Do Central Bank Liquidity Facilities Affect Interbank Lending Rates?

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Abstract

In response to the global financial crisis that started in August 2007, central banks provided extraordinary amounts of liquidity to the financial system. To investigate the effect of central bank liquidity facilities on term interbank lending rates, we estimate a six-factor arbitrage-free model of U.S. Treasury yields, financial corporate bond yields, and term interbank rates. This model can account for fluctuations in the term structure of credit risk and liquidity risk. A significant shift in model estimates after the announcement of the liquidity facilities suggests that these central bank actions did help lower the liquidity premium in term interbank rates.

We thank conference participants at the Federal Reserve Bank of New York, the Federal Deposit Insurance Corporation, the Board of Governors of the Federal Reserve System, the Bank of England, Köc University, and the Federal Reserve Bank of San Francisco—and especially, Pierre Collin-Dufresne and Simon Potter—for helpful comments. The views in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Federal Reserve Bank of San Francisco or the Board of Governors of the Federal Reserve System.

Draft date: June 2, 2009.

1 Introduction

In early August 2007, amidst declining prices and credit ratings for U.S. mortgage-backed securities and other forms of structured credit, international money markets came under severe stress. Short-term funding rates in the interbank market rose sharply relative to yields on comparable-maturity government securities. For example, the three-month U.S. dollar London interbank offered rate (LIBOR) jumped from only 20 basis points higher than the three-month U.S. Treasury yield during the first seven months of 2007 to over 110 basis points higher during the final five months of the year. This enlarged spread was also remarkable for persisting into 2008.

LIBOR rates are widely used as reference rates in financial instruments, including derivatives contracts, variable-rate home mortgages, and corporate notes, so their unusually high levels in 2007 and 2008 appeared likely to have widespread adverse financial and macroeconomic repercussions.[1] To limit these adverse effects, central banks around the world established an extraordinary set of lending facilities that were intended to increase financial market liquidity and ease strains in term interbank funding markets, especially at maturities of a few months or more. Monetary policy operations typically focus on an overnight or very short-term interbank lending rate. However, on December 12, 2007, the Bank of Canada, the Bank of England, the European Central Bank (ECB), the Federal Reserve, and the Swiss National Bank jointly announced a set of measures designed to address elevated pressures in term funding markets. These measures included foreign exchange swap lines established between the Federal Reserve and the ECB and the Swiss National Bank to provide U.S. dollar funding in Europe. The Federal Reserve also announced a new Term Auction Facility, or TAF, to provide depository institutions with a source of term funding. The TAF term loans were secured with various forms of collateral and distributed through an auction.

The TAF and similar term lending facilities by other central banks were not monetary policy actions as traditionally defined.[2] Instead, these central bank actions were meant to improve the distribution of reserves and liquidity by targeting a narrow market-specific funding problem. The press release introducing the TAF described its purpose in this way: "By allowing the Federal Reserve to inject term funds through a broader range of counterparties and against a broader range of collateral than open market operations, this facility could help

[1] As a convenient redundancy, we follow the literature in referring to "LIBOR rates."

[2] The Federal Reserve, in its normal operations, tries to hit a daily target for the federal funds rate, which is the overnight interest rate for interbank lending of bank reserves. The central bank liquidity facilities were not intended to alter the current level or the expected future path for the funds rate or the overall level of bank reserves (i.e., the term lending was sterilized by sales of Treasury securities).

promote the efficient dissemination of liquidity when the unsecured interbank markets are under stress." (Federal Reserve Board, December 12, 2007).

This paper assesses the effect of the establishment of these extraordinary central bank liquidity facilities on the interbank lending market and, in particular, on term LIBOR spreads over Treasury yields. In theory, the provision of central bank liquidity could lower the liquidity premium on interbank debt through a variety of channels. On the supply side, banks that have a greater assurance of meeting their own unforseen liquidity needs over time should be more willing to extend term loans to other banks. In addition, creditors should also be more willing to provide funding to banks that have easy and dependable access to funds, since there is a greater reassurance of timely repayment. On the demand side, with a central bank liquidity backstop, banks should be less inclined to borrow from other banks to satisfy any precautionary demand for liquid funds because their future idiosyncratic demands for liquidity over time can be met via the backstop. However, assessing the relative importance of these channels is difficult. Furthermore, judging the efficacy of central bank liquidity facilities in lowering the liquidity premium is complicated because LIBOR rates, which are for unsecured bank deposits, also include a credit risk premium for the possibility that the borrowing bank may default. The elevated LIBOR spreads during the financial crisis likely reflected both higher credit risk and liquidity premiums, so any assessment of the effect of the recent extraordinary central bank liquidity provision must also control for fluctuations in bank credit risk.

To analyze the effectiveness of the central bank liquidity facilities in reducing interbank lending pressures, we use a multifactor arbitrage-free (AF) representation of the term structure of interest rates and bank credit risk. Specifically, we estimate an affine arbitrage-free term structure representation of U.S. Treasury yields, the yields on bonds issued by financial institutions, and term LIBOR rates using weekly data from 1995 to midyear 2008. For tractability, the model uses the arbitrage-free Nelson-Siegel (AFNS) structure. Christensen, Diebold, and Rudebusch (CDR, 2007) show that a three-factor AFNS model fits and forecasts the Treasury yield curve very well and avoids many of the estimation difficulties encountered with unrestricted AF latent factor models. In this paper, we incorporate three additional factors: two factors that capture bank debt risk dynamics, as in Christensen and Lopez (2008), and a third factor specific to LIBOR rates. The resulting six-factor representation provides arbitrage-free joint pricing of Treasury yields, financial corporate bond yields, and LIBOR rates. This structure allows us to decompose movements in LIBOR rates into changes in bank debt risk premiums and changes in a factor specific to the interbank market that includes a liquidity premium. We can also conduct hypothesis testing and counterfactual analysis related to the introduction of the central bank liquidity facilities.

Our results support the view that the central bank liquidity facilities established in December 2007 helped lower LIBOR rates. Specifically, the parameters governing the term LIBOR factor within the model are shown to change after the introduction of the liquidity facilities. The hypothesis of constant parameters is overwhelmingly rejected, suggesting that the behavior of this factor, and thus of the LIBOR market, was directly affected by the introduction of central bank liquidity facilities. To quantify this effect, we use the model to construct a counterfactual path for the 3-month LIBOR rate by assuming that the LIBOR-specific factor remained constant at its historical average after the introduction of the liquidity facilities. Our analysis suggests that the counterfactual 3-month LIBOR rate averaged significantly higher—on the order of 70 basis points higher—than the observed rate from December 2007 through the middle of 2008. Figure 1 shows the difference between the observed three-month LIBOR rate and our model-implied counterfactual path for that rate during this period. From the start of the financial crisis—which was triggered by an August 9, 2007, announcement by the French bank BNP Paribas—until the TAF and swap joint central bank announcement in mid-December 2007, the observed LIBOR rate averaged 8 basis points higher that the counterfactual rate. Such signs of distress in the interbank market helped spur the announcement of the central bank liquidity facilities. After that announcement, the difference between the observed three-month LIBOR rate and the counterfactual rate quickly turned negative and reached approximately -75 basis points, where it stayed for the remainder of our sample. This result suggests that if the central bank liquidity facilities had not been created, the 3-month LIBOR rate would have been substantially higher.

There are two recent research literatures particularly relevant to our analysis. First, in terms of methodology, our empirical model is similar to earlier factor models of LIBOR rates, notably Collin-Dufresne and Solnik (2001) and Feldhütter and Lando (2008). Feldhütter and Lando (2008), for example, incorporate a LIBOR rate in a six-factor arbitrage-free model of Treasury, swap, and corporate yields. They use two factors to describe Treasury yields, two factors for the credit spreads of financial corporate bonds, one factor for LIBOR rates, and one factor for swap rates—with all factors assumed to be independent. Although similar, our six-factor model allows for complete dynamic interactions among the various factors and includes a broader range of maturities in the estimation. A second relevant literature, of course, is the burgeoning analysis of the recent financial crisis. Notably, with respect to the interbank market, Taylor and Williams (2009), McAndrews, Sarkar, and Wang (2008) and

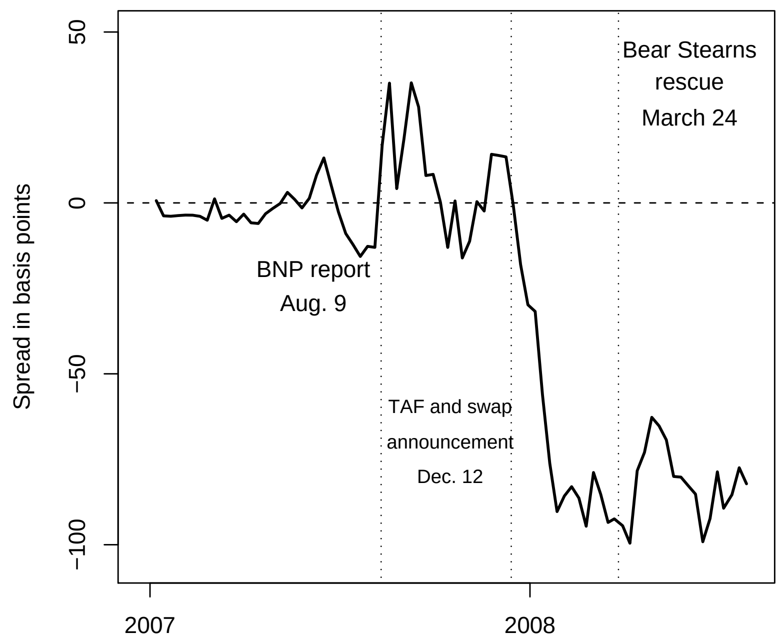

Figure 1: **Difference Between the Three-Month LIBOR Rate and Counterfactual.** This figure shows the observed three-month LIBOR rate minus the model-implied counterfactual path generated by fixing the LIBOR-specific factor at its historical average prior to December 14, 2007, in effect neutralizing the idiosyncratic effects in the LIBOR market. The illustrated period starts at the beginning of 2007, while the model estimation sample covers the period from January 6, 1995 to July 25, 2008.

Wu (2009) examine the effect of central bank liquidity facilities on the liquidity premium in LIBOR by controlling for movements in credit risk as measured by credit default swap prices for the borrowing banks in standard simple event-study regressions.[3] Unfortunately, based on only small differences in the specifications of their regressions, these studies disagree about the effectiveness of the central bank actions; therefore, we employ a very different methodology that provides a complete accounting of the dynamics of credit and liquidity risk.

The remainder of the paper is structured as follows. The next section presents our data and details the structure of our empirical six-factor arbitrage-free term structure model. Section 3 presents our estimation method and model estimates, and Section 4 focuses on the financial crisis that started in August 2007. It describes the central bank liquidity facilities established

[3]There are also recent related theoretical analysis of liquidity in the interbank lending market, as described in Allen, Carletti, and Gale (2009).

and the subsequent interest rate movements through the lens of our estimated model. Various interpretations of our results are considered. Section 6 concludes.

2 An Empirical Model of Treasury, Bank, and LIBOR Yields

In this section, we describe the data from the three financial markets of interest to our analysis and introduce an affine arbitrage-free joint model of Treasury yields, financial bond yields, and LIBOR rates.

