# Optimal Monetary Policy in a Model of the Credit Channel *

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**Abstract**

We consider a simple extension of the basic new-Keynesian setup in which we relax the assumption of frictionless financial markets. In our economy, asymmetric information and default risk lead banks to optimally charge a lending rate above the risk-free rate. Our contribution is threefold. First, we derive analytically the loglinearised equations which characterise aggregate dynamics in our model and show that they nest those of the new-Keynesian model. A key difference is that marginal costs increase not only with the output gap, but also with the credit spread and the nominal interest rate. Second, we find that financial market imperfections imply that exogenous disturbances, including technology shocks, generate a trade-off between output and inflation stabilisation. Third, we show that, in our model, an aggressive easing of policy is optimal in response to adverse financial market shocks.

*Keyworks*: optimal monetary policy, financial markets, asymmetric information

*JEL codes*: E52, E44

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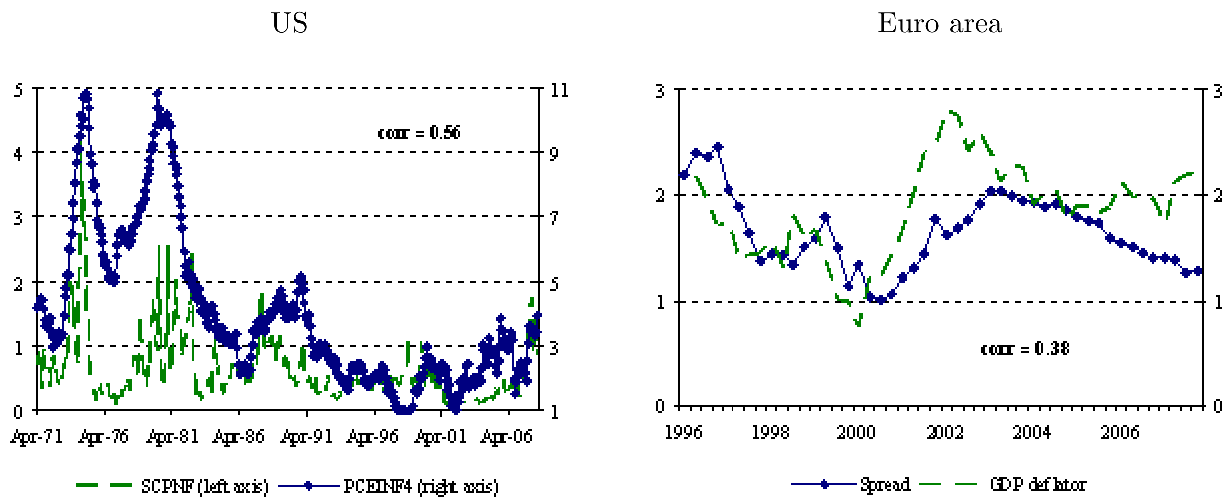
# 1 Introduction

Central banks devote much effort to the analysis of the financial positions of households, firms and financial institutions, and to monitor the evolution of credit aggregates and interest rate spreads. One reason is that financial market conditions are perceived to be factors which contribute to shape the performance of the economy and to affect its inflationary prospects. This perception appears to be consistent with the time series properties of some financial variables. For example, credit spreads in both the US and the euro area display a positive correlation with inflation (see Figure 1). The correlation is suggestive of a possible role for spreads in affecting firms' marginal costs. Higher spreads would then be reflected in a causal way into higher prices.

In several historical episodes, central banks have also reacted sharply to changes in financial conditions. One example are the US developments during the late 1980s, when banks experienced large loan losses as a consequence of the bust in the real estate market. Due to weak financial conditions, banks could not raise new capital and, because of the requirement to comply with the Basel Accord, they were forced to cut back on loans. This led to a slowdown in credit growth and aggregate spending. According to Rudebusch (2006), this slowdown contributed to the FOMC decision to reduce the Federal funds rate well below what suggested by an estimated Taylor rule. A more recent example is provided by the financial market turmoil initiated in 2007 with the deterioration in the performance of nonprime mortgages in the US. In August 2007, the FOMC justified a cut in the discount rate of 50 basis points by expressing concerns about the ongoing deterioration of financial market conditions and tightening of credit conditions, which increased appreciably the downside risks to growth.

To analyze whether financial market conditions ought to have a bearing on monetary policy decisions, we consider a simple extension of the basic New-Keynesian setup, where firms need to borrow funds in advance of production and informational frictions characterize financial markets. As in Bernanke, Gertler and Gilchrist (1989) and Carlstrom and Fuerst (1997, 1998), we assume that firms have private information about the realization of an idiosyncratic productivity shock, which banks can only monitor ex-post at a cost. The presence of asymmetric information introduces the risk of default on loans, so that banks find it optimal to charge a lending rate which is above their marginal cost (the deposit rate). One deviation from the set-up adopted in Bernanke, Gertler and Gilchrist (1989) and Carlstrom and Fuerst (1997, 1998) is that we assume that loans are denominated in nominal – rather than real – terms.

Figure 1: Credit spreads and inflation




Source: Fred database, ECB and AWM database (see Fagan et. al, 1999).
Legend: SCPNF: spread between the 3-month commercial paper rate (to non-financial corporations since 1997) and the 3-month Treasury Bill rate; PCEINF4: year-on-year PCE inflation; Spread: spread between the interest rate on short-term loans to non-financial corporations and the 3-month interbank rate.

The main appealing feature of our model is its analytical simplicity and the possibility to disentangle the role of financial frictions for inflation and output dynamics. We obtain three main sets of results.

First, we show that the loglinear approximation of the aggregate structural equations of our model is similar in structure to the one arising in the new-Keynesian setup with frictionless financial markets. As in the new-Keynesian case, private sector decisions can be characterized by an intertemporal IS equation and a Phillips curve. These relationships, however, include additional terms to reflect the existence of informational asymmetries. The main difference is that firms' marginal costs reflect, on top of the costs of labour input, also the credit spread and the nominal interest rate. The latter two variables matter because they determine the cost of credit in the economy. Thus, they act as endogenous "cost-push" factors.

Second, we find that both technology and financial market shocks operate as exogenous cost-push factors in the model. This is noticeable for technology shocks, which in the benchmark model with frictionless financial markets generate fully efficient fluctuations in output and consumption. In our model, however, these fluctuations produce corresponding variations in firms' exposure to external finance. The ensuing volatility in credit spreads and bankruptcy ratios represent the inefficient implications of technology shocks in the presence of credit frictions.

Third, we show that the presence of financial market imperfections and endogenous variations in credit spreads change the characterization of optimal monetary policy. Using an analytic, second-order approximation of the welfare function, we demonstrate that welfare is directly affected not just by the volatility of inflation and the output gap, as in the benchmark case with frictionless financial markets, but also by the volatility of the nominal interest rate and of the credit spread.

