The Signaling Channel for Federal Reserve Bond Purchases*

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Abstract

Previous research has emphasized the portfolio balance effects of Federal Reserve bond purchases, in which a reduced bond supply lowers term premia. In contrast, we find that such purchases have important signaling effects that lower expected future short-term interest rates. Our evidence comes from dynamic term structure models that decompose declines in yields following Fed announcements into changes in risk premia and expected short rates. To overcome problems in measuring term premia, we consider unbiased model estimation and restricted risk price estimation. We also characterize the estimation uncertainty regarding the relative importance of the signaling and portfolio balance channels.

Keywords: monetary policy, zero lower bound, quantitative easing, LSAP, arbitrage-free

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*The views expressed herein are those of the authors and not necessarily shared by others at the Federal Reserve Bank of San Francisco or in the Federal Reserve System.

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1 Introduction

During the recent financial crisis and ensuing deep recession, the Federal Reserve reduced its target for the federal funds rate—the traditional tool of U.S. monetary policy—essentially to the lower bound of zero. In the face of deteriorating economic conditions and with no scope for further cuts in short-term interest rates, the Fed initiated an unprecedented expansion of its balance sheet by purchasing large amounts of Treasury debt and federal agency securities of medium and long maturity.[1] Other central banks in comparable circumstances have taken broadly similar actions. Notably, the Bank of England also purchased longer-term debt during the financial crisis, and the Bank of Japan, when confronted over a decade ago with stagnation and near-zero short-term rates, purchased debt securities in its program of Quantitative Easing (QE).[2]

The goal of the Fed's large-scale asset purchases (LSAPs) of bonds was to put downward pressure on longer-term yields in order to ease financial conditions and support economic growth. Using a variety of approaches, several studies have concluded that the Fed's LSAP program was effective in lowering yields below levels that otherwise would have prevailed (D'Amico and King, 2011; Gagnon et al., 2011; Hamilton and Wu, 2011; Krishnamurthy and Vissing-Jorgensen, 2011). However, understanding the underlying mechanism and causes for the declines in long-term interest rates remains an open question. Based on the usual decomposition of long rates, there are two potential elements that central bank bond purchases could affect: the term premium and the average level of short-term risk-free interest rates over the maturity of the bond, also known as the risk-neutral rate. The term premium could have fallen because the Fed's LSAPs reduced the amount of longer-term bonds in private-sector portfolios—which is loosely referred to as the *portfolio balance channel*. Alternatively, the LSAP announcements could have led market participants to revise down their expectations for future short-term interest rates, lengthening, for example, the expected period of a near-zero federal funds rate target. Such a *signaling channel* for LSAPs would reduce yields by lowering the average expected short-rate (or risk-neutral) component of long-term rates.

Much discussion of the financial market effects of the Fed's bond purchases treats the portfolio balance channel as the key channel for that impact. For example, Chairman Bernanke (2010) described the effects of the Fed's bond purchases in this way:

> I see the evidence as most favorable to the view that such purchases work primarily

[1] The federal agency securities were debt or mortgage-backed securities that had explicit or implicit credit protection from the U.S. government.

[2] The Fed's actions led to a larger central bank balance sheet and higher bank reserves much like the Bank of Japan's QE; however, the Fed's purchases were focused on longer-maturity assets.

> through the so-called portfolio balance channel, which holds that once short-term interest rates have reached zero, the Federal Reserve's purchases of longer-term securities affect financial conditions by changing the quantity and mix of financial assets held by the public. Specifically, the Fed's strategy relies on the presumption that different financial assets are not perfect substitutes in investors' portfolios, so that changes in the net supply of an asset available to investors affect its yield and those of broadly similar assets.

As well as being highlighted by central bankers, the portfolio balance channel has also found support among researchers in accounting for the effects of LSAPs. The most influential evidence supporting a portfolio balance channel has come from event studies that examine changes in asset prices following announcements of central bank bond purchases. Notably, Gagnon et al. (2011), henceforth GRRS, examine changes in the ten-year Treasury yield and Treasury yield term premium.[3] They document that after eight key LSAP announcements, the ten-year yield fell by a total of 91 basis points (bps), while their measure of the ten-year term premium, which is based on the model of Kim and Wright (2005), fell by 71 bps. Based largely on this evidence, the authors argue that the Fed's LSAPs primarily lowered long-term rates through a portfolio balance channel that reduced term premia.

In this paper, we reexamine the conclusion that the signaling of lower short rates through LSAP announcements played a negligible role in lowering yields. As a first step, we provide model-free evidence suggesting that the Fed's actions lowered yields to a considerable extent by changing policy expectations about the expected future path of the federal funds rate. Under a market segmentation assumption that LSAPs primarily affected security-specific term premia in Treasury markets, changes after LSAP announcements in spreads between Treasury yields and money market and swap rates of comparable maturity illuminate the contribution of the portfolio balance channel. Joyce et al. (2010), for example, argue that increases in spreads between U.K. Treasury and swap yields following Bank of England QE announcements support a portfolio balance channel. In contrast, in the U.S., we find that a large portion of the observed yield changes was also reflected in lower money market and swap rates. This suggests that the expectations component may make an important contribution to the declines in yields.

We next reconsider the GRRS results that are based on the Kim-Wright decompositions of yields into term premia and risk-neutral rates using a conventional arbitrage-free dynamic term structure model (DTSM). Although DTSMs are the workhorse model in empirical fixed income finance, they have been very difficult to estimate and plagued by biased coefficient estimates

[3] Other event studies include Joyce et al. (2010), Neely (2010), Krishnamurthy and Vissing-Jorgensen (2011), and Swanson (2011).

as described by previous studies (e.g., Duffee and Stanton, 2004; Kim and Orphanides, 2005; and Bauer et al., 2011, henceforth BRW). Therefore, to get better measures of the term premium, we examine two alternative estimates of the DTSM. The first is obtained from a novel estimation procedure—following BRW—that directly adjusts for the small-sample bias in estimation of a maximally flexible DTSM. Since conventional biased DTSM estimates—like the Kim-Wright model that GRRS rely on—overstate the speed of mean reversion of the short rate, the model-implied forecast of the short rate is too close to the unconditional mean. Consequently, too much of the variation in forward rates is attributed to the term premium component. Intuitively then, conventional biased DTSM estimates understate the importance of the signaling channel. Indeed, we find that an LSAP event study using term premia obtained from DTSM estimates with reduced bias finds a much larger role for the signaling channel. Our second estimation approach imposes restrictions on the risk pricing as in Bauer (2011). Intuitively, under restricted risk pricing, the cross-sectional interest rate dynamics, which are estimated very precisely, are being used to pin down the time series parameters. This reduces both small-sample bias and statistical uncertainty, so that short rate forecasts and term premium estimates are more reliable (Cochrane and Piazzesi, 2008; Joslin et al., 2010; Bauer, 2011). Here, too, we find a more substantial role for the signaling channel than is commonly acknowledged.

As a final contribution, we also quantify the statistical uncertainty surrounding the DTSM-based estimates of the relative contributions of the portfolio balance and signaling channels. In particular, we take into account the parameter uncertainty that underlies estimates of the term premium and produce confidence intervals that reflect this estimation uncertainty. Our confidence intervals reveal that definitive conclusions about the relative importance of term premia and expectations effects of LSAP are difficult. Both of the extreme views of "only term premia" and "only expectations" effects are statistically plausible. However, under restrictions on the risk pricing in the DTSM, statistical uncertainty is reduced. Consequently, our decompositions of the LSAP effects using DTSM estimates under restricted risk prices not only point to a larger role of the signaling channel, but also allow much more precise inference about the respective contribution of signaling and portfolio balance. Taken together, our results indicate that an important effect of the LSAP announcements was to lower the market's expectation of the future policy path, or, equivalently, to lengthen the expected duration of near-zero policy rates.

The paper is structured as follows. In Section 2, we describe the portfolio balance and signaling channels for LSAP effects on yields and discuss the event study methodology that we use to estimate the effects of the LSAPs. Section 3 presents model-free evidence on the importance of the signaling and portfolio balance channels. Section 4 describes the econometric problems with existing term premium estimates and outlines our two approaches for obtaining more appropriate decompositions of long rates. In Section 5, we present our model-based event study results. Section 6 concludes.

2 Identifying portfolio balance and signaling channels

Here we describe the two key channels through which LSAPs can affect interest rate, and discuss how their respective importance can be quantified, albeit imperfectly, through an event study methodology.

2.1 Portfolio balance channel

In the standard asset-pricing model, changes in the supply of long-term bonds do not affect bond prices. In particular, in a pricing model without frictions, bond premia are determined by the risk characteristics of bonds and the risk aversion of investors, both of which are unaffected by the quantity of bonds available to investors. In contrast, to explain the response of bond yields to central bank purchases of bonds, researchers have focused their attention exactly on the effect that a reduction in bond supply has on the risk premium that investors require for holding those securities. The key avenue proposed for this effect is the *portfolio balance channel*.[4] As described by GRRS: (p. 6)

> By purchasing a particular asset, a central bank reduces the amount of the security that the private sector holds, displacing some investors and reducing the holdings of others, while simultaneously increasing the amount of short-term, risk-free bank reserves held by the private sector. In order for investors to be willing to make those adjustments, the expected return on the purchased security has to fall.

The crucial departure from a frictionless model for the operation of a portfolio balance channel is that bonds of different maturities are not perfect substitutes. Instead, there are "preferred-habitat" investors that have maturity-specific demands for bonds and a less-than-perfect offset to this effect from other "arbitrageurs" in the market.[5] In this setting, the maturity structure

[4] Like most of the literature, we focus on the portfolio balance channel to account for term premia effects of LSAPs. Some recent papers have also discussed a liquidity/market functioning channel through which LSAPs could affect bond premia, including, for example, GRRS, Krishnamurthy and Vissing-Jorgensen (2011), and Joyce et al. (2010). This channel appears most relevant for limited periods of market dislocation.

[5] For example, pension funds, other institutional investors, and foreign central banks might have a specific need to hold Treasury securities. Recent work on theoretical underpinnings of the portfolio balance channel includes Vayanos and Vila (2009) and Hamilton and Wu (2011).

of outstanding debt can affect term premia.

Still, the precise portfolio balance effect of purchases on term premia in different markets will vary depending on the interconnectedness of markets. To be concrete, consider the decomposition of the ten-year Treasury yield, y_t^{10}, into a risk-neutral component,[6] YRN_t^{10}, and a term premium, YTP_t^{10}:

$$\begin{aligned} y_t^{10} &= YRN_t^{10} + YTP_t^{10} && (1)\\ &= YRN_t^{10} + YTP_{risk,t}^{10} + YTP_{instrument,t}^{10}. && (2) \end{aligned}$$

The term premium is further decomposed in equation (2) into a maturity-specific term premium that reflects the pricing of interest risk and an idiosyncratic instrument-specific term premium that captures, for example, demand and supply imbalances for that particular security. Some researchers have focused on a fairly extreme *market segmentation* version of the portfolio balance channel in which an absence of arbitrageurs leads essentially to a complete disconnect between markets (Joyce et al., 2010). Changes in the bond supply then would have direct price effects through $YTP_{instrument,t}^{10}$, and because of market segmentation, the change in the price of a given security would depend on how much of that security was purchased.

Alternatively, markets for securities of different maturities may be somewhat connected because of the presence of arbitrageurs, though with some residual segmentation because of maturity-specific demand and limits to arbitrage. In this case, researchers, including GRRS, have emphasized that changes in the bond supply affect the aggregate amount of duration available in the market and the pricing of the associated interest rate risk term premia, $YTP_{risk,t}^{10}$. In this *duration removal* version of the portfolio balance channel, central bank purchases of even a few specific bonds can affect the the risk pricing and term premia for a wide range of securities. Notably, in the absence of further frictions, all fixed income securities (e.g., swaps and Treasuries) of the same duration would be similarly affected. Furthermore, if the Fed were to remove a given amount of duration risk from the market by purchasing ten-year securities or by purchasing (a smaller amount of) 30-year securities, the effect through the duration removal version of the portfolio balance channel would be the same. Thus, there are two ways in which bond purchases can affect term premia in Treasury yields: First, with some lack of substitutability between Treasuries and other assets, bond purchases can reduce Treasury-specific premia. Second, by lowering aggregate duration risk it can reduce term premia in all fixed-income securities.