2.1 Three Financial Markets

Treasury securities, bank bonds, and interbank term lending contracts are closely related debt instruments but differ in their relative amounts of credit and liquidity risk. Treasury securities are generally considered to be free from credit risk and are the most liquid debt instruments available. In our empirical work, we use 708 weekly observations (Fridays) from January 6, 1995, to July 25, 2008 on zero-coupon Treasury yields with maturities of 3, 6, 12, 24, 36, 60, 84, and 120 months, as described by Gürkaynak, Sack and Wright (2007).[4] Prices for unsecured lending of U.S. dollars at various maturities between banks are given by LIBOR rates, which are determined each business morning by a British Bankers' Association (BBA) survey of a panel of 16 large banks.[5] In the credit risk literature (e.g., Collin-Dufresne and Solnik 2001), LIBOR rates are often considered on par with AA-rated corporate bond rates since the BBA survey panel of banks is reviewed and revised as necessary to maintain high credit quality. Our LIBOR data consist of the 3-, 6-, and 12-month maturities.[6]

Figure 2 illustrates the spread of the three-month LIBOR rate over the three-month Treasury yield. Both the size and duration of this elevated spread in 2007 and 2008 clearly stand out as exceptional. A key date is August 9, 2007, which marks the start of the turmoil in financial markets and the jump in LIBOR rates. An important trigger for the financial

[4] We limit our sample to the first year of the financial crisis for two reasons. During this period, the Fed's liquidity operations were being sterilized, so they altered the composition and not the size of the Fed's balance sheet. Also, after the end of our sample, there were additional policy actions, such as government insurance for bank debt and interbank loans, that have potentially significant implications for bank credit and liquidity risk but do not involve direct injections of liquidity. Therefore, our limited sample allows us to get a clean reading on just the effect of liquidity facilities.

[5] The BBA discards the four highest and four lowest quotes and takes the average of the remaining eight quotes, which becomes the LIBOR rate for that specific term deposit on that day. Currently, the banks in the U.S. dollar LIBOR panel include: Bank of America, Bank of Tokyo-Mitsubishi UFJ Ltd, Barclays Bank plc, Citibank NA, Credit Suisse, Deutsche Bank AG, HBOS, HSBC, JP Morgan Chase, Lloyds TSB Bank plc, Rabobank, Royal Bank of Canada, The Norinchukin Bank, The Royal Bank of Scotland Group, UBS AG, and West LB AG.

[6] Appendix 1 describes the conversion of the quoted LIBOR rates into continuously compounded yields.

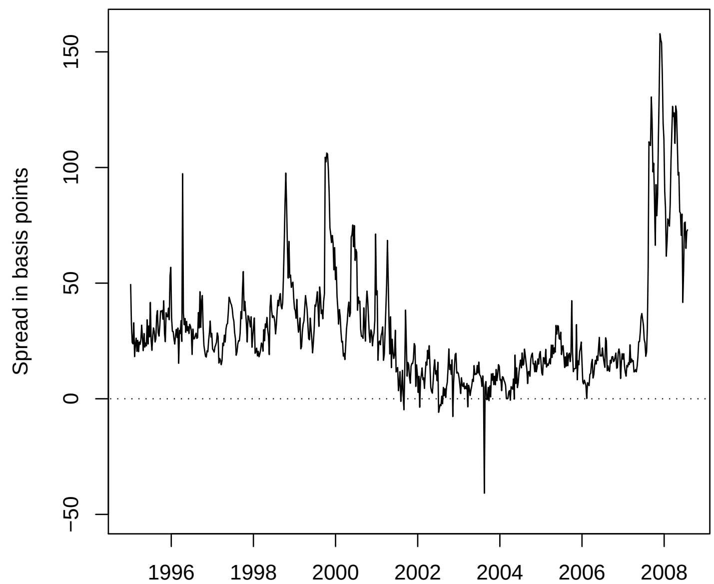

Figure 2: **Spread of Three-Month LIBOR rate over the Treasury Yield.**
This figure shows the weekly spread of the three-month LIBOR rate over the three-month Treasury bond yield from January 6, 1995 to July 25, 2008.

crisis and the tightening of the money markets was the announcement by the French bank BNP Paribas that it would suspend redemptions from three of its investment funds.[7] The mean spread in our sample prior to August 10, 2007, is about 25 basis points, while after that date, the mean spread is 98 basis points.[8] Fluctuations in the LIBOR-Treasury spread are commonly attributed to movements in credit and liquidity risk premiums.[9] The credit risk premium compensates for the possibility that the borrowing bank will default. The

[7]The BNP Paribas press release stated that "the complete evaporation of liquidity in certain market segments of the U.S. securitization market has made it impossible to value certain assets fairly regardless of their quality or credit rating ... during these exceptional times, BNP Paribas has decided to temporarily suspend the calculation of the net asset value as well as subscriptions/redemptions."

[8]Data on the LIBOR-Treasury spread and on a very similar spread, the well-known eurodollar to Treasury (or TED) yield spread, can be obtained earlier than the 1995 start of our estimation sample (which is determined by the availability of bank debt rates). Even in comparison to these earlier periods, the recent episode stands out as extraordinary.

[9]The LIBOR-Treasury spread is also affected by changes in the "convenience yield" for holding Treasury securities; therefore, Feldhütter and Lando (2008) and others use swap rates as an alternative riskless rate benchmark that is free from idiosyncratic Treasury movements. However, because we focus on the dynamic interactions between bank bond yields and LIBOR rates, the choice of the risk-free rate is not an issue for our analysis. Also note that seasonality issues, such as examined by Neely and Winters (2006), should not be an issue for our analysis since our LIBOR rates have maturities greater than one month.

liquidity risk premium is compensation for tying up funds in loans that—unlike liquid Treasury securities—cannot easily be unwound before the loan matures. Importantly, liquidity risk depends on the expected size of the idiosyncratic and aggregate liquidity shocks that effect both the lender and borrower.[10] Specifically, in the interbank market, borrowing and lending banks worry about their ability to obtain ready funds during the term of the loan, and each may desire a precautionary liquidity buffer.

To shed some light on the extent to which the jump in LIBOR rates represented an increase in liquidity risk or in credit risk, our empirical analysis compares these rates to yields on the unsecured bonds of U.S. financial institutions. We obtain zero-coupon yields on the bond debt of U.S. banks and financial corporations from Bloomberg at the eight Treasury maturities listed above. Our empirical model will estimate the amount of risk associated with this financial debt by pooling across five different categories: A-rated and AA-rated financial corporate debt, and BBB-, A-, and AA-rated bank debt.[11] Yields for the first four types of debt are available for our entire 1995-2008 sample, while yields on AA-rated bank debt are only available after August 2001. At comparable maturities, LIBOR rates and yields on AA-rated bank debt should be very close because both represent the cost of lending unsecured funds to similar institutions. Indeed, for much of our sample, these rates are almost identical. As shown in Figure 3, at a three-month maturity, the spread of the AA-rated bank debt yield over the LIBOR rate and the spread of the AA-rated financial corporate debt yield over the LIBOR rate are typically very close to zero. Furthermore, most deviations—say, in 2001 and 2002—were short-lived; therefore, financial bond debt and interbank loans appear to have had very similar credit and liquidity risk characteristics. Of course, there was a persistent and exceptional deviation that started at the end of 2007 during which the LIBOR fell *below* the yield on comparable financial corporate debt. We provide empirical evidence in Section 5 that the *relatively* low rate on interbank borrowing after December 12, 2007, reflected the extraordinary commitment by central banks to provide liquidity to the interbank market.

2.2 Six-factor AFNS Model

In this subsection, we introduce a joint affine AF model of Treasury yields, financial bond yields, and LIBOR rates. Following Duffie and Kan (1996), affine AF term structure models have been very popular, especially because yields are convenient linear functions of underlying

[10]The underlying liquidity risk is systemic in nature, as in Li, et al. (2009); that is, the borrowing or lending bank may be unable to sell sufficient quantities of assets in a timely manner and at a low cost, especially without a significant adverse effect on market prices.

[11]Appendix 1 describes the conversion of the reported interest rates into continuously compounded yields. For more information on the Bloomberg data, see Feldhütter and Lando (2008).

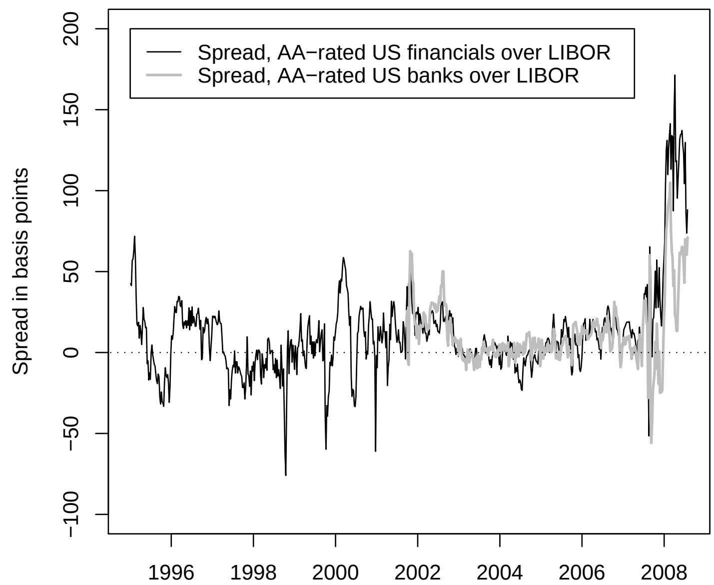

Figure 3: **Spreads of Three-Month Bank Debt Yields over LIBOR Rates.**
This figure shows the yield spread on three-month bonds issued by AA-rated U.S. banks over the three-month LIBOR rate and the similar spread for AA-rated U.S. financial firms. The data for financial firms are from January 6, 1995, to July 25, 2008, while the data for banks start on September 21, 2001.

latent factors with factor loadings that can be calculated from a system of ordinary differential equations. Unfortunately, there are many technical difficulties involved with the estimation of AF latent factor models, which tend to be overparameterized and have numerous likelihood maxima that have essentially identical fit to the data but very different implications for economic behavior (Kim and Orphanides, 2005 and Duffee, 2008). Researchers have employed a variety of techniques to facilitate estimation including the imposition of additional model structure.[12] Notably, CDR impose general level, slope, and curvature factor loadings that are derived from the popular Nelson and Siegel (1987) yield curve to obtain an AFNS model. They show that such a model can closely fit and forecast the term structure of Treasury yields quite well over time and can be estimated in a straightforward and robust fashion.

In this paper, we show that an AFNS model can be readily estimated for a joint rep-

[12]For example, many researchers simply restrict parameters with small t-statistics in a first round of estimation to zero. Duffee (2008) describes the difficulties associated with the canonical model that require "a fairly elaborate hands-on estimation procedure."

resentation of Treasury, bank bond, and LIBOR yields.[13] Researchers have typically found that three factors—typically referred to as level, slope, and curvature—are sufficient to model the time-variation in the cross-section of nominal Treasury bond yields (e.g., Litterman and Scheinkman, 1991). Similarly, we use a three-factor representation for Treasury yields. The most general joint model of Treasury, bank bond, and LIBOR rates would add three more factors for the bank bond yield curve and another three for the LIBOR rates of various maturities. However, this nine-factor model is unlikely to be the most parsimonious empirical representation, for as noted in the previous section, movements in Treasury, bank bond, and LIBOR rates all share common elements.

Some evidence on the number of additional factors required to capture variation in financial bond yields can be obtained from their principal components. We subtract the bond yields for the four categories of debt that are available for our complete sample (i.e., A-rated and AA-rated financial corporate debt and BBB- and A-rated bank debt) from comparable-maturity Treasury yields and calculate the first two principal components for these 32 yield spreads (i.e., four rating-industry categories times eight maturities). The first two principal components account for 85.5 and 8.8 percent, respectively, of the observed variation in the bank debt yield spreads. The associated 32 factor loadings for these principal components are reported in Table 1. The first principal component has very similar loadings across various maturities so it can be viewed as a level factor. In contrast, the loadings of the second principal component monotonically increase with maturity, which suggests a slope factor. Therefore, we include two spread factors in our model to account for differences between bank debt yields and Treasuries, which is also supported by evidence in Driessen (2005) and Christensen and Lopez (2008). Finally, as in Feldhütter and Lando (2008), a single LIBOR factor appears likely to be able to capture the small deviations between LIBOR rates and bank debt yields. Therefore, our joint representation has six factors: three for nominal Treasury bond yields, two additional ones for financial bond rate spreads, and finally, a sixth factor to capture idiosyncratic variation in LIBOR rates.

