

# Leaning Against the Credit Cycle\*

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## Abstract

We study how monetary policy and household debt interact when mortgages are amortized gradually. Slow amortization implies the empirically observed debt persistence and debt-to-GDP swings driven by output and inflation, rather than shifts in current borrowing. Interest hikes only weakly influence household debt, *increasing* debt-to-GDP in the short run, and reducing it in the medium run. Policy rules that responds positively to the debt-to-GDP ratio induce equilibrium indeterminacy and greater volatility of debt itself. Relative to inflation targeting, leaning against debt-to-GDP swings calls for more *expansionary* policy when debt-to-GDP is *high*, and more *contractive* policy when debt-to-GDP is *low*.

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**JEL Classification:** E52, E32, E44.

## 1 Introduction

Credit typically moves in a gradual manner, as highlighted by several recent studies of the *credit cycle*, for instance Aikman, Haldane, and Nelson (2015) and Drehmann, Borio, and Tsatsaronis (2012). Moreover, the historical evolution of household leverage has largely been driven by

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variation in income growth, inflation and interest rates, rather than active changes in borrowing, as documented by Mason and Jayadev (2014). In contrast, macroeconomic models typically assume that households refinance their debt each period, as in the influential work of Iacoviello (2005), with the implication that the entire stock of debt responds swiftly to shocks and policy changes. This simplifying assumption might be useful and innocuous for many purposes, but cannot be relied upon in the current policy debate on *if* and *how* monetary policy should respond to movements in household debt.<sup>1</sup> The likely performance of such policies can only be evaluated within frameworks that realistically account for debt dynamics.

We therefore study monetary policy when debt is gradually amortized and only new loans are constrained by the current value of collateral. There are two specific stylized facts we aim to capture. First, debt is highly persistent. For instance, linearly detrended U.S. real household debt has a first-order autocorrelation coefficient above 0.99. Second, the contemporaneous cross-correlation between household debt and house prices is moderate, and debt lags house prices. Linearly detrended U.S. data reveals a contemporaneous correlation of 0.14 and a correlation between house prices and debt 5 quarters later of 0.22. Our overriding policy questions are: How is a monetary policy tightening likely to affect households' debt relative to GDP? What are the likely consequences of systematically increasing the interest rate when the debt-to-GDP ratio is high? And what characterizes a monetary policy that targets stability of the debt-to-GDP ratio?

Our approach builds on the amortization framework proposed by Kydland, Rupert, and Sustek (2012). Here the amortization rate of debt follows a process calibrated to match the properties of a standard mortgage contract. Importantly, this framework implies a distinction between new loans and pre-existing debt. We embed this debt specification into (i) a relatively simple calibrated DSGE-model with collateral constraints, akin to Monacelli (2009), Iacoviello (2005) and Campbell and Hercowitz (2005), and (ii) the fully-fledged DSGE-model on housing and the macroeconomy by Iacoviello and Neri (2010), which we re-estimate. Importantly, in our framework pre-existing debt is not constrained by current swings in collateral value, but instead follows a gradual amortization process. Only new loans are constrained by collateral. Hence, the evolution of household leverage at the aggregate level, measured as debt relative to GDP or relative to housing value, is decoupled from the evolution of new borrowing.

With gradual amortization, our models imply debt dynamics that are highly persistent. While there is feedback between debt and the macroeconomy via a collateral constraint on new loans, other macroeconomic variables than debt move faster, and revert considerably earlier to steady state than debt does after a shock. In this sense, we capture the coexistence of a

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<sup>1</sup>For instance, the role of household indebtedness in Swedish monetary policy has recently received much attention. See Financial Times, October 29, 2014: "Tactic of 'lean against the wind' has failed in Sweden", Financial Times, May 7, 2014: "Riksbank raises concern on household debt", Laséen and Strid (2013), and the multiple comments by Lars Svensson at <http://larseosvensson.se>.

low-frequency credit cycle together with a conventional business cycle, similar to what recent empirical studies, such as Drehmann, Borio, and Tsatsaronis (2012), emphasize. Moreover, our estimated model features dynamic cross-correlations between debt and house prices that are highly similar to what is seen in the data.

Because only new loans respond on impact, and these constitute a small fraction of total debt, monetary policy shocks cause only a moderate reduction of nominal debt. Hence, inflation and output may potentially respond faster, so that real debt and debt-to-GDP might in principle rise immediately after a monetary policy tightening. Within our estimated model, we find that this actually occurs for debt-to-GDP. However, as inflation and output return to steady state some time after the initial impulse, the debt-to-GDP ratio drops moderately below its steady state level, and stays low for a considerable period. Notably, this medium-run decline is influenced by the amortization process of debt. If mortgage debt is of the annuity loan type, the amortization rate increases over the lifetime of a mortgage. Hence, by reducing the share of new loans in the economy-wide stock of debt, a monetary tightening raises the average amortization rate in the economy. This effect propagates the extent to which debt falls after a monetary policy shock.

To address our main questions about how monetary policy should be designed, we first consider simple interest rate rules. We here find that it is detrimental to mechanically lean against the level of households' debt burden by raising interest rates.<sup>2</sup> A positive response coefficient on steady state-deviations of debt-to-GDP (or the real debt level) in the policy rule induces equilibrium indeterminacy. Notably, the opposite conclusion is reached from a 1-quarter debt model. It is because debt is gradually amortized and persistent, policy should not respond positively to it. When new loans constitute only a small fraction of the aggregate stock of debt, higher inflation expectations reduce the expected and, via contemporaneous inflation, current levels of real debt. Thus, a policy of systematically raising the interest rate when debt-to-GDP is high, indirectly induces a negative response of the interest rate to inflation. Consequently, expectations of higher inflation are likely to turn self-fulfilling. In order to curb this destabilizing influence of reacting to debt, the interest rate must react more strongly to inflation if indeterminacy is to be avoided. Moreover, the persistent nature of debt means that the required coefficient on inflation in the interest rate rule increases sharply with the debt coefficient.

Notably, by raising the interest rate when the debt-to-GDP ratio goes up, policy will destabilize not only inflation, but also debt itself. As one might expect then, for a policy rule with the debt-to-GDP to stabilize that ratio, the response coefficient needs to be *negative*. This holds regardless of whether the interest rate reacts to current or expected future debt, and regardless of whether the interest rate reacts to past interest rates or output growth as well as debt and

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<sup>2</sup>Within our model, we study interest rate reactions to debt in deviation from steady state. The real-world counterpart would naturally be deviations from trend.

inflation. In contrast, for a policy rule with debt *growth*, a positive response coefficient will stabilize debt-to-GDP. Not surprisingly, such a policy increases inflation volatility. If we compare the option of reacting negatively to debt-to-GDP to the alternative of responding positively to debt growth, the former reduces debt-to-GDP volatility more powerfully, but also has a stronger destabilizing effect on inflation.

Finally, we contrast inflation targeting to debt-to-GDP targeting. We here allow the policymaker to freely set the interest rate under commitment, in order to minimize a loss function with debt-to-GDP or inflation. A striking pattern emerges: Whenever a strict inflation targeting policy would imply a reduction in debt-to-GDP, the optimal policy of a debt-to-GDP targeter engineers more of an economic *contraction*. Vice versa, if debt-to-GDP rises under inflation targeting (or under a simple policy rule), optimal stabilization of debt-to-GDP engineers more of an economic *expansion*. These insights are the direct opposite of what is typically assumed in the debate on how monetary policy can contribute to containing households' debt burden.

Our study is closely related to the arguments put forth by Svensson (2013), who challenges the conventional view that tighter policy reduces households' debt burden. He combines estimates of how inflation, output and house prices respond to monetary policy shocks, with an accounting formula for debt dynamics. The key ingredient in the accounting formula is that mortgage contracts are assumed to be refinanced only infrequently. Our approach is different, as we study two models where all variables, including debt, are jointly determined. This allows us to move beyond monetary policy shocks, and study the interplay between debt dynamics and systematic monetary policy. Notably, both Svensson's exercise and our models imply that a monetary policy shock is likely to raise households' debt-to-GDP ratio in the short run. Our results diverge from Svensson's in the medium run, where our model implies that debt-to-GDP will fall. In a simultaneous paper to ours, Alpanda and Zubairy (2014) also distinguish between new loans and old debt. Consistently with our model, their impulse responses indicate that monetary policy shocks increase the debt-to-GDP ratio in the short run, but without the medium-run decline that we find, probably because they assume a constant amortization rate.

On the more empirical side, several recent papers have used vector auto regression (VAR) models to explore how monetary policy shocks affect debt-to-GDP. Bauer and Granziera (2016) study a panel of eighteen advanced countries, and find that the debt-to-GDP ratio rises on impact and then falls moderately in the medium run, as in our structural model. Robstad (2014) finds a similar pattern for Norway, utilizing a host of VAR models. In contrast, Laséen and Strid (2013) use a Bayesian VAR-model on Swedish data, and find that debt-to-GDP drops after a monetary policy shock. Importantly, one should bear in mind that our model's implications for systematic policy do *not* hinge on the short-run sign of the debt-to-GDP response. Rather, what matters is that debt is highly persistent, which is an indisputable feature of the data.

The distinction between new loans and existing debt is key in our analysis. Recent work

supports the importance of this distinction for understanding the debt dynamics in the data. Justiniano, Primiceri, and Tambalotti (2013) highlight that in the recent boom-bust episode of the US housing market, the aggregate ratio of debt over real estate value peaked several quarters after house prices started falling. A standard model where all debt is continuously re-adjusted, can only explain this pattern as the consequence of lending standards being loosened at the onset of the financial crisis. Such an interpretation of the housing bust seems misguided. In contrast, Gelain, Lansing, and Natvik (2015) show that once one takes into account that current collateral constraints primarily matter for new loans, the behavior of the ratio of debt relative to housing value no longer is at odds with a credit contraction at the onset of the housing bust. Within our estimated model in this paper, we find the same pattern.

Calza, Monacelli, and Stracca (2013), Garriga, Kydland, and Sustek (2013) and Rubio (2011) study the role of mortgage finance in the monetary transmission mechanism. All emphasize that monetary policy is likely to be less influential when fixed rate mortgages are prevalent. In our paper we ignore the issue of fixed versus flexible rate mortgages, and focus instead on the distinction between how pre-existing and new loans are affected by current borrowing constraints. This relates our paper to the work by Andrés, Arce, and Thomas (2014), who study structural reforms when households are overhung by debt. The key assumption they make, similarly to Justiniano, Primiceri, and Tambalotti (2013), is that households cannot be forced to deleverage faster than a given amortization rate, even if the collateral value of real estate falls faster than the amortization reduces debt. In our model, the counterpart to that assumption would be to impose a non-negativity constraint on new loans. This would most likely further dampen the extent to which a contractive monetary policy shock can reduce the debt level. Ultimately, all these modeling approaches are reduced form representations of households' liquidity management, aiming to avoid the curse of dimensionality that follows with a deeper modeling of household choice. Iacoviello and Pavan (2013) make substantive progress in modeling the lumpiness of housing purchases. Chen, Michaux, and Roussanov (2013) impressively incorporate the many details relevant for mortgage refinancing at a micro level, and succeed in accounting for the U.S. credit boom in the 2000s. However, these studies are conducted within partial equilibrium models, which treat house prices, inflation and GDP as exogenous, and therefore cannot address the questions we ask in this paper.

While our focus is on the household side, Gomes, Jermann, and Schmid (2014) offer a complementary view focusing on the firm side. They show how inflation movements affect investment and output if firms have long-term debt. Their key mechanism is that inflation shocks propagate through real leverage. Hence, they share our focus on Fisherian debt dynamics. Differently from our study, they consider a model without further nominal rigidities than nominal debt, so that inflation is stimulated by nominal interest rate shocks rather than vice versa. Moreover, they do not study policies that aim to stabilize debt and they do not focus on the

empirical performance of their long-term debt model relative to the short-term alternative.

The evidence that perhaps most convincingly points toward the need for distinguishing between new borrowing and existing debt, is the empirical decomposition of US household debt dynamics by Mason and Jayadev (2014). They account for how the “Fisher” factors of inflation, nominal income growth and nominal interest rates have contributed to the evolution of US debt-to-income, in addition to the changes in borrowing and lending, since 1929. Their findings show how the dynamics of debt-to-income have been strongly influenced by the Fisher factors, and often has gone in the opposite direction of households’ primary deficits.

In this paper we first develop a simple model that allows a transparent assessment of the interplay between household debt and monetary policy. Section 2 presents the simple model and Section 3 analyzes monetary policy in it. In Section 4 we move to a fully-fledged DSGE model suitable for estimation. We first embed gradual amortization into the framework of Iacoviello and Neri (2010) and estimate it on U.S. data, and thereafter explore if the monetary policy implications from the simple model hold here too. Section 5 concludes.

## 2 A Simple Model

We consider a standard New Keynesian model with household debt and collateral constraints. Our novelty is to allow for gradual amortization of the outstanding stock of mortgage debt, and correspondingly modify the borrowing constraint so that it applies to new loans only.

### 2.1 Households

There are two household types: *patient* (indexed by  $l$ ) and *impatient* ( $b$ ), of mass  $1 - n$  and  $n$ , respectively. Both derive utility from a flow of consumption  $c_{j,t}$  and services from housing  $h_{j,t}$ ,  $j = b, l$ . They derive disutility from labor,  $L_{j,t}$ . Each household maximizes:

$$E_t \sum_{t=0}^{\infty} \beta_j^t \left\{ \log(c_{j,t} - \gamma c_{j,t-1}) + \nu_h \log(h_{j,t}) - \nu_{l,j} \frac{L_{j,t}^{1+\varphi_L}}{1+\varphi_L} \right\}, \quad (1)$$

where  $\gamma$  measures habit formation in utility,  $\nu_h$  governs the utility from housing services,  $\nu_{j,L}$  governs the disutility of labor, and  $\varphi_L$  governs the elasticity of labor supply.<sup>3</sup>  $\beta_b$  and  $\beta_l$  are the households’ discount rates, with  $\beta_b < \beta_l$ . The total housing stock is fixed at 1, such that  $(1 - n)h_{l,t} + nh_{b,t} = 1$  for all  $t$ .

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<sup>3</sup>Habits are included for the model’s GDP movements to be reasonably sluggish, which in turn is important for debt-to-GDP dynamics. However, the policy conclusions drawn below do not hinge on the presence of habits.

