Exchange Rates, Aggregate Productivity and the Currency of Invoicing of International Trade*

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Abstract

This paper uses novel customs data from Chile and a model of international prices with nominal rigidities to study how nominal exchange rate movements impact aggregate output and productivity. A special feature of this dataset is that it records the currency in which transactions are invoiced. We exploit this feature to estimate how exchange rate movements generate substitution across goods priced in different currencies. Export prices are rigid in the currency in which they are invoiced, so that, in a given destination, the relative price of products invoiced in different currencies fluctuates with the nominal exchange rate. We show that the relative quantities sold by firms that invoice in different currencies also fluctuate with the nominal exchange rate, with an elasticity that ranges between 1 and 2. By shifting the allocation of production across firms pricing in different currencies, exchange rate movements can affect aggregate productivity. Guided by a quantitative open economy model disciplined by some features of the Chilean data, we show that these effects can be significant. In our baseline calibration, a 10% change in the exchange rate that increases markup dispersion reduces aggregate productivity in the tradable sector by 0.4%. Alternative parameterizations that do not account for the observed heterogeneity in invoicing predict changes in productivity at least four times smaller.

1 Introduction

Nominal exchange rates often experience dramatic fluctuations. How do these movements affect the real economy? A large literature in international economics has emphasized two mechanisms through which nominal exchange rate movements can impact real

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output and productivity. First, exchange rate movements can stimulate (depress) output by inducing a switch in expenditures between goods that are priced in different currencies.[1] Second, exchange rates can affect efficiency in the allocation of factors across firms by inducing price movements that are not related to changes in marginal costs.[2] Understanding the quantitative importance of these two mechanisms is key for the design of exchange rate policy.

This paper sheds light on the strength of these forces guided by a novel dataset from Chilean customs that contains information on the currency in which exports are invoiced. The first contribution of the paper is to use these data to measure how exchange rate movements generates switches in expenditures across goods priced in different currencies. The second contribution is to measure how exchange rate movements impact aggregate productivity by shifting the allocation of production across Chilean firms, using a quantitative model of international prices guided by the data.

A major challenge in identifying expenditure switching effects is that exchange rates respond to shocks that affect supply and demand conditions in the domestic and foreign economies. We exploit the fact that, within some destinations, Chilean firms use different currencies to invoice their exports. Hence, we can compare how Chilean exporters that invoice in different currencies respond to changes in the exchange rate controlling for destination and product fixed effects. Consistent with previous findings in the literature (see for example Gopinath, Itskhoki and Rigobon 2010) we show that export prices are rigid in the currency in which they are invoiced, so the relative price of two products invoiced in different currencies fluctuates almost one-to-one with the exchange rate. The key advantage of the customs data relative to price survey data used in the literature is that it contains information on quantities, which is needed to study how expenditures shift. We estimate that the relative quantities of products invoiced in different currencies comoves with the exchange rate, with an elasticity that is in the range of -1 and -2.[3] These elasticities are in line with those used by the international business cycle literature to match the observed comovements between the terms of trade and the trade balance. In contrast, our estimates are obtained directly from microdata on the response of prices and quantities to changes in the exchange rate. Such low elasticities indicate that the expenditure switching effects are limited, even when exchange rate movements change

[1]This topic has motivated a vast literature on how exchange rate movements affect international relative prices. See Burstein and Gopinath (2013) for a summary.

[2]See Corsetti, Dedola and Leduc (2010).

[3]This assumes that the currency of invoicing is set before the exchange rate changes, and that relative demand shocks affecting both firms are uncorrelated with nominal exchange rates. As we describe in the empirical section, these assumptions are likely to hold in these data.

relative prices.

We then measure how exchange rate movements impact aggregate productivity using a quantitative model of international relative prices. An extensive literature studies how exchange rates movements affect efficiency by inducing price movements that are not driven by changes in marginal costs. This literature has been mainly theoretical in nature, and typically assumes that desired markups are constant and that all firms from a given country are homogenous and invoice in the same currency.[4] In contrast, we design a quantitative model that is consistent with the following three features of the Chilean data. First, within destinations, there is substantial heterogeneity in the currency in which exporters invoice transactions. Second, the invoicing currency is correlated to exporters' size, exporters invoicing in the destination's market currency are larger on average. Third, the relative price of exports invoiced in different currencies displays persistent changes that comove with the exchange rate, which leads to movements in relative quantities. We show that by incorporating these features into the model, we obtain very different measures of efficiency losses due to exchange rate movements than those obtained under the standard assumptions made in the literature.

The model is an open economy model with time-dependent nominal rigidities of the sort that has been used to evaluate optimal exchange rate policy.[5] A continuum of monopolistically competitive producers set prices in each country that are sticky in the currency in which they are invoiced. We expand this framework along two dimensions to accommodate the three features of the Chilean data mentioned above. First, producers are heterogenous in their productivity and can choose in which currency to invoice their exports. Second, tradable goods must be combined in fixed proportions with local distribution services to reach consumers in each country.[6] In this setting, the price elasticity for the producer (and the producer's desired markup) is endogenous and depends on the price of the distribution services relative to the costs of production.[7] Producers with higher productivity have lower costs of production, higher markups, and a higher markup elasticity with respect to the price of services and the exchange rate in the destination's market. Building on results from Engel (2006) and Gopinath, Itskhoki and Rigobon (2010), we show that when facing a discrete invoicing decision, these producers are more

[4] See for example Engel (2002), Engel (2011) and Gali and Monacelli (2005). Duarte and Dotsey (2011) and Gust, Leduc and Sheets (2009) are examples of richer quantitative models that evaluate the role of exchange rate pass-through on aggregate variables such as the trade balance.

[5] See Gali and Monacelli (2005) and Engel (2011).

[6] One interpretation is that all goods use the same "shelf space", regardless the technology used for production.

[7] This way of generating endogenous variables markups was first introduced by Corsetti and Dedola (2005).

likely to invoice in the destination's market currency.[8] Intuitively, producers with a high markup elasticity with respect to the exchange rate want to absorb exchange rate movements into markups and keep prices stable in the destination market. In a context where prices are sticky in the currency in which they are invoiced, this is achieved by invoicing in the destination's currency. Given that high productivity producers set lower prices and have higher sales, the model can account for the observed correlation between exporter's size and invoicing.

Exchange rate fluctuations affect aggregate productivity by affecting markup dispersion across firms within a country. In the model, markup dispersion generates an inefficient allocation of factors across firms. Exchange rate fluctuations affect markup dispersion through three different channels. First, since producer prices are sticky in the currency in which they are invoiced, relative markups across producers invoicing in different currencies fluctuate with the exchange rate. Second, for those producers that are able to reset prices, there is dispersion in desired markups across producers with different productivities. Finally, there is dispersion originating from the staggered price adjustment caused by the price stickiness. The response of productivity to an exchange rate shock depends on whether the shock magnifies or reduces the initial dispersion in markups.

We parameterize a three-country version of the model to quantitatively evaluate how exchange rate movements affect production efficiency. Since the U.S. dollar and the euro are the most prevalent currencies for Chilean exporters, we take the countries to represent Chile, the U.S. and Europe. We use the correlation between firm size and invoicing in the data to put discipline on the relation between desired markups and invoicing in the model. Other key parameters control the share of firms invoicing in each currency in each country and the elasticity of relative quantities to nominal exchange rate shocks. The invoicing choices are directly observable in the data, while the elasticity is estimated in the empirical section of the paper.

We simulate an appreciation of the euro against all currencies and evaluate its effect on Chilean aggregate productivity. In data and in the model, only the largest Chilean exporters invoice in euros when selling into Europe. Since large firms start with higher markups, an appreciation of the Euro increases markup dispersion across Chilean exporters selling in Europe. In contrast, all Chilean exporters invoice in dollars when selling into the U.S., so markup dispersion across Chilean exporters selling in the U.S. is not affected much by the shock. As a result, an appreciation of the euro increases total markup

[8] Gopinath, Itskhoki and Rigobon (2010) provide direct evidence that invoicing is related to the elasticity of desired markups by documenting substantial differences in pass-through into the United States of the average good priced in dollars versus non-dollars after conditioning on a price change.

dispersion across Chilean exporters. We show that the effects are quantitatively important; in our baseline calibration, a 10 percent appreciation of the euro reduces Chilean productivity in the tradable sector by 0.4 percent. To evaluate the role of heterogeneity in invoicing, we reparameterize the model by assuming that all Chilean firms selling in each destination invoice in the same currency. We also conduct counterfactual parameterizations that assume that invoicing is exogenous and uncorrelated to firm characteristics. These alternative parameterizations predict changes in productivity that are at least four times smaller than our baseline results. This indicates that taking heterogeneity in invoicing into account is crucial for understanding how exchange rate fluctuations affect productive efficiency.

Some final considerations are in order. First, firms may respond differently to changes in the exchange rate if they use imported intermediate inputs from different source countries.[9] In such instances, changes in the relative price of these firms would be an efficient response to the changes in input costs generated by the exchange rate. We show, however, that the changes in relative prices that I document arise from movements in relative markups rather than from relative costs. In particular, in estimating how prices respond to the exchange rate, we exploit the fact that Chilean firms sell into more than one destination and use a fixed-effect strategy to control for changes in firm level marginal costs that are common across destinations.[10]

Second, it is well known that the efficiency losses from inflation are smaller when price rigidities are state-dependent rather than time-dependent, as is assumed in our model.[11] In this sense, one could interpret the results from our quantitative exercises as evidence that heterogeneity in invoicing greatly amplifies the effects of exchange rates on productivity rather than focusing on the absolute numbers of the counterfactuals.

This paper is related to three strands of literature. First, there is a growing literature that uses firm or product level data to document how international prices respond to changes in the nominal exchange rate.[12] From this literature, the paper that is the closest to this one is Gopinath, Itskhoki and Rigobon (2010), who document differences in pricing practices by firms importing into the U.S. in dollars vs. non-dollars. Part of the empirical

[9]Using data from Belgium, Amiti, Itskhoki and Konings (Forthcoming) argue that about half of the lack of exchange rate pass-through into prices comes from this channel.

[10]Fitzgerald and Haller (2012*a*) use a similar fixed effects strategy to document pricing-to-market by Irish firms selling in Ireland and the UK, for firms invoicing their exports in pounds.

[11]See Golosov and Lucas (2007) and Burstein and Hellwig (2008). Time dependent (Calvo) pricing is the common assumption in open economy literature with price rigidities.

[12]Some of these papers are Berman, Martin and Mayer (2012), Amiti, Itskhoki and Konings (Forthcoming), Fitzgerald and Haller (2012*a*) , Burstein and Jaimovich (2012), and Burstein and Gopinath (2013). See Goldberg and Knetter (1997) for a summary of an older literature measuring exchange rate pass-through using sector level data.

contribution of our paper is to document how these differences in prices translate into quantities. This is essential for establishing how exchange rate movements affect actual allocations and for measuring expenditure switching effects.

The paper is also related to an extensive literature that uses open economy models with sticky prices to study the transmission of monetary shocks across countries.[13] Our contribution to this literature is to quantitatively evaluate the effects of exchange rate movements on aggregate productivity in a model that incorporates firm heterogeneity, endogenous invoicing choices and endogenous variable markups in a parsimonious way to match key features of the micro data. In doing so, we provide evidence linking invoicing choices to firm size. To our knowledge, this is the first paper linking invoicing to firm characteristics.[14] This evidence can shed light on the determinants of invoicing practices.

Finally, the paper is also related to the literature on the international elasticity puzzle (see Ruhl 2008 and Fitzgerald and Haller 2012*b*). This literature documents that trade flows are very responsive to changes in tariffs, but not to changes in the exchange rate. This paper contributes to this discussion by providing a new micro-estimate on how trade flows respond to exchange rates by exploiting special features of the Chilean data.

The rest of the paper is organized as follows. The empirical evidence is presented in the next section. Section 2 introduces the model. Section 3 describes the parameterization and the quantitative exercises, and the last section concludes.

2 Empirical Evidence

2.1 Data

We use two different datasets from Chilean customs. The first dataset contains all export shipments between the years 2009 and 2011.[15] We refer to this dataset as the "Benchmark" dataset in the analysis below. As a robustness check, we also consider a second dataset which only covers wine export shipments, but spans more years, from 2003 to 2011. We refer to this dataset as the "Wine" dataset. We use both datasets in the empirical section below.

