Risky Mortgages in a DSGE Model*

Chiara Forlati[†] Luisa Lambertini[‡]

EPFL EPFL

September 1, 2010

Abstract

This paper develops a DSGE model with housing, risky mortgages and endogenous default. Housing investment is subject to idiosyncratic risk and some mortgages are defaulted in equilibrium. An unanticipated increase in the volatility of housing investment produces a credit crunch where delinquencies and mortgage interest rates increase, lending is curtailed, and aggregate demand for non-durable goods falls. The economy experiences a recession as a consequence of the credit crunch. We compare economies that differ only in the riskiness of housing investment. Economies with lower risk are characterized by lower mortgage default rates, higher loan-to-value and leverage ratios. The macroeconomic effects of an unexpected increase in mortgage risk are amplified in high-leverage economies. Monetary policy plays an important role in the transmission of mortgage risk, as more inertial interest rate rules deepen the short-run contraction of output.

Keywords: Financial crisis; Housing; Mortgage default

JEL Codes: E32, E44, G01, R31

Address of corresponding author: Luisa Lambertini, EPFL CDM SFI CFI, ODY 2 05, Station 5, CH-1015 Lausanne, Switzerland. E-mail: luisa.lambertini@epfl.ch

*We would like to thank Matteo Iacoviello, Andrew Levin and Eileen Mauskopf for their comments and suggestions and Yankai Shao for excellent research assistance. We are also grateful to seminar participants at Universidad Autonoma de Barcelona, UNIL, the conference on Monetary and Fiscal Policy for Macroeconomic Stability, the workshop on Heterogeneous Nations and Globalized Financial Markets and the 2010 EEA Meeting for useful feedbacks.

[†]École Polytechnique Fédérale de Lausanne, Lausanne, Switzerland, chiara.forlati@epfl.ch.

[‡]École Polytechnique Fédérale de Lausanne, Lausanne, Switzerland, luisa.lambertini@epfl.ch.

1 Introduction

The global financial crisis that began in August 2007 has its roots in increased mortgage delinquencies that put financial institutions into distress. The bursting of the housing bubble in the United States put many borrowers in a difficult financial position with mortgages they could not pay in the long run and larger than the value of the houses against which they were underwritten. As a result the rate of seriously delinquent mortgages[1] increased from 2% in the third quarter of 2006 to 10% by the first quarter of 2010. Banks were forced to write down several hundred billion dollars in bad mortgages. These losses combined with a high degree of opacity surrounding mortgage-backed securities and a complicated web of interconnected obligations among financial institutions triggered a severe liquidity crisis in the interbank market. A credit crunch followed that caused failure of several financial institutions, brought many others close to it, raised interbank rates and drastically reduced household access to borrowing. The fall in housing prices and tightened credit conditions forced many borrowers to quickly reduce leverage, cut consumption and housing investment. The turmoil that started in the mortgage market amplified to the rest of the economy to spark the great recession.

Several factors contributed to the depth of the 2008 recession and no single factor in isolation is likely to explain it. In particular, certain pre-crisis trends in the banking industry have been key in amplifying the effects of an increase in mortgage delinquencies to the second largest recession in U.S. history.

This paper focuses on one of these factors: the increase in mortgage delinquencies and its transmission to the rest of the economy. It introduces endogenous default on mortgages in a DSGE macroeconomic model with housing. The goal of the paper is to understand how aggregate shocks and variations in macroeconomic variables affect the rate of default on mortgages. We are also interested in understanding how an increase in the rate of default on mortgages transmits to the rest of the economy. We are interested in identifying qualitative effects, but we also want to get an idea of how important this channel is and how much it can contribute to a fall in GDP. Driven by recent events, we focus on an unexpected increase in mortgage risk, which we model as an unanticipated increase in the volatility of the idiosyncratic risk of

[1] According to the National Delinquency Survey of the Mortgage Banker Association, seriously delinquent mortgages are those more than 90 days past due or in foreclosure.

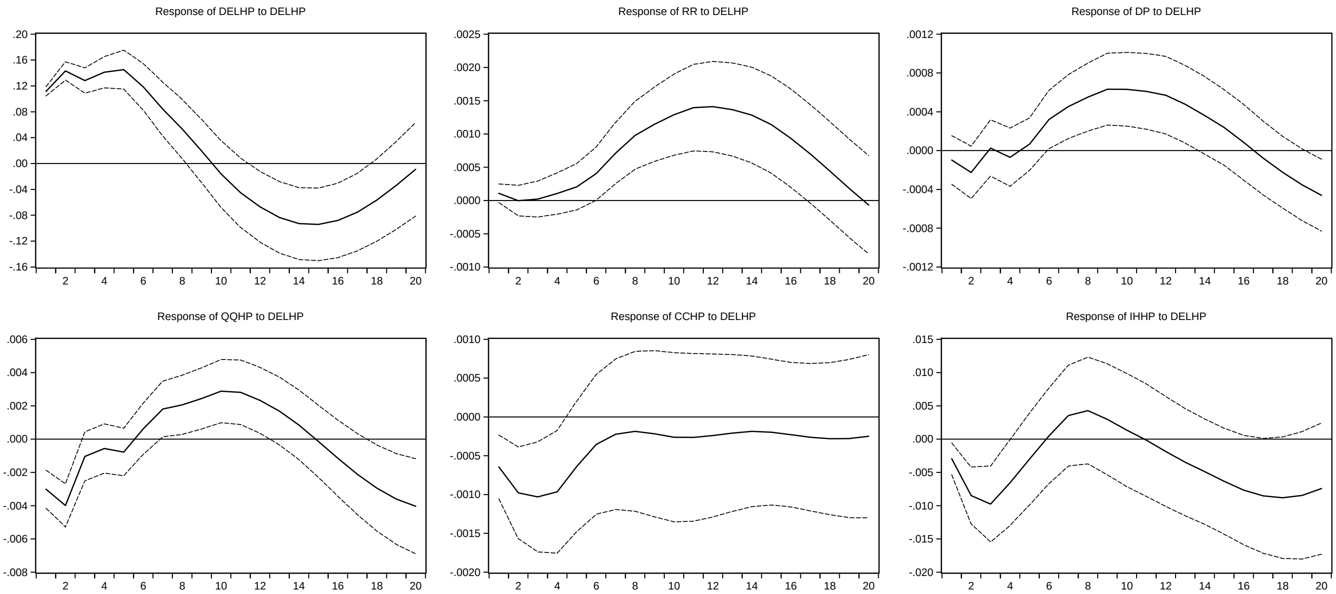
housing investment.

Figure 1 shows the impulse responses from a VAR with de-trended seriously delinquent mortgages (DELHP), nominal short-term interest rate (RR), change in the log of the implicit price deflator for the nonfarm business sector (DP), de-trended real house prices (QQHP), de-trended real per capita consumption (CCHP), and de-trended real per capita residential investment (IHHP) from 1980Q1 to 2009Q4.[2] The top part of Figure 1 illustrates the impulse responses of interest rate, inflation, housing prices, consumption and residential investment to an innovation in serious delinquencies. It suggests a significant negative response of real house prices, private consumption and residential investment to a positive disturbance to mortgage delinquencies. The nominal interest rate and inflation respond positively, but with a delay. This evidence points to a significant transmission mechanism of shocks from the mortgage market to the rest of the economy. The bottom part of Figure 1 shows the impulse responses of seriously delinquent mortgages to innovations to all other variables. These responses suggest that mortgage delinquencies react significantly to developments in the macro-economy. This VAR evidence suggests that mortgage delinquencies are an endogenous variable that affects and is affected by other fundamental macroeconomic variables. Our goal is to build a general equilibrium macroeconomic model that captures these relationships.

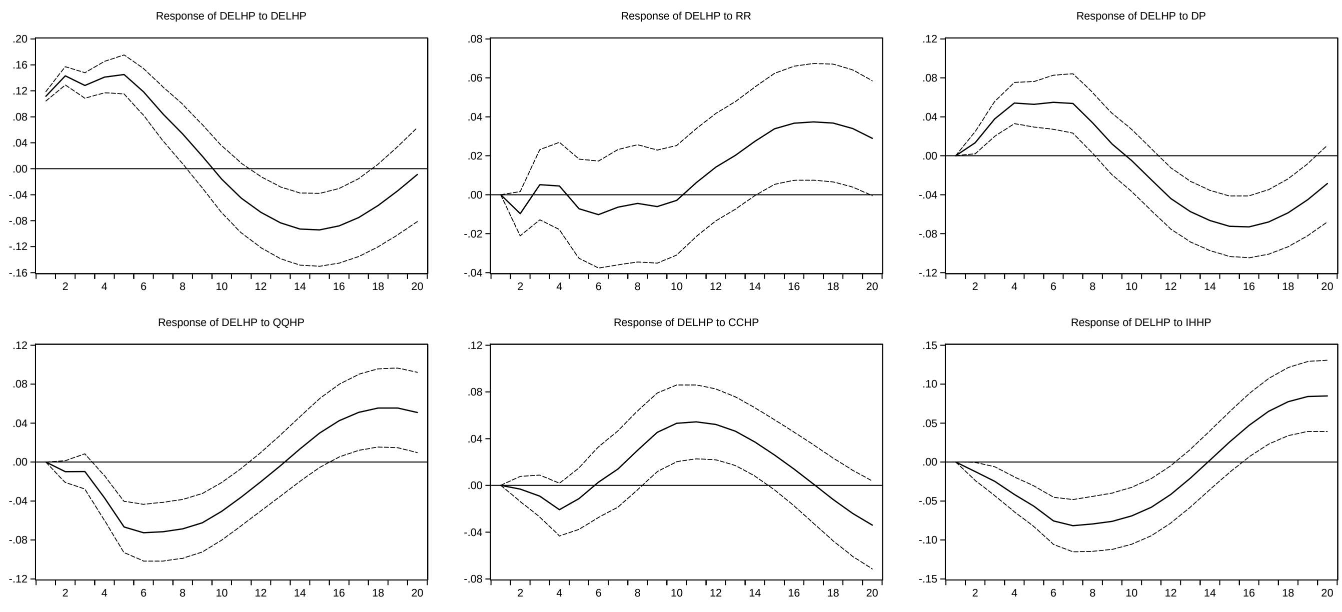
Our model features two households that differ in terms of their discount factor. Savers have a higher discount factor than Borrowers and in equilibrium lend to Borrowers. Preferences are specified over consumption of non-durable goods and housing services and hours worked. Borrowers use their houses as collateral for mortgages and experience idiosyncratic housing investment shocks that can only be observed by the household itself. Lenders must pay a monitoring cost to observe Borrower's realized housing return. Borrowers experiencing low realizations of the idiosyncratic shock default on their mortgages; Borrowers who repay their mortgages pay a state-contingent adjustable mortgage rate that is above the risk-free one. Hence, our model is characterized by endogenous default on mortgages and an external finance, or mortgage, premium.

We study the dynamic response to an unanticipated increase in the idiosyncratic risk to housing investment. Our model produces a credit crunch that generates a recession in the non-durable sector. Mortgage default rates as well as the mortgage premium, namely the spread

[2]See Appendix A for data definition and sources. De-trending is done using the Hodrick-Prescott filter.



Impulse Responses to Cholesky One Standard Deviation Innovation to Delinquencies



Impulse Responses of Delinquencies to Cholesky One Standard Deviation Innovations in DELHP, RR, DP, QQHP, CCHP and IHHP

Figure 1: VAR Evidence

Notes: VAR estimated from 1980Q1 to 2009Q4. The dashed lines indicate the +/- one standard error bands. The Choleski ordering is DELHP, RR, DP, QQHP, CCHP, IHHP. Vertical axis: percent deviation from baseline.

between the adjustable mortgage rate and the risk-free rate, increase significantly. Households with mortgages are particularly hurt. Borrowers experience a deterioration of their financial situation and tighter credit conditions, which force them to reduce their mortgages and cut non-durable consumption and housing investment. Aggregate non-durable consumption and total output in the economy fall. The relative price of houses falls but it quickly rebounds, driven by the fall in non-durable prices. An increase in idiosyncratic housing investment risk generates a recession but it fails to generate a large and persistent fall in housing prices.

We consider two economies that differ only in mortgage risk, namely the volatility of idiosyncratic housing investment risk. The economy with the lower volatility has a lower steady-state rate of default on mortgages. As a result, mortgages are larger and the economy is more leveraged. Economies with higher leverage ratios display more redistribution and more polarized responses to aggregate shocks, as the two households stand at the opposite sides of the lending contract. When mortgage risk increases, Borrowers are hurt more and the effects of the credit crunch are amplified. Hence, more leveraged economies suffer deeper recessions.

We analyze the dynamic response to an increase in mortgage risk under alternative specifications of the interest rate rule that governs monetary policy. More inertial rules feature less aggressive interest rate reductions in response to a mortgage risk shock that ultimately lead to deeper output contractions in the non-durable sector and in the economy in the short run. A non-inertial interest rate rule is successful in smoothing out the contraction in the non-durable sector. Hence, interest rate flexibility is important for responding to a mortgage risk shock.

The rest of the paper is organized as follows. Section 2 discusses the related literature and section 3 presents the model. Section 4 presents two calibrations, benchmark and low leverage, and analyzes the difference in the steady state of the two economies. Section 5 analyzes the transmission mechanism in response to a mortgage risk shock in the benchmark and in the Low-leverage economy. Section 6 considers monetary policy; section 6.1 analyzes the dynamic response of the model to a monetary shock and section 6.2 analyzes the role of interest rate flexibility. Section 7 summarizes and suggests future directions for research.

