

Have We Underestimated the Likelihood and Severity of Zero Lower Bound Events?

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Abstract

Prior to the financial crisis, most economists probably did not view the zero lower bound (ZLB) as a major problem for central banks. Using a range of structural and statistical models, we find that previous research understated the ZLB threat by ignoring uncertainty about model parameters and latent variables; focusing too much on the Great Moderation experience; and relying on structural models whose dynamics cannot generate sustained ZLB episodes. Our analysis also suggests that the Federal Reserve's asset purchases, while materially improving macroeconomic conditions, did not prevent the ZLB constraint from having first-order adverse effects on real activity and inflation.

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I. Introduction

The zero lower bound (ZLB) on nominal interest rates limits the ability of central banks to add monetary stimulus to offset adverse shocks to the real economy and to check unwelcome disinflation. The experience of Japan in the 1990s motivated a great deal of research on both the macroeconomic consequences of the ZLB and on monetary policy strategies to overcome these effects. Economic theory has provided important insights about both the dynamics of the economy in the vicinity of the ZLB and possible policy strategies for mitigating its effects. But theory alone cannot provide a quantitative assessment of the practical importance of the ZLB threat, which depends critically on the frequency and degree to which the lower bound constrains the actions of the central bank as it seeks to stabilize real activity and inflation, thereby impinging on the unconstrained variability and overall distribution of the nominal funds rate that would otherwise arise. These factors in turn depend on the expected magnitude and persistence of adverse shocks to the economy; the dynamic behavior of real activity, inflation, and expectations; and the monetary policy strategy followed by the central bank, including its inflation target. (The latter factor plays a key role in ZLB dynamics, because the mean of the unconstrained distribution of the nominal funds rate equals the inflation target plus the economy's equilibrium real short-term rate of interest.) The quantitative evaluation of these factors requires one to use a model of the economy with sound empirical foundations.

Previous research was generally sanguine about the practical risks posed by the ZLB, as long as the central bank did not target too low an inflation rate. Reifschneider and Williams (2000) used stochastic simulations of the Federal Reserve's large-scale rational-expectations macroeconomic model, FRB/US, to evaluate the frequency and duration of episodes when policy was constrained by the ZLB. They found that if monetary policy followed the prescriptions of the standard Taylor (1993) rule with an inflation target of 2 percent, the federal funds rate would be near zero about 5 percent of the time and the "typical" ZLB episode would last four quarters. Their results also suggested that the ZLB would have relatively minor effects on macroeconomic performance under these policy assumptions. In addition, they found that monetary policy rules with larger responses to output and inflation than the standard Taylor rule encountered the ZLB more frequently, with relatively minor macroeconomic consequences as long as the inflation target did not fall too far below 2 percent. Other studies reported similar findings although they, if anything, tended to find even smaller effects of the ZLB; see, for

example, Coenen (2003), Schmitt-Grohe and Uribe (2007), and other papers in the reference list. Finally, research in this area suggested that monetary policies could be crafted that would greatly mitigate any effect of the ZLB. Proposed strategies to accomplish this goal included responding more aggressively to economic weakness and falling inflation, or promising to run an easier monetary policy for a time once the ZLB is no longer binding (see Reifschneider and Williams (2002) and Eggertsson and Woodford (2003) and references therein).

The events of the past few years call into question the reliability of those analyses. The federal funds rate has been at its effective lower bound for two years, and futures data suggest that market participants currently expect it to remain there until late 2011. The current episode thus is much longer than those typically generated in the simulation analysis of Reifschneider and Williams (2000). The same study suggested that recessions as deep as what we are now experiencing would be exceedingly rare—on the order of once a century or even less frequent. Of course, recent events could be interpreted as just bad luck—after all, five hundred year floods do eventually happen. Alternatively, they could be flashing a warning sign that previous estimates of ZLB effects significantly understated the inherent volatility of the economy that arises from the interaction of macroeconomic disturbances and the economy’s dynamics.

The goal of this paper is to examine and attempt to answer three key questions regarding the frequency and severity of ZLB episodes using a range of econometric models, including structural and time series models. First, how surprising have recent events been and what lessons do we take for the future in terms of the expected frequency, duration, and magnitude of ZLB episodes? Second, to what extent have alternative monetary policy actions been effective at offsetting the effects of the ZLB in the current U.S. situation? And, finally, how severely did the ZLB bind during the recent crisis after taking account of the effects of asset purchases?

In examining these questions, we employ a variety of structural and statistical models, rather than using a single structural model as was done in most past research. Research on the ZLB has generally focused on results from structural models because many of the issues in this field have monetary policy strategy and expectational dynamics at their core. For example, studies have typically employed structural models run under rational expectations to assess expected macro performance under, say, different inflation targets or under price-level targeting in order to ensure consistency between the central bank’s actions and private agents’ beliefs. In this paper, we use two empirical macroeconomic models developed at the Board of Governors—

one a more traditional large-scale model and the other an optimization-based dynamic stochastic general equilibrium model—to analyze the extent that the ZLB is likely to constrain policies. In addition, we use the Smets and Wouters (2007) estimated DSGE model. Because these models have strong empirical foundations, they should provide informative quantitative estimates of the risks posed by the ZLB.

However, a potential drawback to using structural models to quantify the likelihood of the risks confronting policymakers is that such models impose stringent constraints and priors on the data, and such restrictions may inadvertently lead to flawed empirical characterizations of the economy. In particular, they are all constructed to yield “well-behaved” long-run dynamics, as long as the monetary policy rule satisfies certain conditions such as the Taylor principle, and as long as the fiscal authorities (explicitly or implicitly) pursue stable policies that, say, target a fixed debt-to-GDP ratio. In addition, these models tend to abstract from structural change and generally assume that the parameters and the shock processes are constant over time and known by policymakers. As a result of these features, structural models may significantly understate the persistence of episodes of low real interest rates, because they implicitly assume that the medium- to long-run equilibrium real interest rate—a key factor underlying the threat posed by the ZLB—is constant.¹ This is because the asymmetric nature of the ZLB implies that low frequency variation in the equilibrium real interest rate raises the overall probability of hitting the ZLB, all else equal.

Because of these potential limitations of structural models, we include in our analysis three statistical models that impose fewer theoretical constraints on the data and allow for a wider set of sources of uncertainty. One is a vector autoregression model with time-varying parameters (TVP-VAR); the second is a model that allows for unit-root behavior in both potential output growth and the equilibrium real interest rate (Laubach-Williams 2003); and the third is a univariate model that allows for GARCH error processes. In selecting these statistical models, one of our aims is to use models that arguably provide more scope than structural

¹ Whether or not real interest rates are stationary is, admittedly, not obvious. Ex post measures for the United States display no clear trend over the past sixty years, and the fact that U.S. real short-term rates were on average low during the 1970s, and high during the 1980s, is in part an artifact of excessively loose monetary policy in the former period and corrective action during the latter period. But phenomena such as the persistent step-down in Japanese output growth since the early 1990s, the global savings glut of the past decade, and secular trends in government indebtedness illustrate that there are many reasons to view the equilibrium real interest rate as a series that can shift over time.

models do for taking into account uncertainty about the range and persistence of movements in the equilibrium real interest rate.

In summary, our findings are as follows. We find that the decline in economic activity and interest rates in the United States has generally been very far out in the tails of the conditional forecast distributions of many empirical macroeconomic models. In contrast, the decline in inflation has been less surprising. This underestimation of the risk of the ZLB can be traced to a number of sources. For one, uncertainty about model parameters and latent variables, which were typically ignored in past research, significantly increases the probability of hitting the ZLB. Second, models that are based primarily on the Great Moderation period severely understate the incidence and severity of ZLB events. Third, the propagation mechanisms and shocks embedded in standard DSGE models appear to be insufficient to generate sustained periods of policy being stuck at the ZLB, such as we now observe. We conclude that past estimates of the incidence and effects of the ZLB were too low and suggest a need for a general reexamination of the empirical adequacy of standard models. In addition to this statistical analysis, we use model simulations to assess the effectiveness of the Federal Reserve's efforts to mitigate the effects of the ZLB by providing stimulus through large-scale asset purchases, and conclude that these purchases have both significantly checked the rise in unemployment and prevented the U.S. economy from falling into deflation. Despite this additional unconventional stimulus, however, the final section of the paper concludes that ZLB probably had a first-order impact on macroeconomic outcomes in the United States.

II. Models and methodological issues

As noted, we use six different models to evaluate the likely incidence of encountering the ZLB. Each of these models is “off the shelf” in that we have taken models already in use at the Federal Reserve or that are well-established in the academic literature. In some cases, we have made minor modifications to the models to facilitate the comparison of results across models. In this section, we provide brief descriptions of the models and references for more detailed information. Table 1 provides a summary of the key features of the models.

FRB/US

The FRB/US model is a large-scale estimated model of the U.S. economy with a detailed treatment of the monetary transmission mechanism. We include the FRB/US model because it has good empirical foundations and has long been used at the Federal Reserve for forecasting and policy analysis. In addition, FRB/US has the advantage of having been used in previous analyses of the ZLB. Although it is not a DSGE model, the main behavioral equations are based on the optimizing behavior of forward-looking households and firms subject to costs of adjustment. The model displays sluggish adjustment of real activity and inflation in response to shocks (see Brayton et al. 1997 for details).

Most of the 56 behavioral equations in FRB/US are estimated individually using least-squares, rather than system estimation methods. Such equations often incorporate cointegrating relationships that are estimated separately; in addition, many include explicit expectational terms that are generated using the forecasts of a small-scale VAR model. However, some portions of the model, such as the New Keynesian wage-price equations, are estimated using system techniques and rational expectations by augmenting these equations with a simple monetary policy rule and reduced-form equations to capture the dynamics of the real-side of the economy. The complexity of this estimation process limits our ability to carry out some of the exercises discussed below that involve re-estimating the model for different sample periods and controlling for parameter uncertainty.

We assume rational expectations for those parts of our analysis where we explore the macroeconomic effects of substantial changes in monetary policy, such as alternative policy rules or the initiation of large-scale asset purchases. In forecasting exercises, however, we simulate the model using the expectational assumption commonly used at the Fed for this type of work. Under this assumption, agents base their expectations on the forecasts of a small VAR model rather than the full FRB/US model. This approach has the virtue of computational simplicity; it also has a proven track record in forecasting.

Another noteworthy aspect of the FRB/US projections presented in this paper concerns the extrapolation of shocks and exogenous variables. Although shocks to behavioral equations are assumed to be serially uncorrelated with mean zero in the estimation of the model, we do not follow the standard approach used with the other models and set the baseline projected values of the stochastic innovations to zero. Instead, we extrapolate these shocks at their weighted

average value over the preceding sixty quarters, using weights that decline geometrically at a rate of 1 percent per quarter. Analysis at the Federal Reserve indicates that this type of intercept-adjustment procedure—which has been the standard approach to forecasting with FRB/US since the inception of the model in the mid-1990s—increases real-time predictive accuracy. As for exogenous variables, again we follow standard practice in FRB/US forecasting and extrapolate these series using simple univariate time-series equations.

