A Probability-Based Stress Test of Federal Reserve Assets and Income

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Abstract

The Federal Reserve has greatly expanded its balance sheet to provide monetary stimulus to a weak economy. Worries have arisen about the Fed's resulting interest rate risk, which some have assessed using arbitrary interest rate scenarios. We instead measure potential losses to the Fed's Treasury securities holdings and potential declines in the Fed's remittances to the Treasury using probability-based interest rate environments generated from a dynamic term structure model that respects the zero lower bound on yields. We find that the Fed's losses are unlikely to be large, and its remittances are unlikely to show more than a brief cessation.

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1 Introduction

In late 2008, in response to a severe financial crisis and recession, the Federal Reserve reduced its key policy rate, the overnight federal funds rate, to its effective lower bound. To provide additional monetary stimulus to spur economic growth and avoid deflation, the Fed also conducted several rounds of large-scale asset purchases—frequently referred to as quantitative easing (QE). These actions left the Federal Reserve's portfolio of longer-term securities several times larger than its pre-crisis level. Although the Fed's securities portfolio carries essentially no credit risk, its market value can vary over time. Of course, for centuries, central bank balance sheets have contained assets, such as gold, bonds, and foreign currencies, whose market value has varied over time. So financial risks associated with fluctuations in the market prices of central bank assets are nothing new. Still, the greater size of the Fed's portfolio does potentially expose it to unusually large financial gains and losses from market fluctuations. Furthermore, the Fed's purchases have not only produced a larger balance sheet, they have shifted the composition of the Fed's portfolio toward longer-maturity securities. The longer duration of the Fed's portfolio implies that its market value is especially sensitive to changes in interest rates. The combination of a larger securities portfolio with a longer duration implies that the Fed is taking on much more interest rate risk than usual. In part because of this interest rate risk, the Fed's purchases of longer-term securities have been quite controversial. For example, former Fed Governor Frederic Mishkin (2010) argued that "major holdings of long-term securities expose the Fed's balance sheet to potentially large losses if interest rates rise. Such losses would result in severe criticism of the Fed and a weakening of its independence."

In fact, there are two types of interest rate risk that the Fed faces. Besides the risk that increases in *longer-term* interest rates will erode the market value of the Fed's portfolio—that is, balance sheet risk—there is also the risk that increases in *short-term* interest rates, notably the interest rate that the Fed pays on bank reserves, will greatly increase the funding cost of the Fed's securities portfolio—that is, income risk. Because the Fed's interest income is generated from fixed coupon payments on longer-maturity securities, rising short-term interest rates will squeeze the Fed's net interest income, which will in turn lower the Fed's remittances to the U.S. Treasury. In extreme circumstances, remittances could fall to zero, but as is also the case with any capital losses posted by the Fed, the Fed's capital base would remain secure and its monetary policy operations would not be directly impeded.[1] Still, as above, some worry about the attendant political fallout from large (realized or unrealized) capital losses or a cessation in remittances (Rudebusch, 2011).

In order to understand and assess the Fed's balance sheet and income risks, it is crucial to quantify them. Two recent papers—Carpenter et al. (2013) and Greenlaw et al. (2013), henceforth

[1] Regardless of its income expenses or capital losses, the Fed still has the operational control of short-term interest rates. In particular, the Fed's ability to pay interest on bank reserves allows it to conduct monetary policy independently of the size of its balance sheet.

GHHM—have made great progress in doing just that. They have generated detailed projections of the market value and cash flow of the Fed's assets and liabilities under specific baseline and alternative interest rate assumptions. In essence, their analyses are akin to the regulatory oversight financial stress tests that the Fed has applied to commercial banks and large financial institutions. These stress tests consider whether these institutions have enough capital to endure various adverse economic scenarios. Importantly, these stress tests do not place probabilities on the baseline scenario or the alternatives. Similarly, Carpenter et al. (2013) and GHHM consider changes in the Fed's assets and income in various scenarios including, for example, one in which the yield curve is shifted up at all maturities by 100 basis points. However, like the usual stress tests, while informative, it would arguably be of great interest to know the likelihoods to attach to this range of possible outcomes.[2] Attaching likelihoods to the alternative scenarios—or more generally, looking at the entire distributional forecast—would result in what we term "probability-based" stress tests. In this paper, to assess the Fed's financial position, we provide a probability-based stress test of the Fed's interest rate risks, specifically, potential mark-to-market losses for the Fed's Treasury securities and the potential cessation of Fed remittances to the Treasury. This probability-based stress test involves distributional forecasts of interest rates and consideration of the resulting effects on the Fed's financial position, with particular emphasis on the lower tail of the distribution. The addition of distributional information enables us to provide new assessments of the likelihoods of certain events, such as losses on the Fed's securities holdings of more than, say, $200 billion, or a negative net interest rate margin persisting for more than one year.

To construct a probability-based stress test of the Fed's interest rate risk, we use a dynamic term structure model to generate yield curve projections consistent with historical interest rate variation. However, with nominal yields on Treasury debt very near their zero lower bound (ZLB), we use the shadow-rate arbitrage-free Nelson-Siegel (AFNS) model class developed by Christensen and Rudebusch (2013a,b) to generate the requisite potentially asymmetric distributional interest rate forecasts. Shadow-rate models are latent-factor models where the state variables have standard Gaussian dynamics, but the short rate is replaced by a shadow short rate that may be negative in the spirit of Black (1995). The model-generated observed short rate and yield forecasts respect the ZLB and account for the effect on bond pricing stemming from the option to hold currency. However, despite this nonlinearity, our shadow-rate AFNS model, with its underlying Gaussian dynamics, remains as flexible and empirically tractable as a standard AFNS model. Critically for our purposes, we demonstrate that these models are able to accurately price the Fed's portfolio of Treasury securities.

To assess the Fed's balance sheet risk, we examine the distributional forecast of the value of

[2]Berkowitz (2000, p. 6) makes a similar point in the context of bank stress tests, arguing that "to make stress scenarios useful, they must be assigned probabilities."

Fed's Treasury securities. We focus on nominal Treasury securities because they represent almost 60 percent of the securities held outright and the lion's share of the Fed's assets.[3] The next largest share is comprised of agency mortgage-backed securities (MBS), but the valuation of MBS is very difficult because it requires assessing mortgage refinancing and prepayment patterns in interest rate environments without historical parallel.[4] Despite our focus on Treasuries, we view our analysis as an illustrative first step down the road to more complete probability-based stress tests.

In the empirical assessment of the Fed's Treasury portfolio, we use three different ways of generating Treasury yield curve projections. The first approach is based on the shadow-rate AFNS model favored by Christensen and Rudebusch (2013b, henceforth CR) in their analysis of U.S. Treasury yields near the ZLB. The second relies on a straightforward non-parametric ranking of historical Treasury yield curve changes. Finally, the third approach converts the Treasury yield projections from the January 2013 Blue Chip Financial Forecasts survey of professional forecasters into a distribution of future portfolio values. Despite differences in methods, the results are very similar and indicate that in all likelihood the potential losses to the Fed's Treasury securities holdings over the next several years should be modest. Based on our findings, we conclude that potential losses over the next three years on the Fed's enlarged portfolio of long-term securities are manageable.

As a second exercise, we examine the income risk faced by the Fed. To do this, we use the model-based distributional yield curve projections to generate distributional projections of the Fed's remittances to the U.S. Treasury. Again, this is an illustrative but somewhat limited exercise, as we do not consider the distributional projections of all possible relevant conditioning factors (such as inflation or Fed policy actions). Still, even at the lower 5 percentile of the distribution, the cumulative remittance shortfall (the Fed's "deferred asset") peaks at less than $11.0 billion in 2017. As a consequence, the risk of a long cessation of the Fed's remittances to the U.S. Treasury appears remote.

Finally, an important caveat to our analysis should be noted. We are not conducting a comprehensive assessment of the costs and benefits of the Fed's program of QE (Rudebusch, 2011). Indeed, our probability-based stress test captures only part of the financial consequences of the Fed's securities purchases and, notably, excludes three key fiscal benefits. First, the U.S. Treasury has received significant additional remittances from the Fed's purchases of additional securities in the past (notably, from 2009 through 2013). Second, as longer-term interest rates were pushed lower by the Fed's securities purchases, the resulting higher output and household income boosted federal tax revenue and reduced federal outlays.[5] Finally, the lower longer-term interest rates associated with QE also helped

[3]As of the start of 2013, foreign assets and other claims represent about 10 percent of the Fed's assets. In addition, the Fed holds a small amount of TIPS and agency debt.