[6] The risk-neutral yield equals the expected average risk-free rate over the lifetime of the bond plus a negligible convexity term.

2.2 Signaling channel

Despite the recent interest in the portfolio balance channel, which emphasizes the role of *quantities* of securities in asset pricing, it runs counter to at least the past half century of mainstream frictionless finance theory. That theory, which is based on the presence of pervasive, deep-pocketed arbitrageurs, has little or no role for financial market segmentation or movements in idiosyncratic, security-specific term premia like $YTP^{10}_{instrument,t}$. Moreover, the duration removal version of the portfolio balance channel, and shifts in $YTP^{10}_{risk,t}$, would also seem unlikely in conventional models. This is because the scale of the Fed's purchase of $1.725 trillion of debt securities is small relative to the size of bond portfolios. The U.S. fixed income market is on the order of $30 trillion, and the global bond market—arguably, the relevant one—is several times larger. In addition, other assets, such as equities, also bear interest rate risk, thus the total amount of duration risk is even larger.

Instead, the traditional finance view of the Fed's actions would focus on the new information provided to investors about the future path of short-term interest rates. That is, the potential *signaling channel* for central bank bond purchases to affect bond yields by changing the risk-neutral component of interest rates. By late 2008, with the short-term interest rate being essentially zero, many investors were wondering how long the Fed would leave its policy rate unchanged. The extended period language in the FOMC statement provided some guidance, but the zero bound was terra incognita. In such a situation, the Fed's unprecedented announcements of asset purchases with the goal of putting further downward pressure on yields might well have had an important signaling component, in the sense of conveying to market participants how bad the economic situation really was, and that extraordinarily easy monetary policy was going to be in place for some time to come. In general, LSAP announcements may signal to market participants that the central bank has changed its views on current or future economic conditions. Alternatively, they may be thought to convey information about changes in the monetary policy reaction function or policy objectives, such as the inflation target. In such cases, investors may alter their expectations of the future path of the policy rate, perhaps by lengthening the expected period of near-zero short-term interest rates. According to such a signaling channel, announcements of LSAPs would lower the expectations component of long-term yields.

2.3 Event study methodology

The few studies, specifically GRRS and Krishnamurthy and Vissing-Jorgensen (2011) for the U.S. and Joyce et al. (2010) for the U.K., to consider the relative contributions of the portfolio balance and signaling channels have used an event study methodology.[7] This methodology focuses on changes in asset prices over tight windows around discrete events. We also employ such a methodology to assess the effects of LSAPs on fixed income markets.

In the portfolio balance channel described above, it is the quantity of asset purchases that affects prices; however, forward-looking investors will in fact react to *news* of future purchases. Therefore, as changes in the expected maturity structure of outstanding bonds are priced in immediately, credible *announcements* of future LSAPs can have the immediate effect of lowering the term premium component of long-term yields. In our event study, we focus on the eight LSAP announcements that GRRS include in their "baseline" event set, which are described in Table 1.

In calculating the yield responses to these announcements, there are two competing requirements for the size of the event window so that price changes reflect the effects of the announcements. First, the window should be large enough to encompass all of an announcement's effects. Second, the window should be short enough to exclude other events that might significantly affect asset prices. Following GRRS, we use one-day changes in market rates to estimate responses to the Fed's LSAP announcements. (See GRRS for further discussion.) A one-day window appears to be a workable compromise.[8] First, for large, highly liquid markets such as the Treasury bond market, and under the assumption of rational expectations, new information in the announcement about economic fundamentals should quickly be reflected in asset prices. Second, the LSAP announcements appear to be the dominant sources of news for fixed income markets on the days under consideration. On these announcement days, the majority of bond and money market movements appeared to be due to new information that markets received about the Fed's LSAP program.

Of course, if news about LSAPs is leaked or inferred prior to the official announcements, then the event study will underestimate the full effect of the LSAPs. The inability to account for important pre-announcement LSAP news makes us wary of analyzing later LSAP announcements after the eight examined. For example, expectations of a second round of asset purchases (QE2) were incrementally formed before official confirmation in fall 2010, which is a possible reason for why studies like Krishnamurthy and Vissing-Jorgensen (2011) find small effects on financial markets in their event study of QE2. For the events we consider, one can argue that markets mostly did not expect the Fed's purchases ahead of the announcements.[9]

[7]GRRS also provide evidence on the portfolio balance channel from monthly time-series regressions of the Kim-Wright term premium on variables capturing macroeconomic conditions and aggregate uncertainty, as well as a measure of the supply of long-term Treasury securities. However, our experience with these regressions suggests the results are sensitive to specification (see also Rudebusch, 2007).

[8]Our results are robust to using the two-day change following announcements.

[9]On the issue of the surprise component of monetary policy announcements during the recent LSAP period

2.4 Changes in risk-neutral rates and the role of signaling

How can an event study can distinguish between the portfolio balance and signaling channels? A simple view is that these two channels are associated with the changes in term premia and risk neutral rates (measured, say, using a DTSM) after LSAP announcements. However, there is an important subtlety in the empirical assessment of portfolio balance and signaling effects: As a theoretical matter, the split between the portfolio balance and signaling channels is not the same as the decomposition of the long rate into expectations and risk premium components. In fact, because of secondary effects of the portfolio balance and signaling channels, estimated changes of risk-neutral rates are likely a lower bound for the contribution of signaling to changes in long-term interest rates.

To illustrate the mapping between the two channels and the long rate decomposition, consider first a scenario in which the portfolio balance channel is working but there is no signaling. In this case, LSAPs reduce term premia, which would act to boost future economic growth.[10] However, the improved economic outlook will also necessitate less conventional monetary policy stimulus in the future since to achieve the optimal stance of monetary policy, the more policymakers add of one type of stimulus, the less they need to add of the other. Thus, the operation of a portfolio balance channel would cause LSAPs to *increase* risk-neutral rates as well as reducing the term premium. We would measure higher policy expectations despite the absence of any direct signaling effects. The changes in risk-neutral rates following LSAP announcements will include both the direct signaling effects (presumably negative), as well as the indirect portfolio balance effects on future policy expectations (positive). Hence, this would mean that the true signaling effects on risk-neutral rates are likely larger than the estimated decreases in risk-neutral rates.

Conversely, consider the case where there are no portfolio balance effects, but a signaling channel is operational because LSAP announcements contain news about easier monetary policy in the future. This news could take various forms, such as, (1) a longer period of near-zero policy rate, (2) lower risks around a little-changed but more certain policy path, (3) higher medium-term inflation and potentially lower real short-term interest rates, and (4) improved prospects for real activity, including diminished prospects for Depression-like outcomes. Taken together, it seems likely that this news, and the demonstration of the Fed's commitment to act, would reduce the likelihood of future large drops in asset prices and hence lower the risk premia on financial assets. Indeed, although the effects of easier expected monetary policy on term premia could in general go either way, during the previous Fed easing cycle from 2001 to 2003,

see Wright (2011).

[10]On this connection, see Rudebusch et al. (2007).

lower risk-neutral rates were accompanied by lower term premia. Table 2 shows changes in the actual, fitted, and risk-neutral ten-year yield, and in the corresponding yield term premium (according to the Kim-Wright model) for those days with FOMC announcements during 2001 to 2003 when the risk-neutral rate decreased.[11] That is, on days on which the average expected future policy rate was revised downward by market participants—comparable to the potential signaling effects of LSAP announcements—the term premium usually fell as well. Over all such days, the cumulative change in the term premium was -21 bps, which has the same sign and more than half the magnitude of the cumulative change in the risk-neutral yield (-35 bps). Thus, during this episode, easing actions that lowered policy expectations at the same time lowered term premia. Arguably, the signaling effect of LSAPs on term premia would be even larger given the potential curtailment of extreme downside risk.

Both of these effects appear to work in the same direction of making the decomposition into changes in risk-neutral rates and term premia a downwardly biased estimate for the importance of the signaling channel. Conversely, the true portfolio balance effects are probably smaller in magnitude than the estimated decrease in term premia. Therefore, the event study results should be considered conservative ones, with the true signaling effects likely larger than the estimated decreases in risk-neutral rates.

3 Model-free evidence

As a first step in evaluating the effects of an LSAP program on financial markets, it is instructive to consider model-free event-study evidence using data from futures, overnight index swaps (OIS), and Treasury securities markets.

3.1 Market segmentation

What can we learn about changes in policy expectations from considering changes in money market futures rates and OIS rates? Of course these interest rates also contain a term premium and thus do not purely reflect the market's expectations of future short rates. There are, however, two reasons why changes in these market rates can reveal shifts in policy expectations. First, at short maturities the term premium is likely small, so changes in near-term rates are mostly driven by the expectations component. This argument can be used to interpret changes

[11] The data for actual (fitted) yields and the Kim-Wright decomposition of yields are both available at http://www.federalreserve.gov/econresdata/researchdata.htm (accessed August 30, 2011). We only present results for the Kim-Wright term premium since the qualitative conclusions are similar when we use our preferred term premium measures.

at the very short end of the term structure of interest rates (see, for example, GRRS, p. 24). Second, if the portfolio balance effects of LSAPs operate only on instrument-specific premia like $YTP^{10}_{instrument,t}$, then the responses of futures and OIS rates mainly reflect the signaling effects of the announcements. To study the effects of the Bank of England's LSAP program, Joyce et al. (2010) make exactly this market segmentation assumption, and draw inferences about the importance of signaling and portfolio balance purely from observed interest rates in OIS and bond markets. They assume that the asset purchases only affect the term premium specific to government bonds and neither the instrument-specific term premium in OIS rates (which were not part of the asset purchases), nor the general level of the term premium $YTP_{risk,t}$ that reflects compensation for interest rate risk. Under the market segmentation assumption, one can take changes in futures and OIS rates to be predominantly driven by changing policy expectations. Correspondingly, changes in the spreads between these rates and yields are likely due to changes in yield-specific term premia. If the identifying assumption is violated, then the changes in the spreads reflect changes in both YRN^n_t and $YTP^n_{risk,t}$, and thus constitute a lower bound for the contribution of term premium changes, and an upper bound for the magnitude of shifts in policy expectations.

3.2 Money market futures

Money market futures are bets on the future value of a short-term interest rate, and they are used by policymakers, academics, and practitioners to construct implied paths for future policy rates. Federal funds futures settle based on the federal funds rate, and contracts for maturities out to about six months are highly liquid. Eurodollar futures pay off according to the three-month London interbank offered rate (LBOR), and the most liquid contracts have quarterly maturities out to about four years. While LBOR and the fed funds rate do not always move in lockstep, these two types of futures contracts are typically used in combination to construct a policy path over all available horizons.

Consider how the futures-implied policy path has changed around key LSAP dates. Figure 1 shows the futures-implied policy paths around the first five LSAP events, based on futures rates on the end of the previous day and on the end of the event day.[12] On almost all days, the policy paths appear to have shifted down significantly at horizons of one year and longer

[12] The policy paths are derived from the futures rates in the following way: Federal funds futures contracts are used for the current quarter and two quarters beyond that. For longer horizons, we use Eurodollars futures. The Eurodollars futures are adjusted by the difference between the last quarter of the federal funds futures contracts and the overlapping Eurodollar contract. Beginning five months out, a constant term premium adjustment of 1bp per month of additional maturity is applied.

in response to the LSAP announcements.[13]

Table 3 displays the changes at specific horizons on all eight LSAP event days. Also shown are total changes over all event days, as well as cumulative changes and standard deviations of daily changes over the LSAP period. At the short end, the path has shifted down by about 20-40 bps, while at longer horizons of one to three years the total decrease is around 50 bps. Because the decreases in short-term futures rates are arguably driven primarily by expectations, these results indicate that markets revised their near-term policy expectations downward around LSAP announcements by about 20-40 bps.[14]

The last three columns of the table show the changes in the average futures-implied policy path over the next three years, the changes in the three-year yield, and the spread between the yield and the futures-implied rate.[15] With the exception of March 2009, every LSAP announcement had a much larger effect on the futures-implied policy path than on yields. Over all event days, the lower spread contributed around half of the decline in the three-year yield. Under a market segmentation assumption, this evidence suggests that lower policy expectations contributed 37 bps, or 46%, to the decrease in the three-year yield.