SW (Smets-Wouters)

Our second model is a slightly modified version of the DSGE model developed by Smets and Wouters (2007) (SW hereafter). We made four modifications to the original model and refer readers to the original paper for a more complete description of the model. The first modification is that we assume that the monetary policy shocks are independently and normally distributed. Second, we replace the measure of the output gap in SW (2007), which is based on a definition of potential output consistent with an equilibrium where nominal rigidities are absent, by a production function-based measure (see Kiley 2010). Third, we follow the approach of Justiniano, Primiceri, and Tambalotti (2010) and include purchases of consumer durables in the investment series as opposed to the consumption series. Finally, for consistency with the other models, we replaced the GDP deflator with the core personal consumptions expenditures price (PCE) index as the price index in the model.

EDO (Estimated Dynamic Optimization-based model)

The EDO model is a DSGE model of the US economy developed and used at the Board of Governors for forecasting and policy analysis; see Chung, Kiley and Laforge (2010) for documentation on the current version of the model, and Edge, Kiley and Laforge (2008) for additional information. Like FRB/US and the SW model, EDO has strong empirical foundations. And although the model has not been in service long enough at the Federal Reserve to compile a reliable track record, pseudo real-time forecasting exercises carried out by Edge, Kiley and Laforge (2010) suggest that it has good forecasting properties.

EDO builds on the Smets and Wouters (2007) model. Households have preferences over nondurable consumption services, durable consumption services, housing services, and leisure and feature internal habit in each service flow. Production in the model takes place in two

distinct sectors that experience different stochastic rates of technological progress—an assumption that allows the model to match the much faster rate of growth in constant dollar terms observed for some expenditure components, such as nonresidential investment. As a result, growth across sectors is balanced in nominal, rather than real, terms. Expenditures on nondurable consumption, durable consumption, residential investment, nonresidential investment are modeled separately while the remainder of aggregate demand is represented by an exogenous stochastic process.

Wages and prices are sticky in the sense of Rotemberg (1982), with indexation to a weighted average of long-run inflation and lagged inflation. A simple estimated monetary policy reaction function governs monetary policy choices. The exogenous shock processes in the model include the monetary policy shock; the growth rates of economy-wide and investment-specific technologies; financial shocks, such as a stochastic economy-wide risk premium and stochastic risk premia that affect the intermediaries for consumer durables, residential investment, and nonresidential investment; shocks to autonomous aggregate demand; and price and wage markup shocks.

The model is estimated using Bayesian methods over the sample period 1984Q4 to 2009Q4. Accordingly, the model's estimates are guided almost entirely by the Great Moderation period. The data used in estimation include the following: real GDP; real consumption of nondurables and services excluding housing; real consumption of durables; real residential investment; real business investment; aggregate hours worked in the nonfarm business sector (per capita); PCE price inflation; core PCE price inflation; PCE durables inflation; compensation per hour divided by GDP price index; and the federal funds rate. Each expenditure series is measured in per capita terms, using the (smoothed) civilian non-institutional population over the age of 16. We remove a very smooth trend from hours per capita prior to estimation.

TVP-VAR

The specification of the TVP-VAR (time-varying parameter vector autoregression) model follows Primiceri (2005). The VAR model contains a constant and two lags of the four-quarter change in the core PCE price index, the unemployment rate, and the federal funds rate. Let X_t denote the column vector consisting of these variables, ordered as listed. The system obeys

$$A_t^0 X_t = \bar{A}_t + \sum_{s=1}^2 A_t^1 X_{t-s} + B_t \varepsilon_t \quad (1)$$

where A_t^0 is lower triangular and each non-zero element of the A matrices follows an independent Gaussian unit-root process. Consequently, both the equilibrium real interest rate and the variances of the shocks are time-varying. The matrix B_t is diagonal and the logarithm of an entry on the diagonal follows an independent Gaussian unit-root process, i.e., the volatility of structural shocks is stochastic. Estimation is Bayesian, with the prior constructed as in Primiceri (2005), using a 40 quarter training window starting in 1953Q3.²

Laubach-Williams

The Laubach-Williams (LW) model includes estimated equations for the output gap, core PCE price inflation, the funds rate, and relative non-oil import and oil prices. (See Laubach and Williams, 2003). Potential GDP, its growth rate, and the equilibrium real interest rate are all nonstationary unobserved latent variables. The other parameters of the model, including those describing the variances of the shock processes, are assumed to be constant.³ We estimate the LW model by maximum likelihood using the Kalman filter with data starting in 1961.⁴ Unlike FRB/US and EDO, this model implicitly assumes adaptive expectations, features very gradual dynamic responses to shocks, and includes permanent shocks to the equilibrium real interest rate.

GARCH model

We estimate univariate GARCH models of the federal funds rate, the inflation rate of the core PCE price index, and the unemployment rate. Specifically, each series is assumed to follow an auto-regressive process of order two

² The prior setting is identical to Primiceri (2005), with one exception: we have set the prior mean of the covariance matrix for innovations to the log-variances substantially higher than in that paper. Specifically, the prior mean is [0.05, 0.05, 0.001], versus [0.0004, 0.0004, 0.0004] with the original prior. Relative to the original, this prior favors drift in volatilities more so than in VAR coefficients. The estimation algorithm also follows Primiceri (2005) exactly, except that we use the approach of Jacquier, Polson and Rossi (1994) to draw the log-variance states. The MCMC sample was 20000 draws, following a burn-in run of 10000 iterations.

³ In order to conduct stochastic simulations of the model, we append AR(1) equations without constants for relative oil and nonoil import prices to the model and estimate the additional parameters jointly with the other parameters.

⁴ The Kalman gain parameters for the growth rate of potential output and for the latent variable influencing the equilibrium real interest rate are estimated using Stock and Watson's (1998) median unbiased estimator as described in Laubach and Williams (2003). We do not incorporate uncertainty about these gain parameters in our analysis in this paper. Doing so would imply even greater uncertainty about interest rates and would raise the probability of hitting the ZLB.

$$x_t = c + a_1 x_{t-1} + a_2 x_{t-2} + e_t \quad (2)$$

where the conditional variance of the innovation, e_t , is given by

$$\sigma_t^2 = \kappa + \sum_{i=1}^p G_i \sigma_{t-i}^2 + \sum_{j=1}^p A_j e_{t-j}^2 \quad (3)$$

and each equation is estimated subject to the constraints

$$\sum_{i=1}^p G_i + \sum_{j=1}^p A_j < 1, \kappa > 0, G_i \geq 0, A_j \geq 0 \quad (4)$$

The lag structure of the GARCH model was selected on the basis of the Bayesian information criterion over the sample 1968q1-2007q4.⁵ See Engle (2001) for further details on the estimation of GARCH models.

Simulation Methodology

We use stochastic simulations to construct estimated probability distributions. The ultimate goal is to derive the best characterization of future uncertainty using historical data. We report results for the case where all parameters and latent variables are known. For four of the models, we also report results that incorporate uncertainty about parameters, latent variables, and measurement error. In particular, in the EDO and LW simulations, we incorporate both parameter uncertainty and measurement error. In the case of LW, uncertainty about the equilibrium real interest rate and the output gap, two variables that enter in the monetary policy reaction function, is substantial, as discussed in Laubach and Williams (2003). The stochastic simulations of SW and TVP-VAR also take account of parameter uncertainty. The sheer size of FRB/US makes it computationally infeasible to incorporate parameter uncertainty and measurement error into the uncertainty estimates.

Imposing the non-linear ZLB constraint on FRB/US, EDO, and SW imposes no major problems. For forecasting exercises, the zero lower bound was imposed contemporaneously through unanticipated shocks to the monetary policy rule. For the FRB/US alternative policy simulations discussed in sections 4 and 5, however, special code is needed to ensure that the (rational) expectations of private agents are fully consistent with the changes to future economic conditions implied by switching to different policy rules or by undertaking large-scale asset

⁵ The optimal values p and q for the bill rate and inflation innovations are both one. For the unemployment rate, the optimal value of p remains one while the BIC assigns a value of four to q.

purchases. Because LW is a backward-looking model, there is no difficulty in enforcing the ZLB. Imposing the ZLB constraint on the TVP-VAR and the GARCH models can be quite problematic, and so we allow nominal short-term interest rates to fall below zero in our analysis.⁶ Failure to impose the constraint in these two models will bias downward the estimates of the adverse effects of the ZLB on output and inflation that we derive from them. However, such understatement is less of an issue with the GARCH model because the equations are univariate.

We use the monetary policy reaction functions embedded in each structural model or we append an estimated rule to the model as needed. In the EDO, FRB/US, and LW models, the estimated policy reaction functions assume that the federal funds rate depends on core PCE inflation, the assumed inflation target (2 percent under baseline assumptions), and the model-specific estimate of the output gap. The specific reaction functions for these three models are:

$$R_t = .82R_{t-1} + .18 \left[R_t^* + \pi_t + .65(\pi_t - \pi_t^*) + 1.04Y_t \right] \quad \text{FRB/US and LW} \quad (5)$$

$$R_t = .66R_{t-1} + .34 \left[R_t^* + \pi_t + .46(\pi_t - \pi_t^*) + .21Y_t + .33\Delta Y_t \right] \quad \text{EDO} \quad (6)$$

$$R_t = .82R_{t-1} + .18 \left[R_t^* + \pi_t + 1.02(\pi_t - \pi_t^*) + .08Y_t + .22\Delta Y_t \right] \quad \text{SW} \quad (7)$$

For these rules, the concept of potential output underlying Y is not the flex-price level of output but a measure that evolves more smoothly over time—specifically, a production-function measure in the cases of FRB/US and SW, a Beveridge-Nelson measure in the case of EDO, and a Kalman filter estimate in LW. In LW simulations, we assume the policymaker does not know the true value of the equilibrium real interest rate and the output gap, but instead uses the Kalman filter estimates of these objects in the setting of policy.⁷ We do not include shocks to the policy rules, except for those owing to the ZLB, in stochastic simulations of FRB/US, EDO, SW, and LW models but do in the case of the TVP-VAR and GARCH models.

Past research has generally used large sets of stochastic simulations to estimate in an unconditional sense how often the ZLB is likely constrain monetary policy. Such an approach

⁶ Formally, we may regard the ZLB as a shock to the monetary policy rule. Imposing it on a reduced form model therefore requires being able to identify a monetary policy shock—indeed, in principle, to identify a vector of anticipated shocks out to the horizon at which the ZLB is expected to bind. In the case of a univariate GARCH model, no widely accepted benchmark identification exists. The TVP-VAR does assume a triangular structural form at every time, but the resulting “monetary policy shock” does not appear to have reasonable properties over the entire distribution at the dates of interest.