In quantitative terms, the optimal policy reaction to technology shocks is not dramatically different from the case with frictionless financial markets. More specifically, near complete inflation stabilization remains optimal. In reaction to a financial market shock which increases the credit spread, however, a more significant short-term trade-off arises between inflation and output stabilization. In order to help households smooth their consumption path in reaction to the shock, optimal monetary policy is aggressively expansionary for one year.

Our paper is not the first attempt to analyze monetary policy in models with credit frictions. Ravenna and Walsh (2006) characterizes optimal monetary policy when firms need to borrow in advance to finance production. However, there is no default risk in that model and the cost of financing for firms is the risk-free rate. We show that our model nests that of Ravenna and Walsh (2006) in the special case in which the costs of asymmetric information disappear. Faia and Monacelli (2006) compares the welfare losses of optimized simple interest rate rules in models with a structure similar to ours, but it does not characterize fully optimal (Ramsey) monetary policy. Similarly, Christiano, Motto and Rostagno (2003) focus on how monetary aggregates could provide useful information for the central bank in a medium scale DSGE model with several frictions, including credit frictions. Christiano, Motto and Rostagno (2006) argue that the monetary policy reaction to a stock market boom/bust cycle would be superior, in terms of welfare, if liquidity developments were taken into account.

Our paper is closest to recent work by Curdía and Woodford (2008), which also characterizes optimal monetary policy in a model where financial frictions matter, because of heterogeneity in the spending opportunities available to different households. Our work differs in the underlying source of financial frictions. Financial frictions are microfounded in our model and credit spreads arise from an explicit characterization of loan contracts. Curdía and Woodford (2008) assume instead a flexible, reduced-form function linking the credit spread to macroeconomic conditions.

The paper proceeds as follows. In section 2, we describe the environment and derive the conditions characterizing the equilibrium of the economy when financial contracts are written in nominal terms. In section 3, we discuss the log-linearized version of our model, in comparison to the new-Keynesian benchmark. This enables us to highlight the effect of financial market frictions on inflation and output dynamics. In section 4, we characterize the optimal monetary policy. We derive a simple quadratic approximation of the social welfare, which we compare to the one arising under frictionless financial markets. In section 5, we derive the first-order conditions of the social planner problem under discretion and we discuss the role of financial frictions for the optimal conduct of monetary policy. We then characterize numerically optimal monetary policy under commitment. Section 6 concludes.

## 2 The environment

The economy is inhabited by a representative infinitely-lived household, a continuum of wholesale firms producing a homogeneous good and owned by risk-neutral entrepreneurs, a continuum of monopolistically competitive retail firms producing differentiated goods and owned by the households, a zero-profit financial intermediary, and a central bank.

In the first part of the period, households decide how to allocate their nominal wealth among existing nominal assets, namely money, a portfolio of nominal state-contingent bonds, and one-period deposits. In the second part of the period, they receive wage income and purchase consumption goods.

Wholesale firms are endowed with a linear production technology that uses labor as the only input. They need to pay workers in advance of production. At the beginning of the period, each firm receives from the government an exogenous nominal endowment that is used as the firm's internal funds. However, these funds are not sufficient to finance the wage bill, so firms need to raise external finance. In their productive activity, wholesale firms face idiosyncratic productivity shocks and thus default risk. Lending occurs through perfectly competitive banks, which are able to ensure a safe return by providing funds to the continuum of firms facing idiosyncratic shocks. Bank loans take the form of risky debt, which is the optimal contractual arrangement between lenders and borrowers in the presence of asymmetric information and costly state verification.

Firms in the retail sector buy the homogeneous good from wholesale firms in a competitive market and use them to produce differentiated goods at no costs. Because of this product differentiation, retail firms acquire some market power and become price makers. In their price-setting activity, however, they are not free to change their price at will, because prices are subject to Calvo contracts. Retail firms are owned by the households, who receive their profits.

## 2.1 Households

At the beginning of period $t$, the financial market opens. First, the interest on nominal financial assets acquired at time $t-1$ is paid. The households, holding an amount $W_t$ of nominal wealth, choose to allocate it among existing nominal assets, namely money $M_t$, a portfolio of nominal state-contingent bonds $A_{t+1}$ each paying a unit of currency in a particular state in period $t+1$, and one-period deposits denominated in units of currency $D_t$ paying back $R_t^d D_t$ at the end of the period.

In the second part of the period, the goods market opens. Households' money balances are increased by the nominal amount of their revenues and decreased by the value of their expenses. Taxes are also paid or transfers received. The amount of nominal balances brought into period $t+1$ is equal to

$$M_t + P_t w_t h_t + Z_t - P_t c_t - T_t, \tag{1}$$

where $h_t$ is hours worked, $w_t$ is the real wage, $Z_t$ are nominal profits transferred from retail producers to households, and $T_t$ are lump-sum nominal taxes collected by the government. $c_t$ denote a CES aggregator of a continuum $j \in (0,1)$ of differentiated consumption goods produced by retail firms,

$$c_t = \left[ \int_0^1 c_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

with $\varepsilon > 1$. $P_t(j)$ denotes the price of good $j$, and $P_t = \left[ \int_0^1 P_t(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}}$ is the price of the CES aggregator.

Nominal wealth at the beginning of period $t+1$ is given by

$$W_{t+1} = A_{t+1} + R_t^d D_t + R_t^m \left\{ M_t + P_t w_t h_t + Z_t - P_t c_t - T_t \right\}, \tag{2}$$

where $R_t^m$ denotes the interest paid on money holdings.

The household's problem is to maximize preferences, defined as

$$E_o \left\{ \sum_0^\infty \beta^t \left[ u(c_t) + \kappa(m_t) - v(h_t) \right] \right\}, \tag{3}$$

where $u_c > 0$, $u_{cc} < 0$, $\kappa_m \geq 0, \kappa_{mm} < 0$ and $v_h > 0$, $v_{hh} > 0$, and $m_t \equiv M_t/P_t$ denotes real balances. The problem is subject to the budget constraint

$$M_t + D_t + E_t \left[ Q_{t,t+1} A_{t+1} \right] \leq W_t, \tag{4}$$

Define $\pi_t \equiv \frac{P_t}{P_{t-1}}$ and $\Delta_{m,t} \equiv \frac{R_t - R_t^m}{R_t}$. The optimality conditions can be written as

$$\frac{v_h(h_t)}{u_c(c_t)} = w_t \tag{5}$$

$$\frac{1}{R_t} = E_t \left[ Q_{t,t+1} \right] \tag{6}$$

$$R_t = R_t^d$$

$$u_c(c_t) + \kappa_m(m_t) = \beta R_t E_t \left\{ \frac{u_c(c_{t+1}) + \kappa_m(m_{t+1})}{\pi_{t+1}} \right\}$$

$$\frac{\kappa_m(m_t)}{u_c(c_t)} = \frac{\Delta_{m,t}}{1 - \Delta_{m,t}}. \tag{7}$$

Moreover, the optimal allocation of expenditure between the different types of goods leads to the demand functions

$$c_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\varepsilon} c_t, \tag{8}$$

where $P_t(j)$ is the price of good $j$.