The data contain information on each export shipment originating in Chile during

[13] A non-exhaustive list of these papers is Obstfeld and Rogoff (1995), Corsetti and Pesenti (2005), Devereux and Engel (2003), Gali and Monacelli (2005) and Engel (2002) and Engel (2011).

[14] There is a large literature studying the determinants of invoicing practices. See for example Goldberg and Tille (2008), Goldberg and Tille (2009), Bacchetta and van Wincoop (2005), Engel (2006).

[15] Since copper exports are a large fraction of aggregate chilean exports and the main copper exporter is a state owned company, mining shipments (isic codes 10-14) are excluded from the analysis below.

these periods. Before shipping their products abroad, Chilean exporters, to be authorized by customs, must file an export authorization form.[16] This form records, among other information, the date, the value and quantity of the shipment, the exporter tax id, the destination port and country, the HS8 category of the product, and the product brand and description.[17] The form records the currency in which the transaction was settled, for which the exporter must provide the receipt. We refer to this as the currency of invoicing.

As it is typically the case with customs data, we use firm-product-destination level unit values as proxy for prices. A disadvantage of using unit values is that we cannot measure price stickiness directly, because we do not observe the frequency at which firms adjust prices. On the other hand, an important advantage of the data relative to survey data on prices is that it records values and quantities of actual transactions.

Finally, we take the period average nominal exchange rate from the IMF International Financial Statistics. Data on nominal GDP and domestic inflation are taken from the same source.

2.1.1 Summary Statistics

Table 1 provides summary statistics for the "Benchmark" and "Wine" datasets. There were over 3 million shipments between 2009 and 2011, and over a million wine shipments between 2003 and 2011. These were made by 11,596 and 816 exporters respectively. Finally, note that Chilean exporters sold a wide variety of products (almost 6000 HS8 products) to over 170 destinations during this period.

The second and third panels of the table show the distribution of price and nominal exchange rate changes used in the estimations (both computed as log differences). The distributions are plotted in Figure 1. First, note that there is significant heterogeneity in how firms change prices. Second, note that the changes in exchange rates during this period are significant relative to this variation in prices. The median change in the exchange rate in each sample is -0.04, indicating that this was a period during which currencies appreciated relative to the dollar.

Invoicing: This section summarizes some important features of the data before proceeding with the econometric analysis. First, most of the invoicing by Chilean exporters is done using the U.S. dollar, while the Chilean peso is seldom used. This is in line with

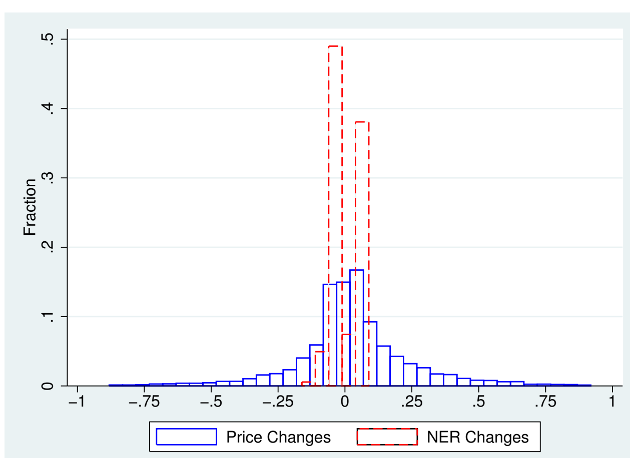
[16] More precisely, exporters need to get a "Documento Unico de Salida" or "DUS" authorized by customs to be able to get their products out of the country.

[17] HS8 is a very detailed classification system that Chilean customs uses to impose tariffs. This classification contains over 6000 products and has a level of dissagregation that is equivalent to the HS10 classification used in the US or the CN8 classification used in Europe.

Table 1: Summary Statistics

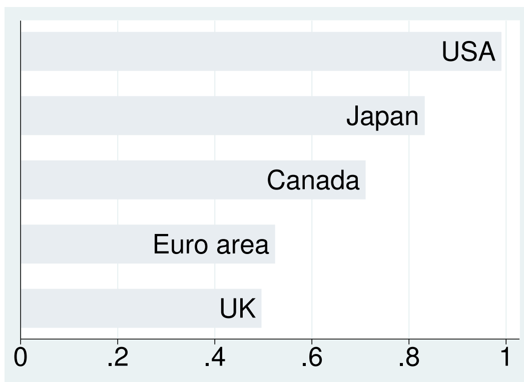
	Benchmark	Wine
Number of shipments	3,142,211	1,388,131
Number of exporters	11,596	816
Number of HS8 products	5,746	26
Number of destinations	171	140
Number of currencies used	27	16
Distribution of price changes, expressed in destination's currency*		
10%	-0.19	-0.21
25%	-0.06	-0.09
50%	0.02	0.00
75%	0.12	0.07
90%	0.29	0.18
Distribution of NER changes, destination's currency per U.S. dollar*		
10%	-0.05	-0.10
25%	-0.05	-0.09
50%	-0.04	-0.04
75%	0.05	0.01
90%	0.05	0.05

Figure 1: Distribution of Price Changes



what previous studies have found in developing countries.[18] Figure 2 shows the predominance of the dollar in selected destination countries. We can see that in some markets a significant fraction of the invoicing is done in the currency of the destination country. For instance, about 50 percent of the exports to the UK and Europe are invoiced in pounds and euros respectively. In a given destination, Chilean exporters typically use either the dollar or the destination's currency, while exports in a third currency are extremely infrequent. In addition, over 85 percent of the exporters in our sample use only one currency in a given destination during the period. This suggests that i) exporters play a major role in determining the currency in which international transactions are invoiced, and ii) exporters rarely switch currencies over time in a particular destination.

Figure 2: Share of Sales Invoiced in U.S. dollars



2.2 Empirical Strategy

This section describes the empirical strategy for estimating how exporters invoicing in different currencies respond to changes in the exchange rate. Assuming that: (i) the invoicing currency is set before the exchange rate changes, and (ii) differences in the shocks to exporters invoicing in different currencies are not correlated with the bilateral exchange rate, the response in relative quantities to changes in relative prices generated by the exchange rate can be used to identify the elasticity. Both assumptions are likely to hold in this setting. The first assumption holds since exporters do not change their invoicing currency during the period. The second assumption is also likely to hold, since exporters from the same country who sell the same product into the same destination are likely to be affected by the same set of aggregate shocks.

[18] See Goldberg and Tille (2008) for some aggregate facts on how countries invoice their exports.

We proceed by estimating the following equation at the firm-product-destination level:

$$\Delta \log Y_{fpd,t} = \beta_{dc} \times D_{fpd} \times \Delta \log NER_{d,t} + \beta_{\$} \times [1 - D_{fpd}] \times \Delta \log NER_{d,t} + \gamma Z'_{d,t} + v_{fp,t} + \gamma_d + \varepsilon_{fd,t}. \quad (1)$$

Here, $\Delta \log Y_{fpd,t}$ is the dependent variable, which can be either the log change in the price (expressed in the destination market's currency) or the quantity sold by firm f of product p into destination d in year t. $\Delta \log NER_{d,t}$ is the log change in the destination market's nominal exchange rate, expressed in units of the destination market's currency per U.S. dollar. D_{fpd} is a dummy that takes the value of 1 if the good is priced in the destination market's currency and zero if it is priced in dollars. $Z'_{d,t}$ includes controls for the change in the destination's price level and nominal GDP. γ_d is a set of destination fixed effects. $v_{fp,t}$ are firm-product-year fixed effects that control for changes in firm-product level marginal costs or demand that are common across destinations. The coefficients of interest are β_{dc} and $\beta_{\$}$, and capture the elasticity of prices or quantities to changes in nominal exchange rates for firms invoicing in the destination's currency and dollars, respectively.

To be able to interpret changes in quantities across periods and to avoid seasonality issues, we aggregate shipments by year. In addition, since we are interested in the differential response of firms that invoice in different currencies, we exclude from the sample those firms that use multiple currencies in the same destination.[19] Finally, we follow Gopinath, Itskhoki and Rigobon (2010) by focusing on product-destination pairs where multiple currencies are used.[20]

Below we present our benchmark results using both the Benchmark and the Wine datasets. To mitigate concerns about selection, the baseline regressions only include exporters that are active in a destination during each year of the sample. Subsection 2.3.3 presents robustness checks using different samples and controlling for different fixed effects.

2.3 Results

2.3.1 Exchange Rates and Prices

The results from estimating equation (1) using the change in price as the dependent variable, $\Delta \log Y_{fpd,t} = \Delta \log P_{fpd,t}$, are presented in Table 2. Columns 1 and 5 show our

[19] In particular, we only include firms-product-destination triplets for which at least 95% of the value of shipments is invoiced in the same currency.

[20] Following the criterion on Gopinath, Itskhoki and Rigobon (2010), we exclude product-destination pairs in which one currency accounts for more than 95% of the value of shipments.

benchmark results using the Benchmark and the Wine datasets respectively. Note first that the coefficient β_{dc} is not statistically different from zero in either sample. This coefficient captures the price elasticity with respect to the exchange rate for exporters that invoice using the destination's currency. Since prices are denominated in the destination's currency, a zero coefficient indicates that these firms do not change nominal prices in response to changes in the destinations' nominal exchange rate.

In contrast, the elasticity for firms invoicing in U.S. dollars, $\beta_{\$}$, is close to one in both the Benchmark and Wine samples. In fact, we cannot reject the null hypothesis that $\beta_{\$} = 1$ in either sample. This implies that nominal prices for these firms are rigid in U.S. dollars, so that these prices move one-to-one with the destination's exchange rate once they are denominated in the destination market's currency. This evidence suggests that prices are very rigid in the currency in which they are invoiced. This rigidity implies that relative prices move one-to-one with the nominal exchange rate. This result is in line with Gopinath, Itskhoki and Rigobon (2010), who document a similar finding for firms importing into the U.S. using dollars vs. non-dollars.[21]

Table 2: Exchange Rate and Prices

	Benchmark				Wine	
	(1)	(2)	(3)	(4)	(5)	(6)
β_{dc}	-0.239	-0.242	-0.062	-0.096	-0.045	-0.043
	(0.189)	(0.188)	(0.119)	(0.116)	(0.087)	(0.066)
$\beta_{\$}$	1.235***	1.112***	1.291***	1.287***	0.889***	0.911***
	(0.218)	(0.210)	(0.117)	(0.112)	(0.10)	(0.061)
Observations	9,100	9,877	9,100	9,877	9,637	21,282
Cty FE	Yes	Yes	No	No	Yes	Yes
FPY FE	Yes	Yes	No	No	Yes	Yes
Continuing	Yes	No	Yes	No	Yes	No

Note: "Cty FE" stands for destination country fixed effects. "FPY FE" stands for firm-product-year fixed effects. "Continuing" indicates that the sample includes only the continuing firms. Standard errors clustered at the firm-product-year level in parentheses. *** signif. diff. from zero at 1% level, ** signif. diff. from zero at 5% level, *** signif. diff. from zero at 10% level.

A possible interpretation for such an extreme difference in relative prices may be that firms invoicing in different currencies use intermediate inputs sourced from different countries, so that exchange rate changes affect relative marginal costs across firms.[22]

[21] Fitzgerald and Haller (2012*a*) also document extreme pricing to market (i.e., an elasticity close to zero) for Irish firms selling in pounds into the UK.

[22] Using data from Belgium, Amiti, Itskhoki and Konings (Forthcoming) argue that about half of the lack

An important characteristic of our data is that we can include fixed effects at the firm-product-year level to control for changes in marginal costs that are common across destinations. Assuming that each firm uses the same set of inputs to source every destination, the difference in the coefficients can be attributed to changes in relative markups rather than to changes in marginal costs brought forth by the exchange rate.[23]

Note also that the rigidity of prices seems to be beyond what can be explained by nominal rigidities. Table 3 repeats the regressions aggregating the Wine dataset over periods of two years.[24] Although the coefficient β_{dc} in these estimations turns out to be positive, there continues to be a significant difference between the response of firms invoicing in U.S. dollars relative to those using the destination's currency. The literature on nominal rigidities documents a median price duration of a year. In contrast, we find a difference in markups that moves one-to-one with the exchange rate over a period of a year, and that is still significant over a period of two years. Such stark responses are in line with those reported by Gopinath, Itskhoki and Rigobon (2010), and Fitzgerald and Haller (2012*a*).

