-74.9 percentile of the wealth distribution, reported in SCF for 1989-2007. The average ratio is 19%. About 80% of the payments are classified as residential by the purpose of debt, yielding an average ratio of 15.2%. A key limitation of this procedure is that the data exclude the 1970s and most of the 1980s—periods that experienced almost twice as high mortgage interest rates, on average, than the period covered by the survey. Another issue is that the information reported in the survey is not exactly for the 3rd and 4th quintiles.

4. Estimate based on Consumer Expenditure Survey

The fourth procedure is based on the Consumer Expenditure Survey (CEX), 1984-2006. This survey reports the average income and mortgage payments (interest and amortization) of homeowners with a mortgage. To the extent that homeowners without a mortgage are likely to belong to the 5th quintile of the wealth distribution—they have 100% of equity in their home and thus have higher net worth than homeowners with a mortgage—the survey's homeowners with a mortgage should closely correspond to the notion of homeowners used in this paper (CEX does not contain data on wealth). The resulting average, for the available data period, for mortgage debt servicing costs of this group (pre-tax income) is 15%. Given that the data do not cover the period of high mortgage rates of the late 1970s and early 1980s, like the third estimate, this estimate probably also underestimates the debt servicing costs for the period used in calibrating the model.

Taken together, the four procedures lead us to use 18.5%, a value in the middle of the range of the estimates, as a target in calibration.