## 2.2 Wholesale firms

The wholesale sector consists of a continuum of competitive firms, indexed by $i$, owned by infinitely lived entrepreneurs. Each firm produces the amount $y_{i,t}$ of a homogeneous good, using a linear technology

$$y_{i,t} = A_t \omega_{i,t} l_{i,t}. \tag{9}$$

Here $A_t$ is an aggregate, serially correlated productivity shock and $\omega_{i,t}$ is an idiosyncratic, iid productivity shock with distribution function $\Phi$ and density function $\phi$.

The production function (9) reflects our choice to abstract from capital accumulation. This is in contrast with the rest of the literature that introduces credit frictions in macro-models, where entrepreneurs are assumed to decide in period $t$ how to allocate their profits to consumption and investment expenditures (see e.g. Carlstrom and Fuerst (1997) and Bernanke, Gertler and Gilchrist (1999)). The value of the stock of capital available to firms in period $t+1$ provides the firm with a certain net worth (internal funds) that can be used in that period production. In that environment, aggregate shocks affect the evolution of firms' net worth, thus creating endogenous persistence. In our model, we assume instead that each firm receives a constant endowment $\tau$ at the beginning of each period, which can be used as internal funds. Since these funds are not sufficient to finance the firm's desired level of production, firms need to raise external finance. As a result, financial frictions affect the economy as in models with capital so that, for example, a spread arises endogenously between the loan rate charged by financial intermediaries to firms and the risk-free rate, to reflect the existence of default risk. At the same time, our simpler set-up enables us to provide an analytical characterization of economic dynamics and of optimal policy in the presence of credit constraints and information asymmetry.

### 2.2.1 Labor demand

Firms need to raise external finance to pay for labor services. Before observing the idiosyncratic productivity shock $\omega_{i,t}$, firms sign a contract with the financial intermediary to raise the amount $P_t\left(x_{i,t} - \tau\right)$, for total funds at hand $P_t x_{i,t}$, where

$$x_{i,t} \geq w_t l_{i,t}. \tag{10}$$

We assume that entrepreneurs sell output only to retailers. Let $\overline{P}_t$ be the price of the wholesale homogenous good, and $\frac{\overline{P}_t}{P_t} = \chi_t^{-1}$ the relative price of wholesale goods to the aggregate price of retail goods. Each firm $i$'s demand for labor is derived by solving the problem

$$\max \quad \left[ \frac{\overline{P}_t}{P_t} \mathcal{E} \left[ A_t \omega_{i,t} l_{i,t} \right] - w_t l_{i,t} \right]$$

subject to the financing constraint (10), where the expectation $\mathcal{E}[\cdot]$ is taken with respect to the idiosyncratic shock unknown at the time of labor hiring decision, and $w_t$ denotes the payment of labor services measured in terms of the final consumption good. Denote the Lagrange multiplier on the financing constraint as $(q_{i,t} - 1)$. Optimality requires that

$$q_{i,t} = q_t = \frac{A_t}{w_t \chi_t} \tag{11}$$

$$x_{i,t} = w_t l_{i,t} \tag{12}$$

implying that

$$\mathcal{E} \left[ y_{i,t} \right] = \chi_t q_t x_{i,t}. \tag{13}$$

Equation (13) states that, as the production function is CRS, wholesale firms must sell at a mark-up $\chi_t q_t$ over firms' production costs. This allows them to cover for the presence of credit frictions and for the monopolistic distortion in the retail sector. This latter matters for firms in the wholesale sector because $P_t$ is the deflator of the nominal wage, and thus affects the real marginal cost faced by wholesale producers.

Equation (12) states that the financing constraint is always binding. Given the contract stipulated by the firm with the financial intermediary (which sets the amount of funds $x_{i,t}$ and the repayment on these funds), the firm always finds it profitable to use the entire amount of funds and to produce, also when expected productivity is low. This way, it can minimize the probability of default.

### 2.2.2 The financial contract

Loans are stipulated in units of currency after all aggregate shocks have occurred, and repaid at the end of the same period. Lending occurs through the financial intermediary, which collects deposits from households and use them to finance loans to firms.

Firms face an idiosyncratic productivity shock, whose realization is observed at no costs only by the entrepreneur. The financial intermediary can monitor its realization but a fraction of the firm's input is consumed in the monitoring activity. If the realization of the idiosyncratic shock is sufficiently low, the value of the firm's production is not sufficient to repay the loan and the firm defaults. Households lend to firms through a financial intermediary, which is able to ensure a safe return. This is possible because by lending to the continuum of firms $i \in (0,1)$ producing the wholesale good, the financial intermediary can differentiate the risk due to the presence of idiosyncratic shocks.

The informational structure corresponds to a costly state verification (CSV) problem. The solution is a standard debt contract (see e.g. Gale and Hellwig, 1985) which is derived in the appendix. The optimality conditions can be written as

$$q_t = \frac{R_t}{1 - \mu_t \Phi\left(\overline{\omega}_t\right) + \frac{\mu_t f(\overline{\omega}_t)\phi(\overline{\omega}_t)}{f_{\overline{\omega}}(\overline{\omega}_t)}} \tag{14}$$

$$x_t = \left\{ \frac{R_t}{R_t - q_t g\left(\overline{\omega}_t; \mu_t\right)} \right\} \tau. \tag{15}$$

where $\overline{\omega}_t$ is a threshold for the distribution of the idiosyncratic productivity shock below which firms go bankrupt, and $f\left(\overline{\omega}_t\right)$ and $g\left(\overline{\omega}_t; \mu_t\right)$ are the expected shares of output accruing to the entrepreneur and the bank, respectively. Moreover, $\mu_t$ denotes the monetary costs of loan monitoring, as a function of the firm's input costs. Given the large time-variation in bankruptcy costs documented by Natalucci et al. (2004), $\mu_t$ is assumed to be subject to serially correlated shocks.

Substituting equation (14) into equation (15), it can be noticed that a change in the nominal interest rate has no direct impact on the amount of real funds borrowed by entrepreneurs; the amount of funds is only modified in general equilibrium, to the extent that it induces changes in the threshold $\overline{\omega}_t$. An increase in the nominal interest rate, however, tends to lead to an increase in the mark-up $q_t$ due to the presence of agency costs.

Notice also that the gross interest rate on the loan extended to firm $i$, $R^l_{i,t}$, can be backed up from the debt repayment. It is given by

$$\overline{P}_t \overline{\omega}_t \chi_t q_t x_t = R^l_{i,t} P_t \left(x_t - \tau\right)$$

implying that $R^l_{i,t} = R^l_t$, for all $i$.