3.3 Overnight index swaps

In an overnight index swap (OIS), one party pays a fixed interest rate on the notional amount and receives the overnight rate, i.e., the federal funds rate, over the entire maturity period. Under absence of arbitrage, OIS rates reflect risk-adjusted expectations of the average policy rate over the horizon corresponding to the maturity of the swap. Intuitively, while futures are bets on the value of the short rate at a future point in time, OIS contracts are essentially bets on the average value of the short rate over a certain horizon.

Table 4 shows the results of an event study analysis of changes in OIS rates with maturities of one, two, five, and ten years, yields of the same maturities, and yield-OIS spreads. We consider the same set of event dates as before.[16] The response of yields to the Fed's LSAP

[13]The FOMC statement for January 28, 2009, contrary to the other announcements, actually caused sizable increases in yields and other market interest rates, as documented in GRRS and in our own results below. Anecdotal evidence indicates that market participants were disappointed by the lack of concrete language regarding the possibility and timing of purchases of longer-dated Treasury securities.

[14]One minor confounding factor is that on December 16, 2008, markets also were surprised by the target rate decision—expectations were for a new target of 25 bps, however the Federal Open Market Committee decided on a target range of 0-25 bps. Changes in short-term rates on this day reflect also reflect the effects of conventional monetary policy.

[15]The yield data source is described below.

[16]OIS rates are taken from Bloomberg, and yields are zero-coupon yields from a smoothed yield curve data set constructed as described in Gürkaynak et al. (2007)(henceforth GSW), which is available on the website of the Federal Reserve Board of Governors (BOG). See http://www.federalreserve.gov/econresdata/researchdata.htm

announcements is largely paralleled by an OIS rate response of similar magnitude. For certain days and maturities, OIS rates respond even more strongly than yields, and at the ten-year maturity the cumulative change of the OIS rate is larger than the yield change, with an increasing OIS spread. In those instances where the OIS spread significantly decreased, its relative contribution to the yield change is typically still much smaller than the contribution of the OIS rate change. The March 2009 announcement is the only one that significantly lowered spreads. On the other event days, yield-OIS spreads barely moved or increased, suggesting that large decreases in term premia are unlikely.

Our evidence in this section is consistent with the finding of GRRS "that LSAPs had widespread effects, beyond those on the securities targeted for purchase" (p. 20). Under a market segmentation identifying assumption, the evidence that spreads moved much less than OIS rates suggests a very important contribution of lower policy expectations to the decreases in Treasury yields. Without this assumption, it just indicates that instrument-specific premia in Treasuries did not move much around announcements.

We now turn to model-based evidence to address the question to which extent Treasuries were affected by the LSAPs through downward shifts in the expected policy path and through shifts in a their term premium.

4 Term premium estimation

A theoretically rigorous decomposition of interest rates into expectations and term premium components requires a DTSM, which have generally proven difficult to estimate. Therefore, we consider several different term premium estimates to overcome important bias and uncertainty problems.

4.1 Econometric problems: bias and uncertainty

To estimate the term premium component in long-term interest rates, researchers typically resort to DTSMs. Such models simultaneously capture the cross section and time series dynamics of interest rates, and impose absence of arbitrage, which ensures that the two be consistent with each other. Term premium estimates are obtained by forecasting the short rate using the estimated time series model, and subtracting the average short rate forecast (i.e., the risk-neutral rate) from the actual interest rate. The very high persistence of interest rates, however, causes major problems with estimating the time series dynamics. The parameter

(accessed July 29, 2011).

estimates typically suffer from small-sample bias and large statistical uncertainty, which makes the resulting estimated risk-neutral rates and term premia inherently unreliable.

The small-sample bias in conventional estimates of DTSMs stems from the fact that the largest root in autoregressive models for persistent time series is generally underestimated. Therefore the speed of mean reversion is overestimated, and the model-implied forecasts for longer horizons are too close to the unconditional mean of the process. Consequently, risk-neutral rates are too stable, and too much of the variation in long-term rates is attributed to the term premium component.[17] In the context of LSAP event studies, this bias works in the direction of attributing too large a share of changes in long-term interest rates to the term premium. Hence, the relative importance of the portfolio balance channel will be overestimated. Because of this concern, we reassess the question of interest using term premium estimates that have smaller or no bias.

Large statistical uncertainty underlies any estimate of the term premium, due to both specification and estimation uncertainty. The specification uncertainty is due to the fact that different specifications of a DTSM, where each might seem plausible in itself, lead to quite different economic implications.[18] We address this issue in a pragmatic way by presenting alternative estimates based on different specifications. The estimation uncertainty results from the parameters governing the time series dynamics in a DTSM being estimated imprecisely because of the high persistence of interest rates.[19] Consequently, large statistical uncertainty underlies short rate forecasts and term premia calculated from such parameter estimates. Despite this fact, studies typically report only point estimates of term premia.[20] In our event study, we report interval estimates of changes in risk-neutral rates and of changes in the term premium.

4.2 Alternative term premium estimates

We now briefly describe the alternative term premium estimates that we include in our event study. Details are provided in Appendix A. The data used in the estimation of our models consist of daily observations of interest rates from January 2, 1985, to December 30, 2009. We include T-bill rates at maturities 3 and 6 months from the Federal Reserve H.15 release and

[17]This problem has been pointed out by Ball and Torous (1996) and discussed in numerous subsequent studies, such as Duffee and Stanton (2004) and Kim and Orphanides (2005).

[18]This issue has been highlighted, for example, by Rudebusch et al. (2007) and Bauer (2011).

[19]The slow speed of mean reversion of interest rates makes it difficult to pin down the unconditional mean and the persistence of the estimated process. See, among others, Kim and Orphanides (2005).

[20]Exceptions are the studies by Bauer (2011) and Joslin et al. (2010), who present measures of statistical uncertainty around estimated risk-neutral rates and term premia.

GSW zero-coupon yields at maturities of 1, 2, 3, 5, 7, and 10 years.

4.2.1 Kim-Wright

The term premium estimates used by GRRS are obtained from the model of Kim and Wright (2005). What distinguishes their model from an unrestricted, i.e., maximally flexible affine Gaussian DTSM is the inclusion of survey-based short rate forecasts and some slight restrictions on the risk pricing. While Kim and Orphanides (2005) argue that incorporating additional information from surveys might help alleviate the problems with DTSM estimation, it is unclear to which extent bias and uncertainty are reduced. Survey expectations are problematic because on the one hand they are available only at low frequencies (monthly/quarterly), and on the other hand they might not represent rational forecasts of short rates (Piazzesi and Schneider, 2008). In terms of risk price restrictions, the model imposes only very few constraints, so the link between cross-sectional dynamics and time series dynamics is likely to be weak.

4.2.2 Ordinary least squares

As a benchmark, we estimate a maximally flexible affine Gaussian DTSM. The risk factors correspond to the first three principal components of yields. We use the normalization of Joslin et al. (2011). The estimation is a two-step procedure: First, the parameters of the vector autoregression (VAR) for the risk factors are estimated using ordinary least squares (OLS). Second, we obtain estimates of the parameters governing the cross-sectional dynamics using the minimum-chi-square method of Hamilton and Wu (2010). Because the model is exactly identified, these are also the maximum likelihood (ML) estimates.

To account for the estimation uncertainty underlying the decompositions of long-term interest rates, we obtain bootstrap distributions of the VAR parameters. We can thus calculate risk-neutral rates and term premia for each bootstrap replication of the parameters, and calculate confidence intervals for all objects of interest. Details on the estimation and the bootstrap procedure are provided in Appendix B.1.

4.2.3 Median-unbiased

One way to deal with the small-sample bias in DTSM estimates is to directly correct the estimates of the dynamic system for bias. Starting from the same model, we perform median-unbiased estimation of the VAR parameters in the first step, and proceed with the second step of finding cross-sectional parameters as before. Our methodology, which closely parallels the one laid out in BRW, is detailed in Appendix B.2. We also obtain bootstrap replications of the VAR parameters.

The resulting estimates imply interest rate dynamics that are more persistent and short rate forecasts that revert to the unconditional mean much more slowly than is implied by the biased OLS estimates. Therefore, one would expect a larger contribution of the expectations component to changes in long-term rates around LSAP announcements. Because this estimation method only addresses the bias problem and not the uncertainty problem, confidence intervals cannot be expected to be any tighter than for OLS.

4.2.4 Restricted risk prices

The no-arbitrage restriction can be a powerful remedy for both the bias and the uncertainty problem if the risk pricing is restricted.[21] The intuition is that cross-sectional dynamics are precisely estimated and can help pin down the parameters governing the time series dynamics, reducing both bias and uncertainty in these parameters and leading to more reliable estimates of risk-neutral rates and term premia. There is a large set of possible restrictions on the risk pricing in DTSMs, and alternative restrictions may lead to different economic implications. To deal with these complications, we use a Bayesian framework parallel to the one suggested in Bauer (2011) for estimating our DTSM with restricted risk prices. This allows us to select those restrictions that are supported by the data and to deal with specification uncertainty by means of Bayesian model averaging. Another advantage is that interval estimates naturally fall out of the estimation procedure, because the Markov chain Monte Carlo (MCMC) sampler that we use for estimation, described in Appendix C.2, produces posterior distributions for any object of interest.

First we estimate a maximally flexible model where risk price restrictions are absent using MCMC sampling. These estimates will be denoted by URP (*U*nrestricted *R*isk *P*rices). The point estimates of the model parameters are almost identical to OLS.[22] With regard to interval estimation, there will however be some numerical differences, because the Bayesian credibility intervals (which we will for simplicity also call confidence intervals) for URP are conceptually different from the bootstrap confidence intervals for OLS. Because of potential differences between OLS and URP we include the URP estimates as a point of reference.

The estimates under *R*estricted *R*isk *P*rices will be denoted by RRP. To be clear, here parameters and the objects of interest such as term premium changes are estimated by means

[21] This has been argued for example by Cochrane and Piazzesi (2008), Bauer (2011), and Joslin et al. (2010).

[22] With uninformative priors the Bayesian posterior parameter means are the same as the OLS/maximum likelihood estimates. In our case differences between the two sets of point estimates, which could result from the priors and from approximation error, turn out to be negligibly small.

of Bayesian model averaging, since in this setting the MCMC sampler provides draws across model and parameter space. Because of the averaging over the set of restricted models, the inference takes into account both estimation and model uncertainty.

Because of the risk price restrictions, and in light of the results in Bauer (2011), one would expect a larger role for the expectations component in driving changes in long-term rates around LSAP announcements, as well as tighter confidence intervals around point estimates, i.e., more precise inference about the respective roles of the signaling and portfolio balance channels.

5 Changes in policy expectations and term premia

We now turn to model-based event study results to assess the effects of the Fed's LSAP announcements on the term structure of interest rates. Here changes in Treasury yields around LSAP events are decomposed into changes in risk-neutral rates, i.e., in policy expectations, and term premia using the alternative DTSM estimates that were described above.

5.1 Cumulative changes in long-term yields

Let us first consider cumulative changes in long-term Treasury yields over the LSAP events, and how they are decomposed into expectations and risk premium components. The results are shown in Table 5. In addition to point estimates, we present 95% confidence intervals for the changes in risk-neutral rates and premia. We decompose changes in the ten-year yield as in GRRS, and also include results for the five-year yield. Cumulatively over these eight days, the ten-year yield decreased by 89 bps, and the five-year yield decreased even more strongly by 97 bps.[23]

The Kim-Wright decomposition of the change in the fitted ten-year yield of -102 bps results in a decrease in the risk-neutral yield (YRN) of 31 bps and a decrease in the yield term premium (YTP) of 71. Notably the cumulative change in the DTSM's fitting error of -13 bps is contained in the term premium, which is calculated as the difference between fitted yield and YRN. This is not made explicit in the GRRS study, and the authors compare the 71 bps decrease in the term premium to the 91 bps decrease in the actual (constant-maturity) ten-year yield. However, based on model-fitted results, the contribution of the term premium is not $\frac{-71}{-91} \approx 78\%$ but instead $\frac{-71}{-102} \approx 70\%$, with the risk-neutral component contributing 30%

[23]GRRS consider the constant-maturity ten-year yield, which decreased by 91 bps, whereas we focus throughout on zero-coupon yields obtained from the GSW dataset.

to the decrease. For the five-year yield the relative contributions of expectations and term premium components are 32% and 68%.