[4]Carpenter et al. (2013) and GHHM provide initial attempts at this effort.

[5]See Gagnon, Raskin, Remache, and Sack (2011), Christensen and Rudebusch (2012), and Bauer and Rudebusch (2013) amongst many others.

lower the Treasury's borrowing costs for issuing new debt. Furthermore, it is important to stress that any kind of financial or fiscal accounting is ancillary to the Fed's mission. The Fed, of course, strives to be a cost-efficient steward of the public purse. But its statutory mandate for conducting monetary policy is to promote maximum employment and price stability. These macroeconomic goals are the key metrics for judging monetary policy. Financial considerations—even potentially large capital losses—are secondary.

The rest of the paper is structured as follows. Section 2 describes the evolution of the Fed's securities portfolio since the onset of the financial crisis and details the Treasury data we use. Section 3 describes the shadow-rate AFNS model class and the specific model favored by CR. Section 4 reports our empirical estimation results, while Section 5 is dedicated to detailing the generation of yield curve projections, their conversion into portfolio stress tests, and the results. Section 6 details our projections of the Fed's remittances to the U.S. Treasury. Section 7 concludes.

2 The Fed's Securities Portfolio

In this section, we describe the Federal Reserve's securities portfolio and how it has evolved since the onset of the financial crisis of 2007-2008. To that end, Figure 1 shows the changes in the major asset classes held by the Federal Reserve since 2008.

In the early stages of the financial crisis, the Fed's balance sheet was expanded through various emergency lending facilities, most notably the Term Auction Facility (TAF).[6] In the figure, this lending appears in the "Other Assets" category, which currently represents less than 10 percent of the Fed's assets. The "Non-Treasury Securities" category is almost exclusively comprised of agency MBS, much of which was purchased during the Fed's first large-scale asset purchase program (QE1). This program ran from late 2008 to early 2010 and included purchases of \$1.25 trillion of agency MBS and \$175 billion in federal agency debt issued by Fannie Mae and Freddie Mac. It also included purchases of \$300 billion in Treasury securities. More recently, in the Fed's third purchase program, here referred to as QE3, the Fed has purchased additional MBS. At the start of 2013, the MBS portfolio totalled \$927 billion and represented 34.7% of the securities held outright. As noted above, this portfolio is difficult to value because it requires assessing mortgage refinancing and prepayment patterns.[7] This category also contains a very small amount—about \$77 billion as of the start of 2013—of federal agency debt.

The "Treasury Securities" category experienced a large expansion during the second purchase program, here referred to as QE2, which operated from November 2010 through June 2011. Some

[6]See Christensen, Lopez, and Rudebusch (2009) for details on the functioning and effectiveness of the TAF.

[7]The MBS portfolio is also quite diverse. About 5.5 percent of the portfolio is spread across 22,164 securities with a holding of each less than \$10 million. Another 9.2 percent is held in 3,932 securities with a maximum face value of \$50 million.

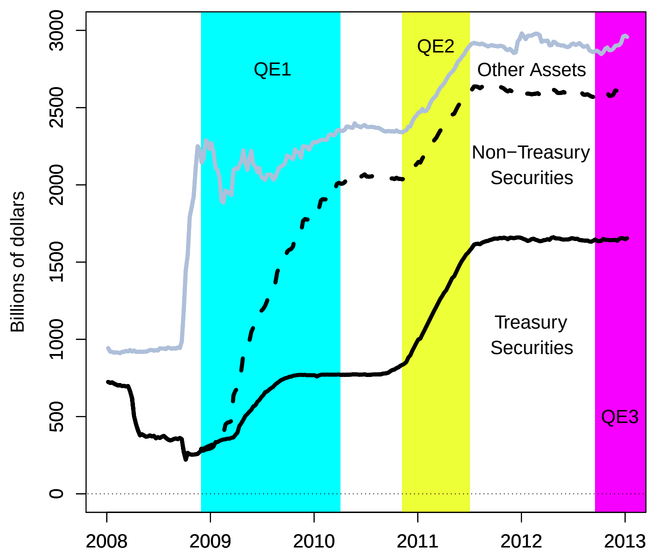


Figure 1: **Assets of the Federal Reserve System.**
Illustration of the total assets of the Federal Reserve System broken down into Treasury securities, non-Treasury securities, and other assets. The data are weekly covering the period from January 2, 2008, to January 2, 2013.

of these purchases were indexed bonds or Treasury Inflation Protected Securities (TIPS). At the start of 2013, TIPS represented $75 billion in principal and another $11 billion in accrued inflation compensation, again a relatively small share.[8] This leaves us with the $1.58 trillion in nominal Treasury notes and bonds representing 59.2% of the Fed's securities held outright as of January 2, 2013, spread out across a total of 241 securities. It is this portfolio that we will focus on in our analysis of the Fed's balance sheet risk. As noted above the duration of this portfolio is also relevant for assessing balance sheet risk. From September 2011 through the end of 2012, the Fed conducted a Maturity Extension Program that sold Treasury securities with remaining maturities of 3 years or less and purchased a similar amount of Treasury securities with remaining maturities of 6 years to 30 years. As a result of this policy, the Fed has sold almost all of its short-term Treasury securities, and Treasuries with less than 3 years to maturity only represented 0.25% of the Treasury securities holdings as of the time of our analysis.

Finally, we note the source of our Treasury yield data. The specific Treasury bond yields we use are zero-coupon yields constructed by the method described in Gürkaynak, Sack, and Wright (2007)

[8]Christensen and Gillan (2013) analyze the effects of TIPS purchases included in QE2.

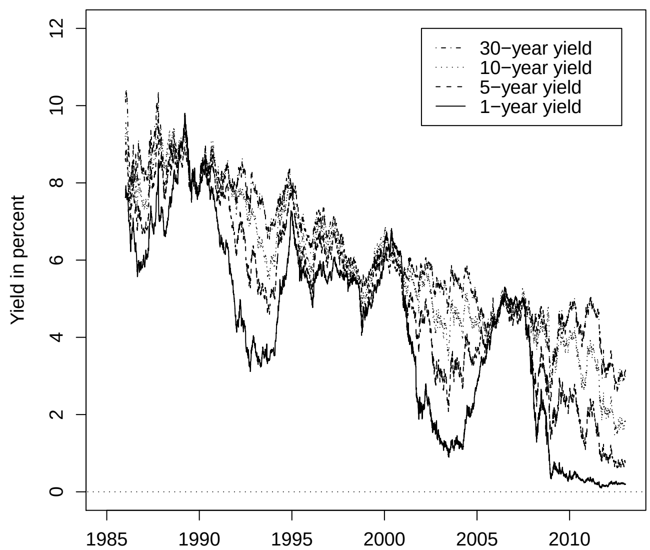


Figure 2: **Time Series of Treasury Bond Yields.**
Illustration of the daily Treasury zero-coupon bond yields covering the period from January 2, 1986, to January 2, 2013. The yields shown have maturities in one year, five years, ten years, and thirty years, respectively.

and briefly detailed here.[9] For each business day, a zero-coupon yield curve is fitted to price a large pool of underlying off-the-run Treasury bonds, which is ideal for our purposes as most Treasuries held by the Fed are seasoned. From this data set, we construct Treasury zero-coupon bond yields for the following maturities: 3-month, 6-month, 1-year, 2-year, 3-year, 5-year, 7-year, 10-year, 15-year, 20-year, and 30-year. We use daily data and limit our sample to the period from January 2, 1986, to January 2, 2013, as the longest maturity Treasury yields are not available prior to November 25, 1985.[10] Figure 2 illustrates the constructed time series of the one-, five-, ten-, and thirty-year Treasury zero-coupon yields. Here, we note that yields were historically low at all maturities towards the end of our sample. Thus, the most recent period from which we want to make future yield projections is the leading candidate for accounting for the ZLB on nominal yields.

[9]The Federal Reserve Board of Governors frequently updates the factors and parameters of this method, see the related website http://www.federalreserve.gov/pubs/feds/2006/index.html

[10]Between October 2001 and February 2006 the U.S. Treasury did not issue any 30-year bonds. Still, the GSW 30-year yield appears reliable for this period. Furthermore, any bias arising from this circumstance would only have a minuscule effect on our estimation results as they are primarily determined by the yields with 10 years or less to maturity.

3 A Shadow-Rate Model of U.S. Treasury Yields

In this section, we describe the option-based approach to the shadow-rate model we use. This is followed by a detailed description of the shadow-rate AFNS model developed in Christensen and Rudebusch (2013a).