We can use the expression above to obtain the spread between the loan rate and the risk-free rate, $\Delta_t \equiv R_t^l / R_t^d$, can be written as an endogenous function of the $\overline{\omega}_t$ threshold

$$\Delta_t = \frac{\overline{\omega}_t}{g(\overline{\omega}_t; \mu_t)}. \tag{16}$$

### 2.2.3 Entrepreneurs

Entrepreneurs have linear preferences on consumption and are infinitely lived. They consume a CES basket of differentiated goods similar to that of households.

At the end of each period, entrepreneurs sell their output to the retail sector and, if they do not default, repay the debt. Profits of entrepreneurs are entirely allocated to final consumption goods

$$\int_0^1 P_t(j)\, e_{i,t}(j)\, dj = \overline{P}_t (\omega_{i,t} - \overline{\omega}_t)\, \chi_t q_t x_t,$$

where $e_{i,t}(j)$ is firm $i$'s consumption of good $j$. Notice that $\int_0^1 P_t(j)\, e_{i,t}(j) = P_t e_{i,t}$,where $e_{i,t}$ is the demand of the final consumption good of entrepreneur $i$. Aggregating across firms, we obtain $e_t = f(\overline{\omega}_t)\, q_t x_t$, where $e_t = \int_0^1 e_{i,t} di$ is the aggregate entrepreneurial consumption of the final consumption good. Using equations (14)-(15), we can rewrite aggregate entrepreneurial consumption as

$$e_t = \tau R_t H(\mu_t, \overline{\omega}_t) \tag{17}$$

where $H(\mu_t, \overline{\omega}_t) \equiv \left(1 + \frac{\mu_t \phi(\overline{\omega}_t)}{f_{\overline{\omega}}(\overline{\omega}_t)}\right)^{-1}$.

Equation (17) shows that entrepreneurial consumption depends only on the nominal interest rate, on the bankruptcy threshold $\overline{\omega}_t$, and on the exogenous shock $\mu_t$. The two endogenous variables, however, affect entrepreneurial consumption through different channels.

As mentioned above, an increase in the nominal interest rate has no direct effect on loans and affects financial conditions mainly by inducing an increase in the mark-up $q_t$. Ceteris paribus, however, a higher $R_t$ leads to an increase in entrepreneurial consumption.

Changes in the threshold $\overline{\omega}_t$ act instead by modifying the output share $f(\overline{\omega}_t)$ (together with $g(\mu_t, \overline{\omega}_t)$). Since entrepreneurs' profits are decreasing in the threshold, an increase in bankruptcy rates tends to depress entrepreneurial consumption.

Finally, the impact of $\mu_t$ on entrepreneurial consumption is relatively complex. An increase in monitoring costs $\mu_t$ implies a larger waste of resources for the economy. For given $\overline{\omega}_t$, this tends to reduce the amount of production available for consumption, including entrepreneurial consumption. At the same time, a higher $\mu_t$ will produce changes in the threshold $\overline{\omega}_t$. If total production changes little, firms will have to pay a higher interest rate spread and $\overline{\omega}_t$ will tend to increase, which implies a reduction in entrepreneurial consumption. If however the shock is sufficiently contractionary, the demand for credit will fall and $\overline{\omega}_t$ will decrease.

## 2.3 Retail firms

As in Bernanke, Gertler and Gilchrist (1999), monopolistic competition occurs at the "retail" level. More specifically, a continuum of monopolistically competitive retailers buy wholesale output from entrepreneurs in a competitive market and then differentiate it at no cost. Because of product differentiation, each retailer has some market power. Profits are distributed to the households, who own firms in the retail sector.

Let $Y_t(j)$ be the quantity of output sold by retailer $j$. This quantity can be used for households' consumption, $c_t(j)$, and for entrepreneurs' consumption, $e_t(j)$. Hence,

$$Y_t(j) = c_t(j) + e_t(j).$$

The final good $Y_t$ is a CES composite of individual retail goods

$$Y_t = \left[\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{18}$$

with $\varepsilon > 1$.

### 2.3.1 Price setting

We assume that each retailer can change its price with probability $1-\theta$, following Calvo (1983). Let $P_t^*(j)$ denote the price for good $j$ set by retailers that can change the price at time $t$, and $Y_t^*(j)$ the demand faced given this price. Then each retailer chooses its price to maximize expected discounted profits, given by

$$E_t\left[\sum_{k=0}^{\infty} \theta^k \overline{Q}_{t,t+k} \frac{P_t^* - \overline{P}_{t+k}}{P_{t+k}} Y_{t+k}^*(j)\right],$$

where $\overline{Q}_{t,t+k} = \beta \frac{u_c(c_{t+1})+\kappa_m(m_{t+1})}{u_c(c_t)+\kappa_m(m_t)}$.

The first-order conditions of the firm's profit maximization problem imply that

$$\frac{P_t^*}{P_t} = \frac{\varepsilon}{\varepsilon-1}\frac{E_t\left\{\sum_{k=0}^{\infty}\theta^k\overline{Q}_{t,t+k}\frac{\overline{P}_{t+k}}{P_{t+k}^{1-\varepsilon}}P_t^{-\varepsilon}Y_{t+k}\right\}}{E_t\left\{\sum_{k=0}^{\infty}\theta^k\overline{Q}_{t,t+k}\frac{P_t^{1-\varepsilon}}{P_{t+k}^{1-\varepsilon}}Y_{t+k}\right\}}$$

Now define

$$\begin{aligned}\Theta_{1,t} &\equiv \frac{\overline{P}_t}{P_t}Y_t + E_t\left\{\sum_{k=1}^{\infty}\theta^k\overline{Q}_{t,t+k}\frac{\overline{P}_{t+k}}{P_{t+k}^{1-\varepsilon}}P_t^{-\varepsilon}Y_{t+k}\right\}\\ \Theta_{2,t} &\equiv Y_t + E_t\left\{\sum_{k=1}^{\infty}\theta^k\overline{Q}_{t,t+k}\frac{P_t^{1-\varepsilon}}{P_{t+k}^{1-\varepsilon}}Y_{t+k}\right\}\end{aligned}$$

Using the expression for the aggregate price index, $P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta)(P_t^*)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$, and substituting out $\frac{P_t^*}{P_t}$, we can recursify the first order condition as

$$\begin{aligned}1 &= \theta\pi_t^{\varepsilon-1} + (1-\theta)\left(\frac{\varepsilon}{\varepsilon-1}\frac{\Theta_{1,t}}{\Theta_{2,t}}\right)^{1-\varepsilon}\\ \Theta_{1,t} &= \frac{1}{\chi_t}Y_t + \theta E_t\left[\pi_{t+1}^{\varepsilon}\overline{Q}_{t,t+1}\Theta_{1,t+1}\right]\\ \Theta_{2,t} &= Y_t + \theta E_t\left[\pi_{t+1}^{\varepsilon-1}\overline{Q}_{t,t+1}\Theta_{2,t+1}\right].\end{aligned}$$

### 2.3.2 Price dispersion

Recall that the aggregate retail price level is given by $P_t = \left[\int_0^1 P_t(j)^{1-\varepsilon}dj\right]^{\frac{1}{1-\varepsilon}}$. Define the relative price of differentiated good $j$ as $p_t(j) \equiv \frac{P_t(j)}{P_t}$ and divide both sides by $P_t$ to express everything in terms of relative prices, $1 = \int_0^1 (p_t(j))^{1-\varepsilon}dj$.