The decomposition based on the OLS estimates leads to a slightly larger contribution of the expectations component than for the Kim-Wright decomposition, particularly for the five-year yield. For the ten-year yield the contributions are 35% and 65%, respectively, and for the five-year yield they are 43% and 57%. The bootstrapped confidence intervals (CIs) reveal the tremendous estimation uncertainty that underlies these point estimates. Based on these estimates it is similarly plausible that the entire yield change was due to changes in the term premium as it is plausible that yields were driven lower almost entirely by the expectations component. While it remains unclear how much uncertainty underlies the Kim-Wright results, these results make it seem likely that the -71 bps change in the Kim-Wright term premium is a very uncertain point estimate.

The median-unbiased (MU) estimates imply a larger role for the expectations component, which now accounts for about 50% of the yield change, both at the five-year and ten-year maturity. The CIs are even wider than for the OLS estimates. Addressing the bias problem in term premium estimation via direct bias correction increases the estimated contribution of the signaling channel, but the inference is still very imprecise, since the uncertainty problem remains.

The last two decompositions are for the URP and RRP estimates. The URP point estimates are naturally very similar to the OLS results.[24] The confidence intervals, being conceptually different as mentioned above, are more narrow for URP than for OLS and indicate that both components contributed to the decrease in yields. However, there still is considerable statistical uncertainty around the point estimates, due to the absence of parameter restrictions: The contribution of risk-neutral rates could plausibly be anywhere between $\frac{-7}{-94} \approx 7\%$ and $\frac{-71}{-94} \approx 76\%$.

Turning to the estimates under restricted risk prices, the point estimates for the five-year yield closely correspond to the MU results, with a contribution of expectations that is slightly larger than the contribution of the term premium. The split between changes in expectations and premia here is 52% and 48%. For the ten-year yield the RRP decomposition is closer to the OLS results than to the MU results, but nevertheless attributes more, if only by a little, to the expectations component than Kim-Wright and OLS – the contributions here are 38% and 62%. Importantly, the confidence intervals around the RRP estimates are much tighter than for unrestricted DTSM estimates. They clearly indicate that both the expectations and

[24]Slight differences are due to the fact that the decompositions for URP are posterior means of the object of interest, whereas for OLS the decompositions are calculated at the point estimates of the parameters.

term premium components have played an important role in lowering yields. For the ten-year yield the relative contribution of risk-neutral rates is estimated to be between $\frac{-29}{-94} \approx 30\%$ and $\frac{-53}{-94} \approx 56\%$.

5.2 Shifts in the forward curve and policy expectations

To understand these decompositions of yield changes and to get a more comprehensive perspective of the effects of the LSAP announcements on the term structure, it is useful to look at forward rates and the expected policy path. We visualize the shift in the forward rate curve in Figures 2 and 3. Based on our four alternative DTSM estimates, the graphs show the cumulative change over the LSAP event days in instantaneous forward rates out to ten years maturity, as well as cumulative changes in expected policy rates with 95% confidence intervals.

The shift in forward rates, shown as a solid line, is common to all four decompositions because fitted rates are essentially identical across DTSM estimates. The shift is hump-shaped, with the largest decrease, about -110 bps, occurring at a horizon of three years. At the short end the decrease is about -45 bps for the six-month horizon, and about -80 bps for the twelve-month horizon. At the long end, forward rates decreased by approximately -80 bps. The decreases at the short end are particularly interesting, because the size of the term premium is presumably small at short horizons. Based on this argument most of the drop in the six-month forward rate, and a significant portion of the drop in the one-year rate, would be attributed to decreasing policy expectations. This is confirmed by our model-based decompositions.

Figure 2 contrasts the OLS (left panel) and MU results (right panel). The decompositions at the short end are very similar, with essentially all of the decrease in the six-month rate and a sizable fraction of the decrease in other near-term rates attributed to the expectations component. The difference between OLS and MU is most evident in the resulting decompositions of changes in long-term rates with horizons of five to ten years. Here, the OLS estimates imply a rather small contribution for the expectations component, whereas under the MU estimates around half of the decrease in forward rates is attributed to lower expectations. The figure also visualizes the very large estimation uncertainty underlying these decompositions. For either decomposition, at horizons longer than five years both the forward rate curve and the zero line are within the confidence bands for the changes in expectations. Neither the "all expectations" hypothesis—that these forward rates decreased solely because of lower policy expectations—nor the "all term premia" hypothesis—that expectations did not change and only term premia drove long rates lower—can be rejected.

Figure 3 shows the decompositions resulting from the URP (left panel) and RRP estimates (right panel). Again, the improved decomposition in the right panel leads to a larger role for expectations than the conventional decomposition in the left panel. However, here the main difference between the two is that under restricted risk prices, a larger share of the decrease in short- and medium-term forward rates is attributed to lower expectations, whereas decompositions of changes in long-term forward rates are rather similar. This exemplifies that the economic implications for changes in term premia are different under our MU and RRP estimates. In our view, this model uncertainty for term premium estimation underlines the need to include more than one set of estimates to draw robust conclusions.

Figure 3 also visualizes how imposing risk price restrictions greatly increases the precision of the inference. In the left panel, the confidence bands around the estimated downward shift in expectations are quite large. They are almost as large as for OLS, and are slightly different only because of conceptual and methodological differences between how they are constructed. In the right panel however, for the RRP estimates, the confidence bands are comparably tight and our conclusions about the role of expectations are a lot more precise. In a maximally flexible DTSM, the estimation uncertainty is so large that we cannot really be sure about the relative contribution of changes in policy expectations, but plausible restrictions on risk prices lead to the conclusion that both components, expectations as well as premia, played an important role for lowering rates around LSAP events.

5.3 Day-by-day results

To further drill down into how these shifts in the term structure came about, Tables 6 and 7 show the decompositions of yields changes on each of the eight event days, for the ten-year and five-year yields, respectively. Here we only present point estimates. The tables each have two panels: in the top panel we compare the Kim-Wright decompositions of daily changes to the OLS and MU results, whereas in the bottom panel we compare Kim-Wright to the URP and RRP results. In the bottom three rows of each panel we show total changes over the event days (which correspond to the point estimates in Table 5), as well as cumulative changes and standard deviations of daily changes over the LSAP period.

The tables show in detail how the event days differ from each other. The first three event days, in 2008, show very similar decreases in yields and decompositions. The LSAP event on January 28, 2009, is different in that rates increased due to markets being disappointed by the lack of concrete announcements of Treasury purchases, as discussed earlier. On March 18, 2009, the most dramatic decrease occurred, with the long-term yield falling by half a percentage point. Again, we see how this announcement seems to have had the largest impact on term premia. The last three days showed only minor movements, which when compared to the standard deviations of daily changes are not significant.

The typical pattern is that the estimated contribution of risk-neutral rates to the changes in yields is larger for MU/RRP than for OLS/URP. Notably, the RRP decompositions always have the same signs as the Kim-Wright decompositions. The OLS and MU decompositions, on the other hand, differ from Kim-Wright and RRP in that they imply decreases in the risk-neutral yield on every day, due to the downward movement of the short-end of the term structure. These differences in the daily decompositions between the MU and URP estimates again exemplify the significant model uncertainty and the need to consider alternative specifications to achieve robustness.

5.4 Summary of model-based evidence

The results in this section have shed more light on the question of why long-term yields decreased around LSAP announcements, by presenting decompositions of rate changes into expectations and term premium components. Previous findings in GRRS were based on the Kim-Wright decomposition of long-term rates and seemed to show a large contribution of term premium changes. In addition to the caveat that the decrease in the estimated term premium also included a sizable pricing error component, there are two other important reasons why these results need to be taken with a large grain of salt. First, in terms of point estimates the decomposition of rate changes based on alternative DTSM estimates imply a larger contribution of the expectations component to rate changes around LSAP announcements than the Kim-Wright decomposition. And second, putting confidence intervals around the estimated changes in risk-neutral rates and term premia reveals that large changes in policy expectations around LSAP announcements are consistent with the data. Increasing the precision by restricting the risk pricing of the DTSM leads to a statistically significant role for both the expectations component and the term premium component in lowering yields.

In terms of quantitative conclusions, one would take away from the GRRS study that only $1 - \frac{-71}{-91} \approx 22\%$ of the cumulative decrease in the ten-year yield around LSAP events was due to changing policy expectations. Taking into account the pricing error, this already rises to 30%. Alternative model estimates and the resulting confidence intervals, however, suggest that this is likely a lower bound and that a reasonable point estimate for the contribution of the expectations component would have to be around 40-50%.

6 Conclusion

In this paper we have challenged the conclusion of GRRS that the Fed's LSAP program has mostly worked through a portfolio balance channel. Evidence from different sources, both model-free and based on DTSM estimates, points to a larger role of the signaling channel than previous studies have acknowledged.[25] Our results indicate that changes in the expectations component of long-term interest rates were sizable. Furthermore, we argue that because of secondary effects of signaling and portfolio balance, the relative contribution of expectations to changes in interest rates are conservative estimates of the importance of the signaling channel.

Therefore, it appears that the Fed affected long rates not only by changing the risk premium in long-term interest rates, but instead to an important extent by altering the market expectations of the future path of monetary policy. The plausible interpretation is that through announcing and implementing LSAPs, the Fed signaled to market participants that it would maintain an easy stance for monetary policy for a much longer time than previously anticipated.

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Appendices

A Model specification

We use a discrete-time affine Gaussian DTSM. A vector of N pricing factors, X_t, follows a first-order Gaussian VAR:

$$X_{t+1} = \mu + \Phi X_t + \Sigma \varepsilon_{t+1}, \tag{3}$$

where $\varepsilon_t \overset{iid}{\sim} N(0, I_N)$ and Σ is lower triangular. The short rate, r_t, is an affine function of the pricing factors:

$$r_t = \delta_0 + \delta_1' X_t. \tag{4}$$

The stochastic discount factor (SDF) is of the form

$$-\log(M_{t+1}) = r_t + \frac{1}{2}\lambda_t'\lambda_t + \lambda_t'\varepsilon_{t+1},$$

where the N-dimensional vector of risk prices is affine in the pricing factors,

$$\Sigma\lambda_t = \lambda_0 + \lambda_1 X_t,$$

for N-vector λ_0 and $N \times N$ matrix λ_1. Under these assumptions X_t follows a first-order Gaussian VAR under the risk-neutral pricing measure $\mathbb{Q}$,

$$X_{t+1} = \mu^{\mathbb{Q}} + \Phi^{\mathbb{Q}} X_t + \Sigma\varepsilon^{\mathbb{Q}}_{t+1}, \tag{5}$$

and the prices of risk determine how VAR parameters under the objective measure and the $\mathbb{Q}$ measure are related:

$$\mu^{\mathbb{Q}} = \mu - \lambda_0 \quad \Phi^{\mathbb{Q}} = \Phi - \lambda_1. \tag{6}$$

Furthermore bond prices are exponentially affine functions of the pricing factors:

$$P_t^m = e^{\mathcal{A}_m + \mathcal{B}_m X_t},$$

and the loadings $\mathcal{A}_m = \mathcal{A}_m(\mu^{\mathbb{Q}}, \Phi^{\mathbb{Q}}, \delta_0, \delta_1, \Sigma)$ and $\mathcal{B}_m = \mathcal{B}_m(\Phi^{\mathbb{Q}}, \delta_1)$ follow the recursions

$$\begin{aligned} \mathcal{A}_{m+1} &= \mathcal{A}_m + (\mu^{\mathbb{Q}})'\mathcal{B}_m + \frac{1}{2}\mathcal{B}_m'\Sigma\Sigma'\mathcal{B}_m - \delta_0 \\ \mathcal{B}_{m+1} &= (\Phi^{\mathbb{Q}})'\mathcal{B}_m - \delta_1 \end{aligned}$$

with starting values $\mathcal{A}_0 = 0$ and $\mathcal{B}_0 = 0$. Model-implied yields are determined by $y_t^m = -m^{-1}\log P_t^m = A_m + B_m X_t$, with $A_m = -m^{-1}\mathcal{A}_m$ and $B_m = -m^{-1}\mathcal{B}_m$. Risk-neutral yields, the yields that would prevail if investors were risk-neutral, can be calculated using

$$\tilde{y}_t^m = \tilde{A}_m + \tilde{B}_m X_t, \quad \tilde{A}_m = -m^{-1}\mathcal{A}_m(\mu, \Phi, \delta_0, \delta_1, \Sigma), \quad \tilde{B}_m = -m^{-1}\mathcal{B}_m(\Phi, \delta_1).$$

Risk-neutral yields reflect policy expectations over the lifetime of the bond, $m^{-1}\sum_{h=0}^{m-1} E_t r_{t+h}$, plus a convexity term. The yield term premium is defined as the difference between actual and risk-neutral yields, $ytp_t^m = y_t^m - \tilde{y}_t^m$.

