Learning From Stock Prices and Economic Growth

Joel Peress*

INSEAD

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Preliminary

Abstract

A competitive stock market with partially revealing prices is embedded into a neoclassical growth economy to analyze the interplay between information acquisition and dissemination through stock prices, capital allocation and income. The stock market contributes to growth by allowing investors to share their costly private signals in an incentive-compatible way when the signals' precision is not contractible, but its impact is only transitory. Several predictions on the evolution of real and financial variables are derived, including capital efficiency, total factor productivity, industrial specialization, stock trading intensity and idiosyncratic return volatility.

JEL classification codes : O16, G11, G14

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*INSEAD, Department of Finance, Boulevard de Constance, 77305 Fontainebleau Cedex, France. Telephone: (33)-1 6072 4035. Fax: (33)-1 6074 5500. E-mail: joel.peress@insead.edu.

1 Introduction

Economic institutions are widely believed to play a crucial role for economic growth. In particular, there is now considerable evidence that financial institutions, once considered a "sideshow" (Robinson (1952)), promote economic growth by relaxing constraints undermining the efficiency of investments. In this paper, we analyze the role of one such institution, the stock market, in alleviating one such constraint, investors' inability to perfectly communicate their private information. Economists have long argued that stock prices improve the allocation of capital by aggregating dispersed information and pointing to the most promising investment opportunities. While several aspects of the relation between the stock market and the real economy have been examined, "existing theories have not yet assembled the links in the chain from the functioning of stock markets, to information acquisition, and finally to aggregate long-run economic growth" (Levine (1997)).[1] This paper assembles these links.

We present a fully integrated model of information acquisition and dissemination through prices, capital allocation and economic growth. A competitive stock market in the spirit of Grossman and Stiglitz (1980) is embedded into a neoclassical growth economy. The economy is composed of firms that raise capital on the stock market, and overlapping generations of workers who invest their labor income in them. Firms' productivity is unknown but agents can collect private signals about it at a cost. Specifically, they are endowed with one unit of free time which they can either use for learning or for leisure. Agents' information is reflected in stock prices, but only partially so because of the presence of noise. Prices in turn guide investors in their portfolio allocations.

The only friction in the model stems from agents' inability to contract on the precisions of their signals (in particular, there is no short-sales constraint, nor minimum investment requirement). If they could, then the first best outcome would be achieved: agents would commit to infinitesimal precisions (arbitrarily close but not equal to zero), pool their signals and discover firms' productivity thanks to the Law of Large Numbers (signal errors are uncorrelated across agents and each generation consists of a continuum of agents).[2] Unfortunately, this outcome is not a Nash equilibrium when precisions are

[1]Page 695. More recently, Levine (2005) confirms this assessment: "While some models hint at the links between efficient markets, information and steady-state growth, existing theories do not draw the connection between market liquidity, information production and economic growth very tightly" (page 9). See Levine (1997, 2005) for reviews of the empirical and theoretical literatures on finance and growth.

[2]Reaching the first best does not require all agents to select non-zero precisions. A randomly chosen subset sufficies.

not contractible, as assumed here. Indeed, agents' best response is to set their precisions to zero and report noise, which results in no learning.

The stock market offers the means to share private information in an incentive-compatible way. For example, when agents receive optimistic signals about a firm, they buy its shares and bid up its stock price. The high stock price in turn indicates that investors collectively believe the firm to have good prospects. Thanks to stock prices, agents are better informed even though no new information is actually produced. Naturally, the effectiveness of the stock market is limited by the very existence of informative prices which undermines the incentive to collect costly information in the first place. Indeed, investors' cannot fully appropriate the benefit of their signals as they are leaked to competitors through prices.[3] Thus, informative stock prices have an impact that is beneficial *ex post* but detrimental *ex ante* to capital efficiency.

To a first approximation, income in the stock market economy is governed by a standard neoclassical law of motion similar to that which obtains under the first best. It grows at a decreasing rate until it reaches a steady-state in which it no longer grows.[4] Hence, the process of learning cannot counter the diminishing returns to capital. It does nevertheless affect the steady-state level of income and its growth rate during the transition to the steady-state – an important effect as consumption and welfare are ultimately determined by the long-run *level* of income. In comparison to the first best, income grows faster during the transition if the precision of information increases with income, but it grows slower otherwise. In the former case for example, wealthier agents are better informed, and allocate their labor income more efficiently across the various firms. This enhances the marginal product of labor and makes the next generation of workers richer. Whether or not information rises along the growth path depends on two competing forces. On the one hand, wealthier investors retire with more goods to consume. This reduces the marginal utility of goods, and hence the usefulness of information. On the other hand, information generates increasing returns to scale – its benefit, unlike its cost, rises with the amount to be invested. Indeed, discriminating across firms is more valuable when one has a lot to invest. The former effect leads wealthier agents to learn less while the latter induces them to learn

[3]This is the well-known Grossman-Stiglitz paradox. Noise trading provides the smoke screen behind which investors can conceal their informed trades and reap some benefit.

[4]There is no technological progress nor population growth in the model.

more. The intratemporal elasticity of substitution between goods and leisure determines which effect dominates. When it is larger than one, the scale effect of information dominates, so the precision of private information rises with income and income grows at an accelerated rate. Otherwise, the precision decreases and the growth rate of income is reduced.

The implications of the model, when the elasticity of substitution between goods and leisure is larger than one, are consistent with several patterns observed in the data. First of all, the stock market develops (e.g., as measured by information expenditures) in tandem with income, and it contributes to economic growth. Empirically, Levine and Zervos (1998), Rousseau and Wachtel (2000) and Carlin and Mayer (2003) document that income grows faster in countries with better functioning stock markets. Atje and Jovanovic (1993) estimate that this growth effect is permanent, but Harris (1997) finds that it is only transitory after controlling for possible endogeneity problems. The model also implies that the stock market processes information only when income exceeds a threshold, again a consequence of the increasing returns to information. This is consistent with the casual observation that financial institutions only emerge once a critical level of income has been reached.

Second, the model implies first that capital is more efficiently allocated across firms as income grows. That is, more (less) capital is channeled to more (less) productive firms when agents are wealthier. This superior efficiency leads to higher total factor productivity (TFP), even though there is no technological progress.[5] TFP is driven here by knowledge *about* technologies rather than by technological knowledge. Empirically, Wurgler (2000) documents that investments are more responsive to value added in more financially developed countries, and in particular in countries with a more informative stock market.[6] Furthermore, Levine and Zervos (1998) show that stock markets promote TFP growth, rather than capital growth.[7]

[5]TFP, also known in the growth literature as the "Solow residual", is defined as the residual from a regression of income growth on factor growth. It encompasses any factor, beyond labor growth and the capital growth, that contributes to output growth. Empirically, most of the differences in income across countries and periods stem from differences in TFP (e.g. Jorgenson (1995, 2000), Prescott (1998), Hall and Jones (1999) and Harberger (1998)).

[6]Wurgler (2000) constructs cross-country estimates of the elasticity of investments to value added by regressing, for each country, growth in industry investment on growth in industry value added. As a proxy for stock market informativeness, he uses a measure developed by Morck, Yeung and Yu (2000) who estimate the extent to which stocks move together and argue (in line with our model) that prices move in a more unsynchronized manner when they incorporate more firm-specific information.

[7]The findings of Levine and Zervos (1998) are consistent with those of Caballero and Hammour (2000), Restuccia and Rogerson (2003) and Hsieh and Klenow (2006) who show that variations in the allocation of resources account for a large fraction of the cross-country differences in TFP. Moreover, Henry (2003) confirms that countries that liberalize their stock market experience a rise in TFP, and Bekaert, Harvey and Lundblad (2001, 2005), Bertrand, Schoar and Thesmar (2005),

Third, we show that the economy specializes as it grows. Indeed, agents invest more selectively, leading capital and profits to become more concentrated across firms. Empirically, Imbs and Wacziarg (2003) report that countries go through two stages of sectoral diversification. Diversification increases at first, but beyond a certain level of income, the process is reversed and economic activity starts concentrating. The pattern of specialization among advanced countries is consistent with our model as we show that private information is collected only once a critical level of income has been reached. Similarly, Kalemli-Ozcan, Sørensen and Yosha (2003) report that industrial specialization in a sample of developed countries is positively related to the share of the financial sector in GDP, a proxy for financial development.

Fourth, we establish that, as the economy grows, stocks' idiosyncratic and total volatility increase, while the market's volatility remains constant. Thus, individual stocks returns fluctuate more, but they fluctuate in a less synchronized manner. This pattern obtains because more information is incorporated into stock prices. Empirically, Morck, Yeung and Yu (2000) show that stock prices are less synchronous in richer economies. In line with this observation, Campbell, Lettau, Malkiel and Xu (2001) document a strong increase in idiosyncratic return volatility in the U.S. from 1962 to 1997, while the volatility of the market remained stable.

Finally, we characterize the dynamics of wealth inequality across agents and trading activity, measured by the share turnover – the ratio of the value of shares traded to the total capitalization of the market. Both decrease at first and then increase as the economy grows. Indeed, disagreement encourages agents to trade and leads them to more unequal terminal wealths through more heterogenous portfolios. Disagreement weakens as the economy grows because agents' private signals tend to be more similar. But it tends to intensify beyond an income threshold because agents with more precise private signals rely more on them. Empirically, Levine and Zervos (1998) and Rousseau and Wachtel (2000) report that the share turnover on the stock market is positively related to output growth.

Our work relates to three main strands of theory. First and foremost, it contributes to the theoretical literature on finance and growth.[8] Most closely related is the seminal paper by Greenwood and Jovanovic

Galindo, Schiantarelli and Weiss (2005) and Chari and Henry (2006) that their allocative efficiency improves.