## Impatient Households

Borrowers face the following budget constraint:

$$c_{b,t} + q_t h_{b,t} + \frac{r_{t-1} + \delta_{t-1}}{\pi_t} b_{b,t-1} = w_{b,t} L_{b,t} + q_t h_{b,t-1} + l_{b,t}, \quad (2)$$

where  $r_{t-1}$  is the net nominal interest rate at the end of period  $t-1$ ,  $\pi_t \equiv P_t/P_{t-1}$  is the gross inflation rate during period  $t$ ,  $w_t$  is the real wage,  $q_t$  is the real price of housing, and  $b_{b,t}$  is the borrower's real debt at the end of period  $t$ . Moreover,  $\delta_t$  is the amortization rate on existing debt, and  $l_{b,t}$  is the new borrowing incurred in period  $t$ . New borrowing and total debt are tied together through the law of motion for debt:

$$b_{b,t} = (1 - \delta_{t-1}) \frac{b_{b,t-1}}{\pi_t} + l_{b,t}. \quad (3)$$

The distinguishing feature of our analysis is to allow for  $\delta_t < 1$ . Our approach here follows Kydland, Rupert, and Sustek (2012), by specifying a process for the amortization rate that can be calibrated to match the profile of a typical annuity loan. The amortization process is:

$$\delta_t = \left(1 - \frac{l_{b,t}}{b_{b,t}}\right) (\delta_{t-1})^\alpha + \frac{l_{b,t}}{b_{b,t}} (1 - \alpha)^\kappa, \quad (4)$$

where  $\alpha \in [0, 1)$  and  $\kappa > 0$  are parameters, and  $l_{b,t}/b_{b,t}$  represents the share of new loans in the end-of-period outstanding stock of debt. When  $\alpha = 0$ , (4) and (3) imply  $\delta_t = 1$  and  $l_{b,t} = b_{b,t}$  for all  $t$ , such that we recover a 1-period mortgage contract where all outstanding debt is repaid each period. When  $\alpha > 0$ , the above law of motion captures how the amortization rate typically is low during the early years of a mortgage (when mortgage payments consist mainly of interest) and thereafter rises during later years as principal is repaid. Kydland, Rupert, and Sustek (2012) show that appropriate settings for the parameters  $\alpha$  and  $\kappa$  can approximately match the amortization schedule of a typical 30-year mortgage.

From (4) and (3), the amortization process can be expressed in terms of the debt stock:

$$\delta_t = (1 - \alpha)^\kappa + \frac{b_{b,t-1}}{\pi_t b_{b,t}} (1 - \delta_{t-1}) [\delta_{t-1}^\alpha - (1 - \alpha)^\kappa], \quad (5)$$

By combining (2) and (3), we arrive at the conventional formulation of the budget constraint:

$$c_{b,t} + q_t (h_{b,t} - h_{b,t-1}) = w_{b,t} L_{b,t} + b_{b,t} - \frac{R_{t-1}}{\pi_t} b_{b,t-1}, \quad (6)$$

where  $R_{t-1}$  is the gross nominal interest rate. Hence, the introduction of long-term debt does not change the nature of impatient households' budget constraint. Amortization still matters in our model, though, due the presence of a borrowing constraint. As in the literature on collateral constraints following Kiyotaki and Moore (1997) and Iacoviello (2005), we assume that debt is constrained by borrowers' housing wealth. However, because we allow for an amortization rate

below one, we must distinguish new loans  $l_t$  from the entire stock of debt  $b_t$  in the borrowing constraint. Logically, a large part of the economy's debt stock is given by decisions made in the past, and will not be directly influenced by the borrowing constraint today. Instead, the constraint in any given period affects only new loans  $l_{b,t}$ . The collateral constraint is then

$$l_{b,t} \leq m \left[ \frac{E_t [q_{t+1} \pi_{t+1}] h_{b,t}}{R_t} - b_{b,t} \right],$$

expressing that new loans cannot exceed a fraction  $m$  of households' net worth. Combined with the law of motion for debt in (3), and imposing that the constraint always binds, the constraint can be expressed in terms of the debt stock rather than new loans:<sup>4</sup>

$$b_{b,t} = \frac{m}{1+m} \frac{E_t [q_{t+1} \pi_{t+1}] h_{b,t}}{R_t} + \frac{1 - \delta_{t-1}}{1+m} \frac{b_{b,t-1}}{\pi_t}. \quad (7)$$

We see that if all debt is amortized quarterly, that is if  $\delta_t = 1$ , the constraint collapses to its conventional formulation where the current stock of debt is determined by collateral value only, with  $\frac{m}{1+m}$  as the loan-to-value ratio. In contrast, if debt is longer-lasting,  $\delta_t < 1$ , the current stock of debt is constrained by existing debt as well, as only the current period's loans are constrained by the current collateral value.

Impatient households maximize lifetime utility (1) subject to the law of motion for amortization (5), the budget constraint (6), and the borrowing constraint (7). The resultant first-order conditions for  $c_{b,t}$ ,  $L_{b,t}$ ,  $h_{b,t}$ ,  $b_{b,t}$ , and  $\delta_t$  are:

$$U_{c_{b,t}} = \lambda_t, \quad (8)$$

$$-U_{L_{b,t}} = U_{c_{b,t}} w_{b,t}, \quad (9)$$

$$U_{h_{b,t}} - U_{c_{b,t}} q_t + U_{c_{b,t}} \mu_t \frac{m}{1+m} \frac{E_t [q_{t+1} \pi_{t+1}]}{R_t} + \beta_b U_{c_{b,t+1}} q_{t+1} = 0, \quad (10)$$

$$U_{c_{b,t}} = \beta_b E_t \left[ \frac{U_{c_{b,t+1}}}{\pi_{t+1}} \right] R_t + U_{c_{b,t}} \mu_t - \beta_b E_t \left[ \frac{U_{c_{b,t+1}} \mu_{t+1}}{(1+m) \pi_{t+1}} \right] (1 - \delta_t) - U_{c_{b,t}} \eta_t \frac{b_{b,t-1}}{\pi_t b_{b,t}^2} (1 - \delta_{t-1}) [\delta_{t-1}^\alpha - (1 - \alpha)^\kappa] + \quad (11)$$

$$\beta_b E_t \left[ \frac{U_{c_{b,t+1}} \eta_{t+1}}{\pi_{t+1}} \right] \frac{1}{b_{b,t+1}} (1 - \delta_t) [\delta_t^\alpha - (1 - \alpha)^\kappa], \quad (12)$$

$$U_{c_{b,t}} \eta_t = \beta_b E_t \left[ \frac{U_{c_{b,t+1}} \mu_{t+1}}{(1+m) \pi_{t+1}} \right] b_{b,t} + \beta_b E_t \left[ \frac{U_{c_{b,t+1}} \eta_{t+1}}{\pi_{t+1} b_{b,t+1}} \right] b_{b,t} \left[ \alpha \delta_t^{(\alpha-1)} (1 - \delta_t) - \delta_t^\alpha + (1 - \alpha)^\kappa \right]. \quad (13)$$

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<sup>4</sup>Because  $\beta_b < \beta_l$ , the borrowing constraint binds in the non-stochastic steady state. In addition, we assume that the constraint holds always in the vicinity of the steady state that we shall explore. As is well-known, this assumption can be rationalized as long as the difference between  $\beta_b$  and  $\beta_l$  is sufficiently high relative to the volatility of the shocks considered. The gap between  $\beta_b$  and  $\beta_l$  in itself has no substantial influence on our results.



Here  $\lambda_t$ ,  $\mu_t$ , and  $\eta_t$  are the Lagrange multipliers associated with the budget constraint (6), the borrowing constraint (7) and the law of motion for amortization (5).

## Patient Households

Patient households lend to the borrowers. They also choose how much to consume, work, and invest in housing. From firms they receive profits,  $Div_t$ . Their budget constraint reads:

$$c_{l,t} + q_t(h_{l,t} - h_{l,t-1}) + \frac{b_{l,t-1}R_{t-1}}{\pi_t} = b_{l,t} + w_{l,t}L_{l,t} + Div_t, \quad (14)$$

where  $(1-n)b_{l,t-1} = -nb_{b,t-1}$ .

Patient household's optimal choices are characterized by the first-order conditions:

$$-U_{L_{l,t}} = U_{c_{l,t}}w_t, \quad (15)$$

$$U_{c_{l,t}} = \beta_l R_t E_t \left[ \frac{U_{c_{l,t+1}}}{\pi_{t+1}} \right], \quad (16)$$

$$U_{c_{l,t}} q_t = U_{h_{l,t}} + \beta_1 E_t [U_{c_{l,t+1}} q_{t+1}], \quad (17)$$

## 2.2 Firms and Price Setting

Firms are owned by the patient households. A sector with perfect competition and flexible prices produce  $y_t$  with the technology  $y_t = \left[ \int_0^1 y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$ , where  $i \in [0, 1]$ . The inputs are a continuum of intermediate goods  $y_t(i)$  and  $\varepsilon > 1$  is the constant elasticity of substitution between these goods. Cost minimization implies that demand for each intermediate good is  $y_t(i) = [P_t(i)/P_t]^{-\varepsilon} y_t$ , where the price index for intermediates is  $P_t = \left[ \int_0^1 P_t(i)^{1-\varepsilon} di \right]^{1/(1-\varepsilon)}$ .

Intermediate goods firms are monopolistically competitive and use labor only:

$$y_t(i) = \exp(z_t) L_t(i)^{1-\xi},$$

where  $z_t$  is an AR(1) productivity shock with autocorrelation coefficient  $\rho_z$  and standard deviation  $\sigma_z$ , and  $L_t(i) = (nL_{b,t}(i))^{\varpi} ((1-n)L_{l,t}(i))^{1-\varpi}$ .

Each period an intermediate firm may reset its price with probability  $1 - \theta$ , as in Calvo (1983), otherwise the price is partially indexed to past inflation with the degree of indexation governed by  $v \in (0, 1)$ , as in Smets and Wouters (2003).<sup>5</sup> If  $v$  is 1, prices are fully indexed to past inflation, and if  $v$  is 0, there is no indexation. With zero steady state inflation, the log-linear Phillips curve reads:

$$\hat{\pi}_t = \frac{\beta}{1 + \beta v} E_t \hat{\pi}_t + \frac{v}{1 + \beta v} \hat{\pi}_{t-1} + \frac{(1 - \beta\theta)(1 - \theta)}{(1 + \beta v)\theta} (\widehat{mc}_t), \quad (18)$$

where  $\hat{\pi}_t$  is inflation and  $\widehat{mc}_t$  is marginal costs in log deviations from steady state.

<sup>5</sup>Price indexation is included for the model's inflation movements to be reasonably sluggish. None of the monetary policy conclusions hinge on the existence of price indexation.

### 2.3 Monetary Policy

As a baseline, we assume that the central bank follows the simple rule

$$R_t = R_{t-1}^{\phi_R} \left[ R^{ss} \pi_t^{\phi_\pi} \right]^{1-\phi_R} \varepsilon_t, \quad (19)$$

where  $R_t$  is the gross nominal interest rate,  $R^{ss} = 1/\beta_1$  is the steady-state real interest rate, and  $\varepsilon_t$  is an i.i.d. monetary policy shock.

### 2.4 Parameter Values

We choose the values of  $n$ ,  $\nu_{l,l}$ ,  $\nu_{l,b}$  and  $\varpi$  so that the model's steady state matches key statistics of the relatively stable 1990s' U.S. economy, documented in Justiniano, Primiceri, and Tambalotti (2013) (JPT). The fraction of borrowers,  $n$ , is set to 0.61, the share of liquidity constrained households reported by JPT. The preference weights  $\nu_{l,l} = 0.106$  and  $\nu_{l,b} = 0.222$  so that borrower households work 1.08 times more than lenders. The labor share parameter  $\varpi$  is 0.5. so that the ratio of borrowers' to lenders' labor income is 0.64.

The housing preference weight,  $\nu_h$ , is chosen to match a ratio of housing wealth to yearly consumption of 2, consistently with Iacoviello and Neri (2010), implying  $\nu_h = 0.084$ .<sup>6</sup> The parameters governing the amortization process,  $\alpha$  and  $\kappa$ , are set so as to minimize the distance between the steady state profiles of amortization and interest rate payments implied by equation (4), and the same profiles implied by an actual 30-year, flexible interest rate, annuity loan contract, as in Kydland, Rupert, and Sustek (2012). This leads to  $\alpha = 0.9959$  and  $\kappa = 1.0487$ . When we later vary the duration of the loan contract, we necessarily recalibrate these parameters with the same procedure. Given the consequent steady state amortization rate, the value of  $m$ , which controls the tightness of the borrowing constraint, is set so that the steady state aggregate loan-to-value ratio of the economy,  $\tilde{m} \equiv \frac{Rb_b}{qh_b} = \frac{m}{m+\delta}$ , equals 0.5, which is its approximate average U.S. value after 1960. As result,  $m = 0.045$

The remaining parameters are simply set to conventional levels in the existing literature. As in Iacoviello and Neri (2010), patient households' discount factor,  $\beta_l$ , is 0.9925, consistently with a 3% annual interest rate, while  $\beta_b = 0.97$ . The labor supply elasticity parameter  $\varphi_L$  is set to 1, while the labor elasticity of production,  $1 - \xi$ , is 0.67. The price adjustment probability  $\theta$  is set 0.75 so that prices change on average once a year. The elasticity of substitution between goods,  $\varepsilon$ , is set to 6, implying a 20% steady state markup. In the monetary policy rule, the weights on the lagged interest rate and inflation,  $\phi_R$  and  $\phi_\pi$ , are 0.75 and 1.5. The price indexation and the habit parameters,  $v$  and  $\gamma$  are both set to 0.5. Finally, the technology shock process has an AR-coefficient,  $\rho_z$ , of 0.95 and a standard deviation,  $\sigma_z$  of 0.0124, chosen so that the model

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<sup>6</sup>Both Iacoviello and Neri (2010) and Justiniano, Primiceri, and Tambalotti (2013) calibrate  $\nu_h$  to match housing wealth relative to output ratio. Because there is no capital accumulation in our model, we instead target housing wealth relative to consumption, in similar spirit as Campbell and Hercowitz (2005).

matches the standard deviation of GDP growth over the period 1960 – 2012 when driven by technology shocks alone.

Unless otherwise stated, we consider a first-order Taylor-approximation of the model, linearized around its steady state.

### 3 Debt Dynamics and Monetary Policy

What does gradual amortization imply for a monetary policy authority concerned about household debt? We address this question from three different angles. First, we explore how a monetary tightening is likely to affect debt-to-GDP. Second, we study the properties of simple policy rules with a weight on credit. Third, we characterize a monetary policy that targets debt-to-GDP stability rather than inflation.