Denote by $\hat{Y}_t$ the vector of observed yields on day t. The number of observed yield maturities is $J = 8$. We take the risk factors X_t to be the first $N = 3$ principal components of observed yields. That is, if W denotes the $N \times J$ matrix with rows corresponding to the first three eigenvectors of the covariance matrix of $\hat{Y}_t$, we have $X_t = W\hat{Y}_t$.

We parameterize the model using the canonical form of Joslin et al. (2011). Thus, the free parameters of the model are $r_\infty^{\mathbb{Q}} = E^{\mathbb{Q}}(r_t)$, the risk-neutral long-run mean of the short rate, $\lambda^{\mathbb{Q}}$, the eigenvalues of $\Phi^{\mathbb{Q}}$, and the VAR parameters μ, Φ, and Σ. For the canonical model this leaves $1 + 3 + 3 + 9 + 6 = 22$ parameters to be estimated, apart from the measurement error specification. To see how $\mu^{\mathbb{Q}}$, $\Phi^{\mathbb{Q}}$, δ_0 and δ_1 are calculated from $(W, \lambda^{\mathbb{Q}}, r_\infty^{\mathbb{Q}}, \Sigma)$ refer to Proposition 2 in Joslin et al. (2011).

B Frequentist estimation

B.1 Ordinary least squares

First we use OLS to obtain the VAR parameters in (3). The mean-reversion matrix Φ is estimated using a demeaned specification without intercept, and then the intercept vector is calculated as $\mu = (I_N - \Phi)\bar{X}$, where $\bar{X}$ is the unconditional sample mean vector. The innovation covariance matrix is estimated from the residuals in the usual way. Denote these OLS estimates by $\hat{\mu}$, $\hat{Phi}$ and $\hat{\Omega}$.

We obtain estimates of the cross-sectional parameters $r_\infty^{\mathbb{Q}}$ and $\lambda^{\mathbb{Q}}$ using the approach of Hamilton & Wu (2010, henceforth HW). As cross-sectional measurements, Y_t^2 in HW's notation, we use the fourth principal component of yields. Write the corresponding eigenvector as the row vector W_2, then we have $Y_t^2 = W_2\hat{Y}_t$. The reduced-form equations in the first step of the HW approach are the VAR for $Y_t^1 = X_t$ and the single measurement equation, which we write here as

$$Y_t^2 = a + bY_t^1 + u_t, \tag{7}$$

for scalar a and row vector b, where u_t is a measurement error. The reduced for parameters are $(\mu, \Phi, \Omega, a, b, \sigma_u^2)$, where $\sigma_u^2 = Var(u_t)$. The second step of the HW approach is to find the structural parameters which result in a close match for the reduced-form parameters, to be found by minimizing a chi-square distance statistic. A simplification is possible because we have exact identification, where the number of reduced-form parameters equals the number of structural parameters. Because the chi-square distance of the HW's second step reaches exactly zero, the weighting matrix is irrelevant and the problem separates into simpler, separate analytical and numerical steps, particularly simple in our case. The parameters for the VAR for Y_t^1 are directly available, namely $(\hat{\mu}, \hat{\Phi}, \hat{\Omega})$, because these parameters are both reduced-form and structural parameters. The parameters for the cross-sectional equation, a and b are found by choosing $r_\infty^{\mathbb{Q}}$ and $\lambda^{\mathbb{Q}}$ so that the distance between the least squares estimates, $(\hat{a}, \hat{b})$, and the model-implied values (W_2A_m, W_2B_m), is small. Here the J-vector A_m and the $J \times N$ matrix B_m contain the model-implied yield loadings. In addition to a dependence on Ω, B_m is determined only by $\lambda^{\mathbb{Q}}$, and A_m depends both on $r_\infty^{\mathbb{Q}}$ and $\lambda^{\mathbb{Q}}$. Therefore we can first search over values for $\lambda^{\mathbb{Q}}$ to minimize the distance between $\hat{b}$ and W_2B_m – we use the Euclidean norm as the distance metric – and then pick $r_\infty^{\mathbb{Q}}$ to minimize the distance between $\hat{a}$ and W_2A_m. Denote the resulting estimates by $\hat{r}_\infty^{\mathbb{Q}}$ and $\hat{\lambda}^{\mathbb{Q}}$.

Because of OLS does most of the work in this estimation procedure, it is very fast even for a daily model. We have 6245 observations and the estimation takes only seconds.

The table shows the OLS estimates in the left column. The estimated intercept and the risk-neutral mean are scaled up by $100n$, where $n = 252$ is the number of periods (business days) per year. Thus these number correspond to annualized percentage points.

The estimated persistence is high: The largest eigenvalue of $\hat{\Phi}$, .999484, is close to one. The half life calculated from $\hat{\Phi}$ of the level factor in response to a level shock is 4.6 years.

	OLS			MU		
$\mu \cdot 100n$	-0.0276	0.0022	0.0076	-0.0223	0.0046	0.0073
Φ	0.9995	-0.0004	0.0251	0.9998	0.0000	0.0249
	-0.0004	0.9982	-0.0168	-0.0003	0.9986	-0.0167
	-0.0001	-0.0001	0.9876	-0.0001	-0.0002	0.9883
λ	0.999484	0.998266	0.987565	0.999770	0.998824	0.988035
$r_\infty^{\mathbb{Q}} \cdot 100n$	12.37			12.38		
$\lambda^{\mathbb{Q}}$	0.999774	0.998069	0.994425	0.999774	0.998069	0.994425

Note: Parameter estimates from frequentist estimation, obtained using OLS and MU. λ are the eigenvalues of Φ, $\lambda^{\mathbb{Q}}$ are the eigenvalues of $\Phi^{\mathbb{Q}}$.

B.2 Median-unbiased estimation

The intuition for our unbiased estimation procedure is to find parameters for the VAR that yield a median of the OLS estimator equal to the OLS estimates from the data. We utilize the inverse bootstrap procedure detailed in our paper Bauer et al. (2011). A residual bootstrap is used for every attempted value of Φ to generate data and find the median of the OLS estimator. In successive iterations, the attempted parameter values are adjusted using an updating scheme based on stochastic approximation, until the median of the OLS estimator on the generated data is sufficiently close $\hat{\Phi}$. Denote the resulting estimate by $\tilde{\Phi}^{unr}$, indicating the unrestricted unbiased estimate.

In working with daily data, where the persistence is extremely high, our unbiased estimation procedure can lead to estimates for Φ with eigenvalues that are either greater than one or below but extremely close to one. This is unsatisfactory because it implies VAR dynamics that are either explosive or display mean reversion that is so slow as to be unnoticeable. Therefore we impose a restriction on our bias-corrected estimates, ensuring that the largest eigenvalue does not exceed *the largest eigenvalue under the pricing measure*. This seems to us a useful and intuitively appealing restriction, since from a finance perspective the far-ahead real-world expectations (under the physical measure) should not be more variable than the far-ahead risk-neutral expectations (under $\mathbb{Q}$).[26] To obtain our bias-corrected estimate of Φ we thus shrink $\tilde{\Phi}^{unr}$ toward $\hat{\Phi}$ using the adjustment procedure of Kilian (1998), until its largest eigenvalue is smaller, in absolute value, than the largest eigenvalue of $\hat{\Phi}$. Denote the restricted estimate by $\tilde{\Phi}$. Of course this is not an unbiased estimate, but nevertheless an estimate with smaller bias than $\hat{\Phi}$.

Based on our estimate $\tilde{\Phi}$ we calculate the intercept $\tilde{\mu}$ and the innovation covariance matrix $\tilde{\Omega}$, as well as the cross-sectional parameters $\tilde{r}_\infty^{\mathbb{Q}}$ and $\tilde{\lambda}^{\mathbb{Q}}$ in analogous fashion as for OLS.

[26]This intuition is also built into other models in the DTSM literature, such as Christensen et al. (2011) where the largest $\mathbb{Q}$-eigenvalue is unity and the VAR is stationary, or Joslin et al. (2010) which explicitly restrict the largest eigenvalue under the two measures to be equal.

B.3 Bootstrap

In order to perform inference about changes in risk-neutral rates and term premia, we construct a bootstrap distribution for the parameters of the DTSM. The focus is on the VAR parameters, since these crucially affect the characteristics of risk-neutral rates and premia. Because the cross-sectional parameters are estimated very precisely and re-estimating them on each bootstrap sample would be computationally costly, we only produce bootstrap distributions for Φ, μ and Ω. As is evident from the estimation results, different values of the VAR parameters essentially have no effect on the estimated values for the cross-sectional parameters, so this simplification is completely innocuous.

By definition of the MU estimate, if we generate bootstrap samples (indexed by $b = 1, \ldots, B$) using $\tilde{\Phi}^{unr}$, the OLS estimator has a median equal to $\hat{\Phi}$. The realizations of the OLS estimator on these samples thus provide a bootstrap distribution around $\hat{\Phi}$, which is conveniently obtained as a by-product of the unbiased estimation procedure. We denote these bootstrap values by $\hat{\Phi}_b$.

To obtain a bootstrap distribution around the MU estimate $\tilde{\Phi}$ we shift the OLS bootstrap distribution by the estimated bias. That is we set $\tilde{\Phi}_b = \hat{\Phi}_b + \tilde{\Phi} - \hat{\Phi}$, with the result that the values of $\tilde{\Phi}_b$ are centered around $\tilde{\Phi}$.

To ensure that the resulting VAR dynamics are stationary for every bootstrap replication, we again apply a stationarity adjustment similar to the one suggested by Kilian (1998). For the MU bootstrap replications, we shrink non-stationary values of $\tilde{\Phi}_b$ toward $\tilde{\Phi}$. We also apply such a stationarity adjustment if values of $\hat{\Phi}_b$ have non-stationary roots, in that case shrinking toward $\hat{\Phi}$. These stationarity adjustments have no impact on the median.

For each value of $\hat{\Phi}_b$ and $\tilde{\Phi}_b$ we calculate the corresponding estimates of μ and Ω as described above.

In terms of computing time, these bootstrap distributions are very quick to obtain. They naturally fall out of the median-unbiased estimation procedure. The only time-consuming task is the stationarity adjustment, which however has very manageable computational cost.

Having available bootstrap distributions for the VAR parameters allows us to obtain bootstrap distributions for every object of interest, for example for the ten-year risk-neutral rate at a specific point in time, or for the cumulative changes in the ten-year yield term premium over a set of days. While our methodology is in some respects *ad hoc*, it has the unique advantage of enabling us to account in a relatively straight-forward and computationally efficient way for the underlying estimation uncertainty of our inference about policy expectations and term premia.

C Bayesian estimation

We employ Markov chain monte carlo (MCMC) methods to perform Bayesian estimation.[27] Specifically, we obtain a sample from the joint posterior distribution of the model parameters using a block-wise Metropolis-Hastings (MH) algorithm. Other papers that have used MCMC methods for estimation of DTSMs include Ang et al. (2007), Ang et al. (2009) and Chib and

[27] On estimation of asset pricing models using MCMC see Johannes and Polson (2009).

Ergashev (2009). Our methodology is heavily borrowing from Bauer (2011), where the focus is specifically on restrictions on risk pricing.

First we estimate the canonical model, and then, in a second step, we estimate over-identified models with zero restrictions on elements of λ_0 and λ_1. For this purpose it is convenient to parameterize the model in terms of $(\lambda_0, \lambda_1, \Omega, r_\infty^\mathbb{Q}, \lambda^\mathbb{Q})$.

The prior for the elements of λ_0 and λ_1 is independent normal, with mean zero and standard deviation .01. This prior cannot be too diffuse because that would affect the model selection exercise in the direction of favoring parsimonious models (the Lindley-Bartlett paradox; see Bartlett, 1957). In light of the magnitude of the frequentist estimates that we have obtained, this prior is not overly informative.