[8]Many papers highlight the different functions fulfilled by financial institutions, such as monitoring managers, improving risk management, mobilizing savings and facilitating the exchange of goods and services. An important function consist in identifying the best investment opportunities, as in our paper. For example, King and Levine (1993), Acemoglu, Aghion

(1990). In their setup, investors choose whether to invest directly in their own project or through a financial intermediary in exchange for a fee. The intermediary pools numerous individual projects and discovers the state of the economy. Thanks to its superior information and its ability to eliminate project-specific risks, it offers a higher return and a lower risk on capital, thereby promoting growth. Greenwood and Jovanovic (1990) show that economic and financial development feed on each other, as in our model. Their dynamics are driven by the cost of financial intermediation which includes a fixed fee akin to our information cost. But they do not specify where investors' private signals (projects) come from nor how they are pooled. In contrast, we explicitly model how investors make their decisions to collect costly signals, and how the stock market aggregates and transmits these signals. Moreover, we can characterize the evolution of several observable features of the stock market as the economy grows, such as the volatility of stock returns and the trading intensity. Finally, we differ from Greenwood and Jovanovic (1990) in that they obtain a permanent growth effect while we do not. But this is only because they assume that capital displays constant returns to scale while we assume that it is subject to diminishing returns.

Second, our work is connected to the endogenous growth literature (e.g. Romer (1986, 1990), Aghion and Howitt (1992), Grossman and Helpman (1991)). This literature models the discovery of technologies by profit-maximizing agents. In contrast to this literature, we *endow* the economy with technologies and focus instead on their selection by investors trading on the stock market. Similar issues arise nonetheless. In particular, technical innovations and information about stocks both give rise to increasing returns to scale, limited by the incomplete appropriability of the rents generated.[9] Whether long-run growth is possible or not depends essentially on the law of motion postulated for technological progress rather than on the structure of the models.[10] When technological progress is assumed away, we find that the

and Zilibotti (2003) and Morales (2003) argue that financial intermediaries such as banks promote growth by selecting the best entrepreneurs. These papers do not deal specifically with stock markets and their information processing role.

[9]Unlike standard goods, information is non-rival, i.e. it is costly to generate but costless to replicate. This property, which applies to financial information (information about stock returns) as well as to technological knowledge (such as the design for a new good), leads to increasing returns: the cost of information is fixed while its benefit rises with the scale of its applications (the number of shares traded or the number of goods sold). See Jones (2004) for an overview of the importance of this insight for endogenous growth theory. While models of endogenous growth and models of stock selection incorporate the scale effects of information, they differ in the way they preserve incentives to do research. The former grant some market power to innovators, while the latter introduce noise into the price system.

[10]For example, if the rate of growth of technological knowledge, dA/dt, increases linearly with the level of technological knowledge, A, as in Romer (1990), then the economy grows without bound. Otherwise, growth is only transitory. As Romer (1990, page 84) puts it, "linearity in A (in the equation for dA/dt) is what makes unbounded growth possible, and, in this sense, unbounded growth is more like an assumption than a result of the model".

information technology cannot generate any permanent growth effect. Finally, our work belongs to the body of research, too large to reference, on trading under endogenous and asymmetric information. A subset emphasizes the real benefits of informational efficiency. Our model contributes to this literature by developing a rational expectations framework in which income and learning interact dynamically.

The remaining of the paper is organized as follows. Section 2 describes the economy. Section 3 studies a benchmark economy in which the first best is achieved. Section 4 characterizes the equilibrium. Section 5 examines the dynamics of income. Section 6 derives predictions concerning the real and financial properties of the economy during its transition to the steady-state. Section 7 shows how the economy can emerge from or fall into a no-information regime. Section 8 concludes. Proofs are featured in the appendix

2 Economic Environment

We embed a competitive stock market *à la* Grossman and Stiglitz (1980) into Diamond's (1965) neo-classical growth economy. The economy is composed of two sectors – a final and an intermediate goods sector, and overlapping generations of agents. Firms in the intermediate goods sector raise capital on the stock market by issuing claims to their future profits. Young agents save by purchasing these claims.

2.1 Agents

The economy is populated by overlapping generations of agents who live for two periods. There is no population growth. Each generation consists of a continuum of agents with mass L indexed by $l \in [0, L]$. Young agents are each endowed with one unit of labor time and one unit of free time. Utility is derived from the consumption of the final good g and leisure j, and displays constant elasticity of substitution (CES):

$$U(g, j) \equiv (\varpi g^{\sigma} + (1 - \varpi) j^{\sigma})^{1/\sigma} ,$$

where ϖ is in $(0, 1)$ and $\sigma < 1$. The elasticity of substitution between goods and leisure is given by $1/(1 - \sigma)$. The case $\sigma = 0$ corresponds to Cobb-Douglas utility ($U(g, j) \equiv g^{\varpi} j^{1-\varpi}$).

Young agents are employed in the final good sector, to which they supply their unit of labor time inelastically for a competitive wage w_t, so aggregate labor supply equals L. They save their entire labor income by investing in the stock market to consume in the next period when they are old.[11] They divide their unit of free time between enjoying leisure and analyzing stocks. There are no short-sales constraints.

2.2 Technologies

2.2.1 Final Good Sector

The final good is produced according to a riskless technology that employs labor and intermediate goods:

$$G_t \equiv L^{1-\beta} \sum_{m=1}^{M} (Y_t^m)^\beta,$$

where G_t is final output, L is labor, M is the number of types of intermediate goods, Y_t^m is the employment of the m'th type and $0 < \beta < 1$ is the factor share of intermediate goods in the production of the final good. The production function follows Spence (1976), Dixit and Stiglitz (1977) and Romer (1987, 1990) among others. The final good is used as the numeraire. Many identical firms compete in the final good sector and aggregate to one representative firm.

2.2.2 Intermediate Good Sector

M firms operate in the intermediate goods sector. Firm m is the exclusive producer of good m. Its production is determined by a risky technology that displays constant returns to capital:

$$\widetilde{Y}_{t+1}^m \equiv \widetilde{A}_t^m K_t^m \qquad \text{for } m = 1, ..., M$$

where $\widetilde{Y}_{t+1}^m$ is the quantity of goods produced in period $t+1$ by firm m net of capital depreciation, $\widetilde{A}_t^m$ is its random productivity and K_t^m is the amount of capital it raises in period t. Tildes denote random variables not yet realized. Firms are liquidated immediately after production.[12]

The productivity shocks $\widetilde{A}_t^m$ are assumed to be log-normally distributed and independent from one another and over time. Because there is no closed-form solution to investors' portfolio choice under

[11] Thus the saving rate is exogenously set to 1. We make this assumption not only to simplify the model but also because the evidence suggests that financial development enhances growth through higher productivity rather than through higher saving rates (Levine and Zervos (1998), Beck, Levine and Loayza (2000)).

[12] Assuming firms are liquidated just after production simplifies the dynamics of the economy and allows to focus on the early stage of a firm's development. It is well known that young firms, because they have little retained earnings, are more dependent on external financing than mature firms. Several empirical studies confirm that financial development fosters growth mainly through the former (Rajan and Zingales (1998), Kumar, Rajan and Zingales (1999), Demirgüç-Kunt and Maksimovic (1998), Beck, Demirgüç-Kunt and Maksimovic (2001), Love (2003), Brown, Fazzari and Petersen (2008)).

CES preferences, we resort to a small-risk expansion to solve the model. We assume that productivity shocks are small and log-linearize the return on investors' portfolio (e.g. Campbell and Viceira (2002)). Specifically, we assume that $\ln \widetilde{A}_t^m \equiv \widetilde{a}_t^m z$ where $\widetilde{a}_t^m z$ is normally distributed with mean $\widetilde{\alpha}_t^m z$ and variance $\sigma_a^2 z$, $\widetilde{\alpha}_t^m$ is normally distributed with mean 0 and variance σ_α^2 and z is a scaling factor. The model is solved in closed-form by driving z toward zero. Throughout the paper, we assume that z is small enough for the approximation to be valid.[13]

Firms raise capital in the stock market. Firm m issues one perfectly divisible share – a claim to its entire future profit, for a price P_t^m. The productivity shock $\widetilde{a}_t^m$ is not observed at the time agents invest but they can learn about its average $\widetilde{\alpha}_t^m$ as we describe next.

2.2.3 Information Technology

At the time they invest, agents do not observe intermediate firms' productivity. Instead, they receive private signals about its average. The private signal $s_{l,t}^m$ received by agent l in period t about firm m's average productivity shock is given by:

$$s_{l,t}^m = \beta \widetilde{\alpha}_t^m + \widetilde{\varepsilon}_{l,t}^m,$$

where $\widetilde{\varepsilon}_{l,t}^m$ is an agent-specific disturbance independent of $\widetilde{a}_t^m$, $\widetilde{\alpha}_t^m$, across firms and time. $\widetilde{\varepsilon}_{l,t}^m$ is normally distributed with mean 0 and variance $1/x_{l,t}^m$ (precision $x_{l,t}^m$). Investors choose the precision of their signals before the stock market opens. Observing a signal of precision $x_{l,t}^m$ costs $C(x_{l,t}^m)z$ units of free time, where C is continuous, increasing, convex and $C(0) = C'(0) = 0$. Without loss of generality, we assume that agents (who are identical at the time they make their decision) choose identical precisions, and drop the subscript l. We emphasize that the information technology does not lead to the discovery of new physical technologies nor improve existing ones. Instead, it allows to allocate capital more efficiently to the physical technologies.

[13] Rational expectations models of competitive stock trading under asymmetric information typically conjecture that equilibrium stock prices are linear functions of random variables. This conjecture is not valid in a neoclassical framework because productivity and capital interact multiplicatively in the production of goods, and capital itself is a function of stock prices.

2.2.4 Noise Trading

Agents know that stock prices reflect other investors' private information in equilibrium, and they learn from them. Some noise is needed to blur price signals and avoid the Grossman-Stiglitz paradox, that is, preserve incentives to collect costly information. We assume that a fraction q of agents form their portfolio guided by exogenous shocks. The source of these shocks is not specified but they could stem from liquidity needs, preference shifts, random stock endowments, private risky investment opportunities, or some form of irrationality. Specifically, noise traders believe that the expected return on stock m equals $\widetilde{\theta}_t^m$, where $\widetilde{\theta}_t^m$ is normally distributed with mean 0 and variance σ_θ^2, and is independent of $\widetilde{a}_t^m$, $\widetilde{\varepsilon}_{l,t}^m$, across firms and time.[14]

2.3 Timing

The timeline is summarized in figure 1. An agent lives one period as a young agent (as a worker, then as an investor) and one period as an old agent (as a consumer). After earning a wage and before the stock market opens, workers choose how much time to spend on analyzing the stock market and on leisure, by setting the precision of their signals. Then, they invest their wage across the different stocks, guided by stock prices and their private signals. In the following period, the young become old, productivity shocks are revealed, final goods are produced and old agents consume their share of profits.