3.1 The Option-Based Approach to the Shadow-Rate Model

The concept of a shadow interest rate as a modeling tool to account for the ZLB can be attributed to Black (1995). He noted that the observed nominal short rate will be nonnegative because currency is a readily available asset to investors that carries a nominal interest rate of zero. Therefore, the existence of currency sets a zero lower bound on yields. To account for this ZLB, Black postulated as a useful modeling tool the use of a shadow short rate, s_t, that is unconstrained by the ZLB. The usual observed instantaneous risk-free rate, r_t, which is used for discounting cash flows when valuing securities, is then given by the greater of the shadow rate or zero:

$$r_t = \max\{0, s_t\}. \tag{1}$$

Accordingly, as s_t falls below zero, the observed r_t simply remains at the zero bound.

While Black (1995) described circumstances under which the zero bound on nominal yields might be relevant, he did not provide specifics for implementation. The small set of empirical research on shadow-rate models has relied on numerical methods for pricing.[11] To overcome the computational burden of numerical-based estimation that limits the use of shadow-rate models, Krippner (2012) suggested an alternative option-based approach that makes shadow-rate models almost as easy to estimate as the corresponding non-shadow-rate model. To illustrate this approach, consider two bond-pricing situations: one without currency as an alternative asset and the other that has a currency in circulation that has a constant nominal value and no transaction costs. In the world without currency, the price of a shadow-rate zero-coupon bond, $P_t(\tau)$, may trade above par, that is, its risk-neutral expected instantaneous return equals the risk-free shadow short rate, s_t, which may be negative. In contrast, in the world with currency, the price at time t for a zero-coupon bond that pays \$1 when it matures in τ years is given by $\underline{P}_t(\tau)$. This price will never rise above par, so nonnegative yields will never be observed.

Now consider the relationship between the two bond prices at time t for the shortest (say, overnight) maturity available, δ. In the presence of currency, investors can either buy the zero-coupon bond at price $P_t(\delta)$ and receive one unit of currency the following day or just hold the currency. As a consequence, this bond price, which would equal the shadow bond price, must be

[11] For example, Kim and Singleton (2012) and Bomfim (2003) use finite-difference methods to calculate bond prices, while Ichiue and Ueno (2007) employ interest rate lattices.

capped at 1:

$$\begin{aligned} \underline{P}_t(\delta) &= \min\{1, P_t(\delta)\} \\ &= P_t(\delta) - \max\{P_t(\delta) - 1, 0\}. \end{aligned}$$

That is, the availability of currency implies that the overnight claim has a value equal to the zero-coupon shadow bond price minus the value of a call option on the zero-coupon shadow bond with a strike price of 1. More generally, we can express the price of a bond in the presence of currency as the price of a shadow bond minus the call option on values of the bond above par:

$$\underline{P}_t(\tau) = P_t(\tau) - C_t^A(\tau, \tau; 1), \tag{2}$$

where $C_t^A(\tau, \tau; 1)$ is the value of an American call option at time t with maturity in τ years and strike price 1 written on the shadow bond maturing in τ years. In essence, in a world with currency, the bond investor has had to sell off the possible gain from the bond rising above par at any time prior to maturity.

Unfortunately, analytically valuing this American option is complicated by the difficulty in determining the early exercise premium. However, Krippner (2012) argues that there is an analytically close approximation based on tractable European options. Specifically, Krippner (2012) shows that the ZLB instantaneous forward rate, $\underline{f}_t(\tau)$, is

$$\underline{f}_t(\tau) = f_t(\tau) + z_t(\tau),$$

where $f_t(\tau)$ is the instantaneous forward rate on the shadow bond, which may go negative, while $z_t(\tau)$ is an add-on term given by

$$z_t(\tau) = \lim_{\delta \to 0} \left[\frac{d}{d\delta} \left\{ \frac{C_t^E(\tau, \tau + \delta; 1)}{P_t(\tau + \delta)} \right\} \right],$$

where $C_t^E(\tau, \tau+\delta; 1)$ is the value of a European call option at time t with maturity $t+\tau$ and strike price 1 written on the shadow discount bond maturing at $t + \tau + \delta$. Thus, the observed yield-to-maturity is

$$\begin{aligned} \underline{y}_t(\tau) &= \frac{1}{\tau} \int_t^{t+\tau} \underline{f}_t(s) ds \\ &= \frac{1}{\tau} \int_t^{t+\tau} f_t(s) ds + \frac{1}{\tau} \int_t^{t+\tau} \lim_{\delta \to 0} \Big[\frac{\partial}{\partial \delta} \frac{C_t^E(s, s + \delta; 1)}{P_t(s + \delta)} \Big] ds \\ &= y_t(\tau) + \frac{1}{\tau} \int_t^{t+\tau} \lim_{\delta \to 0} \Big[\frac{\partial}{\partial \delta} \frac{C_t^E(s, s + \delta; 1)}{P_t(s + \delta)} \Big] ds. \end{aligned}$$

It follows that bond yields constrained at the ZLB can be viewed as the sum of the yield on the unconstrained shadow bond, denoted $y_t(\tau)$, which is modeled using standard tools, and an add-on correction term derived from the price formula for the option written on the shadow bond that provides an upward push to deliver the higher nonnegative yields actually observed.

As Christensen and Rudebusch (2013a,b) stress, the Krippner (2012) framework should be viewed as not fully internally consistent and simply an approximation to an arbitrage-free model.[12] Of course, away from the ZLB, with a negligible call option, the model will match the standard arbitrage-free term structure representation. In addition, the size of the approximation error near the ZLB has been determined via simulation in Christensen and Rudebusch (2013a,b) to be quite modest.[13]

3.2 The Shadow-Rate AFNS Model

In theory, the option-based shadow-rate result is quite general and applies to any assumptions made about the dynamics of the shadow-rate process. However, as implementation requires the calculation of the limit term under the integral, the option-based shadow-rate models are limited practically to the Gaussian model class where option prices are available in analytical form. The arbitrage-free Nelson-Siegel (AFNS) representation developed by Christensen, Diebold, and Rudebusch (2011, henceforth CDR) is well suited for this extension.[14] Its three factors correspond to the level, slope, and curvature factors commonly observed for Treasury yields and are denoted L_t, S_t, and C_t, respectively. The state vector is thus defined as $X_t = (L_t, S_t, C_t)$.

In the shadow-rate AFNS model, the instantaneous risk-free rate is the nonnegative constrained process of the shadow risk-free rate, which is defined as the sum of level and slope as in the original AFNS model class:

$$r_t = \max\{0, s_t\}, s_t = L_t + S_t. \quad (3)$$

Also, the dynamics of the state variables used for pricing under the Q-measure remain as in the regular AFNS model:

$$\begin{pmatrix} dL_t \\ dS_t \\ dC_t \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\lambda & \lambda \\ 0 & 0 & -\lambda \end{pmatrix} \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} dt + \Sigma \begin{pmatrix} dW_t^{L,Q} \\ dW_t^{S,Q} \\ dW_t^{C,Q} \end{pmatrix}, \quad (4)$$

[12] In particular, there is no explicit PDE that bond prices must satisfy, including boundary conditions, for the absence of arbitrage as in Kim and Singleton (2012).

[13] Christensen and Rudebusch (2013a,b) analyze how closely the option-based bond pricing from the estimated B-CR model matches an arbitrage-free bond pricing that is obtained from the same model using Black's (1995) approach based on Monte Carlo simulations. They consider bonds of maturities out to 10 years. We extended these simulation results to consider bond maturities of 30 years (needed for pricing the longest bonds in the Fed's portfolio). At the thirty-year maturity, the approximation errors are understandably larger but still do not exceed 6 basis points, which are notably smaller than the model's fitted errors.

[14] For details of the derivations, see Christensen and Rudebusch (2013a).

where Σ is the constant covariance (or volatility) matrix.[15]

Based on this specification of the Q-dynamics, the yield on the shadow discount bond maintains the popular Nelson and Siegel (1987) factor loading structure

$$y_t(\tau) = L_t + \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau}\right) S_t + \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) C_t - \frac{A(\tau)}{\tau}, \tag{5}$$

where $A(\tau)/\tau$ is a maturity-dependent yield-adjustment term. The corresponding instantaneous shadow forward rate is given by

$$f_t(\tau) = -\frac{\partial}{\partial\tau} \ln P_t(\tau) = L_t + e^{-\lambda\tau} S_t + \lambda\tau e^{-\lambda\tau} C_t + A^f(\tau), \tag{6}$$

where the final term is another maturity-dependent yield-adjustment term.