#### 3.1 Does an Interest Rate Hike Reduce the Debt Burden?

Figures 1 and 2 disentangle the impulse responses to a monetary policy shock in our model. We see that inflation, output and house prices are largely unaffected by the speed of amortization. However, the dynamics of the debt burden, measured either as the stock of real debt or as debt-to-GDP, are starkly affected. With one-quarter debt, real debt and debt-to-GDP fall markedly on impact and thereafter return gradually to their steady state levels. Qualitatively, this behavior is consistent with the conventional view that a policy tightening will reduce the debt burden. In contrast, with a 30-year amortization process the effect of monetary policy is muted, and the debt burden displays a hump-shaped *increase* before decreasing, as best seen in Figure 2. On impact, debt-to-GDP hardly moves, before it rises to a peak response after about a year. It thereafter falls gradually, reaches its steady state level after approximately two years, and then drops further and stays low for an extended period. Figure 2 shows that real debt and debt-to-GDP stay moderately below their state levels for approximately thirty years, reaching a trough around 0.4 percent below steady state after about ten years.

What explains the dynamics of debt under the 30-year amortization profile? Here it is useful to consider the responses of inflation and GDP. With 30-year amortization, debt becomes highly persistent, as revealed by equation (7). Hence, on impact real debt and debt-to-GDP are both largely driven by the responses of inflation and GDP, respectively. The fall in these two variables increase the debt burden. However, since house prices decline as well, fewer new loans will be issued, as seen in the lower right panel of Figure 1. Because the initial drop in house prices is relatively strong, this force counteracts the influence of reduced inflation and output. As house prices revert faster than output and inflation, the debt burden gradually builds up. The peak response of debt is reached when house prices are back to steady state. Thereafter, as inflation and output revert to steady state, the debt burden also falls. However, after the

effects on the other variables have died out, debt-to-GDP keeps falling. The reason is the initial contraction in new loans. This contraction, although modest enough to be dominated by output and inflation dynamics in the short run, has long-lived effects due to the long-term nature of debt. Moreover, because households have annuity loans, the aggregate amortization rate,  $\delta_t$ , rises as new loans constitute a declining fraction of the total debt stock. Hence, when the other macroeconomic variables have settled down at steady state, the initial contraction in new loans causes a persistent fall in the aggregate debt burden.

How important are the dynamics of the amortization rate for these results? Figure 2 gives an answer, comparing the benchmark model with annuity loans, to one where  $\delta$  is constant at the same steady state level as in the benchmark. We see that the amortization dynamics are unimportant for the initial increase in debt, but shape the subsequent decline. The reason is that with annuity loans, amortization is low for new loans. Hence, as the monetary tightening reduces new loans, the total stock of debt becomes “older”, and the aggregate amortization rate increases which brings the total debt stock down. With constant amortization, the debt stock hardly falls below its steady state level even in the medium run.

While not reported here, the shorter is the duration of debt, the smaller and more short-lived is the initial increase, and the greater is the subsequent decline, of debt-to GDP. With all the other parameters left unchanged, debt-to-GDP responds negatively on impact for amortization schemes shorter than 5 years.<sup>7</sup>

With regard to the question of how monetary policy affects the aggregate debt burden, we thus see that the answer depends on the horizon one has in mind. Consistently with the back-of-the-envelope calculation of Svensson (2013), and in contrast to the conventional view, our model implies that tighter policy increases the debt burden in the short run. In the intermediate run, though, monetary tightening is likely to cause a mild, but prolonged reduction of the debt burden, more in line with the conventional view on how debt is affected by monetary tightening. The extent to which this reduction comes about, is heavily influenced by the prevalence of annuity loan contracts in the economy. More generally, and more importantly, the effect of monetary policy on debt is muted and persistent. By assuming that all debt is re-financed and subject to the prevailing tightness of collateral constraints, which existing macroeconomic models typically do, one is likely to greatly exaggerate the extent and speed with which monetary policy affects the debt level.

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<sup>7</sup>Results for shorter amortization schemes, a nominal decomposition and further details on how a monetary policy shock affects debt in our model, are provided in the working paper version of this study, available at <http://www.norges-bank.no/en/Published/Papers/Working-Papers/2015/42015/>.

### 3.2 Debt and Simple Policy Rules

We now turn to the question of how systematic monetary policy reactions to credit swings might affect the economy. To address this issue, we study the consequences of responding to debt via a simple interest rate rule of the type

$$R_t = \pi_t^{\phi_\pi} \left( \frac{b_t/y_t}{b/y} \right)^{\phi_{b/y}} . \quad (20)$$

#### Reacting to the Debt Burden and Equilibrium Determinacy

A fundamental guideline for systematic monetary policy is the “Taylor principle” (Woodford (2001)). The Taylor principle states that the nominal interest rate must react more than one-for-one to changes in inflation. If this principle is not satisfied, expectations of higher inflation might turn self-fulfilling and induce macroeconomic fluctuations, as increased inflation expectations lower the path of real interest rates and boost demand. Should monetary policy also respond to other variables, the critical coefficient on inflation might well vary, so as to ensure that the ultimate response to inflation is greater than one. For instance, several studies have explored how the joint response to output and inflation together determine the scope for equilibrium (in)determinacy, with Bullard and Mitra (2002) as a prominent example. In the current policy debate, it is natural to ask how systematic responses to debt-to-GDP in addition to inflation, alter the scope for equilibrium indeterminacy.

The upper-left panel in Figure 3 plots the determinacy region in the  $(\phi_\pi, \phi_{b/y})$ -space when debt is fully amortized every quarter. With  $\phi_{b/y} = 0$ , the critical value for the inflation coefficient is one, consistently with the Taylor principle. If policy starts responding positively to increases in debt-to-GDP, the required inflation coefficient falls moderately. Hence, in terms of ensuring equilibrium determinacy, responding to inflation and responding to the debt-to-GDP ratio are substitutes. To understand why, consider the effects of an increase in inflation expectations unjustified by fundamentals. All else equal, the real interest rate drops, while the forward-looking Phillips curve implies that current inflation increases. The borrowing constraint (7) with  $\delta_t = 1$  then implies that the real debt level increases. Hence, debt moves in the same direction as inflation, and a positive coefficient on debt-to-GDP has similar stabilizing properties as a positive response to inflation.

The remaining three panels in Figure 3 plot the determinacy region if amortization is gradual, under 10-, 20- and 30-year amortization profiles respectively. Now the relationship between the threshold inflation response and the debt-to-GDP reaction is increasing. Intuitively, this occurs because an expectations driven increase in activity no longer moves the stock of real debt in the same direction as inflation. Instead, if inflation expectations rise without fundamental justification, the inflationary pressure this generates will reduce the stock of real debt. A positive

value of  $\phi_{b/y}$  will then push the nominal interest rate *down*, making the real interest rate fall even further. To counteract this destabilizing force, the response to inflation,  $\phi_\pi$ , must be greater than if debt were not reacted to. The relationship between the critical values of  $\phi_\pi$  and  $\phi_{b/y}$  is steeper, the longer is the horizon over which household debt is amortized.

The relationship between  $\phi_{b/y}$  and  $\phi_\pi$  in Figure 3 is steep due to medium-run debt dynamics. A moderate increase in the real interest rate will induce a reduction of debt in the future, as we saw in the analysis of monetary policy shocks in Figure 1. With a positive value of  $\phi_{b/y}$ , this implies a reduction of the future interest rate, which in itself tends to support a sunspot induced increase of inflation expectations. Thus, responding positively to the debt-to-GDP ratio is destabilizing for two reasons: (i) in the short run the real debt level falls when inflation increases, and (ii) in the medium-run debt-to-GDP falls if the current real interest rate increases.<sup>8</sup>

Reflecting the two forces at play here, the upper-right panel of Figure 3 shows that with 10-year debt, there is a narrow intermediate region of  $(\phi_\pi, \phi_{b/y})$ -combinations over which the equilibrium is determinate. For instance, if  $\phi_{b/y} = 0.1$ , the panels shows that when  $\phi_\pi$  is around 1.2, the equilibrium is determinate, while a slightly higher response to inflation, say  $\phi_\pi = 1.3$ , causes indeterminacy again. The knife-edge region with determinacy is one where the inflation coefficient  $\phi_\pi$  is barely big enough to compensate for the fact that a positive reaction to debt implies a negative contemporaneous response to inflation, and barely small enough to avoid causing a substantive medium-run decline in mortgage debt. This intermediate region is similar to that which can arise when there is investment in productive capital, as emphasized by Benhabib and Eusepi (2005), Carlstrom and Fuerst (2005) and Sveen and Weinke (2005).

### Debt and Inflation Volatility when Monetary Policy Reacts to Debt

Equilibrium determinacy aside, how should monetary policy respond to debt-to-GDP in order to stabilize this ratio? We address this question by considering how the volatility of debt-to-GDP and inflation changes with alternative response coefficients  $\phi_{b/y}$  in (20), holding  $\phi_\pi$  constant at 1.5.

The results from this exercise are displayed in Figure 4. In the upper left plot, we see that with 1-quarter debt, conventional wisdom applies, as debt-to-GDP is stabilized when monetary policy responds positively to its movements. Negative responses induce volatility and eventually indeterminacy. In contrast, with 30-year debt the pattern is the opposite: The standard deviation of debt-to-GDP increases with the coefficient on debt in the interest rate rule. Coefficients above zero induce indeterminacy, as we have seen before. If stability of debt-to-GDP is the objective, a negative coefficient on debt-to-GDP should be applied. Moreover, the lower plot

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<sup>8</sup>Consistently with the intuitions provided above, the effects of reacting to the real debt level are similar to those that arise from reacting to the debt-to-GDP ratio. Results are found in the working paper at <http://www.norges-bank.no/en/Published/Papers/Working-Papers/2015/42015/>

shows that such a policy increases inflation variance somewhat. The quantitative magnitude of the tradeoffs here are better evaluated in the estimated model that follows below.

The upshot is straightforward, and contradicts what follows from an analysis that ignores the long-term nature of household debt: If monetary policy aims to stabilize households' debt-to-GDP ratio, the interest rate should be *cut* when the debt-to-GDP ratio is *high*.

### 3.3 Inflation vs Debt Stabilization

Rather than restricting monetary policy to follow a simple rule, an alternative is to specify the Central Bank's objectives and let it freely choose the interest rate that achieves these objectives best, as advocated by for instance Svensson (2003). We here follow this approach, and study monetary policy as the minimization of the following loss function

$$L_t = \sum_{j=0}^{\infty} \beta_t^j \left[ (1 - \Gamma) \left( (1 - \lambda_y) \pi_{t+j}^2 + \lambda_y \left( \frac{y_{t+j}}{y_{t+j}^f} \right)^2 \right) + \Gamma \left( \frac{b_{b,t+j}/y_{t+j}}{b_b/y} \right)^2 \right]. \quad (21)$$

$\Gamma$  is the Central Bank's weight on stabilizing debt-to-GDP rather than inflation and the output gap, defined as deviations from the output level that would arise if prices were flexible,  $y_t^f$ .

We first characterize the properties of a policy that stabilizes debt rather than inflation, holding concerns for output at zero. To this end, Figures 5 and 6 display impulse responses to a technology shock when policy minimizes (21) under 1-quarter and 30-year debt, respectively. Again we see that the implications for debt-stabilization policy are radically different under the alternative debt durations.

With 30-year debt, strict inflation targeting ( $\Gamma = 0$ ) is associated with a short term fall and a medium run increase in the debt-to-GDP ratio. To prevent these movements, the optimal policy to stabilize debt-to-GDP contracts output and inflation more in the short run, and stimulates them more in the medium run, than under inflation targeting. In a nutshell, as compared to strict inflation targeting, debt-to-GDP targeting implies a more *contractive* policy when debt-to-GDP would otherwise be *low* and a more *expansionary* policy when debt-to-GDP would otherwise be *high*. This is directly opposite to what occurs under 1-quarter debt, where the debt targeting Central Bank engineers a lower path of inflation and output than an inflation targeter, in order to *increase* the debt-to-GDP ratio. Intuitively, under long-term debt the policymaker relies on the "Fisherian" forces of inflation and output dynamics to stabilize the debt-GDP-ratio.

The degree to which a higher weight on debt-to-GDP ( $\Gamma$ ) worsens performance in terms of the conventional monetary policy objective in (21),  $L_1 = (1 - \lambda_y)var(\pi) + \lambda_y var(y_t/y^f)$ , is illustrated in the upper two panels of Figure 7. In each plot, three different scenarios are considered, differing by the weight  $\lambda_y$  on the output gap in the loss function.<sup>9</sup> In the upper

<sup>9</sup>As the weight on output increases,  $\lambda_y$  increases both in the loss function under which the model is solved, and in the computation of  $L_1$ .

left plot,  $\Gamma$  varies from 0.01 to 1, whereas the right plot considers the range from 0 to 1. The purpose with this distinction follows from the plots themselves: if  $\Gamma = 0$ , the debt-to-GDP ratio is extremely volatile, consistently with Figure 6, and hence only a slight preference for stabilizing this ratio implies substantive stabilization of its squared deviations from steady state. More generally, the two plots show that moderate stabilization of debt-to-GDP comes at small costs in terms of output and inflation movements, as the frontiers in the left panel are relatively flat until the debt variance reaches an intermediate level. When the variance of debt-to-GDP is pushed toward zero,  $L_1$  increases sharply. Finally, we see that the more output is weighted in addition to inflation in the loss function, the smaller is the  $L_1$  associated with a given variance of debt-to-GDP. Vice versa, debt-to-GDP is less volatile when output receives weight in addition to inflation.

Finally, the bottom two plots of Figure 7 show the welfare consequences of increasing the emphasis on debt-to-GDP. The metric displayed is the percentage change in non-stochastic steady state consumption that makes borrowers and lenders equally well off as in a regime with  $\Gamma = 0$ .<sup>10</sup> The magnitudes here are moderate, as they typically are when assessing the merits of stabilization policy. For the lenders, we see that their welfare is monotonically decreasing in  $\Gamma$ . For borrowers, the relationship is non-monotonic, as it first increases when  $\Gamma$  increases from zero. This initial increase is related to the pattern in the upper right plot: When  $\Gamma = 0$ , debt-to-GDP is highly volatile, and for borrowers this will be associated with volatile consumption too, due to their budget constraint. For less extreme volatility of debt, however, the usual costs in terms of inefficient swings in activity and price dispersion due to inflation dominate, and also borrower welfare declines. Hence, in this environment, a moderate concern for debt-to-GDP stability, may be advisable on the grounds that it benefits the borrowing constrained part of the population. Yet, one should then carry in mind that such a concern implies more expansionary policy when debt-to-GDP is high, and less expansionary policy when debt-to-GDP is low, in contrast to what is typically advocated in the policy debate.

## 4 Evidence From A Medium Scale Estimated Model

It is natural to question whether the results above are specific to our simple model and parameterization, and how quantitatively relevant the key mechanisms highlighted actually are. We therefore incorporate our multi-period debt framework into a medium scale dynamic stochastic general equilibrium model, estimate it, and re-evaluate the main policy implications from above.