2.4 Notation

For any firm-specific variable ψ_t^m, $\overline{\psi}_t$ denotes its average across firms and $\Delta\psi_t^m$ its deviation from the average:

$$\overline{\psi}_t \equiv \frac{1}{M}\sum_{m=1}^{M}\psi_t^m \quad \text{and} \quad \Delta\psi_t^m \equiv \psi_t^m - \overline{\psi}_t.$$

The variable enclosed in brackets, $\{\psi_t^m\}$, represents the vector of stacked variables for $m = 1$ to M. Finally, we adopt the following notation to keep track of the quality of the approximation: $o(1)$, $o(z)$ and $o(z^2)$ capture respectively terms of an order of magnitude smaller than 1, z and z^2.

[14]The accuracy of noise traders' beliefs can be set arbitrarily. Moreover, including an agent-specific component to noise traders' beliefs about expected stock returns has no incidence on the equilibrium. Under this formulation, noise trading remains commensurate with rational trading as the economy grows. As equation 2 shows, portfolio holdings are scaled by a function of income γ. If $\sigma > 0$ for example, this function decreases with income so trades, both rational and noise-motivated, grow with the economy. If we assumed instead that noise trades equal an exogenous constant, then they would shrink relative to rational trades. This would mechanically make stock prices more informative and the allocation of capital more efficient, and reinforce the results of the paper.

2.5 Equilibrium Concept

We describe the equilibrium concept working backwards from production in period $t+1$, to capital allocation and information acquisition in period t. The gains from trade depend on how much information is collected in aggregate and revealed through prices. We denote X_t^m the average precision of private information about firm m.[15] A rational expectations equilibrium satisfies the following conditions.

1. Market clearing in the intermediate goods sector

Final goods producers maximize their profit. Since labor and intermediate goods trade in competitive markets and aggregate labor supply equals L, the following equilibrium factor prices obtain in period $t+1$:

$$\widetilde{w}_{t+1} = (1-\beta)\sum_{m=1}^{M}(\widetilde{Y}_{t+1}^m/L)^{\beta} \quad \text{and} \quad \widetilde{\rho}_{t+1}^m = \beta(L/\widetilde{Y}_{t+1}^m)^{1-\beta},$$

where $\widetilde{\rho}_{t+1}^m$ denotes the price of intermediate good m in period $t+1$ and $\widetilde{\Pi}_{t+1}^m = \widetilde{\rho}_{t+1}^m \widetilde{Y}_{t+1}^m$ is firm m's profit.

2. Capital allocation

Let $f_{l,t}^m$ denote the fraction of her wage that agent l invests in stock m in period t or her 'portfolio weights'. She sets $\left\{f_{l,t}^m\right\}$ to maximize her expected utility, guided by stock prices and private signals, and taking as given her income w_t, her leisure time j_t, the precision of her signals $\{x_t^m\}$, the average precisions $\{X_t^m\}$, share prices and capital stocks:

$$\max_{\{f_{l,t}^m\}} E\left[U(\widetilde{g}_{l,t+1}, j_t) \mid \mathcal{F}_{l,t}\right] \quad \text{subject to} \quad \begin{cases} \widetilde{g}_{l,t+1} = w_t \widetilde{R}_{l,t+1} \\ \widetilde{R}_{l,t+1} = \sum_{m=1}^{M} f_{l,t}^m \widetilde{R}_{t+1}^m \\ \sum_{m=1}^{M} f_{l,t+1}^m = 1 \end{cases}, \tag{1}$$

where $\mathcal{F}_{l,t} \equiv \{s_{l,t}^m, P_t^m \text{ for } m=1 \text{ to } M\}$, $\widetilde{g}_{l,t+1}$, $\widetilde{R}_{l,t+1}$ and $\widetilde{R}_{t+1}^m = \widetilde{\Pi}_{t+1}^m / P_t^m$ denote respectively agent l's information set, her consumption of the final good, the return on her portfolio and the return on stock m. We call $U_0(\{x_t^m, X_t^m\}, j_t, w_t)$ the value function for this problem.

In equilibrium, prices clear the stock market. Since each firm issues one share, its capital stock

[15] $X_t^m \equiv \int_l x_{l,t}^m / L = x_t^m$ given that $x_{l,t}^m = x_t^m$ for all agents l and stocks m.

coincides with its stock price: Formally,

$$\int_l w_t f_{l,t}^m = K_t^m = P_t^m \quad \text{for } m = 1, ..., M,$$

where the integral sums up the demand emanating from rational and noise traders.

3. Precision choice

An agent's optimal precisions $x_t^m = x(w_t, \{X_t^m\})$ maximize her *ex ante* expected utility subject to her free time budget constraint, taking her income w_t and the average precisions $\{X_t^m\}$ as given:

$$\max_{\{j_t \geq 0,\, x_t^m \geq 0\}} E[U_0(\{x_t^m, X_t^m\}, j_t, w_t)] \quad \text{subject to } \sum_{m=1}^{M} C(x_t^m) z + j_t = 1,$$

where $C(x_t^m)z$ is the time spent investigating stock m and $1 - \sum_{m=1}^{M} C(x_t^m)z$ is the time left for leisure. In equilibrium, the average and optimal precisions must be consistent:

$$X_t^m = x(w_t, \{X_t^m\}) \quad \text{for } m = 1, ..., M.$$

3 First Best

Before we proceed to the general case, we describe the first-best outcome, in which agents perfectly share their private information. It will serve as a benchmark when we examine the role of the stock market. The first-best is achieved when signal precisions are contractible. In that case, agents all commit to infinitesimal precisions – very close but not equal to zero, and share their private signals. They can perfectly infer productivity shocks thanks to the Law of Large Numbers because there is a continuum of signals with finite variances and uncorrelated errors ($\int_l \varepsilon_{l,t+1}^m = 0$). The first best obtains in particular in Greenwood and Jovanovic (1990). In their model, a financial intermediary pools numerous projects (signals) supplied by individuals and discovers the state of the economy. The reason the first best is achieved in equilibrium is that agents incur no penalty for communicating their information: they are endowed with a project rather than produce it at a cost, and technologies display constant returns to scale so the returns to capital do not diminish as firms attract more capital. We assume noise traders ignore the information derived from aggregating other agents' signals so their portfolios are determined by noise as in the stock market equilibrium. The following lemma describes portfolio shares and the capital allocation in this economy.

Lemma 1

In the first-best outcome, portfolio weights are given by:

$$f_{l,t}^{mFB} = \frac{1}{M} + \frac{1}{\gamma(w_t)\beta^2\sigma_a^2 z} E(\Delta \ln R_{t+1}^m \mid \{\widetilde{\alpha}_t^m\}) + o(1), \quad (2)$$

$$\text{where} \quad \gamma(w) \equiv \frac{\sigma\varpi w^\sigma + 1 - \varpi}{\varpi w^\sigma + 1 - \varpi}. \quad (3)$$

Hence,

$$f_{l,t}^{mFB} = \frac{1}{M} + \frac{1}{\gamma(w_t)\beta^2\sigma_a^2}\left(\Delta\beta\widetilde{\alpha}_t^m - (1-\beta)k_t^m\right) + o(1) \text{ for rational traders,}$$

$$\text{and } f_{l,t}^{mFB} = \frac{1}{M} + \frac{1}{\gamma(w_t)\beta^2\sigma_a^2}\Delta\widetilde{\theta}_t^m + o(1) \text{ for noise traders.} \quad (4)$$

Firm m's capital stock equals $K_t^{mFB} = \frac{Lw_t}{M}\exp(k_t^{mFB}z) + o(z)$

$$\text{where} \quad k_t^{mFB} = \frac{1}{1-\beta}\left(\Delta\beta\widetilde{\alpha}_t^m + \frac{q}{1-q}\Delta\widetilde{\theta}_t^m\right). \quad (5)$$

Stock m's portfolio weight equals the weight it would receive if firms were identical, $1/M$, tilted by a measure of the stock's expected excess performance relative to the market, $E(\Delta \ln \widetilde{R}_{t+1}^m \mid \{\widetilde{\alpha}_t^m\}) \equiv E(\ln \widetilde{R}_{t+1}^m - \overline{\ln R_{t+1}} \mid \{\widetilde{\alpha}_t^m\})$.[16] The tilde away from equal portfolio shares is more pronounced when stocks are less risky (lower β or σ_a^2). It also depends on income through the function γ. Its impact depends on the magnitude of the elasticity of substitution between goods and leisure, $1/(1-\sigma)$. If this elasticity is larger than one ($\sigma > 0$), then γ decreases with income, so wealthier investors' portfolio weights deviate more from equal shares. If instead the elasticity is smaller than one ($\sigma < 0$), then γ increases with income and wealthier investors' portfolio weights deviate less from equal shares. If $\sigma = 0$ (Cobb-Douglas utility), then γ is a constant, $1 - \varpi$, so portfolio weights are independent of wealth as in the case of constant relative risk aversion.