The priors for Ω and $r_\infty^\mathbb{Q}$ are taken to be completely uninformative. The elements of $\lambda^\mathbb{Q}$ are a priori assumed to be independent, uniformly distributed over the unit interval.

For the measurement equations we slightly deviate from our previous specification and simply take all J yields individually as the measurements, as in Joslin et al. (2011). The measurement errors are assumed to have equal variance, denoted by σ_u^2. Notably, there are only $J - N$ independent linear combinations of these measurement errors, because N linear combinations of yields, namely the first three principal components, are priced perfectly by the model. We specify the prior for σ_u^2 to be uninformative.

C.1 Maximally-flexible model

Denote the parameters of the model as $\theta = (\lambda_0, \lambda_1, \Omega, r_\infty^\mathbb{Q}, \lambda^\mathbb{Q}, \sigma_u^2)$. There are five blocks of parameters which we draw successively in our MCMC algorithm.

The likelihood of the data factors into the likelihood of the risk factors, denoted by $P(X|\theta)$, and the cross-sectional likelihood, written as $P(Y|X, \theta)$ – X stands for all observations of X_t and Y stands for the data, i.e., all observations of $\hat{Y}_t$. The factor likelihood function is simply the conditional likelihood function of a Gaussian VAR.[28] It depends on the VAR parameters, which in this parameterization are determined by $(\lambda_0, \lambda_1, \Omega, r_\infty^\mathbb{Q}, \lambda^\mathbb{Q})$. The cross-sectional likelihood function depends on $(\Omega, r_\infty^\mathbb{Q}, \lambda^\mathbb{Q}, \sigma_u^2)$. Thus we have

$$\begin{aligned} P(Y|\theta) &= P(X|\theta) \cdot P(Y|X, \theta) \\ &= P(X|\lambda_0, \lambda_1, \Omega, r_\infty^\mathbb{Q}, \lambda^\mathbb{Q}) \cdot P(Y|X, \Omega, r_\infty^\mathbb{Q}, \lambda^\mathbb{Q}, \sigma_u^2). \end{aligned}$$

Our sampler allows us to draw from the joint posterior distribution

$$P(\theta|Y) \propto P(Y|\theta) \cdot P(\theta),$$

where $P(\theta)$ denotes the joint prior over all model parameters, despite the fact that this distribution is only known up to a normalizing constant. This, of course, is the underlying idea of essentially all MCMC algorithms employed in Bayesian statistics.

As starting values of the chain, we use OLS estimates for μ, Φ, and Ω, the sample mean of all yields for $r_\infty^\mathbb{Q}$, the eigenvalues of $\hat{\Phi}$ for $\lambda^\mathbb{Q}$, and a tenth of the standard deviation of all yields for σ_u (since yield pricing errors have smaller variance than yields).

[28] We always condition on the first observation.

We run the sampler for 50,000 iterations. We discard the first half as a burn-in sample and then take every 50'th iteration of the remaining sample. This constitutes our MCMC sample which approximately comes from the joint posterior distribution of the parameters.

To ensure that the MCMC chain has converged, we closely inspect trace plots and make sure that our starting values have no impact on the results. In addition we calculate convergence diagnostics of the type reviewed in Cowles and Carlin (1996).

C.1.1 Drawing (λ_0, λ_1)

Every element of λ_0 and λ_1 is drawn independently, iterating through them in random order, using a random walk (RW) MH step. For the conditional posterior distribution of these parameters we have

$$\begin{aligned} P(\lambda_0, \lambda_1 | \theta_-, X, Y) &\propto P(Y|\theta, X) P(X|\theta) P(\theta) \\ &\propto P(X|\theta) P(\theta), \end{aligned}$$

where θ_- denotes all parameters except for λ_0 and λ_1. The second line follows because the likelihood of the data for given risk-neutral dynamics does not depend on the prices of risk, as noted above. For each parameter, I use a univariate random walk proposal with t_2-distributed innovations that are multiplied by scale factors to tune the acceptance probabilities to be in the range of 20%-50%. After obtaining the candidate draw, the restriction that the physical dynamics are non-explosive is checked, and the draw is rejected if the restriction is violated. Otherwise the acceptance probability for the draw is calculated as the minimum of one and the ratio of the factor likelihood times the ratio of the priors for the new draw relative to the old draw.

C.1.2 Drawing Ω

For the conditional posterior of Ω we have

$$P(\Omega | \theta_-, X, Y) \propto P(Y|\theta, X) P(X|\theta) P(\theta)$$

where θ_- denotes all parameters except Ω. Since we need successive draws of Ω to be close to each other—otherwise the acceptance probabilities will be too small—independence Metropolis is not an option. Element-wise RW MH does not work particularly well either. A better alternative in terms of efficiency and mixing properties is to draw the entire matrix Ω in one step. I choose a proposal density for Ω that is Inverse-Wishart (IW) with mean equal to the value of the previous draw and scale adjusted to tune the acceptance probability, which is equal to

$$\alpha(\Omega^{(g-1)}, \Omega^{(g)}) = \min \left\{ \frac{P(X|\Omega^{(g)}, \theta_-) P(\Omega^{(g)}, \theta_-) q(\Omega^{(g)}, \Omega^{(g-1)})}{P(X|\Omega^{(g-1)}, \theta_-) P(\Omega^{(g-1)}, \theta_-) q(\Omega^{(g-1)}, \Omega^{(g)})}, 1 \right\},$$

where g is the iteration. Here $q(A, B)$ denotes the transition density, which in this case is the density of an IW distribution with mean A. The ratio of priors is equal to one since we assume an uninformative prior, unless the draw would imply non-stationary VAR dynamics, in which case the prior ratio is zero. The reason that some draws of Ω can imply non-stationary VAR dynamics is that in the JSZ normalization, the value of Ω matters for the mapping from $r^{\mathbb{Q}}_{\infty}$ and $\lambda^{\mathbb{Q}}$ into $\mu^{\mathbb{Q}}$ and $\Phi^{\mathbb{Q}}$, which together with λ_0 and λ_1 determine the VAR parameters.

C.1.3 Drawing $r^{\mathbb{Q}}_{\infty}$

Both factor likelihood and cross-sectional likelihood depend on $r^{\mathbb{Q}}_{\infty}$, thus

$$P(r^{\mathbb{Q}}_{\infty}|\theta_{-}, X, Y) \propto P(Y|\theta, X)P(X|\theta)P(\theta),$$

where θ_{-} denotes all parameters except $r^{\mathbb{Q}}_{\infty}$. We use a RW MH step, with proposal innovations from a t-distribution with 2 degrees of freedom, multiplied by a scaling parameter to tune the acceptance probabilities. The ratio of priors is equal to one, because we have a non-informative prior, if the implied VAR dynamics are stationary and zero otherwise, in which case the prior ratio is zero. The acceptance probability is equal to the minimum of one and the product of prior ratio, the ratio of cross-sectional likelihoods, and the ratio of factor likelihoods.

C.1.4 Drawing $\lambda^{\mathbb{Q}}$

Again both likelihoods depend on this parameter, so we have

$$P(\lambda^{\mathbb{Q}}|\theta_{-}, X, Y) \propto P(Y|\theta, X)P(X|\theta)P(\theta),$$

where θ_{-} denotes all parameters except $\lambda^{\mathbb{Q}}$. We draw all three elements in one step, using a RW proposal with independent t-distributed innovations, each with 2 degrees of freedom and multiplied to tune acceptance probabilities. The prior ratio is one if all three proposed values are within the unit interval and the implied VAR dynamics are stationary, and zero otherwise. We implement the requirement that the three elements of $\lambda^{\mathbb{Q}}$ are in descending order by rejecting draws that would change this ordering. Again the acceptance probability is equal to the minimum of one and the product of prior ratio, the ratio of cross-sectional likelihoods, and the ratio of factor likelihoods.

C.1.5 Drawing σ^2_u

In this block the conditional posterior distribution of σ^2_u is known in close form. The problem of drawing this error variance corresponds to drawing the error variance of a pooled regression. The condition posterior distribution is inverse gamma, because an uninformative prior on this parameter is conjugate.

C.2 Restricted risk prices

We closely follow the methodology laid out in Bauer (2011, Appendix C), where Gibbs variable selection (Dellaportas et al., 2002) is applied to the context of DTSM estimation. Let λ denote a vector stacking all elements of λ_0 and λ_1. For the purpose of model selection, we introduce a vector of indicator variables, γ, that describes which risk price parameters, i.e., which elements of λ, are restricted to zero. The parameters of the model are now $(\gamma, \theta) = (\gamma, \lambda, \Omega, r^{\mathbb{Q}}_{\infty}, \lambda^{\mathbb{Q}}, \sigma^2_u)$. The goal of course is to sample from the joint posterior

$$P(\gamma, \theta|Y) \propto P(Y|\gamma, \theta)P(\theta|\gamma)P(\gamma).$$

The likelihood $P(Y|\gamma, \theta)$ is the product of factor likelihood and cross-sectional likelihood, as before. The difference is that here it is evaluated by treating those elements of λ as zero for which the corresponding element in γ is zero. The priors for the parameters conditional on the model indicator $P(\theta|\gamma)$ are specified as before. The prior for the model indicators $P(\gamma)$ is such that all elements are independent Bernoulli random variables with .5 prior probability.

The parameters $\Omega, r^{\mathbb{Q}}_{\infty}, \lambda^{\mathbb{Q}}$, and σ^2_u are drawn exactly as in the estimation algorithm for the URP model. What is different here is we sample the vector indicating the model specification, γ, and the parameter vector γ, which all models have in common.

For each iteration g of the MCMC sampler, we draw the block (γ, λ) by drawing pairs (γ_i, λ_i), going through the $N + N^2 = 12$ risk price parameters in random order.

C.2.1 Drawing λ_i

For each pair we first draw $\lambda_i^{(g)}$ conditional on $\gamma_i^{(g-1)}$ and all other parameters. If the parameter is currently included (unrestricted), i.e., if $\gamma_i = 1$, we draw from the conditional posterior. If the parameter is currently restricted to zero ($\gamma_i = 0$) the data is not informative about the parameter and we draw from a so-called pseudo-prior (Carlin and Chib, 1995; Dellaportas et al., 2002). That is,

$$\begin{aligned} P(\lambda_i|\lambda_{-i}, \gamma_i = 1, \gamma_{-i}, \theta_{-}, X, Y) &\propto P(X|\theta, \gamma)P(\lambda_i|\gamma_i = 1) && (8) \\ P(\lambda_i|\lambda_{-i}, \gamma_i = 0, \gamma_{-i}, \theta_{-}, X, Y) &\propto P(\lambda_i|\gamma_i = 0), && (9) \end{aligned}$$

where θ_{-} denotes all parameters in θ other than λ, and λ_{-i} (γ_{-i}) contains all elements of λ (γ) other than λ_i (γ_i).[29] I assume prior conditional independence of the elements of λ given γ, and the prior for each price of risk parameter, $P(\lambda_i|\gamma_i = 1)$, is taken to be standard normal. The conditional posterior in (8) is not known analytically and we use a RW MH step to obtain the draws, with a fat-tailed RW proposal and scaling factor as before. For the pseudo-prior $P(\lambda_i|\gamma_i = 0)$ we use a normal distribution, with moments corresponding to the marginal posterior moments from our estimation of the URP model.

C.2.2 Drawing γ_i

When we get to the second element of the pair, the indicator γ_i, the conditional posterior distribution is known and we can directly sample from it without MH step. It is Bernoulli, and the success probability is easily calculated based on the ratio:

$$q = \frac{P(\gamma_i = 1|\gamma_{-i}, \theta, X, Y)}{P(\gamma_i = 0|\gamma_{-i}, \theta, X, Y)} = \frac{P(X|\gamma_i = 1, \gamma_{-i}, \theta)}{P(X|\gamma_i = 0, \gamma_{-i}, \theta)} \frac{P(\lambda_i|\gamma_i = 1)}{P(\lambda_i|\gamma_i = 0)} \frac{P(\gamma_i = 1, \gamma_{-i})}{P(\gamma_i = 0, \gamma_{-i})}. \quad (10)$$

[29] These conditional distributions parallel the ones in equations (9) and (10) of Dellaportas et al. (2002).