⁷ In this way we allow for policymaker misperceptions of potential output and the equilibrium real interest rate. See Orphanides et al. (2000) and Orphanides and Williams (2002) for analyses of this issue. We abstract from policymaker misperceptions of this type in the other models analyzed in this paper.

requires that the model yield a stationary steady state with well-behaved long-run dynamics. The particular specification choices made in order to impose these restrictions may inadvertently bias the estimate of the incidence of hitting the ZLB. For example, in the FRB/US and EDO models, the long-run equilibrium real interest rate is constant. In contrast, the LW and TVP-VAR models allow for low-frequency variation in the equilibrium real interest rate. Indeed, the TVP-VAR allows for nonstationary time-variation in all parameters and variances, which implies the absence of any meaningful steady state and unconditional moments.⁸

Given that some of the models we consider do not have well-defined unconditional moments, in this paper we focus primarily on conditional probabilities of policy being constrained by the ZLB. Specifically, we compute five-year-ahead model forecasts conditional on the state of the economy at a given point in time. We then use these simulations to describe the model's prediction regarding the incidence of hitting the ZLB and the resulting macroeconomic outcomes.

III. How surprising have recent events been?

We start our analysis by comparing the actual course of events over the past few years with what each of the models would have predicted prior to the crisis, hopping off from conditions in late 2007.⁹ With the exception of the FRB/US model, the projections are based on model parameters estimated with historical data only through 2007. In addition, we also characterize forecast distributions for the projections, based on the sort of shocks encountered prior to 2008; these shocks extend back to the 1960s for all the models. At this stage, we do not take account of parameter and latent variable uncertainty. By comparing the actual evolution of the economy with these distributions, we can judge whether the models view recent events as especially unlikely.

⁸ In principle, we could modify the TVP-VAR and LW models so that they generate stationary steady states by imposing stationarity on all model parameters. Such an undertaking lies outside the scope of the present paper and we leave this to future research.

⁹ For all models except FRB/US, forecast distributions were computed using 50000 draws from the models' estimated shock distributions; FRB/US simulations use 25000 draws, bootstrapped from residuals between 1968 and 2000:Q4. The initial states for EDO and SW were taken from the posterior mode, while the TVP-VAR initial conditions are from the posterior mean. The initial state for LW is the mean of a distribution calculated using the procedure in Hamilton (1986). When accounting for uncertainty about latent states and parameters, we use the posterior distribution for EDO, SW and the TVP-VAR, and the previously mentioned Hamilton (1986) distribution for LW.

Figure 1 summarizes results from stochastic simulations of the FRB/US model for the output gap, the unemployment rate, core PCE price inflation, and the federal funds rate over the period 2008 to 2012. As can be seen, the model prior to the crisis would have viewed the subsequent evolution of real activity and short-term interest rates as extremely improbable, in that actual conditions by 2010 fall far outside the 95 percent band about the late 2007 projection.¹⁰ In contrast, the model is not surprised by the behavior of inflation during the downturn, given the modest degree of disinflation that has occurred to date.¹¹ Similar results are found in the four of the other five models, although the GARCH model is less surprised by events. The results for the five models are shown in Figures 2 through 6.

The upper panel of Table 2 reports summary statistics of the simulations used in Figures 1 through 6. The models give very different predictions regarding the probability of hitting the ZLB. The three structural models yield very small probabilities of being stuck at the ZLB for four consecutive quarters. In contrast, the three statistical models indicate that such an event would not be as rare, with probabilities between 2 and 9 percent. Only the GARCH model predicts a nontrivial probability of being stuck at the ZLB for eight consecutive quarters. But even the GARCH model is surprised by the rise in the unemployment rate, as seen in Figure 6.

The bottom line of this analysis is that recent events would have been judged very unlikely prior to the crisis, based on analyses using stochastic simulations of a variety of structural and statistical models estimated on U.S. data on conditions over the past several decades. We now consider possible sources of this underestimate of the probability of such events.

One key factor in determining the odds of hitting the ZLB is the period over which the model is estimated: Folding in the events of the past few years into the estimated variability of the economy tends to boost this probability considerably. The middle panel of table 2 reports

¹⁰ With hindsight, FRB/US sees the economy as having been hit primarily by huge shocks to the demand for new houses and to the value of residential real estate. By themselves, these shocks account for about half of the widening of the output gap seen since late 2007. In addition, shocks to risk premiums for corporate bonds, equity and the dollar account for another third of the fall in aggregate output. In contrast, EDO sees the economy as primarily having been hit with a big, persistent risk-premium shock in late 2008 and during the first half of 2009. In 2008Q4, the estimated economy-wide risk premium was two standard deviations away from its mean under the stationary distribution; by the first half of 2009, the premium was three standard deviations away from its mean.

¹¹ This statement that the model has not been surprised by the magnitude of the decline in the inflation rate does not condition on the surprisingly large decline in economic activity as measured by the output gap and the unemployment rate. Conditional on the significant realized decline in economic activity, the model is surprised by the relatively modest decline in inflation that has occurred.

corresponding results for simulations starting from conditions at the end of 2007 but using models where the sample period used in estimating the innovation covariance and other parameters is extended through the middle of 2010.¹² Thus, these simulations take account of the information learned over the past three years regarding both the structure of the economy and, most importantly, the incidence of large shocks during this period. Not surprisingly, the predicted probabilities of hitting the ZLB rise in most cases. More interestingly, the probabilities of being stuck at the ZLB for four or more quarters are now nontrivial in the structural models and sizable in the statistical models. Even so, only the TVP-VAR and GARCH models see more than a very small probability of being stuck at the ZLB for eight consecutive quarters.

Models estimated using data only from the Great Moderation period yield very small probabilities of hitting the ZLB. The lower panel of table 2 reports results from four of the models where the innovation covariances are estimated based on data from 1984-2007. Based on this sample, the three structural models see very low probabilities of hitting the ZLB and (with the exception of SW) trivial probabilities of being stuck there for a year or longer. The probabilities from the LW model are higher, but only about one half as large as those based on the long sample. These results illustrate the sensitivity of quantitative analysis of the effects of the ZLB to the Great Moderation period.

A second key factor is the incorporation of uncertainty about model parameters and latent variables in the simulations. The middle panel of table 3 reports simulation summary statistics from the four models for which we are able to adjust our estimates for uncertainty about parameters and (in the case of LW) latent variables. In all four cases, the probabilities of hitting the ZLB rise significantly once this additional source of uncertainty is taken into account. In fact, for the two statistical models, the probability of being stuck at the ZLB for four consecutive quarters doubled, while the probability of being stuck for eight quarters rose from about 1 percent to 2 or 5 percent. These results highlight the quantitative importance of these forms of uncertainty that have heretofore been neglected in analysis of the ZLB.

Finally, incorporating into the analysis *both* the widest possible type of uncertainty *and* longer samples that include periods of severe macroeconomic disruptions significantly increases the estimated probability and duration of ZLB events. The lower panel of table 3 reports results

¹² In the case of FRB/US, only the innovation covariance is re-estimated using the additional data from 2008 to 2010. In the other models, all model parameters are re-estimated.

where the sample is extended through 2010q2 and parameter and latent variable uncertainty are accounted for. These results demonstrate the critical importance of incorporating these factors in analysis of the ZLB.

An interesting question is whether the estimated probability of hitting the ZLB has changed much over time, conditional on the information available in real time. Additional analysis using our suite of models suggests that it has.¹³ For almost all the models, rolling pseudo real-time estimates of the likelihood of hitting the ZLB at least once in the next 20 quarters declined markedly after the 2001 recession and were quite low by 2007, reflecting the increasing importance of the Great Moderation experience in the information set. As the Great Recession unfolded, however, these probability estimates climbed sharply, coming near or reaching 100 percent by late 2008 and remaining elevated through 2010. In contrast, the various models tell a less uniform story concerning the likelihood of being persistently stuck at the ZLB for two years or more. For example, the EDO and SW models consistently saw virtually no chance of such a prolonged ZLB episode, even during the depths of the financial crisis, while FRB/US shows the odds of such an event shooting up in 2008 and remaining high through 2010. Results from the statistical models are less optimistic than the DSGE model but more optimistic than FRB/US. These results may help to explain why some researchers working with DSGE models in the past have not viewed the ZLB as a serious concern.

IV. Did large-scale asset purchases significantly ease the ZLB constraint?

The results presented in the previous section suggest that the probability of encountering the ZLB may be significantly higher than earlier research indicated; alternatively put, our standard models may err in judging that the depth of the recent recession and the sluggishness of the subsequent recovery constitute an extraordinarily rare event. However, the surprise from the models' viewpoint may be significantly greater than that analysis indicates, because the model simulations do not take account of the unusual policy actions undertaken during the crisis to support real activity and to check disinflationary pressures. In particular, the analysis ignores the efforts of the FOMC to provide additional stimulus through purchases of longer-term Treasury securities and agency debt and mortgage-backed securities (MBS) once the federal funds rate

¹³ For additional details on this analysis, see the working paper version of this study.

had fallen to near zero. In the absence of this unprecedented effort to ease the ZLB constraint (and so unincorporated into the forecast distributions of the previous section), the contraction in real activity and decline in inflation would likely have been even more pronounced.

A primary objective of the FOMC's asset purchases has been to put additional downward pressure on longer-term yields at a time when short-term interest rates had already fallen to their effective lower bound. Because of spillover effects, such a reduction in bond yields should lead to more accommodative financial conditions overall, thereby stimulating real activity and checking disinflationary pressures through reduced borrowing costs, higher stock valuations, and a lower foreign exchange value of the dollar. In many ways, this transmission mechanism is similar to the standard one involved in conventional monetary policy, which primarily operates through the influence on long-term yields of changes to the current and expected future path of the federal funds rate. Because of this similarity, we can use our structural models to obtain a rough estimate of the macroeconomic effects of large-scale asset purchases, specifically by combining the FRB/US model with a simple model of the influence of Federal Reserve holdings of longer-term assets on the term premium embedded in long-term yields. Ideally, we would carry out a similar analysis using the EDO and Smets-Wouters models but the structure of those particular DSGE models unfortunately does not lend itself to such an exercise.¹⁴

One way to view the effects of asset purchases on long-term interest rates is through their implications for the price demanded by market participants to expose themselves to some of the risks involved in lending long-term. In the case of Treasury securities, these risks primarily center on the uncertain path for future inflation and help to explain why the yield curve generally slopes up: Risk-averse investors do not like to assume this risk and so require a compensating term premium for holding longer-term Treasury securities in place of Treasury bills—a premium that might rise if the government or the central bank were to increase the average duration of government debt supplied to the market. Term premiums can also be viewed as arising from the existence of preferred-habit investors whose willingness to buy securities of a given maturity is

¹⁴ In FRB/US, bond yields that embed term premiums play a direct role in influencing real activity through the cost of capital; long-term interest rates also influence real activity indirectly through the stock market and the exchange rate. In contrast, real activity in EDO and Smets-Wouters depends on expectations for the future path of short-term interest rates, not on actual long-term asset prices, and so term premium shocks do not enter the models. As documented by Rudebusch, Sack and Swanson (2007), however, econometric evidence suggests that a reduction in term premiums implies stronger future real activity, independent of other interest rate effects. Other DSGE models, such as the one developed by Andres, Lopez-Salido, and Nelson (2004) that incorporates imperfect substitution among assets into the standard new Keynesian framework, could be used for this type of analysis.

an increasing function of the yield on that asset; Vayanos and Vila (2009) have incorporated such investors into a no-arbitrage model in which yields on securities of different maturities are linked in a manner that depends in part on their relative supplies. An implication of such preferred-habit effects is that the central bank should be able to lower long-term interest rates if it can substantially reduce the stock of long-term debt held by the private sector.