Specifically, Treasury yields depend on a state vector of the three nominal factors (i.e., level, slope, and curvature) denoted as $X_t^T = (L_t^T, S_t^T, C_t^T)$. The instantaneous risk-free rate is given by

$$r_t^T = L_t^T + S_t^T,$$

while the dynamics of the three state variables under the risk-neutral (or Q) pricing measure

[13] In related work, Christensen, Lopez, and Rudebusch (2008) show that a four-factor AFNS model provides a tractable and robust joint empirical model of nominal and real Treasury yield curves.

Maturity in months	U.S. Financials				U.S. Banks			
	A		AA		BBB		A	
	PC1	PC2	PC1	PC2	PC1	PC2	PC1	PC2
3	-0.16	-0.31	-0.16	-0.37	-0.18	-0.26	-0.14	-0.27
6	-0.15	-0.24	-0.14	-0.30	-0.17	-0.22	-0.15	-0.24
12	-0.14	-0.06	-0.13	-0.13	-0.16	-0.05	-0.15	-0.10
24	-0.17	0.01	-0.16	-0.08	-0.20	0.06	-0.18	-0.02
36	-0.20	0.08	-0.19	-0.01	-0.21	0.15	-0.19	0.11
60	-0.20	0.15	-0.18	0.05	-0.21	0.16	-0.20	0.09
84	-0.17	0.19	-0.17	0.12	-0.22	0.30	-0.20	0.19
120	-0.16	0.16	-0.15	0.10	-0.22	0.13	-0.19	0.07

Table 1: **Loadings on the First Two Principal Components of Credit Spreads**
This table reports the loadings of each maturity on the first (PC1) and second (PC2) principal components for the zero-coupon credit spreads for A- and AA-rated U.S. financial firms and BBB- and A-rated U.S. banks covering the period from January 6, 1995, to July 25, 2008. The analysis is based on 32 time series, each with 708 weekly observations.

are given by

$$\begin{pmatrix} dL_t^T \\ dS_t^T \\ dC_t^T \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\lambda^T & \lambda^T \\ 0 & 0 & -\lambda^T \end{pmatrix} \begin{pmatrix} L_t^T \\ S_t^T \\ C_t^T \end{pmatrix} dt + \begin{pmatrix} \sigma_{L^T} & 0 & 0 \\ 0 & \sigma_{S^T} & 0 \\ 0 & 0 & \sigma_{C^T} \end{pmatrix} \begin{pmatrix} dW_t^{Q,L^T} \\ dW_t^{Q,S^T} \\ dW_t^{Q,C^T} \end{pmatrix},$$

where W^Q is a standard Brownian motion in $\mathbf{R}^3$. Given this affine framework, CDR show that the yield on a zero-coupon Treasury bond with maturity τ at time t, $y_t^T(\tau)$, is given by

$$y_t^T(\tau) = L_t^T + \left(\frac{1 - e^{-\lambda^T \tau}}{\lambda^T \tau}\right) S_t^T + \left(\frac{1 - e^{-\lambda^T \tau}}{\lambda^T \tau} - e^{-\lambda^T \tau}\right) C_t^T + \frac{A^T(\tau)}{\tau}.$$

That is, the three factors are given exactly the same level, slope, and curvature factor loadings as in the Nelson-Siegel (1987) yield curve. A shock to L_t^T affects yields at all maturities uniformly; a shock to S_t^T affects yields at short maturities more than long ones; and a shock to C_t^T affects mid-range maturities most.[14] The yield function also contains a yield-adjustment term, $\frac{A^T(\tau)}{\tau}$, that is time-invariant and only depends on the maturity of the bond. CDR provide an analytical formula for this term, which under our identification scheme is entirely determined by the volatility matrix. CDR find that allowing for a maximally flexible parameterization of the volatility matrix diminishes out-of-sample forecast performance, so we

[14] Again, it is this identification of the general *role* of each factor, even though the factors themselves remain unobserved and the precise factor loadings depend on the estimated λ, that ensures the estimation of the AFNS model is straightforward and robust—unlike the maximally flexible affine arbitrage-free model.

restrict it to be diagonal.[15]

To incorporate bond yields for U.S. banks and financial firms into this structure, we follow Christensen and Lopez (2008). Namely, the instantaneous discount rate for corporate bonds from industry i (bank or financial corporation) with rating c (BBB, A, or AA) is assumed to be

$$r_t^{i,c} = \alpha_0^{i,c} + \left(1 + \alpha_{L^T}^{i,c}\right) L_t^T + \left(1 + \alpha_{S^T}^{i,c}\right) S_t^T + \left(\alpha_{L^S}^{i,c}\right) L_t^S + \left(\alpha_{S^S}^{i,c}\right) S_t^S,$$

where $\left(L_t^T, S_t^T\right)$ are the Treasury factors described above and $\left(L_t^S, S_t^S\right)$ are two bank debt yield spread factors. The instantaneous credit spread over the instantaneous risk-free Treasury rate becomes

$$\begin{aligned} s_t^{i,c} &\equiv r_t^{i,c} - r_t^T \\ &= \alpha_0^{i,c} + \left(\alpha_{L^T}^{i,c}\right) L_t^T + \left(\alpha_{S^T}^{i,c}\right) S_t^T + \left(\alpha_{L^S}^{i,c}\right) L_t^S + \left(\alpha_{S^S}^{i,c}\right) S_t^S. \end{aligned}$$

Note that the sensitivity of these risk factors can be adjusted by varying the $\alpha^{i,c}$ parameters, which is consistent with the pattern observed in the principal component analysis of the yield spreads in Table 1.[16]

To obtain the desired Nelson-Siegel level and slope factor-loading structure for the two bank yield spread factors, their dynamics under the pricing measure are given by

$$\begin{pmatrix} dL_t^S \\ dS_t^S \\ dL_t^T \\ dS_t^T \\ dC_t^T \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -\lambda^S & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\lambda^T & \lambda^T \\ 0 & 0 & 0 & 0 & -\lambda^T \end{pmatrix} \begin{pmatrix} L_t^S \\ S_t^S \\ L_t^T \\ S_t^T \\ C_t^T \end{pmatrix} dt + \Sigma^S \begin{pmatrix} dW_t^{Q,L^S} \\ dW_t^{Q,S^S} \\ dW_t^{Q,L^T} \\ dW_t^{Q,S^T} \\ dW_t^{Q,C^T} \end{pmatrix},$$

where Σ^S is a diagonal matrix, since the two common credit risk factors are assumed to be independent of the three factors determining the risk-free rate. This structure delivers the desired Nelson-Siegel factor loadings for all five factors in the corporate bond yield function. As a result, the yield on a corporate zero-coupon bond from industry i with rating c and

[15] We have fixed the mean under the Q-measure at zero, without loss of generality. The AFNS model dynamics under the Q-measure may appear restrictive, but CDR show this structure coupled with general risk pricing provides a very flexible modeling structure.

[16] Note that for each rating category, we do not take rating transitions into consideration. This is a theoretical inconsistency of our approach, but the model will extract common risk factors across rating categories and business sectors if they are present in the data. Taking the rating transitions into consideration will not change our results in a significant way. The model framework does allow for such extensions; for example, the method used by Feldhütter and Lando (2008) can be applied in this setting under the restriction that each rating category has the same factor loading on the two common credit risk factors. We leave this for future research.

maturity τ is given by

$$
\begin{aligned}
y_t^{i,c}(\tau) &= \Big(1+\alpha_{L^T}^{i,c}\Big)L_t^T + \Big(1+\alpha_{S^T}^{i,c}\Big)\Big(\frac{1-e^{-\lambda^T\tau}}{\lambda^T\tau}\Big)S_t^T + \Big(1+\alpha_{S^T}^{i,c}\Big)\Big(\frac{1-e^{-\lambda^T\tau}}{\lambda^T\tau} - e^{-\lambda^T\tau}\Big)C_t^T \\
&\quad +\alpha_0^{i,c} + \Big(\alpha_{L^S}^{i,c}\Big)L_t^S + \Big(\alpha_{S^S}^{i,c}\Big)\Big(\frac{1-e^{-\lambda^S\tau}}{\lambda^S\tau}\Big)S_t^S + \frac{A^{i,c}(\tau)}{\tau},
\end{aligned}
$$

where the yield-adjustment term $\frac{A^{i,c}(\tau)}{\tau}$ is time-invariant and depends only on the maturity of the bond.

Finally, to account for idiosyncratic differences between U.S. dollar LIBOR rates and corporate bond yields paid by AA-rated U.S. financial institutions, we include a sixth factor in the model for the discount rate applied to term loans in the interbank market. This instantaneous discount rate is given by

$$r_t^{Lib} = r_t^{Fin,AA} + \alpha^{Lib} + X_t^{Lib},$$

where the Q dynamics of the LIBOR-specific factor are assumed to be given by

$$dX_t^{Lib} = -\kappa_{Lib}^Q X_t^{Lib}dt + \sigma_{Lib}dW_t^{Q,Lib}.$$

This factor is assumed to be independent of the other five factors under the pricing measure. Thus, the full state vector, $X_t = (L_t^S, S_t^S, L_t^T, S_t^T, C_t^T, X_t^{Lib})$, of the six-factor model has assumed Q-dynamics:

$$
dX_t = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\lambda^S & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\lambda^T & \lambda^T & 0 \\ 0 & 0 & 0 & 0 & -\lambda^T & 0 \\ 0 & 0 & 0 & 0 & 0 & -\kappa_{Lib}^Q \end{pmatrix} X_t dt + \Sigma^{Lib} \begin{pmatrix} dW_t^{Q,L^S} \\ dW_t^{Q,S^S} \\ dW_t^{Q,L^T} \\ dW_t^{Q,S^T} \\ dW_t^{Q,C^T} \\ dW_t^{Q,Lib} \end{pmatrix},
$$

where Σ^{Lib} is a diagonal matrix. The discount rate to be applied to LIBOR contracts is then

$$
\begin{aligned}
r_t^{Lib} &= r_t^{Fin,AA} + \alpha^{Lib} + X_t^{Lib} \\
&= \alpha_0^{Fin,AA} + \Big(1+\alpha_{L^T}^{Fin,AA}\Big)L_t^T + \Big(1+\alpha_{S^T}^{Fin,AA}\Big)S_t^T + \Big(\alpha_{L^S}^{Fin,AA}\Big)L_t^S + \Big(\alpha_{S^S}^{Fin,AA}\Big)S_t^S + \alpha^{Lib} + X_t^{Lib}.
\end{aligned}
$$

