

Optimal Monetary and Prudential Policies*

Fabrice Collard[†] Harris Dellas[‡] Behzad Diba[§] Olivier Loisel[¶]

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Abstract

We develop a New Keynesian model with banks to study the interactions between monetary policy and a prudential policy that sets state-dependent bank-capital requirements. In our model, limited liability and deposit insurance lead to risk-taking incentives that involve the type (not necessarily the volume) of bank credit. This makes monetary policy potentially ineffective in ensuring financial stability. Locally optimal (Ramsey) policy, setting both instruments jointly, dedicates the prudential policy instrument to preventing inefficient risk-taking by banks. Compared to optimal monetary policy under a constant capital requirement, jointly optimal policies involve a more expansionary or less contractionary monetary policy stance when capital requirements are raised to curb risk taking, in order to mitigate the effects of prudential policy on bank lending and output. Our model can capture circumstances that make the optimal conduct of both policies countercyclical, as well as circumstances in which the two policies optimally move in opposite directions over the cycle.

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[†]Department of Economics, University of Bern. fabrice.collard@gmail.com, <http://fabcol.free.fr>

[‡]Department of Economics, University of Bern, CEPR. harris.dellas@vwi.unibe.ch, <http://www.harrisdellas.net>

[§]Department of Economics, Georgetown University. dibab@georgetown.edu, <http://www9.georgetown.edu/faculty/dibab/>

[¶]CREST (ENSAE). olivier.loisel@ensae.fr, <http://www.cepremap.ens.fr/depot/pageperso/LOIOLI/>

1 Introduction

Monetary and prudential policies have traditionally been designed and analyzed in isolation from one another. However, both the recent financial crisis and the regulatory response to this crisis – notably the forthcoming introduction of state-contingent bank capital requirements [Basel Committee on Banking Supervision (2010)] – have aroused interest in the interactions between monetary and prudential policies. Policymakers [e.g., Bernanke (2010), Blanchard et al. (2010)] have commented on the extent to which interest-rate policy can or should address concerns about financial stability. And policy-oriented discussions [e.g., Canuto (2011), Cecchetti and Kohler (2012)] have summarized alternative views about the potential substitutability or complementarities between the new prudential policy tools (like state-contingent bank capital requirements) and the interest rate set by monetary policy. There is a general presumption that both policies will be counter-cyclical most of the time, but policymakers and commentators [e.g., Macklem (2011), Wolf (2012), Yellen (2010)] are also envisaging hypothetical scenarios that may put the two policies at odds with each other over the business cycle.

In this paper, we develop a New Keynesian model with banks to study, from a normative perspective, the interactions between monetary policy and a prudential policy that sets state-dependent bank-capital requirements. Several recent contributions [e.g., Angeloni and Faia (2011), Christensen, Meh, and Moran (2011)] have developed models that address this issue. We depart from this literature in two main ways: first, by making excessive risk taking involve the type (not necessarily the volume) of bank credit; second, by determining some jointly locally Ramsey-optimal monetary and prudential policies.

In the existing literature, excessive risk taking typically involves the aggregate volume of credit in the economy. In Angeloni and Faia (2011), this occurs through the bank leverage ratio that governs the risk of bank runs; in Christensen, Meh and Moran (2011), through an externality that links the riskiness of projects funded by banks to the aggregate credit to GDP ratio. This link between excessive risk taking and the volume of credit can also be found in a number of contributions that abstract from monetary policy [e.g., Bianchi (2010), Bianchi and Mendoza (2010), Jeanne and Korinek (2010)] or from prudential policy [Benigno et al. (2011)]: in these contributions, this occurs through a pecuniary externality associated with a collateral constraint, which makes an asset-price boom increase the value of borrowers' collateral.¹ In all these models, economic expansions – following, for example, a favorable productivity shock – lead to excessive risk taking and call for a policy response that may be either monetary or prudential.

This focus on risk-taking behavior over the credit cycle, which also reflects policy-oriented concerns about the “risk-taking channel” of monetary policy [as discussed, for example, in Borio and Zhu (2008)], seems natural in the aftermath of the recent crisis. However, other perspectives should also serve to inform the design of future regulatory frameworks, as the interactions between monetary and prudential policies deserve scrutiny from alternative theoretical vantage points.

In this paper, we view these interactions through the lens of an older literature on bank-capital

¹Kashyap, Berner and Goodhart (2011) emphasize the relevance of another pecuniary externality (fire sales of assets) for the design of prudential policies.

requirements.² Specifically, we introduce aggregate risk into a variant of Van den Heuvel's (2008) model of optimal capital requirements, and we embed the resulting model into a DSGE framework with aggregate shocks, sticky prices, and monetary policy. In our model, following Van den Heuvel (2008) and others, the need for capital requirements arises from limited liability and deposit insurance. These institutional features truncate the distribution of risky returns facing investors, the banks lending to these investors, and the depositors funding the banks; this is the externality that leads to excessive risk taking in our model. This perspective, unlike models emphasizing the credit cycle, does not automatically link excessive risk taking to the volume of credit. In our model, excessive risk taking involves the type (not necessarily the volume) of credit that banks may be tempted to extend. Therefore, a productivity boom does not lead to excessive risk taking as long as it does not make the funding of risky projects relatively more lucrative than the funding of safe projects.

In this context, a high enough capital requirement can make banks internalize the riskiness of the projects they fund, and tame risk taking. However, in our benchmark model with perfectly competitive banks operating under constant returns to scale, the interest rate has no effect on risk-taking incentives as it affects the cost of funding all (safe or risky) projects equally. From this vantage point, capital requirements and the interest rate are sharply distinct policy tools that do not affect the same margins: monetary policy affects the volume but not the type of credit, while prudential policy affects both the type and the volume of credit. This makes monetary policy potentially ineffective in ensuring financial stability. So, in contrast to existing models that emphasize the credit cycle, our framework does not suggest a strong connection between interest-rate policy and financial stability. As such, our framework accords with policymakers' consensus [expressed, for instance, in Bernanke (2010)] that standard interest-rate policy cannot serve as the first line of defense against financial instability.

We determine, in our model, a monetary policy (numerically) and a prudential policy (analytically) that are jointly locally Ramsey-optimal. In so doing, we improve on the existing literature [e.g., Angeloni and Faia (2011), Christensen, Meh, and Moran (2011)], which determines (numerically) some monetary and prudential policies that are jointly optimal in a more restrictive sense. Indeed, in this literature, the deviations of the interest rate and the capital requirement from their steady-state values are optimized within some parametric families of simple rules, and the steady-state value of the capital requirement is not locally optimal.

In our model, the locally Ramsey-optimal policy sets the capital requirement to the minimum level that prevents inefficient risk taking by banks. Setting the capital requirement just below this threshold level is not optimal because it triggers a discontinuous increase in the amount of inefficient risk taken by banks. This discontinuity is due to our deposit-insurance and limited-liability assumptions, which make banks' expected excess return convex in the amount of risk that they take, so that they choose to take either zero risk or the maximum amount of risk. Setting the capital requirement just above this threshold level is not optimal either, because it has a negative first-order effect on welfare that cannot be offset by any change in the interest rate around its optimal value (as this change would have a zero first-order effect on welfare). This negative first-order effect on welfare, in turn, is due

²There are other perspectives on capital requirements (besides the approach we take) in the literature, but their implications for monetary policy remain to be analyzed. For example, Gete and Tiernan (2011) consider the role of capital requirements in Hachem's (2010) model of overlending (as we elaborate below). They also provide extensive references to earlier work.

to a tax on banks' profits, which makes equity finance more expensive than debt finance for banks: this tax distortion implies that raising the capital requirement above the threshold level decreases the (bank-loan-financed) capital stock, which is already inefficiently low due to monopolistic competition, without reducing the amount of inefficient risk, which is already zero.³

This locally Ramsey-optimal capital requirement is state dependent in our model: it rises (falls) in response to shocks – as, for instance, a decrease (increase) in default risk – that increase (decrease) banks' incentives to fund risky projects rather than safe ones. So our benchmark model, in which the interest rate and the capital requirement do not affect the same margins, implies a clear-cut optimal division of tasks between monetary and prudential policies: in response to shocks that do not affect banks' risk-taking incentives, prudential policy should leave the capital requirement constant, and monetary policy should move the interest rate in a standard way. In response to shocks that increase (decrease) banks' risk-taking incentives, prudential policy should raise (cut) the capital requirement, and monetary policy should cut (raise) the interest rate in order to mitigate the effects of prudential policy on bank lending and output. In the latter case, optimal prudential policy is pro-cyclical (as it is the proximate cause of the contraction), while optimal monetary policy is counter-cyclical. So, with this chain of causality, the two policies move in opposite directions over the cycle – a situation envisaged by some policymakers and commentators [e.g., Macklem (2011), Wolf (2012), Yellen (2010)].

In reality, of course, unlike in our benchmark model, monetary policy may affect banks' risk-taking incentives: a prolonged period of low interest rates may lead banks to make more risky loans, for instance because they run out of less risky lending opportunities. To illustrate this possibility, we develop an extension of our benchmark model in which the cost of originating and monitoring safe loans is an increasing function of the aggregate volume of such loans.⁴ In this extension, all the shocks that affect the volume of safe loans also affect the cost of such loans and, therefore, banks' risk-taking incentives. A favorable productivity shock, for instance, by raising the volume and hence the cost of safe loans, increases banks' risk-taking incentives. Following this shock, optimal prudential policy raises the capital requirement, and optimal monetary policy raises the interest rate by less than in the benchmark model in order to mitigate the effects of the rise in the capital requirement on bank lending and output. In this case, the jointly optimal monetary and prudential policies are therefore both countercyclical.

The rest of the paper is organized as follows. Section 2 presents our benchmark model. Section 3 discusses our analytical results on prudential policy, with proofs relegated to the Appendix. Sections 4 and 5 discuss our calibration and report our numerical results for the locally Ramsey-optimal monetary and prudential policies in the benchmark model. Section 6 presents two extensions (one with an externality in the cost of banking, the other with correlated shocks) that seem relevant for policy concerns. Section 7 contains concluding remarks.

³Instead of considering a tax distortion, Van den Heuvel (2008) models the cost of raising capital requirements as foregone liquidity from holding bank deposits. In his model, liquid deposits and equity are the only sources of funding for bank loans. So, when capital requirements are higher, banks don't issue as much liquid deposits, and households suffer a loss of utility. We don't pursue this track because commercial paper (rather than liquid deposits) is a more likely marginal source of funding for US banks, as Cúrdia and Woodford (2009) point out. For the same reason, following Cúrdia and Woodford (2009) and others, our modeling of optimal monetary policy will abstract from the transactions frictions that motivate the Friedman Rule.

⁴We use this ad-hoc assumption about costs of banking to keep our digression brief. Hachem (2010) develops a full model of this type of externality in bank lending.

2 Benchmark Model

To motivate the role of banks in our model, we assume that households must sell their unfurnished capital stock to capital producers at the end of each period and buy back the furnished capital at the beginning of the next period. These producers have access to two alternative technologies to furnish capital: one is safe and the other risky. The latter technology is less efficient on average, but limited liability tempts the capital producers to use it. Banks are needed to monitor the producers who claim to use the safe technology, to ensure that they do so. Banks themselves, however, may have adverse incentives due to limited liability and deposit insurance, and these adverse incentives give a role to prudential policy.

Each period is divided into two subperiods. At the beginning of the first subperiod, all exogenous shocks are realized, except one, and these realizations are observed by all agents. The only shock that is not realized at the beginning of the first subperiod is the binary shock leading to the success or failure of the risky technology (in the case of failure, forcing any capital producers using this technology to default on their bank loans). This shock is realized at the end of the second subperiod, after households, firms, and banks have made their optimal decisions.

2.1 Households

Preferences are defined by the discount factor $\beta \in (0, 1)$ and the period utility

$$U(c_t, h_t) = \log(c_t) - \frac{1}{1+\chi} h_t^{1+\chi}$$

over consumption c_t and hours of work h_t , where $\chi > 0$. Households maximize $E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, h_t)$. All household decisions are taken in the first subperiod of each period t . We assume that, during this subperiod, households own the furnished capital stock k_t and rent it, at the rental price z_t , to intermediate goods producers. At the end of the subperiod, after production has taken place, households get back $(1 - \delta)k_t$ worn-out capital from intermediate goods producers, where $0 < \delta < 1$, and invest i_t in new capital. Unfurnished capital x_t , made of both worn-out capital and new capital, has to be furnished before it can be used for production next period. So, at this stage, households sell their unfurnished capital

$$x_t = (1 - \delta)k_t + i_t, \tag{1}$$

at the price q_t^x , to capital goods producers, who can furnish it in the second subperiod of period t . At the beginning of the next period, households buy furnished capital k_{t+1} , at a price q_{t+1} , from capital goods producers.

Households also acquire s_t shares in banks at a price q_t^b . These banks are perfectly competitive and last for only one period.⁵ Households face the budget constraint

$$c_t + d_t + q_t^b s_t + q_t k_t + i_t = w_t h_t + \frac{1 + R_{t-1}^D}{\Pi_t} d_{t-1} + s_{t-1} \omega_t^b + z_t k_t + q_t^x x_t + (\omega_t^k + \omega_t^f - \tau_t^h), \tag{2}$$

⁵We do not need to model equity stakes in firms as we assume that the representative household owns these firms forever.

where d_t represents the real value of bank deposits with a gross nominal return R_t^D , $\Pi_t = \frac{P_t}{P_{t-1}}$ is the gross inflation rate in the price index for consumption, e_t is households' equity stake in banks, w_t is the real wage, ω_t^k and ω_t^f represent the profits of capital producers and firms producing intermediate goods, ω_t^b stands for dividends paid by banks, and τ_t^h is a lump-sum tax paid by households.