Define also the relative price dispersion term as

$$s_t \equiv \int_0^1 (p_t(j))^{-\varepsilon}dj.$$

This equation can be written in recursive terms as

$$s_t = (1-\theta)\left(\frac{1-\theta\pi_t^{\varepsilon-1}}{1-\theta}\right)^{-\frac{\varepsilon}{1-\varepsilon}} + \theta\pi_t^{\varepsilon}s_{t-1}.$$

### 2.4 Market clearing

Market clearing conditions are listed below.

Money:

$$M_t^s = M_t,$$

Bonds:

$$A_t = 0$$

Labor:

$$h_t = l_t$$

Loans:

$$d_t = x_t - \tau$$

Wholesale goods:

$$y_t = \int_0^1 Y_t(j)\, dj$$

Retail goods:

$$Y_t(j) = c_t(j) + e_t(j)\,, \text{ for all } j.$$

### 2.5 Competitive equilibrium

The central bank needs to specify an additional rule for either $R_t^m$ or $M_t^s$. It is convenient to express this rule in terms of $\Delta_{m,t}$. In order to facilitate the comparison of our model with the standard New-Keynesian setup, we assume a monetary policy such that

$$\Delta_{m,t} = \Delta_m,$$

for all $i$. Then,

$$\kappa_m(m_t) = \frac{\Delta_m}{1 - \Delta_m} u_c(c_t)$$

and we can define

$$U(c_t, \Delta_{m,t}) \equiv u_c(c_t)\left(1 + \frac{\Delta_m}{1 - \Delta_m}\right).$$

Under a policy of constant $\Delta_{m,t}$, money demand becomes recursive and can therefore be neglected for the solution of the system.

We assume a functional form $U(c_t;\Delta_m) - v(h_t) = \frac{c_t^{1-\sigma^{-1}}}{1-\sigma^{-1}} - \psi \frac{h_t^{1+\varphi}}{1+\varphi}$ and we define $\widehat{\pi}_{t+1} \equiv \log \pi_{t+1}$, $\widehat{p}_t(j) = \log p_t(j)$, $a_t = \log A_t$, and $\widehat{\mu}_t = \log \mu_t$.

The system of equilibrium conditions can be written in log-linearized form as reported in the appendix.

## 2.6 The system in reduced form

The system of equilibrium conditions that characterizes the evolution of the aggregate variables (once a monetary policy rule is specified) can be linearized around a zero-inflation steady state as

$$\begin{aligned} \left(1+\sigma^{-1}\frac{e}{c}\right)\widehat{\chi}_t &= (1+\varphi)\ a_t - \left(\sigma^{-1}\alpha_1+\alpha_2\right)\widehat{\Delta}_t - \left(\sigma^{-1}+\varphi\right)\widehat{Y}_t - \widehat{R}_t \\ &\quad + \left(\alpha_3 - \sigma^{-1}\alpha_1\frac{g_\mu \mu}{g}\right)\widehat{\mu}_t \end{aligned} \tag{19}$$

$$\delta_1\widehat{\Delta}_t = \left(1+\varphi+\sigma^{-1}\frac{Y}{c}\right)\widehat{Y}_t - \sigma^{-1}\frac{e}{c}\widehat{R}_t - (1+\varphi)\ a_t - \delta_2\widehat{\mu}_t \tag{20}$$

$$\begin{aligned} \widehat{Y}_t &= E_t\widehat{Y}_{t+1} - \frac{1}{\sigma^{-1}}\left(\widehat{R}_t - E_t\widehat{\pi}_{t+1}\right) + \alpha_1 E_t\left(\widehat{\Delta}_{t+1} - \widehat{\Delta}_t\right) \\ &\quad + \frac{e}{c}E_t\left(\widehat{\chi}_{t+1} - \widehat{\chi}_t\right) + \alpha_1\frac{g_\mu \mu}{g}E_t\left(\widehat{\mu}_{t+1} - \widehat{\mu}_t\right) \end{aligned} \tag{21}$$

$$\widehat{\pi}_t = -\overline{\kappa}\left(1+\sigma^{-1}\frac{e}{c}\right)\widehat{\chi}_t + \beta E_t\widehat{\pi}_{t+1} \tag{22}$$

where the coefficients $\alpha_1, \alpha_2, \alpha_3, \delta_1$, $\delta_2$ and $\overline{\kappa}$ are defined in the appendix. Notice that $\alpha_1$ and $\alpha_2$ are positive coefficients.

In our economy, the mark-up $\chi_t = P_t/\overline{P}_t$ is inversely related to the marginal costs of retail firms. Indeed, an increase in the input cost of retail production, i.e. a higher price of wholesale goods, generates a fall in the mark-up. Equation (19) shows that the markup is negatively related to three factors. The first is the spread between the loan rate and the policy rate. An increase in the spread implies a higher cost of external finance for wholesale firms, which then need to increase the price of intermediate goods, $\overline{P}_t$. The second is the demand for final goods. In the presence of higher demand for retail goods, and correspondingly of intermediate goods to be used as production inputs, wholesale firms need to pay a higher real wage to workers to induce them to supply the required labor services. This increases the price of wholesale goods, $\overline{P}_t$, relative to the price of retail goods, $P_t$. The third is the nominal interest rate. Wholesale firms must borrow funds to finance production through nominal loans. As a result, any increase in the policy rate represents an additional cost which is covered by charging a higher price of wholesale goods.

Equation (20) summarizes equilibrium conditions in the credit market. Notice that because of the assumption that debt contracts take place period-by-period, only contemporaneous variables enter this equation. It is reasonable to expect that this feature of the credit market equilibrium would change in a richer model.

In section 3, we will express the system (19)-(22) in terms of gaps from the efficient equilibrium and discuss in more details the conditions corresponding to equations (20)-(22). In the rest of this section, we show that our model nests both the cost-channel model of Ravenna and Walsh (2006) and the standard new-Keynesian model.

We consider the special case when monitoring costs are zero, i.e. $\mu_t = 0$, for all $t$. In this case, firms still need to borrow in advance of production. However, the information asymmetry concerning wholesale firms' productivity disappears because banks can monitor at no cost. The system then becomes

$$\widehat{\chi}_t = -\left(\sigma^{-1} + \varphi\right)\widehat{Y}_t - \widehat{R}_t + (1+\varphi)\, a_t$$

$$\widehat{Y}_t = E_t \widehat{Y}_{t+1} - \sigma\left(\widehat{R}_t - E_t \widehat{\pi}_{t+1}\right)$$

$$\widehat{\pi}_t = -\overline{\kappa}\left(1 + \sigma^{-1}\frac{e}{c}\right)\widehat{\chi}_t + \beta E_t \widehat{\pi}_{t+1}$$

The equations above coincide with the reduced-form system of equilibrium conditions obtained by Ravenna and Walsh (2006) in their model of the "cost-channel," where firms borrow in advance of production but, since there is no asymmetric information nor default risk, they simply pay the risk-free rate on these funds.