The continuously compounded LIBOR yield is

$$
\begin{aligned}
y_t^{Lib}(\tau) \;=\;& \alpha_0^{Fin,AA} + \alpha^{Lib} \\
&+\Big(1+\alpha_{L^T}^{Fin,AA}\Big)L_t^T + \Big(1+\alpha_{S^T}^{Fin,AA}\Big)\left(\frac{1-e^{-\lambda^T\tau}}{\lambda^T\tau}\right)S_t^T + \Big(1+\alpha_{S^T}^{Fin,AA}\Big)\left(\frac{1-e^{-\lambda^T\tau}}{\lambda^T\tau} - e^{-\lambda^T\tau}\right)C_t^T \\
&+\Big(\alpha_{L^S}^{Fin,AA}\Big)L_t^S + \Big(\alpha_{S^S}^{Fin,AA}\Big)\left(\frac{1-e^{-\lambda^S\tau}}{\lambda^S\tau}\right)S_t^S + \left(\frac{1-e^{-\kappa_{Lib}^Q\tau}}{\kappa_{Lib}^Q\tau}\right)X_t^{Lib} + \frac{A^{Lib}(\tau)}{\tau},
\end{aligned}
$$

where the yield-adjustment term is

$$
\begin{aligned}
\frac{A^{Lib}(\tau)}{\tau} \;=\;& -\frac{\sigma_{L^T}^2(1+\alpha_{L^T}^{Fin,AA})^2}{6}\tau^2 - \sigma_{S^T}^2(1+\alpha_{S^T}^{Fin,AA})^2\left(\frac{1}{2(\lambda^T)^2} - \frac{1}{(\lambda^T)^3}\frac{1-e^{-\lambda^T\tau}}{\tau} + \frac{1}{4(\lambda^T)^3}\frac{1-e^{-2\lambda^T\tau}}{\tau}\right) \\
&-\sigma_{C^T}^2(1+\alpha_{S^T}^{Fin,AA})^2\left(\frac{1}{2(\lambda^T)^2} + \frac{1}{(\lambda^T)^2}e^{-\lambda^T\tau} - \frac{1}{4\lambda^T}\tau e^{-2\lambda^T\tau} - \frac{3}{4(\lambda^T)^2}e^{-2\lambda^T\tau}\right) \\
&-\sigma_{C^T}^2(1+\alpha_{S^T}^{Fin,AA})^2\left(\frac{5}{8(\lambda^T)^3}\frac{1-e^{-2\lambda^T\tau}}{\tau} - \frac{2}{(\lambda^T)^3}\frac{1-e^{-\lambda^T\tau}}{\tau}\right) \\
&-\frac{\sigma_{L^S}^2(\alpha_{L^S}^{Fin,AA})^2}{6}\tau^2 - \sigma_{S^S}^2(\alpha_{S^S}^{Fin,AA})^2\left(\frac{1}{2(\lambda^S)^2} - \frac{1}{(\lambda^S)^3}\frac{1-e^{-\lambda^S\tau}}{\tau} + \frac{1}{4(\lambda^S)^3}\frac{1-e^{-2\lambda^S\tau}}{\tau}\right) \\
&-\sigma_{Lib}^2\left(\frac{1}{2(\kappa_{Lib}^Q)^2} - \frac{1}{(\kappa_{Lib}^Q)^3}\frac{1-e^{-\kappa_{Lib}^Q\tau}}{\tau} + \frac{1}{4(\kappa_{Lib}^Q)^3}\frac{1-e^{-2\kappa_{Lib}^Q\tau}}{\tau}\right).
\end{aligned}
$$

The description so far has detailed the dynamics under the pricing measure and, by implication, determined the functions that we fit to the observed yields. The above structure places no restrictions on the dynamic drift components under the real-world P-measure beyond the requirement of constant volatility; therefore, to facilitate the empirical implementation, we employ the essentially affine risk premium specification introduced in Duffee (2002). In the Gaussian framework, this specification implies that the risk premiums, Γ_t, depend on the state variables as

$$\Gamma_t = \gamma^0 + \gamma^1 X_t,$$

where $\gamma^0 \in \mathbf{R}^6$ and $\gamma^1 \in \mathbf{R}^{6\times 6}$ contain unrestricted parameters. The relationship between real-world yield curve dynamics under the P-measure and risk-neutral dynamics under the Q-measure is given by

$$dW_t^Q = dW_t^P + \Gamma_t dt.$$

Thus, we can write the P-dynamics of the state variables as

$$dX_t = K^P(\theta^P - X_t)dt + \Sigma dW_t^P,$$

where both K^P and θ^P are allowed to vary freely relative to their counterparts under the Q-measure.

3 Model estimation

This section first describes our Kalman filter estimation procedure for the AFNS joint model of Treasury, bank debt, and LIBOR rates and then provides estimation results.

3.1 Estimation procedure

We estimate the six-factor AFNS model using the Kalman filter, which is an efficient and consistent estimator for our Gaussian model. In addition, the Kalman filter requires a minimum of assumptions about the observed data and easily handles missing data. The measurement equation for estimation is

$$y_t = \begin{pmatrix} y_t^c \\ y_t^T \\ y_t^{Lib} \end{pmatrix} = \begin{pmatrix} A^c \\ A^T \\ A^{Lib} \end{pmatrix} + \begin{pmatrix} B^c \\ B^T \\ B^{Lib} \end{pmatrix} X_t + \varepsilon_t.$$

The data vector y_t is a (51×1) vector consisting of y_t^c with 40 financial bond rates, y_t^T with the eight Treasury yields, and y_t^{Lib} with the three LIBOR yields.[17] Correspondingly, the constant term consists of a (40×1) vector A^c, an (8×1) vector A^T, and a (3×1) vector A^{Lib}. The factor-loading matrix for our six factors consists of a 40x6 matrix B^c, an (8×6) matrix B^T, and a (3×6) matrix B^{Lib}. Note that the λ parameters are included in these parameter matrices.

For identification, we choose the A-rated bond yields to be the benchmark for the financial corporate sector. That is, we set the constant $\alpha_0^{Fin,A}$ equal to zero, and let the factor loadings on the two spread factors have unit sensitivity, i.e., $\alpha_L^{Fin,A} = 1$ and $\alpha_S^{Fin,A} = 1$. This choice is motivated by the availability of a full sample of data for both A-rated banks and financial firms, but it is not restrictive and simply implies that the sensitivities to changes in the two spread factors are measured relative to those of the A-rated financial firms and that the estimated values of those factors represent the absolute sensitivity of the benchmark A-rated financial corporate bond yields.

[17] Note that y_t^c contains 40 rates across our five (industry, rating) categories after September 11, 2001. Before that date, when yields for AA-rated bonds issued by U.S. banks are unavailable, y_t^c contains 32 series across four categories.

For continuous-time Gaussian models, the conditional mean vector and the conditional covariance matrix are given by

$$\begin{aligned} E^P[X_T|\mathcal{F}_t] &= (I - \exp(-K^P \Delta t))\mu^P + \exp(-K^P \Delta t) X_t, \\ V^P[X_T|\mathcal{F}_t] &= \int_0^{\Delta t} e^{-K^P s} \Sigma \Sigma' e^{-(K^P)' s} ds, \end{aligned}$$

where $\Delta t = T - t$ and $\exp(-K^P \Delta t)$ is a matrix exponential. Stationarity of the system under the P-measure is ensured provided the real components of all the eigenvalues of K^P are positive. This condition is imposed in all estimations, so we can start the Kalman filter at the unconditional mean and covariance matrix

$$\widehat{X}_0 = \mu^P \quad \text{and} \quad \widehat{\Sigma}_0 = \int_0^{\infty} e^{-K^P s} \Sigma \Sigma' e^{-(K^P)' s} ds,$$

where the latter is approximated with a 10-year span. The transition state equation for the Kalman filter is given by

$$X_{t_i} = \Phi^0_{\Delta t_i} + \Phi^1_{\Delta t_i} X_{t_{i-1}} + \eta_{t_i},$$

where $\Delta t_i = t_i - t_{i-1}$ and

$$\Phi^0_{\Delta t_i} = (I - \exp(-K^P \Delta t_i))\mu^P, \quad \Phi^1_{\Delta t_i} = \exp(-K^P \Delta t_i), \quad \text{and} \quad \eta_{t_i} \sim N\Big(0, \int_0^{\Delta t_i} e^{-K^P s} \Sigma \Sigma' e^{-(K^P)' s} ds\Big).$$

All measurement errors are assumed to be independently and identically distributed white noise with an error structure given by

$$\begin{pmatrix} \eta_t \\ \varepsilon_t \end{pmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} Q & 0 \\ 0 & H \end{pmatrix} \right].$$

Each maturity of the Treasury bond yields has its own measurement error standard deviation. For parsimony, the measurement errors for the corporate bond yields are assumed to have a uniform standard deviation across all ratings and maturities. Furthermore, we include a separate standard deviation parameter for each of the three maturities in the LIBOR rate data.

3.2 Estimation results

The estimation of our six-factor model requires specification of the P-dynamics of the state variables. We conduct a careful evaluation of various model specifications, as summarized

Alternative	Goodness of fit statistics			
Specifications	$\log L$	k	p-value	BIC
(1) Unrestricted K^P	180,171.90	86	n.a.	-359,779.4
(2) $\kappa_{35}^P = 0$	180,171.86	85	0.7773	-359,785.9
(3) $\kappa_{35}^P = \kappa_{16}^P = 0$	180,171.82	84	0.7773	-359,792.4
(4) $\kappa_{35}^P = \kappa_{16}^P = \kappa_{23}^P = 0$	180,171.80	83	0.8415	-359,798.9
(5) $\kappa_{35}^P = \cdots = \kappa_{41}^P = 0$	180,171.68	82	0.6242	-359,805.2
(6) $\kappa_{35}^P = \cdots = \kappa_{63}^P = 0$	180,171.58	81	0.6547	-359,811.6
(7) $\kappa_{35}^P = \cdots = \kappa_{13}^P = 0$	180,171.47	80	0.6390	-359,817.9
(8) $\kappa_{35}^P = \cdots = \kappa_{24}^P = 0$	180,171.31	79	0.5716	-359,824.2
(9) $\kappa_{35}^P = \cdots = \kappa_{54}^P = 0$	180,171.01	78	0.4386	-359,830.1
(10) $\kappa_{35}^P = \cdots = \kappa_{34}^P = 0$	180,170.51	77	0.3173	-359,835.7
(11) $\kappa_{35}^P = \cdots = \kappa_{32}^P = 0$	180,170.47	76	0.7773	-359,842.2
(12) $\kappa_{35}^P = \cdots = \kappa_{36}^P = 0$	180,170.25	75	0.5071	-359,848.3
(13) $\kappa_{35}^P = \cdots = \kappa_{53}^P = 0$	180,169.37	74	0.1846	-359,853.1
(14) $\kappa_{35}^P = \cdots = \kappa_{14}^P = 0$	180,167.84	73	0.0802	-359,856.6
(15) $\kappa_{35}^P = \cdots = \kappa_{15}^P = 0$	180,167.15	72	0.2401	-359,861.8
(16) $\kappa_{35}^P = \cdots = \kappa_{42}^P = 0$	180,166.20	71	0.1681	-359,866.5
(17) $\kappa_{35}^P = \cdots = \kappa_{52}^P = 0$	180,165.23	70	0.1637	-359,871.1
(18) $\kappa_{35}^P = \cdots = \kappa_{51}^P = 0$	180,162.42	69	0.0178	-359,872.0
(19) $\kappa_{35}^P = \cdots = \kappa_{31}^P = 0$	180,160.46	68	0.0477	**-359,874.7**
(20) $\kappa_{35}^P = \cdots = \kappa_{43}^P = 0$	180,153.73	67	0.0003	-359,867.8
(21) $\kappa_{35}^P = \cdots = \kappa_{21}^P = 0$	180,149.97	66	0.0061	-359,866.8
(22) $\kappa_{35}^P = \cdots = \kappa_{65}^P = 0$	180,146.25	65	0.0064	-359,865.9
(23) $\kappa_{35}^P = \cdots = \kappa_{46}^P = 0$	180,142.61	64	0.0070	-359,865.2
(24) $\kappa_{35}^P = \cdots = \kappa_{56}^P = 0$	180,135.02	63	0.0001	-359,856.6
(25) $\kappa_{35}^P = \cdots = \kappa_{25}^P = 0$	180,125.40	62	< 0.0001	-359,843.9
(26) $\kappa_{35}^P = \cdots = \kappa_{64}^P = 0$	180,111.52	61	< 0.0001	-359,822.7
(27) $\kappa_{35}^P = \cdots = \kappa_{26}^P = 0$	180,083.20	60	< 0.0001	-359,772.7
(28) $\kappa_{35}^P = \cdots = \kappa_{12}^P = 0$	180,079.76	59	0.0087	-359,772.3
(29) $\kappa_{35}^P = \cdots = \kappa_{45}^P = 0$	180,059.29	58	< 0.0001	-359,738.0
(30) $\kappa_{35}^P = \cdots = \kappa_{62}^P = 0$	180,043.08	57	< 0.0001	-359,712.1
(31) $\kappa_{35}^P = \cdots = \kappa_{61}^P = 0$	180,038.57	56	0.0027	-359,709.6