Households choose $(c_t, h_t, d_t, s_t, k_t, i_t, x_t)_{t \geq 0}$ to maximize utility subject to (1) and (2). The first-order conditions for optimality are:

$$\begin{aligned}
\frac{1}{c_t} &= \lambda_t, \\
\lambda_t &= \beta (1 + R_t^D) E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \right\}, \\
h_t^x &= \lambda_t w_t, \\
\lambda_t q_t^x &= \lambda_t^k, \\
\lambda_t &= \lambda_t^k, \\
\lambda_t (q_t - z_t) &= \lambda_t^k (1 - \delta), \\
\lambda_t q_t^b &= \beta E_t \{ \lambda_{t+1} \omega_{t+1}^b \},
\end{aligned} \tag{3}$$

where $E_t \{ \cdot \}$ denotes the expectation operator conditional on the information available in the first subperiod of period t , which includes the realization of all the aggregate shocks except the binary shock leading to the success or failure of the risky technology. The optimality conditions imply in particular

$$\begin{aligned}
q_t^x &= 1, \\
q_t &= 1 - \delta + z_t.
\end{aligned}$$

2.2 Intermediate goods producers

There is a unit mass of monopolistically competitive firms producing intermediate goods. Firm j operates the production function:

$$y_t(j) = h_t(j)^{1-\nu} k_t(j)^\nu \exp(\eta_t^f),$$

where $0 < \nu < 1$, $k_t(j)$ is capital rented by firm j , and η_t^f is an exogenous productivity shock. We assume that firms set their prices facing a Calvo-type price rigidity (with no indexation). Since their optimization problem is standard, we don't present the details. We let α the probability that a firm does not get to set a new price at a given date.

The firms' cost minimization problem implies

$$\frac{z_t}{w_t} = \left(\frac{\nu}{1-\nu} \right) \left[\frac{h_t(j)}{k_t(j)} \right].$$

2.3 Final goods producers

Producers of the final good are perfectly competitive and aggregate the intermediate goods $y_t(j)$ to form the final good y_t . The production function is given by

$$y_t = \left(\int_0^1 y_t(j)^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}}, \quad (4)$$

where $\sigma > 1$. Profit maximization leads to the demand for good j

$$y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\sigma} y_t, \quad (5)$$

and free entry lead to the general price index

$$P_t = \left(\int_0^1 P_t(j)^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}}. \quad (6)$$

The final good may be used for consumption, investment, the monitoring of firms, and government purchases.

2.4 Capital goods producers

The capital producing firms are owned by households and are perfectly competitive. They buy unfurnished capital x_t during the second subperiod of period t to produce furnished capital k_{t+1} that they sell to households at the price q_{t+1} in the first subperiod of period $t+1$. Each capital producer chooses to operate either a safe technology (S) or a risky technology (R). Those choosing technology S use x_t^S units of unfurnished capital to produce k_{t+1}^S units of furnished capital with

$$k_{t+1}^S = x_t^S. \quad (7)$$

Producers choosing technology R are subject to an aggregate shock θ_t that is independent of all the other shocks. When $\theta_t = 0$, they produce nothing. More specifically, they use x_t^R units of unfurnished capital to produce

$$k_{t+1}^R = \theta_t \exp(\eta_t^R) x_t^R$$

units of furnished capital, with

$$\begin{aligned} \theta_t &= 0 \text{ with probability } \phi_t, \\ \theta_t &= 1 \text{ with probability } 1 - \phi_t, \end{aligned}$$

where ϕ_t is the exogenous stochastic probability of failure and η_t^R is the exogenous stochastic productivity if the project is successful. We assume that the realization of η_t^R is always positive ($\eta_t^R > 0$), so that in the absence of failure, the risky technology is more productive than the safe one. Producers choose whether to use technology S or technology R after observing the realization of η_t^R and ϕ_t (which occur at the beginning of the first subperiod), but before observing the realization of θ_t (which occurs at the end of the second subperiod).

We assume that using the risky technology is always inefficient, but capital producers have limited liability and may have an incentive to hide the fact that they use the risky technology. There is therefore a need to monitor capital producers who claim to use the safe technology. We further assume that only banks have the appropriate monitoring skills. This motivates a setup with capital producers borrowing from banks to buy unfurnished capital.⁶

More specifically, the risky technology is inefficient in the sense that, for all realizations of ϕ_t , η_t^R and Ψ_t ,

$$(1 - \phi_t) \exp(\eta_t^R) \leq 1 - \Psi_t, \quad (8)$$

where $\Psi_t > 0$ is the exogenous marginal resource cost of monitoring a capital producer who claims to use the safe technology.⁷ The left-hand side of (8) represents the marginal benefit of allocating one unit of unfurnished capital to the risky technology (the expected output of this technology). The right-hand side is the opportunity cost, which is the output of the safe technology net of the monitoring cost.

A capital producer i choosing technology $j \in \{S, R\}$ borrows

$$q_t^x x_t^j(i) = l_t^j(i) \quad (9)$$

at a nominal interest rate R_t^j .⁸ Since capital producers have limited liability, those using the risky technology will default on their loans in the event of failure (when $\theta_t = 0$).

A producer i using technology S chooses $x_t^S(i)$ to maximize

$$\beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[q_{t+1} k_{t+1}^S(i) - \frac{1 + R_t^S}{\Pi_{t+1}} l_t^S(i) \right] \right\}$$

subject to (7) and (9). The first-order condition of this programme implies

$$E_t \{ \lambda_{t+1} q_{t+1} \} = E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \right\} (1 + R_t^S) q_t^x. \quad (10)$$

A producer i using technology R chooses $x_t^R(i)$ to maximize

$$(1 - \phi_t) \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[q_{t+1} \exp(\eta_t^R) x_t^R(i) - \frac{1 + R_t^R}{\Pi_{t+1}} l_t^R(i) \right] \middle| \theta_t = 1 \right\}$$

subject to (7) and (9), where $E_t \{ \cdot | \theta_t = 1 \}$ denotes the expectation operator conditional on the information available in the first subperiod of period t and on the absence of default in the second subperiod of period t . The first-order condition of this programme implies

$$E_t \{ \lambda_{t+1} q_{t+1} | \theta_t = 1 \} \exp(\eta_t^R) = E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \middle| \theta_t = 1 \right\} (1 + R_t^R) q_t^x. \quad (11)$$

Since our model allows for two distinct interest rates, banks need to monitor the capital producers that borrow at the lower rate to ensure that they use the associated technology. Our model has no

⁶Bank loans are optimal financial contracts in our model because (i) households have no incentive to fund the risky technology and cannot monitor capital goods producers who claim to use the safe technology; and (ii) there is no uncertainty about the output of the safe technology or about the output of the risky technology when it is non-zero.

⁷In Section 6, we will consider an extension of the model in which Ψ_t is endogenous.

⁸There is no need to work with nominal loan contracts in our model. However, since we will assume that monetary policy sets a nominal interest rate, and for the sake of realism, we make loan contracts nominal.

equilibrium with $R_t^R < R_t^S$.⁹ Therefore, there is no need for banks to monitor capital producers that claim to use the risky technology. Accordingly, we will associate a cost with monitoring capital producers that claim to use the safe technology.

As usual, with constant returns to scale, the first-order conditions imply that firms make zero profits. When both (10) and (11) hold, capital producers are indifferent between the two technologies and

$$\frac{1 + R_t^R}{1 + R_t^S} = \frac{E_t \{ \lambda_{t+1} q_{t+1} | \theta_t = 1 \}}{E_t \{ \lambda_{t+1} q_{t+1} \}} \frac{E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \right\}}{E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} | \theta_t = 1 \right\}} \exp(\eta_t^R). \quad (12)$$

If the interest-rate ratio on the left-hand side is strictly higher than the critical value on the right-hand side, then capital producers use only technology S.

2.5 Banks

Banks are owned by households. They are perfectly competitive. They incur a cost $\Psi_t l_t^S$ of monitoring safe loans, where Ψ_t satisfies (8). They can fund their loans by raising equity ($e_t = q_t^b s_t - \Psi_t l_t^S$) or issuing deposits (d_t). They make safe and risky loans (l_t^S and l_t^R). Their balance-sheet identity is

$$l_t^S + l_t^R = e_t + d_t. \quad (13)$$

We assume that banks can hide risky loans in their portfolio from regulators up to a fraction γ_t of their safe loans. The prudential authority imposes risk-weighted capital requirements on risky loans above this fraction. We specify the capital requirement as:

$$e_t \geq \kappa_t (l_t^S + l_t^R) + \bar{\kappa} \max \{ 0, l_t^R - \gamma_t l_t^S \}. \quad (14)$$

Under our assumptions (inefficiency condition (8), risk aversion, and no correlation between θ_t and other shocks), risky projects reduce welfare. Therefore, the prudential authority will optimally choose a sufficiently high $\bar{\kappa}$ for $l_t^R \leq \gamma_t l_t^S$ in equilibrium.¹⁰ Therefore, the capital requirement can be rewritten as

$$e_t \geq \kappa_t (l_t^S + l_t^R). \quad (15)$$

In the first subperiod of period $t + 1$, regulators close the banks that cannot meet their deposit obligations: the banks with

$$\frac{1 + R_t^S}{\Pi_{t+1}} l_t^S + \theta_t \frac{1 + R_t^R}{\Pi_{t+1}} l_t^R - \frac{1 + R_t^D}{\Pi_{t+1}} d_t < 0,$$

or equivalently, using (13), those with

$$e_t < - \left(\frac{R_t^S - R_t^D}{1 + R_t^D} \right) l_t^S - \left(\theta_t \frac{1 + R_t^R}{1 + R_t^D} - 1 \right) l_t^R.$$

When $l_t^R = 0$ or $\theta_t = 1$, the right-hand side of this inequality is negative as long as lending rates are above the deposit rate, which will be the case in equilibrium because loans either incur a monitoring

⁹Indeed, if we had $R_t^R < R_t^S$, then funding the safe projects would strictly dominate funding the risky projects because it would pay more in every state (whatever the realization of θ_t) and incur no monitoring cost.

¹⁰Our model can be extended to allow regulators to choose γ_t by incurring some supervision cost, as in Van den Heuvel (2008), but we do not pursue this dimension of optimal policy.

cost or entail a risk for banks. When $l_t^R > 0$ and $\theta_t = 0$, the right-hand side of this inequality is positive if and only if

$$l_t^R > e_t + \left(\frac{R_t^S - R_t^D}{1 + R_t^D} \right) l_t^S.$$

We want our model to reflect the fact that banks find equity finance more costly than debt finance in reality. We attribute this higher cost to a tax distortion (tax deduction for debt finance), although this interpretation is not essential for our analysis. We take this distortionary tax as a feature of the environment: the model does not explain why this tax is in place, and the policymakers in our model (the monetary and prudential authorities) cannot set this tax optimally.¹¹

The particular way we specify the tax distortion (and the timing of the tax deduction for monitoring costs) ensures that unanticipated changes in the price level cannot cause insolvency.¹² The banks in our model may be insolvent only if they extend too many risky loans, and the risky projects fail. Specifically, we assume that gross revenues from loans are taxed at the constant rate τ after deductions for gross payments on deposits and monitoring costs.

The representative bank chooses e_t , d_t , l_t^R and l_t^S to maximize

$$E_t \left\{ \beta \frac{\lambda_{t+1} (1 - \tau) \omega_{t+1}^b}{\lambda_t} \right\} - e_t - (1 - \tau) \Psi_t l_t^S,$$

where

$$\omega_{t+1}^b = \max \left\{ 0, \frac{1 + R_t^S}{\Pi_{t+1}} l_t^S + \theta_t \frac{1 + R_t^R}{\Pi_{t+1}} l_t^R - \frac{1 + R_t^D}{\Pi_{t+1}} d_t \right\}, \quad (16)$$

subject to (13) and (15).

Note that, for any non-negative value of κ_t (therefore e_t), we have

$$\omega_{t+1}^b = \theta_t \left(\frac{1 + R_t^S}{\Pi_{t+1}} l_t^S + \frac{1 + R_t^R}{\Pi_{t+1}} l_t^R - \frac{1 + R_t^D}{\Pi_{t+1}} d_t \right) + (1 - \theta_t) \max \left\{ 0, \frac{1 + R_t^S}{\Pi_{t+1}} l_t^S - \frac{1 + R_t^D}{\Pi_{t+1}} d_t \right\}.$$

2.6 Government and market-clearing conditions

The government has exogenous purchases G_t and guarantees bank deposits. The lump-sum tax on households balances the budget.¹³

The losses imposed by bank j on the deposit insurance fund amount to

$$\zeta_t(j) = \max \left\{ 0, \frac{1 + R_{t-1}^D}{\Pi_t} d_{t-1}(j) - \frac{1 + R_{t-1}^S}{\Pi_t} l_{t-1}^S(j) - \delta_{t-1} \frac{1 + R_{t-1}^R}{\Pi_t} l_{t-1}^R(j) \right\},$$

and the lump-sum tax paid by households is

$$\tau_t^h = G_t + \int_0^1 \left\{ \zeta_t(j) - \tau [\omega_t^b(j) + \Psi_t l_t^S(j)] \right\} dj.$$

¹¹This feature of the tax code seems to be one of the primary reasons for banks to lobby against higher capital requirements, at least in the US and the euro area. It is commonly invoked in models with both debt and equity finance [e.g. Jermann and Quadrini (2008)], to break the Modigliani-Miller theorem about irrelevance of financial structure. Admati et al. (2011) call for removing this fiscal distortion while possibly preserving the same level of bank tax shields.

¹²In our setting with one-period competitive banks incurring real monitoring costs and extending nominal loans, a change in the price level could lead to insolvency. We don't think this is an interesting feature of the model and have specified our "tax code" to rule it out.

¹³It is harmless to abstract from deposit insurance fees paid by banks and include these in the lump-sum tax paid by households who own the banks.