The capital allocation follows from aggregating rational and noise traders' portfolios. When $z = 0$, capital is equally distributed across firms, each firm raising Lw_t/M units of goods. When $z \neq 0$, relatively more productive firms (higher $\Delta\widetilde{\alpha}_t^m$) receive more capital. The elasticity of investments to productivity shocks, $\partial(\ln K_t^{mFB})/\partial(\ln \widetilde{A}_t^m) = 1/(1-\beta)$, captures the efficiency of the capital allocation. It increases with β, the factor share of capital because a higher β indicates that firms' marginal profits

[16] Firm m's marginal profit, $\partial\Pi_{t+1}^m/\partial K_t^m = \partial[\beta(A_t^m K_t^m)^\beta]/\partial K_t^m = \beta^2 A_t^{m\beta} K_t^{m\beta-1}$, is a decreasing function of K_t^m. Hence, if firms were identical, investors would distribute their wage symmetrically across the M stocks

decline with their stock of capital at a slower rate, so more capital can be invested in the better firms without immediately damaging their return. Firms favored by noise traders (higher $\Delta\widetilde{\theta}_t^m$) also attract more capital. Their impact is stronger if they are more numerous relative to rational traders ($q/(1-q)$ higher).

Given its capital stock, firm m produces $\widetilde{Y}_{t+1}^m = \widetilde{A}_t^m K_t^{mFB}$ intermediate goods. The number of final goods produced is:

$$\widetilde{G}_{t+1} = L w_t^\beta M^{1-\beta} \overline{\exp\left(\beta(\widetilde{a}_t^m z + k_t^{mFB})z\right)},$$

and the wage equals:

$$\widetilde{w}_{t+1} = (1-\beta)\widetilde{G}_{t+1}/L = (1-\beta) w_t^\beta M^{1-\beta} \overline{\exp\left(\beta(\widetilde{a}_t^m z + k_t^{mFB})z\right)}.$$

The wage is random as it depends on the realizations of the productivity shocks. The following lemma characterizes the dynamics of the economy along its average path, i.e. assuming that the wage realized in any period equals its mean. This is a good description of the economy if the number of firms is large.

Lemma 2

In the first-best outcome, average income evolves according to the following equation:

$$E(\widetilde{w}_{t+1}) = \Lambda \exp\left(\lambda^{FB} z^2\right) w_t^\beta, \tag{6}$$

where Λ and λ^{FB} are two positive constants given by:

$$\Lambda \equiv (1-\beta) M^{1-\beta} \exp\left(\frac{1}{2}\beta^2(\sigma_a^2 z + \sigma_\alpha^2 z^2)\right), \tag{7}$$

and

$$\lambda^{FB} \equiv \frac{M-1}{M}\left(\frac{\beta}{1-\beta}\right)^2 \left(\beta\left(1-\frac{\beta}{2}\right)\sigma_\alpha^2 + \frac{1}{2}\left(\frac{q}{1-q}\right)^2 \sigma_\theta^2\right) + o(1). \tag{8}$$

Average income converges to a steady-state, w^{FB}, given by:

$$w^{FB} = \Lambda^{1/(1-\beta)} \exp\left(\frac{\lambda^{FB}}{1-\beta} z^2\right). \tag{9}$$

The average wage evolves according to a standard neoclassical law of motion. The marginal product of labor increases with current income (assuming income is initially below its steady-state value) but at a decreasing rate, until it reaches a steady-state in which it no longer grows. The growth rate of income is given by $\Gamma^{FB}(w_t) \equiv E(\widetilde{w}_{t+1})/w_t = \Lambda w_t^{-(1-\beta)} \exp\left(\lambda^{FB} z^2\right)$. It declines at the rate $-(1-\beta)$, i.e. $d\ln\Gamma^{FB}(w_t)/d\ln w_t = -(1-\beta)$. The steady-state level of income w^{FB} solves

$w^{FB} = \Lambda w^{FB\beta} \exp\left(\lambda^{FB} z^2\right)$, which leads to equation 9. The dashed curves in figures 4 and 5 illustrate the dynamics of income in the first best. Steady-state income increases with the number of intermediate goods M as the production possibility set expands, and with the variance of productivity shocks $\sigma_a^2 z + \sigma_\alpha^2 z^2$ and noise shocks $\sigma_\theta^2 z^2$ because output is a convex function of these shocks – a positive shock increases $\widetilde{G}_{t+1}$ more than a negative shock decreases it. It decreases with the factor share of intermediate goods β as the marginal product of labor is reduced.

The first-best is not achievable if agents cannot commit to strictly positive signal precisions. Indeed, suppose all investors do agree to acquire some information about a stock, however imprecise ($x > 0$). They will collectively learn the stock's productivity shock. Given that the cost of information is not zero, the optimal strategy for an agent is to deviate from the agreement, i.e. to not collect any information and make a random announcement. But if all agents make random announcements, then the productivity shock cannot be learned. Thus, the first-best outcome cannot be reached if signal precisions are not contractible, for example because they are not publicly observable. The remainder of the paper assumes precisions are not contractible. In that case, the stock market offers a way to share information, albeit imperfectly.

4 Equilibrium Characterization

We characterize first investors' portfolios and the allocation of capital, and then information choices. Throughout this section, we take as given investors' income w_t which we endogenize in the next section.

4.1 Capital Allocation

We follow the usual method for solving a noisy rational expectations equilibrium: We guess that capital is a log-linear function of shocks, solve for portfolio, derive the equilibrium capital allocation, and check that the guess is valid. The following lemma displays investors' portfolio composition for the conjectured capital allocation.

Lemma 3

Assume that firm m's capital stock takes the form $K_t^m = \frac{Lw_t}{M}\exp(k_t^m z)$ where $k_t^m \equiv k_{\alpha t}^m(\beta\Delta\widetilde{\alpha}_t^m + \mu_t^m \Delta\widetilde{\theta}_t^m) + o(z)$ and μ_t^m is a deterministic scalar. The portfolio weights for a rational agent l who receives private signals of precision $\{x_t^m\}$ are given by:

$$f_{l,t}^m = \frac{1}{M} + \frac{1}{\gamma(w_t)\beta^2\sigma_a^2}\left\{\frac{x_t^m}{h(x_t^m)}\Delta s_{l,t}^m - \left(\frac{1}{h(x_t^m)\mu_t^{m2}\sigma_\theta^2 k_{\alpha t}^m} - (1-\beta)\right)k_t^m\right\} + o(1). \quad (10)$$

$$\text{where} \quad h_0(\mu) \equiv \frac{1}{\beta^2\sigma_\alpha^2} + \frac{1}{\mu^2\sigma_\theta^2} \quad \text{and} \quad h(x,\mu) \equiv h_0(\mu) + x. \tag{11}$$

The portfolio weights for a noise trader are given in equation 4.

As in the first-best, portfolio weights deviate from equal shares, $1/M$, by a function of the stock's expected excess performance relative to the market, $E(\Delta \ln \widetilde{R}^m_{t+1} \mid \mathcal{F}_{l,t})$, which equation 11 expresses as a combination of the stock price (the k^m_t term) and the relative private signal (the $\Delta s^m_{l,t}$ term).[17] In this expression, the stock price plays a dual role: it clears the stock market and provides information about productivity. Given our conjecture, observing stock prices is equivalent to observing $\beta\Delta\alpha^m_t + \mu^m_t\Delta\theta^m_t$ for each firm, a signal about $\beta\Delta\alpha^m_t$ with error $\mu^m_t\Delta\theta^m_t$. Thus, μ^m_t represents the noisiness of the stock m's price. The function $h(x^m_t) = 1/Var(\beta\alpha^m_t \mid \mathcal{F}_{l,t})$ measures the total precision of an investor's signals about a stock. She receives signals from three sources: her priors (the $1/(\beta^2\sigma^2_\alpha)$ term), the price (the $1/(\mu^{m2}_t\sigma^2_\theta)$ term) and her private signal (the x^m_t term), and their precisions simply add up (equation 11). The following proposition describes the equilibrium allocation of capital.

Proposition 4

Let X^m_t and μ^m_t be the average precision about stock m and the noisiness of its price. There exists a log-linear rational expectations equilibrium in which firm m's capital stock and its share price equal $K^m_t = P^m_t = \frac{Lw_t}{M}\exp(k^m_t z) + o(z)$ where:

$$k^m_t \equiv k_\alpha(X^m_t, \mu^m_t)(\beta\Delta\widetilde{\alpha}^m_t + \mu^m_t\Delta\widetilde{\theta}^m_t), \tag{12}$$

$$k_\alpha(X,\mu) \equiv \frac{1}{1-\beta}\left(1 - \frac{1}{\beta^2\sigma^2_\alpha h(X,\mu)}\right) > 0, \tag{13}$$

and X^m_t and μ^m_t are related through

$$X^m_t\left(\frac{1-q}{q}\mu^m_t - 1\right) = h_0(\mu^m_t). \tag{14}$$

The proposition establishes that capital and stock prices are approximately log-linear functions of productivity and noise shocks. As in the first-best, they equal those that would obtain if firms were identical (Lw_t/M), disturbed by an order-z function of shocks. Productivity shocks appear directly in the price function though they are not known by any agent, because individual signals, $\widetilde{s}^m_{l,t}$, once aggregated, collapse to their mean, $\beta\widetilde{\alpha}^m_t$. Noise traders' introduce noise $\widetilde{\theta}^m_t$ into the price system

[17] Firm m' profit equals $\widetilde{\Pi}^m_{t+1} = \widetilde{\rho}^m_{t+1}\widetilde{Y}^m_{t+1}$ so its log stock return equals $\ln \widetilde{R}^m_{t+1} = \ln(\widetilde{\Pi}^m_{t+1}/K^m_t) = \ln(\beta L^{1-\beta}) + \beta a^m_t z - (1-\beta)[\ln(Lw_t/M) + k^m_t z] + o(z)$.

through their trades. Stock prices are defined up to a period t-measurable multiplicative constant of order z. We choose a normalization that preserves symmetry.[18] In the perfect-information limit (X_t^m infinite), the capital allocation coincides with the first-best: $k_\alpha = 1/(1-\beta)$ and $\mu_t^m = q/(1-q)$.

Proposition 4 outlines the allocative role played by the stock market. Equation 12 implies that capital and technology shocks are positively correlated. The key parameter is $k_\alpha \equiv \partial(\ln K_t^m)/\partial(\ln \widetilde{A}_t^m)$, the elasticity of investments to productivity shocks. A positive k_α means that funds tend to flow to the most productive firms. Moreover, it increases with the quality of information. When there is no information ($X_t^m = 0$ so μ_t^m is infinite), $k_\alpha = 0$ so capital is allocated independently from productivity shocks. It increases with X_t^m until it reaches $1/(1-\beta)$ under perfect information as in the first best. k_α also decreases with σ_θ^2, as noise blurs price signals less. Thus, better-informed economies allocate capital more efficiently, and achieve the first best allocation through the stock market in the perfect-information limit.