Finally, the system would boil down to the new-Keynesian model in the absence of a nominal interest rate effect on marginal costs.

# 3 The system in deviation from the efficient equilibrium

In order to characterize the optimal response of monetary policy, it is convenient to write the reduced form of the system (19)-(22) in terms of gaps from the efficient equilibrium, in which $\mu_t = 0$, $\tau \approx 0$ and prices are flexible. We denote a variable with a hat and a superscript $e$ as the log-deviation of the variable from its steady state in the efficient equilibrium, which is characterized by

$$\begin{aligned}
\widehat{Y}_t^e &= E_t \widehat{Y}_{t+1}^e - \sigma \widehat{r}_t^e \\
\left(\sigma^{-1} + \varphi\right) \widehat{Y}_t^e &= (1+\varphi) a_t,
\end{aligned}$$

and where $\widehat{r}_t^e$ denotes the real interest rate.

We find it useful to define the output gap, $\widetilde{Y}_t$, as actual output in deviation from efficient output, when both variables are linearized around the actual steady state $Y$. Note that under this definition the output gap will not be zero in steady state, but equal to the difference between the two steady states $y^* \equiv \log Y - \log Y^e$.

We can now rewrite the system as

$$\widehat{\Delta}_t = \frac{1+\varphi+\sigma^{-1}\frac{Y}{c}}{\delta_1} \widetilde{Y}_t - \frac{\sigma^{-1}\frac{e}{c}}{\delta_1} \widehat{R}_t + \frac{1}{\delta_1} \widehat{\xi}_{2,t} \tag{23}$$

$$\begin{aligned}
\widetilde{Y}_t &= E_t \widetilde{Y}_{t+1} - \sigma \frac{1+\sigma^{-1}\frac{e}{c}}{1-\varphi\frac{e}{c}} \left(\widehat{R}_t - E_t \widehat{\pi}_{t+1} - \widehat{R}_t^e\right) \\
&\quad - \frac{\alpha_1 - \alpha_2 \frac{e}{c}}{1-\varphi\frac{e}{c}} \left(\widehat{\Delta}_t - E_t \widehat{\Delta}_{t+1}\right) + \frac{\frac{e}{c}}{1-\varphi\frac{e}{c}} \left(\widehat{R}_t - E_t \widehat{R}_{t+1}\right) + \upsilon_t
\end{aligned} \tag{24}$$

$$\widehat{\pi}_t = \overline{\kappa} \left(\sigma^{-1} + \varphi\right) \widetilde{Y}_t + \overline{\kappa} \widehat{R}_t + \overline{\kappa} \left(\sigma^{-1}\alpha_1 + \alpha_2\right) \widehat{\Delta}_t + \beta E_t \widehat{\pi}_{t+1} - \overline{\kappa} \widehat{\xi}_{1,t} \tag{25}$$

where

$$\widehat{\xi}_{1,t} \equiv -\sigma^{-1} \frac{1+\varphi}{\sigma^{-1}+\varphi} \left(E_t a_{t+1} - a_t\right) + \left(\alpha_3 - \sigma^{-1}\alpha_1 \frac{g_\mu \mu}{g}\right) \widehat{\mu}_t \tag{26}$$

$$\widehat{\xi}_{2,t} \equiv -\sigma^{-2} \frac{e}{c} \frac{1+\varphi}{\sigma^{-1}+\varphi} \left(E_t a_{t+1} - a_t\right) + \left(1 + \sigma^{-1}\frac{e}{c}\right) \frac{1+\varphi}{\sigma^{-1}+\varphi} a_t - \delta_2 \widehat{\mu}_t \tag{27}$$

$$\upsilon \equiv \frac{\frac{e}{c}}{1-\varphi\frac{e}{c}} E_t \left(\widehat{\xi}_{1,t+1} - \widehat{\xi}_{1,t}\right) + \alpha_1 \frac{1+\sigma^{-1}\frac{e}{c}}{1-\varphi\frac{e}{c}} \frac{g_\mu \mu}{g} E_t \left(\widehat{\mu}_{t+1} - \widehat{\mu}_t\right) \tag{28}$$

and $g_\mu$ denotes the partial derivative of $g\left(\overline{\omega}_t; \mu_t\right)$ with respect to $\mu$.

Equation (23) shows that the spread between the loan rate and the policy rate increases with excess aggregate demand. An increase in the demand of retail (and thus also of wholesale) goods implies an implicit tightening of the credit constraint, since the exogenously given amount of internal funds must now be used to finance a higher level of debt. The increased default risk generates a larger spread. For the same reasons, the spread decreases with the nominal interest rate. An increase in the latter variable generates a reduction in the demand for final goods and thus in the demand for input in their production (wholesale goods). For a given amount of internal funds, leverage and the risk of default fall, reducing the spread.

Equation (24) is a forward-looking IS-curve describing the determinants of the gap between actual output and its efficient level. The first line of the expression shows that, as in the standard new-Keynesian model, the gap is affected by its expected future value and by the real interest rate. In our model, however, the output gap also depends on the expected change in the nominal interest rate and in the credit spread, as well as on the shock $\upsilon_t$. Note that this dependence is not present in a cost channel model: it would disappear in the absence of monitoring costs.

A higher spread between loan and deposit rates is contractionary in our model, because it induces an increase in bankruptcy rates and a fall in entrepreneurial consumption. In our calibration, an expected increase in the spread between periods $t$ and $t+1$ tends instead to be expansionary, in spite of the fact that entrepreneurs are myopic in their consumption patterns. The transmission of this effect operates through households' consumption. Through the aggregate resource constraint, the reduction in $t+1$ entrepreneurial consumption, which is due to the higher expected spread, also tends to imply an increase in future households' consumption. Since households are forward looking, this effect will feed through to current households' consumption, thereby leading to an expansionary effect on output.

On top of the standard real interest rate effect, changes in the nominal interest rate have an impact on output which operates through similar, but opposite, channels to those of the spread. A higher nominal interest rate will in fact have a small expansionary effect, as it will increase the financial mark-up and entrepreneurial consumption. However, an expected increase in the nominal interest rate will be contractionary, as it will lead to an expected fall in households' future consumption.

Equation (25) represents an extended Phillips curve. The first determinant of inflation in this equation is an output gap term. This term is standard, but it enters here with a different coefficient reflecting the presence of entrepreneurs in the economy. As in the cost channel model, equation (25) also includes a nominal interest rate term, whose increase pushes up marginal costs. Finally, the novel feature of our model is the presence of a credit spread in the equation. A higher credit spread also increases firms' costs and therefore exerts independent pressure on inflation. The credit spread and the nominal interest rate therefore act as endogenous "cost-push" terms on inflation.