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<sup>10</sup>Welfare is here computed by using a second-order approximation of the model, accounting for the inflation costs of inefficient price dispersion. The reported metric of welfare difference, is the value of  $\lambda$  such that  $\sum_{t=0}^{\infty} \beta_j^t [u(c_{j,t}^{\Gamma=0}, h_{j,t}^{\Gamma=0}, n_{j,t}^{\Gamma=0}) - u(c^{ss}, h^{ss}, n^{ss})] = \sum_{t=0}^{\infty} \beta_j^t [u(c_{j,t}^{\Gamma=1}, h_{j,t}^{\Gamma=1}, n_{j,t}^{\Gamma=1}) - u((1+\lambda)c^{ss}, h^{ss}, n^{ss})]$ , where the superscripts  $\Gamma = 0$  and  $\Gamma = 1$  index outcomes under inflation and debt-to-GDP targeting respectively, and the superscript *ss* denotes the non-stochastic steady state.



## 4.1 Model and Empirical Approach

We build on the model developed and estimated by Iacoviello and Neri (2010) (IN, hereafter). This is a well-documented quantitative model on the inter-linkages between housing and the macroeconomy, originally with the assumption that all debt is amortized quarterly, which will facilitate transparency about the role played by multiperiod debt. Because our model and approach to estimating it are direct extensions of the work by IN, we limit ourselves to a brief exposition of what is common with IN, and highlight the extensions we make.

### The Model

As in our simple model, some households are patient and lend, while others are impatient and borrow subject to a collateral constraint tied to housing value. All households work, consume, and accumulate housing. In addition, patient households own the productive capital of the economy, and lend to firms and the impatient households. On the supply side there are two sectors. A housing sector uses capital, labor and land to produce new homes. A non-housing sector uses capital and labor to produce consumption and new business capital.<sup>11</sup>

There are 10 structural shocks, affecting intertemporal preferences, labor supply, housing preferences, nonhousing sector productivity, housing sector productivity, investment specific technology, final good costs, the inflation target, the monetary policy rate, and lending standards.

Relative to IN's original framework there are two extensions. First, we add long term debt, which gives the exact same the model block for impatient households as in our simple model described in Section 2. Borrowers' collateral constraint is given by (7), their first-order conditions are (10) and (13), and the amortization rate follows (5). Second, for estimation purposes explained below, we introduce a stochastic process for lending standards,  $m_t$ :

$$\ln m_t = (1 - \rho_m) \ln m + \rho_m \ln m_{t-1} + u_t^m, \quad (22)$$

where  $u_t^m$  is an independently and identically distributed innovation, with zero mean and standard deviation  $\sigma_m$ .

### Data and Empirical Strategy

The model is estimated with a Bayesian approach using 11 U.S. time series: real consumption, real residential investments, real business investments, real house prices, nominal interest rates, inflation, hours and wage inflation in the consumption sector, hours and wage inflation in the housing sector and household debt. Data sources and transformations are reported in Appendix B, and graphically displayed in an online appendix.

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<sup>11</sup>All agents' maximization problems are described in Appendix A.

Of the 11 observables, our only addition relative to IN is household debt. It is this addition that motivates our aforementioned introduction of lending standard shocks.

We update IN’s data set to span the period 1965q1 to 2014q1, rather than ending in 2006q4. New data have been revised, and business and housing investment series are not available for the full period. Hence there are some minor differences, but as shown in online Appendix F they do not affect the parameter estimates notably.

A further difference between our and IN’s approach is that we detrend non-stationary variables, each with its own linear trend, prior to estimation.<sup>12</sup> The alternative would be to formulate the theoretical model in stationary balanced growth terms and estimate trend parameters together with all the other structural parameters of the model. Our strategy is motivated by the fact that a theoretical balanced growth formulation of our model would assume that household debt and consumption grow at the same rate over time, which clearly does not hold in the data. This observation is beyond the scope of our paper to explain. Hence, we instead remove trends prior to estimation and do not include growth in the model. Iacoviello (2015) adopts the same strategy for the exact same reason.

Several parameters are calibrated, and we impose priors on those that are estimated. We here follow IN wherever possible. Calibrated parameter values are reported in Table 1 and all the prior distributions are reported in Tables 2 and 3. For the parameters that govern debt dynamics,  $m$ ,  $\alpha$  and  $\kappa$ , we necessarily depart from IN. Our approach here rests on two building blocks. First, we introduce the auxiliary parameter  $\chi = \frac{1}{1+m}$ . To see why, recall equation (7). We see that  $\chi$  would enter here as an autoregressive coefficient on lagged debt net of amortization. Second, we impose that the steady state rate of household debt to real estate,  $\tilde{m} \equiv \frac{Rb_b}{qh_b} = \frac{m}{m+\delta}$ , equals 0.5, which approximately is its mean value in our sample.<sup>13</sup> Our approach is to estimate the value of  $\chi$ , and thereby the value of  $m$  and thus  $\delta$  via the steady state restriction. Note that the values of  $\alpha$  and  $\kappa$  that govern the amortization dynamics, are then determined by the steady state value of  $\delta$ . Finally, as with all the other estimated parameters, we impose a prior distribution on  $\chi$ . Based on the studies emphasizing that credit moves slowly, such as Drehmann, Borio, and Tsatsaronis (2012), and the high observed persistence of debt in our data, we choose a normal distribution with mean 0.95.<sup>14</sup>

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<sup>12</sup>We treat stationary variables in the same way as IN.

<sup>13</sup>In our framework, there will generally be a distinction between the aggregate ratio of household debt to real estate value and the loan-to-value ratio on new loans. This distinction disappears in the special case where all debt is refinanced each quarter.

<sup>14</sup>A further technical complication arises because we impose that debt durations must be of an integer number of quarters. Hence, we consider only a discrete grid of possible values for  $\delta$ ,  $m$  and  $\chi$ . To implement this discretization, any draw of  $\chi$  from its continuous prior is translated into the closest value of  $\chi$  in the discrete set imposed.

## 4.2 Estimation Results and Model Properties

We estimate two versions of our model. A baseline version following the approach outlined above, and a comparison version where debt is restricted to be fully amortized each quarter.

The median of the posterior distributions of the estimated parameters for the two models, and their 90% High Posterior Density (HPD) regions, are reported in Tables 2 and 3.<sup>15</sup> Most of the estimated parameters are very similar in the two estimated versions of the model and have overlapping HPD regions.<sup>16</sup> Notably, the estimated posterior median for  $\chi$  is 0.97 in the model that allows for long-term debt. This implies a mortgage duration of 73 quarters, approximately 18 years, with a steady state amortization rate of 0.03 per quarter. The other main difference between the two models is the autoregressive coefficient,  $\rho_m$ , in the process for lending standards. With long-term debt, the posterior median is 0.73, while in the one-quarter model it is 0.98. The HPD regions do not overlap. This is a straightforward reflection of how the one-quarter debt model lacks intrinsic debt persistence, in contrast to the framework with long-term debt.

Table 2 reports the log data density for both estimated models. The 73-quarter model fits the data considerably better than the one-quarter model: the log data density of the former is 6418.04, while it is 6128.12 for the latter. These imply a logarithmic Bayes factor of 289.92 which is sufficiently high to conclude that there is a decisive evidence in favor of the 73-quarter debt model by the criteria of for instance Kass and Raftery (1995).<sup>17</sup>

Why is the fit substantially improved? The answer lies in certain key moments of real household debt. The top panels of Figure 8 show the first to the fifth order autocorrelations of linearly detrended real household debt in the data compared to the 5th and 95th percentiles derived from model simulations.<sup>18</sup> They highlight that the 73-quarter debt model matches the autocorrelation structure of real household debt extremely well, thus capturing the high-persistent nature of credit as emphasized by recent studies of the credit cycle, such as Drehmann, Borio, and Tsatsaronis (2012). In contrast, the one-quarter debt model fails in this dimension, implying too low persistence.

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<sup>15</sup>Draws from the posterior distribution of the parameters are obtained using the random walk Metropolis algorithm. We sampled 1000000 draws and discarded half of them before computing posterior statistics. Details on prior and posterior distributions and on the convergence properties of our estimations are reported in online Appendix C.

<sup>16</sup>The estimates are also highly similar to the parameters originally estimated by IN, as shown in the online appendix.

<sup>17</sup>Kass and Raftery (1995) suggest the following evaluation guidelines. Define  $B_{10}$  as the Bayes factor for model 1 (73-quarter debt model, in our case) compared to the alternative model 0 (one-quarter debt model). Values of  $2\log_e(B_{10}) > 10$ ,  $6 < 2\log_e(B_{10}) < 10$ , or  $2 < 2\log_e(B_{10}) < 6$  respectively suggest decisive, strong, or substantial evidence against the null hypothesis that model 0 is better than model 1, and hence decisive, strong or substantial support for model 1.

<sup>18</sup>Model simulations are computed using a random selection of 1000 draws from the posterior distribution. For each of them, 100 artificial time series of the main variables of length equal to that of the data are generated, giving a sample of 100000 series. We apply to those series the same treatment that we apply to data (i.e. either linear detrending or demeaning) and for each simulation we compute moments of interest, obtaining in that way distributions of moments.

Panels in the middle of Figure 8 highlight another important dimension where the long-term debt model performs well. They display the correlograms between house prices at time  $k = 0$  with debt at different leads and lags up to 5 quarters away in the data and in the model. Model results are reported as the 5th and 95th percentiles from model simulations. The comovement between debt and house prices is interesting, as it reflects the contrasting mechanics of the models with one-quarter and gradual amortization. When a house price increase raises collateral value, the entire stock of debt follows immediately in the one-quarter model, while only new loans respond on impact in the long-term debt model. Hence, the contemporaneous correlation ( $k = 0$  in Figure 8) between debt and house prices is particularly high with one-quarter debt, while it is not in the data.<sup>19</sup> Moreover, we see that in the data, house prices lead debt, reflected by the positive slope of the correlogram after  $k = 0$ . The long-term debt model captures this well. In contrast, if all debt is assumed to be amortized each period, the model implies that the correlation between debt and house prices declines sharply after  $k = 0$ .<sup>20</sup>

Finally, the bottom panel of Figure 8 displays the estimated series of the shock to lending standards, which we have introduced since we are bringing debt into the model as an observable. Conventional narratives of the last U.S. housing market boom-bust episode are that lending standards were loosened in the boom phase and tightened thereafter, see for instance Mian and Sufi (2014). The long-term debt model implies the same. The one-quarter debt model, in contrast, implies the exact opposite, as its structure makes debt moves in tandem with house prices. Gelain, Lansing, and Natvik (2015) elaborate on this specific issue when they account for the recent U.S. boom-bust dynamics.

The above-mentioned moments are the main dimensions where the 73-quarter debt model performs differently than the one-quarter debt model.<sup>21</sup>

### 4.3 Impulse Responses

How does a monetary policy shock affect the debt burden in our estimated model? Figure 9 gives an answer, displaying the 90 % probability bands of impulse responses in the estimated one-quarter and multi-period debt models. Unlike in the simple model considered before, we see that the impact effect on real household debt is no longer positive, but mildly negative. The medium run effects are very similar to those present in the simple model. The debt-to-GDP

<sup>19</sup>One might find the extremely low correlation between debt and GDP in the data surprising, given that Iacoviello and Pavan (2013) report a contemporaneous correlation of 0.78 for their sample. This dramatic difference is due to the filtering of the series. By detrending linearly, we remove little of the low-frequency developments in credit before computing the correlations, whereas Iacoviello and Pavan (2013) use a Hodrick-Prescott filter with a smoothing parameter of 1600. Using such a filter would necessarily remove the substantial low-frequency credit swings that we want our models to capture, and hence entirely contradicts our purposes.

<sup>20</sup>While we here focus on the real debt level, as this is the series used in the estimation, the correlation structure of debt-to-GDP exhibits similar properties, both in the data and in the model. Results are given in the online appendix, Section E.

<sup>21</sup>More moments are reported in the online appendix.

ratio increases on impact and falls persistently in the medium run, as in the simple model. There are two reasons why the real debt level no longer increases in the short run. First, debt is of shorter maturity than assumed in Figure 1. Second, the estimated degree of price stickiness is considerably higher (posterior median  $\theta = 0.89$ ) than in the parameterized simple model ( $\theta = 0.75$ ), so that inflation and thereby the price level drops less.

As in the simple model, we see that how the other variables than debt itself respond to a monetary policy shock, is relatively insensitive to the speed of amortization. Does this pattern hold for all shocks? For the three disturbances that most directly affect house prices and credit, it does not. These are the housing productivity shock, the housing preference shock and the lending standards shock.<sup>22</sup> Figure 10 displays how the responses of debt and consumption differ with debt duration for these three shocks.<sup>23</sup>

Starting with the housing preference shock, we see that borrowers' consumption would rise on impact with 1-quarter debt, whereas it initially drops with long-term debt. This difference is due to the two forces at play here. All else equal, a higher preference for housing motivates all households to cut non-housing consumption in favor of housing. On the other hand, the house price hike that follows enables more borrowing. With one-quarter debt, the latter effect is strong and borrowers' consumption rises. With long-term debt, the link from collateral value to debt is tempered in the short run, and consumption rises only after some time.

For housing productivity shocks, we have a similar scenario, but with the opposite sign. As housing costs, and thus house prices, are exogenously pushed down, borrowers' consumption follows the fall in collateral value. Long-term debt weakens this effect in the short run. Finally, we have the shock to lending standards. When only a limited fraction of debt is immediately affected by the altered collateral value, the total stock of debt increases little on impact, but builds up over time. Hence, consumption will increase less in the short run and more in the long run, than if all debt were refinanced every quarter. As we see, under long term debt borrowers' consumption even drops for the first quarters after the shock. This drop occurs because the shock makes housing more valuable as collateral, both today and in the future, motivating the borrowers to shift resources toward housing in the short run. If the shock were not persistent, this effect would be considerably weaker and borrower consumption would not fall.

#### 4.4 Monetary Policy

We now return to our main question: If monetary policy aims to stabilize household debt, how should it be conducted? In particular, we wish to assess if the policy implications from the stylized framework in Section 3 carry over to our estimated model with its larger set of shocks.

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<sup>22</sup>A variance decomposition reveals that these three shocks account for 70% of household debt's asymptotic variance, and 87% at the business cycle frequency.

<sup>23</sup>Impulse responses to all the shocks in the model are provided in the online appendix.