We consider two policy instruments: the deposit rate R_t^D for monetary policy and the capital requirement κ_t for prudential policy. We will discuss our specifications of prudential policy in Sections 3 and 5. For each specification, our monetary policy will be the Ramsey-optimal policy.

Firms producing intermediate goods rent their capital from the representative household; in equilibrium, their choices must satisfy

$$\int_0^1 k_t(j) dj = k_t.$$

Similarly obvious market-clearing conditions must be satisfied in the markets for labor, loans, and unfurnished capital. The market-clearing condition for goods is

$$c_t + i_t + G_t + \Psi_t l_t^S = y_t.$$

3 Prudential Policy

This section derives conditions for prudential policy to rule out equilibria with risk taking and ensure the existence of equilibria without risk taking. We first show that our model can only have equilibria at the two corners with $l_t^R = 0$ and $l_t^R = \gamma_t l_t^S$, and that the capital constraint is binding in any equilibrium. Next, we consider a benchmark prudential policy that internalizes the externality (arising from limited liability) by making banks the residual claimants to any losses they may incur. We then characterize the least stringent prudential policy that rules out risk taking, and show that it is locally Ramsey-optimal.

3.1 Ruling out candidate equilibria

We focus on symmetric equilibria in which all banks have the same loan portfolio. We will also assume throughout that the following condition holds:

$$\frac{E_t \{ \lambda_{t+1} q_{t+1} | \theta_t = 1 \}}{E_t \{ \lambda_{t+1} q_{t+1} \}} \leq 1. \quad (17)$$

This condition seems plausible because failure of risky projects at date t leads to destruction of the capital stock at date $t + 1$, and this by itself should increase both the price of capital (q_{t+1}) and the marginal utility of consumption (λ_{t+1}). However, this condition amounts to an implicit restriction on the set of policies that we consider, as it presumes that policies will not overturn the qualitative effects of the failure of risky projects.

We first show that the banks' optimization problem rules out the existence of equilibria with $0 < l_t^R < \gamma_t l_t^S$. The basic insight follows Van den Heuvel (2008), but since we have added aggregate risk and made other changes to his model, we prove the following proposition in the Appendix.

Proposition 1: *There are no equilibria with $0 < l_t^R < \gamma_t l_t^S$. When $0 < l_t^R < \gamma_t l_t^S$, (a) if banks go bankrupt ($\omega_{t+1}^b = 0$) when risky projects fail ($\theta_t = 0$), then banks can increase their market value by tilting the loan portfolio towards more risky loans; (b) if banks do not go bankrupt ($\omega_{t+1}^b > 0$) when risky projects fail ($\theta_t = 0$), then they can increase their market value by tilting the loan portfolio towards more safe loans.*

The intuition follows. If, given the loan portfolio, bank equity is sufficiently small to be wiped out when risky projects fail, then banks do not internalize the cost of additional risk taking. Additional losses from increasing l_t^R , if risky projects fail, are truncated by deposit insurance and limited liability. Consequently, the only candidate for an equilibrium with the possibility of bank failure involves the corner solution $l_t^R = \gamma_t l_t^S$.

Alternatively, if bank equity is sufficiently large for banks to remain solvent even when risky projects fail, then banks internalize the cost of additional risk taking. In that case, since we assume that the risky technology is inefficient, banks can increase their market value by reducing l_t^R . Accordingly, the only candidate for an equilibrium without the possibility of bank failure involves the corner solution $l_t^R = 0$. In particular, if bank equity is large enough to make banks residual claimants on their risky loans when $l_t^R = \gamma_t l_t^S$, then there does not exist an equilibrium with $l_t^R = \gamma_t l_t^S$.

Next, we show that there are no equilibria in which the capital constraint is lax:

Proposition 2: *In equilibrium, the capital constraint is binding:*

$$e_t = \kappa_t (l_t^S + l_t^R). \quad (18)$$

This Proposition follows almost directly from our assumption about the tax advantage of debt finance over equity finance, but we provide a proof in the Appendix.

3.2 A benchmark policy

Proposition 1 leads to a sufficient condition for prudential policy to rule out equilibria with $l_t^R > 0$ and ensure the existence of an equilibrium with $l_t^R = 0$: the capital requirement can be sufficiently high to make any bank the residual claimant to the potential losses arising from funding risky projects. This benchmark policy is characterized by the following proposition:

Proposition 3: *(a) A sufficient condition for existence and uniqueness of an equilibrium and for $l_t^R = 0$ in this equilibrium is that*

$$\kappa_t > \tilde{\kappa}(R_t^D, R_t^S) \equiv 1 - \frac{1}{1 + \gamma_t} \frac{1 + R_t^S}{1 + R_t^D}; \quad (19)$$

(b) in this equilibrium,

$$\tilde{\kappa}(R_t^D, R_t^S) = \tilde{\kappa}_t \equiv \frac{(1 - \tau)(\gamma_t - \Psi_t)}{\tau + (1 - \tau)(1 + \gamma_t)}; \quad (20)$$

(c) $\tilde{\kappa}_t$ is increasing in γ_t , and decreasing in Ψ_t .

We prove this proposition in the Appendix, by considering a given bank j that takes the maximum amount of risk ($l_t^R(j) = \gamma_t l_t^S(j)$). We show that this bank will remain solvent when risky projects fail ($\theta_t = 0$) if and only if (19) holds. We then use the banks' optimality conditions at the equilibrium with $l_t^R = 0$ to express $\tilde{\kappa}(R_t^D, R_t^S)$ in terms of parameters and exogenous shocks and obtain (20).

We assume $\gamma_t > \Psi_t$, which implies $\tilde{\kappa}_t > 0$, so that condition (20) may or may not be met depending on the value of κ_t . This restriction states that the temptation to take risk would be present if banks were not subject to any (positive) capital requirements. The threshold $\tilde{\kappa}_t$ is increasing in γ_t : the higher the fraction of risky loans that a deviating bank can hide, the riskier this bank, and the higher the capital requirement needed to make it remain solvent in case of failure. And $\tilde{\kappa}_t$ is decreasing in Ψ_t : the higher the cost of monitoring safe loans, the higher the spread between the interest rate on safe loans and that on deposits; thus, the larger the cash flow from safe loans that is available to redeem the deposits, and the lower the capital requirement needed to make a deviating bank remain solvent in case of failure.

Although this benchmark policy suffices to ensure the existence of an equilibrium without risk taking, we show next that it is more stringent than necessary and that the least stringent policy ensuring the existence of an equilibrium without risk taking is locally Ramsey-optimal in our model.

3.3 The locally optimal policy

We now derive a necessary and sufficient condition for prudential policy to ensure the existence of an equilibrium with $l_t^R = 0$, and then show that the least stringent policy satisfying this condition is locally Ramsey-optimal.

Consider a bank j that deviates from a candidate equilibrium with $l_t^R = 0$ to take the maximum amount of risk ($l_t^R(j) = \gamma_t l_t^S(j)$). There exists an equilibrium with $l_t^R = 0$ if and only if this deviating bank has a negative expected excess return. In the Appendix, we derive the threshold value of κ_t that makes its expected excess return negative, and we prove the following proposition:

Proposition 4: (a) A necessary and sufficient condition for existence of an equilibrium with $l_t^R = 0$ is $\kappa_t \geq \kappa_t^*$, where

$$\kappa_t^* \equiv (1 - \tau) \frac{(1 - \phi_t) \gamma_t [\exp(\eta_t^R) - 1] + \Psi_t [(1 - \phi_t) \gamma_t \exp(\eta_t^R) - \phi_t]}{\phi_t (1 + \gamma_t) - \gamma_t \tau (1 - \phi_t) [\exp(\eta_t^R) - 1]}, \quad (21)$$

(b) $\kappa_t^* < \tilde{\kappa}_t$; (c) κ_t^* is decreasing in the probability of default ϕ_t , and increasing in the productivity of the risky technology η_t^R (conditionally on the absence of default).

The derivations in the Appendix consider a bank j that contemplates a deviation from a candidate equilibrium with $l_t^R = 0$. The same intuition we gave for Proposition 1 (roughly) applies: if there are profitable deviations, the most profitable one is at the corner with maximum risk ($l_t^R(j) = \gamma_t l_t^S(j)$). To derive the value of κ_t^* , we make bank j indifferent between staying at the safe corner and moving to the risky corner. The bank turns indifferent with less equity at stake than what would make it residual claimant (i.e., we have $\kappa_t^* < \tilde{\kappa}_t$) because the bank has incurred monitoring costs and has a vested interest in remaining solvent to recoup these costs. In a way, monitoring costs in our model work like giving the banks some charter value that they would like to preserve by avoiding bankruptcy.

The preceding intuition also helps us understand the nature of the state dependence, in our model, of the constraint $\kappa_t \geq \kappa_t^*$. Macro-prudential policy must be tight enough to allow for no risk taking

in equilibrium. The threshold κ_t^* depends negatively on the probability of default ϕ_t because default risk, by itself, makes risk-taking less attractive. Similarly, κ_t^* rises with the productivity of the risky technology η_t^R (conditionally on the absence of default) because a higher η_t^R increases the temptation to finance risky, rather than safe, projects.

Perhaps a more surprising feature of (21) is that κ_t^* does not depend on the monetary policy instrument R_t^D . This is because, in our model, the deposit rate R_t^D does not affect banks' incentives for risk taking. In particular, it does not affect the spread between the interest rate on risky loans R_t^R and the interest rate on safe loans R_t^S . In a way, this is not a surprising feature for a model with perfect competition and constant returns. Our banks never run out of safe projects to fund and always end up making zero profits. This is the opposite extreme from arguments that (explicitly or implicitly) postulate a fixed number of potential projects and thereby link more lending with more risk taking (as banks run out of safe lending opportunities). We will revisit this contrast between extreme modeling assumptions in Section 6.

Let $(R_\tau^{D*})_{\tau \geq 0}$ denote the monetary policy that is Ramsey-optimal when the prudential policy is $(\kappa_\tau^*)_{\tau \geq 0}$. The following proposition states that, under a certain condition, setting jointly $(R_\tau^D)_{\tau \geq 0}$ to $(R_\tau^{D*})_{\tau \geq 0}$ and $(\kappa_\tau)_{\tau \geq 0}$ to $(\kappa_\tau^*)_{\tau \geq 0}$ is locally Ramsey-optimal:

Proposition 5: *If the right derivative of welfare with respect to κ_t at $(R_\tau^D, \kappa_\tau)_{\tau \geq 0} = (R_\tau^{D*}, \kappa_\tau^*)_{\tau \geq 0}$ is strictly negative for all $t \geq 0$, then the policy $(R_\tau^D, \kappa_\tau)_{\tau \geq 0} = (R_\tau^{D*}, \kappa_\tau^*)_{\tau \geq 0}$ is locally Ramsey-optimal.*

We prove this proposition in the Appendix. The intuition is the following. First, whatever R_t^D in the neighborhood of R_t^{D*} , setting κ_t just below κ_t^* is not optimal, because it triggers a discontinuous increase in the amount of risk taken by banks. Under our assumptions (inefficiency condition (8), risk aversion, and no correlation between θ_t and other shocks), this discontinuous increase in the amount of risk has a discontinuous negative effect on welfare. Any other effect on welfare is continuous and, therefore, dominated by this discontinuous negative effect provided that (R_t^D, κ_t) is close enough to (R_t^{D*}, κ_t^*) . Second, if the right derivative of welfare with respect to κ_t at (R_t^{D*}, κ_t^*) is strictly negative, then setting κ_t just above κ_t^* is not optimal either, because it has a negative first-order effect on welfare that cannot be offset by any change in R_t^D around its optimal value R_t^{D*} (as this change would have a zero first-order effect on welfare).

The right derivative of welfare with respect to κ_t at (R_t^{D*}, κ_t^*) can be expected to be strictly negative because increasing κ_t from κ_t^* decreases the capital stock, which is already inefficiently low due to the monopoly and tax distortions, without reducing the amount of risk, which is already zero. We check numerically, for the calibration considered in the next section, that this derivative is indeed strictly negative. This derivative is equal to the Lagrange multiplier associated to the constraint $\kappa_t = \kappa_t^*$ in the optimization program that determines R_τ^{D*} . We first use the program Get Ramsey developed by Levin and López-Salido (2004) and used in Levin, Onatski, Williams and Williams (2005) to get analytically the non-linear first-order conditions of this optimization program. We then use Dynare to solve numerically, at the first order, the resulting system of constraints and first-order conditions, and thus get the first-order approximation of this Lagrange multiplier (among other variables). We check that this Lagrange multiplier is strictly negative at the steady state, which implies that it is

strictly negative for small enough shocks. We also check that it is strictly negative at the first order in the presence of shocks of a standard size.

3.4 Ruling out equilibria with $l_t^R = \gamma_t l_t^S$

We next formulate a prudential feedback rule that precludes equilibria with $l_t^R = \gamma_t l_t^S$, and coincides with $\kappa_t = \kappa_t^*$ in equilibrium. That is, under this rule, there is a unique equilibrium and, in this equilibrium, $l_t^R = 0$ and κ_t takes the minimum value that is consistent with $l_t^R = 0$.