To this end, the Federal Reserve purchased about \$1.25 trillion in agency MBS, \$170 billion in agency debt, and \$300 billion in longer-term Treasury securities over the course of 2009 and early 2010, with the purchases reflected in an increase in banks' reserve balances held at the Federal Reserve Banks. The FOMC subsequently expanded this program by purchasing another \$600 billion in longer-term Treasury securities between November 2010 and June 2011. Finally, the Federal Reserve implemented a policy in August 2010 of preventing its security holdings from passively shrinking by reinvesting principal payments in longer-term Treasury securities. Because of these actions, and taking account of earlier redemptions and MBS principal payments, security holdings in the Federal Reserve's System Open Market Account (SOMA) climbed to \$2.6 trillion by the middle of 2011, with essentially all of these assets having an original maturity of greater than one year. In contrast, SOMA security holdings prior to the crisis in mid-2007 were only \$790 billion, \$280 billion of which were in Treasury bills.

Several recent studies have attempted to estimate the quantitative effect of large-scale asset purchases by the Federal Reserve on U.S. long-term interest rates, including Gagnon et al. (2010), D'Amico and King (2010), Doh (2010), Hamilton and Wu (2011), and Krishnamurthy and Vissing-Jorgensen (2011). Other researchers have examined the quantitative effects of similar unconventional policy actions recently carried out abroad, such as the Bank of England's quantitative-easing program; see Meier (2009) and Joyce et al. (2010). In the case of the U.S. estimates, some are derived from event studies following FOMC announcements about asset purchases in late 2008 and 2009 (Gagnon et al.), while others are based on the estimated effects of purchases during the same period using time-series regressions of reduced-form bond rate models (Gagnon et al., Doh). In contrast, Hamilton and Wu base their estimates on a version of the term-structure model proposed by Vayanos and Vila, estimated using pre-crisis data from 1990 to 2007 and adapted to predict how long-term yields should respond to large-scale asset

purchases when short-term interest rates are at zero.¹⁵ The general conclusion from this research is that the first phase of the Federal Reserve’s asset purchases reduced the general level of long-term interest rates by around 50 basis points, with estimates running as low as 30 basis points and as high as 100 basis points.¹⁶ To put the 50-basis-point figure into perspective, a back-of-the-envelope calculation suggests that achieving such a reduction in long-term yields would require something on the order of a 200 basis point cut in the federal funds rate.¹⁷ Thus, the overall stimulus provided by the FOMC during 2009 is better characterized as a (virtual) reduction in nominal short-term interest rates to -2 percent, not to zero.¹⁸

For a quantitative assessment of the macroeconomic benefits of the Federal Reserve’s elevated asset holdings, we now turn to simulations of the FRB/US model. For this exercise, we need to specify how the evolution of SOMA holdings of longer-term securities influences long-term interest rates over time, not just initially. To do this, we specify a simple model of portfolio-balance effects in which the size of the term premium effect on yields is proportional to the discounted present value of expected future SOMA holdings of longer-term securities in excess of what the Federal Reserve would normally hold, relative to the level of nominal GDP. Specifically, we assume that the portfolio-balance effect on the term premium embedded in 10-year Treasury yields, denoted by θ_t , is given by:

¹⁵ Another estimate based on pre-crisis data comes from Swanson (2011), who uses an event study to reexamine the effects of “Operation Twist”, the attempt by the U.S. government in the early 1960s to lower long-term yields by altering the relative supplies of longer-term and short-term Treasury debt. He concludes that the program lowered long-term yields by about 15 basis points—a finding that, after adjusting for scale, is in line with the effects reported by Gagnon et al. for the Federal Reserve’s 2009 program. Bernanke et al (2004) also find large effects from pre-crisis changes in the relative supplies of Treasury securities.

¹⁶ Krishnamurthy and Vissing-Jorgensen (2011) find significant spillovers to yields on “safe” U.S. assets such as high-grade corporate bonds but smaller effects on other long-term interest rates. With regards to the foreign experience, estimates reported in Meier (2009) and Joyce et al. (2010) for the effects of the Bank of England’s quantitative easing program are somewhat larger at around 100 basis points. In addition, Neely (2010) finds that the Federal Reserve’s asset purchases had effects that spilled over to yields on medium-term sovereign debt in several other countries and led to a decline in the exchange value of the U.S. dollar.

¹⁷ Regressing quarterly changes in the 10-year Treasury yield on quarterly changes in the federal funds rate for the period 1987 through 2007 yields a coefficient of about 0.25, implying that a 100 basis point reduction in short-term interest rates is typically associated with a 25 basis point decline in long-term yields. This estimate is similar to that of Poole (2005), who finds a coefficient of 0.32.

¹⁸ As discussed in Gagnon et al. (2010) and Hancock and Passmore (2011), the Federal Reserve’s MBS purchases also improved the functioning of the mortgage market, thereby reducing spreads of residential mortgage rates over 10-year Treasury yields by about 50 basis points during 2009. The model simulations reported below incorporate this effect on mortgage rate spreads but assume that it fades away during the first half of 2010, on the grounds that market conditions would have eventually improved on their own as the financial crisis passed and the recovery proceeded.

$$\theta_t = \lambda E_t \sum_{j=0}^{\infty} \beta^j \left[\frac{A_{t+j}}{X_{t+j}} - \phi_{t+j} \right] \quad (8)$$

where A denotes the amount of securities held in the SOMA portfolio with an initial maturity of greater than one year; X is a scaling factor set equal to nominal GDP; ϕ denotes the trend ratio of SOMA longer-term security holdings to the scaling factor, β is a quarterly discount factor assumed to equal 0.99, and λ is calibrated to equal -33.4, a figure consistent with the estimate that the first-round of asset purchases reduced long-term interest rates 50 basis points.¹⁹

An implication of this model is that the magnitude of the term-premium effect (and the calibration of λ) depends on both the volume of longer-term assets held by the Federal Reserve and investors' expectations for the evolution of those holdings over time. Such forward-looking behavior is consistent with the response of market interest rates to the original program, in that yields moved sharply in response to announcements about future purchases that would not be completed for many months. Similar behavior was observed again between late August 2010 and the November 2010 FOMC meeting, when market participants gradually came to anticipate a second round of asset purchases—an anticipation that led to noticeable decline in long-term yields in advance of the actual FOMC announcement.

Figure 7 summarizes the predictions of this simple model for the evolution of portfolio-balance effects arising from the Federal Reserve's large-scale asset purchases. Under the first phase of the program, which effectively began in early 2009 and is assumed to have lasted into the first half of 2010, the overall size of the SOMA portfolio rose from around \$750 billion to around \$2 trillion by the first half of 2010 (solid line, upper panel). For illustrative purposes, we assume that agents during this first phase anticipated the actual trajectory of the portfolio through mid-2010 and additionally expected that the Federal Reserve would thereafter renormalize its size and composition gradually, bringing excess SOMA holdings of securities with a maturity of longer than one year back to zero by early 2016 (solid line, middle panel). As shown in the bottom panel, this expectation implies an expected trajectory for term premium effects under the first phase of the program that starts out at minus 50 basis points and then steadily declines.

¹⁹ The value of λ also depends on assumptions for the path of the SOMA portfolio expected by market participants when the first phase of asset purchases was announced. Further details on the model and its calibration are provided in the working paper version of this study.

The asset-purchase program later went through two additional phases. The second began in August 2010 when the FOMC announced that it would reinvest principal payments on agency securities in longer-term Treasury securities to prevent the portfolio from shrinking passively. We assume that this announcement led investors to project that the size of the SOMA portfolio would remain near \$2 trillion through mid-2012 and then decline gradually back to its long-run trend over the next five years (dotted line); such an expectation would have been consistent with comments by various FOMC officials. As shown in the bottom panel, we estimate that this policy action increased the downward pressure on term premiums modestly. Later in the fall, the FOMC expanded the program yet again by announcing that it intended to purchase an additional \$600 billion in longer-term Treasury securities by June 2011, increasing the overall size of the SOMA portfolio to about \$2.6 trillion (dashed line). Again assuming that investors expected the portfolio to be back to normal by 2017, this action increased the downward pressure on term premiums to 65 basis points in late 2010, with the effect thereafter falling back to zero.

Figures 8 and 9 summarize the macroeconomic effects of the asset purchase program, as simulated by the FRB/US model under various assumptions for monetary policy and wage-price dynamics. In these simulations, we shock bond term premiums in the model by the amount shown in the bottom panel of figure 7, with agents—who have rational expectations but not perfect foresight about monetary policy—revising their expectations for the future with each new phase of the program. In particular, we assume that, from 2009Q2 to 2010Q2, expectations for the future path of the portfolio and associated term premium effects are consistent with phase 1 of the program. Expectations then shift in 2010Q3 and again in 2010Q4 as agents revise their views about the future path of SOMA holdings to be consistent with phases 2 and 3 of the program, respectively.

In the simulations, the response of bond yields to asset purchases depends not only on the trajectory of portfolio-balance effects but also on any accompanying changes to the expected path of the federal funds rate. For example, agents may view the program as a way to provide additional monetary stimulus over the medium term, beyond what could be attained by keeping the funds rate very low for an extended time. If so, they might expect little or no change to the average stance of conventional monetary policy over the next few years, relative to their expectations prior to the announcement of the program. Alternatively, agents might view asset purchases as primarily a short-term measure that effectively acts as a substitute for keeping the

funds rate low in the future when the FOMC is no longer constrained by the ZLB, thereby prompting them to raise their expectations for the funds rate and so partially offsetting the downward pressure on long-term interest rates. Finally, the program could have the opposite effect and lower expectations for the path of the funds rate if, say, agents saw asset purchases as a sign that the FOMC was going to respond more aggressively to high unemployment and low inflation than had been previously thought. Some support for this last possibility is provided by movements in Eurodollar futures prices in 2009 following FOMC announcements about asset purchases.