We start by considering simple rules. Holding the other coefficients in the interest rate rule constant at their estimated posterior medians ( $\phi_\pi = 1.42$ ,  $\phi_y = 0.53$ ,  $\phi_R = 0.62$ ), we introduce a reaction to either the current debt-to-GDP ratio ( $\phi_{(b/y)}$ ), debt growth ( $\phi_{\Delta b}$ ) or the expected future debt-to-GDP ratio ( $\phi_{E(b/y)}$ ). The class of interest rate rules we consider is given by

$$R_t = (R_{t-1})^{\phi_R} \left[ R\pi_t^{\phi_\pi} \left( \frac{GDP_t}{GDP_{t-1}} \right)^{\phi_y} D_j^{\phi_{D_j}} \right]^{1-\phi_R}$$

where  $D_j$  is either  $b_{b,t}/GDP_t$ ,  $b_{b,t}/b_{b,t-4}$ , or  $E_t(b_{b,t+i}/GDP_{t+i})$ . We report results for  $i = 4, 8$ .

Figure 11 displays the theoretical standard deviations of debt and inflation for alternative debt-responses when the economy is subject to the full set of shocks used in the estimation, except those stemming from monetary policy itself (interest rate and inflation target shocks).<sup>24</sup> The upper and lower plots to the left show that a debt-to-GDP response ( $\phi_{b/y}$ ) has qualitatively similar effects as in the simple model. First, responding positively to the debt-to-GDP ratio leads to equilibrium indeterminacy. With the estimated responses to inflation, output and the lagged interest rate, the threshold debt-to-GDP coefficient is given by the solid vertical line marginally above zero.<sup>25</sup> Second, it is a negative debt-to-GDP response that stabilizes debt.

We next move beyond debt-to-GDP and consider a policy rule with debt-growth. Figure 11 shows that here the conventional wisdom of raising the interest rate when debt grows more quickly has some merit, as a moderate positive response coefficient reduces the volatility of debt-to-GDP. We see that a positive debt growth coefficient reduces debt-to-GDP by less than a negative response to debt-to-GDP does, but that the costs in terms of inflation volatility are smaller with the debt growth rule.

The third and fourth columns of plots in Figure 11 show that responding to expected, rather than current, debt-to-GDP, or responding more or less strongly to output growth, does not alter our main conclusions on the consequences of responding to debt-to-GDP. The main effect of increasing the debt-to-GDP forecast horizon is to raise the scope for equilibrium indeterminacy, as not only positive, but also negative response coefficients are associated with this phenomenon, illustrated by the two vertical lines for  $\phi_{b/y}$ -values below zero in the forecast rule plots.<sup>26</sup>

Next, we consider targeting policies, defined as the minimization of the loss function in equation (21) under commitment. Figures 12 and 13 characterize how outcomes under strict debt-to-GDP targeting deviate from strict inflation targeting and the estimated policy rule. The figures display impulse responses to each of the eight shocks in the model. We see that for each

<sup>24</sup>The standard deviations of all the shocks are as estimated, see Table 3.

<sup>25</sup>We do not plot the determinacy frontier in the  $(\phi_{b/y}, \phi_\pi)$ -space here. The resulting determinacy frontier, both for responses to debt-to-GDP and the real debt level, are highly similar to that generated from the simple model, and reported in the online appendix.

<sup>26</sup>Increasing the output response ( $\phi_y$ ) alone, holding all the other response coefficients constant, has a non-monotonic, quantitatively moderate, effect on debt-to-GDP volatility. As  $\phi_y$  increases from zero, debt-to-GDP volatility first falls, but thereafter increases. For details, see the online appendix.

shock that causes a fall in debt-to-GDP under inflation targeting, a monetary authority that targets the debt-to-GDP ratio engineers a lower inflation and output level. Likewise, if debt-to-GDP rises under inflation targeting (or the estimated policy rule), the debt-to-GDP targeter will boost inflation and generate a greater economic expansion.

In short, our main conclusion found previously is upheld: stabilization of the debt-to-GDP ratio implies a more contractive policy when the debt-to-GDP ratio otherwise would be low, and a more expansive policy when the debt-to-GDP ratio otherwise would be high.

## 5 Conclusion

After the 2007-2009 financial crisis, household indebtedness has been high on the policy agenda. Unfortunately, discussions of household debt tend to implicitly assume that variation in debt-to-income ratios reflect active shifts in borrowing and lending, which is misguided. By introducing a reasonable distinction between new loans and pre-existing debt, we offer a framework with debt dynamics that lie close to empirical patterns. For the analysis of how monetary policy can act to stabilize debt, empirical performance along this dimension is of paramount importance.

Our results show that the persistence of household debt matters for monetary policy, and that policy advice from models with one-quarter debt should be treated with caution. First, with plausible debt dynamics, interest rate changes have far weaker influence on household debt than a typical one-quarter debt model implies. Moreover, with long-term debt the qualitative effect of a policy tightening on household debt-to-GDP is likely to be positive in the short run. Second, and more importantly, a policy that systematically increases the interest rate in response to high debt-to-GDP is destabilizing. Not only does such a policy induce greater inflation volatility, but it will destabilize debt itself. Moreover, by reacting positively to the debt-to-GDP level, monetary policy induces equilibrium indeterminacy, so that economic fluctuations driven solely by expectations become possible. Instead it is a negative interest rate response to debt-to-GDP that actually stabilizes debt. Relative to a strict inflation targeter, a central bank aiming to stabilize debt-to-GDP should always stimulate the economy *more* when inflation targeting is associated with a boom in the debt-to-GDP ratio, and stimulate the economy *less* when inflation targeting is associated with a fall in the debt-to-GDP ratio.

This paper is part of a broader agenda to establish the principles behind “leaning against the wind” policies. It is notable that our findings go somewhat against the conventional wisdom. Here there is a parallel to Galí (2014), who shows how a monetary policy that systematically responds to rational asset price bubbles might actually raise the volatility of the bubbles themselves. Of course, these types of results cannot necessarily be taken at face value in actual policy design, as they stem from models that abstract from many of the aspects that feature in the policy debate. For instance, in our model there is no mechanism through which debt

accumulation might trigger a financial crisis or raise the cost of recovering from one. But we would argue that these potential mechanisms are exactly why it so important that the policy debate is based on frameworks which can account for the debt dynamics that are empirically observed. From our results, it is natural to speculate that “leaning against the wind” policy, in the sense of raising interest rates to curb debt-to-GDP growth, might work as intended in the medium run, but at the cost raising the probability of a crisis in the short run. Hence, there would seem to be a trade-off here that policymakers should be aware of.

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## Appendix

We here summarize the model from Iacoviello and Neri (2010), which we estimate with long term debt, and the data we use in our estimation.

### A. The Estimated Model

Patient households maximize lifetime utility

$$V_{l,t} = E_0 \sum_{t=0}^{\infty} \beta_l^t \varepsilon_{c,t} \left[ \frac{1 - \gamma_l}{1 - \beta_l \gamma_l} \ln(c_{l,t} - \gamma_l c_{l,t-1}) + \nu_{h,t} \ln(h_{l,t}) - \frac{\nu_{l,t}}{1 + \varphi_l} \left( L_{l,c,t}^{1+\mu_l} + L_{l,h,t}^{1+\mu_l} \right)^{\frac{1+\varphi_l}{1+\mu_l}} \right],$$

with respect to  $c_{l,t}$ ,  $h_{l,t}$ ,  $L_{l,c,t}$  (hours in the consumption sector), and  $L_{l,h,t}$  (hours in the housing sector), where  $\mu_l$  measures the degree of labor mobility between the two sectors. Moreover,  $\ln \varepsilon_{c,t} = \rho_c \ln \varepsilon_{c,t-1} + u_t^c$ ,  $\ln \nu_{l,t} = \rho_{\nu_l} \ln \nu_{l,t-1} + u_t^{\nu_l}$ , and  $\ln \nu_{h,t} = (1 - \rho_{\nu_h}) \ln \nu_h + \rho_{\nu_h} \ln \nu_{h,t-1} + u_t^{\nu_h}$  are processes for intertemporal preferences, labour supply and housing preference respectively, where  $u_t^c$ ,  $u_t^{\nu_h}$ , and  $u_t^{\nu_l}$  are independently and identically distributed innovations with standard deviations  $\sigma_c$ ,  $\sigma_{\nu_h}$ , and  $\sigma_{\nu_l}$ . Patient households' budget constraint is:

$$\begin{aligned} & c_{l,t} + \frac{k_{c,t}}{A_{k,t}} + k_{h,t} + k_{ib,t} + q_t h_{l,t} + p_{l,t} l_t + \frac{R_{t-1} b_{l,t-1}}{\pi_t} + a(z_{h,t}) k_{h,t-1} \\ = & \frac{w_{l,c,t}}{X_{l,wc,t}} L_{l,c,t} + \frac{w_{l,h,t}}{X_{l,wh,t}} L_{l,h,t} - \phi_t + \left( R_{c,t} z_{c,t} + \frac{1 - \delta_{kc}}{A_{k,t}} \right) k_{c,t-1} + (R_{h,t} z_{h,t} + 1 - \delta_{kh}) k_{h,t-1} \\ & + p_{ib,t} k_{ib,t} + (p_{l,t} + R_{l,t}) l_{t-1} + q_t (1 - \delta_h) h_{l,t-1} + Div_{l,t} - \frac{a(z_{c,t})}{A_{k,t}} k_{c,t-1} + b_{l,t}, \end{aligned}$$

where  $k_{c,t}$  is capital in the consumption good sector,  $k_{h,t}$  is capital in the housing sector,  $k_{ib,t}$  is intermediate goods (priced at  $p_{ib,t}$ ) rented to the housing sector,  $l_t$  is land priced at  $p_{l,t}$ ,  $z_{c,t}$  and  $z_{h,t}$  are capital utilization rates, and  $b_{l,t}$  is debt.  $A_{k,t}$  captures investment-specific technology, and evolves as  $\ln A_{k,t} = \rho_{AK} \ln A_{k,t-1} + u_t^K$ , where  $u_t^K$  are independently and identically distributed with zero mean and standard deviation  $\sigma_{AK}$ . Loans are set in nominal terms and yield a nominally riskless return  $R_t$ . Real wages are denoted by  $w_{l,c,t}$  and  $w_{l,h,t}$ , real rental rates by  $R_{c,t}$ ,  $R_{h,t}$ , and  $R_{l,t}$ , and depreciation rates by  $\delta_{kc}$  and  $\delta_{kh}$ .  $X_{l,wc,t}$  and  $X_{l,wh,t}$  denote wage markups, which arise due to monopolistic competition in the labor market and are collected by labor unions.  $Div_{l,t} = \frac{X_t - 1}{X_t} Y_t + \frac{X_{l,wc,t} - 1}{X_{l,wc,t}} w_{l,c,t} L_{l,c,t} + \frac{X_{l,wh,t} - 1}{X_{l,wh,t}} w_{l,h,t} L_{l,h,t}$  (with  $X_t$  and  $Y_t$  to be defined below) are lump-sum profits from final good firms and from labor unions. Finally,  $\phi_t$  denotes adjustment costs for capital, and  $a(\cdot)$  are the costs of capital utilization:

$$\begin{aligned} \phi_t &= \frac{\phi_{k,c}}{2} \left( \frac{k_{c,t}}{k_{c,t-1}} - 1 \right)^2 k_{c,t-1} + \frac{\phi_{k,h}}{2} \left( \frac{k_{h,t}}{k_{h,t-1}} - 1 \right)^2 k_{h,t-1}, \\ a(z_{c,t}) &= R_c \left[ \varrho \frac{z_{c,t}^2}{2} + (1 - \varrho) z_{c,t} + \left( \frac{\varrho}{2} - 1 \right) \right], \end{aligned}$$

$$a(z_{h,t}) = R_h \left[ \varrho \frac{z_{h,t}^2}{2} + (1 - \varrho) z_{h,t} + \left( \frac{\varrho}{2} - 1 \right) \right],$$

were  $R_c$  and  $R_h$  are the steady state rental rates in sectors  $c$  and  $h$ . In the estimation of the model, a prior is specified for the curvature of the capacity utilization function,  $\zeta = \varrho / (1 + \varrho)$ .

Impatient households choose  $c_{b,t}$ ,  $h_{b,t}$ ,  $L_{b,c,t}$ , and  $L_{b,h,t}$  to maximize

$$V_{b,t} = E_0 \sum_{t=0}^{\infty} \beta_b^t \varepsilon_{c,t} \left[ \frac{1 - \gamma_b}{1 - \beta_b \gamma_b} \ln(c_{b,t} - \gamma_b c_{b,t-1}) + \nu_{h,t} \ln(h_{b,t}) - \frac{\nu_{l,t}}{1 + \varphi_b} \left( L_{b,c,t}^{1+\mu_b} + L_{b,h,t}^{1+\mu_b} \right)^{\frac{1+\varphi_b}{1+\mu_b}} \right],$$

subject to the budget constraint

$$c_{b,t} + q_t h_{b,t} = \frac{w_{b,c,t}}{X_{b,wc,t}} L_{b,c,t} + \frac{w_{b,h,t}}{X_{b,wh,t}} L_{b,h,t} + b_{b,t} - \frac{R_{t-1} b_{b,t-1}}{\pi_t} + q_t (1 - \delta_h) h_{b,t-1} + Div_{b,t}$$

the collateral constraint in equation (7) with  $m_t$  instead of  $m$ , and the law of motion for amortization in (5). As stated in the main text,  $m_t$  follows the process (22).  $Div_{b,t} = \frac{X_{b,wc,t}-1}{X_{b,wc,t}} w_{b,c,t} L_{b,c,t} + \frac{X_{b,wh,t}-1}{X_{b,wh,t}} w_{b,h,t} L_{b,h,t}$  are dividends from labor unions.

Wholesale firms hire labor and capital services and purchase intermediate goods to produce wholesale goods  $Y_t$  and new houses  $IH_t$ . They maximize profits

$$\max \frac{Y_t}{X_t} + q_t IH_t - \left( \sum_{i=c,h} w_{l,i,t} L_{l,i,t} + \sum_{i=c,h} w_{b,i,t} L_{b,i,t} + \sum_{i=c,h} R_{i,t} z_{i,t} k_{i,t-1} + R_{l,t} l_{t-1} + p_{ib,t} k_{ib,t} \right),$$

where  $X_t$  is the mark-up of final goods over wholesale goods prices. The production technologies are:

$$Y_t = \left[ z_t \left( L_{l,c,t}^{\varpi} L_{b,c,t}^{1-\varpi} \right) \right]^{1-\xi} (z_{c,t} k_{c,t-1})^{\xi},$$

$$IH_t = \left[ A_{h,t} \left( L_{l,h,t}^{\varpi} L_{b,h,t}^{1-\varpi} \right) \right]^{1-\mu_h - \mu_{ib} - \mu_{la}} (z_{h,t} k_{h,t-1})^{\mu_h} k_{ib,t}^{\mu_{ib}} l_{t-1}^{\mu_{la}},$$

where  $\ln z_t = \rho_z \ln z_{t-1} + u_t^z$  and  $\ln A_{h,t} = \rho_{AH} \ln A_{h,t-1} + u_t^{AH}$ , with  $u_t^z$  and  $u_t^{AH}$  independently and identically distributed innovations with zero means and standard deviations  $\sigma_z$  and  $\sigma_{AH}$ , respectively. The capital share in the goods production function is  $\xi$ , the capital share in the housing production function is  $\mu_h$ , the land share is  $\mu_{la}$ , and the intermediate goods share is  $\mu_{ib}$ .