We will assume throughout that the following condition holds:

$$\frac{E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \mid \theta_t = 1 \right\}}{E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \right\}} \leq 1. \quad (22)$$

This condition seems plausible but, as we noted in our discussion of (17), it amounts to an implicit restriction on the set of policies that we consider.¹⁴

We prove the following proposition in the Appendix:

Proposition 6: *Under the macro-prudential policy rule*

$$\kappa_t = \frac{1 - \phi_t}{\phi_t} \frac{\gamma_t}{1 + \gamma_t} \frac{R_t^R - R_t^S}{1 + R_t^D} + \frac{1}{\phi_t} \frac{\gamma_t}{1 + \gamma_t} \Psi_t - \frac{R_t^S - R_t^D}{1 + R_t^D}, \quad (23)$$

there exists a unique equilibrium and, in this equilibrium, $l_t^R = 0$ and $\kappa_t = \kappa_t^$.*

Although the formal proof in the Appendix takes a different approach, a heuristic rendition is to start with the equilibrium at the safe corner and define R_t^R as the highest rate that a deviating bank could charge on a loan to a risky firm. In this case, (23) just states κ_t^* as a function of interest-rate spreads. It gives the critical value of κ_t for making the bank indifferent between staying at the safe corner (where all the other banks are) and jumping to the risky corner. The critical value is fairly intuitive. The first two terms represent the temptation to deviate from the safe corner to the risky corner: a deviating bank will pocket $R_t^R - R_t^S$ if risky projects succeed (with probability $1 - \phi_t$) and save monitoring costs. The third term represents the opportunity cost $R_t^S - R_t^D$ of this deviation when risky projects fail (with probability ϕ_t).

So, this feedback rule suffices for keeping banks at the safe corner. In the Appendix we show that it also suffices to rule out an equilibrium at the risky corner, because the safe corner becomes even more attractive to an individual bank if there is a mass of banks at the risky corner (in which case the risk is priced).

¹⁴The condition seems plausible when we consider the pricing of a bond with default risk— a bond that pays \$1 when risky projects succeed and pays nothing when they fail. The inequality (22) says this risky bond has a higher expected real return, compared to a nominal bond with no default risk, in the equilibria we consider.

4 Calibration

The parameters pertaining to households and firms are standard. The period of time is a quarter. The discount rate is such that the household discounts the future at the deposit rate, 2.76% per year. The labor supply elasticity is set to 1. Markups are set to 10%, which implies a value of the elasticity of substitution between intermediate goods of 11. The capital elasticity in the intermediate-good technology is set such that the labor share is 0.66, implying a value for ν of 0.34. The depreciation rate, δ , is set to 0.025 which corresponds to a 10% annual depreciation rate. Firms are assumed to reset their prices every 4 quarters on average, implying the value 0.75 for the Calvo parameter α .

Table 1: Calibration

Parameter	Description	Value
Preferences		
β	Discount factor	0.993
χ	Inverse of labor supply elasticity	1.000
Technology		
ν	Capital elasticity	0.340
σ	Elasticity of substitution	11.00
δ	Depreciation rate	0.025
Nominal rigidities		
α	Price stickiness	0.750
Banking (steady state)		
τ	Tax rate	0.023
κ^*	Capital requirement	0.080
Ψ	Marginal monitoring cost	0.006
ϕ	Failure probability	0.031
γ	Maximal risky/safe loans ratio	0.356
η^R	Risk premium	1.005
Shock processes		
ρ	Persistence	0.950

The parameters pertaining to the banking system are set as follows. Using variables without a time subscript to denote steady-state values, η^R is set such that the annualized lending rate on risky projects is 2% higher than that on safe projects, in the steady state. Following Van den Heuvel (2008), we assume that the steady-state yield differential $R^S - R^D$ is 3.16% per annum. The tax rate on bank profits is set to 0.023. This value is chosen to equate the after-tax return on bank equity in our model to the after-tax return in US data.¹⁵ Our calibration of the optimal steady-state capital requirement, κ^* , is 0.08, which corresponds to the value used by Van den Heuvel (2008). The steady-state monitoring cost, Ψ , is set such that the first-order condition of the representative bank is satisfied

$$\Psi = \frac{R^S - R^D}{1 + R^D} - \frac{\tau \kappa}{1 - \tau}.$$

This yields the value $\Psi = 0.006$. The steady-state failure probability of risky projects is set such that

¹⁵In the data, the after-tax return on equity is given by $(1 - \tau^c)\pi/e$ where τ^c , π and e respectively denote corporate tax rate, profits and equity. In our model, this quantity is given by $(1 - \tau)(\pi + e)/e - 1$ where τ denotes the proper tax rate that applies in our model. By equating these two quantities, and using the fact that the average return on equity is 7% and the tax rate on corporate profits is 35%, we obtain the number reported in Table 1.

the model matches the average failure rate of the US economy (0.86% per quarter). This leads to a steady-state value for ϕ_t of 0.031. The maximal risky/safe loans ratio is then obtained by solving the optimal capital requirement equation, $\kappa = \kappa^*$, yielding

$$\gamma = \frac{\phi(1 - \tau)\Psi + \kappa}{(1 - \phi)(1 - \tau(1 - \kappa) + (1 - \tau)\Psi)(R^R - R^S) - \phi\kappa} = 0.356.$$

The persistence of all the shocks is set to $\rho = 0.95$.¹⁶ For the impulse-response functions presented in the next section, we set the innovations to the technology shock η_t^f and the fiscal shock G_t equal to 1%, and the innovation to Ψ_t to 10%. We set the innovation to η_t^R such that the annualized risk premium increases from 2% to 3%. And we set the innovation to ϕ_t such that the probability of failure increases by 1/3 of a percent.

5 Numerical Results

We consider two alternative prudential policies. Our benchmark macro-prudential policy sets κ_t equal to its locally Ramsey-optimal value κ_t^* , which is 0.08 at the steady state. The other policy keeps κ_t constant at 0.10. This value is high enough, given the size of our shocks, to keep the economy in the safe equilibrium.

We solve for the Ramsey monetary policy using Dynare and the program Get Ramsey developed by Levin and López-Salido (2004) and used in Levin, Onatski, Williams and Williams (2005).

Figure 1 displays the optimal responses to a favorable productivity shock (positive innovation to η_t^f). The responses, with the exception of those of the interest rates, are expressed as percentage deviations from each steady state. The response of the interest rates is measured in terms of the level of the interest rate as a percentage per annum (rather than a deviation from the steady state). The horizontal dashed line corresponds to the steady-state level of the interest rate, so values below this line represent accommodative monetary policy following the shock, and values above represent restrictive monetary policy.

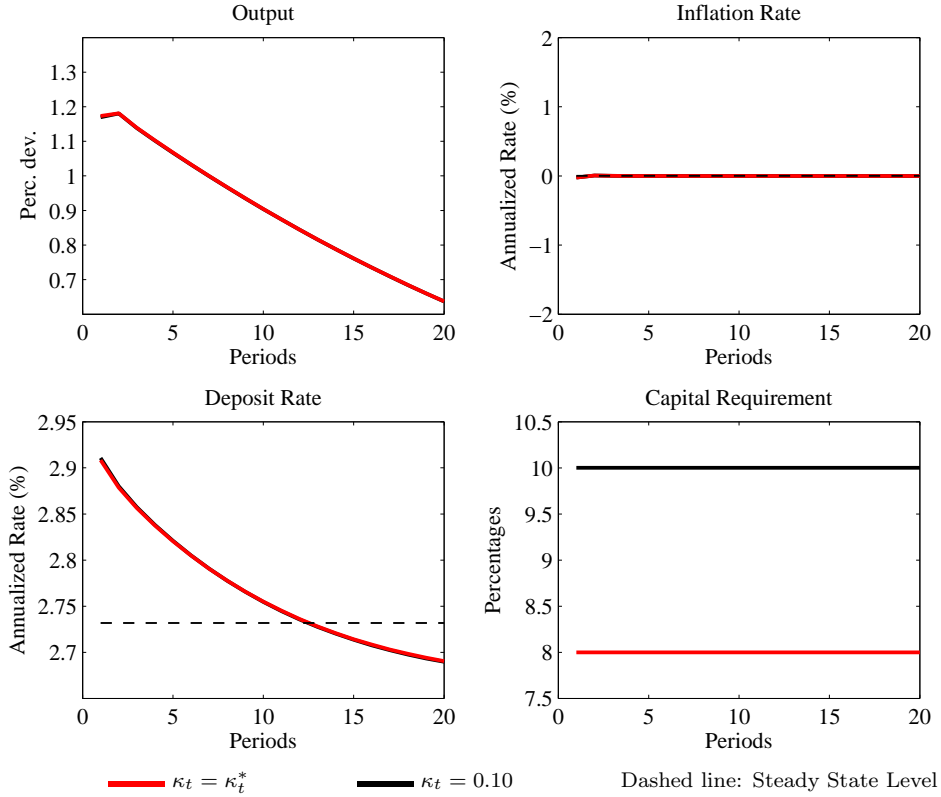
Since a productivity shock does not create a temptation to take more risk in our model, it does not affect the optimal capital requirement κ_t^* . So the optimal responses of the policy rate, output and inflation are the same, regardless of the prudential policy in place ($\kappa_t = \kappa_t^*$ or $\kappa_t = 0.10$). These optimal responses to a productivity shock are qualitatively similar to optimal responses in the benchmark New-Keynesian (NK) model with capital. Optimal policy essentially keeps inflation at zero. This requires an increase in the deposit rate for a while, because the natural real interest rate rises in the model with capital.¹⁷ Optimal responses to an increase in government purchases (not reported here) are also similar to those from the NK model, and independent of prudential policy.

In our model, a positive shock to η_t^R is a pure temptation for banks and firms to deviate from the safe equilibrium; it increases the return on risky projects in case they succeed. Figure 2 shows that this shock increases the capital requirement under the optimal prudential policy ($\kappa_t = \kappa_t^*$). By itself, the

¹⁶Note that in order to study the response of the economy to shocks to the failure rate, we assume that $\phi_t = (1 + \exp(-(u_t - v)))^{-1}$ where u_t is assumed to follow a zero-mean AR(1) process and $v = 3.3253$.

¹⁷Both the favorable productivity shock and the resulting increase in employment increase the marginal product of capital.

Figure 1: Response to a favorable Productivity Shock (η_t^f)



tightening of capital requirements increases the cost of banking in our model. The optimal monetary-policy response is to cut the deposit rate in order to curb the increase in bank lending rates. The overall effects on output are small, and inflation is essentially zero under optimal policy.

We find this thought experiment worth some consideration in the context of policy-oriented discussions [e.g., Canuto (2011), Cecchetti and Kohler (2012), Macklem (2011), Wolf (2012), Yellen (2010)] of how monetary and prudential policies may be substitutes for each other or move to offset each other's effects. In our thought experiment, one policy turns contractionary and the other expansionary to manage risk-taking incentives with the smaller adverse effect on investment.

The same observations apply to optimal responses to shocks to the probability of default (ϕ_t) and the maximal risky/safe loans ratio (γ_t). These shocks affect the economy only through their effect on the optimal capital requirement κ_t^* , which in turn calls for a monetary-policy response to mitigate the macroeconomic effects. Instead of presenting these responses, which are qualitatively the same as those of Figure 2, we present the effects of an exogenous tightening of the capital requirement. Figure 3 shows the responses to an increase in κ_t by one percent (from 0.080 to 0.088). The optimal monetary-policy response is to cut the annualized deposit rate by about 10 basis points. Again, the overall decrease in output is small (less than 0.1% at the trough) and inflation remains at zero under optimal policy.

Figure 4 shows responses to a change in the marginal cost of making safe loans Ψ_t . In contrast to the other shocks in our model of the banking system, this shock has direct macroeconomic effects