Table 3: Wine Sample: Two Year Periods

	(1)	(2)
β_{dc}	0.134	0.167**
	(0.096)	(0.078)
$\beta_{\$}$	0.929***	0.961***
	(0.097)	(0.069)
Observations	6,172	11,067
Cty FE	Yes	Yes
FPT FE	Yes	Yes
Continuing	Yes	No

Note: "Cty FE" stands for destination country fixed effects. "FPT FE" stands for firm-product-time fixed effects. "Continuing" indicates that the sample includes only the continuing firms. Standard errors clustered at the firm-product-year level in parentheses. *** signif. diff. from zero at 1% level, ** signif. diff. from zero at 5% level, *** signif. diff. from zero at 10% level.

2.3.2 Exchange Rates and Quantities

We now present the results for quantities. Table 4 displays the results from estimating equation (1) using the change in quantities as the dependent variable, $\Delta \log Y_{fpd,t}$ =

of exchange rate pass-through into prices comes from this channel.

[23] Fitzgerald and Haller (2012*a*) use a similar fixed effects strategy to document pricing to market by Irish firms.

[24] Unfortunately, the Benchmark dataset does not span enough years to do this exercise.

$\Delta \log Q_{fpd,t}$. Our benchmark results are presented in columns 1 and 5 for the Benchmark and Wine samples, respectively. There is no significant response in quantities for firms invoicing with the destination market's currency, and we cannot reject that the null that the elasticity β_{dc} equals zero in either sample. These are the firms whose price in the destination market did not change in response to the exchange rate. On the other hand, the coefficient for the firms that invoice in dollars, $\beta_{\$}$, comes out negative and significant as expected. These are the firms whose price was rigid in dollars and increased in the destination market's currency when the destination's currency depreciated, as shown in Table 2. The difference in the coefficients is statistically significant and equals 1.7 in our benchmark specification. Note that although relative quantities move in the expected direction, the implied elasticity is low. As mentioned above, such low elasticities are in line with those used by the international business cycle literature to match the observed comovements between the terms of trade and the trade balance. Here, the elasticity is identified from the variation in prices across firms invoicing in different currencies in response to a change in the exchange rate.

Table 4: Exchange Rates and Quantities

	Benchmark				Wine	
	(1)	(2)	(3)	(4)	(5)	(6)
β_{dc}	-0.179	-0.400	0.291	0.446	-0.408	-0.366
	(0.751)	(0.739)	(0.453)	(0.444)	(0.348)	(0.287)
$\beta_{\$}$	-1.882**	-1.324*	-1.377***	-1.428***	-1.784***	-0.996***
	(0.826)	(0.799)	(0.435)	(0.415)	(0.464)	(0.290)
$\beta_{\$} - \beta_{dc}$	1.70**	0.92	1.668***	1.874***	1.37**	0.63
Observations	9,100	9,877	9,100	9,877	9,637	21,282
Cty FE	Yes	Yes	No	No	Yes	Yes
FPY FE	Yes	Yes	No	No	Yes	Yes
Continuing	Yes	No	Yes	No	Yes	No

Note: "Cty FE" stands for destination country fixed effects. "FPT FE" stands for firm-product-time fixed effects. "Continuing" indicates that the sample includes only the continuing firms. Standard errors clustered at the firm-product-year level in parentheses. *** signif. diff. from zero at 1% level, ** signif. diff. from zero at 5% level, *** signif. diff. from zero at 10% level.

2.3.3 Robustness

This section conducts several robustness checks for the results established in the previous two sections. In particular, we conduct the following exercises: First, we run the regressions including the entire sample of firms, instead of using only continuing firms. These results are displayed in columns 2 and 6 of Tables 2 and 4 for the Benchmark and Wine datasets, respectively. The results are robust to these alternative samples, although the difference in coefficients for the quantity regression is somewhat smaller. Second, we run the regressions controlling for different types of fixed effects. The results of these estimates are presented in columns 3 and 4 of the tables. None of these changes modify the conclusions of the previous subsections.

2.4 Invoicing and firm size

Finally, we compare the volume of exports across firms selling in different currencies by estimating the following equation:

$$logQ_{fpd} = \underbrace{D_{fpd}}_{0.67^{***}} + v_p + \gamma_d + \mu_{fpd}$$

Here, Q_{fpd} is the quantity exported by firm f of product p into destination d, while v_p and γ_d are product and destination fixed effects. As in the previous section, D_{fpd} is a dummy that takes the value of 1 if the good is priced in the destination market's currency and zero if it is priced in dollars. We estimate this equation for a cross section of firms in 2010, focusing on firms that use only one currency-per product destination, as defined above. The coefficient on D_{fpd} equals 0.67 and is highly significant. This coefficient implies that, after controlling for product and destination effects, firm's invoicing in the destination market's currency are larger on average than firms invoicing in U.S. dollars.[25]

2.5 Discussion

The data presented in this section establish that: i) relative prices across firms that invoice using different currencies fluctuate one-to-one with the exchange rate, ii) these price fluctuations can be attributed to variations in destination specific markups, as opposed to changes in firm level marginal costs that are common across destinations, and iii) relative quantities respond to the exchange rate in the expected direction, with an implied elasticity that is between -1 and -2. Such low elasticities are indicative of limited substitution following exchange rate movements. The elasticities estimated in this section are for goods in the same product category that were sourced from the same country (Chile). To the

[25]This result is related to Berman, Martin and Mayer (2012), who document that higher firm performance is associated with lower exchange rate pass-through. Our results suggest that the low pass-through may be driven by high performing firms invoicing in the destination's currency.

extent that goods from different source countries are less substitutable than goods from the same country, the expenditure switching effects across goods from different source countries would be even weaker than those implied by these estimates.

In addition, the evidence provided so far shows that exchange rate changes affect relative markups and the allocation of production across firms invoicing in different currencies. The model developed in the next section provides a framework for evaluating how these changes in nominal exchange rates translate to aggregate output per worker.

3 The Model

This section introduces a quantitative open economy model of international relative prices to study how exchange rates affect aggregate productivity.

3.1 Setup

Preliminaries: There are three countries indexed by $i = c, s, e$. Each country is inhabited by n_i agents. We normalize the world population to 1. Identical households in each country consume a final good and supply labor. In each country there is a continuum of n_i monopolistically competitive intermediate producers, each producing a differentiated tradeable good. These producers use labor as their sole input of production and differ in their productivities. The output of these intermediate producers is aggregated by consumers with a Dixit-Stiglitz CES aggregator. Differences in the producers' productivities lead to differences in how producers invoice their exports and respond to nominal exchange rate shocks. The link between exporter productivity, markups and invoicing choices is explained in detail below. In addition, in each country there is a continuum of n_i monopolistically competitive intermediate producers producing differentiated non-tradeable goods. The model is closed by assuming a rule for the path of nominal aggregate demand.[26] Exchange rates movements in the model are driven by nominal shocks. This approach has been used by Chari, Kehoe and McGrattan (2002), Kehoe and Midrigan (2008), Carvalho and Nechio (2011) and Burstein and Gopinath (2013) among others.

[26]This assumption is standard in the literature (see for example, Carvalho and Nechio 2011). All the results below can be derived for the case of an economy with a binding Cash in Advance constraint.

Households: The utility function of a household in country i is given by

$$U_{i,t} = E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{C_{i,t}^{1-\sigma}}{1-\sigma} - \frac{N_{i,t}^{1+\phi}}{1+\phi} \right],$$

where $C_{i,t}$ is consumption of the final good and $N_{i,t}$ denotes labor effort. The parameters σ and ϕ control the intertemporal elasticity of substitution for consumption and the Frisch elasticity of the labor supply, respectively. We assume that households in each country can trade a full set of state contingent nominal bonds. We omit explicit reference to different states of nature to avoid cluttering the notation. The budget constraint of the household is given by:

$$W_{i,t}N_{i,t} + \Pi_{i,t} + E_{i\$,t}B_{it} = P_{i,t}C_{i,t} + E_{i\$,t}E_t\Theta_{t,t+1}B_{i,t+1}.$$

Here, $W_{i,t}$, $P_{i,t}$ and $\Pi_{i,t}$ denote the nominal wage, the consumption price index, and aggregate profits in country i, respectively. B_{it} is a bond that pays one dollar in period t, and $\Theta_{t,t+1}$ is the nominal stochastic discount factor. $E_{ij,t}$ denotes the bilateral nominal exchange rate, expressed as units of country $i's$ currency per currency unit of country $j's$ currency. This problem gives rise to two familiar optimality conditions: an intratemporal consumption-leisure condition,

$$C_{i,t}^{\sigma} N_{i,t}^{\phi} = W_{i,t}/P_{i,t}, \tag{2}$$

and a risk sharing condition

$$\left(\frac{C_{i,t}}{C_{j,t}} \right)^{\sigma} = \kappa_0 \frac{E_{ij,t}P_{j,t}}{P_{i,t}} \equiv Q_{ij,t}, \tag{3}$$

where $\kappa_0 = \frac{P_{i,0}}{P_{j,0}} \frac{C_{i,0}^{\sigma}}{C_{j,0}^{\sigma}} \frac{1}{E_{ij,0}}$. [27]

The first condition states that households equalize the ratio of the marginal utilities between consumption and leisure to the real wage. The risk sharing condition states that the marginal utility of a dollar is equalized across countries. This means that the ratio of marginal utilities between countries i and j must equal the real exchange rate between country i and country j, denoted by $Q_{ij,t}$.

Preferences and Demands: Aggregate consumption in each country is a composite of nontradable and tradable goods. We break down this bundle in steps. First, the final good is given by $C_{i,t} = C_{iT,t}^{\alpha} C_{iN,t}^{1-\alpha}$, where $C_{iT,t}$ and $C_{iN,t}$ are bundles of tradable and nontradable

[27] The risk sharing condition is obtained by equating on the first order conditions for bond holdings between countries i and j, $\frac{P_{i,t}}{P_{i,t+1}} \frac{C_{i,t}^{\sigma}}{C_{i,t+1}^{\sigma}} \frac{E_{ij,t+1}}{E_{ij,t}} = \frac{P_{j,t}}{P_{j,t+1}} \frac{C_{j,t}^{\sigma}}{C_{j,t+1}^{\sigma}}$, and iterating backwards.

goods, respectively. The tradable good bundle is a composite of intermediate tradable goods produced in each country given by:

$$C_{iT,t} = \sum_j \left[\nu_{ji}^{\frac{1}{\xi}} C_{jiT,t}^{\frac{\xi-1}{\xi}} \right]^{\frac{\xi}{\xi-1}}.$$

Here, $C_{jiT,t}$ is a composite of intermediate tradeable goods sold from country j into country i, and ξ is the elasticity of substitution across tradable varieties. The weights that composites from different source countries receive in the aggregate bundle are given by $\nu_{ji} \equiv n_j \lambda$ and $\nu_{ii} \equiv 1 - (1 - n_i)\lambda$. This specification allows weights ν_{ji} to depend on a parameter λ that determines home bias, and on the number of goods n_j produced in the source country j. Hence, consumption shares depend directly on country size n_j. This is a tractable way of making the size of the Chilean economy arbitrarily small in the quantitative exercises below.[28] In addition, the composites $C_{jiT,t}$ are aggregate bundles of the intermediate goods produced in each country. These bundles are given by

$$C_{jiT,t} = \left[\left(\frac{1}{n_j} \right)^{\frac{1}{\xi}} \int_0^{n_j} C_{jiT,t}(f)^{\frac{\xi-1}{\xi}} df \right]^{\frac{\xi}{\xi-1}}, \tag{4}$$

where, $C_{jiT,t}(f)$ denotes consumption of good f. Finally, the nontradable bundle in each country is a composite of domestically produced intermediate goods, given by:

$$C_{iN,t} = \left[\left(\frac{1}{n_i} \right)^{\frac{1}{\rho}} \int_0^{n_i} C_{iN,t}(f)^{\frac{\rho-1}{\rho}} df \right]^{\frac{\rho}{\rho-1}},$$

where ρ is the elasticity of substitution across nontradable intermediate goods.

Cost minimization implies that demands for tradable and non-tradable goods are given by:

$$C_{iT,t} = \frac{\alpha P_{i,t} C_{i,t}}{P_{iT,t}}; \quad C_{iN,t} = \frac{(1-\alpha) P_{i,t} C_{i,t}}{P_{iN,t}};$$

where $P_{iT,t}$ and $P_{iN,t}$ are the price indexes for tradable and nontradable consumption. Demand for goods originating in country j is:

$$C_{jiT,t} = \nu_{ji} \left[\frac{P_{jiT,t}}{P_{iT,t}} \right]^{-\xi} C_{iT,t},$$

[28]This specification has previously been used in Sutherland (2005) and Paoli (2009).