Proposition 4 also makes clear the informational function performed by the stock market. This can best be understood by comparison to an economy in which prices do not reveal any of the information collected by agents. In such an economy, the average investor's total precision is reduced to $1/(\beta^2\sigma_\alpha^2) + X_t^m < h(X_t^m)$ and the elasticity of investments to productivity shocks falls to to $(1 - 1/(1+\beta^2\sigma_a^2 X_t^m))/(1-\beta)$ which is below $k_\alpha(X_t^m)$ (the precision of the price signal, $1/(\mu_t^{m2}\sigma_\theta^2)$, is lost). The allocation of capital is not as efficient though the same private signals were produced. Thanks to the stock market, private signals do not only serve the agents who observe them but benefit all through prices. Investors who collect private signals of precision X_t^m actually receive signals of precision $X_t^m + 1/(\mu_t^{m2}\sigma_\theta^2)$. Thus, the stock market allows investors to share their information (the "*ex post* information sharing effect"). Importantly, since investors communicate their private information through their trades, its transmission is incentive-compatible. This is an essential quality for an information sharing mechanism when signals are costly to acquire and privately observed.[19]

[18] If $\{K_t^m\}$ clears the market for the M stocks, so does $\{K_t^m \times \exp(\upsilon_t z)\}$ for any t-measurable scalar υ_t. Indeed as lemma 1 and 3 make clear, stock demands do not depend on absolute returns but on returns relative to the market. We normalize stock prices such that the geometric average stock price is independent of the realized shocks, i.e. by setting $\left(\prod_{m=1}^M K_t^m\right)^{1/M} = Lw_t/M$.

[19] Incentives are not an issue in Greenwood and Jovanovic (1990), because agents are endowed with a private signal about the state of the economy (a project) and asset returns are independent of the amount of capital they attract. Hence there is no cost to revealing one's private signals.

4.2 Information Acquisition

The following proposition characterizes how much free time investors devote to learning about productivity shocks, given their income w_t.

Proposition 5

In equilibrium, the precision of private information about stock m, X_t^m, and the noisiness of its stock price, μ_t^m, are the solution to:

$$C'(X_t^m) = \frac{M-1}{2M\beta^2\sigma_a^2}\frac{1}{(h_0(\mu_t^m)+X_t^m)^2}\frac{w_t^\sigma}{\gamma(w_t)} \qquad \text{for } m = 1, ..., M, \tag{15}$$

where X_t^m and μ_t^m are related through equation 14.

Investors choose precisions that equate the marginal benefit of information to its marginal cost, taking into account how much is revealed through stock prices. The equilibrium precisions are obtained by first solving for an agent's precision, $x_t^m = x(w_t, \{X_t^m\})$, taking as given the average precisions $\{X_t^m\}$. Then, we search for a fixed point to the system of equations $\{X_t^m\} = x(w_t, \{X_t^m\})$. The resulting precisions and noisiness are identical across stocks and denoted X_t and μ_t. Substituting equation 14 into equation 15 leads to the following equation in μ_t which admits a unique solution for any given income w_t:

$$C'\left(\frac{h_0(\mu_t)}{(1-q)/q\mu_t - 1}\right) = \frac{M-1}{2M\beta^2\sigma_a^2}\frac{[1-q/((1-q)\mu_t)]^2}{h_0(\mu_t)^2}\frac{w_t^\sigma}{\gamma(w_t)}.$$

The equilibrium precision of private information X_t has the following properties. First, it rises when priors are less informative (σ_α^2 larger). Thus, private information acts as a substitute for public information. Second, X_t rises with σ_θ^2. Indeed, stock prices reveal private signals, albeit partially, thereby limiting investors' ability to appropriate the full benefit from their information expenditures. Agents collect more private information when it is easier to conceal, i.e. when the price system is more noisy (σ_θ^2 or the fraction of noise traders q larger). Thus, the stock market undermines incentives to learn (the "*ex ante* incentive effect"). Third, X_t decreases with the conditional variance of productivity shocks σ_a^2 because agents tilt less their portfolio weights away from symmetric shares. Fourth, X_t decreases when the marginal cost of information C' is larger. Most of these properties obtain in the usual framework with exponential utility, normally distributed random variables and a riskless asset (e.g. Verrecchia (1982)).[20]

[20] In an economy similar to ours except that i) preferences display constant absolute risk aversion with a coefficient of absolute risk aversion $\widehat{\gamma}$, ii) stocks have normally distributed payoffs with variance σ_Π^2 and iii) a riskless asset with

Finally, the impact of β, the factor share of intermediate goods, on X_t is non-monotonic. On one hand (for high values of β), a lower β implies that stocks are less risky so agents use their private signals more aggressively, which makes them more valuable. On the other hand (for low values of β), it means that firms' marginal profits decline with their stock of capital at a faster rate. Hence investors cannot channel large amounts of capital to the firms which they have identified as the most productive without quickly reducing their return. This depresses the usefulness of information. The influence of income on X_t is discussed in the next section.

5 The Dynamics of Income

In this section, we tie together investments, learning and income and analyze the evolution of income along the economy's average path. We describe first qualitatively the interplay between learning and income. We start with the impact of a generation's signal accuracy on the next generation's income.

Lemma 6

The wage is larger on average when agents receive more accurate private signals. Formally, $\partial E(w_{t+1})/\partial X_t > 0$.

More accurate signals lead to more efficient investments and a larger supply of intermediate goods on average. This in turn increases the marginal product of labor. The next lemma considers the reverse relationship, from income to signal accuracy.

Lemma 7

If the elasticity of substitution between goods and leisure, $1/(1-\sigma)$, *is larger than one (i.e.* $\sigma > 0$*), then the precision of private information rises with investors' income. Formally,* $\partial X_t/\partial w_t > 0$. *Otherwise, it declines with income.*

The impact of income on learning depends on the elasticity of substitution between goods and leisure. X_t increases with w_t when $\sigma > 0$ (elasticity above one), whereas it decreases with w_t when $\sigma < 0$ (elasticity below one). Two competing forces determine the impact of income. On one hand, wealthier investors end up with more goods to consume since they save more. This reduces the marginal utility of goods and hence the usefulness of information. On the other hand, the marginal benefit of

gross return R is available, the equilibrium precision of private signals solves $2R\widehat{\gamma}C'(X_t) = 1/(h_{0t} + X_t)$ where $h_{0t} \equiv 1/\sigma_\Pi^2 + 1/(\mu_t^2\sigma_\Theta^2)$ and σ_Θ^2 is the variance of noise trading. From this equation, X_t rises when σ_Π^2 or $\mu_t^2\sigma_\Theta^2$ increase or when C' and $\widehat{\gamma}$ decrease.

information rises with the scale of agents' investments. Indeed, discriminating across firms is more valuable when one has a lot to invest. Thanks to its non-rival nature, information can be applied to every dollar of investment without requiring its cost to be incurred repeatedly. Putting it differently, information generates increasing returns with respect to the scale of investments. The former effect leads wealthier agents to learn less while the latter induces them to learn more. The elasticity of substitution σ determines which effect dominates. When σ is positive, the scale effect of information dominates and learning intensifies with income. On the other hand, when σ is negative, the effect on the marginal utility of goods dominates and the precision of private information deteriorates.[21] The two effects offset each other exactly under Cobb-Douglas utility ($\sigma = 0$). In that case, income has no impact on learning. Figure 2 illustrates this lemma 7. The following proposition ties together lemmas 6 and 7 to describe the dynamics of income.

Proposition 8

- *Average income evolves according to the following equation:*

$$E(\widetilde{w}_{t+1}) = \Lambda \exp\left(\lambda(w_t)z^2\right) w_t^{\beta}, \tag{16}$$

where

$$\lambda(w_t) \equiv \frac{M-1}{M}\beta^2 k_\alpha(X_t,\mu_t)\left[\beta\sigma_\alpha^2 + \frac{1}{2}k_\alpha(X_t,\mu_t)(\beta^2\sigma_\alpha^2 + \mu_t^2\sigma_\theta^2)\right] + o(1) > 0, \tag{17}$$

and Λ*,* k_α $X_t = X(w_t)$ *and* $\mu_t = \mu(w_t)$ *are defined respectively in equations 7, 13, 15 and 14.*

- *The economy converges to a steady-state in which it no longer grows. The steady-state level of income* w^* *is given by:*

$$w^* = \Lambda^{1/(1-\beta)} \exp\left(\frac{\lambda\left((1-\beta)^{1/(1-\beta)}M\right)}{1-\beta}z^2\right). \tag{18}$$

- *If the elasticity of substitution between goods and leisure is larger than one (i.e.* $\sigma > 0$*), then* λ *increases with income from* $\lim_{w_t 0}\lambda(w_t) = \frac{M-1}{2M}\left(\frac{\beta}{1-\beta}\frac{q}{1-q}\right)^2\sigma_\theta^2 \equiv \lambda_0$ *to* $\lim_{w_t\infty}\lambda(w_t) = \lambda^{FB}$*. Otherwise, it decreases with income from* $\lim_{w_t 0}\lambda(w_t) = \lambda^{FB}$ *to* $\lim_{w_t\infty}\lambda(w_t) = \lambda_0$*.*

To a first-order approximation (at the order 0 in z), the dynamics of income are similar to those obtained when the first-best is achievable: income grows at a declining rate until it reaches a steady-state

[21] When σ is positive, γ decreases with w so wealthier investors' portfolio weights deviate more from equal shares. Wealthier investors collect more information since they use their signals more aggressively. When instead σ is negative, γ increases with w so portfolio weights deviate less from equal shares and information is less useful.

w^* (assuming the wage is initially below w^*). Thus, the dynamics of income continue to be dominated by the neoclassical force of diminishing returns to capital – learning only generates a deviation of order z^2 from the neoclassical path. This is the case by construction in our model. Indeed, learning about productivity shocks generates benefits that are small since we *assume* these shocks to be small. We conjecture that this property extends to large shocks since income admits the first-best as an upper bound – that is, starting from the same arbitrary level of income, income in the next period is lower than in the first-best in which capital is more efficiently allocated – and income in the first-best eventually reaches a steady-state.