Figure 2: Response to a η_t^R Shock

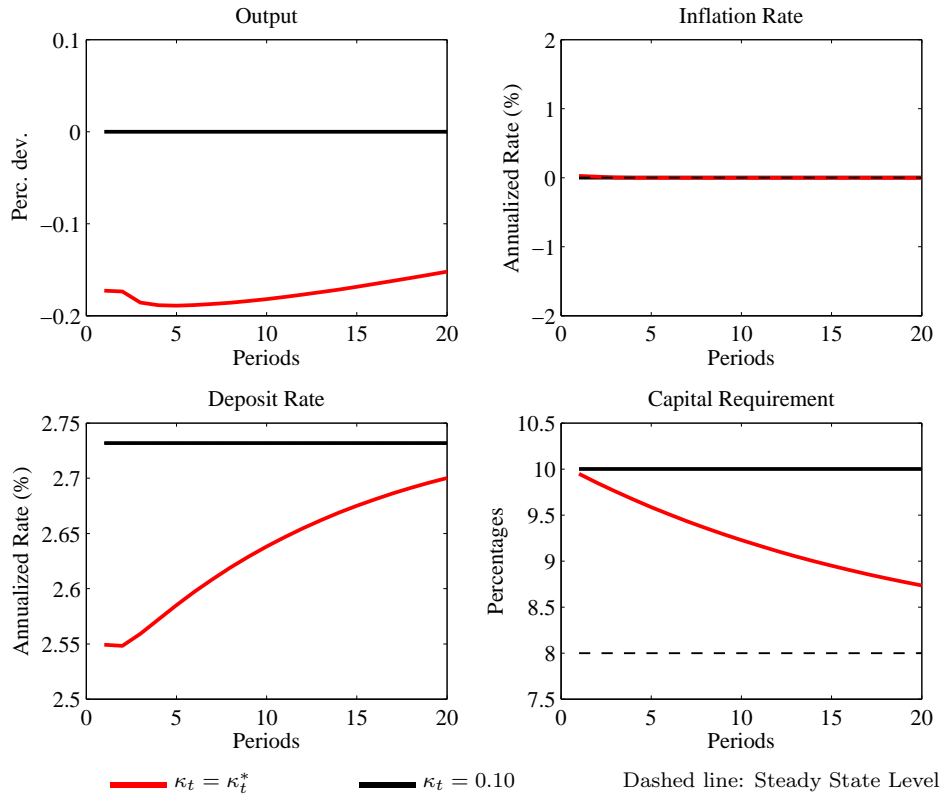


Figure 3: Response to Shock to κ_t

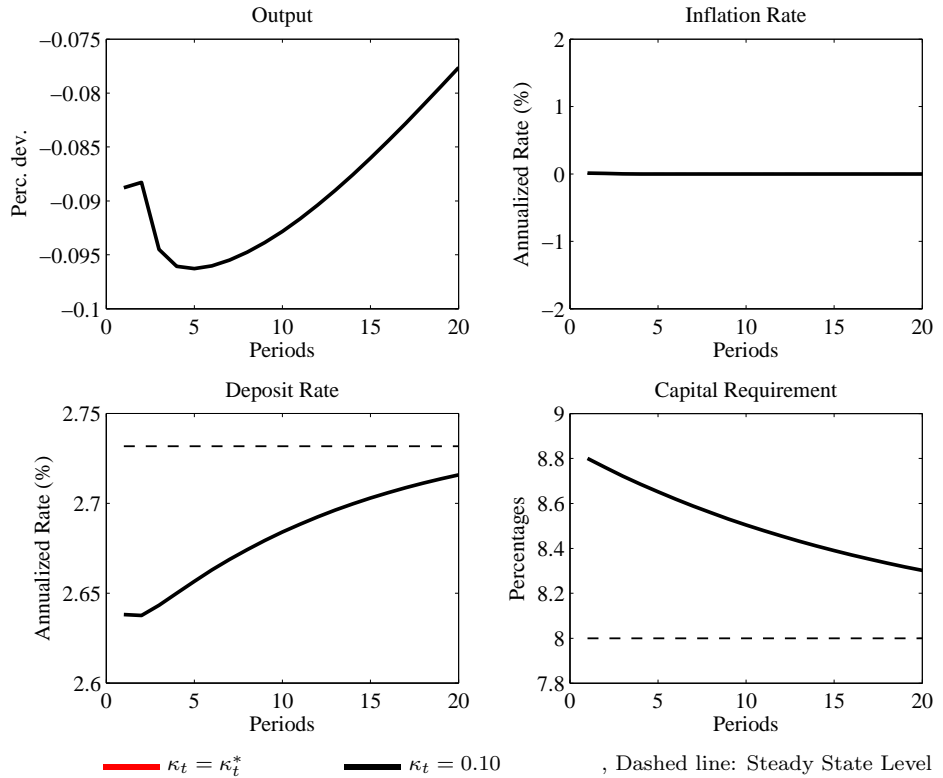
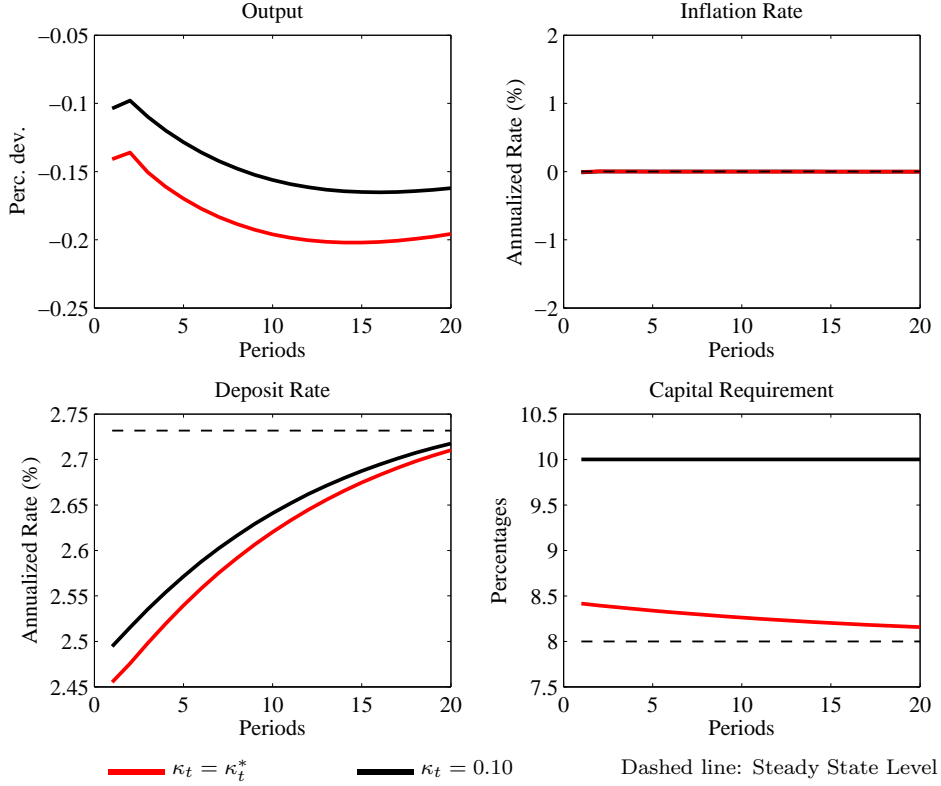


Figure 4: Response to a Shock to Marginal Cost of Making Safe Loans (Ψ_t)



in addition on its effects on the risk-taking incentives of banks. Under a prudential policy keeping κ_t constant, this shock reduces output in our model, and monetary policy cuts the deposit rate to mitigate this effect. Under the optimal prudential policy ($\kappa_t = \kappa_t^*$), output falls more because, as we explained earlier, the increase in κ_t (needed to prevent risk taking) increases bank lending rates. Monetary policy reacts to the tighter capital requirements by cutting the deposit rate further.

Our model highlights a distinction across policy instruments that we think deserves more emphasis than it gets in the existing literature: changes in the capital requirement can directly manage risk-taking incentives, while changes in the policy interest rate cannot. When the capital requirement rises to curb risk taking, a contraction ensues, and the policy interest rate is cut. With this chain of causality, optimal prudential policy is pro-cyclical, and optimal monetary policy is counter-cyclical.

Nonetheless, our model also provides a framework for thinking about some scenarios (or extensions) that can make optimal prudential policy counter-cyclical, as we discuss below.

6 Extensions and Policy Concerns

Our benchmark model is stylized and does not rely on shortcuts that are usually necessary to address policy concerns. Nonetheless, as it stands, this model may still highlight some relevant ideas. For example, as Angeloni and Faia (2011) elaborate, the leading argument for Basel III-type counter-cyclical capital requirements is the observation that default risk rises during recessions; and risk-

weighted (Basel II-type) capital requirements automatically tighten policy in recessions, unless the regulatory rate is lowered.¹⁸ In our model, as it stands, default risk is exogenous; so, we can't address the relevance of cyclical variation in default risk. Our model, however, suggests a further reason for cutting the capital requirement when default risk is high: the risk by itself makes banks more prudent and less inclined to fund risky projects; prudential policy can afford to be less tight and still ensure that risky projects are not funded.

In this subsection, we illustrate (admittedly ad hoc) extensions that may bring the model closer to policy-oriented concerns and may provide casual insights. The underlying mechanisms may involve externalities or the endogenous evolution of the variables that appear in our solution for κ_t^* , or they may involve plausible correlations between these variables and those that drive the business cycle. We will consider one short example of each sort.

6.1 An externality

Our model assumes perfect competition and constant returns in the banking sector. As we noted earlier these assumptions matter for the model's implications for policy interaction issues. Under this specification, optimal monetary policy responds to any shock that leads to a change in the capital requirement; a higher capital requirement increases the cost of banking, and the optimal monetary response cuts the policy rate. As the model stands, however, shocks that directly affect the optimal-policy interest rate (like standard productivity or fiscal shocks) do not affect the optimal bank-capital requirement. This is because the risk taking incentive depends on interest rate spreads that are not affected by the level of the policy rate in our model. When the monetary authority cuts R_t^D , the interactions of competitive banks and firms with constant marginal costs preclude changes in the relevant spreads in (23); and in the safe equilibrium, this leads to setting the requirement at κ_t^* , which is independent of monetary policy, in (21).

In this subsection, we consider a simple (and ad-hoc) extension that links the cost of banking to the aggregate volume of safe loans. Hachem (2010) develops a model with an externality in banking costs. In her model, banks ignore the effect of their own lending decision on the pool of borrowers, with heterogeneous levels of risk, that is available to other banks. Here, we will only consider a simple example of such an externality— we keep the example simple to preserve our earlier derivations that treated Ψ_t as exogenous to the banks' decisions. Specifically, we assume

$$\log(\Psi_t) = \log(\Psi) + \varrho [\log(l_t^S) - \log(l^S)] \quad (24)$$

where the term $\log(l_t^S) - \log(l^S)$ is the log-deviation of the aggregate volume of safe loans from its steady-state value, and $\varrho = 0$ corresponds to our benchmark model. We show the impulse responses for $\varrho = 0, 1$, and 5. Figure 5 illustrates the effects of a favorable productivity shock. Following this shock, optimal prudential policy raises the capital requirement, while optimal monetary policy is less restrictive (raises the deposit rate by less, and later on cuts it by more) than in the benchmark model. The reason why optimal monetary policy is more accommodative during a productivity boom is that optimal prudential policy turns more contractionary.

¹⁸See Covas and Fujita (2010) for a quantitative assessment of the procyclical effects of bank capital requirements under Basel II.

Figure 5: Response to a Favorable Productivity Shock

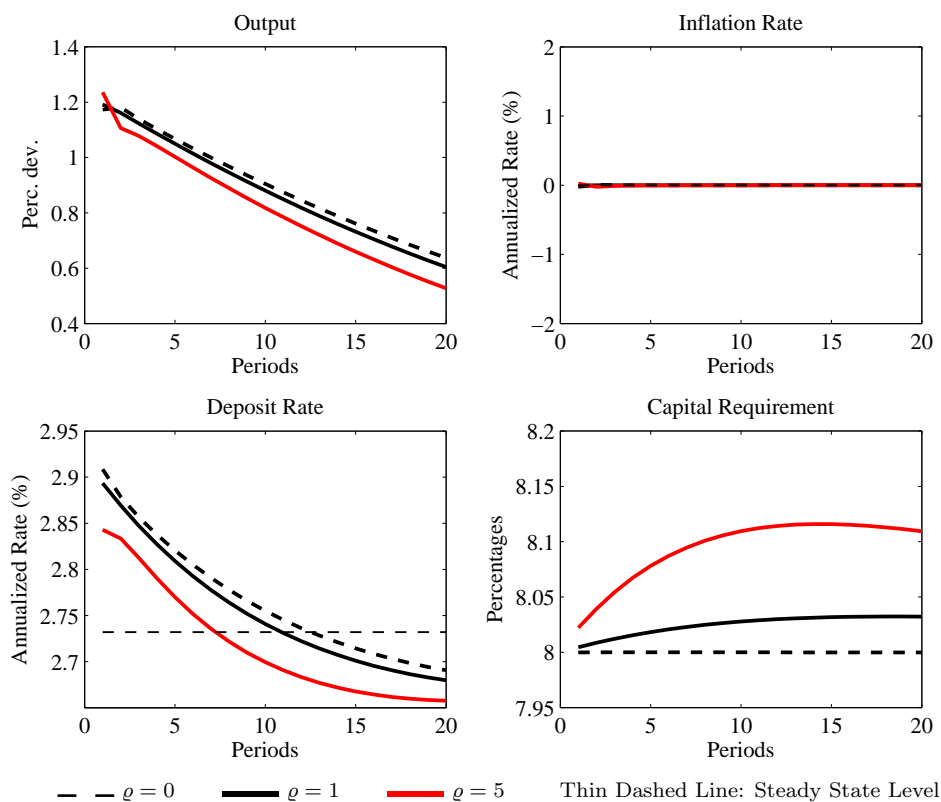


Figure 6 shows the optimal responses to an increase in default risk. Absent the externality (looking at the dashed lines in the figure), optimal prudential policy cuts the capital requirement because banks are naturally less tempted to take risk, while optimal monetary policy raises the deposit rate to curb the expansionary effects of prudential policy. With the externality, the expansion creates a temptation to take more risk (as the cost of making safe loans increases). So, optimal prudential policy cuts the capital requirement by less, and optimal monetary policy raises the deposit rate by less. Figure 7, which is the analogue to Figure 2, makes a similar point about responses to an increase in η_t^R : with the externality, optimal prudential policy increases the capital requirement by less, and optimal monetary policy cuts the deposit rate by less. In terms of optimal output fluctuations in Figures 5–7, the externality always dampens the optimal response (expansion or contraction) of output.