Finally, the three equations are affected by all exogenous disturbances, which therefore act as exogenous "cost-push" factors in the Phillips curve. More specifically, technology shocks are also partly inefficient through their effect on the credit market. This is in contrast with the standard new-Keynesian model, in which they only generate efficient variations in output. The reason is that the output expansion which will typically follow a positive technology shock generates the need for an increase in external finance and in leverage, hence leading to an increase in the credit spread. In turn, the higher credit spread will affect output and inflation through the channels described above.

### 3.1 Impulse responses

As a benchmark for comparison with the optimal policy case, we provide some evidence on the quantitative implications of the model through an impulse response analysis. For this purpose, we close the model with a simple monetary policy rule of the Taylor-type with interest rate smoothing. The parameters of the rule are chosen in line with the values estimated in Smets and Wouters (2007) for the US, namely an inflation response coefficient of 2.0, an output gap response coefficient of 0.1 and an interest rate smoothing equal to 0.8.

The structural parameters are set in line with the literature. Following Levin, Natalucci and Zakrajsek (2004) we set long-run monitoring costs at 15% of the firm's output (i.e. $\mu = 0.15$). We then calibrate the standard deviations of idiosyncratic shocks ($\sigma_\omega$) and the subsidy $\tau$ so that that the annualized steady state spread $\Delta$ is equal to 2% and roughly 1% of firms go bankrupt each quarter. As to monopolistic competition and retail pricing, we assume $\varepsilon = 7$, leading to a steady-state mark-up of 17%, and a probability of not being able to re-optimize prices $\theta = 0.66$, implying that prices are changed on average every 3 quarters. Finally, we set the persistence of technology and monitoring cost shocks to 0.9.

Figure 2 displays the impulse responses of output, the output gap (defined as the difference between actual output and the efficient level of output), the policy interest rate and inflation to a positive 1% technology shock under the Taylor rule.[1] The impulse responses under our

[1] Since the steady states of output $y$ and of the efficient level of output $y^e$ are different, the output gap term in the Taylor rule is written as $gap = \widehat{y}_t - \widehat{y}_t^e - y^*$.

model – denoted as "credit channel model" – are compared to two well-known benchmarks: a model with the cost channel, which is obtained when $\mu_t = 0$ and $\tau = 0$; and a standard new-Keynesian model.

The most notable feature of Figure 2 is that the three models with nominal rigidities produce extremely similar impulse responses under the Taylor rule. As is typically the case, a technology shock exerts downward pressure on inflation (denoted as "inf") and on the interest rate on deposits ("i_dep"). The fall in inflation corresponds to almost the same negative output gap ("ygap") in our model and in the standard new-Keynesian model. It is slightly less pronounced, and turns positive after a few quarters, in the model with the cost channel. In the latter model, the fall in the policy interest rate has an expansionary effect through the ensuing reduction in marginal costs. In our model, the same effect is counteracted by an increase in the credit spread so that the output gap remains negative as in the new-Keynesian model. The responses of households' consumption ("cons_h") are equally very similar.

Of course, our model also has implications for the stock of credit and the spread between loan and deposit rates. Credit expands almost one-to-one with households' consumption, but this also implies an increase in leverage, as firms' net worth is constant. As a result, the bankruptcy rate in the economy increases and so does the credit spread.

A pro-cyclical response of the credit spread to technology shocks is standard in models adopting the Carlstrom and Fuerst (1997) set-up, but the data show that spreads tend to increase during recessions – this is the case, for example, for the difference between lowest and highest rates on corporate bond yields in the US (see e.g. Figure 1 in Levin et al., 2004). This is a problem in terms of the ability of our model to replicate a key feature of the credit market through fluctuations in technology shocks. Nevertheless, it is not crucial for our results in two important respects. The first is that the specific cyclical properties of credit spreads should not change the incentives of the central bank to smooth out fluctuations in such spreads. Our general characterization of the central bank's objectives should therefore remain valid also within a model in which spreads were anticyclical in response to technology shocks. The second reason is that we are particularly interested in characterizing optimal policy in the face of shocks which originate in the financial market. We show below that these shocks do give rise to a realistic, countercyclical response of the credit spread.

Figure 3 presents impulse responses to a policy shock. The similarity of three models is even more striking in this figure. The contraction in the output gap and the corresponding fall in inflation is virtually indistinguishable in the three models, and so is the monetary policy response. As in the case of technology shocks, the quantity of credit, leverage, and the spread between loan and deposit rates all move downwards with output, after a policy tightening.

It should be emphasized that the specific results in Figures 2 and 3 depend on the exact specification of the policy rule. With the original Taylor rule (with response coefficient of 1.5 on inflation and 0.5 to the output gap), for example, the responses of some variables – notably the output gap and inflation – would be more different across models. Other features which are often employed to increase the realism of models with nominal rigidities, e.g. habit formation, could generate further differences across models.

Nevertheless, Figures 2 and 3 suggest that it may be very difficult to discriminate empirically across models without looking also at financial variables, such as interest rate spreads or the stock of loans. They also suggest that the existence of credit frictions is not a sufficient ingredient for financial variables to play a quantitatively important role in shaping the monetary policy transmission mechanism. At least in our set-up, even if financial variables do react endogenously to economic developments and do play a direct role in the way shocks are transmitted through the economy, they modify little the reaction of output and inflation to "standard" macroeconomic shocks.

In spite of the results in Figures 2 and 3, however, credit frictions turn out to be important in two respects. First, they modify the objective of monetary policy compared to the case of frictionless financial markets. Second, they become important when shocks which affect the macroeconomy originate in financial markets. We analyze these two implications of credit frictions in the remainder of the paper.

## 4 Second order welfare approximation

Following Woodford (2003), we obtain a policy objective function by taking a second order approximation to the utility of the economy's representative agents. Since our economy is populated by households and entrepreneurs, the policy objective function will be a weighted average of the (approximate) utility functions of these two agents. The approximation to the objective function takes a form which nests the one in the benchmark new-Keynesian model (see Woordford, 2003) as a special case.

The appendix shows that the present discounted value of social welfare can be approximated by

$$W_{t_0} \simeq \varsigma c^{1-\sigma^{-1}} \left[ \varkappa - \frac{1}{2} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} L_t \right] + t.i.p. \tag{29}$$

where $\varsigma$ is the weight assigned to households' utility, $t.i.p.$ denotes terms independent of policy and

$$L_t \equiv \kappa_\pi \widehat{\pi}_t^2 + \frac{1}{2}\frac{Y}{c}\varphi \left( \widetilde{Y}_t - y^* \right)^2 + \frac{1}{2}\sigma^{-1} \left( \widetilde{c}_t - y^* \right)^2 - \sigma^{-1}\frac{e}{c} \left( \widehat{Y}_t^e + y^* \right) \widehat{e}_t + \frac{e}{c} \left( 1 - \frac{1-\varsigma}{\varsigma} c^{\sigma^{-1}} \right) \left( \widehat{e}_t + \frac{1}{2} \widehat{e}_t^2 \right) \tag{30}$$

where $\widehat{e}_t$ is log-entrepreneurial consumption (in deviation from the steady state) and $\varkappa$, $\kappa_\pi$, $\delta_3$, $\delta_4$ are parameters defined in the appendix.