Table 2: **Evaluation of Alternative Specifications of the Six-Factor LIBOR Model.** There are 31 alternative estimated specifications of the six-factor LIBOR rate model with full 6×6 K^P matrix. Each specification is listed with its maximum log likelihood ($\log L$), number of parameters (k), the p-value from a likelihood ratio test of the hypothesis that it differs from the specification above with one more free parameter, and the BIC information criterion. The period analyzed covers January 6, 1995 to July 25, 2008, a total of 708 weekly observations.

in Table 2. The first column of this table describes the alternative specifications considered. Specification (1) at the top corresponds to an unrestricted 6×6 mean-reversion matrix K^P, which provides maximum flexibility in fitting the dynamic interactions between the six state variables. We then pare down this matrix using a general-to-specific strategy that restricts the

K^P	$K^P_{\cdot,1}$	$K^P_{\cdot,2}$	$K^P_{\cdot,3}$	$K^P_{\cdot,4}$	$K^P_{\cdot,5}$	$K^P_{\cdot,6}$	θ^P	Σ
$K^P_{1,\cdot}$	-1.084	-1.232	0	0	0	0	0.01332	0.001876
	(0.152)	(0.200)					(0.00766)	(0.000111)
$K^P_{2,\cdot}$	0.6535	0.3596	0	0	0.1560	-1.170	-0.009623	0.002006
	(0.303)	(0.295)			(0.0498)	(0.541)	(0.00672)	(0.000180)
$K^P_{3,\cdot}$	0	0	0.05506	0	0	0	0.06669	0.004781
			(0.188)				(0.0194)	(0.000121)
$K^P_{4,\cdot}$	0	0	1.155	0.9203	-1.127	-2.807	-0.03003	0.008242
			(0.583)	(0.185)	(0.173)	(1.38)	(0.0194)	(0.000235)
$K^P_{5,\cdot}$	0	0	0	0	0.7677	13.31	-0.01803	0.02647
					(0.503)	(4.14)	(0.00842)	(0.000631)
$K^P_{6,\cdot}$	3.763	4.574	0	-0.3537	-0.2235	8.939	0.05616	0.004704
	(0.688)	(0.728)		(0.141)	(0.0991)	(1.35)	(0.118)	(0.000239)

Table 3: **Parameter Estimates for the Preferred Six-Factor Specification.** This table shows the estimated parameters and standard deviations (in parentheses) of the K^P matrix, θ^P vector, and diagonal Σ volatility matrix for the six-factor model. The data used are weekly covering the period from January 6, 1995, to July 25, 2008. λ^T is estimated at 0.6407 (0.0034), λ^S is estimated at 0.3914 (0.0095), and κ^Q_{Lib} is estimated at 0.0366 (0.0783). Finally, the constant α^{Lib} is estimated at -0.05695 (0.118).

least significant parameter (as measured by the ratio of the parameter value to its standard error) to zero and then re-estimate the model. Therefore, specification (2) sets $\kappa^P_{35} = 0$, so it has one fewer estimated parameters, and so on. This strategy of eliminating the least significant coefficients continues to the final specification, which has a diagonal K^P matrix.

Each estimated specification is listed with its log likelihood ($\log L$), its number of estimated parameters (k), and the p-value from a likelihood ratio test of the hypothesis that it differs from the specification with one more free parameter—that is, comparing specification (s) with specification ($s-1$). We also report the Bayes information criterion (BIC), which is commonly used for model selection (see, e.g., Harvey, 1989) and is defined as BIC$= -2\log L + k \log T$, where T is the number of data observations, which in our sample is 708. The BIC is minimized by specification (19) (the boldface entry). Although this specification is our preferred one in terms of parsimony and consistency, we should stress that our conclusions in the next section regarding the effectiveness of the central bank liquidity facilities are robust to the specification of the K^P matrix.[18]

[18]In particular, we obtained similar results using the Akaike information criterion.

Based on the BIC results in Table 2, our preferred specification of the K^P matrix is

$$K^P = \begin{pmatrix} \kappa_{11}^P & \kappa_{12}^P & 0 & 0 & 0 & 0 \\ \kappa_{21}^P & \kappa_{22}^P & 0 & 0 & \kappa_{25}^P & \kappa_{26}^P \\ 0 & 0 & \kappa_{33}^P & 0 & 0 & 0 \\ 0 & 0 & \kappa_{43}^P & \kappa_{44}^P & \kappa_{45}^P & \kappa_{46}^P \\ 0 & 0 & 0 & 0 & \kappa_{55}^P & \kappa_{56}^P \\ \kappa_{61}^P & \kappa_{62}^P & 0 & \kappa_{64}^P & \kappa_{65}^P & \kappa_{66}^P \end{pmatrix}.$$

This specification imposes 18 restrictions on the K^P matrix,[19] and the estimated parameter values are presented in Table 3.

These estimated parameters suggest several interesting results. First, the Treasury level factor is not impacted by any of the other factors, supporting the empirical results in Christensen, Lopez and Rudebusch (2008) as well as Christensen and Lopez (2008). The intuition here is that monetary policy is a key driver of the entire interest rate environment. Second, the dynamics of the Treasury slope factor are affected by all the Treasury factors, again as found in the aforementioned studies. Third, the dynamics of the two credit risk factors are interrelated, but only slightly affected by the Treasury factors. Finally, in contrast to Feldhütter and Lando (2008), the dynamics of the LIBOR factor is found to be affected by both credit risk factors as well as the Treasury slope and curvature factors. In addition, the LIBOR factor influences the dynamics of the corporate slope factor, the Treasury slope factor and the Treasury curvature factor. This result suggests that short-term credit rates, and LIBOR rates in particular, contain useful information regarding the dynamics of the overall interest rate environment. This result further highlights how important the functioning of the interbank market appears to be for the broader capital markets.

Table 4 reports the estimated factor loadings of the state variables in the corporate bond yield function for each rating category represented in the data sample. Note that for both U.S. banks and financial firms, lower credit quality tends to imply higher sensitivities to the two common credit risk factors. The exception is the sensitivity of AA-rated U.S. financials to the common credit risk slope factor, which is marginally higher than the value observed for A-rated U.S. financials. Generally speaking, this implies that the credit spreads of bonds issued by firms with lower credit quality tend to have higher and steeper credit spread curves. Furthermore, we can compare the risk sensitivities for U.S. banks and financial firms. For the

[19] The likelihood ratio test of the significance of the 18 parameter restrictions jointly is 22.88. This is χ^2 distributed with 18 degrees of freedom which gives a p-value of 0.1952.

Rating	U.S. Financials				
	α_0^C	α_{LT}^C	α_{ST}^C	α_{LS}^C	α_{SS}^C
A	0	-0.02583	-0.06438	1	1
		(0.0238)	(0.00552)		
AA	0.003369	-0.08189	-0.07158	0.9033	1.040
	(0.000244)	(0.0212)	(0.00616)	(0.00576)	(0.0113)

Rating	U.S. Banks				
	α_0^C	α_{LT}^C	α_{ST}^C	α_{LS}^C	α_{SS}^C
BBB	0.0001667	-0.03335	-0.07316	1.153	1.072
	(0.000262)	(0.0270)	(0.00605)	(0.00514)	(0.00965)
A	-0.00002714	-0.03256	-0.05644	1.032	1.024
	(0.00299)	(0.0241)	(0.00659)	(0.00554)	(0.00947)
AA	-0.0002488	-0.009247	-0.04037	0.8239	0.8702
	(0.000492)	(0.0200)	(0.00684)	(0.00852)	(0.0118)

Table 4: **Estimated Factor Loadings in the Corporate Bond Yield Functions.** The estimated factor loadings for each of the rating categories for the preferred six-factor model. The data used are weekly, covering the period from January 6, 1995 to July 25, 2008. The numbers in parentheses are the estimated standard deviations of the parameter estimates.

benchmark A-rating category, we see that bonds with this rating have nearly identical risk sensitivities across the two sectors. For the AA-rating category, we see greater sensitivities in financial bonds than AA-rated bonds issued by U.S. banks. A partial explanation for this difference is the different data sample periods, where yields for AA-rated U.S. banks do not enter the sample until September 2001. Thus, the previous downturn in the credit cycle is only partially represented for AA-rated banks, while the very calm period from mid-2003 until mid-2007 is fully represented.

Finally, Table 5 details the fit of the model for Treasury, bank bond, and LIBOR rates. The fit of the Treasury rates is quite good and only slightly worse than in models of only Treasury yields (see CDR, for example). For the corporate bond yields, the root mean squared errors (RMSEs) of the fitted errors are in line with the estimated standard deviation for the fitted errors that we obtain from the Kalman filter, which is estimated at $\widehat{\sigma}_{\varepsilon^c} = 11.3$ basis points. Overall, given the fact that we are fitting a sizeable number of corporate bond yields jointly with only five state variables, the achieved fit of the corporate bond yields appears quite good. The model fits the six-month LIBOR rate perfectly, while the fit of the other LIBOR rates is well within the range considered acceptable when it comes to regular Treasury bond yield term structure models. Figure 4 illustrates the time series of the fitted errors for the three-month LIBOR rates. Note that there is little deterioration in the model's ability to fit the LIBOR rates during the financial crisis in 2007 and 2008; thus, the model appears

Maturity in months	Treasury yields	LIBOR rates	Bank bond yields BBB	A	AA	Financial bond yields A	AA
			Mean				
3	-5.52	-0.50	-6.33	-4.36	-3.90	-1.47	0.90
6	-3.53	0.00	3.91	5.18	0.64	3.29	5.08
12	0.27	-0.15	0.18	0.75	1.61	-1.19	0.90
24	2.04	–	1.29	0.05	2.01	-1.30	0.18
36	-0.30	–	-1.12	-0.06	1.85	-0.80	-1.31
60	-3.21	–	-3.59	-6.36	-9.16	-3.37	-3.98
84	0.49	–	5.14	3.39	7.74	1.43	-1.30
120	12.61	–	0.52	1.37	-0.74	3.30	-0.49
			RMSE				
3	15.67	10.18	13.28	12.37	11.49	11.66	12.67
6	6.58	0.00	10.15	10.58	11.48	10.15	11.90
12	3.16	8.64	11.58	9.35	11.19	10.72	9.48
24	2.64	–	11.22	7.34	8.50	6.52	7.80
36	1.70	–	12.05	8.94	12.47	8.55	9.57
60	3.91	–	11.58	10.01	13.17	8.88	9.83
84	3.57	–	14.50	9.56	14.87	8.76	8.85
120	14.87	–	14.67	14.15	13.55	12.74	12.83
No. obs	708	708	708	708	358	708	708

Table 5: **Summary Statistics for Six-Factor Model Fitted Errors.**
This table provides the mean and RMSE of the model fitted errors in basis points for Treasury bond yields, LIBOR rates, and corporate bond yields for U.S. banks rated BBB, A, and AA and U.S. financial firms rated A and AA. The model used is the preferred six-factor model.

flexible enough to capture the turmoil in the LIBOR market.