In the consumption sector there is monopolistic competition and Calvo-type price rigidity at the retail level. Retailers buy wholesale goods  $Y_t$  from wholesale firms at the price  $P_t^w$  in a competitive market, differentiate the goods at no cost, and sell them at a markup  $X_t = P_t / P_t^w$  over the marginal cost. Those goods are aggregated through a constant elasticity of substitution technology and converted into homogeneous consumption and investment goods by households. Each period, a fraction of retailers set prices optimally, while the remaining firms partially index prices to past inflation rate. These assumptions deliver the same Phillips curve for the consumption sector as in equation (18). Cost shocks are added to (18). These are independently and identically distributed with zero mean and standard deviation  $\sigma_p$ .

As for the labor market, patient and impatient households supply homogeneous labor services to unions. The unions differentiate labor services, set wages subject to a Calvo scheme and offer labor services to wholesale labor packers who reassemble these services into the homogeneous labor composites  $L_{l,c,t}$ ,  $L_{b,c,t}$ ,  $L_{l,h,t}$ ,  $L_{b,h,t}$ . Wholesale firms hire labor from these packers. The following four relations result for nominal wage inflation,  $\omega_t$ :

$$\ln \omega_{l,c,t} - \iota_{w,c} \ln \pi_{t-1} = \beta_l (E_t [\ln \omega_{l,c,t+1}] - \iota_{w,c} \ln \pi_t) - \frac{(1 - \beta_l \theta_{w,c})(1 - \theta_{w,c})}{\theta_{w,c}} \ln (X_{l,wc,t} / X_{l,wc})$$

$$\ln \omega_{b,c,t} - \iota_{w,c} \ln \pi_{t-1} = \beta_b (E_t [\ln \omega_{b,c,t+1}] - \iota_{w,c} \ln \pi_t) - \frac{(1 - \beta_b \theta_{w,c})(1 - \theta_{w,c})}{\theta_{w,c}} \ln (X_{b,wc,t} / X_{b,wc})$$

$$\ln \omega_{l,h,t} - \iota_{w,h} \ln \pi_{t-1} = \beta_l (E_t [\ln \omega_{l,h,t+1}] - \iota_{w,h} \ln \pi_t) - \frac{(1 - \beta_l \theta_{w,h})(1 - \theta_{w,h})}{\theta_{w,h}} \ln (X_{l,wh,t} / X_{l,wh})$$

$$\ln \omega_{b,h,t} - \iota_{w,h} \ln \pi_{t-1} = \beta_b (E_t [\ln \omega_{b,h,t+1}] - \iota_{w,h} \ln \pi_t) - \frac{(1 - \beta_b \theta_{w,h})(1 - \theta_{w,h})}{\theta_{w,h}} \ln (X_{b,wh,t} / X_{b,wh}).$$

The nominal interest rate is set according to the simple rule

$$R_t = (R_{t-1})^{\phi_R} \left[ R \pi_t^{\phi_\pi} \left( \frac{GDP_t}{GDP_{t-1}} \right)^{\phi_y} \right]^{1-\phi_R} \frac{\varepsilon_t^R}{A_t^s}$$

where  $\varepsilon_t^R$  are independently and identically distributed monetary policy shocks with zero mean and standard deviation  $\sigma_R$ , and  $\ln A_t^s = \rho_s \ln A_{t-1}^s + u_t^s$  is an inflation target shock with independently and identically distributed innovations  $u_t^s$  with zero mean and standard deviation  $\sigma_s$ .  $GDP_t = Y_t + \bar{q}IH_t + IK_t$ .

The market clearing conditions are:

$$C_t + \frac{IK_{c,t}}{A_{k,t}} + IK_{k,t} + k_{ib,t} = Y_t - \frac{\phi_{k,c}}{2} \left( \frac{k_{c,t}}{k_{c,t-1}} - 1 \right)^2 k_{c,t-1} - \frac{\phi_{k,h}}{2} \left( \frac{k_{h,t}}{k_{h,t-1}} - 1 \right)^2 k_{h,t-1}$$

$$h_{l,t} + h_{b,t} - (1 - \delta_h)(h_{l,t-1} + h_{b,t-1}) = IH_t$$

$$b_{l,t} + b_{b,t} = 0$$

## B. Data

*Aggregate consumption:* Real Personal Consumption Expenditure (seasonally adjusted, billions of chained 2009 dollars, table 1.1.6), divided by the Civilian Noninstitutional Population (CNP16OV, source: Bureau of Labor Statistics, retrieved from FRED). Source: Bureau of Economic Analysis (BEA). Linearly detrended.

*Business fixed investment:* Real Private Nonresidential Fixed Investment (seasonally adjusted, billions of chained 2009 dollars, table 1.1.6), divided by CNP16OV. Source: BEA. This series is available from 1999q1 only. To obtain values for 1965q1 to 1989q4, we first recovered

the original Iacoviello and Neri (2010) (IN) series in levels, starting from the transformed series available at the AEJM website. The first value of the series has been imputed from vintages available in FRED2. Then we linearly regressed new data on IN's data for the period 1999q1-2006q4. With the resultant coefficients, we extrapolated our data backwards to get the full new series. Linearly detrended.

*Residential investment:* Real Private Residential Fixed Investment (seasonally adjusted, billions of chained 2009 dollars, table 1.1.6.), divided by CNP16OV. Source: BEA. This series is available from 1999q1 only. We followed the same procedure as for Business Fixed Investment to get the full new series. Linearly detrended.

*Inflation:* Quarter on quarter log differences in the implicit price deflator for the nonfarm business sector. Source: Bureau of Labor Statistics (BLS), retrieved from FRED: IPDNBS. Demeaned.

*The nominal short-term interest rate:* 3-month Treasury Bill Rate (Secondary Market Rate), expressed in quarterly units. Source: Board of Governors of the Federal Reserve System, retrieved from FRED: TB3MS. Demeaned.

*Real house prices:* Census Bureau House Price Index (new one-family houses sold including value of lot) deflated with the implicit price deflator for the nonfarm business sector. Source: Census Bureau, <https://www.census.gov/construction/cpi/>. Linearly detrended.

*Hours in the consumption sector:* Total Nonfarm Payrolls (retrieved from FRED: PAYEMS) less all employees in the construction sector (retrieved from FRED: USCONS), times Average Weekly Hours of Production Workers (retrieved from FRED: AWHNONAG), divided by CNP16OV. Source: BLS. Demeaned.

*Hours in the housing sector:* All Employees in the Construction Sector (retrieved from FRED: USCONS), times Average Weekly Hours of Construction Workers (retrieved from FRED: CES2000000007), divided by CNP16OV. Source: BLS. Demeaned.

*Wage Inflation in the consumption sector:* The Quarterly rate of change in Average Hourly Earnings of Production/Nonsupervisory Workers on Private Nonfarm Payrolls, Total Private (retrieved from FRED: AHETPI). Source: BLS. Demeaned.

*Wage inflation in the housing sector:* Quarterly changes in Average Hourly Earnings of Production/Nonsupervisory Workers in the Construction Industry (retrieved from FRED: CES2000000008). Source: BLS. Demeaned.

*Household debt:* Households and Nonprofit Organizations; Home Mortgages; Liability in billions of dollars (Source: Board of Governors of the Federal Reserve System, retrieved from FRED: HMLBSHNO), deflated with the implicit price deflator for the nonfarm business sector and divided by CNP16OV. Linearly detrended.

Table 1: Calibrated Parameters in the Medium Scale Model

Parameter	Target	Value
$\beta_l$	Steady state annual real interest rate 3%	0.9925
$\beta_b$		0.97
$\nu_h$	Ratio of business capital to annual GDP of 2.1%	0.12
	Ratio of housing wealth to GDP of about 1.5%	
$\xi$		0.35
$\mu_h$		0.10
$\mu_{la}$	Ratio of value of residential land to annual output of 50%	0.10
$\mu_{ib}$		0.10
$\delta_h$	Ratio of residential investments to total output of about 6%	0.01
$\delta_{kc}$	Ratio of nonresidential investments to GDP of about 27%	0.025
$\delta_{kh}$	Ratio of nonresidential investments to GDP of about 27%	0.03
$X, X_{wc}, X_{wh}$	Steady state mark-up of 15%	1.15
$\tilde{m} = b_b/qh_b$	Ratio of debt to real estate	0.50
$\rho_s$	Annual autocorrelation of trend inflation around 0.9	0.975

Notes: All parameter values follow from Iacoviello and Neri (2010).

Table 2: Estimation: Prior and Posterior Distribution of the Structural Parameters

Parameter	Prior distribution				Posterior distribution			
	Distribution	Mean	SD	1-quarter model		73-quarter model		
				Median	90% HPD	Median	90% HPD	
$\gamma_l$	Beta	0.5	0.075	0.28	0.22 – 0.34	0.26	0.19 – 0.32	
$\gamma_b$	Beta	0.5	0.1	0.42	0.31 – 0.52	0.52	0.41 – 0.63	
$\varphi_{L,l}$	Gamma	0.5	0.1	0.42	0.28 – 0.57	0.39	0.28 – 0.52	
$\varphi_{L,b}$	Gamma	0.5	0.1	0.56	0.40 – 0.72	0.52	0.37 – 0.67	
$\mu_l$	Normal	1	0.1	-0.04	-0.07 – -0.02	-0.05	-0.08 – -0.03	
$\mu_b$	Normal	1	0.1	1.14	0.95 – 1.29	1.16	1.01 – 1.32	
$\phi_{k,c}$	Gamma	10	2.5	19.92	16.32 – 23.49	21.18	17.91 – 24.87	
$\phi_{k,h}$	Gamma	10	2.5	11.45	11.54 – 7.91	12.62	8.57 – 17.91	
$\varpi$	Beta	0.65	0.05	0.66	0.58 – 0.74	0.61	0.53 – 0.69	
$\phi_R$	Beta	0.75	0.1	0.60	0.54 – 0.66	0.62	0.57 – 0.68	
$\phi_\pi$	Normal	1.5	0.1	1.41	1.31 – 1.51	1.42	1.31 – 1.52	
$\phi_y$	Normal	0	0.1	0.54	0.46 – 0.63	0.53	0.44 – 0.62	
$\theta$	Beta	0.667	0.05	0.89	0.87 – 0.91	0.89	0.87 – 0.91	
$v$	Beta	0.5	0.2	0.53	0.44 – 0.63	0.54	0.44 – 0.65	
$\theta_{w,c}$	Beta	0.667	0.05	0.76	0.73 – 0.79	0.76	0.72 – 0.79	
$\iota_{w,c}$	Beta	0.5	0.2	0.07	0.02 – 0.14	0.08	0.02 – 0.14	
$\theta_{w,h}$	Beta	0.667	0.05	0.76	0.71 – 0.81	0.74	0.69 – 0.79	
$\iota_{w,h}$	Beta	0.5	0.2	0.37	0.18 – 0.55	0.41	0.23 – 0.60	
$\zeta$	Beta	0.5	0.2	0.76	0.63 – 0.88	0.81	0.68 – 0.93	
$\chi$	Normal*	0.95	0.1	0.5	–	0.97	0.96 – 0.98	
Log data density					6128.12		6418.04	

Notes: \* The prior distribution for  $\chi$  refers only to the 73-quarter debt model because  $\chi$  is 0.5 with 1-quarter debt. The sample is 1965q1-2014q1.

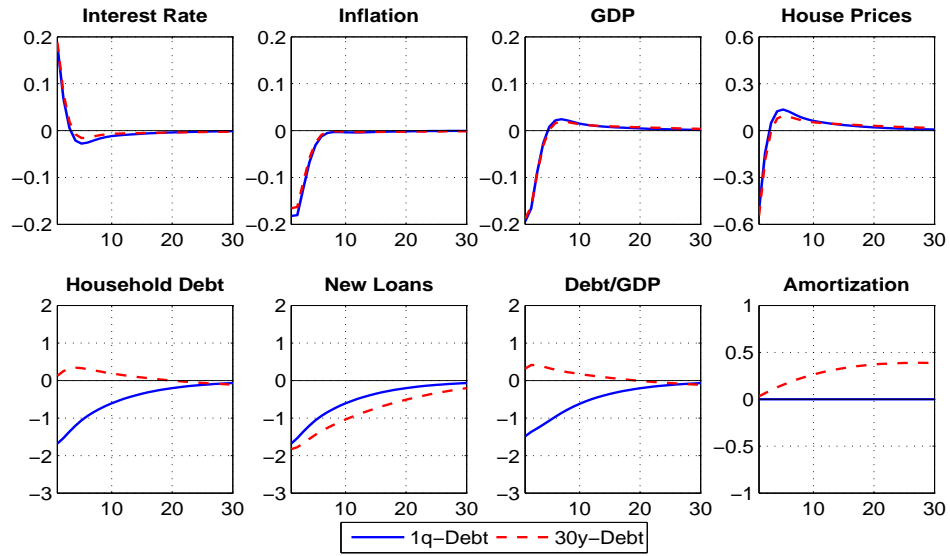
Table 3: Estimation: Prior and Posterior Distribution of the Shock Processes

Parameter	Prior distribution				Posterior distribution			
	Distribution	Mean	SD	1-quarter model		73-quarter model		
				Median	90% HPD	Median	90% HPD	
$\rho_z$	Beta	0.8	0.1	0.95	0.93 – 0.97	0.96	0.94 – 0.98	
$\rho_{AH}$	Beta	0.8	0.1	0.996	0.992 – 0.999	0.996	0.992 – 0.999	
$\rho_{AK}$	Beta	0.8	0.1	0.93	0.90 – 0.95	0.93	0.90 – 0.95	
$\rho_{v_h}$	Beta	0.8	0.1	0.98	0.96 – 0.99	0.98	0.97 – 0.99	
$\rho_c$	Beta	0.8	0.1	0.98	0.96 – 0.99	0.97	0.94 – 0.99	
$\rho_{v_l}$	Beta	0.8	0.1	0.97	0.95 – 0.99	0.97	0.95 – 0.99	
$\rho_m$	Beta	0.8	0.1	0.98	0.96 – 0.99	0.73	0.65 – 0.80	
$\sigma_z$	Inv. Gamma	0.001	0.01	0.0100	0.0091 – 0.0109	0.0100	0.0091 – 0.0109	
$\sigma_{AH}$	Inv. Gamma	0.001	0.01	0.0213	0.0195 – 0.0232	0.0216	0.0197 – 0.0236	
$\sigma_{AK}$	Inv. Gamma	0.001	0.01	0.0104	0.0084 – 0.0123	0.0111	0.0090 – 0.0131	
$\sigma_{v_h}$	Inv. Gamma	0.001	0.01	0.0341	0.0267 – 0.0416	0.0323	0.0261 – 0.0419	
$\sigma_R$	Inv. Gamma	0.001	0.01	0.0032	0.0027 – 0.0037	0.0031	0.0027 – 0.0035	
$\sigma_c$	Inv. Gamma	0.001	0.01	0.0158	0.0096 – 0.0244	0.0127	0.0073 – 0.0252	
$\sigma_{v_l}$	Inv. Gamma	0.001	0.01	0.0192	0.0161 – 0.0226	0.0186	0.0157 – 0.0217	
$\sigma_p$	Inv. Gamma	0.001	0.01	0.0039	0.0035 – 0.0043	0.0039	0.0035 – 0.0044	
$\sigma_s$	Inv. Gamma	0.001	0.01	0.0289	0.0224 – 0.0357	0.0288	0.0218 – 0.0361	
$\sigma_m$	Inv. Gamma	0.001	0.01	0.0361	0.0332 – 0.0392	0.2598	0.2138 – 0.3038	
$\sigma_{L,h}$	Inv. Gamma	0.001	0.01	0.1648	0.1501 – 0.1792	0.1619	0.1447 – 0.1778	
$\sigma_{\omega,h}$	Inv. Gamma	0.001	0.01	0.0051	0.0047 – 0.0056	0.0051	0.0047 – 0.0056	

Notes:  $\sigma_{L,h}$  and  $\sigma_{\omega,h}$  are standard deviations for measurement errors in hours worked and wages in the housing sector. The sample is 1960q2-2014q1.