2 Related Literature

A growing literature has been incorporating housing in economic models. Iacoviello (2005) builds on Kiyotaki and Moore (1997) to model an economy with two households with different discount factors and housing as a durable good. To ensure the existence of an equilibrium, Iacoviello (2005) features an exogenous borrowing constraint according to which impatient agents can borrow a fraction of the expected discounted future value of their houses. Loans are always repaid in this model. Iacoviello and Neri (2010) extend the work of Iacoviello (2005) and write a DSGE model with housing that is estimated using U.S. data for the period 1965Q1 to 2006Q3. Calza, Monacelli and Stracca (2009) analyze how the transmission mechanism of monetary shocks in a housing model á la Iacoviello is affected by alternative values of the down-payment rate and the interest rate mortgage structure. Monacelli (2009) documents positive co-movement in durable and non-durable consumption in response to a monetary policy shock and shows that a DSGE model with an exogenous borrowing constraint is consistent with the empirical evidence. Our housing model draws a number of features from these contributions. As in Iacoviello (2005) and Iacoviello and Neri (2010), we build a model with two household groups and housing as a durable good. The novelty of our paper is to introduce idiosyncratic risk in housing investment and the possibility for loans to be defaulted on, which results in an endogenous borrowing constraint.

The literature on the financial accelerator is vast. Starting with Bernanke, Gertler and Gilchrist (1999) and then Carlstrom and Fuerst (1997), many papers have introduced this credit friction in DSGE models to analyze its effects on the transmission of shocks. We do not present an exhaustive review of this literature here but rather focus on some recent applications that are most relevant for our study. Christiano, Motto and Rostagno (2009) augment a standard monetary DSGE model to include financial markets and a financial accelerator and fit the model to European and U.S. data. Cohen-Cole and Martinez-Garcia (2008) consider a model with a financial accelerator as in Bernanke et al. (1999) and introduce systemic risk, namely an aggregate variable that affects the variance of idiosyncratic risk, and banking regulation. Our paper is the first, to our knowledge, to introduce a financial accelerator originating from default on mortgages.

Dellas, Diba and Loisel (2010) add a banking sector to an otherwise standard New-Keynesian model and consider some financial shocks, among them an increase in the exogenous rate of default of firms on bank loans. They argue that price stability is all that matter for monetary policy, even when financial factors are present. Our work differs from Dellas et al. (2010) in a number of ways. Our model features a housing sector but no financial sector; default on loans is endogenous in our model while it is exogenous in Dellas et al. (2010). Iacoviello (2010) introduces a banking sector in a model with housing and studies an exogenous shock to how much borrowers repay. This repayment shock is exogenous and different from default because borrowers do not lose their houses following a negative repayment shock.

We analyze the dynamic response of the model to an increase in idiosyncratic housing investment risk. Other papers model an increase in risk. Christiano et al. (2009) introduce a financial accelerator as in Bernanke et al. (1999) in a model of the financial sector and analyze an increase in the standard deviation of idiosyncratic risk in loans to entrepreneurs. In our setting, idiosyncratic risk is in mortgage loans. Cohen-Cole and Martinez-Garcia (2008) also consider idiosyncratic risk in loans to entrepreneurs and introduce systemic risk as a correlation between the mean of the idiosyncratic risk and the aggregate shock, as in Faia and Monacelli (2007). We do not consider systemic risk but allow for exogenous shocks to the standard deviation of the distribution of idiosyncratic risk.

3 The Model

Our starting point is a model with patient and impatient households that consume non-durable goods and housing service and work. Many features of our model draw from the housing model of Iacoviello (2005), Iacoviello and Neri (2010) and Monacelli (2009). Our focus, however, is on the mortgage contract and on how its features matter for the transmission of shocks. We do not rely on an exogenous borrowing constraint but rather derive it endogenously from the lenders' participation constraint after explicitly introducing idiosyncratic risk and default.

3.1 Households

The economy is populated by a continuum of households distributed over the $[0, 1]$ interval. A fraction ψ of identical households has discount factor β while the remaining fraction $1 - \psi$

has discount factor $\gamma > \beta$. We are going to refer to the households with the lower discount factor as Borrowers, as these households value current consumption relatively more than the other agents and therefore want to borrow. We are going to refer to households with the higher discount factor as Savers.

Borrowers

Borrowers have a lifetime utility function given by

$$\max \sum_{t=0}^{\infty} \beta^t E_0 \left\{ U\left(X_t, N_{C,t}, N_{H,t}\right) \right\} \quad 0 < \beta < 1 \tag{1}$$

where $N_{C,t}$ is hours worked in the non-durable sector, $N_{H,t}$ is hours worked in the housing sector, and X_t is an index of non-durable and durable consumption services defined as

$$X_t \equiv \left[(1-\alpha)^{\frac{1}{\eta}} C_t^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} H_{t+1}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{2}$$

where C_t denotes consumption of non-durable goods, H_t denotes consumption of housing services, α is the share of housing in the consumption index and $\eta \geq 0$ is the elasticity of substitution between housing and non-durable services. We assume that housing services in period t are equal to the housing stock at the beginning of period t. Assuming that services are a fraction of the stock is not going to change our results qualitatively.

We assume the following utility function:

$$U(X_t, N_t) \equiv \ln X_t - \frac{\nu}{1+\varphi} \left[N_{C,t}^{1+\xi} + N_{H,t}^{1+\xi} \right]^{\frac{1+\varphi}{1+\xi}}, \qquad \varphi, \xi \geq 0. \tag{3}$$

Our specification for the disutility of labor follows Iacoviello and Neri (2010) in allowing that hours in the non-durable and housing sector are imperfect substitutes, as consistent with the evidence found by Horvath (2000). For $\xi = 0$ hours in the non-durable and housing sector are perfect substitutes. On the other hand, positive values of ξ result in wages not being equalized in the two sectors and the substitution of hours across sectors in response to wage differentials being reduced. The parameter φ is the inverse of the Frisch labor supply elasticity.

Borrowers are subject to the sequence of budget constraints:

$$P_{C,t}C_t + P_{H,t}H_{t+1} + [1 - F(\bar{\omega}_t)](1 + R_{Z,t})L_t = L_{t+1} + W_{C,t}N_{C,t} + W_{H,t}N_{H,t} +$$

$$(1 - \delta)\,[1 - G(\bar{\omega}_t)]\,P_{H,t}H_t, \tag{4}$$

where $P_{C,t}$ is the price of non-durable goods, $P_{H,t}$ is the price of housing, L_{t+1} are the loans taken from Savers at t to be repaid in period $t+1$. $R_{Z,t}$ is the state-contingent interest rate that non-defaulting Borrowers pay at t on the loans L_t taken at time $t-1$ and $W_{j,t}$ is the nominal wage in sector $j = C, H$. In equilibrium some loans are going to be defaulted on. The term $[1 - G(\bar{\omega}_t)]$ represents Borrowers' housing stock at the end of period t and after a fraction of the loans has been defaulted on; $[1 - F(\bar{\omega}_t)]$ is the fraction of loans that is repaid to lenders. We explicitly derive and explain these two terms later. The housing stock depreciates at the rate δ. Each household decides non-durable good consumption, housing investment (and consumption), working hours in the two sectors and loans.

It is worth noticing at this point that the state-contingent interest rate $R_{Z,t}$ is determined at time t after the realization of shocks and in order to satisfy the participation constraint of lenders, which we explain in detail later. Hence, our mortgage contract is characterized by adjustable interest rates.

Each household consists of many members. The household decides total housing investment H_{t+1}, the state-contingent mortgage rates to be paid next period on the contracts signed this period, and then assigns the housing stock H_{t+1}^i to the i-th member, where $\int_i H_{t+1}^i di = H_{t+1}$. The $i-$th member finalizes the mortgage contract connected to the housing stock H_{t+1}^i following the instructions of the household and manages his housing stock. All members are ex-ante identical. After the mortgage contract is finalized, the i-th household member experiences an idiosyncratic shock ω_{t+1}^i such that ex-post housing is $\omega_{t+1}^i H_{t+1}^i$. This assumption captures the idea that housing investment is risky. The random variable ω_{t+1}^i is i.i.d. across members of the same household and log-normally distributed with a cumulative distribution function $F_t(\omega)$, which obeys standard regularity conditions.[3] The mean and variance of $\ln \omega$ are chosen so that

[3]The c.d.f. is continuous, at least once-differentiable, and it satisfies

$$\frac{\partial \omega h(\omega)}{\partial \omega} > 0,$$

$E_t(\omega_{t+1}^i) = 1$ at all times. This implies that while there is idiosyncratic risk at the household-member level, there is no risk at the household level and $E_t(\omega_{t+1}^i H_{t+1}^i) = H_{t+1}$. We are going to assume that housing investment riskiness can change over time, namely that the variance $\sigma_{\omega,t}$ of $\ln \omega$ is subject to an exogenous shock and therefore displays time variation. The random variable ω_{t+1}^i is observed by the household but can only be observed by lenders after paying a monitoring cost.

After idiosyncratic shocks are realized, the household decides which loans to repay and which to default. Loans connected to housing stocks that experienced high realizations of the idiosyncratic shock ω_{t+1}^i are repaid while loans connected to housing stocks with low realizations are defaulted on. Let $\bar{\omega}_{t+1}$ be the threshold value of the idiosyncratic shock for which the member is able to repay the loan at the contractual rate $R_{Z,t+1}$, namely

$$\bar{\omega}_{t+1}(1-\delta)P_{H,t+1}H_{t+1} = (1 + R_{Z,t+1})L_{t+1}. \tag{5}$$

Loans connected to $\omega_{t+1}^i \in [\bar{\omega}_{t+1}, \infty]$ are repaid and lenders receive $(1 + R_{Z,t+1})L_{t+1}$. On the other hand, loans connected to $\omega_{t+1}^i \in [0, \bar{\omega}_{t+1}]$ are defaulted. For the defaulted loans lenders pay a monitoring cost to assess and seize the collateral. It is exactly the presence of monitoring that induces Borrowers to truthfully reveal their idiosyncratic shock and justifies the incentive compatibility constraint (5). As in Bernanke et al. (1999), we assume that the monitoring cost is equal to the fraction μ of the housing value, $\omega_{t+1}^i P_{H,t} H_{t+1}$, and that the defaulting household member looses his housing stock. Regarding the defaulting household members we follow the literature on matching and assume that there is perfect insurance among household members so that consumption of non-durable goods and housing services are ex-post equal across all members. Hence, Borrower household members are ex-post identical.

Following Bernanke et al. (1999) we consider a one-period mortgage contract that guarantees lenders a pre-determined rate of return on their total loans. At time t Savers make total loans L_{t+1} to Borrowers and demand the gross rate of return $1 + R_{L,t}$. This rate of return is pre-determined at t and non-state contingent. Hence, the time t participation constraint of lenders

where $h(\omega)$ is the hazard rate.

is given by:

$$(1+R_{L,t})L_{t+1} = \int_0^{\bar{\omega}_{t+1}} \omega_{t+1}(1-\mu)(1-\delta)P_{H,t+1}H_{t+1}f(\omega)d\omega + \int_{\bar{\omega}_{t+1}}^{\infty} (1+R_{Z,t+1})L_{t+1}f(\omega)d\omega, \quad (6)$$

where $f(\omega)$ is the probability density function of ω. The return on total loans is equal to the housing stock net of monitoring costs and depreciation of defaulting Borrower members (the first term on the right-hand side of (6)) and the repayment of non-defaulting members (the second term on the right-hand side of (6)). After idiosyncratic and aggregate shocks have realized, the threshold value $\bar{\omega}_{t+1}$ and the state-contingent mortgage rate $R_{Z,t+1}$ are determined so as to satisfy the participation constraint above. Other things equal, an aggregate shock that raises $\bar{\omega}_{t+1}$ and the rate of default on mortgages generates an increase in the adjustable rate $R_{Z,t+1}$ paid by non-defaulting members in order to satisfy the participation constraint (6). This implies that periods characterized by rising mortgage default rates are also accompanied by rising mortgage interest rates in our model.

We use one-period debt contracts as in Bernanke et al. (1999) for tractability reasons. In reality, conventional U.S. mortgages typically have a fixed 30-year term and about 70% of mortgages have fixed rates, even though this percentage has changed a lot in recent years. Moreover, subprime mortgages with nontraditional ARM features were at the heart of the recent crisis. Our model does not consider these alternative mortgage instruments and therefore it cannot capture their role at the onset of the crisis. Nevertheless, our general equilibrium model captures the effect of a fall in housing prices on mortgage default rates and its transmission to the other sector of the economy.

We simplify the Borrower problem as follows. Let

$$G(\bar{\omega}_{t+1}) \equiv \int_0^{\bar{\omega}_{t+1}} \omega_{t+1}f(\omega)d\omega \quad (7)$$

be the expected value of the idiosyncratic shock conditional on the shock being less than or equal to the threshold value $\bar{\omega}_{t+1}$ and let

$$\Gamma(\bar{\omega}_{t+1}) \equiv \bar{\omega}_{t+1}\int_{\bar{\omega}_{t+1}}^{\infty} f(\omega)d\omega + G(\bar{\omega}_{t+1}) \quad (8)$$

be the expected gross share of housing value that goes to lenders. Then the participation constraint in terms of non-durable consumption prices can be written more compactly as

$$(1 + R_{L,t})l_{t+1} = (\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1}))(1 - \delta)p_{H,t+1}\pi_{C,t+1}H_{t+1}, \tag{9}$$

where $p_{H,t+1}$ is the relative price of houses in terms of non-durable consumption and l_{t+1} are real loans, namely $L_{t+1}/P_{C,t}$. The loan-to-value ratio is given by

$$\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1}), \tag{10}$$

and it measures the size of the loan (principal plus interests) as a fraction of the housing value net of depreciation. The loan-to-value ratio also measures the net share of the housing value that goes to the lender for repayment.