4 The financial crisis and central bank actions

In this section, we use the estimated model to illuminate the effect on LIBOR rates of the financial crisis and the central bank liquidity facilities. Figure 5 focuses on movements in the spread between the three-month LIBOR rate and the three-month Treasury yield during the last 18 months of our sample, from the beginning of 2007 through July 25, 2008. There are two key dates during this period. The first, August 9, 2007, marks the start of the turmoil in many financial markets and the jump in LIBOR rates. The second, December 12, 2007, marks the announcement by the Federal Reserve and other central banks of a strong new commitment to improve liquidity and the functioning of the interbank market.[20] Specifically, the Fed

[20]The Federal Reserve's initial response to the dislocations in the interbank lending market in the fall of 2007 was to promote and enhance the availability of its discount window as a source of funding. In particular, the Federal Reserve reduced the spread between the discount rate (or primary credit rate) and the target federal funds rate. However, through the end of 2007, discount window borrowing remained relatively low and

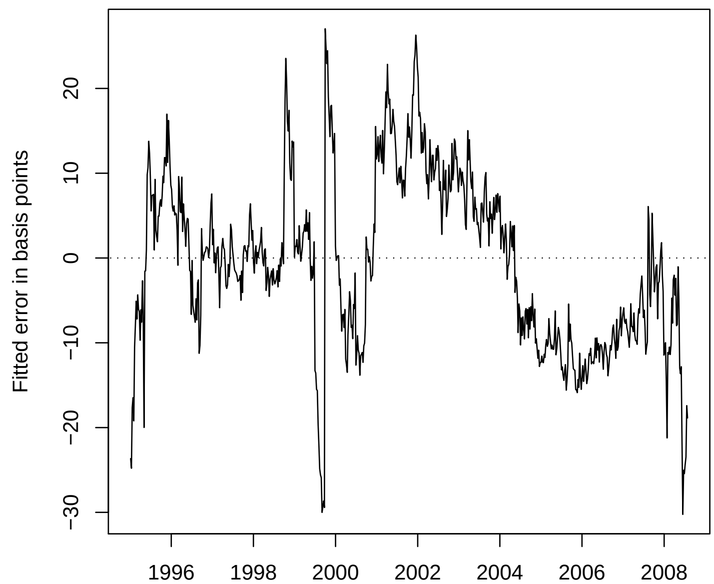

Figure 4: **Fitted Model Errors of Three-Month LIBOR**
This figure shows the fitted errors of three-month LIBOR rates in the six-factor model with the preferred specification of K^P. The data used in the estimation are from January 6, 1995, to July 25, 2008.

announced the creation of the TAF, which consisted of periodic auctions of fixed quantities of term funding to sound depository institutions,[21] and the establishment of coordinated dollar liquidity actions with the European Central Bank and the Swiss National Bank. The latter involved reciprocal foreign exchange swap lines, in which dollars were passed through to foreign central banks so they could extend term lending in dollars abroad. The TAF and the swap lines were meant to alleviate the dollar liquidity risk by making cash loans to banks that were secured by those banks' illiquid but sound assets, and many interpreted the initial mid-December 2007 announcements and actions by central banks as the key events signalling a change in the bank liquidity regime.[22] In particular, the initial announcements of the new liquidity facilities were accompanied by a widespread realization that the Federal Reserve and

interbank lending rates remained quite elevated.

[21]The first TAF auction occurred on December 17 for $20 billion in 28-day credit and was greatly oversubscribed.

[22]Both the TAF and the swap lines were scaled up in size during 2008, and the Federal Reserve subsequently also established several other liquidity facilities that provide loans to financial institutions other than banks, such as the Primary Dealer Credit Facility.

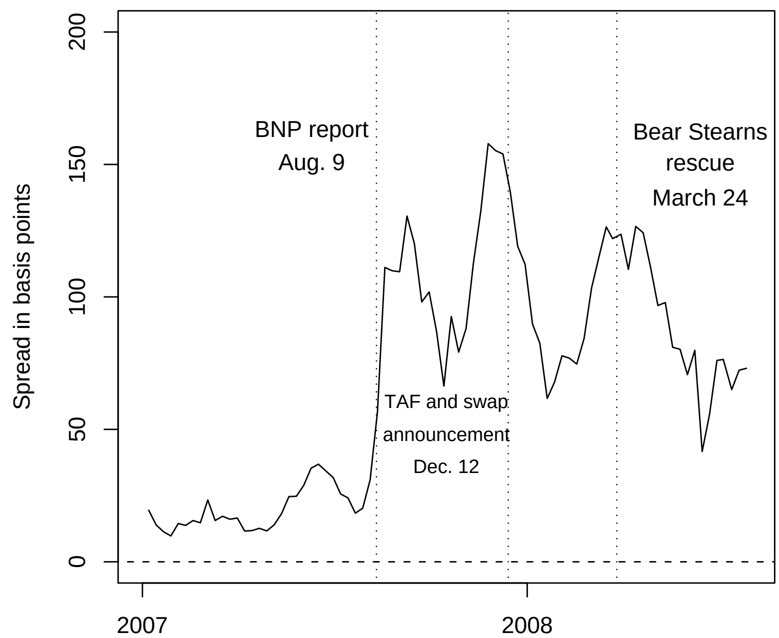

Figure 5: **Spread of Three-Month LIBOR over Three-Month Treasury Yield.** This figure shows the spread of the three-month LIBOR rate over the three-month Treasury bond yield since the beginning of 2007.

other central banks would provide forceful and innovative responses to bank liquidity needs going forward. Therefore, we consider mid-December 2007 as an a priori potential breakpoint in our analysis.

After the central bank announcements and actions in December 2007, the LIBOR-Treasury spread did fall, but not permanently, and it did not revert to its pre-August level. Accordingly, there has been much debate about the extent to which the central bank liquidity facilities alleviated stress in the interbank market (e.g., Taylor and Williams 2009, McAndrews, Sarkar, and Wang 2008, and Wu 2009). We investigate this question with our estimated model. Figure 6 shows the estimated path of our sixth factor, which is specific to the LIBOR market. Deviations of this factor from its mean (shown by a horizontal dashed line) indicate the direction and approximate size of the difference between the yield on AA-rated U.S. financial bonds and term LIBOR rates of the same maturity. Until December 2007, this factor moved within a fairly close range around its mean. However, following the introduction of the

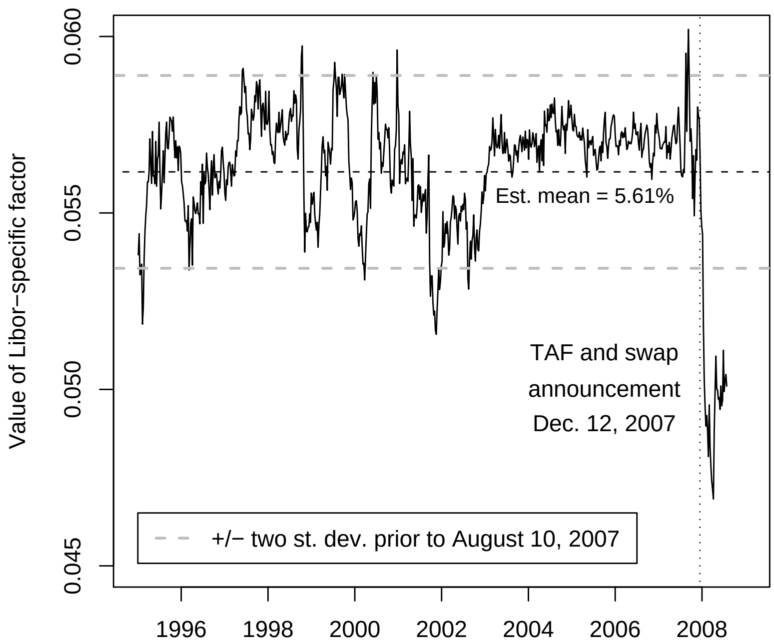

Figure 6: **Estimated LIBOR Factor from Preferred Six-Factor Model.**
This figure shows the estimated LIBOR-specific factor from the preferred six-factor model. The bond yields and LIBOR rates used in the estimation are weekly data from January 6, 1995 to July 25, 2008.

central bank liquidity facilities, it dropped quite low through the end of the sample. The figure appears quite consistent with the presence of a regime change in the dynamic behavior of X_t^{Lib} following the introduction of the TAF and other central bank liquidity operations.

To statistically test for changes in the dynamic properties of X_t^{Lib}, we investigate whether its parameters prior to December 14, 2007, denoted

$$\psi_{Lib}^{pre} = (\kappa_{26}^P, \kappa_{46}^P, \kappa_{56}^P, \kappa_{61}^P, \kappa_{62}^P, \kappa_{64}^P, \kappa_{65}^P, \kappa_{66}^P, \theta_{Lib}^P, \sigma_{Lib}, \kappa_{Lib}^Q, \alpha^{Lib})$$

in our preferred specification, changed to a new set of parameters, denoted

$$\psi_{Lib}^{post} = (\widetilde{\kappa}_{26}^P, \widetilde{\kappa}_{46}^P, \widetilde{\kappa}_{56}^P, \widetilde{\kappa}_{61}^P, \widetilde{\kappa}_{62}^P, \widetilde{\kappa}_{64}^P, \widetilde{\kappa}_{65}^P, \widetilde{\kappa}_{66}^P, \widetilde{\theta}_{Lib}^P, \widetilde{\sigma}_{Lib}, \widetilde{\kappa}_{Lib}^Q, \widetilde{\alpha}^{Lib}).$$

All other parameters in the model are assumed to remain unchanged. As the Kalman filter

K^P	$K^P_{\cdot,1}$	$K^P_{\cdot,2}$	$K^P_{\cdot,3}$	$K^P_{\cdot,4}$	$K^P_{\cdot,5}$	θ^P	Σ
$K^P_{1,\cdot}$	-1.072	-1.208	0	0	0	0.01132	0.001884
	(0.165)	(0.209)				(0.0275)	(0.000116)
$K^P_{2,\cdot}$	0.8645	0.5975	0	0	0.1343	-0.007720	0.002026
	(0.372)	(0.396)			(0.0580)	(0.0239)	(0.000191)
$K^P_{3,\cdot}$	0	0	0.0002158	0	0	0.07634	0.004784
			(0.0995)			(0.120)	(0.000127)
$K^P_{4,\cdot}$	0	0	1.034	0.9684	-1.179	-0.03393	0.008224
			(0.528)	(0.199)	(0.187)	(0.0803)	(0.000247)
$K^P_{5,\cdot}$	0	0	0	0	0.8492	-0.01265	0.02641
					(0.547)	(0.0525)	(0.000656)

Table 6: **Parameter Estimates for Preferred Specification With Regime Switch.** This table provides the estimated parameters and standard deviations (in parentheses) of the K^P matrix, θ^P vector, and Σ volatility matrix for first five factors in the preferred joint six-factor model with a regime switch as of December 14, 2007. The data are weekly from January 6, 1995, to July 25, 2008. λ^T is estimated at 0.6412 (0.00357), λ^S is estimated at 0.3914 (0.00976). The maximum log-likelihood value is 180,174.56.

can handle time-varying parameters, we can test this hypothesis using the likelihood ratio test. The estimated dynamic parameters for the non-LIBOR factors in the estimation of our preferred specification with a regime switch are not meaningfully different from before, as shown in Table 6. Table 7 reports the estimated parameters for the LIBOR-specific factor and compares them to those for the model without a regime switch. The likelihood ratio test of the hypothesis that no regime switch has taken place is

$$LR = 2[180,174.56 - 180,160.46] = 28.2 \sim \chi^2(12),$$

which is highly significant with a p-value of 0.0052. This test suggests that the hypothesis of unchanged parameters can be rejected and that there was a regime change during the week before December 14.