The demands intermediate goods are:

$$C_{jiT,t}(f) = \frac{1}{n_j}\left[\frac{P_{jiT,t}(f)}{P_{jiT,t}}\right]^{-\xi} C_{jiT,t}; \quad C_{iN,t}(f) = \frac{1}{n_i}\left[\frac{P_{iN,t}(f)}{P_{iN,t}}\right]^{-\rho} C_{iN,t};$$

where $P_{jiT,t}(f)$ is the retail price in country i of good f produced in country j, $P_{jiT,t}$ is the ideal consumer price index for goods sold in from country i into country j, and $P_{iN,t}(f)$ is the price of nontradable intermediate f in country i. The relative consumption between goods being sourced from different countries is given by:

$$C_{ijT,t}/C_{jjT,t} = \left(P_{ijT,t}/P_{jjT,t}\right)^{-\xi}.$$

I refer to changes in this ratio following a change in the nominal exchange rate $E_{ij,t}$ as expenditure switching effects. These are determined by changes in retail prices and the elasticity of substitution ξ.

Pricing: There are two sources of price rigidities in the model. First, producer prices are sticky in the currency in which they are invoiced. Second, in the tradeable sector, markups are endogenous and can respond to shocks. Differences in how desired markups respond to exchange rate shocks lead to differences in invoicing choices. We introduce endogenous variable markups in the tradable sector in a tractable way by assuming that competitive retailers must combine the intermediate tradeable good with nontradable distribution services in fixed proportions to deliver the good to consumers.

Retailers: The retailer needs η units of a non-tradeable good composite to deliver each intermediate tradeable good to the consumers.[29] This composite of distribution services is given by $D_{iN,t} = \left[\left(\frac{1}{n_i}\right)^{\frac{1}{\rho}} \int_0^{n_i} D_{iN,t}(f)^{\frac{\rho-1}{\rho}} df\right]^{\frac{\rho}{\rho-1}}$, and its price is given by $P_{iN,t}$. The amount of each nontradable good f used for distribution services in country i is given by $D_{iN,t}(f) = \frac{1}{n_i}\left[\frac{P_{iN,t}(f)}{P_{iN,t}}\right]^{-\rho} D_{iN,t}$.

The retail price in country j for tradable good f produced in country i is given by:

$$P_{ijT,t}(f) = P^{p}_{ijT,t}(f) + \eta P_{jNt}. \tag{5}$$

Here, $P_{ijT,t}(f)$ is the consumer price of good f in country j, $P^{p}_{ijT,t}(f)$ is the producer price of good f denoted in $j's$ currency, and η controls the share of distribution costs in the consumer's price. Note that distribution services use nontradables from the destination

[29]One interpretation is that all goods use the same "shelf space", regardless the technology used for production. This way of generating endogenous variables markups was first introduced by Corsetti and Dedola (2005).

country. Since the response of markups in the non-tradeable sector is beyond the scope of the paper, we assume that distribution services are not required for non-tradeables, so that markups in the nontradeable sector are constant. Below we describe how the introduction of distribution costs affects the pricing problem of the producer in the tradeable sector.

Flexible prices: We start by solving for the optimal price set by producers in the tradeable sector when tradeable prices are flexible. Intermediate producers operate a linear technology $Y_{iT,t}(f) = z(f) N_{iT,t}(f)$, where $Y_{iT,t}(f)$, $N_{iT,t}(f)$ and $z(f)$ denote output, labor inputs, and the productivity of producer f in country i respectively. Since the production function is CRS, the pricing problems in each location are separable. The problem of producer f from country i selling into country j is given by:

$$\max_{P^{pl}_{ijT,t}(f)} \left[P^{p}_{ijT,t}(f) E_{ij,t} - \frac{W_{i,t}}{z(f)} \right] \left[\frac{P_{ijT,t}(f)}{P_{ijT,t}} \right]^{-\xi} C_{ijT,t}.$$

The optimal flexible price expressed in the producer's currency is a markup over the marginal costs

$$P^{p}_{ijT,t}(f) E_{ij,t} = \mu_{ij,t}(f) \frac{W_{i,t}}{z(f)},$$

where the markup is given by:

$$\mu_{ij,t}(f) \equiv \frac{\xi}{\xi - 1} \left[1 + \frac{\eta}{\xi} \frac{z(f) P_{jN,t} E_{ij,t}}{W_{i,t}} \right].$$

To provide intuition on why markups $\mu_{ij,t}(f)$ vary by source country, destination and producer, note that the elasticity of demand faced by the producer is:

$$\varepsilon_{ij,t}(f) \equiv -\frac{d \log C_{ijT,t}(f)}{d \log P^{p}_{ijT,t}(f)} = \xi \left(1 - s_{ij,t}(f) \right),$$

where $s_{ij,t}(f) \equiv \frac{\eta P_{jNt}}{P^{p}_{ijT,t}(f) + \eta P_{jNt}}$ is the share of distribution services in the consumer price. Note also that the elasticity of markups with respect to the relative producer price to distribution services is given by:

$$\Gamma_{ij,t}(f) \equiv -\frac{d \log \mu_{ij,t}(f)}{d \log P^{p}_{ijT,t}(f) - dlog P_{jN,t} E_{ij,t}} = \left[(\xi - 1) \frac{1 - s_{ij,t}(f)}{s_{ij,t}(f)} - 1 \right]^{-1}.$$

Markups depend on the demand elasticity with respect to the producer price, $\varepsilon_{ij,t}(f)$, which depends on the share that the producer price has in the price paid by consumers $s_{ij,t}(f)$. This share is determined by the firm's productivity $z(f)$. More productive firms have lower marginal costs, higher $s_{ij,t}(f)$, and higher desired markups $\mu_{ij,t}(f)$. In addition, these firms have a higher markup sensitivity to changes in the producer price relative to distribution services, $\Gamma_{ij,t}(f)$. How differences in $\Gamma_{ij,t}(f)$ affect the invoicing decision is described below.

Finally, with flexible prices, firms set the same price regardless of the currency that is used for invoicing. That means that the flexible price for a producer that invoices in currency l, $P^{pl}_{ijT,t}(f)$, is given by:

$$P^{pl}_{ijT,t}(f)\,E_{lj,t} = \mu_{ij,t}(f)\,\frac{W_{i,t}}{z(f)}.$$

Nominal rigidities: We now introduce time-dependent nominal rigidities. In particular, intermediate producers in the tradable sector can reset their price with probability $1-\theta_T$, and producers in the nontradable sector reset their prices with probability $1-\theta_N$. We assume that producer prices are rigid in the currency in which they are invoiced. This gives rise to the familiar log-approximation to the pricing equation:

$$\bar{p}^{pl}_{ijT,t}(f) = (1-\beta\theta_T)\sum_{k=0}^{\infty}(\beta\theta_T)^k\,E_t\left[\tilde{p}^{pl}_{ijT,t+k}(f)\right].$$

Here $\bar{p}^{pl}_{ijT,t}(f)$ is the log of the reset producer price of firm f selling from country i to country j invoicing in currency l. $\tilde{p}^{pl}_{ijT,t+k}(f)$ is the log of the price that the firm would set if prices were flexible, which to a first order approximation is given by:

$$\tilde{p}^{pl}_{ijT,t+k}(f) = \frac{1}{1+\Gamma_{ij}(f)}\left[w_{i,t+k} - e_{il,t+k} + \Gamma_{ij}(f)\left[p_{jN,t+k} + e_{lj,t+k}\right]\right] + const_{ijT}. \quad (6)$$

Where we used lowercase to denote the log of a variable and $const_{ijT}$ contains steady state values. Finally, retail prices are flexible.

Currency Choice: We now derive the optimal currency choice of an exporter selling abroad. We follow Engel (2006) and Gopinath, Itskhoki and Rigobon (2010) and take a second order approximation to the exporter's profit function to write the difference in the value of a firm that is choosing between two currencies. Whereas these two papers have focused on the choice between invoicing in the producer currency (PCP) and local currency (LCP), we present a more general formulation in which firms can choose between any set of currencies.

We evaluate the problem of a firm from country i that is selling into county j, and is choosing between invoicing currencies a and b. The difference in discounted profits from choosing currency a vs. currency b, $L_{ij,t}^{a-b}$, is given by:

$$L_{ij,t}^{a-b} = E_t \sum_{l=0}^{\infty} (\beta\theta)^l \left[\Pi_{i,t+l}\left(\bar{p}_{ijT,t}^{pa}\right) - \Pi_{i,t+l}\left(\bar{p}_{ijT,t}^{pb} + e_{ab,t+l}\right)\right]. \tag{7}$$

Here, $\Pi_{i,t+l}$ denotes real profits, weighted by the marginal utility of consumption in country i, in period $t+l$. The firm will choose currency a over currency b when this difference is greater than zero. Following the steps in Gopinath, Itskhoki and Rigobon (2010), in Appendix B we characterize this decision when nominal exchange rates follow a random walk. In particular, if nominal exchange rates follow a random walk, the difference in discounted profits between invoicing in currency a and currency b can be written as:

$$L_{ij,t}^{a-b} = -\frac{1}{2}\tilde{\Pi}_{pp} \sum_{l=0}^{\infty} (\beta\theta)^l \, l var_t\left(\Delta e_{ba,t+l}\right) \left[1 - \frac{2cov\left(\tilde{p}_{ijT,t+l}^{pa}(f), e_{ab,t+l}\right)}{var_t\left(\Delta e_{ab,t+l}\right)}\right].$$

Where $\tilde{\Pi}_{pp}$ is the second derivative of profits evaluated at the date t flexible price. Since, $-\tilde{\Pi}_{pp} > 0$, the firm will choose currency a over currency b iff $L_{ij,t}^{a-b} > 0$, that is:

$$\bar{\Psi}_{ij,t}^{a-b}(f) = \quad (1-\beta\theta)\sum_{l=1}^{\infty}(\beta\theta)^{l-1} \, l \frac{cov\left(\tilde{p}_{ijT,t+l}^{pa}(f), \Delta e_{ab,t+l}\right)}{var_t\left(\Delta e_{ba,t+l}\right)} < \frac{1}{2} \quad . \tag{8}$$

This results is a natural generalization of Proposition 2 in Gopinath, Itskhoki and Rigobon (2010) to a choice between any pair of currencies. Condition (8) states that firms fix prices in currency a when a weighted average of the desired responses of prices in currency a to changes in the exchange rate between a and b, $cov\left(\tilde{p}_{ijT,t+l}^{pa}(f), \Delta e_{ab,t+l}\right)$, is low. As noted in Engel (2006), invoicing is a discrete binary decision. Intuitively, the condition states that if, absent nominal rigidities in the tradeable sector, the exporter's optimal price in expressed in currency a would not respond to change in the exchange rate -$cov\left(\tilde{p}_{ijT,t+l}^{pa}(f), \Delta e_{ab,t+l}\right)$ is low-, then the firm will set prices in currency a. Note from equation (6) that $\tilde{p}_{ijT,t+l}^{pa}(f)$ depends on firm's productivity through $\Gamma(f)$. In Section 4.1, we write $\bar{\Psi}_{ij,t}^{a-b}(f)$ and the currency choice as a function of the model's parameters under stronger assumptions.

Non-Tradeable goods: Producers in the non-tradeable sector are homogeneous and operate a linear technology $Y_{iN,t}(f) = N_{iN,t}(f)$. We assume that non-tradeable goods do not require distribution services, so the optimal flexible price for non tradeable producers is a constant markup over the marginal costs $P_{iN,t}(f) = \mu_N W_{i,t}$. Producers in the nontradable sector reset their prices with probability $1-\theta_N$. Hence, in the non-tradable sector the pricing rule in logs given by:

$$\bar{p}_{iN,t}(f) = (1-\beta\theta_N)\sum_{k=0}^{\infty}(\beta\theta_N)^k E_t\left[\tilde{p}_{iN,t+k}(f)\right],$$

where $\tilde{p}_{iN,t+k}(f) = w_{i,t+k} + const_{iN}$, where $const_{ijN}$ contains steady state values.