Following the decision to default at time t, Borrowers are left with the following stock of housing:

$$\int_{\bar{\omega}_{t+1}}^{\infty} \omega_{t+1}(1 - \delta)P_{H,t}H_{t+1}f(\omega)d\omega = [1 - G(\bar{\omega}_{t+1})]\,(1 - \delta)P_{H,t}H_{t+1}, \tag{11}$$

where the second equality makes use of the fact that $E_t(\omega_{t+1}) = 1$. This is the expression used in the Borrowers budget constraint (4).

Using the relationship between $\bar{\omega}_{t+1}$ and $R_{Z,t+1}$ we can eliminate $R_{Z,t}$ from the Borrower budget constraint and rewrite it in real terms as

$$C_t + p_{H,t}H_{t+1} + (1 + R_{L,t-1})\frac{l_t}{\pi_{C,t}} = l_{t+1} + (1 - \delta)\,[1 - \mu G(\bar{\omega}_t)]\,p_{H,t}H_t + w_{C,t}N_{C,t} + w_{H,t}N_{H,t}, \tag{12}$$

where $\pi_{C,t}$ is non-durable-good inflation and $w_{C,t}, w_{H,t}$ are real wages in the C and H sector, respectively, in terms of $P_{C,t}$. Borrowers maximize (1) subject to the budget constraint (12) and the participation constraint (9) with respect to the variables $C_t, H_{t+1}, N_{C,t}, N_{H,t}, l_{t+1}, \bar{\omega}_{t+1}$. The respective first-order conditions are:

$$U_{C,t} - \lambda_{BC,t} = 0, \tag{13}$$

$$U_{H,t+1} - \lambda_{BC,t}p_{H,t} + \beta(1 - \delta)E_t\left\{[1 - \mu G(\bar{\omega}_{t+1})]\,p_{H,t+1}\lambda_{BC,t+1} + \right. \tag{14}$$

$$\left. \lambda_{PC,t+1}p_{H,t+1}\pi_{C,t+1}\,[\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1})]\right\} = 0,$$

$$U_{N_j,t} + \lambda_{BC,t} w_{j,t} = 0 \quad j \in [C, H] \tag{15}$$

$$\lambda_{BC,t} - (1 + R_{L,t}) E_t \left[\lambda_{PC,t+1} + \beta \frac{\lambda_{BC,t+1}}{\pi_{C,t+1}} \right] = 0, \tag{16}$$

$$-\beta \lambda_{BC,t+1} \mu G'(\bar{\omega}_{t+1}) + \lambda_{PC,t+1} \pi_{C,t+1} \left[\Gamma'(\bar{\omega}_{t+1}) - \mu G'(\bar{\omega}_{t+1}) \right] = 0, \tag{17}$$

where $\lambda_{BC,t}$ is the Lagrangian multiplier on Borrowers budget constraint (12) and $\lambda_{PC,t+1}$ is the Lagrangian multiplier on the participation constraint (9). Notice that the first-order condition with respect to $\bar{\omega}_{t+1}$ is state-by-state and not in expected terms.

Savers

We denote Savers' variables with a ˜, except for loans. Savers maximize lifetime utility

$$\max \sum_{t=0}^{\infty} \gamma^t E_0 \left\{ U(\widetilde{X}_t, \widetilde{N}_{C,t}, \widetilde{N}_{H,t}) \right\} \quad 0 < \beta < \gamma < 1 \tag{18}$$

where $\widetilde{X}_t$ is defined similarly to (2). We assume for Savers the same utility function as for Borrowers. Savers maximize lifetime utility subject to the sequence of budget constraints:

$$\widetilde{C}_t + p_{H,t} \widetilde{H}_{t+1} + \widetilde{l}_{t+1} = (1 - \delta) p_{H,t} \widetilde{H}_t + (1 + R_{L,t-1}) \frac{\widetilde{l}_t}{\pi_{C,t}} + w_{C,t} \widetilde{N}_{C,t} + w_{H,t} \widetilde{N}_{H,t} + \widetilde{\Delta}_t, \tag{19}$$

where $\widetilde{\Delta}_t$ are profits in the intermediate goods sector, which are taken as given.

Savers maximize (18) subject to the budget constraint (19) with respect to $\widetilde{C}_t, \widetilde{H}_{t+1}, \widetilde{N}_{C,t}, \widetilde{N}_{H,t}, \widetilde{l}_{t+1}$. The first-order conditions, respectively, are

$$U_{\widetilde{C},t} - \widetilde{\lambda}_{BC,t} = 0, \tag{20}$$

$$U_{\widetilde{H},t+1} - \widetilde{\lambda}_{BC,t} p_{H,t} + \gamma (1 - \delta) E_t \left[\widetilde{\lambda}_{BC,t+1} p_{H,t+1} \right] = 0, \tag{21}$$

$$U_{\widetilde{N},j,t} + \widetilde{\lambda}_{BC,t} w_{j,t} = 0 \quad j \in [C, H], \tag{22}$$

$$-\widetilde{\lambda}_{BC,t} + \gamma (1 + R_{L,t}) E_t \left[\frac{\widetilde{\lambda}_{BC,t+1}}{\pi_{C,t+1}} \right] = 0, \tag{23}$$

where $\widetilde{\lambda}_{BC,t}$ is the Lagrangian multiplier on Savers budget constraint (19).

3.2 Firms and Technology

Both the non-durable C and the housing H sector have intermediate and final good producers.

Final Good Producers

Final good producers are perfectly competitive and produce $Y_{j,t}$, $j = C, H$. The technology in the $j-$th final good sector is given by

$$Y_{j,t} = \left(\int_0^1 Y_{j,t}(i)^{\frac{\varepsilon_j - 1}{\varepsilon_j}} di \right)^{\frac{\varepsilon_j}{\varepsilon_j - 1}}, \tag{24}$$

where $\varepsilon_j > 1$ is the elasticity of substitution among intermediate goods in sector j. Standard profit maximization implies that the demand for intermediate good i is given by

$$Y_{j,t}(i) = \left(\frac{P_{j,t}(i)}{P_{j,t}} \right)^{-\varepsilon_j} Y_{j,t}, \ \forall i \tag{25}$$

where the price index is

$$P_{j,t} = \left(\int_0^1 P_{j,t}(i)^{1-\varepsilon_j} di \right)^{\frac{1}{1-\varepsilon_j}}.$$

Intermediate Good Sectors

There are two intermediate good sectors $j \in [C, H]$ and in each intermediate sector there is a continuum of firms each producing a differentiated good $i \in [0, 1]$. These firms are monopolistically competitive. We assume that intermediate good firms readjust their price according to a Calvo-type mechanism. Hence, in any given period a firm in sector j may reset its price with probability $1 - \theta_j$.

Intermediate good firm i produces according to the following production function

$$Y_{j,t}(i) = A_{j,t} \left[\zeta^{\frac{1}{\varsigma}} N_{j,t}(i)^{\frac{\varsigma - 1}{\varsigma}} + (1 - \zeta)^{\frac{1}{\varsigma}} \widetilde{N}_{j,t}(i)^{\frac{\varsigma - 1}{\varsigma}} \right]^{\frac{\varsigma}{\varsigma - 1}}, \tag{26}$$

where $A_{j,t}$ is the stochastic level of technology in sector j and $N_{j,t}(i)$ and $\widetilde{N}_{j,t}(i)$ are the two labor types respectively supplied by Borrowers and Savers. $\zeta \in (0, 1)$ is the labor share of Borrowers in the production function and $\varsigma > 0$ is the elasticity of substitution across labor inputs. For simplicity these two parameters are assumed to be equal across sectors.

A firm i re-optimizing in period t chooses labor and its nominal price $P^*_{j,t}(i)$ so as to maximize the expected discount sum of nominal profits over the period during which its price remains unchanged. Thus, the maximization problem for firm i is given by

$$\max_{P^*_{j,t}(i),N_{j,t+k|t}(i),\widetilde{N}_{j,t+k|t}(i)} E_t\left\{\sum_{k=t}^{\infty}\theta_j^k\Lambda_{t,t+k}\left[P^*_{j,t}(i)Y_{j,t+k|t}(i)-W_{j,t+k}N_{j,t+k|t}(i)-\widetilde{W}_{j,t+k}\widetilde{N}_{j,t+k|t}(i)\right.\right.$$
$$\left.\left.+mc_{j,t+k|t}(i)P_{j,t}\left[A_{j,t}\left[\zeta^{\frac{1}{\varsigma}}N_{j,t+k|t}(i)^{\frac{\varsigma-1}{\varsigma}}+(1-\zeta)^{\frac{1}{\varsigma}}\widetilde{N}_{j,t+k|t}(i)^{\frac{\varsigma-1}{\varsigma}}\right]^{\frac{\varsigma}{\varsigma-1}}-Y_{j,t+k|t}(i)\right]\right]\right\}, \tag{27}$$

where $Y_{j,t+k|t}(i)$ denotes output in period $t+k$ for a firm i that last changed its price in period t. A similar interpretation applies to $N_{j,t+k|t}(i)$ and $\widetilde{N}_{j,t+k|t}(i)$. $mc_{j,t+k|t}(i)$ is the real marginal cost of firm i.

In (27) the demand and the stochastic discount are respectively given by

$$Y_{j,t+k|t}(i)=\left(\frac{P^*_{j,t}(i)}{P_{j,t+k}}\right)^{-\varepsilon_j}Y_{j,t+k}, \qquad \Lambda_{t,t+k}\equiv\frac{\gamma^k\widetilde{\lambda}_{BC,t+k}}{\widetilde{\lambda}_{BC,t}}.$$

The first-order condition relative to is $N_{j,t+k|t}(i)$ and $\widetilde{N}_{j,t+k|t}(i)$ are

$$-W_{j,t+k}+mc_{j,t+k|t}(i)P_{j,t+k}Y_{j,t+k|t}(i)^{-\frac{1}{\varsigma}}\zeta^{\frac{1}{\varsigma}}N_{j,t+k|t}(i)^{-\frac{1}{\varsigma}}=0, \tag{28}$$

$$-\widetilde{W}_{j,t+k}+mc_{j,t+k|t}(i)P_{j,t+k}Y_{j,t+k|t}(i)^{-\frac{1}{\varsigma}}(1-\zeta)^{\frac{1}{\varsigma}}\widetilde{N}_{j,t+k|t}(i)^{-\frac{1}{\varsigma}}=0, \tag{29}$$

which state that the nominal marginal cost equals the ratio of the nominal wage to the marginal product of each type of labor input. By rearranging (28) and (29) we obtain:

$$mc_{j,t+k|t}(i)=\frac{1}{A_{j,t+k}P_{j,t+k}}\left[\zeta W_{j,t+k}^{1-\varsigma}+(1-\zeta)\widetilde{W}_{j,t+k}^{1-\varsigma}\right]^{\frac{1}{1-\varsigma}}. \tag{30}$$

According to (30), $mc_{j,t+k|t}(i)=mc_{j,t+k}$. Marginal costs are equal across firms because wages are the same across all firms.

The first-order condition relative to the price is given by

$$E_t\left\{\sum_{k=t}^{\infty}\theta_j^k\Lambda_{t,t+k}\left[\left(\frac{P^*_{j,t}(i)}{P_{j,t+k}}\right)^{(-\varepsilon_j-1)}Y_{j,t+k}\left(\frac{P^*_{j,t}(i)}{P_{j,t+k}}-\frac{\varepsilon}{\varepsilon-1}mc_{t+k}\right)\right]\right\}=0. \tag{31}$$

Finally, it can be shown[4] that, under Calvo price setting, the optimal price set by re-optimizing firms is linked to the aggregate price behavior by the following condition:

$$\left(\frac{P_{j,t}^{*}}{P_{j,t}}\right)^{(1-\varepsilon)} = \frac{1-\theta_j \pi_{j,t}^{\varepsilon-1}}{1-\theta_j}, \tag{32}$$

where $\pi_{j,t}$ denotes gross inflation in sector j prices.

3.3 Monetary Policy

We assume that monetary policy follows a Taylor-type rule for the nominal interest rate:

$$\frac{1+R_{L,t}}{1+R_L} = A_{M,t}\left[\pi_{C,t}^{\phi_\pi}\right]^{1-\phi_r}\left[\frac{1+R_{L,t-1}}{1+R_L}\right]^{\phi_r}, \quad \phi_\pi > 1,\ \phi_r < 1. \tag{33}$$

where R_L is the steady-state nominal interest rate, ϕ_π is the coefficient on the inflation target, ϕ_r is the coefficient on the lagged interest rate, and $A_{M,t}$ is a monetary policy shock. In our benchmark calibration monetary policy targets inflation in the non-durable goods sector and implements interest-rate smoothing. Assuming that monetary policy targets inflation in both sectors does not affect our results.

3.4 Market Clearing

Equilibrium in the non-durable goods market requires that production of the final non-durable good net of adjustment costs equals aggregate demand:

$$Y_{C,t} = \psi C_t + (1-\psi)\widetilde{C}_t. \tag{34}$$

Similarly, equilibrium in the housing market requires

$$Y_{H,t} = \psi\left[H_{t+1} - (1-\delta)(1-\mu G(\bar{\omega}_t))H_t\right] + (1-\psi)\left[\widetilde{H}_{t+1} - (1-\delta)\widetilde{H}_t\right]. \tag{35}$$

[4]For a formal proof see for instance Woodford (2003).