The first factor in the numerator and the denominator is the factor likelihood. The second factor in the numerator is the parameter prior, and in the denominator it is the pseudo-prior. The third factor cancels out, since we use an independent, uninformative prior with prior inclusion probability of each element of 0.5, putting equal weight on $\gamma_i = 1$ and $\gamma_i = 0$. The conditional posterior probability for drawing $\gamma_i = 1$ is given by $q/(q+1)$.[30]

C.2.3 Bayesian model averaging

As output from the MCMC algorithm we have available a sample that comes approximately from the joint posterior distribution of (γ, θ). When we want to calculate the posterior distribution of any object of interest, such as for the value of the ten-year term premium on a certain day, we simply calculate it for every iteration of the MCMC sample. In each iteration that we use from this sample – as before we discard the first half and then only use every 50'th iteration – different elements might be restricted to zero. We are effectively sampling across models and parameter values, that we are taking into account model uncertainty in our posterior inference. This technique is called Bayesian model averaging: the model specification is effectively averaged out, and the inference is not conditional on a specific model but instead taking into account model uncertainty.

[30] A subtlety, which is ignored in the above notation, is that the joint prior $P(\gamma, \theta)$ imposes that the physical dynamics resulting from any choice of γ and λ_1 can never be explosive. This is easily implemented in the algorithm: If including a previously excluded element would lead to explosive dynamics then we simply do not include it, i.e., I set $\gamma_i = 0$, and vice versa.

Table 1: LSAP announcements

Date	Announcement	Description
25 November 2008	initial LSAP announcement	Federal Reserve announces purchases of up to $100 billion in agency debt and up to $500 billion in agency MBS
1 December 2008	Chairman's speech	Chairman states that the Federal Reserve "could purchase longer-term Treasury securities [...] in substantial quantities"
16 December 2008	FOMC statement	Statement indicates that the FOMC is considering expanding purchases of agency securities and initiating purchases of Treasury securities
28 January 2009	FOMC statement	Statement indicates that the FOMC "is prepared to purchase longer-term Treasury securities."
18 March 2009	FOMC statement	Statement announces purchases "up to an additional $750 billion of agency [MBS]," $100 billion in agency debt, and $300 billion in Treasury securities.
12 August 2009	FOMC statement	Statement drops "up to" language and announces slowing pace for purchases of Treasury securities.
23 September 2009	FOMC statement	Statement drops "up to" language for purchases of agency MBS and announces gradual slowing pace for purchases of agency debt and MBS.
4 November 2009	FOMC statement	Statement declares that the FOMC would purchase "around $175 billion of agency debt."

Table 2: Changes around selected policy actions, 2001-2003

Date	Change in FFR target	Change in ten-year yield Actual	Fitted	YRN	YTP
01/31/2001	-50	-4	-3	-3	0
03/20/2001	-50	-3	-3	-2	-1
04/18/2001	-50	-6	-6	-5	-1
08/21/2001	-25	-3	-2	-2	-1
10/02/2001	-50	-2	-1	-2	1
11/06/2001	-50	-2	-2	-3	1
12/11/2001	-25	-3	-4	-2	-2
05/07/2002	0	0	-1	-1	0
06/26/2002	0	-12	-11	-4	-7
08/13/2002	0	-9	-8	-4	-5
09/24/2002	0	-1	-2	-1	-1
11/06/2002	-50	-3	-2	-3	1
05/06/2003	0	-8	-9	-3	-6
Cumulative	-350	-56	-54	-35	-21

Note: Changes, in basis points, in fed funds rate (FFR) target, actual ten-year yield, fitted yield, risk-neutral yield, and yield term premium based on the Kim-Wright estimates, on FOMC announcement days with a negative change in the risk-neutral yield during the 2001-2003 easing cycle.

Table 3: Changes in futures-implied policy paths around LSAP announcements

Date	1m	6m	1y	2y	3y	avg. 3y	3y yld.	diff.
11/25/2008	-5	-6	-10	-13	-22	-12	-18	-7
12/1/2008	1	-4	-7	-18	-21	-11	-16	-5
12/16/2008	-17	-16	-12	-11	-16	-12	-13	-1
1/28/2009	0	0	5	11	15	7	8	0
3/18/2009	-1	-4	-11	-10	-11	-8	-35	-27
8/12/2009	-1	-6	-8	-3	-1	-4	-1	3
9/23/2009	0	-3	-5	-6	-2	-4	-4	0
11/4/2009	0	-2	-1	1	5	1	0	-1
Total	-23	-40	-49	-49	-53	-43	-80	-37
Cum. changes	-33	-27	28	107	122	62	24	-38
Std. dev.	1	2	5	8	9	6	7	4

Note: Changes, in basis points, of futures-implied policy paths at fixed horizons. Paths are linearly interpolated if no futures contract is available for required horizon. The last three columns show the change of the average policy path over the next three years, the change in the three-year zero coupon yield, and the difference between the yield change and the change in the average policy path. Also shown are cumulative changes and standard deviations of daily changes over the period 11/24/08-12/30/09.

Table 4: Changes in yields, OIS rates, and spreads around LSAP announcements

	OIS rates				yields				yield-OIS spreads			
Date	1y	2y	5y	10y	1y	2y	5y	10y	1y	2y	5y	10y
11/25/2008	-7	-14	-25	-28	-9	-14	-22	-21	-2	-1	2	7
12/1/2008	-5	-13	-21	-19	-6	-12	-21	-22	-1	1	-1	-2
12/16/2008	-17	-15	-29	-32	-8	-11	-16	-17	9	5	12	14
1/28/2009	4	6	11	14	-1	5	10	12	-4	-1	-1	-2
3/18/2009	-5	-12	-27	-38	-17	-26	-47	-52	-12	-14	-20	-14
8/12/2009	-2	-1	-2	1	0	-1	1	6	1	0	3	5
9/23/2009	-3	-5	-6	-5	-2	-4	-4	-2	1	1	3	3
11/4/2009	-2	-3	1	5	-1	-1	3	7	1	2	2	2
Total	-37	-58	-97	-102	-45	-65	-97	-89	-8	-7	0	14
Cum. changes	-22	-8	19	59	-45	2	31	16	-23	10	11	-43
Std. dev.	3	5	8	10	4	6	8	9	2	3	3	4

Note: Changes, in basis points, in OIS rates, zero-coupon yields, and yield-OIS spreads around LSAP announcements. Also shown are cumulative changes and standard deviations of daily changes over the period 11/24/08-12/30/09.

Table 5: Decomposition of LSAP effect on long-term yields

	ten-year yield			five-year yield		
	yield	YRN	YTP	yield	YRN	YTP
actual	-89			-97		
Kim-Wright	-102	-31	-71	-94	-30	-64
OLS	-93	-33	-60	-93	-40	-53
OLS UB		-90	-3		-85	-9
OLS LB		9	-102		0	-94
MU	-93	-46	-47	-93	-48	-46
MU UB		-141	48		-112	19
MU LB		0	-93		-3	-90
URP	-94	-31	-62	-93	-39	-53
URP UB		-71	-23		-69	-24
URP LB		-7	-86		-14	-78
RRP	-94	-36	-58	-93	-48	-44
RRP UB		-53	-40		-59	-33
RRP LB		-29	-65		-41	-51

Note: Alternative decompositions of yield changes, in basis points, on announcement days. The first line shows actual yield changes, the following lines show changes in fitted yields, risk-neutral yields (YRN) and yield term premia (YTP) for alternative DTSM estimates. Also shown are upper bounds (UB) and lower bounds (LB) for the change in the term premium, based on bootstrap confidence intervals (for OLS and MU) or quantiles of posterior distributions (for RP and RRP).

Table 6: Ten-year yield, decompositions of day-by-day changes

		Kim-Wright			OLS			MU		
Date	act.	yld.	YRN	YTP	yld.	YRN	YTP	yld.	YRN	YTP
11/25/2008	-21	-24	-7	-17	-23	-6	-17	-23	-8	-15
12/1/2008	-22	-24	-7	-17	-22	-5	-17	-22	-7	-15
12/16/2008	-17	-18	-7	-12	-17	-5	-13	-17	-6	-11
1/28/2009	12	12	3	9	13	-2	15	13	-2	15
3/18/2009	-52	-56	-16	-40	-53	-7	-46	-53	-10	-43
8/12/2009	6	4	1	3	5	-3	8	5	-4	8
9/23/2009	-2	-2	-1	-1	-2	-3	1	-2	-4	3
11/4/2009	7	7	2	5	7	-3	10	7	-4	11
Total	-89	-102	-31	-71	-93	-33	-60	-93	-46	-47
Cum. changes	16	24	-7	31	30	-10	40	30	-12	42
Std. dev.	9	9	3	7	9	4	9	9	5	9

		Kim-Wright			URP			RRP		
Date	act.	yld.	YRN	YTP	yld.	YRN	YTP	yld.	YRN	YTP
11/25/2008	-21	-24	-7	-17	-23	-6	-17	-23	-9	-14
12/1/2008	-22	-24	-7	-17	-22	-6	-17	-22	-9	-14
12/16/2008	-17	-18	-7	-12	-17	-5	-13	-17	-7	-10
1/28/2009	12	12	3	9	13	-1	14	13	5	8
3/18/2009	-52	-56	-16	-40	-54	-9	-44	-54	-21	-32
8/12/2009	6	4	1	3	5	-2	7	5	2	3
9/23/2009	-2	-2	-1	-1	-2	-3	1	-2	-1	-1
11/4/2009	7	7	2	5	7	-2	9	7	2	4
Total	-89	-102	-31	-71	-94	-34	-60	-94	-37	-56
Cum. changes	16	24	-7	31	30	-7	37	30	10	20
Std. dev.	9	9	3	7	9	3	8	9	4	6

Note: Decompositions of yield changes, in basis points, on each LSAP announcement day. The first column shows actual yield changes, the following columns show changes in fitted yields, risk-neutral yields (YRN) and yield term premia (YTP) for alternative DTSM estimates. Also shown are total changes over all events, as well as cumulative changes and standard deviations of daily changes over the period 11/24/08 - 12/30/09.

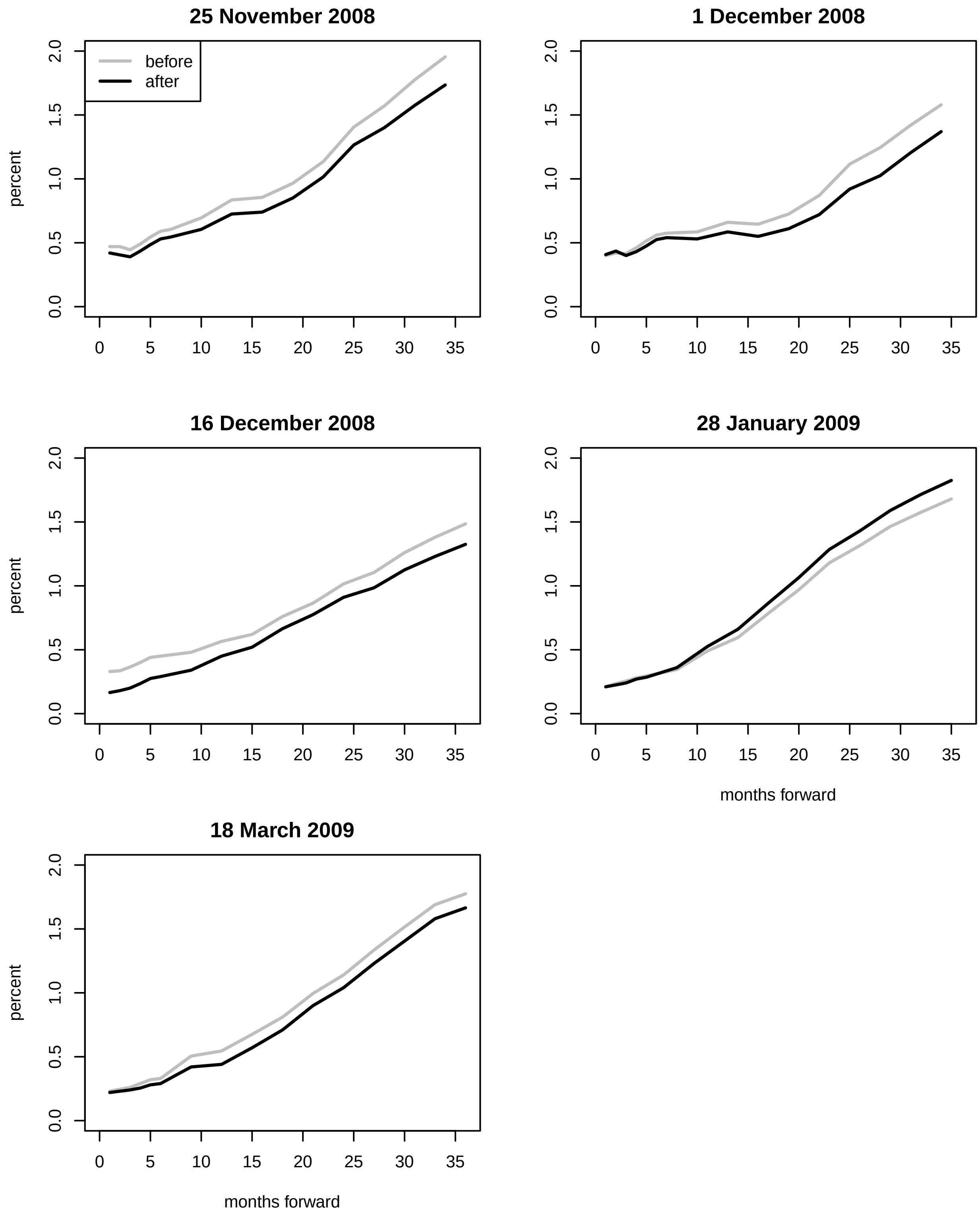
Table 7: Five-year yield, decompositions of day-by-day changes