Money supply: The law of motion for the path of nominal consumption follows: $\log P_{i,t}C_{i,t} = m_{it}$, where $\Delta m_{it} \sim N(0, \sigma_{m_i})$. This assumption is common in the literature (see for example, Carvalho and Nechio 2011).[30]

Market clearing: Goods market clearing in intermediate tradeable goods implies:

$$Y_{iT,t}(f) = \sum_j n_j C_{ijT,t}(f).$$

Goods market clearing in intermediate non-tradeable goods implies:

$$Y_{iN,t}(f) = n_i C_{iN,t}(f) + D_{iN,t}(f),$$

where the total use of the distribution service bundle is given by $D_{iN,t} = \frac{1}{\eta}\sum_j \int_0^{n_j} n_i C_{ji,t}(f)\,df$.

The amount of labor used in the tradable sector is given by:

$$n_i N_{iT,t} = \int_0^{n_i} N_{iT,t}(f)\,df,$$

Using the production function and market clearing in intermediate tradeable goods, we can write this condition as:

$$N_{iT,t} = \frac{1}{n_i}\sum_j n_j C_{ijT,t} V_{ijT,t}, \tag{9}$$

where $V_{ijT,t} \equiv \left[\frac{1}{n_i}\int_0^{n_i}\left[\frac{p_{ijT,t}(f)}{P_{ijT,t}}\right]^{-\xi}\frac{1}{z(f)}df\right]$ is a term capturing the dispersion in tradable prices. The amount of labor used in the non-tradable sector is given by:

$$n_i N_{iN,t} = \int_0^{n_i} N_{iN,t}(f)\,df,$$

[30] Alternatively, the results below can be derived for the case of log utility and a binding Cash in Advance constraint.

we can rewrite this condition as:

$$N_{iN,t} = \left[C_{iN,t} + D_{iN,t}\right] V_{iN,t},$$

with $V_{iN,t} \equiv \left[\frac{1}{n_i}\int_0^{n_i}\left[\frac{p_{iN,t}(f)}{P_{iN,t}}\right]^{-\rho} df\right]$. Labor market clearing implies:

$$N_{i,t} = N_{iT,t} + N_{iN,t}.$$

Equilibrium: An equilibrium in this economy is a set of allocations for households in each country $\{C_{i,t}, N_{i,t}, B_{i,t}\}_{\forall_i}$, $\{C_{iT,t}, C_{iN,t}, C_{jiT,t}\}_{\forall_{i,j}}$ and $\{C_{jiT,t}(f), C_{iN,t}(f)\}_{\forall_{i,j,f}}$; allocations and prices and currency choices for intermediate producers in each country, $\{Y_{iT,t}(f), Y_{iN,t}(f), N_{ijT,t}(f), N_{iN,t}(f)\}_{\forall_{i,j,f}}$ and $\left\{\bar{p}^{pl}_{ijT,t}(f), P_{iN,t}(f)\right\}_{\forall_{i,j,f,l}}$; allocations and prices for retailers in each country, $\{D_{iN,t}\}_{\forall_i}$, $\{D_{iN,t}(f)\}_{\forall_{i,f}}$ and $\{P_{ijT,t}(f)\}_{\forall_{i,j,f}}$; aggregate prices $\{P_{i,t},\}_{\forall_i}$, $\{P_{iT,t}, P_{ijT,t}, P_{iN,t}\}_{\forall_{i,j}}$, $\Theta_{t,t+1}$, $\{W_{i,t}\}_{\forall_i}$ and $\{E_{ij,t}\}_{\forall_{i,j}}$ such that: i) households allocations maximize utility subject to their budget constraint, ii) competitive retailers set prices according to (5), iii) intermediate producers in the tradable and non tradable sector set their prices and choose the invoicing currency to maximize profits, iv) markets clear and, vi) the path for the nominal consumption satisfies the specification above.

The model is solved by log-linearizing the equilibrium conditions around the steady-state and solving the resulting system of linear difference equations.

3.2 Exchange Rates and Productivity

This section describes how exchange rate fluctuations affect markup dispersion and productivity in the model.

3.2.1 Measuring Aggregate Productivity

We define the the change in productivity as the change in output per worker in the tradable sector, given by:

$$TFP_{iT,t}/TFP_{iT,0} = \left[RGDP_{iT,t}/N_{iT,t}\right] / \left[RGDP_{iT,0}/N_{iT,0}\right],$$

where $RGDP_{iT,t}$ is real GDP in the tradable sector. We compute $RGDP_{iT,t}$ following as closely as possible the procedures used in the United States' National Income and Product

Accounts (NIPA) by the Bureau of Economic Analysis to compute real GDP.[31] In particular, we use a Fisher formula, which is a geometric average of a Laspeyres and a Paasche quantity index. For example, real GDP in period t relative to period $t-1$ is given by

$$\frac{RGDP_{iT,t}}{RGDP_{iT,t-1}} = \left[\frac{\sum_j n_j \int_0^{n_i} p_{ijT,t-1}(f)\, c_{ijT,t}(f)\, df}{\sum_j n_j \int_0^{n_i} p_{ijT,t-1}(f)\, c_{ijT,t-1}(f)\, df} \times \frac{\sum_j n_j \int_0^{n_i} p_{ijT,t}(f)\, c_{ijT,t}(f)\, df}{\sum_j n_j \int_0^{n_i} p_{ijT,t}(f)\, c_{ijT,t-1}(f)\, df} \right]^{0.5}, \tag{10}$$

where $p_{ijT,t-1}(f)$ and $c_{ijT,t}(f)$ denote prices and quantities in period t of the detailed components of GDP.[32] The first term in expression (10) is a Laspeyres quantity index (based on $t-1$ prices), while the second term is a Paasche quantity index (based on t prices).[33] Real GDP in period L relative to period 0 is given by:

$$\frac{RGDP_{iT,L}}{RGDP_{iT,0}} = \prod_{t=1}^{L} \frac{RGDP_{iT,t}}{RGDP_{iT,t-1}}. \tag{11}$$

We assume there are two types of firms; z_H and z_L. Using the equilibrium conditions, we show in Appendix A that the log-linearized versions of (9) and (10) can be combined as:

$$\hat{a}_{iT,t} = r\hat{g}dp_{iT,t} - \hat{n}_{iT,t} = -\hat{v}_{iT,t},$$

where $\hat{x}$ denotes log deviations from the non-stochastic steady state. Here $\hat{a}_{iT,t}$ denotes the log change in tradable productivity. In addition, in the Appendix A we show that $\hat{v}_{iT,t}$ can be written as:

$$\hat{v}_{iT,t} \equiv \sum_j \nu_{ij} \hat{v}_{ijT,t}. \tag{12}$$

where

$$\hat{v}_{ijT,t} \equiv -\xi \left(\omega^v - \omega\right) \nu_{ij} \left[\hat{p}_{ijT,t}(H) - \hat{p}_{ijT,t}(L)\right]. \tag{13}$$

Here $\omega \equiv \left[1 + \left(\frac{1-\kappa}{\kappa}\right)\left[\frac{P_{ijT}(L)}{P_{ijT}(H)}\right]^{1-\xi}\right]^{-1}$ and $\omega^v \equiv \left[1 + \left(\frac{1-\kappa}{\kappa}\right)\left[\frac{P_{ijT}(L)}{P_{ijT}(H)}\right]^{1-\xi} \frac{P_{ijT}(H)}{P_{ijT}(L)} \frac{z(H)}{z(L)}\right]^{-1}$, where κ is the share of $z(H)$ firms.

[31] See, e.g. Concepts and Methods of the U.S. National Income and Product Accounts (2009). The procedures that we consider are broadly consistent with the recommendations by the United Nations in their System of National Accounts.

[32] See Burstein and Cravino (2013) for a more detailed discussion of these measures.

[33] The implicit GDP deflator is calculated as the ratio of current-dollar GDP to real GDP, $\left(\sum p_t q_t / \sum p_{t-1} q_{t-1}\right) / \left(RGDP_t / RGDP_{t-1}\right)$, which is equal to a geometric average of a Laspeyres and a Paasche *price* index.

3.2.2 Discussion

Exchange rate fluctuations affect aggregate productivity by generating changes in relative prices across firms within a country that are not related to changes in marginal costs. Exchange rate fluctuations affects relative prices through three different channels. First, since producer prices are sticky in the currency in which they are invoiced, relative prices across firms invoicing in different currencies fluctuate with the exchange rate. Second, the reset price $\bar{p}$ differs across exporters with different markup elasticity Γ. Finally, there is dispersion originating from the staggered price adjustment caused by the price stickiness.

Equation (12) shows how this change in prices translates into aggregate productivity. The effect of a change in relative prices driven by an exchange rate shock depends on trade shares, ν_{ij}, the share of firms of each type, κ, the initial dispersion in prices relative to productivities, $\frac{P_{ijT}(L)}{P_{ijT}(H)}\frac{z(H)}{z(L)}$, and how price changes affects relative quantities (captured in ξ).

Consider an appreciation of the euro against all currencies. Distribution costs in Europe increase relative to production costs in Chile following the appreciation, so all Chilean firms exporting to Europe increase markups. The effects are larger for more productive firms, since they have a higher markup elasticity Γ with respect to the exchange rate. In addition, since prices are sticky and firms invoice in different currencies, an appreciation of the euro increases relative markups of firms invoicing in euros relative to those invoicing in U.S. dollars. How exchange rate movements affect productivity depends on how invoicing and desired markups correlate with the initial markup dispersion. This implies that productivity can move in either direction in response to an exchange rate shock depending on whether the shock magnifies or reduces the initial markup dispersion. In the next section, we calibrate the model using the Chilean data and evaluate the strength of these mechanisms and quantify on how changes in nominal exchange rates affect productivity in the tradable sector, $\hat{a}_{iT,t}$, in Chile.

4 Quantitative Results

In this section, we parameterize the model using the Chilean data and evaluate the impact of exchange rate movements on aggregate output per worker. In what follows, we describe what aspects of the data identify the key parameters in our model. We next present our baseline quantitative results. Finally, we conduct alternative parameterizations and sensitivity analyses to show the importance of different assumptions regarding invoicing for the effects of exchange rates in productivity.

4.1 Parameterization

We parameterize the model assuming that there are two types of firms, z_H and z_L. Then, the parameters that we must choose are the elasticity of substitution across varieties, ξ, the ratio of productivities across firms, $z_r = z_H/z_L$, the share of z_H firms, the steady state share of distribution costs in the retail price, $\frac{\eta P_N}{P_T}$, the degree of price stickiness in the tradable and nontradable sectors, θ_T and θ_N, the share of goods that are exported, λ, and the relative country sizes, n_i. We also need to assign values for the parameters in the utility function σ and ϕ, the share of nontradables in consumption α, and the discount factor β. We now provide an overview of our baseline parameterization procedure, the results of which are summarized in Table 5.

The calibration of most of these parameters is standard. We take the consumption, output, and trade shares in manufacturing for Chile from the OECD-STAN Input-Output Database. This results in setting $\alpha = 0.37$ and $\lambda = 0.42$. We set the country sizes to $n_c \to 0$, and $n_s = 0.52$, $n_e = 0.48$, so that the size of Chile in the world economy is negligible and to match the share of Chilean manufacturing exports to the US and Europe respectively. Since we use unit values as proxy for prices, we cannot observe the frequency of price changes in our data. We set the price stickiness parameters θ_N and θ_T equal 0.80, which imply a median price greater than two years.[34]

There are 3 key parameters that need to be jointly calibrated. These are the elasticity of substitution ξ, the relative productivities between firms, z_r, and the share of distribution costs in the final price $\frac{\eta P_N}{P_T}$. We choose these parameters to target the following three moments: i) the response of relative quantities to the exchange rate, ii) the relative size of firms invoicing in different currencies, and iii) the average share of distribution costs in the retail price. We set the first moment to 1.5, which is within the range of our estimates on Section 2.3.2. The relative size of firms invoicing in different currencies is set so that $lnQ_H/Q_L = 0.67$ in the non-stochastic steady state, in line with our estimate on Section 2.4. Finally, we take the average share of distribution costs in the retail price from the literature and set it equal to 0.5.