Given this range of possibilities, we begin by assuming that conventional monetary policy adjusts in a relatively “neutral” fashion to any economic changes brought about by asset purchases. Specifically, we assume that federal funds rate follows its baseline path through 2014 but thereafter responds to deviations of output and inflation from baseline as prescribed by the estimated inertial rule; we also assume that private agents anticipate this policy. Under these assumptions, the 10-year Treasury yield initially drops by almost as much as the shock to term premiums because the average expected level of future short-term interest rates increases very little (solid line, upper left panel of figure 8). Yields subsequently begin to move back towards baseline until pushed lower again by the announcements of the next two phases of the program (dotted and dashed lines, respectively). Thereafter, the effect of the program on long-term yields fades quickly as the date of portfolio renormalization draws nearer and portfolio-balance effects fade.

Lower long-term interest rates, coupled with higher stock market valuations and a lower foreign exchange value of the dollar, provide a considerable stimulus to real activity over time. Phase 1 of the program by itself is estimated to boost the level of real GDP almost 2 percent above baseline by early 2012, while the full program raises the level of real GDP almost 3 percent by the second half of 2012. This boost to real output in turn helps to keep labor market conditions noticeably better than they would have been without large-scale asset purchases. In particular, the model simulations suggest that private payroll employment is currently 1.8 million higher, and the unemployment rate $\frac{3}{4}$ percentage point lower, than would otherwise be the case; by 2012, the incremental contribution of the program is estimated to grow to 3 million jobs. Finally, the simulations indicate that inflation is currently a percentage point higher than would have been the case if the FOMC had never initiated the program, implying that the economy

would now be close to deflation in the absence of asset purchases. These results also suggest that the longer-run inflationary consequences of the program are likely to be minimal, as portfolio-balance effects rapidly fall to zero and conventional monetary policy adjusts to bring conditions back to baseline in an environment in which agents are assumed to have complete confidence in the FOMC's desire and ability to maintain price stability.

In the simulation results shown in figure 8, part of the stimulus to real activity arises through an interaction between nominal interest rates and inflation. With the nominal funds rate assumed to follow its baseline path through 2014, a virtuous circle temporarily arises in which stronger real activity reduces deflationary pressures, which in turn keeps real interest rates lower than they otherwise would be and so boosts real activity further. Figure 9 illustrates the importance of this effect by comparing the effects of asset purchases under the model's standard inflation dynamics (solid line) and when wage and price setting is more inertial (dotted lines).²⁰ When inflation responds only gradually to changes in economic conditions, asset purchases are less effective at offsetting disinflationary pressures and supporting real activity.

Figure 9 also illustrates that the stimulus from asset purchases is smaller if agents expect that conventional monetary policy over the medium term will partially offset the effects of the program by being appreciably tighter than would have been the case without asset purchases. The dashed lines show the simulated effects of the program when agents understand that the federal funds rate will begin to rise significantly above baseline in 2012; as a result, nominal Treasury yields fall by appreciably less. Moreover, because inflation is little changed from baseline under this policy assumption, the rise in real short-term interest rates is greater, and thus the fall in real long-term interest rates is less, appreciably reducing the net stimulus to real activity. As noted earlier, however, the asset-purchase program may have led agents to expect that conventional monetary policy would be *easier* on average over the medium term; if so, the macroeconomic effects of the asset-purchase program would be *larger* than those shown in figure 8.

Overall, these simulation results suggest that the Federal Reserve's program of large-scale asset purchases has provided significant support to real activity and the labor market; moreover, the program likely has helped to appreciably offset undesirable deflationary pressures.

²⁰ Specifically, in FRB/US' wage-price equations (whose specification is based on a Calvo-style New Keynesian Phillips curve), the model-consistent expectational terms are replaced with the forecasts of a small VAR model. The latter "limited-information" expectational assumption is regularly used in forecasting with the FRB/US model.

These results are consistent with the findings of Baumeister and Benati (2010), who also find large effects on real activity and inflation from reduced bond yields using a Bayesian VAR framework. Of course, considerable uncertainty attends these results; for example, based on the lower-end estimates of the effect on bond yields of the initial purchase program, the macroeconomic effects could be half as great as those reported here. This uncertainty notwithstanding, we think that asset purchases have likely reduced the severity of the ZLB constraint appreciably.

V. How severely did the ZLB bind during the crisis?

Although the Federal Reserve's large scale asset purchases appear to have had a sizable effect on the economy, the question remains as to whether the monetary policy as a whole remained constrained relative to what it could do absent the zero lower bound. To address this issue, we now consider results from counterfactual simulations of FRB/US in which we explore how conditions over the past two years might have evolved had it been possible to push nominal interest rates below zero.

What monetary policy would have done in the absence of the zero lower bound constraint depends, of course, on policymakers' judgments about how best to respond to changes in current and projected economic conditions in order to promote price stability and maximum sustainable employment. Such judgments would have depended on many factors, including assessments of overall resource utilization, the outlook for employment growth and inflation, the risks to that outlook, and the perceived responsiveness of real activity and prices to additional monetary stimulus. Because we cannot hope to account for all the factors that influence the FOMC's decision process, we restrict ourselves to reporting results from counterfactual simulations of the FRB/US model in which the federal funds rate is allowed to follow the unconstrained prescriptions of three different simple policy rules. Specifically, we use the standard Taylor (1993) rule, the more-aggressive rule described in Taylor (1999), and the estimated inertial rule specified previously in equation 1.5. In all three rules, the inflation target is expressed in terms of core PCE inflation and is assumed to equal 2 percent—consistent with the November 2010 long-run inflation projections of a majority of FOMC participants.

The baseline used in these counterfactual simulations matches the actual evolution of real activity and inflation through the early fall of 2010. Beyond this point, we extrapolate shocks to

the model's equation in order to generate a baseline that matches the extended Blue Chip consensus outlook published in October 2010, on the grounds that this projection approximates private-sector expectations for the economy at the time.²¹ As illustrated by the solid lines in the panels of figure 10, this baseline shows the economy emerging only slowly from a deep recession, accompanied by an undesirably low rate of inflation for several years and a prolonged period of near-zero short-term interest rates. Specifically, the baseline unemployment rate peaks at about 10 percent in late 2009 and then slowly drifts down over the next seven years, stabilizing at 6 percent towards the end of the current decade. Inflation, after bottoming out at 1 percent in late 2010, gradually drifts up as the recovery proceeds, settling in at 2 percent after 2012. And the federal funds rate remains at its effective lower bound until the second half of 2011 and then gradually returns to a more normal level of about 4 percent by the middle of the decade.²²

Unfortunately, the Blue Chip survey does not provide explicit information about private forecasters' assumptions for the supply side of the economy—a necessary input for our analysis because all three policy rules respond to movements in economic slack. However, we can infer survey participants' estimates for the NAIRU—whether explicit or implicit—because the long-run consensus forecast shows the unemployment rate stabilizing late in the decade at 6 percent, accompanied by stable output growth and inflation.²³ Using Okun's Law, we adjust this figure to approximate the output gap implicit in the baseline by assuming that $Y_t \approx (6 - U_t) * 2$, or approximately minus 8 percent in the fourth quarter of 2010. The extended Blue Chip forecast also provides information about private forecasters' implicit estimates of the equilibrium real

²¹ Basing the outlook portion of the baseline on the FOMC's own "central-tendency" forecasts might have been preferable but these are insufficiently detailed for our purposes; for example, they do not include projections of the federal funds rate. Alternatively, we could have used the FRB/US model's projections; if we had, they would have suggested a greater need for financial restraint than the Blue Chip consensus forecast.

²² Participants in the October 2010 Blue Chip survey provided forecasts of quarterly real GDP growth, the unemployment rate, overall CPI inflation, the rate on 3-month Treasury bills, and the yield on 10-year Treasury bonds through the end of 2010; in addition, participants provided annual projections for these series for the period 2011 through 2020. Because the policy rules use the federal funds rate instead of the Treasury bill rate, we translate the Blue Chip projections of the latter into forecasts of the former by assuming they were equal. In the case of inflation, we translate Blue Chip forecasts for the CPI into projections for core PCE prices using the projections of the spread between these two series that were reported in the 2010Q3 Survey of Professional Forecasters.

²³ This assumption is reasonably consistent with results from the 2010Q3 Survey of Professional Forecasters, in which the consensus estimate of the NAIRU was reported to be 5¾ percent. An interesting implication of these figures is that private forecasters apparently believe that the financial crisis and the accompanying deep recession have had highly persistent adverse consequences for labor market functioning, given that the Blue Chip long-run projections made prior to the crisis showed the unemployment rate stabilizing at 4¾ percent.

interest rate R^* that appears in the three policy rules, in that the consensus forecast shows the real federal funds rate settling down at about 2 percent in the long run.

The short-dash, long-dash, and dotted lines of figure 10 show simulated outcomes when the federal funds rate follows the prescriptions of the three unconstrained policy rules. Under the Taylor (1993) rule, the prescribed path of the nominal federal funds rate differs little from the baseline path, and accordingly the paths for the unemployment rate and core inflation are only slightly better (short-dash lines). Accordingly, a policymaker who wished to follow this rule would not have felt constrained by the ZLB, after taking account of the additional stimulus the FOMC was able to provide through the Federal Reserve's large-scale purchases of longer-term assets (whose effects are reflected in the baseline paths for the output gap and inflation). However, a policymaker who wished to respond more aggressively to economic slack and low inflation, and so wanted to go beyond the stimulus provided by a zero nominal funds rate and the FOMC's asset-purchase program, would have felt constrained. Under the estimated inertial rule, for example, the counterfactual simulation shows the nominal federal funds rate moving down to minus 1 percent by late 2009 and then remaining persistently below zero until 2012 (dotted lines). As a result of this more accommodative policy, FRB/US predicts that the unemployment rate would have peaked at a somewhat lower level and would have been expected to decline considerably faster over the next few years than anticipated in the Blue Chip consensus forecast. In addition, inflation under the inertial policy rule would not have fallen as low and would be expected to run in the vicinity of $2\frac{1}{2}$ percent over the next few years. As indicated by the long-dash lines, broadly similar results for real activity and inflation are also generated under the Taylor (1999) rule, although the contour of the federal funds rate path is somewhat different. In particular, the Taylor (1999) rule calls for a steeper decline in nominal short-term interest rates in 2009 but a somewhat faster renormalization of monetary policy starting in 2011.

On balance, these counterfactual simulation results suggest that the severity of the ZLB constraint has been considerable over the past few years, even after taking account of the macroeconomic effects of the asset purchase program as manifested in actual and projected output and inflation. Indeed, under some assumptions, a policymaker could have arguably

wished to push the nominal funds rate almost 300 basis points below zero.²⁴ These results are consistent with those of Williams (2009). That said, we should note several caveats to this conclusion.

First, our results are conditioned on the dynamics of the FRB/US model, and other models might yield different results. Indeed, exploratory simulations carried out with the EDO and the SW models suggest that these models would have called for a less dramatic easing in unconstrained monetary policy than FRB/US. The source of this difference between FRB/US and these models appears to be a greater sensitivity of real activity and inflation in the DSGE models to anticipated monetary policy shocks—a property that in turn reflects a higher relative degree of intrinsic persistence in FRB/US. Even assuming that this aspect of the dynamics of the DSGE models is empirically valid, the actual effectiveness of such promissory policies for mitigating the effects of the ZLB would depend critically on the ability of policymakers to credibly commit to future policy actions.