To quantify the impact that the introduction of the liquidity facilities had on the interbank market, we use a counterfactual analysis of what would have happened had they *not* been introduced. We use the full-sample model without the regime switch to generate a counterfactual path for the 3-month LIBOR rate that suggests what that rate *might* have been if it had been priced in accordance with prevailing conditions in the Treasury and corporate bond markets for U.S. financial firms. To quantify this effect, we "turn off" the LIBOR-specific factor by fixing it at its mean prior to December 14, 2007, and leaving the remaining factors unchanged at their previously estimated values. Thus, the counterfactual path provides a LI-

Parameter	Full sample	Regime switch	
		pre 12/07	post 12/07
κ^P_{26}	-1.170	-0.8762	-1.871
	(0.541)	(0.641)	(3.00)
κ^P_{46}	-2.807	-4.806	-3.710
	(1.38)	(2.27)	(9.00)
κ^P_{56}	13.31	16.17	40.44
	(4.14)	(6.15)	(16.1)
κ^P_{61}	3.768	3.200	0.7395
	(0.688)	(0.780)	(4.09)
κ^P_{62}	4.574	3.768	2.456
	(0.728)	(0.800)	(25.7)
κ^P_{64}	-0.3537	-0.3410	-0.08937
	(0.141)	(0.137)	(4.65)
κ^P_{65}	-0.2235	-0.1206	-0.5169
	(0.0991)	(0.108)	(2.15)
κ^P_{66}	8.939	8.158	15.16
	(1.35)	(1.51)	(15.2)
θ^P_{Lib}	0.05616	0.05621	0.05059
	(0.118)	(0.142)	(0.144)
σ_{Lib}	0.004704	0.004525	0.006325
	(0.000239)	(0.000260)	(0.00230)
κ^Q_{Lib}	0.0366	0.03725	0.01783
	(0.0783)	(0.0947)	(0.0590)
α^{Lib}	-0.05695	-0.05711	-0.05654
	(0.118)	(0.142)	(0.144)

Table 7: **Estimated Parameters for the LIBOR Factor with Regime Switch.** This table provides the estimated parameters and standard deviations (in parentheses) associated with the LIBOR-specific factor with and without a regime switch included following the establishment of central bank liquidity facilities. The model used is the preferred six-factor model estimated with Treasury bond yields and corporate bond yields for U.S. banks and U.S. financial firms in addition to LIBOR rates. The data used are weekly covering the period from January 6, 1995, to July 25, 2008.

BOR rate consistent with the risk factors reflected in the yields of bonds issued by AA-rated U.S. financial institutions.

Figure 7 illustrates the effect of the counterfactual path on the three-month LIBOR spread over the three-month Treasury rate since the beginning of 2007. Note that the model-implied three-month LIBOR spread is close to the observed spread over this period. Until December 2007, the counterfactual spread was tracking the observed spread relatively closely. However, by the end of 2007, a significant wedge developed between the two. As of the end of our sample on July 25, 2008, the difference between the counterfactual spread and the observed three-month LIBOR spread was 82 basis points. Therefore, our analysis suggests that, the

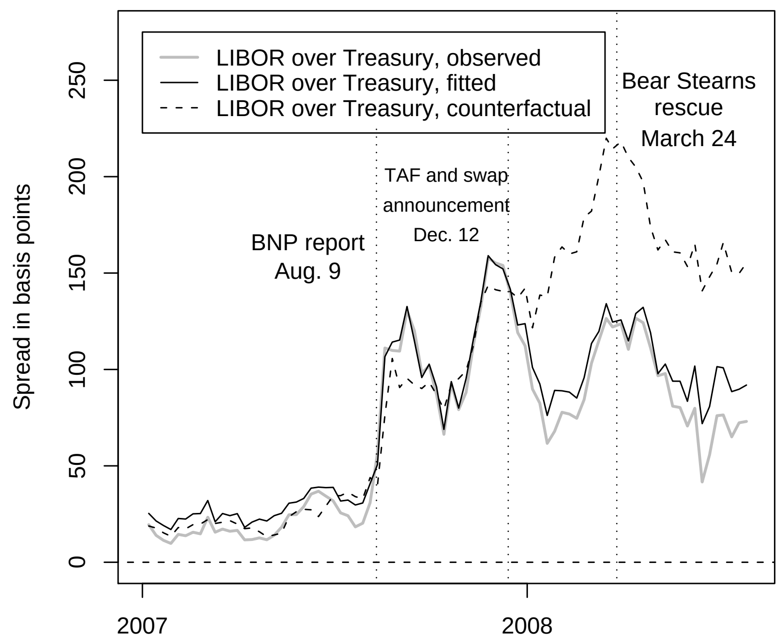

Figure 7: **Spread of the LIBOR Rate over Treasury Yield.**
This figure shows the spread of the observed and fitted three-month LIBOR rate over the three-month Treasury bond yield in the preferred six-factor model. The figure also illustrates the spread of the fitted three-month LIBOR rate when the LIBOR-specific factor is fixed at its historical average prior to December 14, 2007, in effect neutralizing the idiosyncratic effects in the LIBOR market. The illustrated period starts at the beginning of 2007, while the model estimation sample covers the period from January 6, 1995 to July 25, 2008.

three-month LIBOR rate would have been *higher* in the absence of the central bank liquidity facilities.

Our empirical results suggest that the announcement of the central bank liquidity facilities on December 12, 2007 altered the dynamics of the interbank lending market in the intended way; that is, the increased provision of bank liquidity by central banks lowered LIBOR rates relative to where they might have been in the absence of these actions. The abnormally large and persistent spread between bank debt and LIBOR yields that opened up after mid-December 2007 most likely reflects different liquidity concerns between the lender classes in these two markets. The LIBOR rate and interbank market rate are based on banks providing other banks with short-term funding. In contrast, the bank bond rates are derived from debt obligations issued to a broad class of investors that overwhelmingly consists of nonbank institutions. While these two classes of lenders most likely attach similar probabilities and prices to credit risk, they likely have different tolerances to liquidity problems. The different degrees to which central bank liquidity operations lowered the liquidity concerns of lenders in the interbank market by more than those in the bank bond market would be translated directly into the spread between these two markets. (Appendix 2 provides a simple conceptual framework that illustrates this effect.)

There are two other explanations that could also account for the increased spread between bank debt yields and LIBOR rates, but these alternatives do not convincingly fit this episode. The first explanation centers on changes in the nature or the quality of the data. In mid-April 2008, there were news reports that the 16 banks surveyed as part of the daily fixing of the LIBOR rates on U.S. dollar-denominated term deposits were underreporting their actual borrowing costs. If such underreporting were new, the distress in the interbank market would be more severe than reflected in LIBOR rates, and those rates would be low relative to the bank bond yields. However, the persistence of the high LIBOR spread through the end of our sample period despite a speedy investigation and resolution of these underreporting accusations seems to undermine this possible explanation. Alternatively, the quality of the corporate bond data, especially since August 2007, could be questioned due perhaps to reduced bond trading. Yet, the persistence of the larger spread over several months weakens this possible explanation as well. Also, it is hard to see why these data considerations would be linked to a mid-December regime shift.

The second alternative explanation for the larger spread is the possibility of a change in the relative credit risk characteristics of the bank debt and interbank loan markets, for example, through changes in perceived recovery rates.[23] Again, during our sample—and notably even during the 2001 recession—there were no substantial similar differences in relative credit risk. Furthermore, it is difficult to date any changes a priori to mid-December 2007. Still, conceivably, changes could have occurred in the relative credit risk between the LIBOR panel of international AA-rated banks and the domestic AA-rated banks and financial firms used to construct the Bloomberg bank debt curves. To examine this possibility within the context of our model, we generated synthetic five-year credit default swap (CDS) rates for the AA-rated U.S. financial firms and compared these to the median five-year CDS rate for the banks in the

[23] An unsecured deposit (e.g., an interbank loan) is more senior in the liability structure of a bank than senior unsecured debt. McAndrews, Sarkar, and Wang (2008) mention a recovery rate of 91.25% for unsecured deposits at banks with assets larger than $5 billion, as per the work of Kuritzkes, Schuermann, and Weiner (2005). On the other hand, the data provider Markit typically works with a recovery rate as low as 40% in its pricing of credit default swap contracts. However, it is not clear why this difference in recovery rates would have changed dramatically in December 2007.

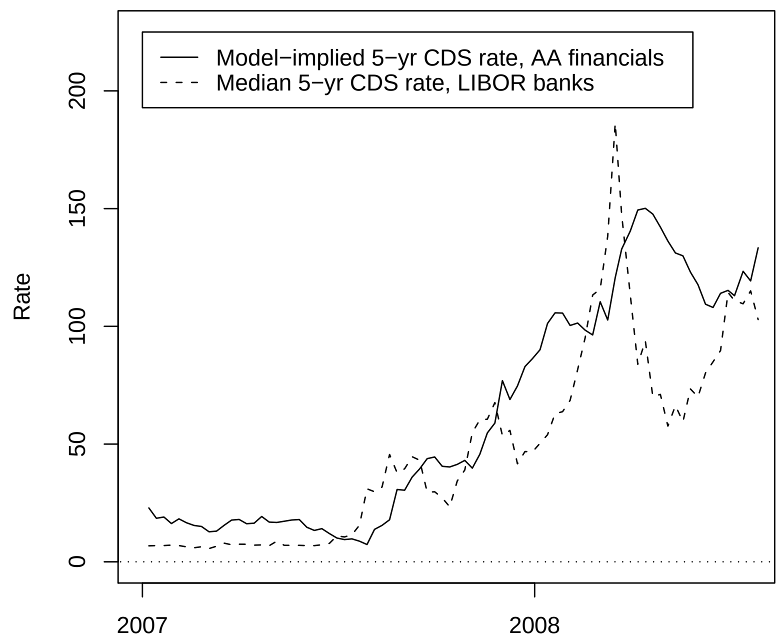

Figure 8: **Model-generated CDS spreads.**
This figure shows the implied five-year CDS rates for AA-rated U.S. financial firms based on the estimated parameters and factor paths from the preferred six-factor LIBOR model. In addition, the median of the five-year CDS rates of the 16 LIBOR panel banks on each observation date are shown. To align the level of the model-implied estimates with the observed CDS rates, the difference between the five-year Treasury par bond yield and the five-year swap rate has been added. The illustrated period starts at the beginning of 2007, while the model estimation sample covers the period from January 6, 1995 to July 25, 2008.

LIBOR panel. CDS rates are readily calculated from our model using the instantaneous credit spread for AA-rated U.S. financial firms, as presented earlier, and a recovery rate assumption of 50%; see Appendix 3 for further details. Figure 8 presents these model-implied five-year CDS rates relative to the median of the corresponding observed CDS rates for the banks in the LIBOR panel. The series have a correlation of nearly 90%, suggesting that the underlying credit dynamics for AA-rated financial institutions estimated by our model are very similar to those observed in the CDS market. This result supports our assumption of common credit characteristics across the LIBOR and bank debt panels and our view that this relationship did not materially change around the announcement of the central bank liquidity facilities.