Currency Choice: We choose the remaining parameters to match the proportion of Chilean firms invoicing in each currency in the US and Europe. We first set the parameters in the utility function to $\sigma = 1$ and $\phi = 0$ to be able to characterize the optimal invoicing

[34] While these values are high relative to the literature, a well-known feature of models with nominal price stickiness is that persistent movements in international relative prices are hard to generate (see Chari, Kehoe and McGrattan 2002). Note, however, that we will be matching the response of relative quantities to exchange rate movements directly, which is what determines the movement in productivity in equation (12). The values of θ_N and θ_T will affect the persistence of the effect of the exchange rate shock on productivity.

Table 5: Parameterization

Parameter	Value	Parameter	Value
n_c, n_s, n_e	$0, 0.52, 0.48$	$\kappa_{cs}, \kappa_{ce}, \kappa_{cc}$	$1, 0.38, 1$
σ	1	ξ	3.38
ϕ	0	z_r	2
θ	0.80	λ	0.42
θ_N	0.80	α	0.37
$\frac{\eta \bar{P}_N}{P_T(L)}$	0.45	$\frac{\eta \bar{P}_N}{P_T(H)}$	0.55

choices analytically. These choices ensure that exchange rates and nominal wages are determined by the paths for nominal consumption (and that, if the monetary shocks are random walks, then the exchange rates are random walks). Given the invoicing choices, the values of these parameters do not affect the response of productivity for a given change in relative prices (see equation (12)).

We show in Appendix B that in this case, we can write $\bar{\Psi}_{ij,t}^{a-b}(f)$, and hence the currency choice, as a function of the properties of the path for nominal consumption, $m_{i,t}$, and the markup elasticity $\Gamma(f)$. In particular, if $\phi = 0$ and $\sigma = 1$, $\bar{\Psi}_{ij,t}^{a-b}(f)$ is constant and is given by:

$$\begin{aligned} \tilde{\Psi}_{cj}^{a-b}(f) &= \frac{cov\left(\Delta m_{j,t}, \Delta m_{a,t} - \Delta m_{b,t}\right)}{\sigma_{ma}^2 + \sigma_{mb}^2} - \frac{\theta\Gamma(f)}{1+\Gamma(f)} \frac{cov\left(\Delta m_{j,t}, \Delta m_{a,t} - \Delta m_{b,t}\right)}{\sigma_{ma}^2 + \sigma_{mb}^2} \\ &+ \frac{\sigma_{ma}^2}{\sigma_{ma}^2 + \sigma_{mb}^2} - \frac{cov\left(\Delta m_{j,t}, \Delta m_{a,t}\right) - cov\left(\Delta m_{j,t}, \Delta m_{b,t}\right)}{\sigma_{ma}^2 + \sigma_{mb}^2}. \end{aligned} \tag{14}$$

Equation (14) states that, under the specified assumptions, the currency choice is a function of aggregate factors (the variance of the money supply in each country, σ_{mi}^2) and firm specific factors (captured in $\Gamma(f)$). All things equal, firms will tend to invoice in currencies that have lower variance. Also, other things equal, firms with larger productivity (and hence larger $\Gamma(f)$) are more likely to invoice in the destination's market currency. In the customs data, the share of Chilean firms using the dollar to sell into the U.S. equals one, while the share of firms using euros when exporting to Europe equals 0.38. We show in Appendix B that, given the values of $\Gamma(f)$, we can choose the volatility of the money supply in each country so that all Chilean firms price in dollars in the US, while in Europe the z_H firms price in Euros and the z_L firms price in dollars.[35] We set the share of firms

[35] This amounts to setting $\sigma_{m\$}^2 < \sigma_{mp}^2$, and $\left[1 - \frac{2\theta\Gamma(H)}{1+\Gamma(H)}\right] < \sigma_{m\$}^2 / \sigma_{me}^2 < \left[1 - \frac{2\theta\Gamma(L)}{1+\Gamma(L)}\right]$.

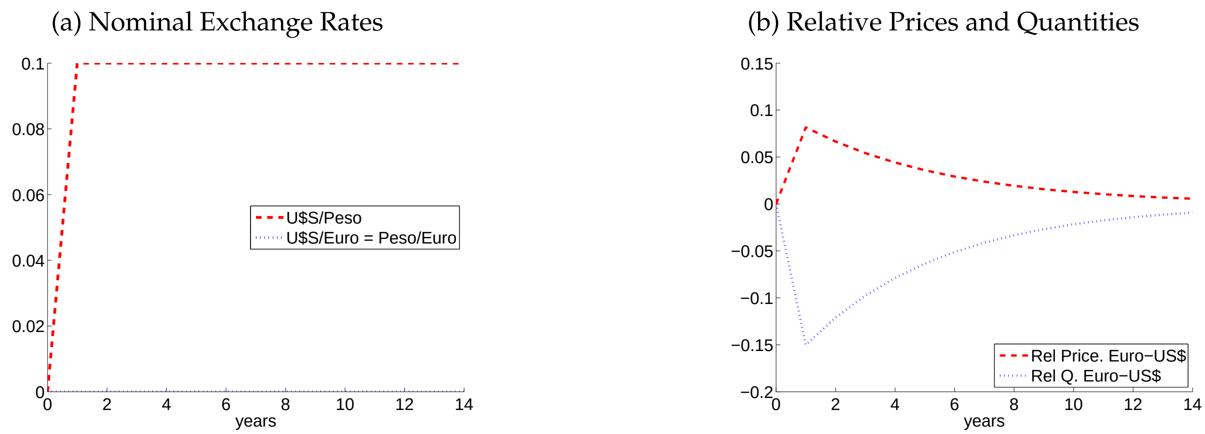
with productivity z_H, κ_{ce}, equal to 0.38 to match the share of firms invoicing in euros when exporting to Europe in the data. Since we do not observe the currency used to sell into the domestic country, we impose that Chilean firms use the Chilean peso when selling into Chile. We also impose that all U.S. firms invoice in dollars and that all European firms invoice in euros in every destination. We will not be focusing on how invoicing affects productivity in these countries.

4.2 Baseline Results

We simulate a change in the European money supply m_e that generates a permanent appreciation of the euro against all other currencies. The first panel of Figure 3 depicts the nominal exchange rate shock. The responses of relative prices and quantities of Chilean firms invoicing in different currencies are displayed in the second panel. The dashed red line displays the change in the relative price of firms invoicing in euros relative to those invoicing in U.S. dollars. The dotted blue line shows the corresponding relative quantities. The shock increases the relative price of firms invoicing in euros relative to those invoicing in U.S. dollars, and decreases the relative quantities between these two types of firms. Since the elasticity is low in the baseline calibration, the resulting change in quantities is small. This corresponds to the limited expenditure switching effect described in first part of the paper. Note that the persistence of the change in prices is lower than in the data. This is a common feature of models with sticky prices. One way to generate higher persistence would be to introduce an even larger degree of price stickiness. In that case, the resulting change in productivity would be even larger and more persistent than those in the results reported below.

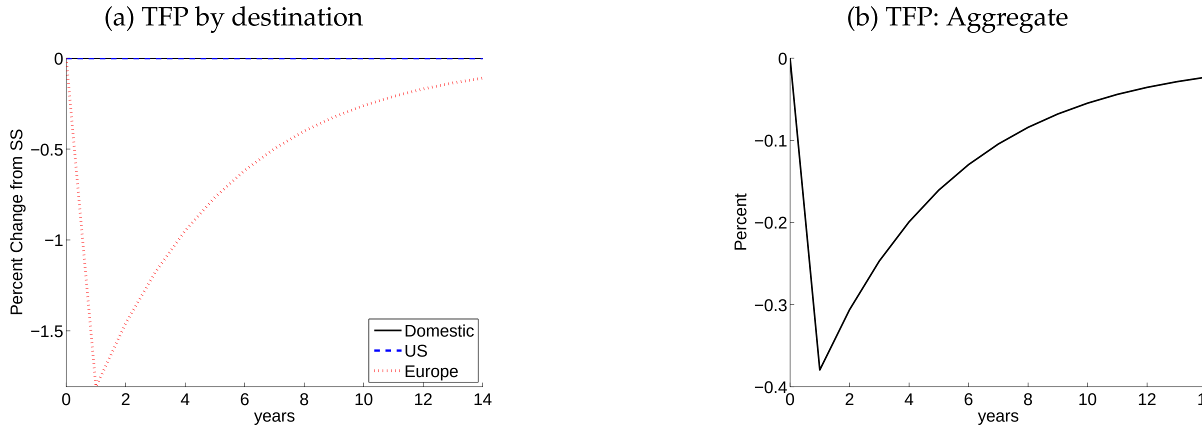
Exchange rates and productivity: Figure 4 shows the response of Chilean productivity in the tradable sector to the change in the European exchange rate. The left panel decomposes the change in productivity corresponding to sales into each destination j, given by $\hat{v}_{cj}$ in equation (13). The y-axis shows the deviation in productivity from the initial steady state. Consider first the change in productivity corresponding to exports into Europe, $\hat{v}_{ce}$. Following an appreciation of the euro there are two effects. First, a fraction $1 - \theta_T$ of exporters cannot change their nominal price, so the markups of firms invoicing in U.S. dollars decline relative to the markups of the firms invoicing in euros. Second, as the appreciation of the euro increases the share of distribution costs in Europe, all Chilean firms increase markups, in particular large firms. Since large firms are precisely those invoicing in euros, these two effects reinforce each other in increasing the relative markups of the larger firms. Since large firms had higher markups before the exchange rate shock, the

Figure 3: Exchange Rate Shock, Baseline Calibration



shock increases the initial dispersion in markups, generating the drop in productivity as shown in the dotted red line depicted in the Figure. In contrast, markup dispersion in the U.S. and Chile is not affected much, since all Chilean exporters invoice in the same currencies in those countries (i.e. all exporters invoice in dollars in the US and in Pesos in Chile). The resulting aggregate effect is obtained from equation (12) and is depicted in the right side panel of the figure. Output per worker falls by almost 0.4 percent on impact and is still 0.3 percent below the initial steady a year after the shock. Note that a depreciation of the euro would close the initial dispersion in markups and have the opposite effect on productivity.

Figure 4: Exchange Rates and Productivity



4.3 Alternative Parameterizations

We now evaluate the role of variable markups and heterogeneity in invoicing in driving these results. For each of the following exercises, we recalibrate the entire model to be consistent with the corresponding assumptions.

Endogenous variable markups: We first solve a version of the model with multiplicative distribution costs to analyze the importance of variable markups.[36] Under this assumption, markups are constant, with the only effects of the monetary shock on productivity being those arising from the staggered price setting. We repeat the counterfactual exercise and show the results in Figure 5. This is the case depicted by the dashed light blue line labeled "no variable markups." Note that the productivity losses in this case are minuscule. It is worth emphasizing that these are the losses typically studied in the literature. By starting from an inefficient allocation, heterogeneity in markups makes the effects of exchange rates in productivity an order of magnitude larger.

Flexible tradable prices: We now analyze the role of sticky prices in the tradable sector. Even without sticky prices in the tradable sector, changes in the nominal exchange rate generate a change in productivity because of the markup dispersion induced by the

[36] In particular, we assume that the consumer price is given by $P_{ijT,t}(f) = \left[P^{p}_{ijT,t}(f)\right]^{1-\eta}\left[P_{jNt}\right]^{\eta}$. In this case, the share of distribution costs in the retail price is constant and all firms set the same constant markup $\xi/(\xi-1)$.

endogenous markup channel. This is depicted by the dotted blue line in Figure 5. The figure shows, however, that price stickiness in the currency of invoicing is important: the change in productivity when tradeable prices are non-sticky is only a third of those in the baseline calibration.

Heterogeneity in invoicing: Figure 5 shows the case in which there is no heterogeneity in invoicing. We consider the cases in which all Chilean firms export using either euros or dollars.[37] These correspond to the circled and crossed lines in Figure 5. The figure shows that the response in either case is about four times smaller than in the benchmark scenario. As in the case of flexible tradable prices, exchange rates still affect markup dispersion through the endogenous markup channel despite their lack of heterogeneity in invoicing. However, the effects are even smaller than with flexible tradable prices, since only a small fraction of the firms have the opportunity to reset markups in response to the exchange rate under this specification.

Random invoicing: Finally, we repeat the counterfactual in an environment of multiple invoicing currencies, but where invoicing is uncorrelated to desired markups. That is, the calibration ignores the optimal invoicing choice and randomly assigns firms into one of the two currencies. This case is shown by the dashed red line in Figure 5. The response of productivity under this scenario is significantly smaller, as firms that increase markups are not necessarily those that had higher markups before the change in the exchange rate.