Equilibrium in the labor market requires

$$\int_0^1 L_{C,t}(i)di = \psi N_{C,t} + (1-\psi)\widetilde{N}_{C,t}, \tag{36}$$

$$\int_0^1 L_{H,t}(i)di = \psi N_{H,t} + (1-\psi)\widetilde{N}_{H,t}, \tag{37}$$

while the equilibrium in the credit market requires

$$\psi l_t = (1-\psi)\widetilde{l}_t. \tag{38}$$

We define total output as

$$Y_t = Y_{C,t} + p_{h,t} Y_{H,t}. \tag{39}$$

Notice that our measurement of total output reflects variations in the relative price of housing. National account statistics, on the other hand, measure GDP at constant relative prices.

3.5 Exogenous Shocks

There are five exogenous shocks in our model. Aggregate productivity in the two sectors and the monetary policy shock evolve according to the following first-order autoregressive processes

$$\ln A_{C,t} = \rho_C \ln A_{C,t-1} + \epsilon_{C,t}, \quad \rho_C \in (-1,1), \tag{40}$$

$$\ln A_{H,t} = \rho_H \ln A_{H,t-1} + \epsilon_{H,t}, \quad \rho_H \in (-1,1), \tag{41}$$

$$\ln A_{M,t} = \rho_M \ln A_{M,t-1} + \epsilon_{M,t}, \quad \rho_M \in (-1,1), \tag{42}$$

where $\epsilon_C, \epsilon_H, \epsilon_M$ are i.i.d. innovations with mean zero and standard deviation $\sigma_C, \sigma_H, \sigma_M$, respectively, and ρ_C, ρ_H, ρ_M are persistence parameters.

As for the idiosyncratic risk in the housing sector, we follow Bernanke et al. (1999) and assume that ω is distributed log-normally:

$$\ln \omega \sim N(-\frac{\sigma_\omega^2}{2}, \sigma_\omega^2). \tag{43}$$

As stated earlier, the mean of the distribution is chosen so that $E(\omega) = 1$. Later we are going to analyze the case where idiosyncratic risk exogenously increases. To do this, we assume that the standard deviation of $\ln \omega$ is itself an exogenous shock subject to a first-order autoregressive process:

$$\ln \frac{\sigma_{\omega,t}}{\sigma_\omega} = \rho_\sigma \ln \frac{\sigma_{\omega,t-1}}{\sigma_\omega} + \epsilon_{\sigma_{\omega,t}}, \tag{44}$$

where ϵ_{σ_ω} is an i.i.d. shock with mean zero and finite standard deviation σ_{σ_ω} and ρ_σ is the serial correlation coefficient.

4 Steady-State Analysis

4.1 Benchmark Calibration

The parameters values for our benchmark calibration are specified in Table 1. We follow Monacelli (2009) in choosing the values for the discount factors for Borrowers and Savers, the rate of depreciation for housing and the elasticity of substitution between non-durable goods and housing services. The Saver's discount factor γ is set equal to 0.99 and Borrower's β is set equal to 0.98. We choose an annual depreciation rate for housing of 4 percentage points, implying $\delta = 0.01$. The elasticity of substitution between non-durable consumption and housing is $\eta = 1$, which implies a Cobb-Douglas specification for the composite consumption index X_t.

U.S. private fixed investment in structures, residential and nonresidential, has been on average 5 percent of GDP from 1960 to 2009, but during the period 2000 to 2007 it averaged 8 percent of GDP. We set the parameter α that measures the share of housing in the consumption bundle equal to 0.16 so that the housing sector represents 8 percent of total output at the steady state. The Saver discount factor pins down the steady-state interest rate at $R_L = 0.0101$ on a quarterly basis. This implies an annual interest rate equal of 4.1 percentage points. The inverse of the Frisch elasticity of labor supply φ is set equal to one, as in Barsky, House and Kimball (2007) and as typical in the macro literature. As for the parameter ξ that measures the degree of substitutability between hours worked in the two sectors, we set it equal to 0.871. This is the appropriate weighted average of the ξ for Borrowers and Savers estimated by Iacoviello and Neri (2010).

For the degree of price stickiness, we assume that housing prices are fully flexible and set

Parameter	Value	Description
γ	0.99	Discount factor of Savers
β	0.98	Discount factor of Borrowers
ψ	0.5	Relative size of Borrower group
δ	0.01	Rate of depreciation for housing
ε_C	7.5	Elasticity of substitution for C goods
ε_H	7.5	Elasticity of substitution for H goods
ς	3	Elasticity of substitution across labor inputs
ζ	0.5	Share of Borrower labor in the production function
ξ	0.871	Elasticity of substitution across labor types
α	0.16	Share of housing in consumption bundle
ν	2.5	Disutility from work
η	1	Elasticity of substitution between C and H goods
φ	1	Inverse of elasticity of labor supply
θ_C	0.67	Calvo probability in C
θ_H	0	Calvo probability in H
ϕ_π	1.5	Taylor-rule coefficient on inflation
ϕ_r	0.9	Taylor-rule coefficient on past nominal interest rate
ρ_C	0.9	Serial correlation of productivity shocks in C
ρ_H	0.9	Serial correlation of productivity shocks in H
ρ_M	0	Serial correlation of monetary policy shocks
σ_ω	0.20	Standard deviation of idiosyncratic shocks
μ	0.12	Monitoring cost

Table 1: Benchmark Calibration

$\theta_H = 0$, which is in line with the empirical estimation of Iacoviello and Neri (2010) and the empirical evidence on price stickiness for durable goods. For non-durable goods, θ_C is set equal to 0.67 to imply that firms in the non-durable sector change their prices on average every nine months. For monetary policy, we set $\phi_\pi = 1.5$ and $\phi_r = 0.9$, which are standard values in the literature. The serial correlation of the monetary policy shock is $\rho_M = 0$. We assume that the Borrower and Saver groups have equal size so that $\psi = 0.5$.

For technology, we follow Calza et al. (2009) and set the elasticity of substitution among intermediate goods ε_j equal to 7.5 in each sector. Labor inputs are imperfect substitutes in production and the elasticity of substitution across Borrower's and Saver's labor is $\varsigma = 3$. We also assume that the share of Borrower's labor in the production function ζ is equal to 0.5. The serial correlation of the productivity shocks in the non-durable and housing sectors are chosen to be $\rho_C = 0.9$ and $\rho_H = 0.9$, respectively.

Table 2 reports the steady-state values for the benchmark calibration. The loan-to-value ratio is defined in equation (10) and the leverage ratio for Borrowers at the steady state is calculated as

$$\text{Leverage Ratio} \; = \; \frac{l}{l + w_C N_c + w_H N_H},$$

which measures the fraction of total expenses financed by loans, namely consumption of C and H plus loan repayment over loans. The leverage ratio captures the dependence of Borrowers from external funding.

We choose the variance of the distribution of $\ln \omega$ to match the pre-crisis delinquency rates for prime loans. In the fourth quarter of 2006 seriously delinquent mortgages represented 2.21 percentage points of all mortgages.[5] We set $\sigma_\omega = 0.2$ and obtain an annual default rate of 2.34 percentage points. As for the shocks that raise the volatility of idiosyncratic housing risk, we believe they are persistent but there is no previous work we can rely on. Christiano et al. (2009) estimate the persistence of the idiosyncratic productivity shock for the United States to be 0.85. We set $\rho_\sigma = 0.9$. The median foreclosure price for single-family residencies, condominiums, and townhouses in California in the first half of 2006 was 12% lower than the median market price of homes sold within the same area without having been previously foreclosed – see Cagan (2006). Hence, we set $\mu = 0.12$ and monitoring costs are 12% of the housing value. Interestingly, this is also the value for monitoring cost in Bernanke et al. (1999). At the steady state, the loan-to-value ratio in our benchmark calibration is almost 60 percentage points. This is lower than 75.7 percentage points, which is the average U.S. loan-to-value ratio between 1973 and 2008. We can raise the steady-state loan-to-value ratio by reducing σ_ω, the riskiness of loans; however this is going to reduce the steady-state rate of default. For this reason, we prefer to match the delinquency rate but have a lower loan-to-value ratio than suggested by the data.

At the steady state, the quarterly mortgage rate rate paid by non-defaulting Borrowers is $R_Z = 0.0111$, which corresponds to an annual rate of 4.51 percentage points. We define the external finance premium at t as $R_{Z,t} - R_{L,t}$, namely the difference between the ex-post state-contingent rate paid by non-defaulting household members and the risk-free interest rate, which in our setting is equivalent to the pre-determined rate received by lenders on aggregate loans.[6]

[5] See the National Delinquency Survey conducted by the Mortgage Bankers Association.

[6] Our definition of external finance premium differs from that in Bernanke et al. (1999), where the premium is the difference between the costs of funds raised externally and the opportunity costs of funds internal to the firm.

Variable	
Consumption, Borrowers*	44.28
Housing Demand, Borrowers*	39.40
Hours Worked in C Sector, Borrowers*	54.30
Hours Worked in H Sector, Borrowers*	54.30
p_h Output H/Total Output	8.09
Loans	2.1747
Loan-to-Value Ratio	59.17
Leverage Ratio	80.12
Default Rate on Mortgages†	2.34
External Finance Premium†	0.41
Mortgage Interest Rate†	4.51

* Expressed as percentage of aggregate level, e.g. Consumption, Borrowers = $\psi C/(\psi C + (1-\psi)\tilde{C})$.
†Annual, percentage points.
Note: The Leverage Ratio is calculated as $l/(l + w_C N_c + w_H N_H)$.

Table 2: Steady State under the Benchmark Calibration

This premium captures the additional cost that Borrowers must pay for their risky mortgages relative to risk-free borrowing. At the steady state, the external finance premium is equal to 0.41 percentage points on an annual basis. We calculate the empirical counterpart to our external finance premium as the difference between the 30-Year Conventional Mortgage Rate[7] and the interest rate on the U.S. Treasury 30-Year bonds.[8] The average difference between these two annual interest rates between 1977 and 2009 was 1.5 percentage points. This makes the finance premium of our benchmark model one percentage point lower than its empirical counterpart.

4.2 Low-Leverage Calibration

We additionally consider an economy characterized by higher idiosyncratic risk but otherwise identical to the benchmark economy described above. For this economy the steady-state standard deviation of the distribution of idiosyncratic housing risk is set to $\sigma_\omega = 0.6$. All the other parameters are as in Table 1. Higher idiosyncratic housing risk implies that more household members are in the left tail of the distribution and that the steady-state rate of default is higher. This raises the external finance premium and reduces mortgage loans, the loan-to-

[7]See the H-15 Release of the Federal Reserve Economic Data
[8]See the Economic Report to the President, Table B73.

Variable	Benchmark	Low Leverage	% Difference
Output C	0.5407	0.5399	0.15
Output H	0.1465	0.1419	3.24
Consumption, Borrowers	0.4789	0.4887	-2.01
Consumption, Savers	0.6026	0.5912	1.93
Housing Demand, Borrowers	11.5421	10.5337	9.57
Housing Demand, Savers	17.7524	17.8431	-0.51
Hours Worked, Borrowers in C Sector	0.5879	0.5789	1.55
Hours Worked, Borrowers in H Sector	0.1617	0.1549	4.41
Hours Worked, Savers in C Sector	0.4948	0.5019	-1.41
Hours Worked, Savers in H Sector	0.1361	0.1343	1.37
Loans	2.1747	0.7980	**172.54**
Loan-to-Value Ratio*	59.17	24.37	**142.80**
Leverage Ratio*	80.12	60.01	**33.51**
Default Rate on Mortgages†	2.36	8.21	**-71.22**
External Finance Premium†	0.41	2.44	-83.20
Mortgage Interest Rate†	4.51	6.54	-31.04

* Percentage points.
†Annual, percentage points.

Table 3: Benchmark and Low-Leverage Economies: Steady-state Comparison

value ratio and leverage ratio relative to the benchmark economy. We label this calibration as "Low-Leverage" and compare it with the benchmark, "High-Leverage" calibration.

Table 3 reports the steady-state values of some endogenous variables in the benchmark and the Low-Leverage economy. The last column of Table 3 reports the percentage point difference of steady-state values between benchmark and Low-Leverage economies. Because loans are larger when idiosyncratic housing risk is lower, steady-state loans are 173 percentage points higher in the benchmark economy, the loan-to-value ratio is 143 percentage points higher and the leverage ratio is 34 percentage points higher. At the same time, the mortgage interest rate and the external finance premium are considerably lower in the benchmark economy. As a result, overall economic activity is higher in economies with high leverage ratios.