Christensen and Rudebusch (2013a) show that, within the shadow-rate AFNS model, the zero-coupon bond yields that observe the zero lower bound, denoted $\underline{y}_t(\tau)$, are easily calculated as

$$\underline{y}_t(\tau) = \frac{1}{\tau} \int_t^{t+\tau} \left[f_t(s)\Phi\Big(\frac{f_t(s)}{\omega(s)}\Big) + \omega(s)\frac{1}{\sqrt{2\pi}} \exp\Big(-\frac{1}{2}\Big[\frac{f_t(s)}{\omega(s)}\Big]^2\Big)\right] ds, \tag{7}$$

where $\Phi(\cdot)$ is the cumulative probability function for the standard normal distribution, $f_t(\tau)$ is the shadow forward rate, and $\omega(\tau)$ takes the following simple form

$$\omega(\tau)^2 = \sigma_{11}^2\tau + \sigma_{22}^2\Big(\frac{1 - e^{-2\lambda\tau}}{2\lambda}\Big) + \sigma_{33}^2\Big(\frac{1 - e^{-2\lambda\tau}}{4\lambda} - \frac{1}{2}\tau e^{-2\lambda\tau} - \frac{1}{2}\lambda\tau^2 e^{-2\lambda\tau}\Big),$$

when the volatility matrix Σ is assumed diagonal.

As in the affine AFNS model, the shadow-rate AFNS model is completed by specifying the price of risk using the essentially affine risk premium specification introduced by Duffee (2002), so the risk premium Γ_t is defined by the measure change

$$dW_t^Q = dW_t^P + \Gamma_t dt,$$

with $\Gamma_t = \gamma^0 + \gamma^1 X_t$, $\gamma^0 \in \mathbf{R}^3$, and $\gamma^1 \in \mathbf{R}^{3\times 3}$. Therefore, the real-world dynamics of the state variables can be expressed as

$$dX_t = K^P(\theta^P - X_t)dt + \Sigma dW_t^P. \tag{8}$$

In the unrestricted case, both K^P and θ^P are allowed to vary freely relative to their counterparts under the Q-measure just as in the original AFNS model.

[15] As per CDR, Σ is a diagonal matrix, and θ^Q is set to zero without loss of generality.

Finally, we note that, due to the non-linear measurement equation for the yields in the shadow-rate AFNS models, their estimation is based on the extended Kalman filter as described in Christensen and Rudebusch (2013a).

4 Model Estimation and Yield Curve Fit

In this section, we describe the in-sample estimation results for our preferred model, its fit to the yield curve, and its ability to price the Treasury securities in the Fed's portfolio.

4.1 Estimation

In this subsection, we briefly describe the shadow-rate AFNS model favored by CR, which is the shadow-rate equivalent of the AFNS model preferred by Christensen and Rudebusch (2012) in their study of the 1987-2010 weekly subsample of our data and briefly detailed in the following.

Using both in- and out-of-sample performance measures, Christensen and Rudebusch (2012) went through a careful empirical analysis to justify various zero-value restrictions on the K^P matrix. Imposing these restrictions results in the following dynamic system for the P-dynamics:

$$\begin{pmatrix} dL_t \\ dS_t \\ dC_t \end{pmatrix} = \begin{pmatrix} 10^{-7} & 0 & 0 \\ \kappa_{21}^P & \kappa_{22}^P & \kappa_{23}^P \\ 0 & 0 & \kappa_{33}^P \end{pmatrix} \left(\begin{pmatrix} 0 \\ \theta_2^P \\ \theta_3^P \end{pmatrix} - \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} \right) dt + \Sigma \begin{pmatrix} dW_t^{L,P} \\ dW_t^{S,P} \\ dW_t^{C,P} \end{pmatrix}, \tag{9}$$

where the covariance matrix Σ is assumed diagonal and constant. Throughout, we refer to the shadow-rate AFNS model given by equations (3), (4), and (9) as the B-CR model.[16]

There are two things to note regarding this specification. First, the Nelson-Siegel level factor is restricted to be an independent unit-root process under both probability measures.[17] As discussed in Christensen and Rudebusch (2012), this restriction helps improve forecast performance independent of the specification of the remaining elements of K^P.[18] Second, we test the significance of the four parameter restrictions imposed on K^P in the model relative to the corresponding model with an unrestricted K^P matrix.[19] Unreported results show that the four parameter restrictions are either statistically insignificant or at most borderline significant throughout our sample period. Thus, the B-CR model is flexible enough to capture the relevant information in the data. Finally, we note that, in the model estimation, we handle the non-stationarity of the factor dynamics in equation (9) in

[16] Following Kim and Singleton (2012), the prefix "B-" refers to a shadow-rate model in the spirit of Black (1995).

[17] Due to the unit-root property of the first factor, we can arbitrarily fix its mean at $\theta_1^P = 0$.

[18] As described in detail in Bauer, Rudebusch, and Wu (2012), bias-corrected K^P estimates are typically very close to a unit-root process, so we view the imposition of the unit-root restriction as a simple shortcut to overcome small-sample estimation bias.

[19] That is, a test of the hypotheses $\kappa_{12}^P = \kappa_{13}^P = \kappa_{31}^P = \kappa_{32}^P = 0$ jointly using a standard likelihood ratio test.

K^P	$K^P_{\cdot,1}$	$K^P_{\cdot,2}$	$K^P_{\cdot,3}$	θ^P		Σ
$K^P_{1,\cdot}$	10^{-7}	0	0	0	σ_{11}	0.0043
						(0.0000)
$K^P_{2,\cdot}$	0.4240	0.3914	-0.4799	0.0386	σ_{22}	0.0086
	(0.1695)	(0.1182)	(0.1059)	(0.0355)		(0.0001)
$K^P_{3,\cdot}$	0	0	0.4249	-0.0296	σ_{33}	0.0264
			(0.1661)	(0.0119)		(0.0002)

Table 1: **Parameter Estimates for the B-CR Model.**
The estimated parameters of the K^P matrix, θ^P vector, and diagonal Σ matrix are shown for the B-CR model. The estimated value of λ is 0.4868 (0.0010). The numbers in parentheses are the estimated parameter standard deviations.

Maturity in months	B-CR model		
	Mean	RMSE	$\widehat{\sigma}_\varepsilon(\tau_i)$
3	-2.62	10.28	10.31
6	-0.04	0.17	0.68
12	2.80	6.63	6.64
24	2.94	5.21	5.29
36	0.01	0.74	1.46
60	-5.35	8.18	8.28
84	-6.61	11.05	11.08
120	-3.66	9.32	9.30
180	1.74	4.70	4.69
240	1.49	11.19	11.22
360	-10.23	33.69	33.73
Max log L	417,381.9		

Table 2: **Summary Statistics of the Fitted Errors.**
The mean and root mean squared fitted errors (RMSE) as well as the estimated yield error standard deviations for the B-CR model are shown. All numbers are measured in basis points. The data covers the period from January 2, 1986, to January 2, 2013.

the way described in Christensen and Rudebusch (2012).

The estimated model parameters are reported in Table 1. Comparing the estimated parameters to the results reported by CR, we note that the parameters are all very similar to theirs, and we have the usual result for AFNS models that the level factor is the most persistent (by construction) and least volatile factor, while the curvature factor is the least persistent, but most volatile factor. Finally, the slope factor have time-series properties in between these two extremes.

If we turn to the summary statistics of the model fit reported in Table 2, we note the very good fit to the entire maturity range up to 20 years with some deterioration in the fit for the 30-year yield. The good fit of the B-CR model documented in Table 2 is also apparent in Figure 3, which shows

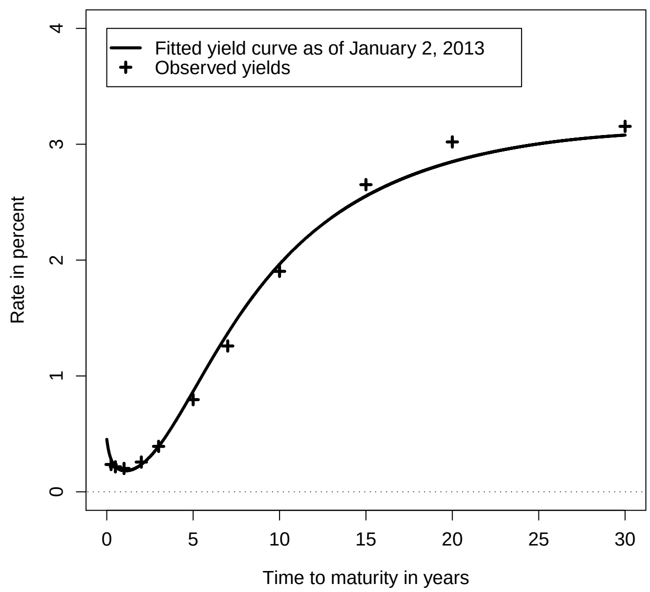


Figure 3: **Fitted Yield Curve from the B-CR Model.**
Illustration of the fitted yield curve as of January 2, 2013, based on the B-CR model. Included are the eleven observed Treasury yields on that day. The data used in the model estimation cover the period from January 2, 1986, to January 2, 2013.

the fitted yield curve as of January 2, 2013, with a comparison to the 11 observed yields on that day. The main weakness of the model on this particular day is a tendency to underestimate the 15- and 20-year yields. As we will see in the following section, this converts into a slight overestimation of the market value of the Fed's portfolio.