Role of elasticity: Finally, we recalibrate the model assuming an elasticity of quantities to the exchange rate equal to 4. Such high elasticities are common in the international trade literature (typically using trade elasticities in the range of 5 and 10, see Eaton and Kortum 2002). Figure 6 shows the results. Not surprisingly, the first panel shows that the expenditure switching effects of exchange rates would be much higher under a higher elasticity. The second panel shows the response of aggregate output per worker. A higher trade elasticity implies a higher elasticity of substitution ξ. Then, for a given change in relative quantities, the response in productivity would be smaller.

4.4 Discussion

The counterfactual parameterizations in this section show that models that ignore heterogeneity in invoicing, endogenous variable markups, or the correlation between markups and invoicing, greatly understate how changes in the exchange rate affect productivity. The intuition is that if firms invoice in different currencies and prices are sticky, changes

[37] That is, we override the optimal currency choice and force firms to invoice in one of the two currencies.

Figure 5: Alternative Parameterizations

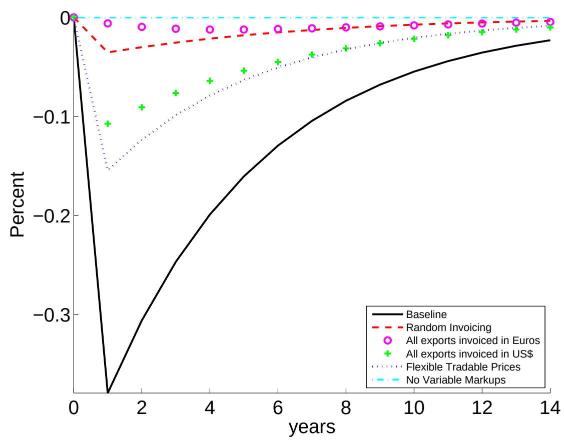
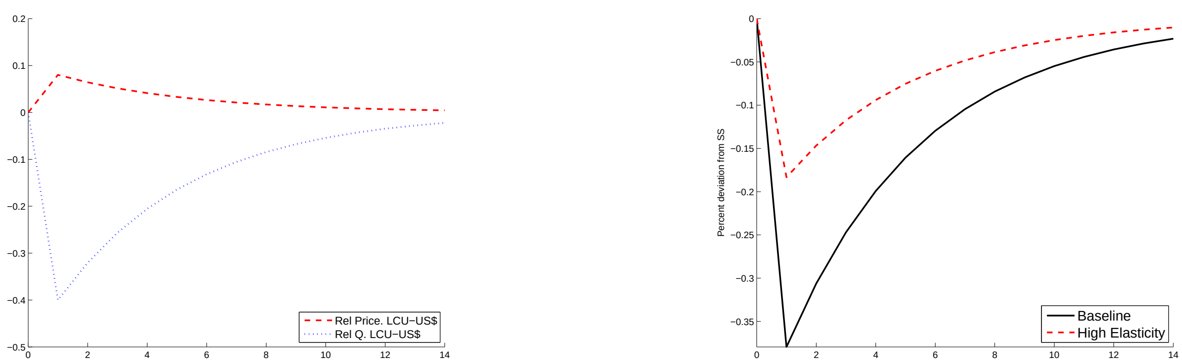


Figure 6: Exchange Rate Shocks, High Elasticity

(a) Nominal Exchange Rates

(b) Relative Prices and Quantities



in the exchange rate have a dramatic impact on relative markups. This effect is reinforced when the invoicing currency is correlated with the initial dispersion in markups. As mentioned above, these features are typically absent in models used to evaluate optimal exchange rate policy.

Some final considerations on how to interpret these results are in order: First, in the model, all changes in relative prices arise from changes in markups rather than from changes in marginal costs. This is consistent with the fixed effects method for estimating changes in relative prices in the empirical section of the paper. Second, the nominal rigidities in the model arise from Calvo pricing. A more realistic assumption is that firms must incur in menu costs to be able to reset prices. The nominal rigidities literature indicates that losses from inflation are smaller when price rigidities are state-dependent rather than time-dependent. In light of this concern, our results can be interpreted as evidence that heterogeneity in invoicing greatly amplifies the effects of exchange rates on productivity.

5 Conclusions

A large literature in international economics has emphasized expenditure switching and misallocation effects as mechanisms through which nominal exchange rates can affect real output and productivity. This paper provides a quantitative exploration of these mechanisms guided by a novel dataset from Chilean customs and a quantitative model of international prices with nominal rigidities. We exploited differences in the response of Chilean firms invoicing exports in different currencies to identify an elasticity of export quantities in response to the exchange rate that is in the range of -1 to -2. Such a low elasticity indicates that the expenditure switching effects of exchange rates are limited not only because price rigidities limit exchange rate pass-through into prices, but also because quantities are not very responsive to these changes in prices.

We then designed a quantitative model of international relative prices that is consistent with the salient features of the Chilean data to measure how exchange rates affect aggregate productivity. We have shown that by incorporating these features, we obtain very different measures of efficiency losses due to exchange rate movements than those obtained under the standard assumptions made in the literature. The results presented here show that taking heterogeneity in invoicing and endogenous variable markups into account is key for the discussion in optimal exchange rate policy.

Finally, in light of our results, a natural question is how developing countries should design exchange rate policy. This question has received surprisingly little attention in the literature of optimal exchange rate policy, which typically focuses on cases where all the invoicing is done either in the producer's currency (PCP) or the destination's currency (LCP). The available evidence suggests that, as in Chile, developing countries use the dollar to invoice a large fraction of their exports. The results in this paper suggest that this is a fruitful area for future research.

References

Amiti, Mary, Oleg Itskhoki, and Jozef Konings. Forthcoming. "Importers, Exporters and Exchange Rate Disconnect." *American Economic Review*.

Bacchetta, Philippe, and Eric van Wincoop. 2005. "A Theory of the Currency Denomination of International Trade." *Journal of International Economics*, 67(2): 295–319.

Berman, Nicolas, Philippe Martin, and Thierry Mayer. 2012. "How do Different Exporters React to Exchange Rate Changes?" *The Quarterly Journal of Economics*, 127(1): 437–492.

Burstein, Ariel, and Christian Hellwig. 2008. "Welfare Costs of Inflation in a Menu Cost Model." *American Economic Review*, 98(2): 438–43.

Burstein, Ariel, and Gita Gopinath. 2013. "International Prices and Exchange Rates." NBER Working Paper No. 18829.

Burstein, Ariel, and Javier Cravino. 2013. "Measured Aggregate Gains from International Trade." NBER Working Paper No. 17767.

Burstein, Ariel, and Nir Jaimovich. 2012. "Understanding Movements in Aggregate and Product-level Real Exchange rates." *Mimeo, UCLA and Duke University*.

Carvalho, Carlos, and Fernanda Nechio. 2011. "Aggregation and the PPP Puzzle in a Sticky-Price Model." *American Economic Review*, 101(6): 2391–2424.

Chari, V.V., Patrick Kehoe, and Ellen McGrattan. 2002. "Can Sticky Price Models Generate Volatile and Persistent Exchange Rates?" *Review of Economic Studies*, 69(3): 533–63.

Corsetti, Giancarlo, and Luca Dedola. 2005. "A Macroeconomic Model of International Price Discrimination." *Journal of International Economics*, 67(1): 129–155.

Corsetti, Giancarlo, and Paolo Pesenti. 2005. "International Dimensions of Optimal Monetary Policy." *Journal of Monetary Economics*, 52(2): 281–305.

Corsetti, Giancarlo, Luca Dedola, and Sylvain Leduc. 2010. "Optimal Monetary Policy in Open Economies." In *Handbook of Monetary Economics*. Vol. 3 of *Handbook of Monetary Economics*, , ed. Benjamin M. Friedman and Michael Woodford, Chapter 16, 861–933. Elsevier.

Devereux, Michael, and Charles Engel. 2003. "Monetary Policy in the Open Economy Revisited: Price Setting and Exchange Rate Flexibility." *Review of Economic Studies*, 70: 765–84.

Duarte, Margarida, and Michael Dotsey. 2011. *Mimeo, University of Toronto and Federal Reserve Bank of Philadelphia*. How Important is the Currency Denomination of Exports in Open Economy Models?

Eaton, Jonathan, and Samuel Kortum. 2002. "Technology, Geography, and Trade." *Econometrica*, 70(5): 1741–1779.

Engel, Charles. 2002. "Expenditure Switching and Exchange Rate Policy." *NBER Macroeconomics Annual*, 17: 231–72.

Engel, Charles. 2006. "Equivalence Results for Optimal Pass-Through, Optimal Indexing to Exchange Rates, and Optimal Choice of Currency for Export Pricing." *Journal of European Economic Association*, 4(6): 1249–60.

Engel, Charles. 2011. "Currency Misalignments and Optimal Monetary Policy: A Reexamination." *American Economic Review*, 101(6): 2796–2822.

Fitzgerald, Doireann, and Stefanie Haller. 2012*a*. "Exchange Rates and Producer Prices: Evidence from Micro Data." Mimeo, Federal Reserve Bank of Minneapolis and University College Dublin.

Fitzgerald, Doireean, and Stephanie Haller. 2012*b*. "Exporters and Shocks." Mimeo, Federal Reserve Bank of Minneapolis and University College Dublin.

Gali, Jordi, and Tommaso Monacelli. 2005. "Monetary Policy and Exchange Rate Volatility in a Small Open Economy." *Review of Economic Studies*, 72(3): 707–734.

Goldberg, Linda S., and Cedric Tille. 2008. "Vehicle currency use in international trade." *Journal of International Economics*, 76(2): 177–192.

Goldberg, Linda S., and Cédric Tille. 2009. "Micro, Macro, and Strategic Forces in International Trade Invoicing." NBER Working Paper No. 15470.

Goldberg, Pinelopi Koujianou, and Michael M. Knetter. 1997. "Goods Prices and Exchange Rates: What Have We Learned?" *Journal of Economic Literature*, 35(3): 1243–1272.

Golosov, Mikhail, and Robert E. Lucas, Jr. 2007. "Menu Costs and Phillips Curves." *Journal of Political Economy*, 115: 171–99.

Gopinath, Gita, Oleg Itskhoki, and Roberto Rigobon. 2010. "Currency Choice and Exchange Rate Pass-through." *American Economic Review*, 100(1): 306–336.

Gust, Christopher, Sylvain Leduc, and Nathan Sheets. 2009. "The Adjustment of Global External Balances: Does Partial Exchange-Rate Pass-Through to Trade Prices Matter?" *Journal of International Economics*, 79(2): 173–185.

Kehoe, Patrick, and Virgiliu Midrigan. 2008. "Sticky Prices and Real Exchange Rates in the Cross-Section." Mimeo, University of Minnesota and New York University.

Obstfeld, Maurice, and Kenneth Rogoff. 1995. "Exchange Rate Dynamics Redux." *Journal of Political Economy*, 103: 624–60.

Paoli, Bianca De. 2009. "Monetary Policy under Alternative Asset Market Structures: The Case of a Small Open Economy." *Journal of Money, Credit and Banking*, 41(7): 1301–1330.

Ruhl, Kim J. 2008. "The International Elasticity Puzzle." Mimeo, New York University.

Sutherland, Alan. 2005. "Incomplete Pass-Through and the Welfare Effects of Exchange Rate Variability." *Journal of International Economics*, 65(2): 375–399.