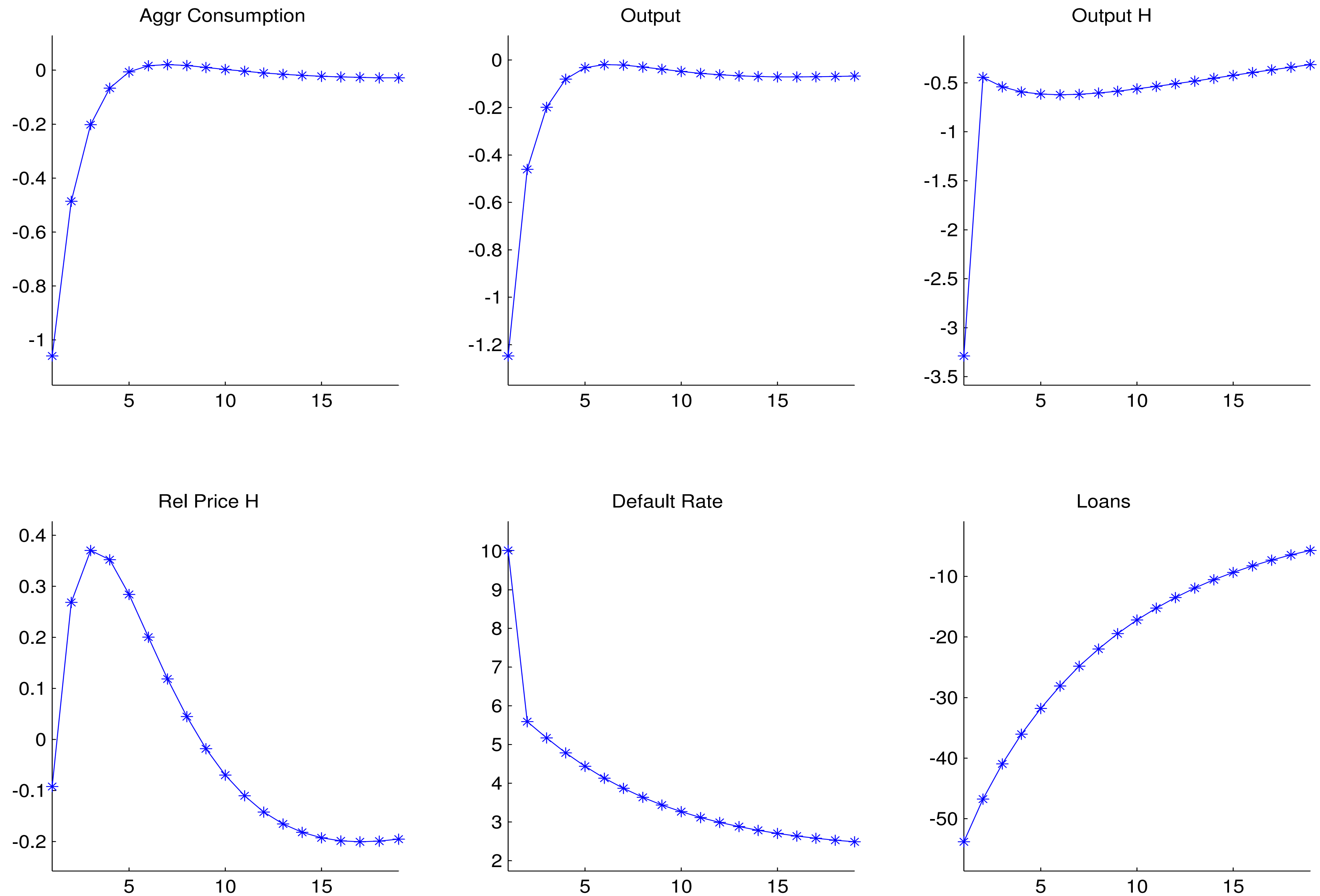
5 Credit Crunch

5.1 Benchmark Calibration

This section analyzes the dynamic response of the model to an unexpected increase in $\sigma_{\omega,t}$, the standard deviation of the distribution of idiosyncratic housing investment risk. This increase in risk wants to capture the situation in which loans are made on the basis of an expected distribution for idiosyncratic risk, but the actual distribution turns out to be characterized by a higher standard deviation. In other words, the riskiness of mortgages changes over time and these changes are persistent. More broadly, exogenous shocks to $\sigma_{\omega,t}$ are an admittedly reduced-form way to capture the entrance in the mortgage market of subprime, or higher risk, debtors. Notice that an increase in $\sigma_{\omega,t}$ has two effects on the distribution of $\ln \omega$: it increases the variance and lowers the mean, which results in a leftward shift of the distribution. See Figure 10 in Appendix B. Since the log-normal distribution does not take negative values, a fall in the mean implies a thicker lower tail of the distribution. Thus, for the same value of $\bar{\omega}$, a higher standard deviation implies a higher cumulative distribution function and therefore a higher default rate on mortgages.

From the fourth quarter of 2006 to the first quarter of 2010 the delinquency rate on U.S. real estate loans increased from 2.21 to 10.44 percentage points, an increase of 823 basis points.[9] Figure 2 shows the impulse responses of six selected variables to a 40% increase in the standard deviation $\sigma_{\omega,t}$. The size of the shock is chosen so as to generate an increase in the default rate of about 800 points. The impulse responses to the same shock but of a larger set of variables are shown in Figure 6 in Appendix B. The default rate increases sharply and, together with it, the monitoring costs. Since the mortgage contract guarantees the risk-free interest rate to aggregate loans, the state-contingent mortgage interest rate must increase to satisfy the lenders' participation constraint, which implies an increase in the external finance premium paid by Borrowers. Our model predicts a 150 basis points increase for the mortgage interest rate and the external finance premium. Borrowers financial conditions worsen significantly. More Borrower household members default on their loans and loose their housing stock, while the non-defaulting household members pay a higher mortgage interest rate. The overall effect

[9]See the National Delinquency Survey of the Mortgage Bankers Association.



Note: Default rate is annual and in percentage points. Loans are difference from steady state, multiplied by 100. All other variables are percentage point deviation from steady state.

Figure 2: Impulse Responses to a 40% Increase in $\sigma_{\omega,t}$

on the budget constraint of the Borrower household is large and negative because the mortgage interest rate paid by non-defaulting household members raises enough to cover the increase in monitoring costs paid by the lenders – see the participation constraint (6). In addition to this, Borrowers experience a tightening in credit conditions due to a reduction in the loan-to-value ratio that reduces their capacity to borrow out of their housing stock. Worsening financial conditions force Borrowers to cut consumption of non-durable goods, housing investment, and loans and to raise hours worked. Savers, on the other hand, are consumption smoothers. In response to a lower real interest rate they reduce lending and consume more non-durable goods. However, Savers' increase in consumption is small relative to the large cut by Borrowers and aggregate consumption of non-durable goods falls.

The non-durable good sector experiences a sharp fall in demand that reduces output and prices. Because prices are sticky in the C sector and cannot fully adjust, the output reduction is significant and more persistent relative to the H sector, where prices are fully flexible. Nevertheless, inflation of non-durable goods falls. Because monetary policy responds to inflation in the C sector, lower inflation in C causes a reduction in the nominal interest rate. The housing sector is also hit by a reduction in demand stemming from Borrowers' housing downsizing, which leads to a fall in housing output.[10] Nominal housing prices are flexible and they adjust instantaneously, causing nominal deflation in housing. As for the relative price of houses, namely the price of housing in terms of non-durable goods, it falls on impact because nominal house prices fall more than nominal non-durable good ones. A lower relative price of housing raises the demand for housing by Savers, who substitute out of loans into housing to smooth their inter-temporal consumption profile. Housing prices bottom out instantaneously and grow afterward returning to their steady-state value, whereas non-durable prices continue falling for two more periods. Two factors contribute to this difference. First, non-durable good prices are sticky while housing prices are flexible; second, housing supply expands faster than the supply of non-durable goods. Borrowers need to replenish their housing stock and this contributes to a quick rebound of housing demand and supply. In fact, housing output gross of monitoring costs (not shown in the graphs) increases following the shock.

Because of imperfect substitutability of hours in the utility function and in the production function, wage differentials emerge in our model. Wage for Borrowers in the housing sector

[10] Output H is measured net of monitoring costs.

falls but less than his wage in the non-durable sector. This wage differential encourages partial switching of hours by Borrowers out of the non-durable into the housing sector and, as a result, hours supplied by Borrowers in the housing sector increase. Wages fall in the C sector for both groups and remain below steady state. Borrowers, who are constrained, still increase hours in the C sector, which lowers their wage in the sector well below those of Savers. As a result, the overall wage index for Borrowers falls more than the wage index for Savers. Total output in the economy falls, independently of whether housing output is measured net or gross of monitoring costs. For consistency, our graphs report output net of monitoring costs. The non-durable sector represents more than 90% of the economy and its dynamics drives that of total output.

In the model of Barsky et al. (2007), a shock that brings down the non-durable sticky-price sector generates an offsetting response in the durable flexible-price sector and GDP is left unchanged, independently of the relative size of the durable good sector. Our model shares some features with that of Barsky et al. (2007), in particular a durable flexible-price sector and a non-durable sticky-price one. The response to a monetary shock (analyzed in the next section) shows hints of a similar behavior. Nevertheless, our results differ. In our model, total output and GDP fall[11] in response to an increase in housing investment risk independently of whether we measure output in the durable sector net or gross of monitoring costs. In our model labor does not move across sectors so as to close wage differentials due to imperfect substitutability both on the supply and demand side. In addition to that, the Borrower household in our model is constrained and fails to behave as a consumption smoother. In particular, Borrower's housing demand is driven by the fact that housing can be used as collateral for borrowing, which makes the shadow value of housing different between Savers and Borrowers.

Our model generates a 1.2 percentage point fall of total output from steady state and a small and short-lived reduction in real housing prices following an increase in housing investment risk. In both these dimensions the model fails to replicate the data. The rapid rebound in housing demand due to the replacement of monitoring costs seems particularly counterfactual. We speculate that adding a housing rental market may improve the fit of our model. We also believe that the credit crunch that followed the liquidity crisis has been quantitatively important in the recession of 2008. Our model, however, has no capital and therefore no role

[11]We do not report GDP measured as total output at steady-state relative prices. Nevertheless, GDP would fall in our model.

for financial intermediation for providing capital to firms. Better modeling of the financial sector and a financial accelerator in loans to firms are indeed interesting extensions of this framework. Nevertheless, our model captures well the transmission of shocks across sectors, as the non-durable sector is significantly affected by a risk shock in the durable sector.

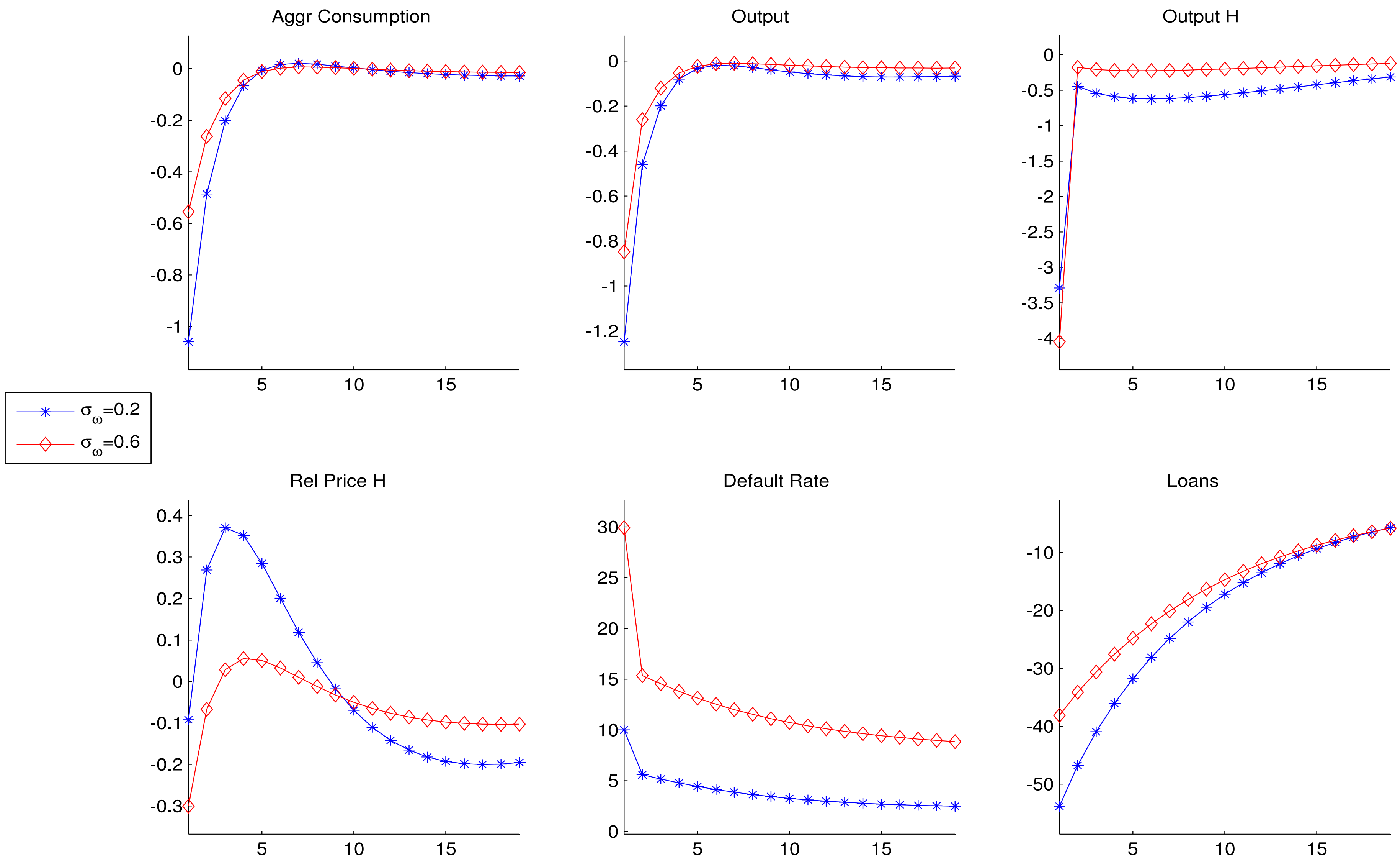
5.2 Low-Leverage Calibration

Next we analyze how differences in steady-state leverage ratios imply different dynamic responses to a housing investment risk shock. Figure 3 plots the impulse responses of six selected variables to a 40% increase in $\sigma_{\omega,t}$ for the Low-Leverage economy (red line with diamonds) and for the benchmark economy (blue starred line). Figure 7 in Appendix B plots the responses for a larger set of variables. The effects of an increase in the idiosyncratic risk in housing investment are amplified in the highly-leveraged benchmark economy, which is the economy with the lower steady-state standard deviation and default rate. In particular, the credit crunch is deeper and the adverse effects on Borrowers stronger. Loans fall more in the High-Leverage economy, which implies that Borrowers must de-leverage more in these economies. As a result, Borrowers consumption of non-durable goods and housing fall more in the economy with high leverage; aggregate consumption and prices of non-durable goods also fall more. The deeper contraction in non-durable demand generates a deeper reduction in total output.