6.2 Correlated shocks

Correlations across shocks may also link the risk-taking incentive to shocks that have direct business-cycle effects. As an example, we replace (7) by

$$k_{t+1}^S = \exp(\eta_t^S) x_t^S$$

adding a shock to the safe technology for producing capital goods, and we allow for the possibility that η_t^S is correlated with η_t^R (the shock to the risky technology). This modification changes our

Figure 6: Response to a Default Shock (ϕ_t)

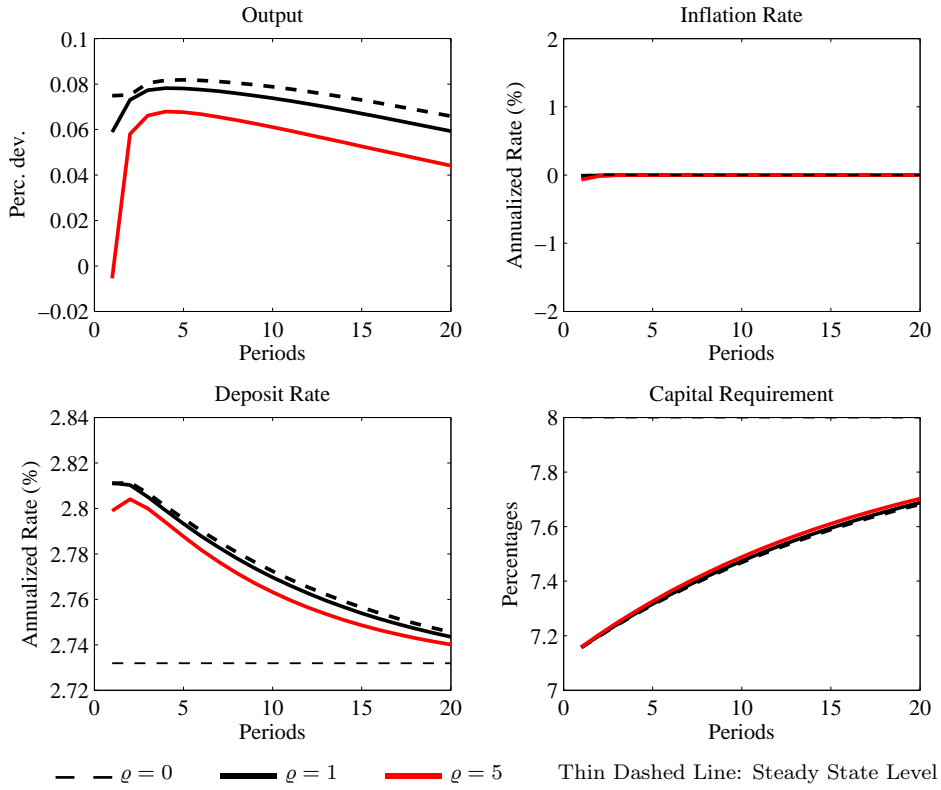
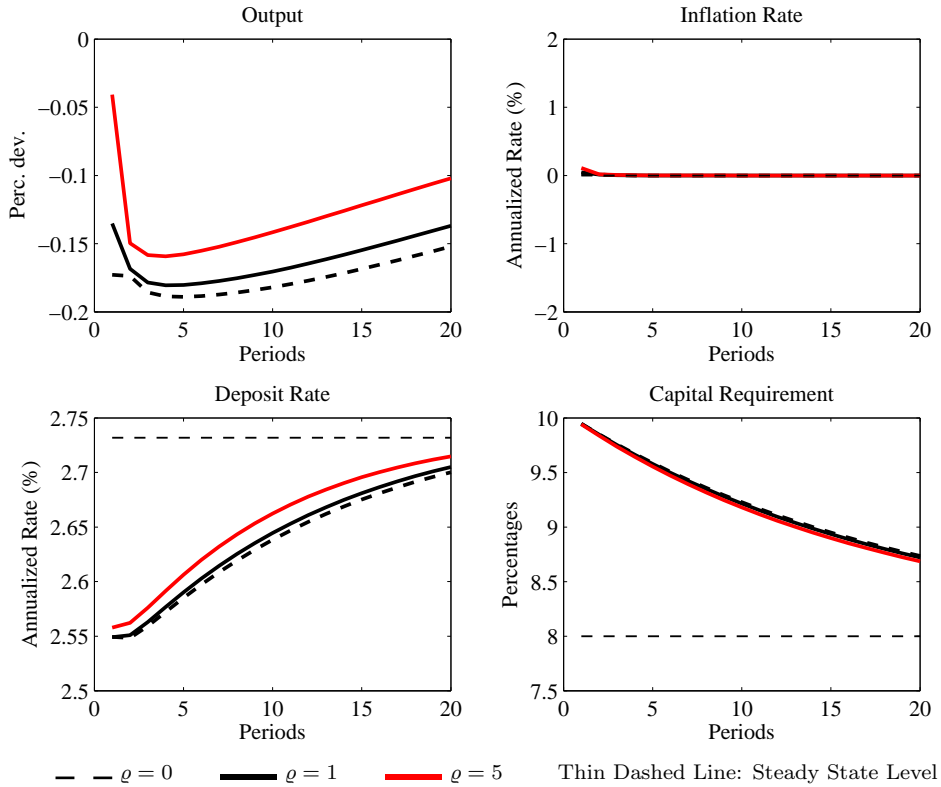


Figure 7: Response to a η_t^R Shock



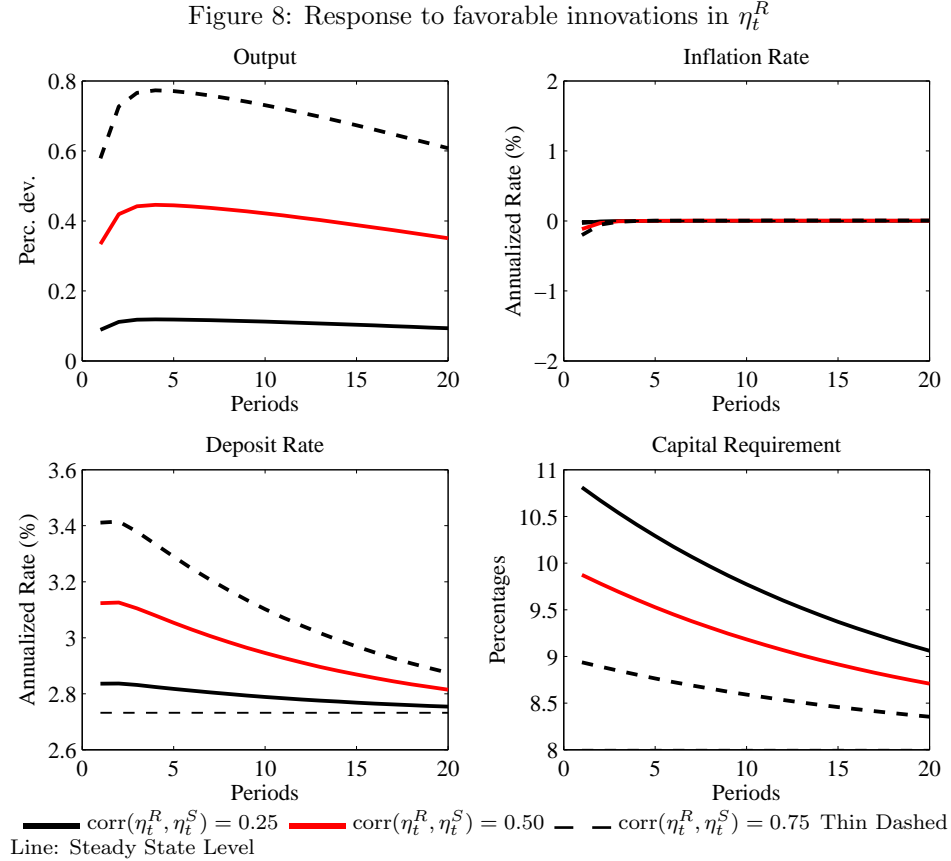
solution for κ_t^* to

$$\kappa_t^* \equiv (1 - \tau) \frac{(1 - \phi_t) \gamma_t [\exp(\eta_t^R - \eta_t^S) - 1] + \Psi_t [(1 - \phi_t) \gamma_t \exp(\eta_t^R - \eta_t^S) - \phi_t]}{\phi_t (1 + \gamma_t) - \gamma_t \tau (1 - \phi_t) [\exp(\eta_t^R - \eta_t^S) - 1]},$$

and changes (10) to

$$E_t \{ \lambda_{t+1} q_{t+1} \} = E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \right\} (1 + R_t^S) \exp(-\eta_t^S) q_t^x.$$

Figure 8 is the analogue of Figure 2; it shows the optimal responses to a positive innovation in η_t^R for three values of its correlation with the innovation to η_t^S : 0.25, 0.50, and 0.75. The correlation makes both optimal policies act in a counter-cyclical way. Optimal prudential policy raises the capital requirement to tame risk taking, and optimal monetary policy raises the deposit rate to tame the inflationary effect of the investment boom. The counter-cyclical tendency of the policies is stronger when the correlation across shocks is higher.



7 Concluding Remarks

The interactions of monetary policy with prudential policies pose urgent questions for the design of future regulatory environments and deserve scrutiny from alternative theoretical vantage points. The perspective we adopted in this paper (based on earlier work) views bank capital requirements as a

tool for addressing the risk-taking incentives created by limited liability and deposit insurance. Our model takes limited liability and deposit insurance as institutional features (which we don't seek to rationalize within the model) and highlights their interactions with another institutional feature: the fact that a tax distortion makes equity finance more expensive than debt finance. We think the latter distortion merits more attention in models of how the banking sector matters for monetary-policy analysis.¹⁹ In our model, it makes an increase in the capital requirement contractionary, as bank lending rates rise.

Our benchmark model with perfectly competitive banks and constant marginal costs leads to a one sided view of how prudential and monetary policies should interact. The locally optimal mandate of prudential policy in our model is to ensure that banks never fund inefficient risky projects, and to accomplish this objective with minimal damage in terms of increased bank lending rates and decreased capital stock. The distortion is minimized if capital requirements are state dependent. The interaction across policies then boils down to cutting (raising) interest rates to moderate the contractions (expansions) caused by changes in the capital requirement. The model also serves to illustrate how time variation in the capital requirement may be in response to shocks that affect the relative attractiveness of risky and safe projects, without necessarily affecting the aggregate volume of bank credit.

Our example with an externality in the cost of banking, however, illustrates how optimal policy interactions may be more complex. In this example, an increase in the aggregate volume of safe loans increases the costs of originating and monitoring safe loans. This feature matters for the policy interactions. Compared to the model with no externality, the optimal expansion of output in response to a productivity shock is smaller. Moreover, because the optimal capital requirement rises in order to prevent excessive risk taking, the optimal monetary policy response does not fight the boom as much (and cuts the policy rate more aggressively later).

Two extensions of the model seem worth pursuing. The first involves allowing for increasing, endogenously determined costs of banking. The second involves a setting where some risk taking by the banks might be desirable. As the model stands now, it has the property that none of “the” risky projects should ever be funded. While this serves to simplify the analysis, it limits the range of prudential policies that are optimal.

¹⁹For one thing, this may account for the fact that banks extend credit using loan contracts in reality, even though loan contracts are not optimal according to most formal models (with the notable exception of models with costly state verification).

8 Appendix

8.1 Proof of Proposition 1

To show that there is no equilibrium with $0 < l_t^R < \gamma_t l_t^S$, we suppose that there is such an equilibrium and consider a perturbation satisfying $dl_t^S(j) = -dl_t^R(j)$ in the loan portfolio of a given bank j . Note that this perturbation neither tightens nor loosens bank j 's balance-sheet identity

$$l_t^S(j) + l_t^R(j) = e_t(j) + d_t(j) \quad (25)$$

and its capital requirement

$$e_t(j) \geq \kappa_t [l_t^S(j) + l_t^R(j)],$$

given that $l_t^S(j) + l_t^R(j)$ is left unchanged. So this perturbation should not increase bank j 's expected excess return. The derivations of the effect of this perturbation on bank j 's expected excess return involves two cases, depending on whether firms' default leads to bank j 's default.

If firms' default leads to bank j 's default, then the change in bank j 's expected excess return is

$$(1 - \tau) \left[\beta(1 - \phi_t) E_t \left\{ \left(\frac{\lambda_{t+1}}{\Pi_{t+1}} \right) \middle| \theta_t = 1 \right\} \frac{R_t^R - R_t^S}{\lambda_t} + \Psi_t \right] dl_t^R(j),$$

since bank j ignores the effect of its loan portfolio change on aggregate variables like λ_{t+1} or Π_{t+1} . As discussed in the main text, we must have $R_t^R \geq R_t^S$ in equilibrium. Therefore, bank j 's expected excess return is increasing in $l_t^R(j)$. This means that bank j would like to take more risk, contradicting our conjecture about the existence of an equilibrium with $l_t^R < \gamma_t l_t^S$. This proves Part (a) of the Proposition.

If firms' default does not lead to bank j 's default, then the change in bank j 's expected excess return is

$$(1 - \tau) \left[\beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t \Pi_{t+1}} [\theta_t (1 + R_t^R) - (1 + R_t^S)] \right\} + \Psi_t \right] dl_t^R(j) \equiv M dl_t^R(j).$$

Now,

$$\begin{aligned} \frac{M}{1 - \tau} &= \beta(1 - \phi_t) \frac{(1 + R_t^R) - (1 + R_t^S)}{\lambda_t} E_t \left\{ \left(\frac{\lambda_{t+1}}{\Pi_{t+1}} \right) \middle| \theta_t = 1 \right\} \\ &\quad - \beta \phi_t \frac{1 + R_t^S}{\lambda_t} E_t \left\{ \left(\frac{\lambda_{t+1}}{\Pi_{t+1}} \right) \middle| \theta_t = 0 \right\} + \Psi_t \\ &= \beta(1 - \phi_t) \frac{1 + R_t^S}{\lambda_t} \left(\frac{1 + R_t^R}{1 + R_t^S} - 1 \right) E_t \left\{ \left(\frac{\lambda_{t+1}}{\Pi_{t+1}} \right) \middle| \theta_t = 1 \right\} \\ &\quad - \beta \phi_t \frac{1 + R_t^S}{\lambda_t} E_t \left\{ \left(\frac{\lambda_{t+1}}{\Pi_{t+1}} \right) \middle| \theta_t = 0 \right\} + \Psi_t \\ &\leq \beta(1 - \phi_t) \frac{1 + R_t^S}{\lambda_t} \left[\frac{E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \right\}}{E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \middle| \theta_t = 1 \right\}} \exp(\eta_t^R) - 1 \right] E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \middle| \right. \\ &\quad \left. \theta_t = 1 \right\} - \beta \phi_t \frac{1 + R_t^S}{\lambda_t} E_t \left\{ \left(\frac{\lambda_{t+1}}{\Pi_{t+1}} \right) \middle| \theta_t = 0 \right\} + \Psi_t \end{aligned}$$

where the last inequality comes from (12) and (17).