4.2 Market Value of Fed's Treasury Securities

To further validate the performance of the B-CR model, we calculate the model-implied value of the 241 Treasury securities that were in the Fed's portfolio as of January 2, 2013, and compare the result to the bond prices downloaded from Bloomberg on that same day.[20] Table 3 reports the total value of the Fed's Treasury securities portfolio and its distribution across maturity buckets. The first column shows the number of securities in each maturity bucket. The second and third columns report the official account based on the face value of the securities. The following two columns reflect the market value of the portfolio based on bond prices from Bloomberg. The last two columns contain the market value of the portfolio implied by the B-CR model as of January 2, 2013.

[20]The Fed's portfolio is available at: http://www.newyorkfed.org/markets/soma/sysopen_accholdings.html

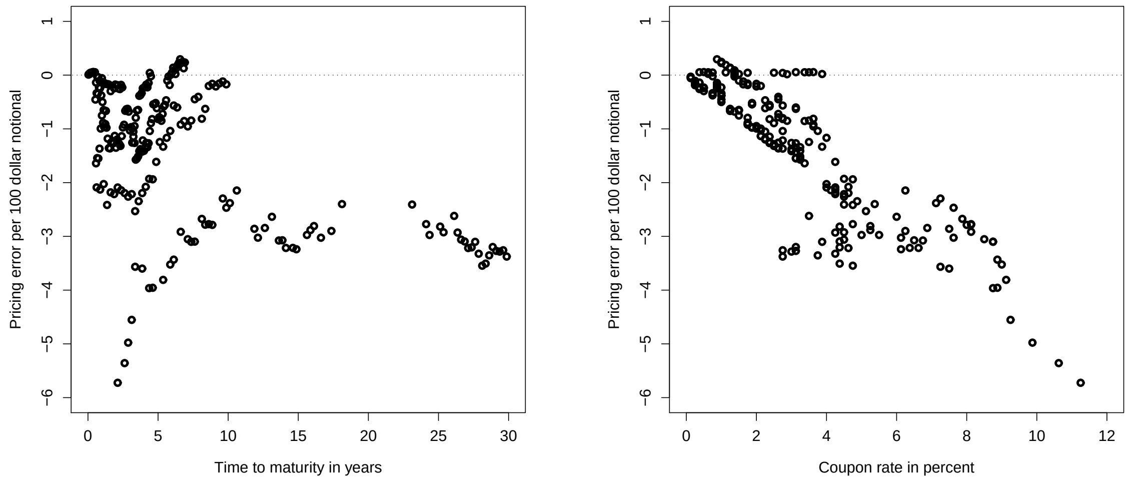
Maturity	No.	Official account		Market value as of January 2, 2013			
		Face value	Percent	Bloomberg	Percent	B-CR model	Percent
All	241	1,580	100.00	1,847	100.00	1,871	100.00
3 years or less	93	4	0.25	4	0.24	5	0.24
4-6 years	72	630	39.87	699	37.82	706	37.74
7-10 years	38	577	36.53	663	35.87	667	35.67
11 or more years	38	369	23.35	482	26.07	493	26.34

Table 3: **Value of Fed's Treasury Securities Portfolio.**
The table reports the distribution of the Fed's Treasury securities portfolio across maturity buckets as of January 2, 2013, using three different valuation methods. The first method is the official account based on the bonds principal value. The second method is to calculate the market value based on bond prices from Bloomberg. The third method is to calculate the market value based on the estimated B-CR model. The reported bond values are measured in billions of dollars.

First, we note the important fact that the Fed's Treasury securities holdings have a market value that is more than $250 billion above their face value according to both methods. Second, we note that the B-CR model's portfolio value is greater by some $22 billion, or 1.27 percent, than the market value calculated based on Bloomberg data. Overall, we consider this difference to be acceptable as our objective is to identify conditions that would entail losses many orders of magnitude larger than the fitted errors above.

Now, we study the B-CR model's pricing performance in greater detail. Figure 4 presents the distribution of pricing errors across maturities and coupon rates. First, we note that 69 of the 241 bonds have fitted prices that are within a quarter of the Bloomberg price. These are bonds that are either close to maturity or have been issued fairly recently. Importantly, the latter group includes bonds with up to ten years to maturity. Thus, the model is clearly able to price medium- and long-term bonds accurately. On the other hand, in Figure 4(a), it is noted that the model overvalues the long-maturity bonds as the fitted long-term yields are below those observed in that maturity range; see Figure 3. Still, on a yield basis, these pricing errors are reasonable. Also, as noted in Figure 4(b), the model tends to overvalue bonds with large coupons, but this could be a consequence of the fact that these bonds are very seasoned and illiquid for that reason. Furthermore, the bonds with the largest coupons are close to maturity and were sold in large amounts by the Fed during its maturity extension program that ended in December 2012. This is likely to have depressed their prices further.

Despite some minor weaknesses in its fit, we emphasize that this is a very strong test of the B-CR model as we are trying to match data that was not used in the model estimation. Obviously, in an ideal world, the GSW yield curve construction would extract all systematic information from the underlying pool of raw bond prices (which presumably is close to the universe of bonds in the Fed's portfolio that we are studying here) and the eleven yield maturities we use in the model estimation would reflect this. Finally, again in an ideal world, the B-CR model would be able to accurately



(a) Price error across maturities.

(b) Price error across coupon rates.

Figure 4: **Price Errors.**
Illustration of the pricing error in dollars per $100 notional based on the B-CR model relative to Bloomberg data for the 241 Treasury securities in the Fed's portfolio as of January 2, 2013. Panel (a) shows the pricing errors as a function of bond maturity, while panel (b) shows the pricing errors as a function of the bond coupon rate.

extract this systematic pricing information from the observed yields. However, in reality, each of these steps is likely to introduce errors. Thus, the fact that the model is able to stay on track and deliver reasonably precise prices across the entire maturity range present in the Fed's portfolio is testament to its accuracy and usefulness for our purposes.

5 Stress Testing the Fed's Portfolio of Treasuries

In this section, we describe how we proceed to stress test the Fed's portfolio of Treasury securities using the entire distribution of future yields. Our main focus in this section considers what could "reasonably" happen to the market value of the existing portfolio of Treasury securities within a three-year horizon.[21] We limit the forecast horizon in this way because the exercise is meant to be simple and illustrative. This exercise also allows us to consider some non-model-based techniques that can be used to generate probability-based scenarios.

The key ingredient in our stress test exercise is to make future projections of the Treasury yield

[21] To be clear, the Fed values its securities at acquisition cost and registers capital gains and losses only when securities are sold. Such historical-cost accounting is considered appropriate given the Fed's macroeconomic policy objectives and is consistent with the buy-and-hold securities strategy the Fed has traditionally followed. However, the Fed also does report unrealized capital gains and losses on its securities portfolio, which mimics private-sector mark-to-market accounting on holdings of longer-term securities. As noted above, large realized or unrealized capital losses could subject the Fed to criticism.

curve. Towards that goal, we use three quite different approaches to illustrate the range of possible probability-based stress tests. The first approach is based on yield curve projections from the B-CR model. The second approach relies on historical Treasury yield curve changes for its projections. The third approach converts the Treasury yield projections in the Blue Chip Financial Forecasts survey of professional forecasters from January 2013 into a distribution of future portfolio values.

Each approach has advantages and disadvantages. The model-based approach is flexible and may be able to capture, for example, any changes in the dynamics of the Treasury yield curve as a consequence of changes in the monetary policy communications strategy of the FOMC. Still, any model-based approach may suffer from model mis-specification. In addition to being easy to implement, the projections based on historical yield changes are clearly grounded in observed outcomes. However, this can be a disadvantage, as conditioning on the current situation is difficult. The approach based on the Blue Chip survey has the advantage that it reflects the views of professional forecasters, who are thought to rely on a mix of theoretical and statistical methods for their submitted forecasts. The disadvantage of this approach is twofold. First, there are only a small number of respondents, which limits the dispersion of possible outcomes. Second, and more importantly, each forecaster presumably submits only a point forecast. Thus, this approach may not provide reliable information about the tails of the yield curve distribution. As such, this approach could be less useful from a risk management perspective.