Proposition 8 is illustrated in figure 4 which displays the law of motion for income along the economy's average path (equation 16, solid curve for $\sigma = 0.5$ and dotted curve for $\sigma = -0.5$). The steady-state is located at their intersection with the 45° line (solid line). If initial income w_0 is below (above) w^*, then the wage increases (decreases) until it reaches w^*.

The effect of learning on income is captured by the function λ, illustrated in figure 3. The steady-state level of income is lower than in the first-best by a factor $w^*/w^{FB} = \exp\{[\lambda((1-\beta)^{1/(1-\beta)}M) - \lambda^{FB}]/(1-\beta)z^2\} < 1$. Its growth rate during the transition to the steady-state is given by $\Gamma(w_t) \equiv E(\widetilde{w}_{t+1})/w_t = \Gamma^{FB}(w_t)\exp\left[(\lambda(w_t) - \lambda^{FB})z^2\right]$. Figure 5 depicts $\Gamma(w_t)$ for various values of σ as well as in the first-best economy. When $\sigma > 0$ (elasticity of substitution between goods and leisure above one), investors collect more information as the economy grows, which contributes to growth further. As a result, the growth rate of income, $\Gamma(w_t) \equiv E(\widetilde{w}_{t+1})/w_t = \Gamma^{FB}(w_t)\exp\left[(\lambda(w_t) - \lambda^{FB})z^2\right]$, declines less quickly than in the first best:

$$\frac{d\ln\Gamma(w_t)}{d\ln w_t} = -(1-\beta) + \frac{d\lambda(w_t)}{d\ln w_t}z^2 > -(1-\beta),$$

where $-(1-\beta) = d\ln\Gamma^{FB}(w_t)/d\ln w_t$ is the change in the growth rate of income in the first-best. Thus in this case, learning has a transitory beneficial effect on growth, that mitigates the negative neoclassical force. When $\sigma < 0$ (elasticity of substitution below one), investors collect less information as the economy grows, which slows down growth. So, the growth rate of income falls at a faster rate than in the first best:

$$\frac{d\ln\Gamma(w_t)}{d\ln w_t} = -(1-\beta) + \frac{d\lambda(w_t)}{d\ln w_t}z^2 < -(1-\beta).$$

6 Properties of the Growth Path

In this section, we derive various observable properties of the economy during its transition to the steady-state (for an initial wage below its steady-state level). Throughout, we assume that $\sigma > 0$ so information about firms improves as the economy grows, in line with the evidence discussed below. We start with the real side of the economy and then proceed to the financial side. The following two propositions characterize the efficiency and concentration of the capital allocation.

Proposition 9
The elasticity of investments to productivity shocks, $\partial(\ln K_t^m)/\partial(\ln \widetilde{A}_t^m)$, *and TFP increase as the economy grows.*

Better-informed agents distribute capital more efficiently across firms, leading to a higher elasticity of investments to productivity shocks. This superior efficiency translates into higher TFP. We define TFP from the following economy-wide production function:

$$E(\widetilde{G}_{t+1}) = ML^{1-\beta}E[(\widetilde{A}_t^m K_t^m)^\beta] = L^{1-\beta}E(\widetilde{A}_t^{m\beta})E(K_t^{m\beta})\exp[Cov(\beta\widetilde{a}_t^m z, \beta k_t^m z)], \quad (19)$$

where we interpret the term $\exp[Cov(\beta\widetilde{a}_t^m z, \beta k_t^m z)]$ as TFP. It captures the additional output obtained from distributing capital in relation to productivity shocks, in comparison to an economy in which capital is arbitrarily allocated. From equation 12, TFP equals $\exp[k_\alpha \beta^2 \sigma_\alpha^2 z^2 (M-1)/M]$, exceeds 1 and increases with the precision of private signals X_t. We stress that technological progress is not required to generate growth in TFP. TFP grows thanks to a more efficient allocation of capital, keeping stationary the distribution of productivity shocks and the cost of information.

The empirical evidence is consistent with proposition 9. Wurgler (2000) constructs cross-country estimates of the elasticity of investments to value added, our parameter k_α. He finds that this elasticity increases with the country's degree of financial development, and in particular with the informativenenss of its stock market. That is, countries with more informative stock markets increase investments more in their growing industries, and decrease investments more in their declining industries, than countries with less informative stock markets.[22] These countries also tend to display higher TFP. Indeed, Levine

[22] Wurgler (2000) uses a proxy for informativeness developed by Morck, Yeung and Yu (2000). They measure the extent to which stocks move together and argue that prices move in a more unsynchronized manner when they incorporate more firm-specific information. This is indeed the case in the present model (see proposition 12). Durnev, Morck and Yeung (2004) and Durnev, Morck, Yeung and Zarowin (2003) report that the synchronicity measure is related to accounting estimates of stock price informativeness as well as to the efficiency of corporate investments captured by Tobin's q.

and Zervos (1998) show that stock markets promote growth in total factor productivity.[23]

We examine next the concentration of economic activity, measured using Herfindhal indices, $Her(K_t^m) \equiv E(K_t^{m2})/[E(K_t^m)]^2$ and $Her(\widetilde{\Pi}_{t+1}^m) \equiv E(\widetilde{\Pi}_{t+1}^{m2})/[E(\widetilde{\Pi}_{t+1}^m)]^2$.

Proposition 10

Capital and profits are more concentrated across firms as the economy grows. Formally, $dHer(K_t^m)/dw_t > 0$ *and* $dHer(\widetilde{\Pi}_{t+1}^m)/dw_t > 0$.

Agents become more selective in their investments as their income grows. They channel increasingly more (less) capital to the more (less) productive firms, so fewer firms account for a larger fraction of the economy's stock of capital. Profits tend to be even more concentrated than capital because they compound the effect of a high productivity shock with that of a large capital stock. Thus, the economy grows more specialized by both measures of economic activity.

Empirically, Imbs and Wacziarg (2003) report that countries go through two stages of sectoral diversification. Diversification increases at first, but beyond a certain level of income, the process is reversed and economic activity starts concentrating. This pattern is consistent with our model to the extent that the model applies to more advanced economies.[24] Similarly, Kalemli-Ozcan, Sørensen and Yosha (2003) report that industrial specialization in a sample of developed countries is positively related to the share of the financial sector in GDP. This fact is consistent with proposition 10 to the extent that this share is positively related with information expenditures about public companies.

We discuss next the evolution of inequality. Given that agents are *ex ante* identical, we consider here the distribution of final wealth or consumption, $\widetilde{g}_{l,t+1}$.

Proposition 11

As the economy grows, income inequality narrows at first and then widens.

Final wealth is unequal because agents, guided by their private signals, choose different portfolios. Two forces work in opposite directions when the precision of private signals rises. On one hand, agents

[23] Levine and Zervos (1998) measure stock market development using the ratio of market capitalization to GDP, the ratio of the value of trades to GDP and the ratio of the value of trades to market capitalization. Their finding is consistent with those of Caballero and Hammour (2000), Restuccia and Rogerson (2003) and Hsieh and Klenow (2006) who show that variations in the allocation of resources account for a large fraction of the cross-country differences in total factor productivity. Moreover, Henry (2003) confirms that countries that liberalize their stock market experience a rise in total factor productivity, and Bertrand, Schoar and Thesmar (2005), Galindo, Schiantarelli and Weiss (2005) and Chari and Henry (2006) that their allocative efficiency improves.

[24] An extension of the model presented in the next section shows that the economy produces more information as it grows, only if income is above a threshold.

put more weight on private information relative to public information, which increases disagreement. On the other hand, idiosyncratic errors shrink so private signals are more similar. This tends to reduce portfolio heterogeneity.[25] The second effect tends to dominate for low private precision X_t and the first for high private precision, so inequality narrows first and then widens.

We conclude with two financial variables, the volatility of stock returns and the trading intensity. We assume a "pre-opening trading session" takes place before private signals are observed. No information is revealed during this session so agents, including noise traders, assign the same portfolio weight to all stocks, $1/M$. Prices (P_t^0) that equate the supply of shares to their demand emerge but trades do not actually take place. Trade occurs during the second round once agents receive their private signals – they set their portfolio weights according to equation 10 (substituting X_t for x_t^m to obtain equilibrium portfolio weights). Stock returns are computed by dividing stock prices in the first and second rounds ($\ln(P_t^0/P_t^m)$). Trades are based on the difference between portfolio weights in the first and second rounds. The value of shares traded equals $\sum_{m=1}^{M}\int_l |(f_{l,t}^m - 1/M)w_t|/2$ where the factor 2 avoids double counting matching buys and sells. We measure the trading intensity as the share turnover, defined as the ratio of the value of shares traded to the total capitalization of the market, $\sum_{m=1}^{M} K_t^m$.

Proposition 12

As the economy grows, stocks' idiosyncratic and total volatility increase, while the market volatility remains constant. Formally, $dVar(\Delta\widetilde{r}_{t+1}^m)/dw_t > 0$, $dVar(\widetilde{r}_{t+1}^m)/dw_t > 0$ *and* $dVar(\overline{r}_{t+1})/dw_t = 0$.

The proposition establishes that individual stocks returns fluctuate more – whether fluctuations are measured as total or idiosyncratic volatility – as the economy grows and prices incorporate more information. Since the market in contrast does not, the decline in the cross-correlation of returns offsets the rise in individual stock volatility. Thus, stock prices vary in a less synchronized manner. Empirically, Morck, Yeung and Yu (2000) show that stock prices are less synchronous in richer economies. In line with this observation, Campbell, Lettau, Malkiel and Xu (2001) document a strong increase in idiosyncratic return volatility in the U.S. from 1962 to 1997, while the volatility of the market remained stable.[26] The following proposition describes the trading activity.