5 Conclusion

In this paper, we address the question of whether interbank lending rates have responded to central bank liquidity operations by using a six-factor AFNS model that encompasses Treasury yields, financial corporate debt yields, and LIBOR rates. Our results provide support for the view that these operations, such as the introduction of the TAF, did lower LIBOR rates starting in December 2007 and through the end of our sample in July 2008. We find that the parameters governing the LIBOR factor in our model appear to change after the introduction of the liquidity facilities; i.e., the hypothesis of constant parameters over the full sample period is rejected. This result suggests that the behavior of this factor, and thus of the LIBOR market, was directly affected by these central bank liquidity operations. To quantify this effect, we use the model to construct a counterfactual path for the three-month LIBOR rate. The counterfactual three-month LIBOR rate averaged significantly higher than the observed rate from December 2007 into midyear 2008, which suggests that if the central bank liquidity operations had not occurred, the three-month LIBOR spread over Treasuries would have been even higher than the observed historical spread.

Appendix 1: Conversion of interest rate data

We convert the Bloomberg data for financial corporate bond rates into continuously compounded yields. The n-year yield at time t, $r_t(n)$, the corresponding zero-coupon bond price, $P_t(n)$, and the continuously compounded yield, $y_t(n)$, are related by

$$P_t(n) = \frac{1}{(1+r_t(n))^n} = e^{-y_t(n)n} \iff y_t(n) = -\frac{1}{n}\ln\frac{1}{(1+r_t(n))^n} = \ln(1+r_t(n)).$$

For maturities shorter than one year, we assume the standard convention of linear interest rates. For example, the zero-coupon bond price corresponding to the six-month yield is calculated as

$$P_t(6m) = \frac{1}{1+0.5r_t(6m)} = e^{-0.5y_t(6m)},$$

and the corresponding continuously compounded yield as

$$y_t(6m) = -2\ln\frac{1}{1+0.5r_t(6m)} = 2\ln(1+0.5r_t(6m)).$$

We also convert the LIBOR rates into continuously compounded yields, as in Feldhütter and Lando (2008). To facilitate this conversion, we approximate the day count ratio assuming that the LIBOR curve is smooth. Therefore, the net present value of the three-month LIBOR contract is

$$NPV_t^{Lib} = \frac{1}{1+\frac{1}{4}L(t,t+0.25)} = e^{-0.25y^{Lib}(t,t+0.25)},$$

where $L(t,t+0.25)$ denotes the quoted three-month LIBOR rate. The continuously compounded equivalent to the quoted three-month LIBOR rates is then

$$y^{Lib}(t,t+0.25) = -4\log\left[\frac{1}{1+\frac{1}{4}L(t,t+0.25)}\right] = 4\log(1+0.25L(t,t+0.25)).$$

Similarly, the six-month and twelve-month LIBOR rates can be converted into continuously compounded zero-coupon yields by the following formulas:

$$\begin{aligned} y^{Lib}(t,t+0.5) &= 2\log(1+0.5L(t,t+0.5)), \\ y^{Lib}(t,t+1) &= \log(1+L(t,t+1)). \end{aligned}$$

Appendix 2: Conceptual framework to illustrate liquidity risk effects

To help interpret the relative movements in Treasury, bank bond, and interbank rates and to motivate our empirical analysis, we present a very simple framework to illustrate differential credit and liquidity risks across different debt obligations and, by extension, how the provision of central bank liquidity can have differential effects on their associated yields. We assume a simple three-period setting in which at date zero, lenders must choose among three different 2-period securities as to where to invest their funds. The first investment option is a liquid Treasury security, which pays the risk-free rate of interest, which we normalize to zero, so a dollar invested in the liquid asset at date zero returns a dollar at date two. The second investment option is a bank-issued bond, in which a dollar invested at date zero will return $1 + r_B$ dollars at date two. The third investment option is an interbank loan, which will return $1 + r_L$ dollars at date two for a dollar invested at time zero. The rates of return, r_B and r_L, will be positive to account for credit and liquidity risk.

We assume that the markets for bank bonds and interbank loans are segmented to some degree, with differing market microstructures and lender preferences; in which case, r_B is not always identical to r_L, which is consistent with the observed data. Specifically, the interbank market investors are predominantly banks providing other banks with short-term funding. In contrast, bank bonds are issued to a much broader class of investors that overwhelmingly consists of nonbank institutions. We assume these two classes of lenders share the same perception of and attach the same price to credit risk. However, regarding liquidity risk, we assume that the two classes of lenders may face different liquidity shocks during the term of the debt at date one and may price that liquidity risk differently. Formally, we assume that at date one, bank bond investors are subject to a liquidity shock, such as an unexpected demand for funds, that induces an adjustment cost of α_1^B with a probability λ_1^B. In addition, the interbank lenders are subject to a liquidity shock that induces an adjustment cost of α_1^L with a probability λ_1^L. Investors in Treasuries can costlessly satisfy any liquidity shocks. Furthermore, at date two, the two bank investment options are also subject to a common default event, in which the borrowing bank declares insolvency, with probability λ_2 and cost δ, which is less than one to reflect only partial repayment of the principal. Any such partial recovery is shared equally by bondholders and interbank creditors.

Given this structure, the rate of return on the Treasury bond at date zero for date two is zero. Assuming that bond investors are indifferent between bank and Treasury bonds and that interbank lenders are indifferent between the Treasury bond and the interbank loan, the expected returns on the illiquid assets are:

$$r_B = \lambda_1^B \alpha_1^B + \lambda_2 \delta \qquad \text{and} \qquad r_L = \lambda_1^L \alpha_1^L + \lambda_2 \delta.$$

That is, the returns compensate for the costs associated with the date-one liquidity shocks and the date-two default, as weighted by the respective probabilities. Note that r_B and r_L, and their corresponding spreads over the Treasury rate, will move together with changes in the borrowing bank's default risk, so the jumps in the spread of LIBOR over the Treasury rate can reflect both credit and liquidity risk.

In contrast, in our simple theoretical structure, the spread between the bank bond rate and the LIBOR rate only reflects liquidity risk:

$$r_B - r_L = \lambda_1^B \alpha_1^B - \lambda_1^L \alpha_1^L.$$

Therefore, fluctuations in counterparty risk will not affect this spread, which in our data is the spread of the AA-rated bank debt yield over the corresponding LIBOR rate. In the early stage of the crisis, the spread was negative as the LIBOR rate rose relative to the bank debt rates. In our framework, this outcome would suggest that $\lambda_1^L \alpha_1^L$, the liquidity risk for interbank lenders, initially rose during the financial turmoil relative to the corresponding cost for the bank bond investors. The spread remained negative until roughly mid-December when the Federal Reserve and other central banks announced liquidity operations concentrating on the interbank market. After that, LIBOR declined sharply relative to the corresponding bank debt rates, and the spread rose. Again, our framework suggests that the central bank liquidity operations lowered the illiquidity cost and probability parameters for banks.

Appendix 3: The swap premium of a plain vanilla CDS

In this appendix, the reduced-form pricing of credit default swaps is described. It is assumed that there is a model for the instantaneous risk-free interest rate r_t, a model for the default intensity of the representative firm considered λ_t, and a model for the loss rate given default L_t. Let T denote the time to maturity of the CDS contract and let $t_1, \ldots, t_N$ denote the swap premium payment dates. In case of default before T, the payment on the default leg is assumed to be the loss rate L_t times the size of the notional. Given these assumptions the value of the default leg of a plain vanilla CDS contract per \$1 notional can be calculated as

$$\begin{aligned} V_{def}^{CDS}(0,T) &= E^Q\Big[\int_0^T L_s \mathbf{1}_{\{s<\tau\leq s+ds\}} e^{-\int_0^s r_u du} ds \Big| \mathcal{F}_0\Big] \\ &= E^Q\Big[\int_0^T L_s \lambda_s e^{-\int_0^s (r_u+\lambda_u) du} ds \Big| \mathcal{F}_0\Big], \end{aligned}$$

where τ is the unpredictable time of the first jump of the point process which indicates the default time in reduced-form credit risk models (for details see Lando, 1998).

In return for the loss protection, the protection buyer has to pay a premium, here denoted by $S^{CDS}(0,T)$ and quoted at an annual rate. If we define $\delta_i = t_i - t_{i-1}$ to be the time between the ith and the (i-1)th payment dates, the contractual payment on the ith payment date equals $\delta_i S^{CDS}(0,T)$. In addition, the market convention requires the accrued swap premium since the last payment date to be paid immediately upon default in exchange for the default leg payment. Given this convention, the value of the premium leg can be calculated as

$$\begin{aligned} V_{prem}^{CDS}(0,T) &= E^Q\Big[S^{CDS}(0,T)\sum_{i=1}^N \delta_i \mathbf{1}_{\{\tau>t_i\}} e^{-\int_0^{t_i} r_u du} | \mathcal{F}_0\Big] \\ &+ E^Q\Big[\sum_{i=1}^N \int_{t_{i-1}}^{t_i} S^{CDS}(0,T)(s-t_{i-1}) \mathbf{1}_{\{s<\tau\leq s+ds\}} e^{-\int_0^s r_u du} ds | \mathcal{F}_0\Big] \\ &= S^{CDS}(0,T) E^Q\Big[\sum_{i=1}^N \delta_i e^{-\int_0^{t_i}(r_u+\lambda_u)du} + \sum_{i=1}^N \int_{t_{i-1}}^{t_i} (s-t_{i-1})\lambda_s e^{-\int_0^s (r_u+\lambda_u)du} ds | \mathcal{F}_0\Big]. \end{aligned}$$

At inception, the swap premium is set to give the contract a value of zero:

$$S^{CDS}(0,T) = \frac{E^Q\Big[\int_0^T L_s \lambda_s e^{-\int_0^s (r_u+\lambda_u)du} ds | \mathcal{F}_0\Big]}{E^Q\Big[\sum_{i=1}^N \delta_i e^{-\int_0^{t_i}(r_u+\lambda_u)du} + \sum_{i=1}^N \int_{t_{i-1}}^{t_i} (s-t_{i-1})\lambda_s e^{-\int_0^s (r_u+\lambda_u)du} ds | \mathcal{F}_0\Big]}.$$

In the six-factor LIBOR model, the instantaneous risk-free rate is given by

$$r_t = L_t^T + S_t^T,$$

while the instantaneous credit spread of the representative AA-rated U.S. financial firm, which is the category we map to LIBOR, is given by

$$s_t^{AA,Fin} = \alpha_0^{AA,Fin} + \alpha_{L^T}^{AA,Fin} L_t^T + \alpha_{S^T}^{AA,Fin} S_t^T + \alpha_{L^S}^{AA,Fin} L_t^S + \alpha_{S^S}^{AA,Fin} S_t^S.$$

If we fix the expected loss rate at $L = 50\%$, which is an assumption frequently made in the credit risk literature, we can solve for the default intensity process consistent with the estimated instantaneous credit spread process of the representative AA-rated U.S. financial firm by using the following no-arbitrage restriction that must hold for the instantaneous credit spread

$$s_t^{AA,Fin} = L\lambda_t^{AA,Fin} \quad \Rightarrow \quad \lambda_t^{AA,Fin} = \frac{1}{L} s_t^{AA,Fin}.$$

With the assumption of $L = 50\%$, this translates into

$$\lambda_t^{AA,Fin} = 2\alpha_0^{AA,Fin} + 2\alpha_{L^T}^{AA,Fin} L_t^T + 2\alpha_{S^T}^{AA,Fin} S_t^T + 2\alpha_{L^S}^{AA,Fin} L_t^S + 2\alpha_{S^S}^{AA,Fin} S_t^S.$$

Combining the Q-dynamics of the AFNS state variables with the general asset pricing result for affine models provided in Duffie, Pan, and Singleton (2000), CDS rates for the representative AA-rated U.S. financial firm can be calculated by solving straightforward systems of ordinary differential equations.

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