5.1 Projections Based on the B-CR Model of the Treasury Yield Curve

The first approach we use to generate Treasury yield curve projections is based on the estimated B-CR model. The advantage of this approach is that we can assign probabilities to specific outcomes as estimated by the model. The obvious drawback is that the results will contain some element of model risk, and it might be difficult to repeat for other data and other types of models as it involves a careful model validation process that we avoid here by relying on the shadow-rate AFNS model structure and the specification thereof favored by CR.

For a start, Figure 5 shows the shadow rate projection from the B-CR model as of January 2, 2013, along with the corresponding short rate projection.[22] We note that the short rate is projected to first come down over the nearest 6 months, then stay flat at zero for the following 18 months, before it starts its gradual reversal towards steady state over the following many years.

As the yield function in the B-CR model given in equation (7) is non-linear in the state variables, we have to use Monte Carlo simulations of the model in the stress test. Specifically, we simulate N = 10,000 sample paths of the state variables up to three years ahead, each starting from the filtered values at the end of our sample, denoted $\widehat{X}_t = (\widehat{L}_t, \widehat{S}_t, \widehat{C}_t)$. For each projection N months ahead,

[22] Note that the shown projected short rates are the realizations consistent with the projected shadow rates, i.e., $\max\{0, E_t^P[s_t]\}$, rather than the expected path of the short rate, which is given by $E_t^P[\max\{0, s_t\}]$.

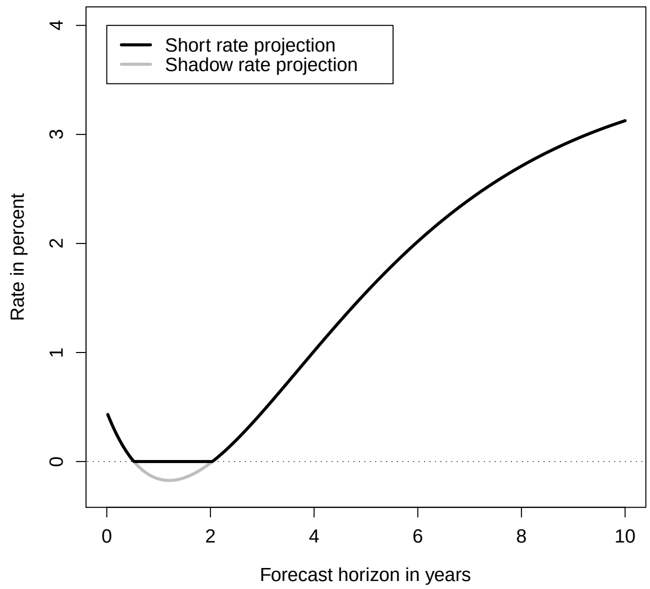


Figure 5: **Short Rate Projections.**
Illustration of shadow rate projection from the B-CR model as of January 2, 2013, along with the corresponding short rate projection.

Projection in months	Percentiles in portfolio value distribution					
	0.1%	1%	5%	10%	25%	50%
3	1,754	1,781	1,807	1,821	1,842	1,864
6	1,699	1,740	1,775	1,795	1,825	1,856
9	1,656	1,700	1,748	1,773	1,811	1,848
12	1,611	1,672	1,723	1,750	1,796	1,839
15	1,592	1,641	1,701	1,731	1,780	1,829
18	**1,569**	1,621	1,683	1,714	1,768	1,821
21	**1,532**	1,599	1,666	1,699	1,756	1,810
24	**1,523**	1,583	1,649	1,683	1,742	1,801
27	**1,505**	**1,569**	1,635	1,671	1,730	1,792
30	**1,490**	**1,558**	1,620	1,659	1,718	1,780
33	**1,478**	**1,545**	1,610	1,646	1,707	1,768
36	**1,466**	**1,531**	1,600	1,636	1,695	1,756

Table 4: **Model-Based Projected Market Value of the Fed's Treasury Securities.**
The table shows percentiles ranging from 0.1% to 50% in the distribution of the market value of the Fed's Treasury securities portfolio projected between 3 and 36 months ahead based on $N = 10{,}000$ Monte Carlo simulations of the B-CR model as described in the main text. All portfolio values are measured in billions of dollars. Projections with portfolio values below the face value as of January 2, 2013, are shown in bold.

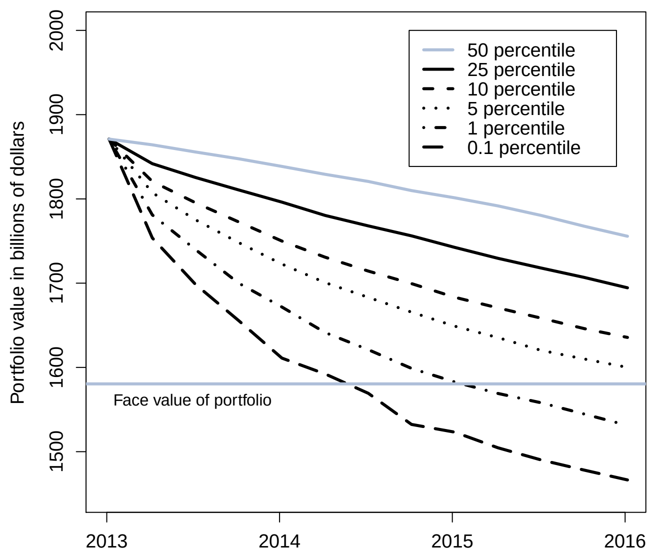


Figure 6: **Model-Based Projected Market Value of the Fed's Treasury Securities.** Illustration of the percentiles ranging from 0.1% to 50% in the distribution of the market value of the Fed's Treasury securities portfolio projected between 3 and 36 months ahead based on N = 10,000 Monte Carlo simulations of the B-CR model as described in the main text.

the simulated state variables are converted into a full yield curve and we calculate the corresponding portfolio values. Table 4 reports the lowest percentiles, 0.1%, 1%, 5%, 10%, and 25%, as well as the median of the projected portfolio values for each forecast horizon, while Figure 6 illustrates the percentiles as a function of the forecast horizon.

In Figure 5 we saw that the short rate is projected to move up over our forecast horizon. This explains why the median portfolio value in our projections trend lower as the forecast horizon is increased. Still, when the portfolio is kept fixed as in this exercise, extending the forecast horizon has two effects that go in opposite directions. On the one hand, with a longer projection horizon a wider range of outcomes are likely, so the potential yield changes are larger, in particular in the tail of the distribution. On the other hand, this is offset by a reduction in the time to maturity on all securities in the portfolio. Bonds with shorter time to maturity have lower duration and, as a consequence, their prices are less sensitive to changes in the interest rate environment. For this exercise, the results in Figure 6 indicate that the former effect dominates at all forecast horizons considered.

5.2 Projections Based on Historical Treasury Yield Curve Changes

In this section, we use the historical yield curve changes in our sample of daily Treasury yields since 1986 to generate Treasury yield curve projections up to three years ahead.

The basic idea is to look to the past for guidance on what might happen N months ahead in the current situation. The first step is to estimate the B-CR model on the full sample that ends in January 2, 2013. In the second step, for the N-month projection, we go through all the N-month yield curve changes observed in the data as measured by changes in the three state variables within the B-CR model.[23] We denote the total number of such yield curve or, equivalently, state variable changes by m_N. In the third step, we take the estimated state variables as of January 2, 2013, denoted $\widehat{X}_t = (\widehat{L}_t, \widehat{S}_t, \widehat{C}_t)$, and add each of the $i = 1, \ldots, m_N$ factor shock constellations identified in the previous step. This gives us m_N new state variable constellations each of which has the property that exact yield curve shock has happened once before. In the fourth step, we use the estimated yield function in equation (7) to convert each new state variable constellation into a full yield curve from 0 to 30 years in maturity that we use to calculate the value of the Fed's portfolio N months hence. In the fifth step, we rank all the estimated portfolio values from the lowest to the highest and focus on the lowest percentiles: 0.1%, 1%, 5%, 10%, and 25% as well as the median. In particular, the tail of these values is interesting as they are based on yield curve changes that have previously occurred. Finally, this is repeated for the following forecast horizons $N \in \{3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36\}$ measured in months.