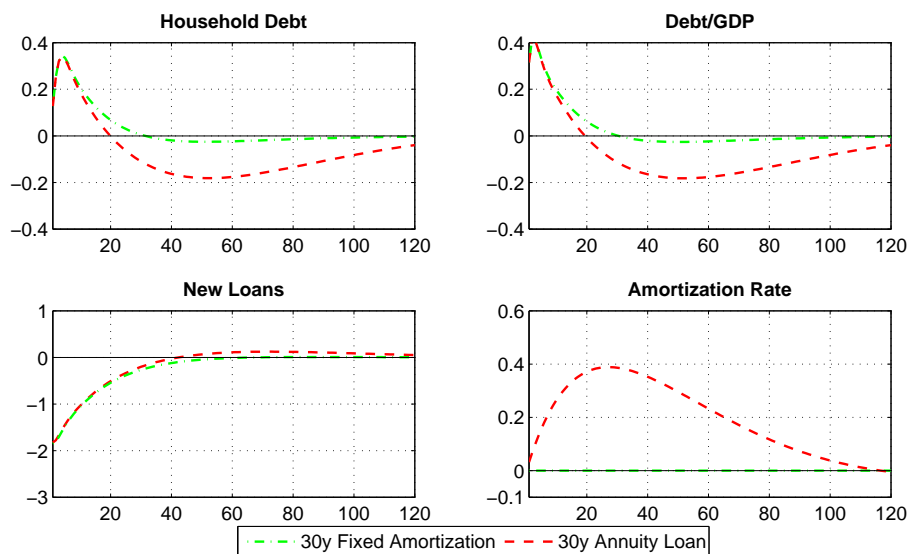


Figure 1: Contractionary Monetary Policy Shock



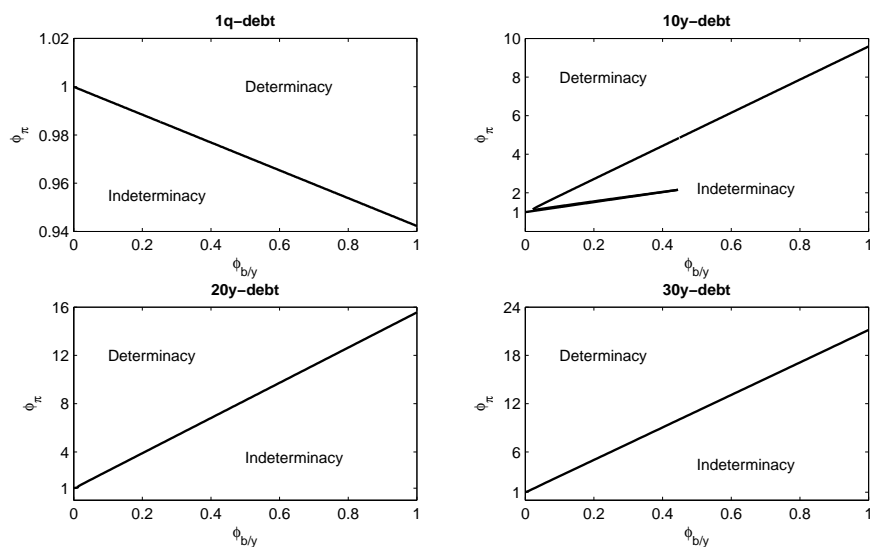
Notes: Impulse responses to a 25 basis point increase in the quarterly interest rate. For output, house prices, debt, new loans, and amortization, the vertical axes report percent deviations from steady state. For the interest rate and inflation, deviations are in percentage points.

Figure 2: Contractionary Monetary Policy Shock – Debt Dynamics under Annuity Loans and Fixed Amortization Loans



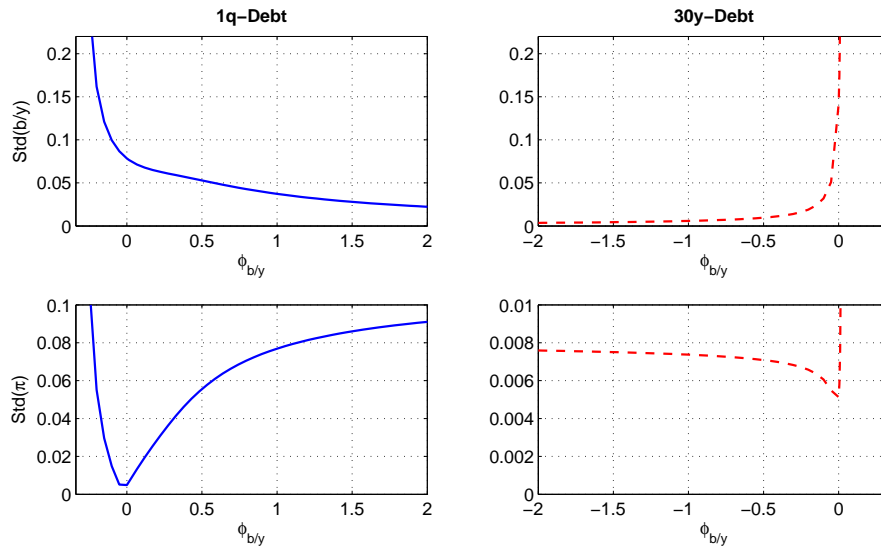
Notes: Impulse responses to a 25 basis point increase in the quarterly interest rate, percent deviations from steady state.

Figure 3: Equilibrium Determinacy with Debt-to-GDP in the Interest Rate Rule



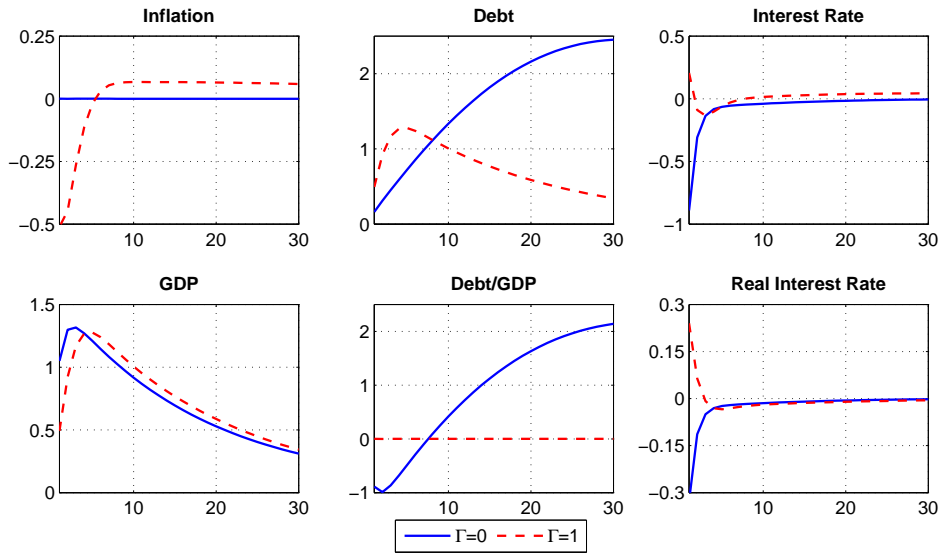
Notes: Regions of equilibrium determinacy for alternative constellations of the interest rate response to debt-to-GDP ( $\phi_{b/y}$ ) and inflation ( $\phi_{\pi}$ ).

Figure 4: Debt and Inflation Volatility under Simple Policy Rules



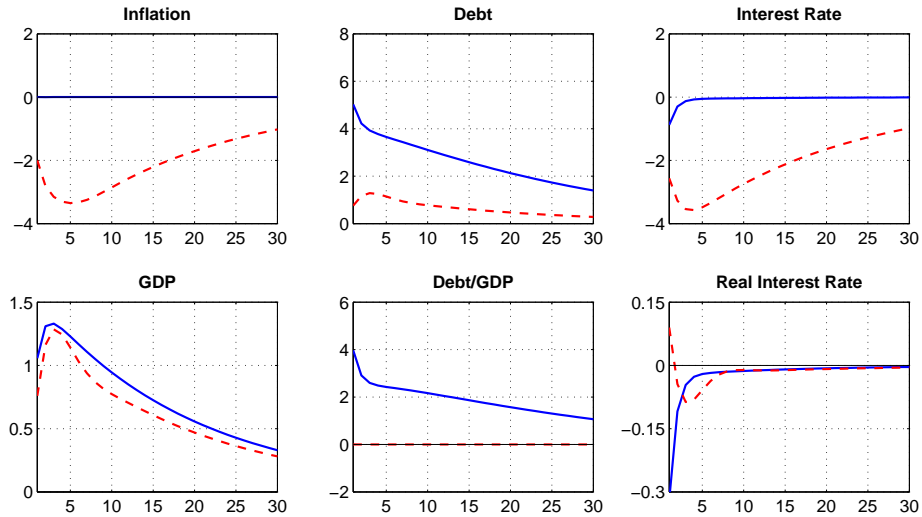
Notes: Standard deviations of debt-to-GDP and inflation for different response coefficients on debt-to-GDP and debt growth. Monetary policy follows either  $R_t = R^{ss} \pi_t^{\phi_\pi} \left(\frac{b_t}{y_t}\right)^{\phi_{b/y}}$ , or  $R_t = R^{ss} \pi_t^{\phi_\pi} \left(\frac{b_t}{b_t - 4}\right)^{\phi_{\Delta b}}$ . The two plots farthest to the left refer to the model where all debt is repaid within a quarter, the other four plots refer to the model where households follows a 30-year amortization schedule.

Figure 5: Debt-to-GDP and Inflation Targeting with 30-Year Debt



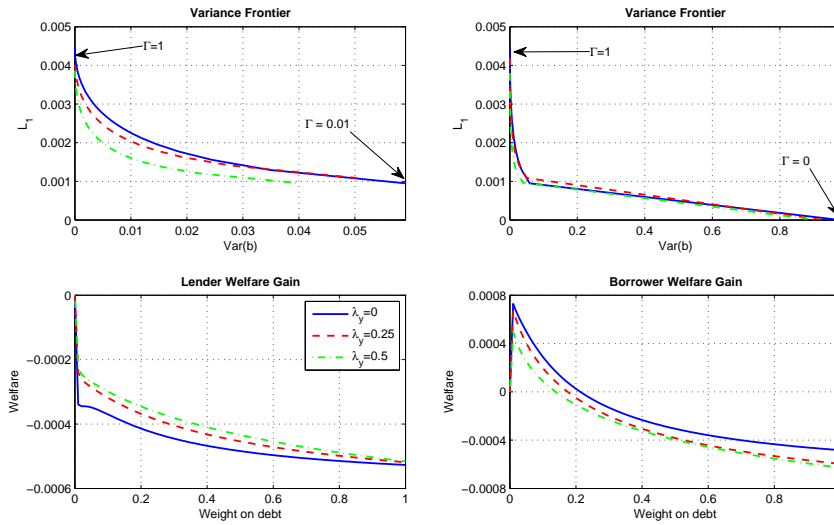
Notes: Impulse responses under optimal policy aiming to stabilize inflation ( $\Gamma = 0$ ) or debt ( $\Gamma = 1$ ).