[25] Formally, according to equation 10 (substituting X_t for x_t^m to obtain equilibrium portfolio weights), an agent's portfolio weights are a function of $(X_t/h(X_t))\Delta s_{l,t}^m = (X_t/h(X_t))\Delta\widetilde{\varepsilon}_{l,t}^m$+other terms. When X_t grows, on one hand the ratio of the precision of the private signal to the total precision, $X_t/h(X_t)$, rises, but on the other hand $var(\widetilde{\varepsilon}_{l,t}^m) \equiv 1/X_t$ falls.

[26] Explanations, other than information-based, have been suggested for these volatility patterns. See for example Thesmar and Thoenig (2004) for an alternative view based on firms' changing risk profile.

Proposition 13

As the economy grows, trading on the equity market weakens at first and then intensifies.

The logic of Proposition 13 is identical to that of Proposition 11 on wealth inequality. Agents trade because they disagree. On one hand, disagreement rises with the precision of private signals because agents use them more aggressively, but on the other hand, it declines because idiosyncratic signal errors shrink. The trading intensity declines first and then rises beyond a threshold. Empirically, Levine and Zervos (1998) and Rousseau and Wachtel (2000) report that the share turnover on the stock market is positively related to output growth.

7 No-Information Trap

In the model, agents always collect private signals. This is because the cost of learning is assumed to satisfy $C'(0) = 0$, i.e. an infinitesimal amount of private information is costless. Empirically however, financial institutions only emerge once a critical level of income has been reached. In this section, we assume that $C'(0) > 0$ and show that information production only takes place for sufficiently developed economies. The following proposition describes how investors' learning decisions are altered.

Proposition 14

Suppose that $C'(0) > 0$. Let $\underline{w}$ be the unique income level such that

$$C'(0) = \frac{(M-1)\beta^2}{2M\sigma_a^2(\sigma_\alpha^2)^2}\frac{\underline{w}^\sigma}{\gamma(\underline{w})}.$$

When the elasticity of substitution between goods and leisure is larger than one (i.e. $\sigma > 0$), then agents collect private signals if and only if their income exceeds the threshold $\underline{w}$. When instead the elasticity is lower than one (i.e. $\sigma < 0$), then they collect private signals if and only if their income is below the threshold $\underline{w}$. When agents collect private signals, the precision of their signals is identical across stocks and given by equation 15.

If $C'(0) > 0$, then equation 15 that determines the equilibrium precision may admit no solution. For example, when income is close to 0 and $\sigma > 0$, the marginal cost of information (the left-hand side of equation 15) exceeds its marginal benefit (the right-hand side) for all precision choices. In that case, agents choose not to learn because information is too costly to be profitable. Since information is more valuable to wealthier investors, they learn if w_t is large enough. The effect is reversed for $\sigma < 0$:

investors stop collecting information when their income exceeds $\underline{w}$. The properties of $\underline{w}$ mirror those of the equilibrium precision X_t : the factors that increase (decrease) X_t tend to decrease (increase) $\underline{w}$.

Assuming that $\sigma > 0$ and that $w^* > \underline{w} > w_0$ where w_0 is the initial level of income, the economy goes through two stages of development. At first, it behaves as the standard neoclassical economy with no information. Once income reaches a threshold, agents start collecting privates signals and growth accelerates by a factor $\exp\left(\lambda(w_t)z^2\right)$. Thus, the stock market only operates as an information processor if the economy is sufficiently developed. If instead $w_0 < w^* < \underline{w}$, then no information is ever collected.

8 Conclusion

A competitive stock market with partially revealing prices is embedded into a neoclassical growth economy to analyze the interplay between information acquisition and dissemination through stock prices, capital allocation and income. The stock market contributes to growth by allowing investors to share their costly private signals in an incentive-compatible way when the signals' precision is not contractible, but its impact is only transitory. Several predictions on the evolution of real and financial variables are derived, including capital efficiency, total factor productivity, industrial specialization, stock trading intensity and idiosyncratic return volatility.

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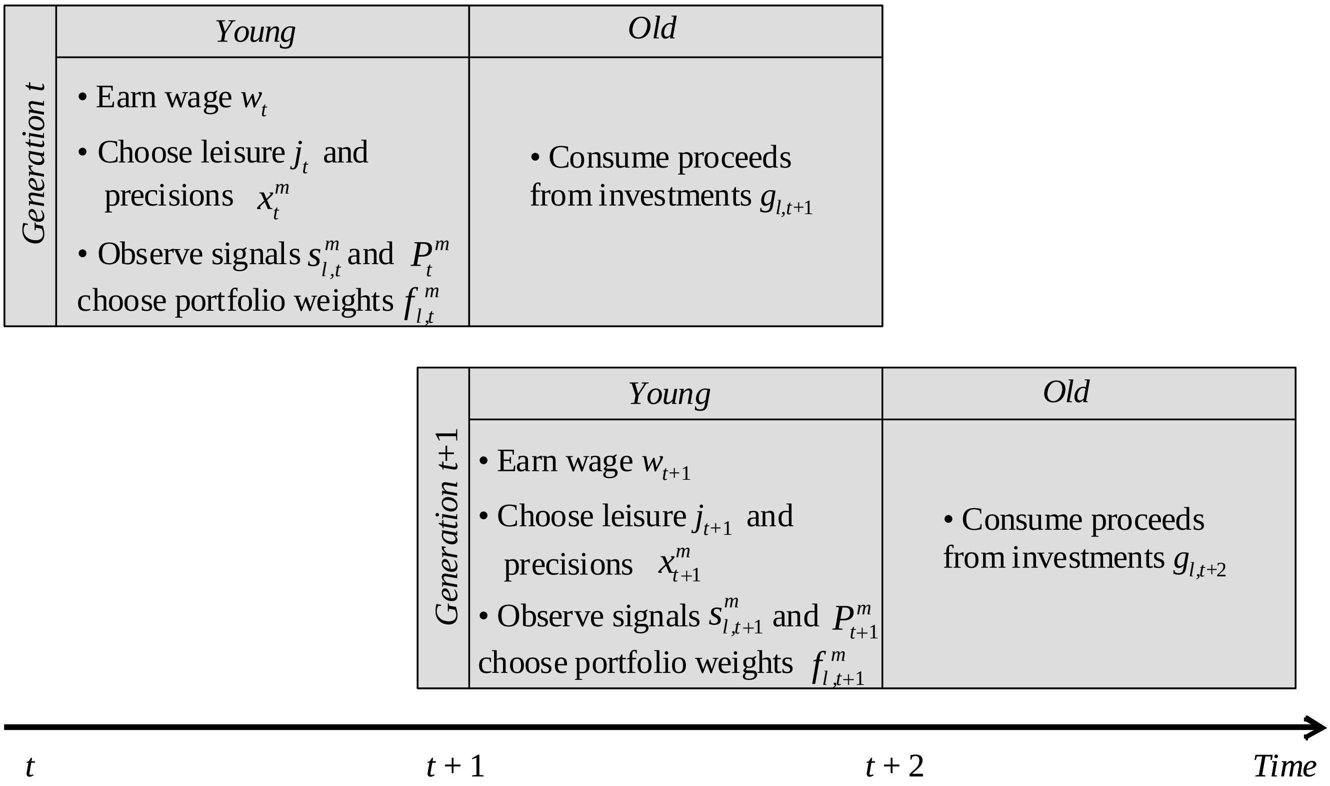

Figure 1: Timing.

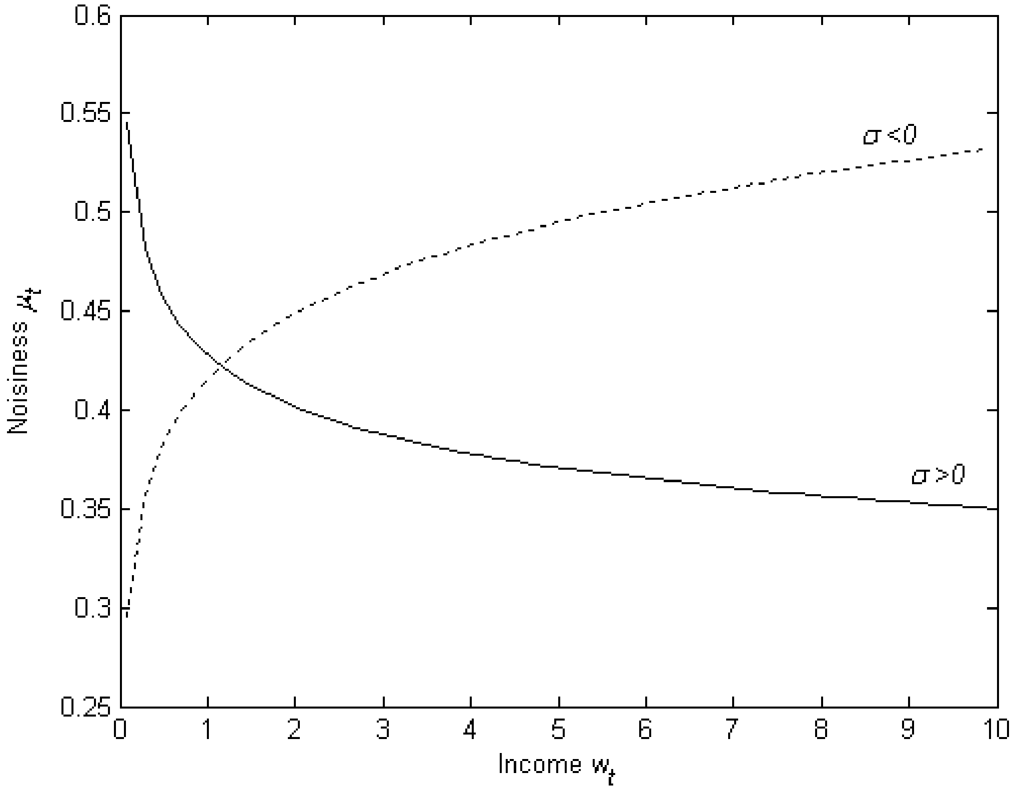

Figure 2: Income and the noisiness of the price system. The picture depicts the equilibrium noisiness μ_t as a function of current income w_t. The solid curve corresponds to $\sigma = 0.5$ and the dotted curve to $\sigma = -0.5$. The other parameters are $\beta = 2/3$, $C(x) = x^2$, $q = 0.1$, $\sigma_a^2 = 0.01$, $\sigma_\theta^2 = \sigma_\alpha^2 = 1$, $\omega = 0.5$, $M = 50$ and $z = 0.5$.