6 Monetary Policy

6.1 Monetary Policy Shock

Figure 4 illustrates the impulse responses of the model under the benchmark calibration in response to a monetary shock, namely a 25 basis point increase in the nominal interest rate $R_{L,t}$. Figure 8 in Appendix B displays the responses of a larger set of variables. Savers, who are consumption smoothers, reduce consumption of non-durable goods in response to higher interest rates. Borrowers are adversely affected in two ways. First, they experience an increase in the cost of borrowing. Second, deflation in non-durable goods raises their real debt via the Fisher effect. As a result, Borrowers reduce mortgage loans and consumption of non-durable goods and housing services. Borrowers also raise internal funds by increasing their labor supply in the



Note: Default rate is annual and in percentage points. Loans are difference from steady state, multiplied by 100. All other variables are percentage point deviation from steady state.

Figure 3: Impulse Responses to a 40% Increase in σ_ω: Low-Leverage Calibration

housing sector, which depresses their real wage in the housing sector fall and thereby the relative price of houses. As the real housing prices falls, more Borrower household members default. Technically, $\bar{\omega}_t$, the threshold value of the idiosyncratic shock below which household members do not repay their mortgages, goes up and with it the default rate and monitoring costs. Since fewer Borrower household members repay their loans, the state-contingent mortgage interest rate rises on impact. The nominal interest rate $R_{L,t}$, which is the risk-free rate guaranteed to Savers on outstanding mortgages to be repaid at time t, was set at time $t-1$ and cannot change on impact. As a result the external financial premium jumps up on impact. The mortgage interest rate increases more than the nominal interest rate and the external finance premium remains above steady state. The negative wealth effect stemming from higher default, higher mortgage rates and lower housing value makes Borrowers further cut their consumption of non-durable goods. Aggregate consumption and production of non-durable goods fall. The fall in relative housing prices raises housing demand by Savers, who substitute out of loans into housing. Monitoring costs reduce the existing stock of housing, thereby raising housing demand. As a result, production in the housing sector increases slightly.

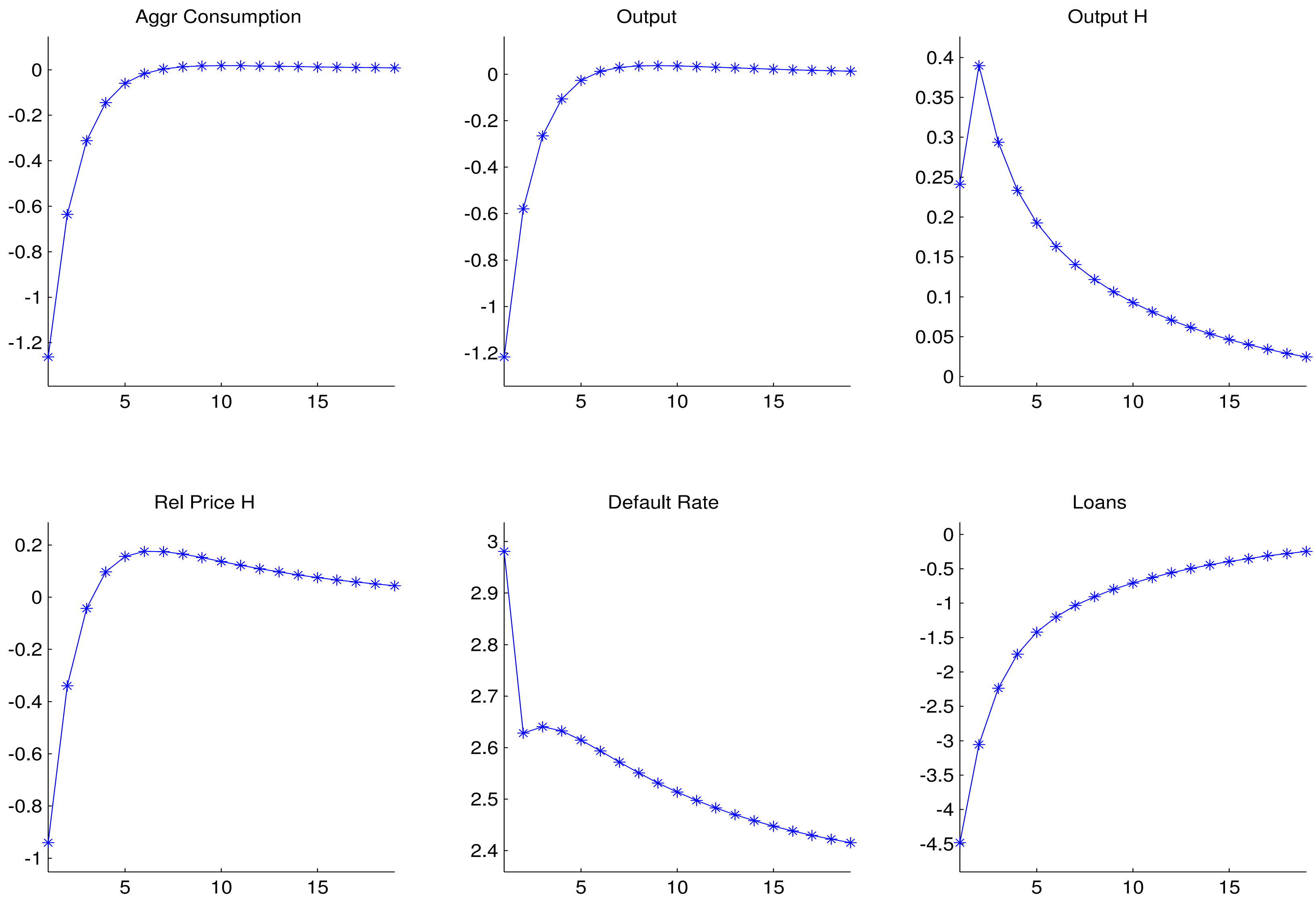
A monetary policy shock causes changes of output of opposite sign in the two sectors, along the lines of Barsky et al. (2007). However, the expansion in the construction sector is small and the contraction in the non-durable sector more than compensates it and total output falls.

6.2 The Importance of Interest Rate Flexibility

Our benchmark calibration features a large inertia term in the interest rate rule. Here we analyze the implications of adopting less inertial interest rate rules.[12] Figure 5 shows the impulse responses of six selected variables to a 40% increase in the standard deviation $\sigma_{\omega,t}$, the same shock analyzed in section 5, under alternative values for the inertial term in the interest rate rule. The starred blue line is the benchmark specification where $\phi_r = 0.9$; the red line with diamonds is the specification for $\phi_r = 0.5$; the light blue line with triangles features no interest-rate smoothing, $\phi_r = 0$. For all specifications we keep the coefficient on inflation constant and equal to $\phi_\pi = 1.5$. Figure 9 in Appendix B reports the response of a larger set of variables.

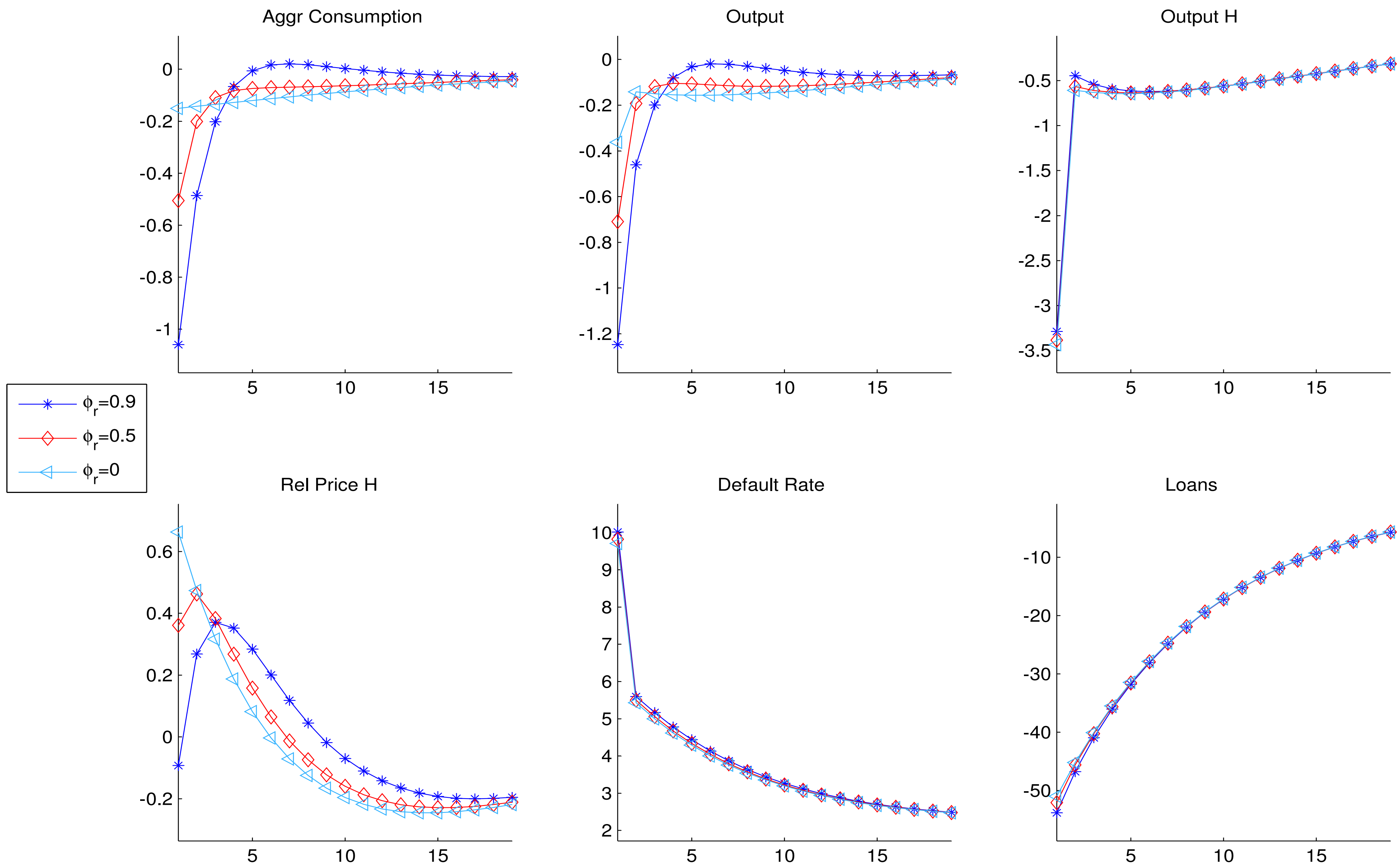
The negative contemporaneous response of the nominal interest rate is dampened down under

[12] The previous version of this paper also considered interest rate rules that respond to output. Since the differences are small, we decided to drop the analysis of the response to output in the current version.



Note: Default rate is annual and in percentage points. Loans are difference from steady state, multiplied by 100. All other variables are percentage point deviation from steady state.

Figure 4: Impulse Responses to a 25 basis points Monetary Shock



Default rate is annual and in percentage points. Loans are difference from steady state, multiplied by 100. All other variables are percentage point deviation from steady state.

Figure 5: Impulse Responses to a 40% Increase in σ_ω and Interest Rate Inertia

inertial interest rate rules. While the interest rate falls less than 60 basis points when $\phi_r = 0.9$, it falls more than 300 basis points when $\phi_r = 0$. The different response of the nominal interest rate has important implications for non-durable consumption. Under inertial interest rate rules, Borrowers cut their non-durable goods consumption more and Savers increase it by less, thereby making the negative response of aggregate consumption stronger. Since the nominal interest rate falls less under inertial rules, Borrowers enjoy a smaller reduction in their interest rate payments and therefore reduce non-durable consumption more and raise hours worked more, especially in the construction sector. Higher real interest rates under inertial rules relative to non-inertial ones also make Savers raise their non-durable consumption less. As a result, output in the non-durable sector as well as total output display a deeper contraction under inertial interest rate rules. Because supply more hours in the housing sector when ϕ_r is high, wages in the sector fall more and housing prices are more depressed. This explains why the real price of houses falls more with inertial rules. Interestingly, Housing demand, the default rate, monitoring costs, the external finance premium, the loan-to-value ratio and loans display almost identical responses under different degree of interest rate smoothing.

Interest rate rules featuring no inertia display a large degree of interest rate flexibility. The nominal interest rate is cut aggressively in response to higher volatility of the idiosyncratic housing investment risk so that the recession in the non-durable sector is significantly softened. On the other hand, inertial interest rate rules feature small interest rate responses that deepen the recession. In a sense, strong inertial rules mimic a zero bound scenario where interest rate cannot be lowered further and the negative effects of a risk shock are amplified.