		Kim-Wright			OLS			MU		
Date	act.	yld.	YRN	YTP	yld.	YRN	YTP	yld.	YRN	YTP
11/25/2008	-22	-22	-7	-15	-21	-7	-15	-21	-8	-13
12/1/2008	-21	-21	-6	-15	-21	-6	-15	-21	-7	-14
12/16/2008	-16	-16	-6	-10	-16	-5	-11	-16	-6	-10
1/28/2009	10	9	3	7	9	-2	12	9	-3	12
3/18/2009	-47	-47	-13	-34	-46	-8	-39	-46	-9	-37
8/12/2009	1	2	0	2	2	-4	6	2	-4	7
9/23/2009	-4	-3	-1	-2	-3	-4	1	-3	-5	1
11/4/2009	3	4	1	3	4	-4	8	4	-5	8
Total	-97	-94	-30	-64	-93	-40	-53	-93	-48	-46
Cum. changes	31	20	-10	29	19	-14	33	19	-16	35
Std. dev.	8	8	3	6	8	5	7	8	5	7

		Kim-Wright			URP			RRP		
Date	act.	yld.	YRN	YTP	yld.	YRN	YTP	yld.	YRN	YTP
11/25/2008	-22	-22	-7	-15	-21	-7	-14	-21	-11	-10
12/1/2008	-21	-21	-6	-15	-21	-6	-14	-21	-10	-10
12/16/2008	-16	-16	-6	-10	-16	-6	-11	-16	-9	-8
1/28/2009	10	9	3	7	9	-1	10	9	5	5
3/18/2009	-47	-47	-13	-34	-46	-10	-36	-46	-24	-22
8/12/2009	1	2	0	2	2	-3	5	2	1	1
9/23/2009	-4	-3	-1	-2	-3	-4	0	-3	-2	-1
11/4/2009	3	4	1	3	4	-3	7	4	1	2
Total	-97	-94	-30	-64	-93	-40	-53	-93	-49	-44
Cum. changes	31	20	-10	29	19	-11	30	19	7	24
Std. dev.	8	8	3	6	8	4	7	8	4	5

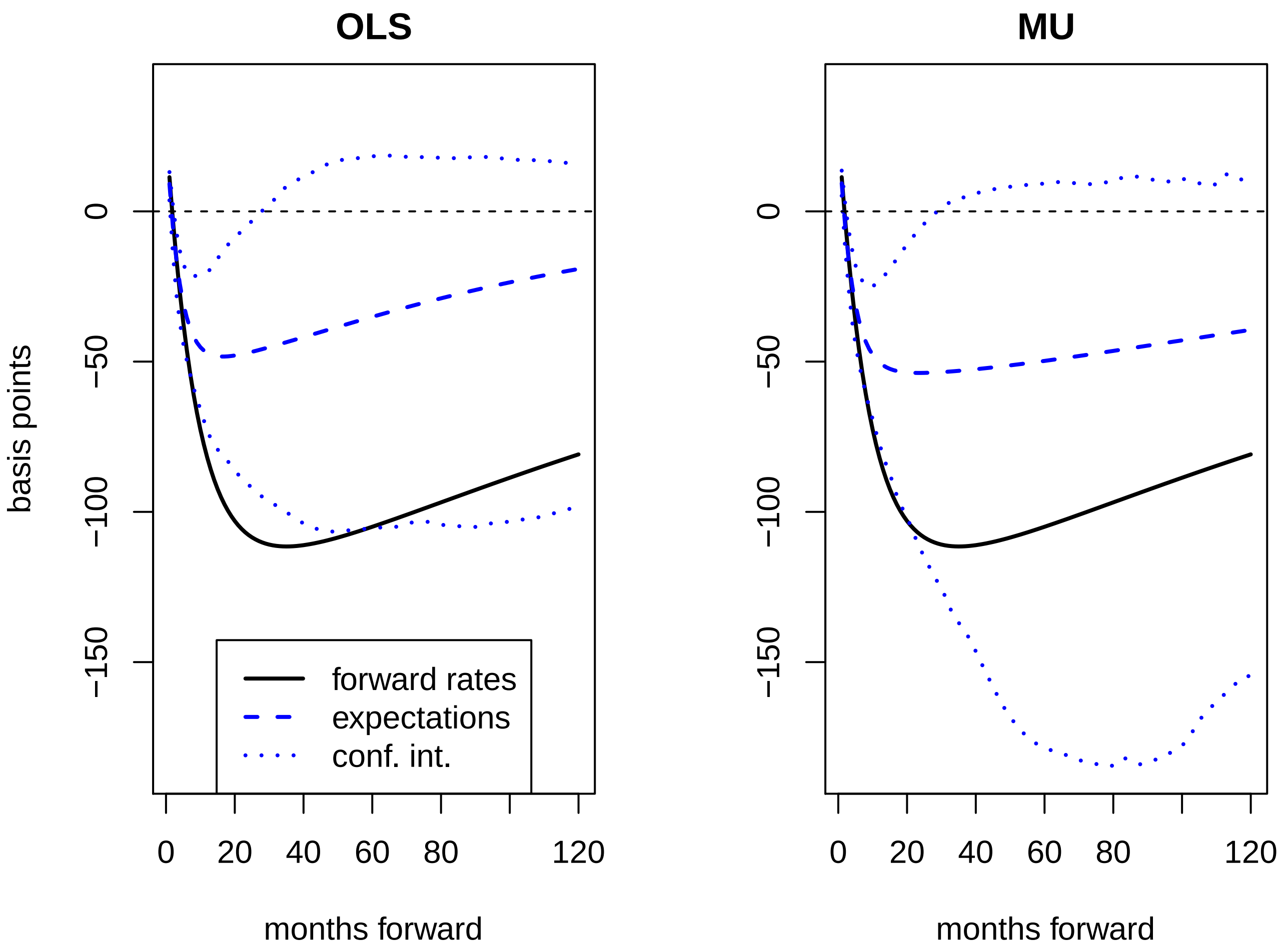
Note: See Table 6.

Figure 1: Shifts of futures-implied policy paths around key LSAP dates



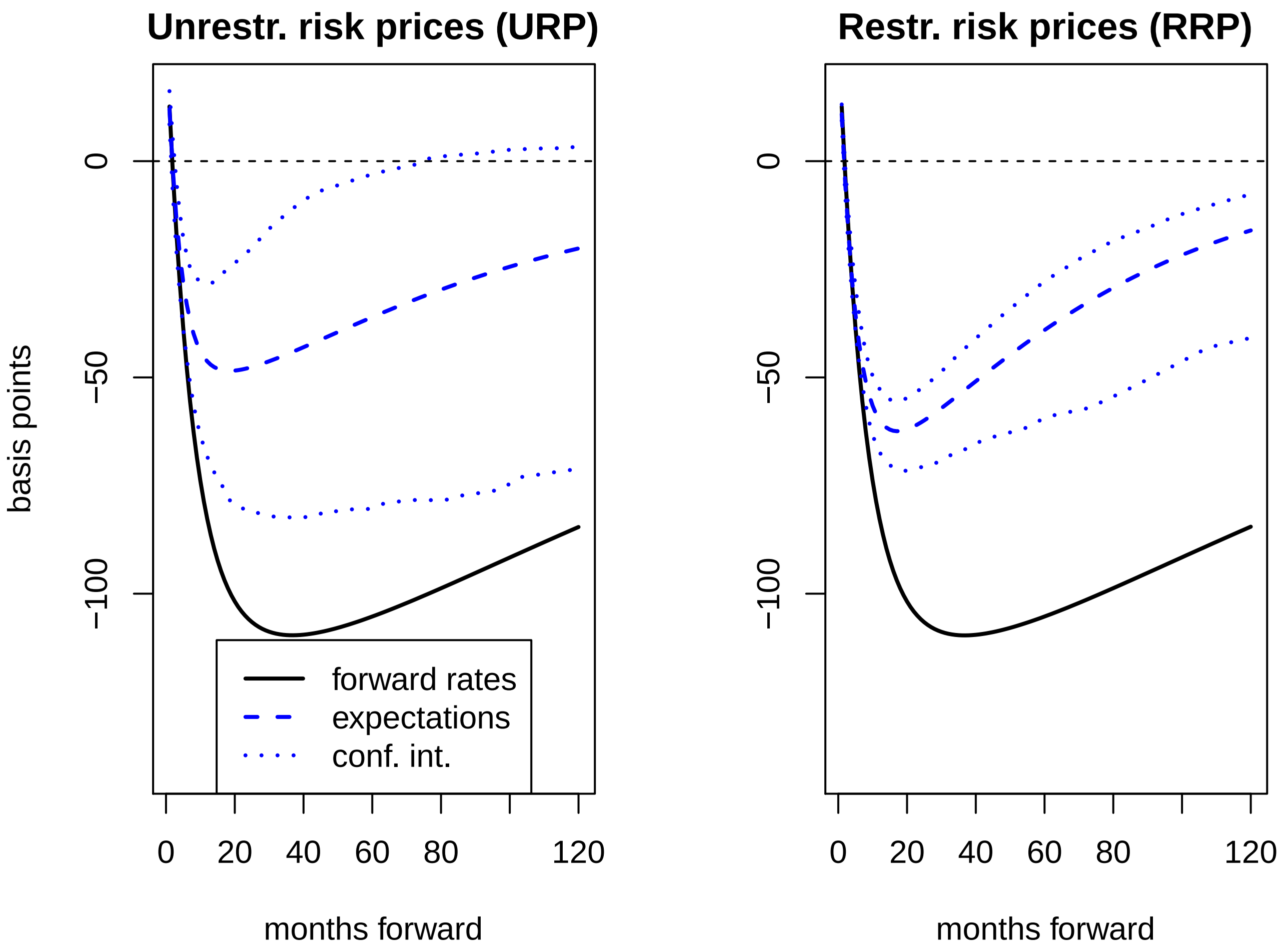
Note: Policy paths before and after five key LSAP announcements that are implied by market rates of federal funds futures and Eurodollar futures. For details on calculation refer to main text.

Figure 2: Shift of forward curve and policy path: OLS vs. MU



Note: Cumulative changes, in basis points, on announcement days in fitted forward rates (solid line) and policy expectations (dashed line) together with 95% confidence intervals for changes in expectations (dotted lines). Left panel shows decomposition based on OLS estimates, right panel for MU estimates.

Figure 3: Shift of forward curve and policy path: URP vs. RRP



Note: Cumulative changes, in basis points, on announcement days in fitted forward rates (solid line) and policy expectations (dashed line) together with 95% confidence intervals for changes in expectations (dotted lines). Left panel shows decomposition based on URP estimates, right panel for RRP estimates.