Therefore,

$$\begin{aligned} \frac{M}{1-\tau} &\leq \beta(1-\phi_t) \frac{1+R_t^S}{\lambda_t} E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \right\} \exp(\eta_t^R) \\ &\quad - \beta(1-\phi_t) \frac{1+R_t^S}{\lambda_t} E_t \left\{ \left(\frac{\lambda_{t+1}}{\Pi_{t+1}} \right) \middle| \theta_t = 1 \right\} \\ &\quad - \beta\phi_t \frac{1+R_t^S}{\lambda_t} E_t \left\{ \left(\frac{\lambda_{t+1}}{\Pi_{t+1}} \right) \middle| \theta_t = 0 \right\} + \Psi_t, \end{aligned}$$

which implies

$$\frac{M}{1-\tau} \leq \beta(1-\phi_t) \frac{1+R_t^S}{\lambda_t} E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \right\} \exp(\eta_t^R) - \beta \frac{1+R_t^S}{\lambda_t} E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \right\} + \Psi_t$$

and

$$\frac{M}{1-\tau} \leq \beta \frac{1+R_t^S}{\lambda_t} E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \right\} [(1-\phi_t) \exp(\eta_t^R) - 1] + \Psi_t.$$

Using (3), we get

$$\frac{M}{1-\tau} \leq \frac{1+R_t^S}{1+R_t^D} [(1-\phi_t) \exp(\eta_t^R) - 1] + \Psi_t.$$

and, using (8),

$$\frac{M}{1-\tau} \leq \Psi_t \left(1 - \frac{1+R_t^S}{1+R_t^D} \right),$$

which implies $M < 0$ because monitoring costs make $R_t^S > R_t^D$ in equilibrium. Therefore, bank j 's expected excess return is decreasing in $l_t^R(j)$. This means that bank j would like to take less risk, contradicting our conjecture about the existence of an equilibrium with $0 < l_t^R$. This proves Part (b) of the Proposition.

8.2 Proof of Proposition 2

This appendix proves Proposition 2 by establishing a more general result that will serve us in subsequent appendices. We show that the capital constraint is always binding for a bank j that deviates from either the candidate equilibrium with $l_t^R = 0$, or the candidate equilibrium with $l_t^R = \gamma_t l_t^S$ (the only two candidate equilibria left, given Proposition 1). As a consequence, for a zero deviation, the capital constraint is binding -i.e. (18) holds- in any of the two candidate equilibria of our model, which leads to Proposition 2.

In general, using (25), bank j 's expected excess return can be written

$$(1-\tau) E_t \left\{ \beta \frac{\lambda_{t+1} \omega_{t+1}^b(j)}{\lambda_t} \right\} - e_t(j) - (1-\tau) \Psi_t l_t^S(j),$$

where

$$\omega_{t+1}^b(j) = \max \left\{ 0, \frac{R_t^S - R_t^D}{\Pi_{t+1}} l_t^S(j) + \left[\theta_t \frac{1+R_t^R}{\Pi_{t+1}} - \frac{1+R_t^D}{\Pi_{t+1}} \right] l_t^R(j) + \frac{1+R_t^D}{\Pi_{t+1}} e_t(j) \right\}.$$

In the case where $\omega_{t+1}^b(j) > 0$ when $\theta_t = 0$, using (3), bank j 's expected excess return can be rewritten

$$\begin{aligned} (1-\tau) \left\{ \frac{R_t^S - R_t^D}{1+R_t^D} l_t^S(j) + \left[(1-\phi_t) \left(\frac{1+R_t^R}{1+R_t^D} \right) - 1 \right] l_t^R(j) + e_t(j) \right\} \\ - e_t(j) - (1-\tau) \Psi_t l_t^S(j). \end{aligned}$$

Since this expression is strictly decreasing in $e_t(j)$, it is maximized when $e_t(j)$ is minimal, that is to say when $e_t(j)$ satisfies

$$e_t(j) = \kappa_t [l_t^S(j) + l_t^R(j)]. \quad (26)$$

In the alternative case where $\omega_{t+1}^b(j) = 0$ when $\theta_t = 0$, consider first the candidate equilibrium with $l_t^R = 0$. Bank j 's expected excess return can then be written

$$(1 - \tau)(1 - \phi_t) \left\{ \frac{R_t^S - R_t^D}{1 + R_t^D} l_t^S(j) + \left[\frac{1 + R_t^R}{1 + R_t^D} - 1 \right] l_t^R(j) + e_t(j) \right\} - e_t(j) - (1 - \tau) \Psi_t l_t^S(j).$$

Since this expression is strictly decreasing in $e_t(j)$, it is maximized for $e_t(j)$ given by (26). Consider next the candidate equilibrium with $l_t^R = \gamma_t l_t^S$. Bank j 's expected excess return can then be written

$$(1 - \tau)(1 - \phi_t) \frac{\beta}{\lambda_t} E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \middle| \theta_t = 1 \right\} [(R_t^S - R_t^D) l_t^S(j) + (R_t^R - R_t^D) l_t^R(j) + (1 + R_t^D) e_t(j)] - e_t(j) - (1 - \tau) \Psi_t l_t^S(j).$$

This expression is strictly decreasing in $e_t(j)$, since its derivative with respect to $e_t(j)$ is strictly negative:

$$\begin{aligned} & (1 - \tau)(1 - \phi_t) \frac{\beta}{\lambda_t} E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \middle| \theta_t = 1 \right\} (1 + R_t^D) - 1 \\ = & (1 - \tau)(1 - \phi_t) \frac{E_t \{ \lambda_{t+1} q_{t+1} | \theta_t = 1 \}}{E_t \{ \lambda_{t+1} q_{t+1} \}} \exp(\eta_t^R) \frac{1 + R_t^S}{1 + R_t^R} - 1 \\ < & (1 - \tau)(1 - \phi_t) \exp(\eta_t^R) \frac{1 + R_t^S}{1 + R_t^R} - 1 \\ < & (1 - \tau)(1 - \Psi_t) \frac{1 + R_t^S}{1 + R_t^R} - 1 \\ < & 0, \end{aligned}$$

where the equality comes from (12) and (3), the first inequality from (17), the second inequality from (8), and the third inequality from the fact that $R_t^R > R_t^S$ in equilibrium. Therefore, bank j will choose the minimal capital requirement, i.e. $e_t(j)$ satisfying (26). To sum up, the capital constraint is always binding for a bank j that deviates from either the candidate equilibrium with $l_t^R = 0$, or the candidate equilibrium with $l_t^R = \gamma_t l_t^S$. In particular, for a zero deviation, the capital constraint is binding –i.e. (18) holds– in any of the two candidate equilibria of our model. This establishes Proposition 2.

8.3 Proof of Proposition 3

Consider a bank j that takes the maximum amount of risk by setting $l_t^R(j) = \gamma_t l_t^S(j)$. Using (25) and (26) to eliminate $d_t(j)$ from

$$\omega_{t+1}^b(j) = \max \left\{ 0, \frac{1 + R_t^S}{\Pi_{t+1}} l_t^S(j) + \theta_t \frac{1 + R_t^R}{\Pi_{t+1}} l_t^R(j) - \frac{1 + R_t^D}{\Pi_{t+1}} d_t(j) \right\},$$

it is straightforward to show that this bank remains solvent ($\omega_{t+1}^b(j) > 0$) when risky projects fail ($\theta_t = 0$) if and only if (20) holds. Part (a) of Proposition 3 follows.

Then, consider a candidate equilibrium with $l_t^R = 0$. Using (13) to eliminate d_t and (18) to eliminate e_t , the representative bank's expected excess return can be rewritten

$$(1 - \tau) E_t \left\{ \beta \frac{\lambda_{t+1} \omega_{t+1}^b}{\lambda_t} \right\} - [\kappa_t + (1 - \tau) \Psi_t] l_t^S,$$

where

$$\omega_{t+1}^b = \left[\frac{R_t^S - R_t^D}{\Pi_{t+1}} + \frac{1 + R_t^D}{\Pi_{t+1}} \kappa_t \right] l_t^S.$$

The representative bank chooses l_t^S so as to maximize its expected excess return. Using (3), the first-order condition of this programme can be written

$$(1 - \tau) \frac{R_t^S - R_t^D}{1 + R_t^D} - \tau \kappa_t - (1 - \tau) \Psi_t = 0. \quad (27)$$

We can then use this first-order condition to rewrite $\tilde{\kappa}(R_t^D, R_t^S)$, at the candidate equilibrium with $l_t^R = 0$, as (19). Parts (b) and (c) of Proposition 3 follow.

8.4 Proof of Proposition 4

To prove Part (a) of Proposition 4, we look for a necessary and sufficient condition on policy instruments for the existence of an equilibrium with $l_t^R = 0$. This amounts to looking for a necessary and sufficient condition on policy instruments for the demand and supply curves on the risky-loans market to intersect at one or several points (R_t^R, l_t^R) with $R_t^R \geq 0$ and $l_t^R = 0$. We proceed in several steps.

Step 1: condition for zero demand for risky loans. Given capital producers' programme, the portion of the demand curve that is consistent with $l_t^R = 0$ is characterized by

$$\frac{1 + R_t^R}{1 + R_t^S} \geq \frac{E_t \{ \lambda_{t+1} q_{t+1} | \theta_t = 1 \}}{E_t \{ \lambda_{t+1} q_{t+1} \}} \frac{E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \right\}}{E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} | \theta_t = 1 \right\}} \exp(\eta_t^R).$$

Because θ_t is independent of any other shock and because the realization of θ_t does not affect the aggregate outcome when $l_t^R = 0$, the latter inequality can be rewritten

$$\frac{1 + R_t^R}{1 + R_t^S} \geq \exp(\eta_t^R). \quad (28)$$

Step 2: condition for zero supply of risky loans. The portion of the supply curve that is consistent with $l_t^R = 0$ can be characterized by a necessary and sufficient condition for an individual bank j not to deviate from the candidate equilibrium with $l_t^R = 0$. We now look for such a condition. Appendix 8.1 implies that, if some deviations are profitable, then the most profitable deviation is $l_t^R(j) = \gamma_t l_t^S(j)$. If bank j makes this deviation, then, using (25) to eliminate $d_t(j)$ and (26) to eliminate $e_t(j)$, its expected excess return can be rewritten

$$(1 - \tau) E_t \left\{ \beta \frac{\lambda_{t+1} \omega_{t+1}^b(j)}{\lambda_t} \right\} - [\kappa_t (1 + \gamma_t) + (1 - \tau) \Psi_t] l_t^S(j),$$

where

$$\omega_{t+1}^b(j) = \max \left\{ 0, \left[\frac{R_t^S - R_t^D}{\Pi_{t+1}} + \theta_t \gamma_t \frac{1 + R_t^R}{\Pi_{t+1}} - \gamma_t \frac{1 + R_t^D}{\Pi_{t+1}} + \frac{1 + R_t^D}{\Pi_{t+1}} \kappa_t (1 + \gamma_t) \right] l_t^S(j) \right\}.$$

Because θ_t is independent of any other shock and because the realization of θ_t does not affect the aggregate outcome in equilibrium (given that $l_t^R = 0$), bank j 's expected excess return can be rewritten, using (3),

$$(1 - \tau) E_t \left\{ \max \left\{ 0, \left[\frac{R_t^S - R_t^D}{1 + R_t^D} + \theta_t \gamma_t \frac{1 + R_t^R}{1 + R_t^D} - \gamma_t + \kappa_t (1 + \gamma_t) \right] l_t^S(j) \right\} \right\} - [\kappa_t (1 + \gamma_t) + (1 - \tau) \Psi_t] l_t^S(j).$$

Note that the ‘max’ that features in this expression is strictly higher than zero when $\theta_t = 1$, because both R_t^R and R_t^S are strictly higher than R_t^D in equilibrium. So we will have to consider two cases, depending on whether this ‘max’ is strictly higher than zero or equal to zero when $\theta_t = 0$.

In the case where this ‘max’ is strictly higher than zero when $\theta_t = 0$, that is to say in the case where $\kappa_t > \tilde{\kappa}_t$, we know from Proposition 1 that bank j 's deviation is not profitable.

In the alternative case where the ‘max’ is equal to zero when $\theta_t = 0$, that is to say in the case where $\kappa_t \leq \tilde{\kappa}_t$, bank j 's expected excess return is

$$\left\{ (1 - \tau) (1 - \phi_t) \left[\frac{R_t^S - R_t^D}{1 + R_t^D} + \gamma_t \frac{1 + R_t^R}{1 + R_t^D} - \gamma_t + \kappa_t (1 + \gamma_t) \right] - \kappa_t (1 + \gamma_t) - (1 - \tau) \Psi_t \right\} l_t^S(j).$$

Using (27) to eliminate R_t^S , we can then rewrite bank j 's expected excess return as

$$\left\{ (1 - \tau) (1 - \phi_t) \gamma_t \frac{R_t^R - R_t^D}{1 + R_t^D} - [\phi_t (1 + \gamma_t) + \gamma_t \tau (1 - \phi_t)] \kappa_t - \phi_t (1 - \tau) \Psi_t \right\} l_t^S(j).$$

Therefore, a necessary and sufficient condition for the deviation not to be profitable is then

$$[\phi_t (1 + \gamma_t) + \gamma_t \tau (1 - \phi_t)] \kappa_t + \phi_t (1 - \tau) \Psi_t \geq (1 - \tau) (1 - \phi_t) \gamma_t \frac{R_t^R - R_t^D}{1 + R_t^D}. \quad (29)$$

To sum up, the portion of the supply curve that is consistent with $l_t^R = 0$ is characterized by the condition that either $\kappa_t > \tilde{\kappa}_t$, or $\kappa_t \leq \tilde{\kappa}_t$ and (29) holds.

Step 3: condition for zero risky loans in equilibrium. The demand and supply curves on the risky-loans market intersect at one or several points (R_t^R, l_t^R) with $R_t^R \geq 0$ and $l_t^R = 0$ if and only if either (i) $\kappa_t > \tilde{\kappa}_t$, or (ii) $\kappa_t \leq \tilde{\kappa}_t$, and (29) holds when (28) holds with equality.