7 Conclusions

This paper introduces endogenous default in DGSE model with housing and analyzes the dynamic response to an increase in the volatility of the distribution of idiosyncratic housing investment risk, i.e. mortgage risk. We calibrate the size of the shock so as to generate an increase in the default rate of the same magnitude as seen in the data. Under the benchmark calibration, which features a inertial interest rate rule, our model predicts a smaller fall in output, both in the durable and non-durable sector, relative to what we have seen in the data. At the trough of the recession, per capita U.S. real GDP was below trend by 3% while our model predicts a fall of 1.25% below steady state. Our model predicts a very small reduction in real housing prices followed by quick rebound. In the data, real house prices have been below baseline since 2008Q3 and were below baseline by more than 7 percentage points in 2009Q1. The calibration without inertial monetary policy predicts even smaller output reductions, while the relative price of housing increases on impact and falls below steady state only after 5 quarters after the shock. We speculate that some features and assumptions of our model play a key role in this respect. We discuss them below.

If we measure output in the housing sector gross of monitoring costs, the housing sector expands following an increase in mortgage risk. This happens because monitoring costs cause a reduction in the housing stock of Borrowers that they want to replenish quickly. A model featuring a rental market may help reducing strong and counterfactual responses in the construction sector. Alternatively, the housing sector could be characterized by lags in production or the use of capital that, together with investment adjustment costs, makes changes in output smoother.

In standard DSGE models with a non-durable sticky price and a durable flexible price sector, a disturbance that generates a recession in the non-durable sector causes an expansion in the durable sector that leaves GDP unchanged. While our model is non-standard in a number of ways, the housing sector displays some tendency to move in the opposite direction from the non-durable sector. Imperfect substitutability of hours in the two sectors reduces. In fact, an earlier version of our model that did not feature imperfect substitutability of hours displayed a large positive response of the housing sector to an increase in mortgage risk, irrespective of whether output in the sector was measured net or gross of monitoring costs. We believe, however, that wage stickiness will also improve bringing the model closer to reality and dampening out the output response in the H sector.

Our model features financial intermediation and a financial accelerator only for the purpose of housing investment. There is no capital in our simple model and therefore no financial intermediation for providing capital to firms. We believe that this specific financial intermediation has been quantitatively important in the great recession. In particular, we believe that the credit crunch, i.e. the tightening of lending conditions faced by firms, that followed the liquidity crisis has been an important element of the recent crisis. At the same time, introducing a formal banking sector with capital requirements is likely to amplify the output effect of financial shocks like the mortgage risk shock considered here.

The mortgage contract in our model is a one-period adjustable rate mortgage contract. Lenders do not take any risk, as they are guaranteed the risk-free rate on the total amount of loans they give out to Borrowers. Mortgage contracts in reality are more complex than the contract we analyze here. In particular, fixed-rate multi-year contracts and ARM contracts with nonstandard features may have played a role in the persistence and amplification of the crisis.

We believe these are important directions for future research.

References

Barsky, Robert B., Christofer L. House, and Miles S. Kimball, "Sticky-Price Models and Durable Goods," *American Economic Review*, 2007, *97* (3), 984–98.

Bernanke, Ben, Mark Gertler, and Simon Gilchrist, *The Financial Accelerator in a Quantitative Business Cycle Framework*, Amsterdam: North Holland, 1999.

Cagan, Christopher, "A Ripple, Not A Tidal Wave: Foreclosure Prevalence and Foreclosure Discount," *First American Real Estate Solutions*, 2006, *November*, 1–17.

Calza, Alessandro, Tommaso Monacelli, and Livio Stracca, "Housing Finance and Monetary Policy," *ECB Working Paper 1069*, 2009.

Carlstrom, Charles T. and Timothy S. Fuerst, "Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis," *American Economic Review*, 1997, *87* (5), 893–910.

Christiano, Lawrence, Roberto Motto, and Massimo Rostagno, "Financial Factors in Business Cycles," *Manuscript*, 2009.

Cohen-Cole, Ethan and Enrique Martinez-Garcia, "Systemic Risk, Banking Regulation and Optimal Monetary Policy," *Manuscript*, 2008.

Dellas, Harris, Behzad Diba, and Olivier Loisel, "Optimal Policy Responses to Financial Shocks," *Manuscript*, 2010.

Faia, Ester and Tommaso Monacelli, "Optimal Interest Rate Rules, Asset Prices, and Credit Frictions," *Journal of Economic Dynamics and Control*, 2007, *31* (10), 3228–54.

Horvath, Michael, "Sectoral Shocks and Aggregate Fluctuations," *Journal of Monetary Economics*, 2000, *45* (1), 69–106.

Iacoviello, Matteo, "House Prices, Borrowing Constraints and Monetary Policy in the Business Cycle," *American Economic Review*, 2005, *95* (3), 739–764.

_ , "Financial Business Cycles," *Manuscript*, 2010.

— **and Stefano Neri**, "Housing Market Spillovers: Evidence from an Estimated DSGE Model," *American Economic Journal: Macroeconomics*, 2010, (forthcoming).

Kiyotaki, Nobuiro and John Moore, "Credit Cycles," *Journal of Political Economy*, 1997, (105(2)).

Monacelli, Tommaso, "New Keynesian Models, Durable Goods, and Collateral Constraints," *Journal of Monetary Economics*, 2009, *56* (2), 242–254.

Woodford, Michael, *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press, 2003.

A Data and Sources

CC : Aggregate Consumption. Real Personal Consumption Expenditure (seasonally adjusted, billions of chained 2005 dollars, Table 1.1.6), divided by the Civilian Noninstitutional Population (CNP16OV, source: Bureau of Labor Statistics). Source: Bureau of Economic Analysis (BEA).

IH : Residential Investment. Real Private Residential Fixed Investment (seasonally adjusted, billions of chained 2005 dollars, Table 1.1.6.), divided by CNP16OV. Source: BEA.

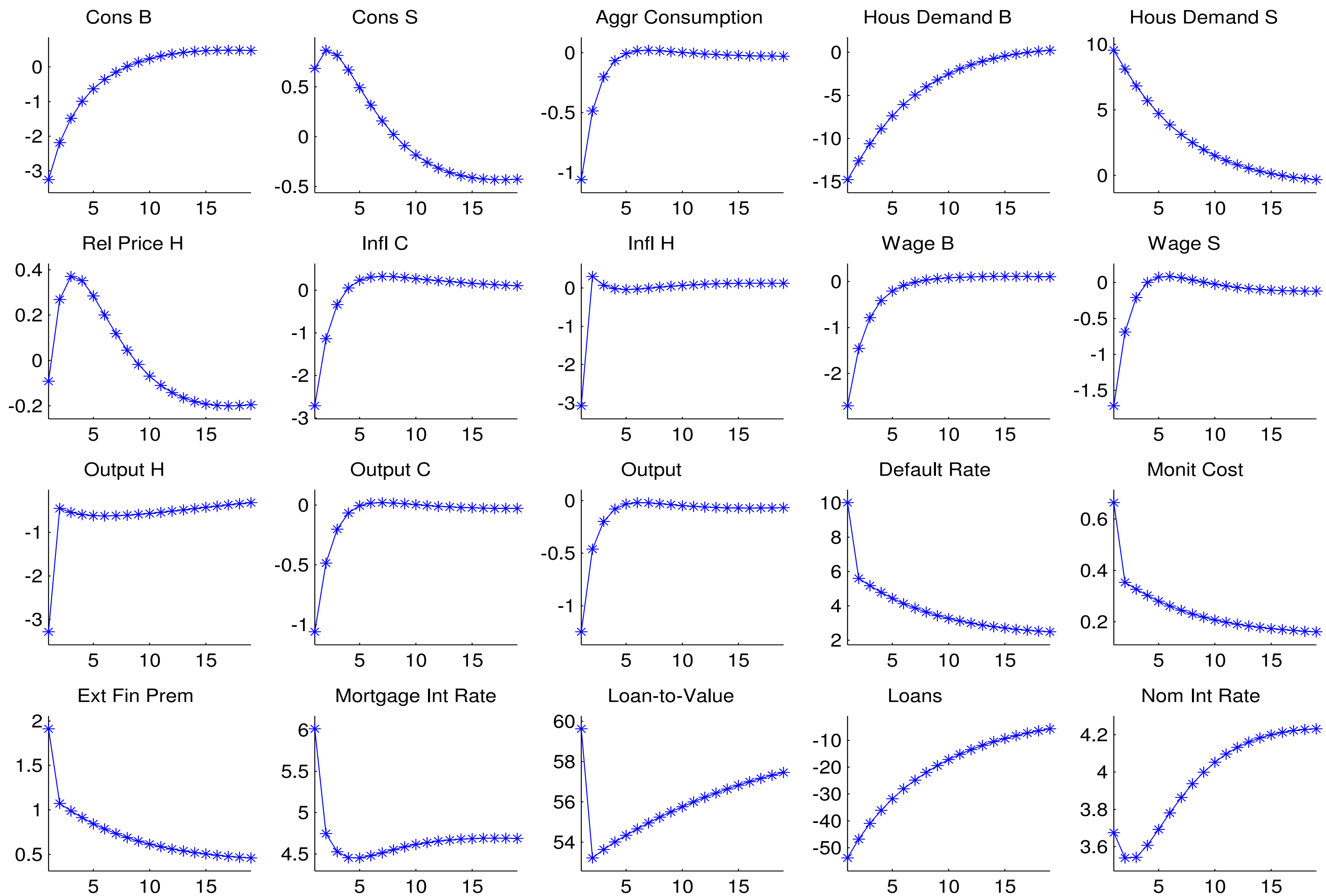
DP : Inflation. Quarter on quarter log differences in the implicit price deflator for the nonfarm business sector, demeaned. Source: Bureau of Labor Statistics (BLS).

RR : Nominal Short-term Interest Rate. 3-month Treasury Bill Rate (Secondary Market Rate), expressed in quarterly units. (Series ID: H15/RIFSGFSM03_NM). Source: Board of Governors of the Federal Reserve System.

QQ : Real House Prices. Census Bureau House Price Index (new one-family houses sold including value of lot) deflated with the implicit price deflator for the nonfarm business sector. Source: Census Bureau, http://www.census.gov/const/price_sold_cust.xls.

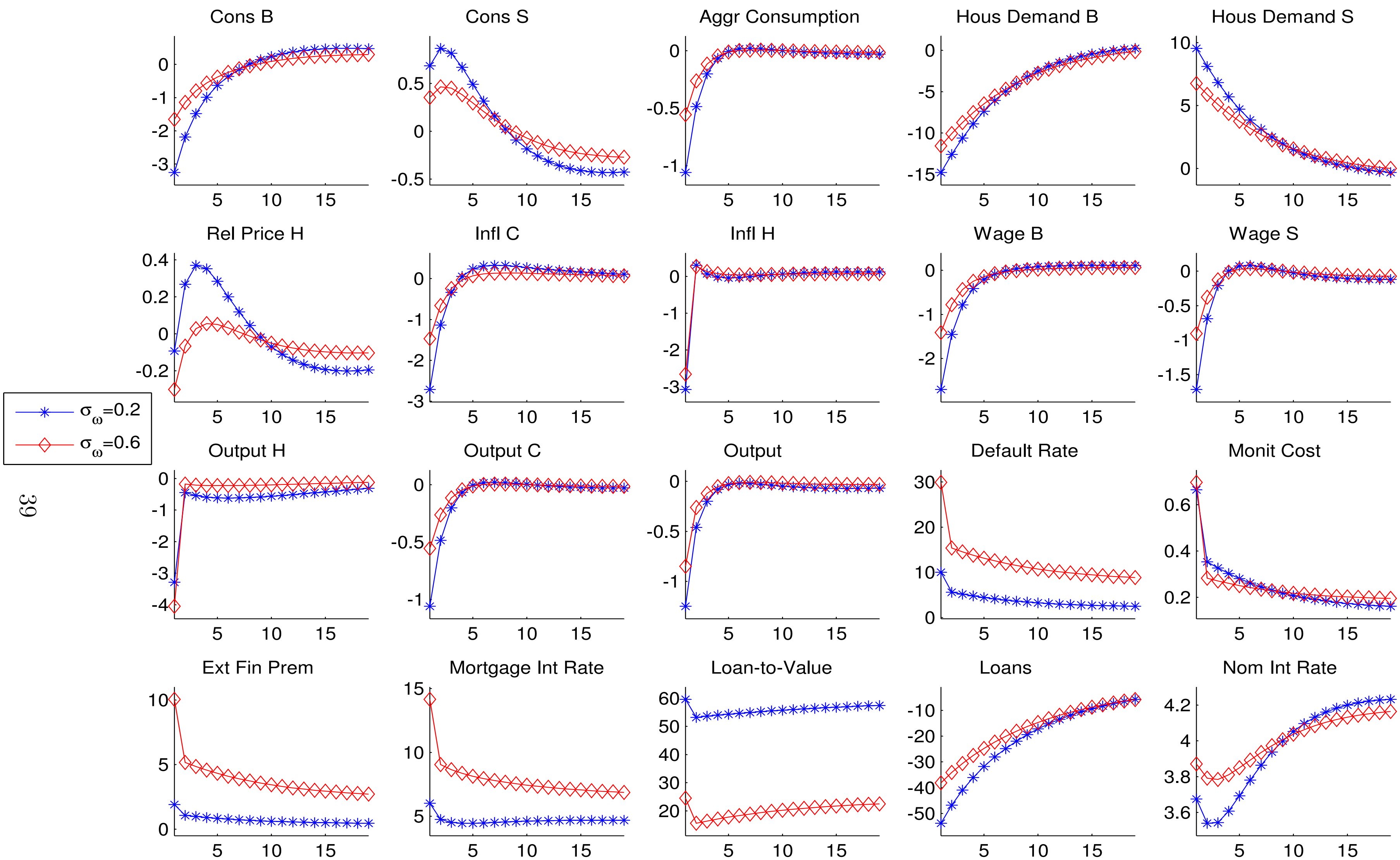
DEL : Seriously delinquent mortgages, not seasonally adjusted, percentage of total mortgages. Source: Mortgage Bankers Association, National Delinquency Survey.