Appendix

A Measuring Output per Worker

In this Appendix we derive equation (12) in the paper. We start by taking a first order approximation to $RGDP_{iT,L}/RGDP_{iT,0}$ around the non stochastic steady state. To a first order approximation, we can write equation (10) as:

$$\begin{aligned} rgdp_{iT,t} - rgdp_{iT,t-1} &= \frac{\sum_j n_j P_{ijT} C_{ijT} \int_0^{n_i} p_{ijT,t}(f)\, c_{ijT,t}(f)}{\sum_j n_j P_{ijT} C_{ijT} \int_0^{n_i} p_{ijT}(f)\, c_{ijT}(f)} \left(c_{ijT,t}(f) - c_{ijT,t-1}(f)\right) df \\ &= \sum_j \nu_{ij} \left(c_{ijT,t} - c_{ijT,t-1}\right), \end{aligned}$$

where we used lowercase to indicate the log of a variable. The second line follows the definition of ν_{ij} and the approximation $c_{ijT,t} = \int_0^{n_i} \frac{\bar{P}_{ijT}(f)\bar{C}_{ijT}(f)}{\bar{P}_{ijT}\bar{C}_{ijT}} c_{ijT,t}(f)$, which follows from equation (4). In addition, equation (9) can be approximated as:

$$\hat{n}_{iT,t} = \sum_j \frac{n_j \bar{C}_{ijT} \bar{V}_{ijT}}{\sum_j n_j \bar{C}_{ijT} \bar{V}_{ijT}} \left(\hat{c}_{ijT,t} + \hat{v}_{ijT,t}\right).$$

In a symmetric SS, $\bar{V}_{ij,T} = \bar{V}_T$ and $\bar{P}_{ijT} = \bar{P}_T$. Then,

$$\hat{n}_{iT,t} = \sum_j \nu_{ij} \left(\hat{c}_{ijT,t} + \hat{v}_{ijT,t}\right) = r\hat{g}dp_{iT,t} + \hat{v}_{iT,t},$$

where: $\hat{v}_{iT} \equiv \sum_j \nu_{ij} \hat{v}_{ijT,t}$. Then, to a first order approximation, the change in output per worker is given by:

$$\hat{a}_{iT,t} = r\hat{g}dp_{iT,t} - \hat{n}_{iT,t} = -\hat{v}_{iT,t},$$

Finally, approximating $V_{ij,T}$ we obtain:

$$V_{ij,T} \equiv \left[\frac{1}{n_i} \int_0^{n_i} \left[\frac{p_{ijT,t}(f)}{P_{ijT,t}}\right]^{-\xi} \frac{1}{z(f)} df\right]$$

then:

$$\begin{aligned}
\hat{v}_{ij,t} &= -\xi\left[\int_0^{n_i}\frac{\left[\bar{p}_{ijT}(f)\right]^{-\xi}\frac{1}{z(f)}}{\int_0^{n_i}\left[\bar{p}_{ijT}(f)\right]^{-\xi}\frac{1}{z(f)}df}\left[\hat{p}_{ijT,t}(f)-\hat{P}_{ijT,t}\right]df\right] \\
&= -\xi\int_0^{n_i}\left[\frac{\left[\bar{p}_{ijT}(f)\right]^{-\xi}\frac{1}{z(f)}}{\int_0^{n_i}\left[\bar{p}_{ijT}(f)\right]^{-\xi}\frac{1}{z(f)}df}-\int_0^{n_i}\frac{\left[\bar{p}_{ijT}(f)\right]^{1-\xi}}{\int_0^{n_i}\left[\bar{p}_{ijT}(f)\right]^{1-\xi}df}\right]\hat{p}_{ijT,t}(f) \\
&= -\xi\left[\begin{array}{c}\frac{\kappa\left[\bar{p}_{ijT}(H)\right]^{-\xi}\frac{1}{z(H)}}{\kappa\left[\bar{p}_{ijT}(H)\right]^{-\xi}\frac{1}{z(H)}+(1-\kappa)\left[\bar{p}_{ijT}(L)\right]^{-\xi}\frac{1}{z(L)}} \\ -\frac{\kappa\left[\bar{p}_{ijT}(H)\right]^{1-\xi}}{\kappa\left[\bar{p}_{ijT}(H)\right]^{-\xi}\frac{1}{z(H)}+(1-\kappa)\left[\bar{p}_{ijT}(L)\right]^{-\xi}\frac{1}{z(L)}}\end{array}\right]\hat{p}_{ijT,t}(H) \\
&\quad+\left[\begin{array}{c}\frac{(1-\kappa)\left[\bar{p}_{ijT}(L)\right]^{-\xi}\frac{1}{z(L)}}{\kappa\left[\bar{p}_{ijT}(H)\right]^{-\xi}\frac{1}{z(H)}+(1-\kappa)\left[\bar{p}_{ijT}(L)\right]^{-\xi}\frac{1}{z(L)}} \\ -\frac{(1-\kappa)\left[\bar{p}_{ijT}(L)\right]^{1-\xi}}{\kappa\left[\bar{p}_{ijT}(H)\right]^{-\xi}\frac{1}{z(H)}+(1-\kappa)\left[\bar{p}_{ijT}(L)\right]^{-\xi}\frac{1}{z(L)}}\end{array}\right]\hat{p}_{ijT,t}(L) \\
&= -\xi\left[\begin{array}{c}\left[1+\left(\frac{1-\kappa}{\kappa}\right)\left[\frac{\bar{p}_{ijT}(L)}{\bar{p}_{ijT}(H)}\right]^{-\xi}\frac{z(H)}{z(L)}\right]^{-1} \\ -\left[1+\left(\frac{1-\kappa}{\kappa}\right)\left[\frac{\bar{p}_{ijT}(L)}{\bar{p}_{ijT}(H)}\right]^{1-\xi}\right]^{-1}\end{array}\right]\left[\hat{p}_{ijT,t}(H)-\hat{p}_{ijT,t}(L)\right]
\end{aligned}$$

then,

$$\hat{v}_{iT,t}=-\xi\left(\omega^{v}-\omega\right)\sum_j \nu_{ij}\left[\hat{p}^{H}_{ijT,t}-\hat{p}^{L}_{ijT,t}\right],$$

where ω and ω^v are defined in the text.

B Derivation of equations (8) and (14)

This section derives equations (8) and (14) in the paper. It also shows how it is possible to calibrate the σ^2_{mi} s to obtain the shares of Chilean firms invoicing in each currency. As stated above, the derivation of equation (8) follows the proof of Proposition 2 in Gopinath, Itskhoki and Rigobon (2010) for the case of two arbitrary currencies, a and b. The difference in discounted profits of choosing currency a vs. currency b, L^{a-b}_t, is given by:

$$L^{a-b}_{ij,t}\equiv E_t\sum_{l=0}^{\infty}(\beta\theta)^l\left[\Pi_{i,t+l}\left(\bar{p}^{pa}_{ijT,t}\right)-\Pi_{i,t+l}\left(\bar{p}^{pb}_{ijT,t}-e_{ba,t+l}\right)\right], \tag{B.1}$$

we take a second order Taylor approximation of the profit differential around the optimal flexible price expressed in currency a, $\tilde{p}^{pa}_{ijT,t+l}$:

$$\Pi_{i,t+l}\left(\bar{p}^{pa}_{ijT,t}\right) - \Pi_{i,t+l}\left(\bar{p}^{pb}_{ijT,t} - e_{ba,t+l}\right) \approx \frac{1}{2}\tilde{\Pi}_{pp}[\left(\bar{p}^{pa}_{ijT,t} - \tilde{p}^{pa}_{ijT,t+l}\right)^2 \tag{B.2}$$

$$- \left(\bar{p}^{pb}_{ijT,t} - e_{ba,t+l} - \tilde{p}^{pa}_{ijT,t+l}\right)^2] \tag{B.3}$$

where $\tilde{\Pi}_{pp} = \Pi_{it,pp}\left(\tilde{p}^{pa}_{ijT,t}\right)$. If the nominal exchange rate is a random walk, then, to a first order approximation, the reset prices are given by: $\bar{p}^{pa}_{ijT,t} = \bar{p}^{pb}_{ijT,t} - e_{ba,t}$. Substituting equation (B.2) in (B.1) we can write:

$$L^{a-b}_t = \frac{1}{2}\tilde{\Pi}_{pp}\sum_{l=0}^{\infty}(\beta\theta)^l E_t\left(e_{ba,t+l} - e_{ba,t}\right)\left(\bar{p}^{pa}_{ijT,t} + \bar{p}^{pb}_{ijT,t} - e_{ba,t+l} - 2\tilde{p}^{pa}_{ijT,t+l}\right) \tag{B.4}$$

or:

$$L^{a-b}_t = -\frac{1}{2}\tilde{\Pi}_{pp}\sum_{l=0}^{\infty}(\beta\theta)^l\left(var_t\left(e_{ba,t+l}\right) + 2cov\left(\tilde{p}^{pa}_{ijT,t+l}, e_{ba,t+l}\right)\right).$$

Equation (8) follows from using the random walk property of the exchange rate to substitute $var_t\left(e_{ba,t+l}\right) = l\,var_t\left(\Delta e_{ba,t+l}\right)$. To derive equation (14) we need to compute $var_t\left(e_{ba,t+l}\right)$ and $cov\left(\tilde{p}^{pa}_{ijT,t+l}, e_{ba,t+l}\right)$ under the parameterization of Section 4.1. Taking logs in equations (3) and (2) and setting $\phi = 0$ and $\sigma = 1$ we can write $\tilde{p}^{pa}_{ijT,t+l}$ as:

$$\tilde{p}^{pa}_{ijT,t+l} = \frac{1}{1+\Gamma(f)}\left[m_{j,t+l} + \Gamma(f)\,p_{jN,t+l}\right] + m_{a,t+l} - m_{j,t+l} + const_{ijT}. \tag{B.5}$$

Note that the reset price in the non tradable sector is given by:

$$\bar{p}_{jN,t+l} = (1-\beta\theta_N)\sum_{k=0}^{\infty}(\beta\theta_N)^k E_{t+l}\left[m_{j,t+l+k}\right] = m_{j,t+l} + const_{jN},$$

where the last equality uses the random walk property of the money supply. Then, we can write:

$$cov\left(p_{jN,t+l}, e_{ba,t+l}\right) = (1-\theta)\,cov\left(\bar{p}_{jN,t+l}, \Delta m_{b,t+l} - \Delta m_{a,t+l}\right) = (1-\theta)\,\sigma_{mj}$$

$$= (1-\theta)\,cov\left(m_{j,t+l}, \Delta m_{b,t+l} - \Delta m_{a,t+l}\right). \tag{B.6}$$

Substituting equations (3), (B.5), and (B.6) into equation (8) we obtain equation (14) in the paper.

We now use equation (14) to obtain the invoicing decisions of Chilean firms selling abroad. To make sure all Chilean firms invoice in US$ into the US we need: $\tilde{\Psi}_{cs}^{\$l}(f) > -\frac{1}{2}$ for $l = p, e$. From equation (14), this implies:

$$-\tilde{\Psi}_{cs}^{sl}(f) = \left[1 - \frac{\theta\Gamma(f)}{1+\Gamma(f)}\right] \frac{\sigma_{ms}^2}{\sigma_{ml}^2 + \sigma_{ms}^2} < \frac{1}{2} \tag{B.7}$$

for $l = p, e$. That is, all firms will invoice in US$ if σ_{ms}^2 is sufficiently small. To make sure that Chilean firms would rather use the US$ instead of the Peso when selling into Europe we need $\tilde{\Psi}_{ce}^{\$p}(f) > -\frac{1}{2}$, this implies:

$$\tilde{\Psi}_{ce}^{\$p}(f) = \frac{\sigma_{m\$}^2}{\sigma_{m\$}^2 + \sigma_{mp}^2} < \frac{1}{2}, \tag{B.8}$$

or $\sigma_{m\$}^2 < \sigma_{mp}^2$. Firms will choose the dollar if the dollar is less volatile than the peso. Note from equation (B.7) that, since $\Gamma(f) \geq 0$, then this condition is sufficient to ensure that Chilean firms invoice in dollars in the US. Finally, we derive the conditions under which high productivity firms choose the euro vs the dollar in Europe. In this case we can write:

$$\tilde{\Psi}_{ce}^{e\$}(f) = \left[1 - \frac{\theta\Gamma(f)}{1+\Gamma(f)}\right] \frac{\sigma_{me}^2}{\sigma_{me}^2 + \sigma_{m\$}^2} < \frac{1}{2}.$$

Note that, given σ_{me}^2 and $\sigma_{m\$}^2$ firms are more likely to invoice in euros the higher $\Gamma(f)$. To make sure that only the high productivity firms use the euros, while low productivity firms use the dollar, we need

$$\tilde{\Psi}_{ce}^{e\$}(H) < \frac{1}{2} < \tilde{\Psi}_{ce}^{e\$}(L). \tag{B.9}$$

We choose the σ_{mi}^2s to ensure that conditions (B.7) , (B.8) and (B.7) hold. It is clear that such combination exists as long as $\Gamma(H) > \Gamma(L)$. For example, we could set $\sigma_{m\$}^2 < \sigma_{mp}^2$, and $\left[1 - \frac{2\theta\Gamma(H)}{1+\Gamma(H)}\right] < \sigma_{m\$}^2/\sigma_{me}^2 < \left[1 - \frac{2\theta\Gamma(L)}{1+\Gamma(L)}\right]$.