This approach has several advantages. First, the projections are predominantly data-driven, not model-based. Second, realism is embedded in the exercise by construction as these are yield curve changes that have occurred in the past. Finally, it is an exercise that other researchers can easily repeat and apply to other data samples and other models. In particular, it is an approach that can be applied to analyze potential stress of the securities portfolios of other central banks that have engaged in quantitative easing, notably the Bank of England and the Bank of Japan.

Table 5 reports percentiles in the projected distribution of the market value of the Fed's Treasury securities portfolio, while Figure 7 provides a graphical representation of the same data. This allows us to contrast the model-based approach with the result from the projections based on historical yield changes. For the latter, the declines in the median and the lower 25 percentile are much more modest as the projection period lengthens. This is caused by the fact that yields have trended lower on average since 1986, see Figure 2.

In principle, if the two approaches are capturing the same underlying fundamental yield curve dynamics, the results should be comparable, but obviously it is two rather different ways of generating

[23] In the exercise, one month is defined as 21 observation dates, which is close to the average for the entire sample. Thus, for example, the six-month projections are based on the observed yield curve changes 126 observation dates apart.

Projection in months	No.	Percentiles in portfolio value distribution					
		0.1%	1%	5%	10%	25%	50%
3	6,675	1,717	1,744	1,794	1,813	1,844	1,873
6	6,612	1,653	1,709	1,759	1,790	1,832	1,877
9	6,550	1,611	1,668	1,742	1,781	1,833	1,876
12	6,487	1,617	1,653	1,726	1,774	1,828	1,873
15	6,425	1,613	1,647	1,728	1,766	1,822	1,872
18	6,363	1,622	1,673	1,725	1,772	1,818	1,866
21	6,300	1,667	1,690	1,739	1,770	1,814	1,860
24	6,238	1,657	1,690	1,748	1,775	1,816	1,853
27	6,176	1,665	1,691	1,745	1,781	1,816	1,849
30	6,113	1,664	1,709	1,744	1,774	1,811	1,845
33	6,051	1,674	1,699	1,746	1,770	1,806	1,836
36	5,989	1,658	1,690	1,746	1,770	1,797	1,830

Table 5: **History-Based Projected Market Value of the Fed's Treasury Securities.** The table shows percentiles ranging from 0.1% to 50% in the distribution of the market value of the Fed's Treasury securities portfolio projected between 3 and 36 months ahead based on historical yield curve changes as described in the main text. All portfolio values are measured in billions of dollars.

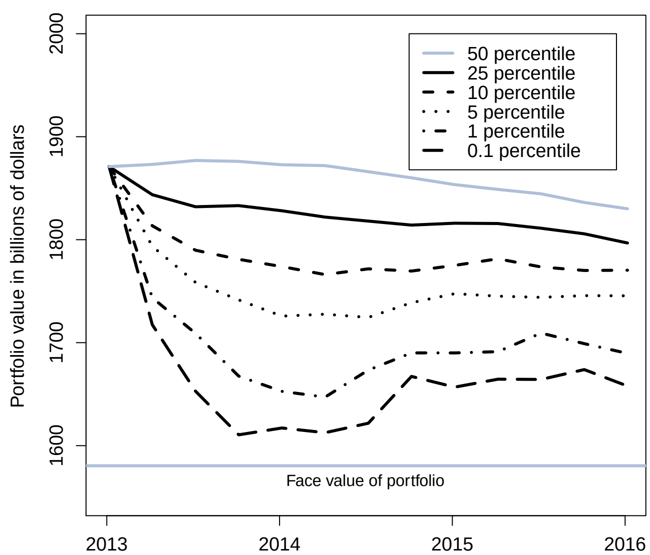


Figure 7: **History-Based Projected Market Value of the Fed's Treasury Securities.** Illustration of the percentiles ranging from 0.1% to 50% in the distribution of the market value of the Fed's Treasury securities portfolio projected between 3 and 36 months ahead based on historical yield curve changes as described in the main text.

Projection in months	No.	Percentiles in portfolio value distribution				
		2%	5%	10%	25%	50%
3	49	1,815	1,822	1,858	1,873	1,885
6	49	1,791	1,799	1,819	1,849	1,870
9	49	1,758	1,769	1,774	1,827	1,845
12	48	1,734	1,734	1,772	1,798	1,831
15	43	1,700	1,702	1,749	1,776	1,816

Table 6: **Survey-Based Projected Market Value of the Fed's Treasury Securities.** The table shows percentiles ranging from 2% to 50% in the distribution of the market value of the Fed's Treasury securities portfolio projected between 3 and 15 months ahead based on the Blue Chip Financial Forecast survey of professional forecasters as described in the main text. All portfolio values are measured in billions of dollars.

yield curve projections. An important difference is that the second approach reflects all yield curve changes in the past, but does not take the potentially very unusual conditions currently into consideration. The advantage of the first, model-based approach is that it is conditioning its projections on where the economy was at the end of the sample period, but still the model has no more experience with exiting a ZLB period than what is reflected in the data from past tightening episodes.

5.3 Projections From the Blue Chip Survey of Professional Forecasters

As a third approach and for robustness, we convert the Treasury yield projections in the January 2013 Blue Chip Financial Forecasts survey of professional forecasters into projections for the value of the Fed's Treasury portfolio.

In the January 2013 Blue Chip survey performed during the last week of 2012, the participants submitted projections for the three- and six-month T-Bill rates and one-, two-, five-, ten-, and thirty-year Treasury yields from one to five quarters ahead. In the survey, the submitted values are supposed to be averages for the quarter. However, we will interpret them as projected values for the yields prevailing at the end of each quarter.

We map the projected yields from each forecaster into a constellation of the three state variables in the B-CR model by minimizing the equal-weighted squared error between the submitted yields and the model-implied yields from equation (7). In the second step, we convert those projected state variables into a full yield curve and calculate the associated value of the Fed's Treasury securities holdings. The relevant percentiles of the distribution of portfolio values are reported in Table 6, while Figure 8 provides a graphical representation of the same data.[24]

In comparing the results to the previous two exercises, we note that the stress test based on

[24]Note that the number of forecasters included decline as the forecast horizon is increased as not all forecasters submit 12-month or longer yield projections.

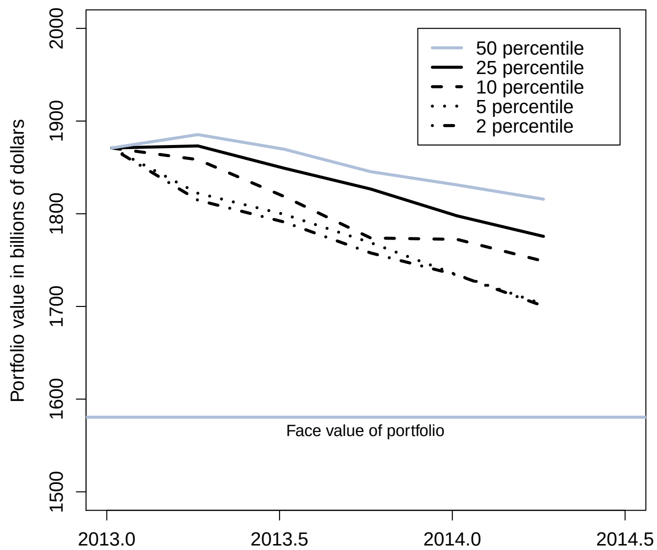


Figure 8: **Survey-Based Projected Market Value of the Fed's Treasury Securities.** Illustration of the percentiles ranging from 2% to 50% in the distribution of the market value of the Fed's Treasury securities portfolio projected between 3 and 15 months ahead based on the Blue Chip Financial Forecast survey of professional forecasters as described in the main text.

historical yield changes is benign at the 15-month forecast horizon even relative to the Blue Chip survey based approach. As already mentioned, this is likely caused by the fact that yields have been trending lower on average since 1986. As a consequence, any upward shifts in the yield curve have been relatively modest in size. Still, the relative closeness in results from these two exercises indicate that the forecasters in the Blue Chip survey anticipate a gradual increase in the yield level that is not significantly different from what has been the experience the most recent 20 years.

More surprisingly, at the 15-month forecast horizon, the 5 to 50 percentiles of the portfolio values from the B-CR model-based approach and the Blue Chip survey based values are very close. Hence, the uncertainty about interest rate outcomes in the short- to medium-term as captured by the B-CR model is not that different from the uncertainty across panelists in the Blue Chip survey even though each surveyed forecaster presumably submit his or hers point estimates.