Second, the FRB/US simulation results may overstate the severity of the ZLB constraint during the current downturn because of our flaws in the baseline supply-side assumptions. As noted above, we calibrated the NAIRU in the baseline to the long-run Blue Chip projections of the unemployment rate, implying an output gap of roughly minus 8 percent in mid-2010. However, estimates of the output gap reported by official institutions such as the International Monetary Fund (2010), the OECD (2010), and the Congressional Budget Office (2010) are somewhat smaller; some estimates reported by Weidner and Williams (2009) indicate even less slack than the official estimates. Using these less-pronounced estimates of slack in place of the baseline assumption would noticeably shrink the estimated severity of the ZLB constraint under the various monetary policy assumptions.

Finally, the dynamics of the FRB/US model may not provide an accurate characterization of the likely responses of real activity and inflation to changes in the federal funds rate in current circumstances. For example, the interest-sensitivity of aggregate demand may now be unusually low because of reduced access to credit or heightened uncertainty about the economic outlook. If so, an unconstrained policy response might conceivably call for larger and more persistent declines in the nominal funds rate.

²⁴ As discussed in the working paper version of this study, an optimal-control exercise that uses a policymaker loss function similar to that regularly employed in FOMC-related policy analysis at the Federal Reserve Board prescribes pushing the nominal funds rate even further into negative territory.

VI. Conclusions and further research

The zero lower bound has been an important constraint on monetary policy in many countries over the past several years. The fact that many central banks have encountered the ZLB should not have come as a surprise—previous research using empirical models that were not based primarily on the Great Moderation period did predict that the ZLB would be a relatively frequent constraint on monetary policy in a low inflation environment. What has been a surprise is the magnitude and duration of the constraint imposed by the ZLB in the United States and in some other countries.

Our analysis has identified a number of factors that may have led to an underestimation of the extent to which the ZLB may affect macroeconomic outcomes. Going forward, researchers who seek to assess the probability and effects of hitting the ZLB will need to confront the issues raised by these factors. First, relying on model stochastic simulations that assume constant parameters and variances, and so abstract from data and parameter uncertainty, contributes to an underestimate of the probability of encountering the ZLB. Our results indicate that time-varying parameters, measurement error, and parameter uncertainty can noticeably raise the estimated probability of hitting the zero lower bound, indicating that future research should incorporate these factors in the analysis. Second, researchers need to find ways to ensure that model-generated probability distributions adequately account for relatively rare tail events, even if the data in the model's estimation sample does not include any such events. This adjustment can be accomplished by using long samples in estimating the shock variances, or by using methods that incorporate a prior on tail events and making the distribution of these events less sensitive to recent data. Finally, our analysis shows that one can obtain quite different answers depending on the model used in the analysis. For example, we find that an estimated DSGE model estimated over the Great Moderation period predicts that it is extremely unlikely that the Federal Reserve could get stuck at the ZLB for a year or longer, while other models that feature stronger intrinsic persistence view such outcomes as much more likely. This range of results indicates that research on the ZLB should explicitly integrate a range of models, including models that allow for structural change.

We also find that the asset purchases undertaken by the Federal Reserve during 2009 and early 2010 were roughly equivalent to a 200 basis point reduction in the short-term interest rate; the additional expansion of the SOMA portfolio that began in late 2010 implies yet more

monetary stimulus. Model simulations suggest that the overall stimulus provided by these purchases have kept the deterioration in labor market conditions from being noticeably worse than it otherwise would be; the asset purchase program may also have kept the economy from falling into deflation. Future analysis of the effects of the ZLB will need to take account of the potential use of such alternative tools. Nonetheless, based on counterfactual model simulations run using a variety of monetary policy rules, we find that the ZLB has importantly constrained the ability of monetary policy to limit the depth and duration of the current slump, even after taking account of the effects of the Federal Reserve's asset purchases.

Finally, we should note that we have ignored a number of factors in our analysis. First, the effects of the ZLB can depend on the assumption regarding expectations formation. Roberts and Reifschneider (2006) and Williams (2006) find that deviations from rational expectations magnify the effects of the ZLB. Second, in most cases we abstracted from the problems of real-time measurement of data and natural rates that add additional uncertainty to the setting of policy and macroeconomic outcomes. Third, our analysis of ZLB risks in the United States has not taken account of the experience of other countries; future studies should put more weight on the cross-country evidence, which shows a frequent pattern of financial crises followed by deep recessions.²⁵ And fourth, we did not take into account any historically-unusual fiscal policy actions that may step in when the economy is constrained by the ZLB.

²⁵ For example, a recent BIS review of the cross-country evidence suggests that the frequency of financial crises is roughly 4 to 5 percent per year—that is, about four or five times a century. The BIS review also found that, following a financial crisis, the cumulative shortfall in output relative to pre-crisis trends was about 20 percent of GDP on average, assuming no permanent reduction in potential output following the crisis. By contrast, no such event occurred in the United States during the postwar period.

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Table 1: Summary of Models

	FRB/US	EDO	SW	TVP-VAR	LW	GARCH
Estimation sample start	1968	1984	1968	1964	1961	1968
Estimation method	OLS	Bayes	Bayes	Bayes	ML	ML
Estimated equations	56	28	13	3	8	3
Time-varying r^*	No	No	No	Yes	Yes	No
Time-varying parameters	No	No	No	Yes	No	No
Time-varying variances	No	No	No	Yes	No	Yes

Table 2: Effect of Altering the Shock-Process Sample Period on Estimated Probabilities of ZLB Events and Confidence Intervals for Projections of Interest Rates, Inflation and Real Activity, Hopping off from Conditions in 2007Q4

	FRB/US	EDO	SW	LW	TVP-VAR	GARCH
<i>Based on long sample ending in 2007^a</i>						
Probability of a ZLB event on or before 2012Q4	0.03	0.10	0.13	0.09	0.07	0.20
Probability of a 4-quarter ZLB event on or before 2012Q4	0.01	<0.01	<0.01	0.05	0.02	0.09
Probability of a 8-quarter ZLB event on or before 2012Q4	<0.01	<0.01	<0.01	0.01	<0.01	0.02
95 percent distribution bands for conditions in 2012Q4						
Federal funds rate	0.9, 8.8	0.3, 9.8	0.6, 10.0	0.1, 10.1	0.1, 8.2	-1.9, 11.7
Inflation rate	0.6, 4.0	-1.1, 6.2	-0.7, 6.8	-0.7, 5.7	-0.5, 5.0	-1.1, 6.3
Output gap	-4.8, 4.9	-3.3, 2.9	-4.4, 4.4	-6.2, 4.6		
Unemployment rate	2.3, 7.8				3.1, 6.7	-0.5, 11.7
<i>Based on long sample ending in 2010^a</i>						
Probability of a ZLB event on or before 2012Q4	0.09	0.23	0.19	0.09	0.25	0.25
Probability of a 4-quarter ZLB event on or before 2012Q4	0.04	0.02	0.02	0.05	0.13	0.13
Probability of a 8-quarter ZLB event on or before 2012Q4	0.01	<0.01	<0.01	0.01	0.04	0.04
95 percent distribution bands for conditions in 2012Q4						
Federal funds rate	0.2, 9.0	0.3, 9.4	0.4, 9.5	0.1, 10.6	-4.1, 12.4	-2.8, 11.8
Inflation rate	0.4, 4.2	-1.7, 5.8	-1.3, 6.5	-0.8, 5.7	-1.2, 5.1	-1.1, 6.0
Output gap	-5.3, 5.0	-3.4, 3.3	-5.0, 4.7	-6.4, 5.0		
Unemployment rate	2.1, 8.1				1.4, 8.4	-0.8, 12.0
<i>Based on 1984-2007 sample</i>						
Probability of a ZLB event on or before 2012Q4	0.01	0.02	0.09	0.05		
Probability of a 4-quarter ZLB event on or before 2012Q4	<0.01	<0.01	0.02	0.02		
Probability of a 8-quarter ZLB event on or before 2012Q4	<0.01	<0.01	<0.01	<0.01		
95 percent distribution bands for conditions in 2012Q4						
Federal funds rate	1.3, 8.1	0.8, 7.0	0.4, 8.1	0.1, 9.1		
Inflation rate	0.8, 3.7	0.9, 3.7	-0.2, 5.1	0.0, 5.0		
Output gap	-3.8, 3.9	-2.7, 1.5	-4.5, 3.6	-5.3, 3.8		
Unemployment rate	2.9, 7.3					

Note. In all models, the funds rate follows an estimated equation. Estimates do not include the effects of uncertainty about parameters and latent variables.

a. The sample starts in 1968 except for LW, in which case the sample starts in 1961.

Table 3: Effect of Uncertainty about Parameters and Latent Variables on Estimated Probabilities of ZLB Events and Confidence Intervals for Projections of Interest Rates, Inflation and Real Activity, Hopping off from Conditions in 2007Q4

	EDO	SW	LW	TVP-VAR
<i>Excluding parameter and latent variable uncertainty (long sample ending in 2007)</i>				
Probability of a ZLB event on or before 2012Q4	0.02	0.13	0.09	0.07
Probability of a 4-quarter ZLB event on or before 2012Q4	<0.01	<0.01	0.05	0.02
Probability of a 8-quarter ZLB event on or before 2012Q4	<0.01	<0.01	0.01	<0.01
95 percent distribution bands for conditions in 2012Q4				
Federal funds rate	0.8, 7.0	0.6, 10.0	0.1, 10.1	-0.1, 8.2
Inflation rate	0.9, 3.7	-0.7, 6.8	-0.7, 5.7	-0.5, 5.0
Output gap	-2.7, 1.5	-4.4, 4.4	-6.2, 4.6	
Unemployment rate				3.1, 6.7
<i>Including parameter and latent variable uncertainty (long sample ending in 2007)</i>				
Probability of a ZLB event on or before 2012Q4	0.09	0.15	0.16	0.14
Probability of a 4-quarter ZLB event on or before 2012Q4	<0.01	0.01	0.11	0.07
Probability of a 8-quarter ZLB event on or before 2012Q4	<0.01	<0.01	0.05	0.02
95 percent distribution bands for conditions in 2012Q4				
Federal funds rate	0.4, 7.3	0.4, 10.3	0.1, 11.8	-1.4, 9.4
Inflation rate	-0.3, 4.1	-0.9, 7.2	-1.9, 6.8	-1.6, 6.1
Output gap	-2.0, 1.7	-5.0, 4.5	-9.0, 6.7	
Unemployment rate				2.1, 7.4
<i>Including parameter and latent variable uncertainty (long sample ending in 2010)</i>				
Probability of a ZLB event on or before 2012Q4	0.17	0.21	0.17	0.29
Probability of a 4-quarter ZLB event on or before 2012Q4	0.01	0.03	0.12	0.16
Probability of a 8-quarter ZLB event on or before 2012Q4	<0.01	<0.01	0.06	0.06
95 percent distribution bands for conditions in 2012Q4				
Federal funds rate	0.3, 7.1	0.3, 9.8	0.1, 12.5	-4.0, 13.5
Inflation rate	-0.1, 3.3	-1.5, 6.9	-2.2, 6.9	-2.3, 5.9
Output gap	-3.4, 3.2	-5.6, 5.1	-9.1, 7.0	
Unemployment rate				0.2, 9.1

Note. In all the models, the federal funds rate follows its estimated rule. All estimation samples start in 1968 for the SW and TVP-VAR models, in 1961 for the LW model, and in 1984 for the EDO model.