Note that, if (28) holds with equality, then, using (27), we can rewrite (29) as

$$\kappa_t \geq \kappa_t^* \equiv (1 - \tau) \frac{(1 - \phi_t) \gamma_t [\exp(\eta_t^R) - 1] + \Psi_t [(1 - \phi_t) \gamma_t \exp(\eta_t^R) - \phi_t]}{\phi_t (1 + \gamma_t) - \gamma_t \tau (1 - \phi_t) [\exp(\eta_t^R) - 1]}, \quad (30)$$

since the denominator on the right-hand side of this inequality is strictly positive:

$$\begin{aligned} & \phi_t (1 + \gamma_t) - \gamma_t \tau (1 - \phi_t) [\exp(\eta_t^R) - 1] \\ &= \phi_t [1 + \gamma_t (1 - \tau)] + \gamma_t \tau - \gamma_t \tau (1 - \phi_t) \exp(\eta_t^R) \\ &> \phi_t [1 + \gamma_t (1 - \tau)] + \gamma_t \tau \Psi_t \\ &> 0, \end{aligned}$$

where the last but one inequality comes from (8). As a consequence, a necessary and sufficient condition on policy instruments for the existence of an equilibrium with $l_t^R = 0$ is that either $\kappa_t > \tilde{\kappa}_t$,

or $\kappa_t^* \leq \kappa_t \leq \tilde{\kappa}_t$. This condition can be equivalently rewritten $\kappa_t \geq \min\{\tilde{\kappa}_t, \kappa_t^*\}$. Now, using (8) to replace $(1 - \phi_t) \exp(\eta_t^R)$ by $1 - \Psi_t$ on the right-hand side of (30), we get

$$\begin{aligned} \kappa_t^* &\leq (1 - \tau) \frac{-\gamma_t \Psi_t^2 + \phi_t (\gamma_t - \Psi_t)}{\gamma_t \tau \Psi_t + \phi_t (1 + \gamma_t - \gamma_t \tau)} \\ &= \tilde{\kappa}_t \left\{ 1 - \frac{\gamma_t \Psi_t}{\gamma_t - \Psi_t} \frac{\gamma_t \tau + \Psi_t (1 + \gamma_t) (1 - \tau)}{\gamma_t \Psi_t \tau + \phi_t (1 + \gamma_t - \gamma_t \tau)} \right\} \\ &< \tilde{\kappa}_t, \end{aligned}$$

where the last inequality comes from our assumption that $\gamma_t > \Psi_t$. Therefore, a necessary and sufficient condition on policy instruments for the existence of an equilibrium with $l_t^R = 0$ is simply $\kappa_t \geq \kappa_t^*$. Parts (a) and (b) of Proposition 4 follow.

Finally, Part (c) of Proposition 4 follows straightforwardly from the fact that the denominator on the right-hand side of (30) is strictly positive, as shown above.

8.5 Proof of Proposition 5

Define welfare as the representative household's expected utility at date 0, $E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, h_t)$. For any policy $(R_\tau^D, \kappa_\tau)_{\tau \geq 0}$, define the distance from $(R_\tau^{D*}, \kappa_\tau^*)_{\tau \geq 0}$ as

$$\varepsilon \equiv \max \left[\max_{\tau \geq 0} (|R_\tau^D - R_\tau^{D*}|), \max_{\tau \geq 0} (|\kappa_\tau - \kappa_\tau^*|) \right].$$

Let us first compare $(R_\tau^{D*}, \kappa_\tau^*)_{\tau \geq 0}$ to policies $(R_\tau^D, \kappa_\tau)_{\tau \geq 0}$ such that ε is arbitrarily small and $\exists t \geq 0$, $\kappa_t < \kappa_t^*$. Moving from $(R_\tau^{D*}, \kappa_\tau^*)_{\tau \geq 0}$ to any such policy triggers a discontinuous increase in the amount of risk, as it makes banks' risky loans l_t^R move from 0 to $\gamma_t l_t^S > 0$ at some date $t \geq 0$. Under our assumptions (inefficiency condition (8), risk aversion, and no correlation between θ_t and other shocks), this discontinuous increase in the amount of risk has a discontinuous negative effect on welfare. Any other effect on welfare is continuous and, therefore, dominated by this discontinuous negative effect provided that ε is small enough. As a consequence, welfare is strictly higher under $(R_\tau^{D*}, \kappa_\tau^*)_{\tau \geq 0}$ than under any such policy provided that ε is small enough.

Let us then compare $(R_\tau^{D*}, \kappa_\tau^*)_{\tau \geq 0}$ to policies $(R_\tau^D, \kappa_\tau)_{\tau \geq 0}$ such that ε is arbitrarily small, $\forall \tau \geq 0$, $\kappa_\tau \geq \kappa_\tau^*$, and $\exists t \geq 0$, $\kappa_t > \kappa_t^*$. Using the equilibrium conditions that are independent of policies, rewrite welfare as

$$W \left[(R_\tau^D)_{\tau \geq 0}, (\kappa_\tau)_{\tau \geq 0}, H_0 \right],$$

where H_0 captures initial conditions (endogenous variables until date -1 , exogenous shocks until date 0). Since $(R_\tau^{D*})_{\tau \geq 0}$ is the monetary policy that is Ramsey-optimal when $(\kappa_\tau)_{\tau \geq 0} = (\kappa_\tau^*)_{\tau \geq 0}$, we have

$$\forall t \geq 0, \frac{\partial W}{\partial R_t^D} \left[(R_\tau^{D*})_{\tau \geq 0}, (\kappa_\tau^*)_{\tau \geq 0}, H_0 \right] = 0.$$

Therefore, the first-order Taylor approximation of $W \left[(R_\tau^D)_{\tau \geq 0}, (\kappa_\tau)_{\tau \geq 0}, H_0 \right]$ in a neighborhood of

$\left[(R_\tau^{D*})_{\tau \geq 0}, (\kappa_\tau^*)_{\tau \geq 0}, H_0 \right]$ such that $\forall \tau \geq 0, \kappa_\tau \geq \kappa_\tau^*$, is

$$W \left[(R_\tau^D)_{\tau \geq 0}, (\kappa_\tau)_{\tau \geq 0}, H_0 \right] = W \left[(R_\tau^{D*})_{\tau \geq 0}, (\kappa_\tau^*)_{\tau \geq 0}, H_0 \right] + \sum_{t=0}^{+\infty} \frac{\partial W}{\partial \kappa_t} \left[(R_\tau^{D*})_{\tau \geq 0}, (\kappa_\tau^*)_{\tau \geq 0}, H_0 \right] (\kappa_t - \kappa_t^*) + O(\varepsilon^2),$$

where $\frac{\partial W}{\partial \kappa_t}$ is the right derivative of welfare with respect to κ_t and $O(\varepsilon^2)$ is a term of second order in ε . As a consequence, if

$$\forall t \geq 0, \frac{\partial W}{\partial \kappa_t} \left[(R_\tau^{D*})_{\tau \geq 0}, (\kappa_\tau^*)_{\tau \geq 0}, H_0 \right] < 0,$$

then welfare is strictly higher under $(R_\tau^{D*}, \kappa_\tau^*)_{\tau \geq 0}$ than under any policy $(R_\tau^D, \kappa_\tau)_{\tau \geq 0}$ such that $\forall \tau \geq 0, \kappa_\tau \geq \kappa_\tau^*$ and $\exists t \geq 0, \kappa_t > \kappa_t^*$, provided that ε is small enough. Proposition 5 follows.

8.6 Proof of Proposition 6

Using (27) and (28), it is easy to show that, at any candidate equilibrium with $l_t^R = 0$, the macroprudential policy rule (23) implies (i) $\kappa_t \geq \kappa_t^*$ and (ii) $\kappa_t = \kappa_t^*$ if and only if (28) holds with equality. Therefore, given Proposition 4, there exists a unique equilibrium with $l_t^R = 0$ under (23) and, at this equilibrium, $\kappa_t = \kappa_t^*$ and (28) holds with equality.

We now show that there exists no equilibrium with $l_t^R = \gamma_t l_t^S$ under (23). To that aim, consider a candidate equilibrium with $l_t^R = \gamma_t l_t^S$. Proposition 1 implies that, if $\omega_{t+1}^b > 0$ when $\theta_t = 0$, then this candidate equilibrium is not an equilibrium. We focus therefore on the case where $\omega_{t+1}^b = 0$ when $\theta_t = 0$. Consider a given bank j , whose expected excess return is

$$E_t \left\{ \beta \frac{\lambda_{t+1} (1 - \tau) \omega_{t+1}^b(j)}{\lambda_t} \right\} - e_t(j) - (1 - \tau) \Psi_t l_t^S(j),$$

where

$$\omega_{t+1}^b(j) = \max \left\{ 0, \frac{1 + R_t^S}{\Pi_{t+1}} l_t^S(j) + \theta_t \frac{1 + R_t^R}{\Pi_{t+1}} l_t^R(j) - \frac{1 + R_t^D}{\Pi_{t+1}} d_t(j) \right\}.$$

Using (25) to eliminate $d_t(j)$ and (26) to eliminate $e_t(j)$, its expected excess return can be rewritten

$$E_t \left\{ \beta \frac{\lambda_{t+1} (1 - \tau) \omega_{t+1}^b(j)}{\lambda_t} \right\} - \kappa_t [l_t^S(j) + l_t^R(j)] - (1 - \tau) \Psi_t l_t^S(j),$$

where

$$\omega_{t+1}^b(j) = \max \left\{ 0, \frac{1 + R_t^S}{\Pi_{t+1}} l_t^S(j) + \theta_t \frac{1 + R_t^R}{\Pi_{t+1}} l_t^R(j) - \frac{1 + R_t^D}{\Pi_{t+1}} (1 - \kappa_t) [l_t^S(j) + l_t^R(j)] \right\}.$$

If bank j does not deviate from the candidate equilibrium with $l_t^R = \gamma_t l_t^S$, then its expected excess return is equal to

$$\left[(1 - \phi_t) \beta \frac{(1 - \tau) [(1 + R_t^S) + \gamma_t (1 + R_t^R) - (1 + \gamma_t) (1 - \kappa_t) (1 + R_t^D)]}{\lambda_t} \right. \\ \left. E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \middle| \theta_t = 1 \right\} - \kappa_t (1 + \gamma_t) - (1 - \tau) \Psi_t \right] \frac{1}{1 + \gamma_t} l_t(j),$$

where $l_t(j) \equiv l_t^S(j) + l_t^R(j)$, since $\omega_{t+1}^b(j) = 0$ when $\theta_t = 0$. Appendix 8.1 implies that, if some deviations from the candidate equilibrium with $l_t^R = \gamma_t l_t^S$ are profitable, then the most profitable deviation is to provide zero risky loans. If bank j makes this deviation, then its expected excess return becomes

$$\left[\phi_t \beta \frac{(1-\tau) [(R_t^S - R_t^D) + \kappa_t (1 + R_t^D)]}{\lambda_t} E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \middle| \theta_t = 0 \right\} \right. \\ \left. + (1 - \phi_t) \beta \frac{(1-\tau) [(R_t^S - R_t^D) + \kappa_t (1 + R_t^D)]}{\lambda_t} E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \middle| \theta_t = 1 \right\} - \kappa_t - (1 - \tau) \Psi_t \right] l_t(j).$$

The change in bank j 's expected excess return, from $l_t^R(j) = \gamma_t l_t^S(j)$ to $l_t^R(j) = 0$, is

$$\left[\phi_t \beta \frac{(1-\tau) [(R_t^S - R_t^D) + \kappa_t (1 + R_t^D)]}{\lambda_t} E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \middle| \theta_t = 0 \right\} \right. \\ \left. - (1 - \phi_t) \beta \frac{(1-\tau)}{\lambda_t} \frac{\gamma_t}{1 + \gamma_t} (R_t^R - R_t^S) E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \middle| \theta_t = 1 \right\} - (1 - \tau) \frac{\gamma_t}{1 + \gamma_t} \Psi_t \right] l_t(j).$$

It is easy to show that this change is strictly positive, and therefore that bank j gains from deviating from the candidate equilibrium with $l_t^R = \gamma_t l_t^S$, if and only if

$$\kappa_t > -\frac{R_t^S - R_t^D}{1 + R_t^D} + \frac{\gamma_t}{1 + \gamma_t} \frac{(1 - \phi_t) \beta \frac{R_t^R - R_t^S}{\lambda_t} E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \middle| \theta_t = 1 \right\} + \Psi_t}{\phi_t \beta \frac{(1 + R_t^D)}{\lambda_t} E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \middle| \theta_t = 0 \right\}} \\ \equiv \hat{\kappa} \left(R_t^D, R_t^S, R_t^R, \lambda_t, \frac{\lambda_{t+1}}{\Pi_{t+1}}, \phi_t \right).$$

Therefore, there exists no equilibrium with $l_t^R = \gamma_t l_t^S$ under the macro-prudential policy rule (23) if

$$\kappa^* (R_t^D, R_t^S, R_t^R, \phi_t) > \hat{\kappa} \left(R_t^D, R_t^S, R_t^R, \lambda_t, \frac{\lambda_{t+1}}{\Pi_{t+1}}, \phi_t \right),$$

where $\kappa^* (R_t^D, R_t^S, R_t^R, \phi_t)$ is the expression on the right-hand side of (23). Using (3), the latter inequality is easily shown to be equivalent to

$$\frac{1 - \phi_t}{\phi_t^2} \frac{E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \right\}}{E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \middle| \theta_t = 0 \right\}} \left[1 - \frac{E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \middle| \theta_t = 1 \right\}}{E_t \left\{ \frac{\lambda_{t+1}}{\Pi_{t+1}} \right\}} \right] \left(\frac{R_t^R - R_t^S}{1 + R_t^D} + \Psi_t \right) > 0$$

and is therefore satisfied, given (22) and $R_t^R \geq R_t^S$. This establishes Proposition 6.

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