6 Stress Testing the Fed's Income

A key contribution of this paper is to introduce a probability-based approach to policy questions associated with the Federal Reserve's balance sheet and particularly with respect to "stress testing"

Variable	GHHM	CLR
1. Asset purchases	Continue at current pace through December 2013, slow to maintenance levels (stock stable) through 2014, stop (stock declines) in 2015.	Purchases through 2014 match Primary Dealer Survey as of June 2013 and end in 2014.
2. Asset sales	No Treasury sales. MBS sales start in late 2015 and are completed in 2019.	No Treasury or MBS sales.
3. MBS prepayment	Follows market models.	Same.
4. Liabilities	Currency grows at 7 % annual rate (2 percentage points above Blue Chip forecast for nominal GDP growth per historical experience); required reserves grow at 4 % annual rate.	Same.
5. Interest rates	Driven by Blue Chip forecast.	B-CR model projections.
6. Fed capital	Grows at 10 % annual rate per historical average.	Same.
7. Operating expenses	Grow on historical trend.	Same.

Table 7: **Assumptions Underlying Balance Sheet and Income Projections.**

scenarios that provide insight on the range of possible adverse outcomes. This is illustrated above with several approaches to value the Fed's Treasury holdings, which encompass the bulk of the Fed's assets. However, a more comprehensive perspective that considers other assets is also of interest. Here, we conduct such an analysis with a focus on the Fed's income risk. Again, the primary concern is that particular combinations of planned asset purchases and sales and interest rate outcomes could lead the Federal Reserve's net interest income and balance sheet values to decline sufficiently to halt remittances.

Carpenter et al. (2013) and GHHM also consider prospects for the Fed's remittances to the Treasury under several scenarios. In this section, we take a probabilistic simulation-based approach to this issue using the B-CR model to generate interest rate forecast distributions. To translate these into remittances, we use the structure of GHHM.[25] As shown in Table 7, the GHHM calculations are based on several assumptions regarding the Federal Reserve's balance sheet and net income projections. For our analysis, we do not alter these assumptions regarding MBS prepayment, currency (or liability) growth, capital accretion, operating expenses, and the absence of asset re-investment. However, we do alter several of their other assumptions. The first modification we made was to link the asset purchase path through 2014 directly to the publicly-announced results of the New York Fed Primary Dealer Survey as of June 2013.[26] This path is less aggressive than assumed in the GHHM baseline, with purchases ending in 2014 as opposed to 2015. The second modification is that we assume no MBS, agency, or Treasury sales are conducted through 2020. This change in

[25] We greatly appreciate the authors' sharing of their code with us for the purposes of this analysis.

[26] The survey is publicly available at: http://www.newyorkfed.org/markets/survey/2013/June_result.pdf

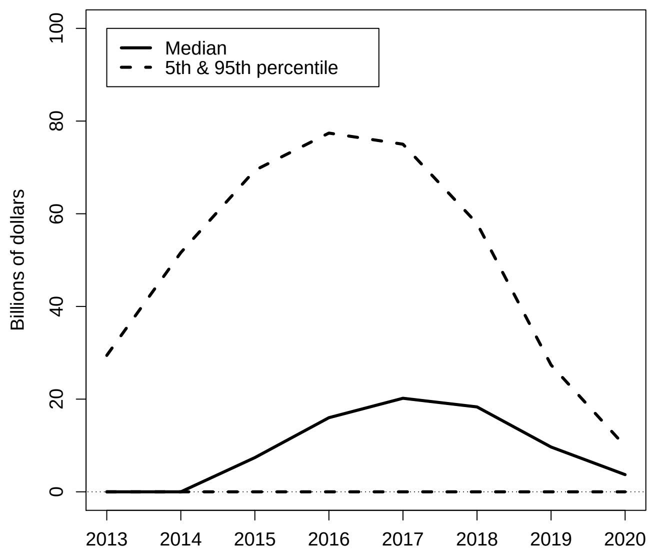


Figure 9: **Projections of Interest Expenses.**
Illustration of the median and the 5th and 95th percentiles of the projected interest expenses based on the CLR baseline scenario combined with $N = 10{,}000$ Monte Carlo simulations of the B-CR model.

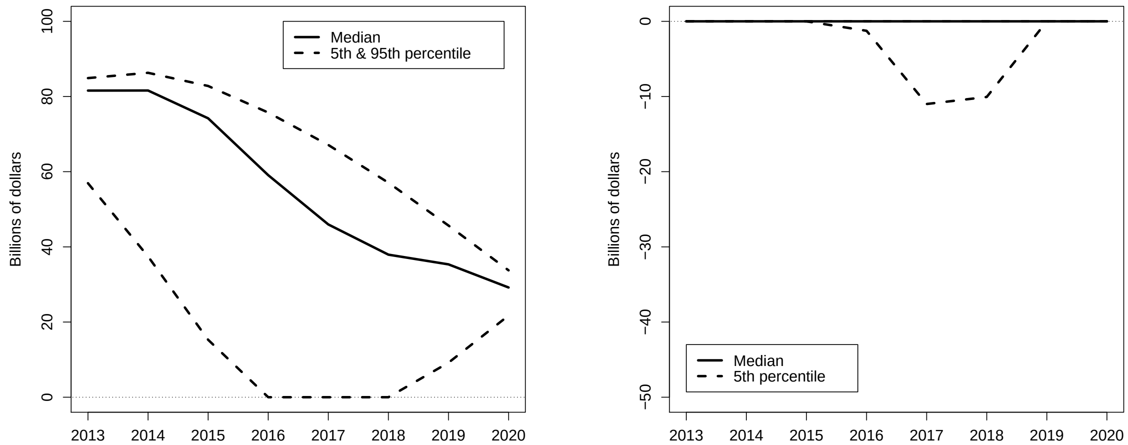
assumptions is motivated by the developing consensus reflected in the minutes from the June 2013 FOMC meeting where most members indicated that they anticipate the Fed not to sell any MBS as part of the policy normalization process.[27] The third modification is that we set the path for the interest on excess reserves (IOER) rate equal to the appropriate overnight rate implied by our yield curve simulation.[28] This assumption is an important source of variation shown in Figure 9, which illustrates the projections of interest expenses under our set of modeling assumptions.

Figure 10 presents the key results of our simulation-based approach for the CLR baseline scenario summarized in Table 7. We focus on positive remittances to the Treasury and the Federal Reserve "deferred asset."[29] Our results imply zero remittances for the lower 5th or so percentile of the outcomes. Even in these cases, the low point is only from 2016 to 2018 and the deferred assets do not fall below minus $11.0 billion as shown in Figure 10(b).

[27]See p. 2 of the minutes at http://www.federalreserve.gov/monetarypolicy/files/fomcminutes20130619.pdf

[28]The overnight rate is approximated by the instantaneous short rate $r_t = \max\{0, s_t\}$.

[29]The Federal Reserve, under its remittance policy, remits all net income to the U.S. Treasury—after expenses, dividends, and additions to capital. If earnings are insufficient to cover these costs, the Fed creates new reserves against a "deferred asset," which represents a claim on future earnings and remittances to the Treasury.



(a) Payments to Treasury.

(b) Deferred assets.

Figure 10: **Projections of Payments to the U.S. Treasury and the Fed's Deferred Assets Using the CLR Baseline Scenario.**
Panel (a) shows the median and the 5th and 95th percentiles of the projected payments to the U.S. Treasury, while panel (b) illustrates the median and the 5th percentile of projections of the Fed's deferred assets based on the CLR baseline scenario combined with $N = 10{,}000$ Monte Carlo simulations of the B-CR model.

7 Conclusion

Our methodological contribution in this paper is to introduce a probabilistic structure into stress tests or scenario analysis of the Fed's balance sheet and income prospects. We argue that attaching likelihoods to adverse outcomes is a crucially important addition to the policy debate. Of course, our analysis is merely a first step in this direction, and more research can aim to expand and refine the probabilistic structure.

In terms of substantive results, we use three different ways of generating Treasury yield curve projections. The first approach is based on the specific shadow-rate AFNS model favored by CR. The second approach relies on historical Treasury yield curve changes, and the third leverages the Blue Chip Financial Forecasts survey of professional forecasters. Despite differences in methods, the results are similar and indicate that in all likelihood the potential losses to the Fed's Treasury securities holdings over the next several years are relatively modest. We also generate more comprehensive projections of the Fed's future income and find a small chance of a temporary halt of the remittances to the Treasury. In summary, our probabilistic scenario approach provides additional and generally reassuring guidance regarding questions related to the financial costs of the Federal Reserve's balance sheet policy.

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