Figure 1: FRB/US Assessment of the Likelihood of Recent Events
History Versus 2007Q4 Model Projection and Associated Forecast Distribution
(Shaded areas: 68, 90 and 95 percent probability envelopes)

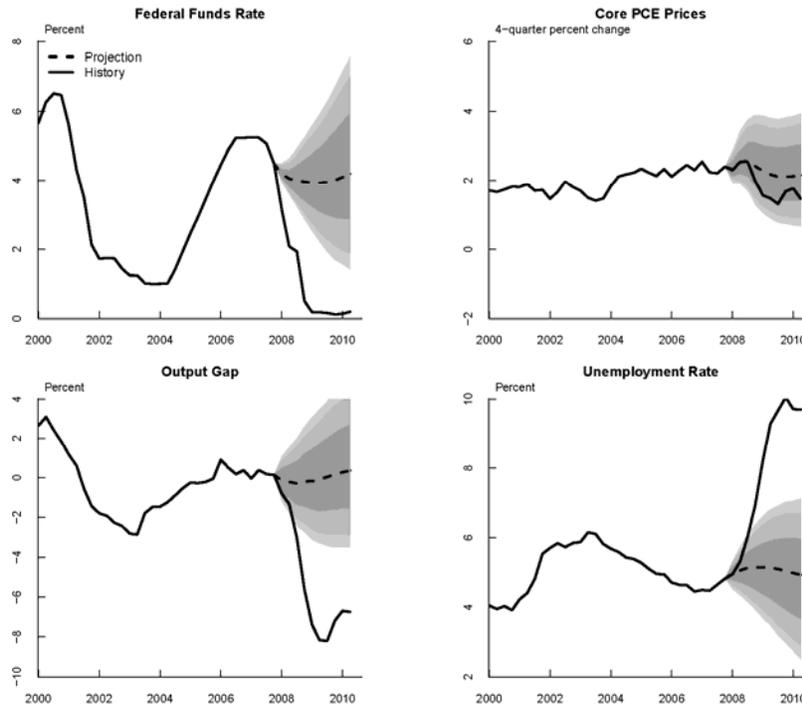


Figure 2: EDO Assessment of the Likelihood of Recent Events
History Versus 2007Q4 Model Projection and Associated Forecast Distribution
Initial Condition (Posterior Mean) Known with Certainty
(Shaded areas: 68, 90 and 95 percent posterior credible sets)

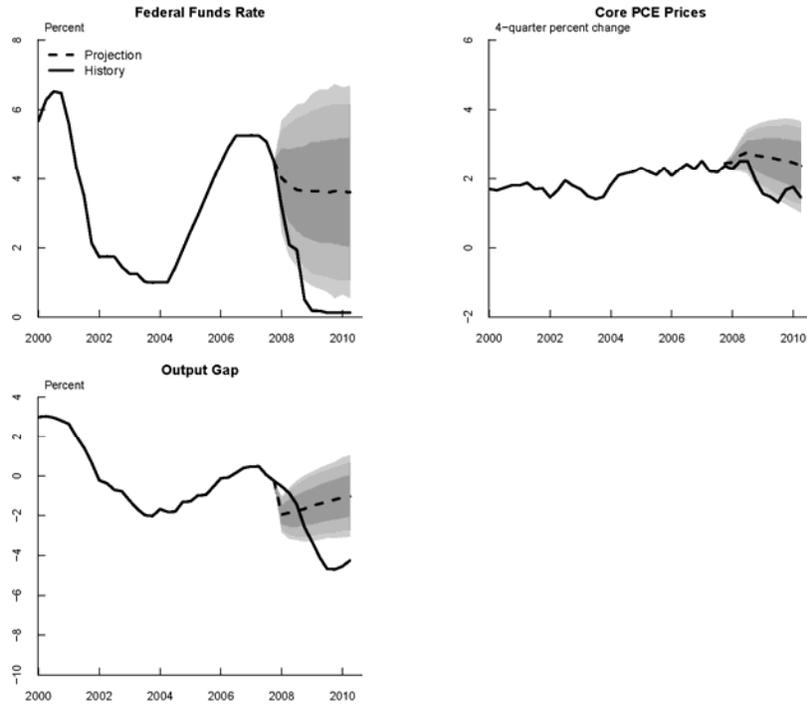


Figure 3: SW Assessment of the Likelihood of Recent Events
 History Versus 2007Q4 Model Projection and Associated Forecast Distribution
 Initial Condition (Posterior Mean) Known with Certainty
 (Shaded areas: 68, 90 and 95 percent posterior credible sets)

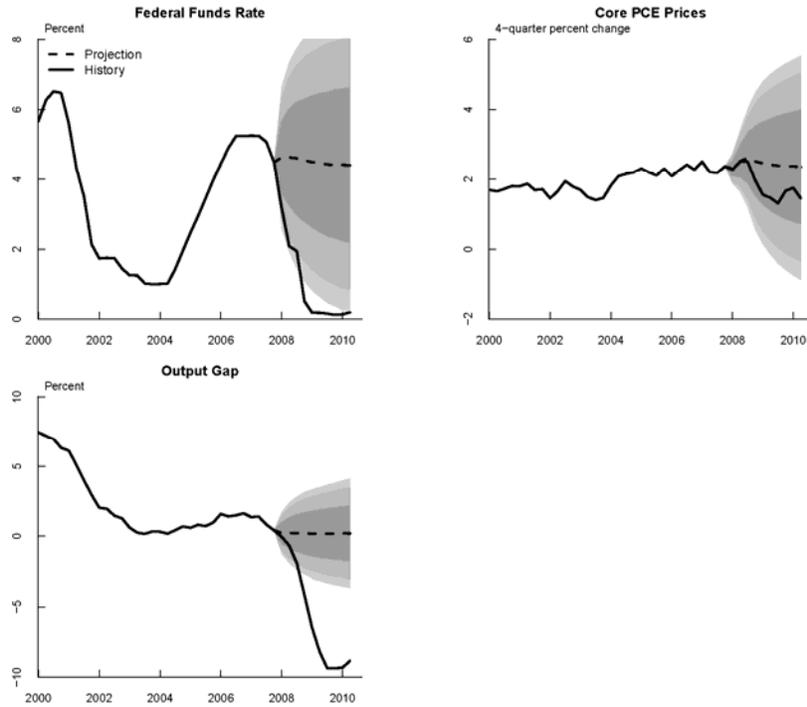


Figure 4: TVP-VAR Assessment of the Likelihood of Recent Events
History Versus 2007Q4 Model Projection and Associated Forecast Distribution
Initial Condition (Posterior Mean) Known with Certainty
(Shaded areas: 68, 90 and 95 percent posterior credible sets)

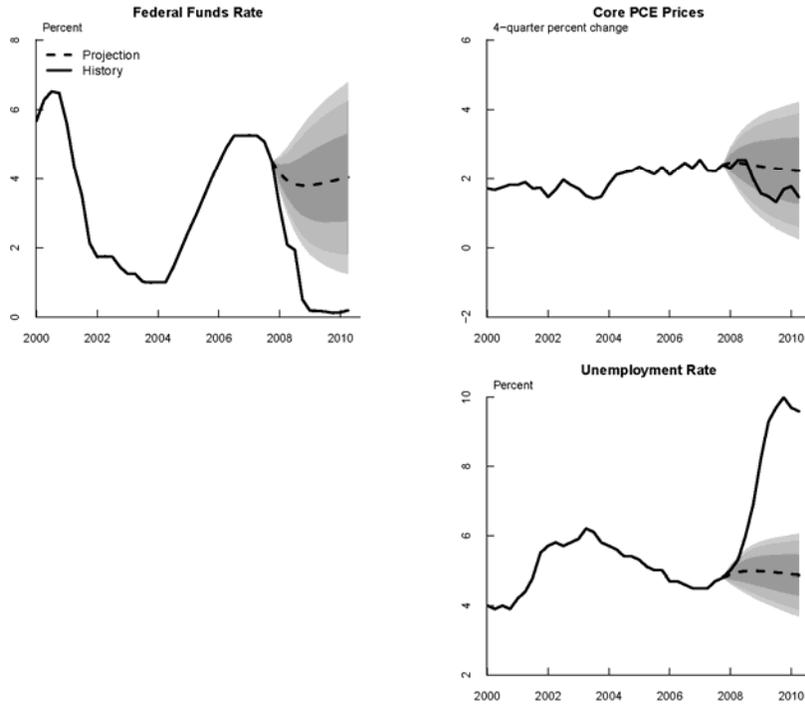


Figure 5: Laubach-Williams Assessment of the Likelihood of Recent Events
History Versus 2007Q4 Model Projection and Associated Forecast Distribution
Initial Condition (Median) Known with Certainty
(Shaded areas: 68, 90 and 95 percent probability envelopes)

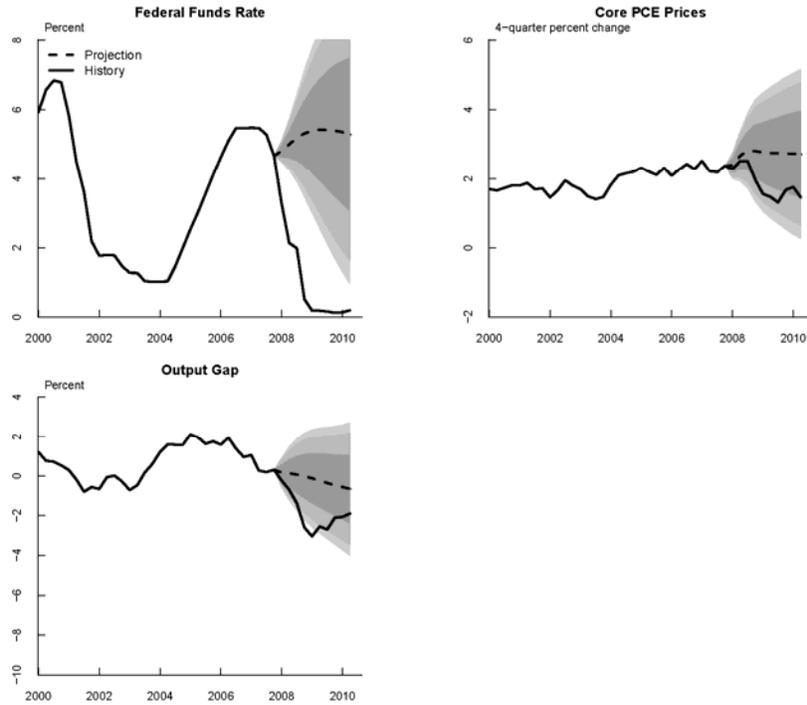


Figure 6: GARCH Assessment of the Likelihood of Recent Events
History Versus 2007Q4 Model Projection and Associated Forecast Distribution
(Shaded areas: 68, 90 and 95 percent probability envelopes)