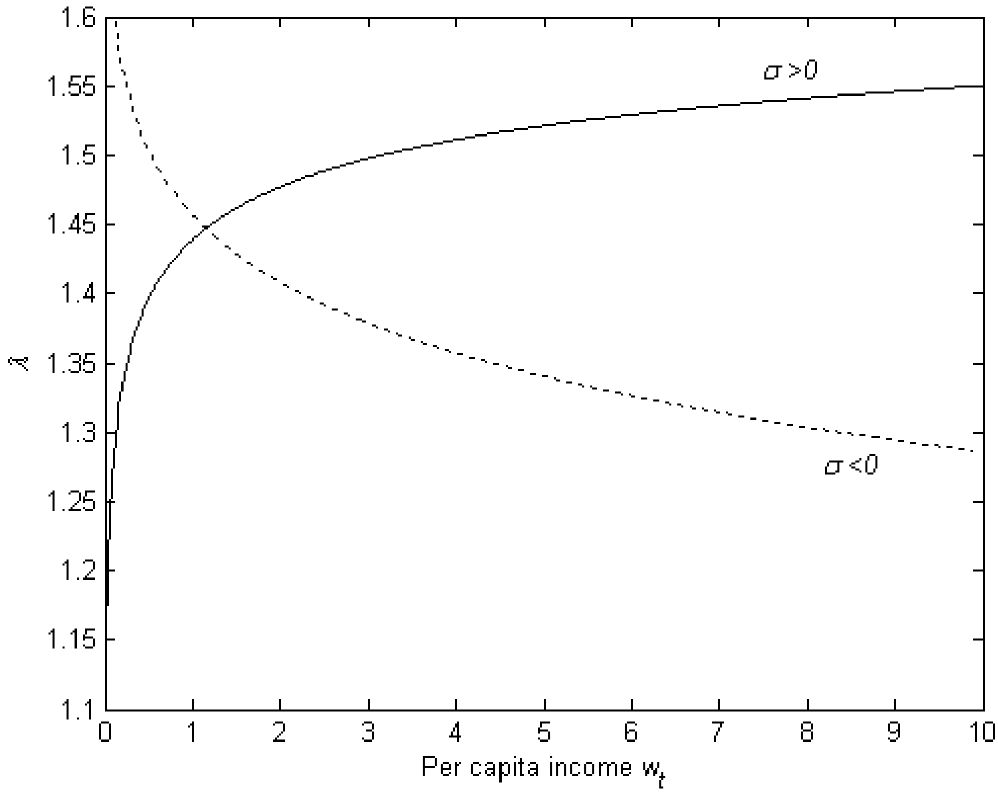

Figure 3: The function $\lambda(w_t)$. The picture depicts λ which captures the effect of learning on income. The solid curve corresponds to $\sigma = 0.5$ and the dotted curve to $\sigma = -0.5$. The other parameters are $\beta = 2/3$, $C(x) = x^2$, $q = 0.1$, $\sigma_a^2 = 0.01$, $\sigma_\theta^2 = \sigma_\alpha^2 = 1$, $\omega = 0.5$, $M = 50$ and $z = 0.5$.

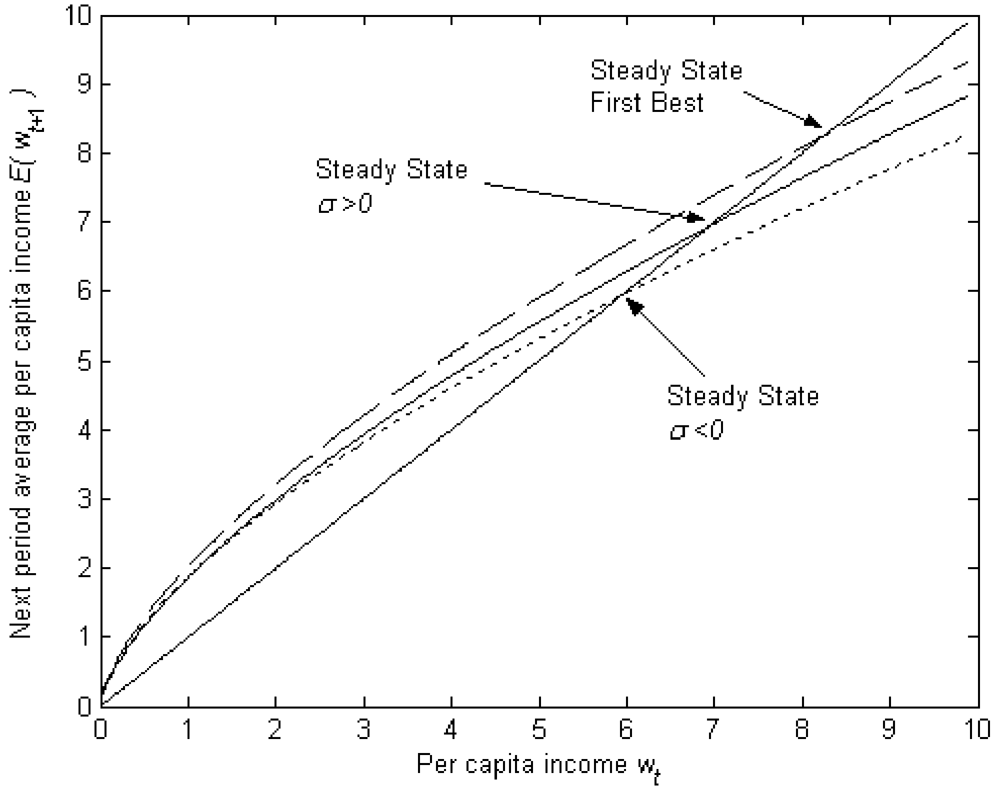

Figure 4: The dynamics of income in an economy along its average path. The curves represent the average income in period $t+1$, $E(w_{t+1})$, as a function of income in period t, w_t. The solid curve corresponds to $\sigma = 0.5$ and the dotted curve to $\sigma = -0.5$. The dashed curve corresponds to the first-best economy. The economies' steady-states are located at the intersections of these curves with the 45° line, represented as a solid line. The other parameters are $\beta = 2/3$, $C(x) = x^2$, $q = 0.1$, $\sigma_a^2 = 0.01$, $\sigma_\theta^2 = \sigma_\alpha^2 = 1$, $\omega = 0.5$, $M = 50$ and $z = 0.5$.

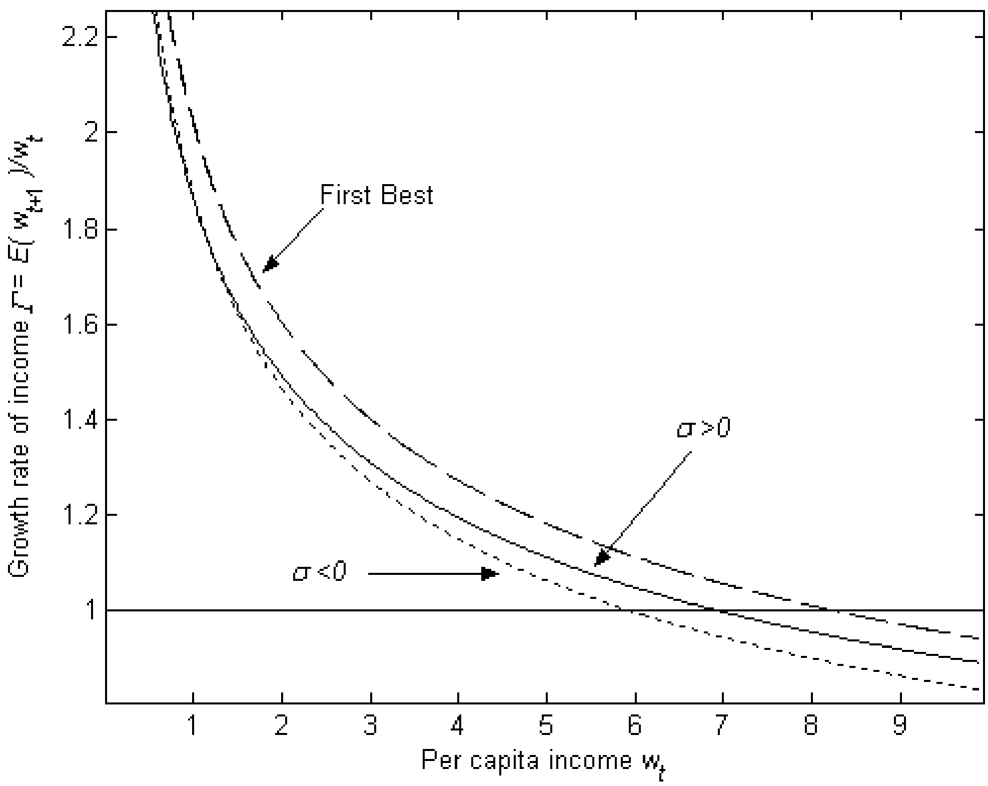

Figure 5: The growth rate of income. The picture depicts the growth rate of income, $\Gamma(w_t) \equiv E(\widetilde{w}_{t+1})/w_t$, during the transition to the steady-state. The solid curve corresponds to $\sigma = 0.5$ and the dotted curve to $\sigma = -0.5$. The dashed curve corresponds to the first-best economy. The other parameters are $\beta = 2/3$, $C(x) = x^2$, $q = 0.1$, $\sigma_a^2 = 0.01$, $\sigma_\theta^2 = \sigma_\alpha^2 = 1$, $\omega = 0.5$, $M = 50$ and $z = 0.5$.