Figure 6: Optimal Debt-to-GDP and Inflation Targeting with 1-Quarter Debt



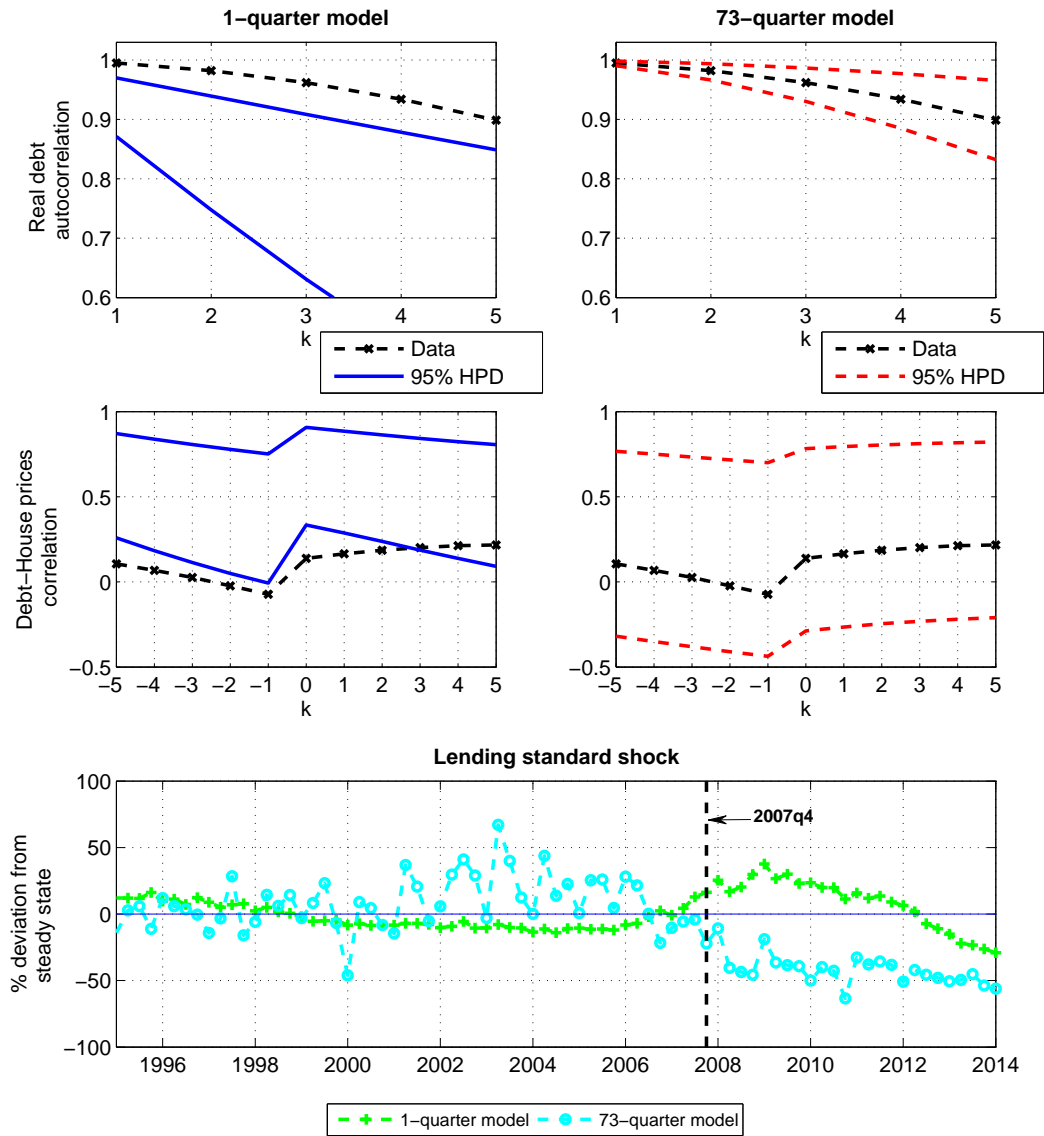
Notes: Impulse responses under optimal policy aiming to stabilize inflation ( $\Gamma = 0$ ) or debt ( $\Gamma = 1$ ).

Figure 7: Variance Frontiers and Welfare under Targeting Policies



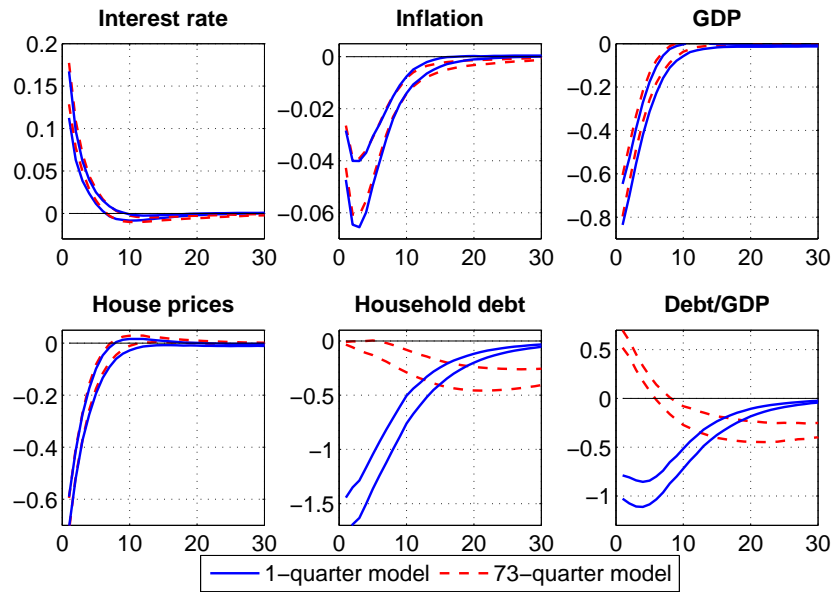
Notes: The upper panels display the possibility frontiers when policy is set to minimize the loss function in equation (21), where  $\Gamma$  is the weight on debt-to-GDP. The vertical axes display  $L_1 = (1 - \lambda_y) \text{var}(\pi) + \lambda_y \text{var}(y)$ . The upper left panel considers  $\Gamma \in [0.01, 1]$ , the upper right panel considers  $\Gamma \in [0, 1]$ . The bottom panels display the welfare gains of increasing the weight on debt-to-GDP rather than inflation ( $\Gamma$ ) for lender and borrower households separately. Welfare measured as percentage share of steady state consumption.

Figure 8: Model Fit



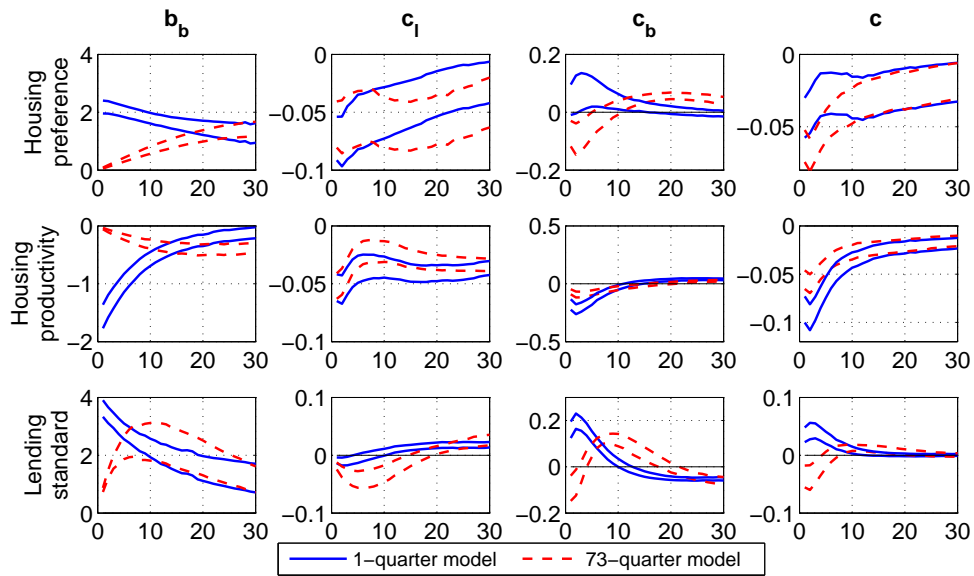
Notes: The top two panels report the  $k$ -th order autocorrelation of real household debt in the data and in the estimated 1-quarter and multiperiod debt models. The middle two panels report the cross correlation between house prices at time  $t$  and real household debt at time  $t + k$ , in the data and in the two versions of the model. From the models we always report the 95% high posterior density region of the simulated distributions of moments. The bottom panel plots the estimated sequence of lending standard shocks in the two versions of the model, with the vertical line indicating the onset of the Great Recession.

Figure 9: Monetary Policy Shock - Impulse Responses



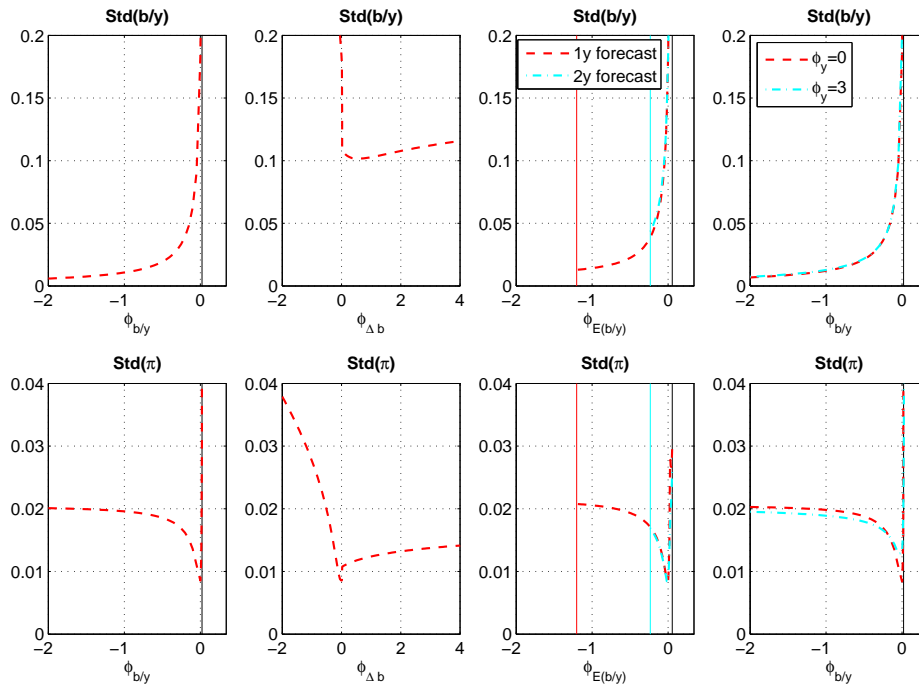
Notes: The 90% high posterior density regions of the impulse responses to a monetary policy shock in the estimated model.

Figure 10: Credit and Housing Shocks - Impulse Responses of Debt and Consumption



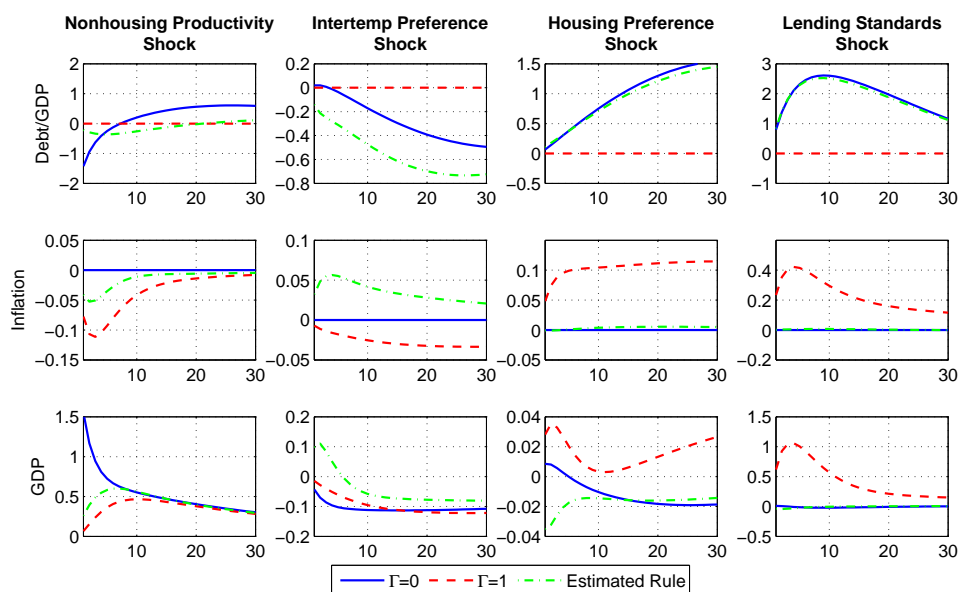
Notes: The 90% high posterior density regions of the impulse responses to housing preference, housing technology, and lending standards shocks in the estimated model.

Figure 11: Debt and Inflation Volatility under Simple Policy Rules in the Estimated Model with long-term debt



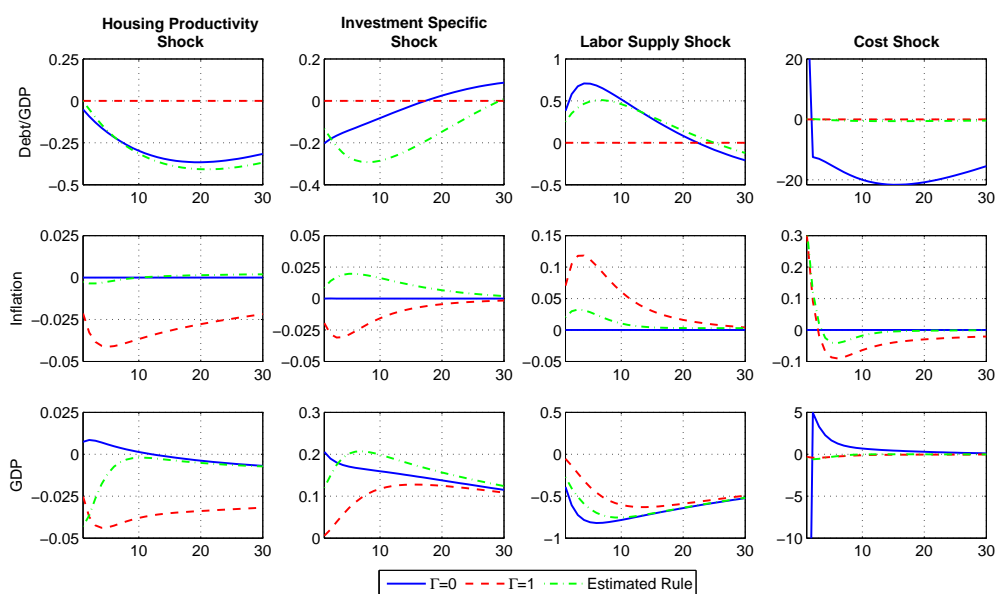
Notes: Standard deviations of debt-to-GDP and inflation when monetary policy responds to debt-to-GDP ( $\phi_{b/y}$ ), debt growth ( $\phi_{\Delta b}$ ) or expected debt-to-GDP ( $\phi_{E(b/y)}$ ). For the latter, 1-year-ahead and 2-year-ahead expected debt-to-GDP are considered. In all plots, the response coefficients on inflation and lagged interest rates are held constant at their estimated values. The upper and lower right-most plots consider two alternative response coefficients on output growth. In all the other plots, the response coefficient on output growth is held constant at its estimated value. The vertical solid lines indicate threshold levels of the response coefficients beyond which the equilibrium is indeterminate.

Figure 12: Optimal Debt-to-GDP and Inflation Targeting in the Estimated Model with Long Term Debt



Notes: Impulse responses under optimal policy aiming to stabilize inflation ( $\Gamma = 0$ ) or debt ( $\Gamma = 1$ ) and when the interest rate follows the estimated simple rule.

Figure 13: Optimal Debt-to-GDP and Inflation Targeting in the Estimated Model with Long Term Debt



Notes: Impulse responses under optimal policy aiming to stabilize inflation ( $\Gamma = 0$ ) or debt ( $\Gamma = 1$ ) and when the interest rate follows the estimated simple rule.