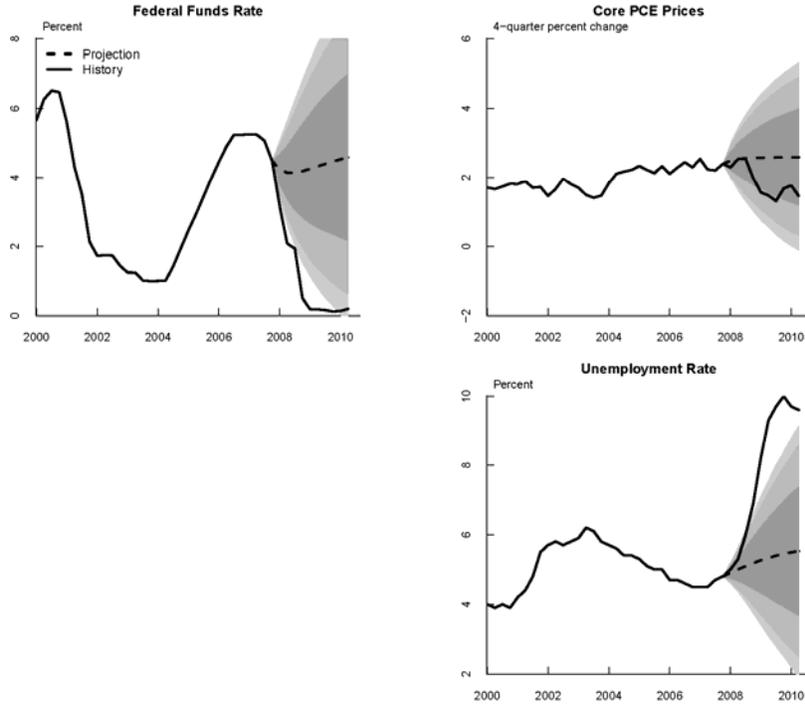


Figure 7: Phases of the Expansion of the SOMA Securities Portfolio and Implied Portfolio Balance Effects on Term Premiums

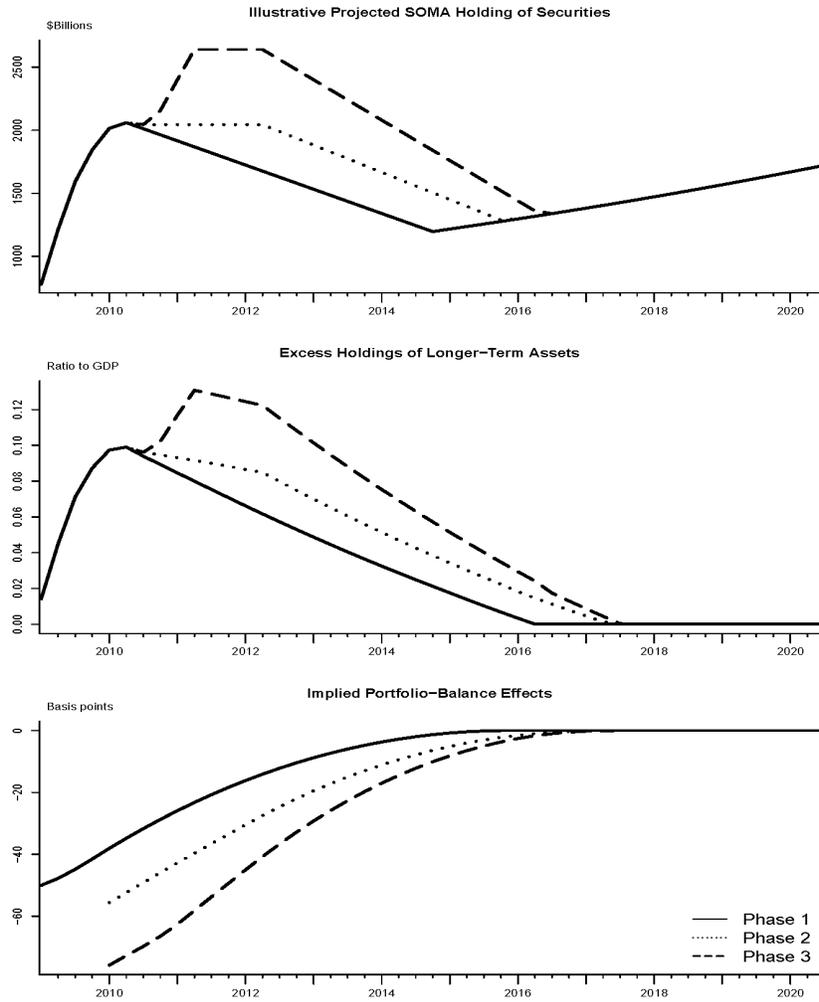


Figure 8: Macroeconomic Effects of Various Phases of the Expansion of the SOMA Portfolio, Assuming No Change in Medium-Term Expectations for Conventional Monetary Policy (results reported as change from baseline, unless otherwise noted)

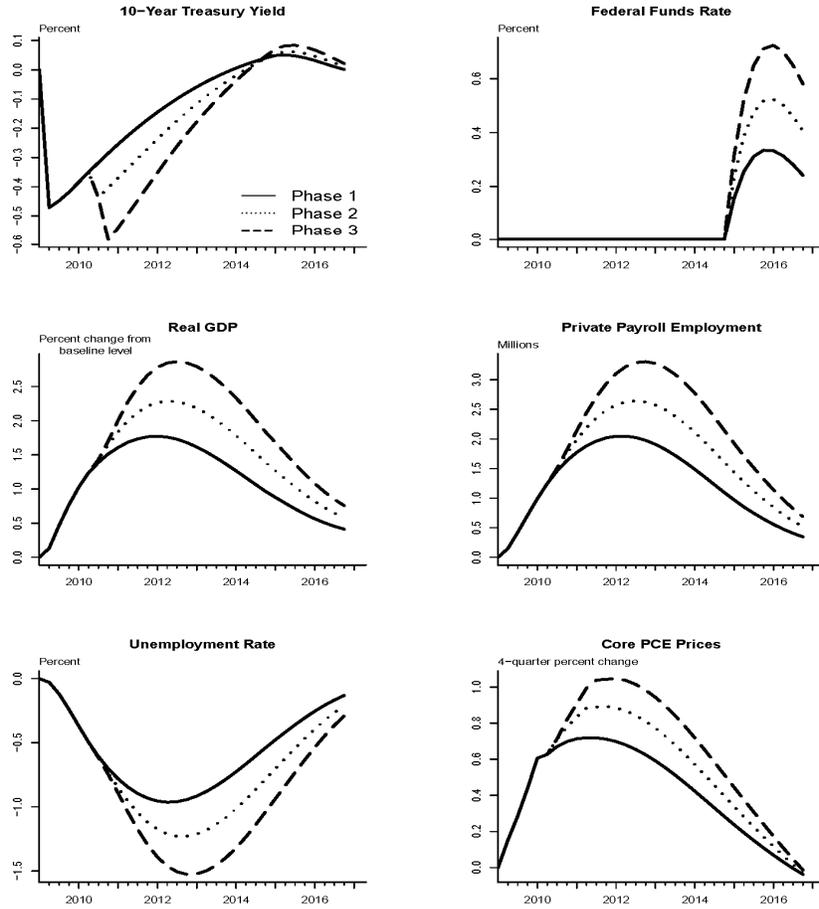


Figure 9: Macroeconomic Effects of Various Phases of the Expansion of the SOMA Portfolio, Under Different Assumptions for the Expected Onset of Offsetting Adjustments to Conventional Monetary Policy and the Inertia in Wage-Price Setting (results reported as change from baseline, unless otherwise noted)

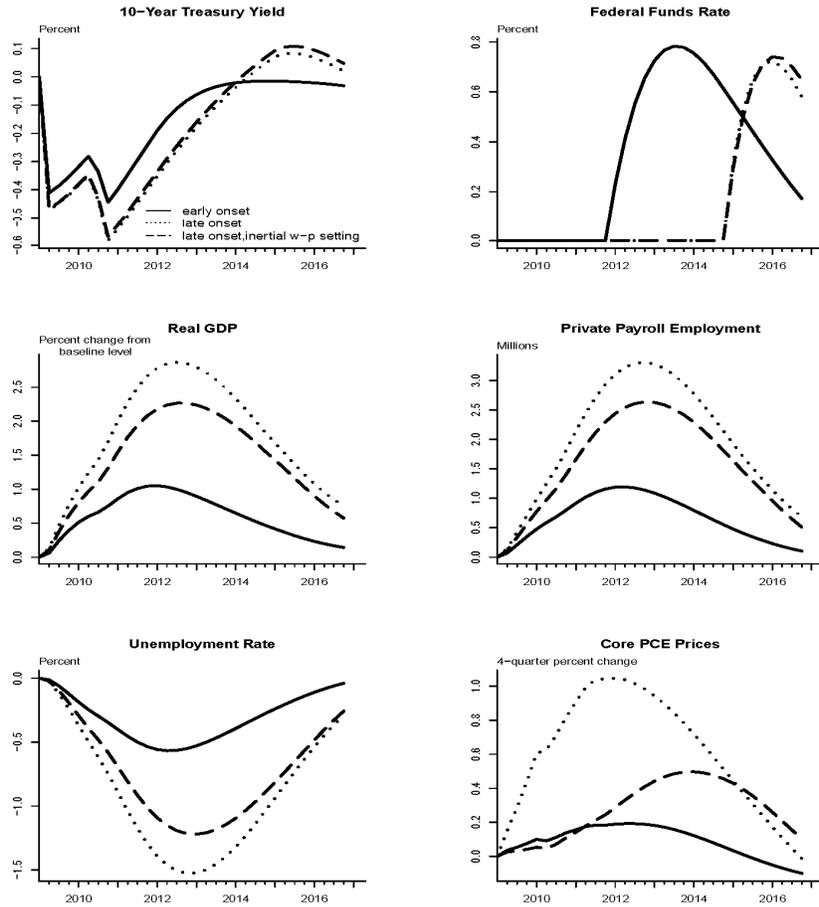


Figure 10: FRB/US Simulations of How the Economy Might Have Evolved Since Early 2009 If Conventional Monetary Policy Had Not Been Constrained by the ZLB