B Figures



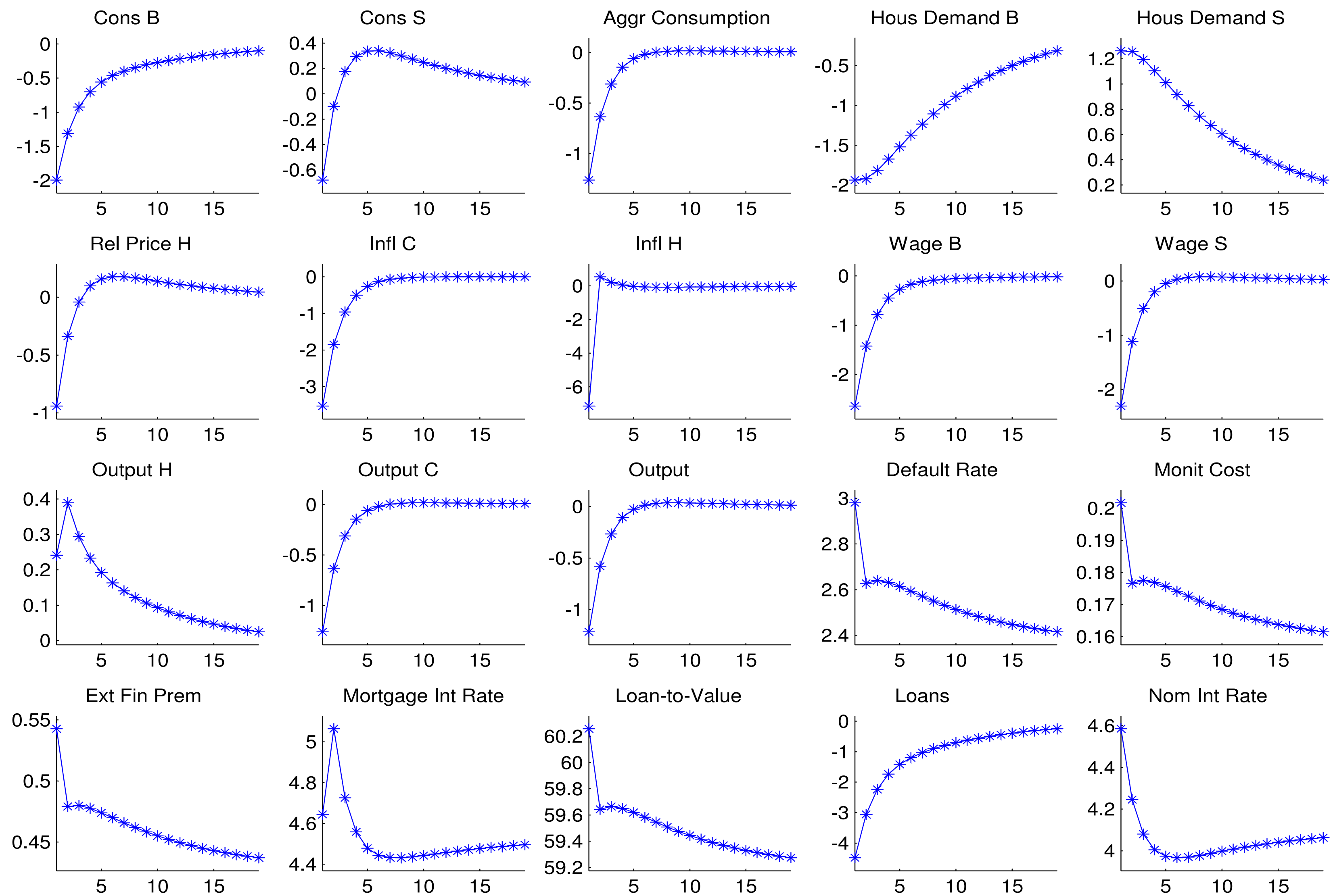
Note: Default rate, inflation in the H and C sector, external finance premium, mortgage interest rate and nominal interest rate are annual and in percentage points. Monitoring costs and loan-to-value ratio are in percentage points. Loans are difference from steady state, multiplied by 100. All other variables are percentage point deviation from steady state.

Figure 6: Impulse Responses to a 40% Increase in $\sigma_{\omega,t}$: Large Set of Variables



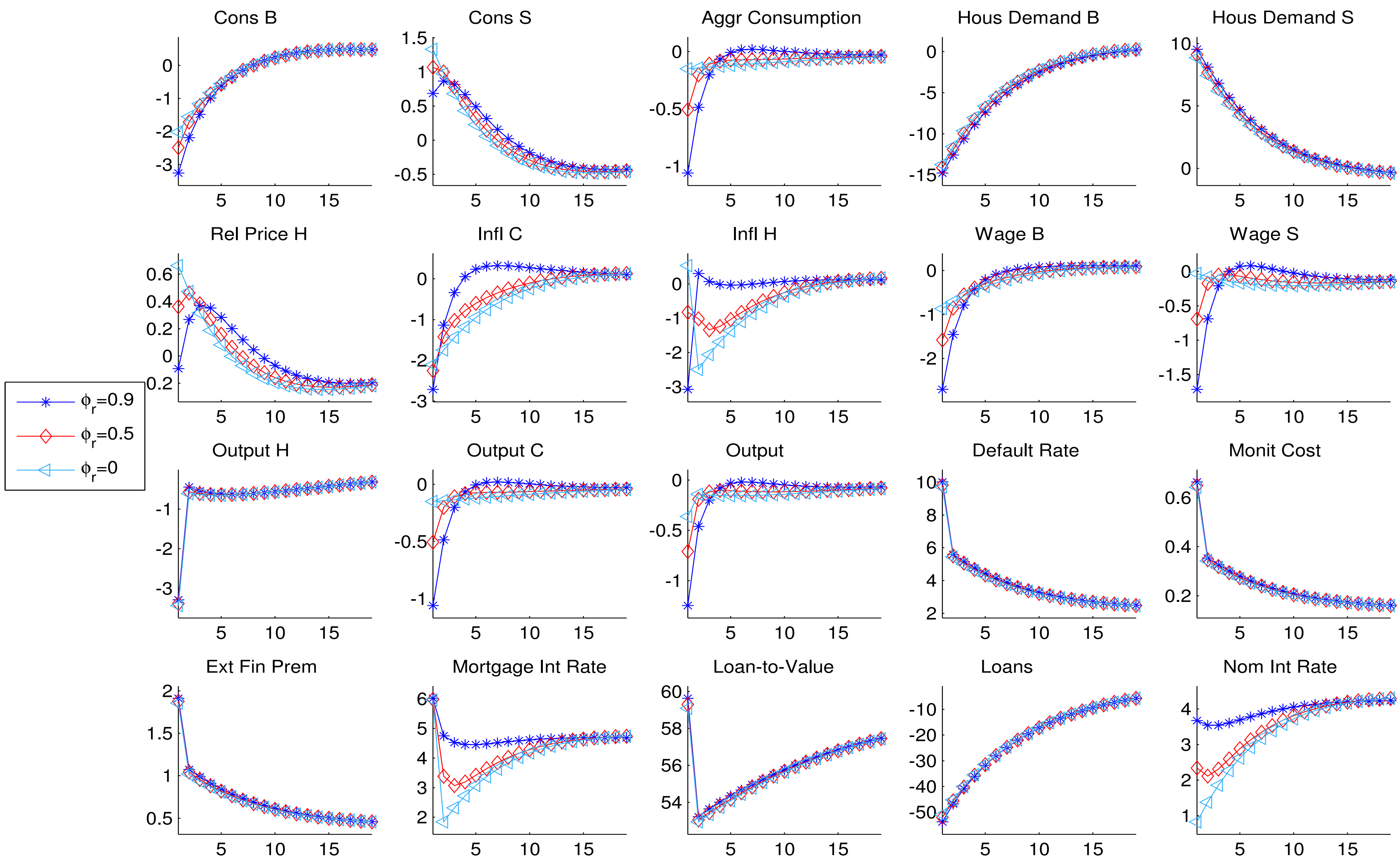
Note: Default rate, inflation in the H and C sector, external finance premium, mortgage interest rate and nominal interest rate are annual and in percentage points. Monitoring costs and loan-to-value ratio are in percentage points. Loans are difference from steady state, multiplied by 100. All other variables are percentage point deviation from steady state.

Figure 7: Impulse Responses to a 40% Increase in $\sigma_{\omega,t}$: Large Set of Variables, Low-Leverage Economy



Note: Default rate, inflation in the H and C sector, external finance premium, mortgage interest rate and nominal interest rate are annual and in percentage points. Monitoring costs and loan-to-value ratio are in percentage points. Loans are difference from steady state, multiplied by 100. All other variables are percentage point deviation from steady state.

Figure 8: Impulse Responses to a 25 basis points Monetary Shock: Larger Set of Variables



Note: Default rate, inflation in the H and C sector, external finance premium, mortgage interest rate and nominal interest rate are annual and in percentage points. Monitoring costs and loan-to-value ratio are in percentage points. Loans are difference from steady state, multiplied by 100. All other variables are percentage point deviation from steady state.

Figure 9: Impulse Responses to a 40% Increase in σ_ω and Interest Rate Inertia: Large Set of Variables

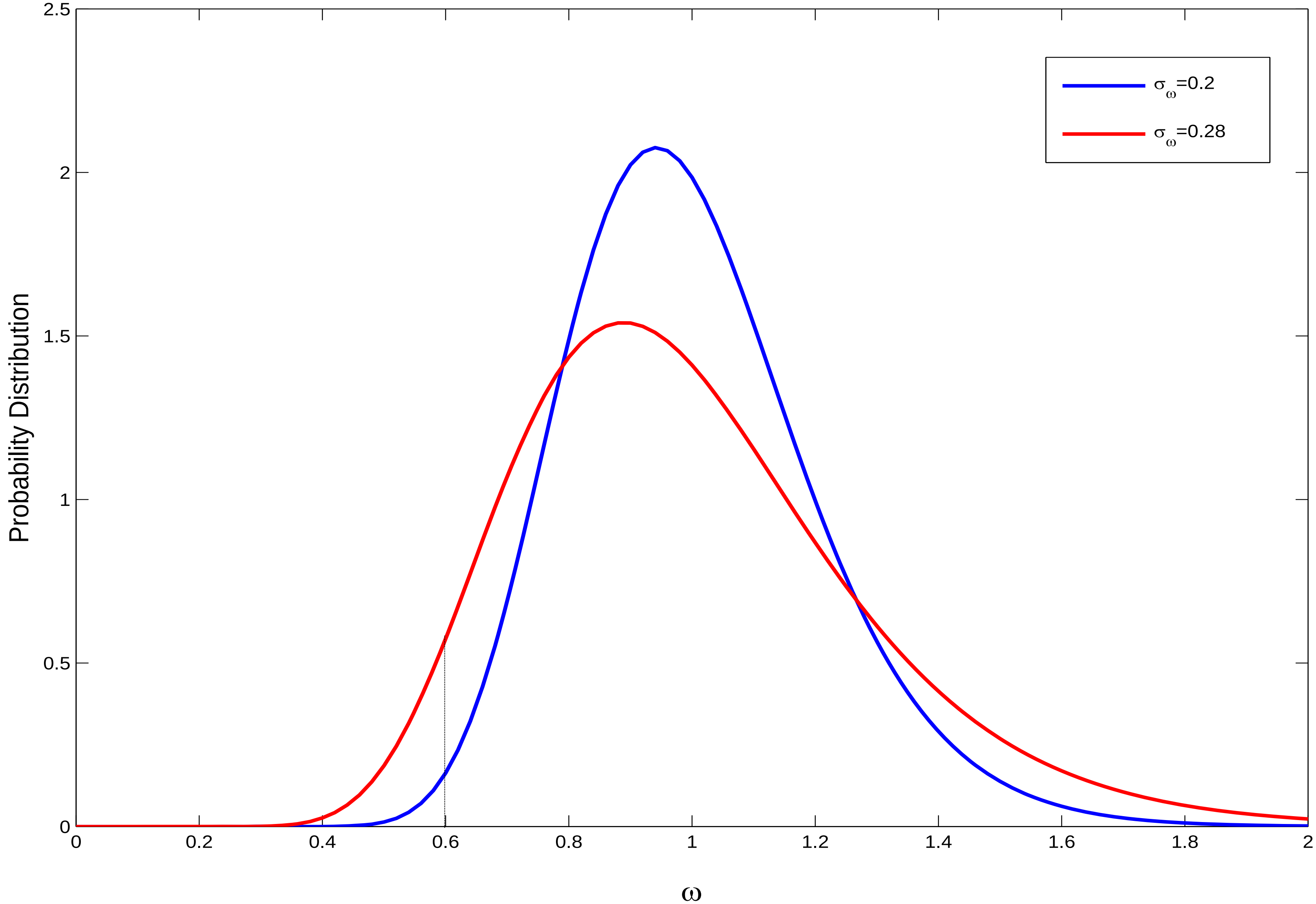


Figure 10: Probability Distribution: An Increase in $\sigma_{\omega,t}$