The first three terms in equation (29) are common to the new-Keynesian model. Intuitively, social welfare decreases with variations of inflation around its target, and of the output gap around its (non-zero) steady state level. The first reason for disliking variations in the output gap is that households wish to smooth their supply labour. The second reason is that households also wish to have a smooth consumption pattern over time. Unlike in the benchmark new-Keynesian model, the consumption smoothing motive only applies to households' consumption, $\widetilde{c}_t^2$, rather than to total output, because entrepreneurs are indifferent about the timing of their consumption.

The main difference relative to the benchmark New-Keynesian model with frictionless financial markets is in the additional terms now appearing in the welfare approximation.

The first of the additional terms, which is proportional to $\left( \widehat{Y}_t^e + y^* \right) \widehat{e}_t$, contributes positively to welfare. The presence of this term is again related to households' consumption smoothing motive (this term would disappear if households utility were linear, i.e. when $\sigma^{-1} = 0$). Under an output expansion induced by a technology shock, an increase in entrepreneurial consumption absorbs aggregate resources and thus contributes to smooth the path of households' consumption over time.

The last two terms in equation (30), which are proportional to $\widehat{e}_t$ and $\widehat{e}_t^2$, have an ambiguous impact on welfare, depending on whether the weight of households in social welfare is larger or smaller than a certain threshold $\varsigma = \left( 1 + c^{-\sigma^{-1}} \right)^{-1}$.

The quadratic term in entrepreneurial consumption is due to two reasons. On the one hand, from the aggregate resource constraint households' consumption is a concave function of entrepreneurial consumption, so that households dislike fluctuations in $\widehat{e}_t$. In a second order approximation of the resource constraint, this shows up in a negative quadratic term in $\widehat{e}_t$. On the other hand, entrepreneurial utility is, by construction, a convex function of the logarithm of entrepreneurial consumption. This implies that entrepreneurs will have a preference for volatile log-consumption. The sign of the term proportional to $\widehat{e}_t^2$ on social welfare depends on which one of these two effects prevails, which is in turn determined by the relative importance of households in social utility.

The linear term in entrepreneurial consumption is also the result of contrasting forces. A higher value of entrepreneurial consumption is obviously beneficial for entrepreneurial welfare. At the same time, any entrepreneurial consumption is detrimental for households' welfare, as it subtracts from the economy resources which could be consumed by households. The net effect of this term on welfare is again determined by the relative weight of households in social utility.

The linear term in entrepreneurial consumption can have strong effects on optimal policy. Their dependence on the relative weight of households and entrepreneurs in social welfare is, however, unappealing. For this reason, in the rest of our derivations we select the particular weight $\varsigma = \left(1 + c^{-\sigma^{-1}}\right)^{-1}$, such that the contrasting motives of households and entrepreneurs cancel out and first order terms disappear entirely from social welfare.

Under this special weight $\varsigma$, the loss function simplifies to

$$\begin{aligned} L_t \equiv & \kappa_\pi \widehat{\pi}_t^2 + \frac{1}{2}\frac{Y}{c}\varphi\left(\widetilde{Y}_t - y^*\right)^2 + \frac{1}{2}\sigma^{-1}\left[\frac{Y}{c}\left(\widetilde{Y}_t - y^*\right) - \frac{e}{c}\left(\widehat{R}_t + \delta_3\widehat{\Delta}_t - \delta_4\widehat{\mu}_t\right)\right]^2 \\ & + \sigma^{-1}\frac{e}{c}\frac{Y}{c}\left(\widehat{Y}_t^e + y^*\right)\left(\widetilde{Y}_t - \widehat{R}_t - \delta_3\widehat{\Delta}_t\right) \end{aligned} \quad (31)$$

where equation (17) was used to write entrepreneurial consumption in terms of the nominal interest rate $\widehat{R}_t$, the credit spread $\widehat{\Delta}_t$ and the exogenous shock $\widehat{\mu}_t$. This expression allows us to perform a complete derivation of optimal policy using the linearized policy equations (23)-(25).

Compared to the case of the standard new-Keynesian model, the novel terms in expression (31) are those with a coefficient proportional to $e/c$ (notice that these terms vanish when entrepreneurs disappear from the economy and $Y = c$).

These novel terms include first, within square brackets, elements proportional to the squared nominal interest rate and the squared loan-deposit rate spread. Hence, the presence of asymmetric information in the economy introduces directly both an interest rate smoothing and a "spread smoothing" motive for optimal policy. At the same time, these terms are relatively small in our calibration, where households' consumption takes up the lion share of output. Under normal circumstances, therefore, the interest rate smoothing concern is unlikely to be predominant compared to the objective of maintaining price stability.

The term within square brackets in equation (31) also include a number of cross products between endogenous variables. More specifically, a planner would be averse to a positive covariance between the nominal interest rate and the loan-deposit rates spread. A high covariance would increase the volatility of entrepreneurial consumption, with negative spillovers on households' consumption-smoothing motive. The planner would however not be averse to a positive covariance between, on the one side, the output gap, on the other side, either the interest rate or the loan-deposit rates spread. These covariances would be welcome to the extent that they help to smooth fluctuations in the output gap.

Finally, the last term in equation (31) shows that increases in the nominal interest rate and in the credit spread have a positive effect on welfare, if they are accompanied by an increase in the efficient level of output – i.e. an increase in productivity. The reason is that households are willing to reap the benefits of the higher productivity on real wages, but wish to smooth their consumption pattern over time. A higher incidence of monitoring costs in the economy at a time of high productivity helps to achieve the latter objective.

# 5 Optimal policy

## 5.1 Discretion

When the welfare function can be approximated as in (29) and (31), the problem of the central bank is to maximize that objective, subject to the system of equilibrium conditions (23)-(25).

The appendix shows that, in the special case in which $\varphi = 0$ and $\sigma^{-1} = 1$, the target rule which characterizes the discretionary equilibrium takes the simple form

$$\widehat{\pi}_t = \nu_e \left(\widehat{Y}_t^e + y^*\right) - \nu_\pi \left[\left(\widetilde{Y}_t - y^*\right) - \frac{e}{Y}\left(\widehat{R}_t + \delta_3 \widehat{\Delta}_t\right)\right] - \nu_\pi \frac{e}{Y} \delta_4 \widehat{\mu}_t$$

for parameters $\nu_\pi$ and $\nu_e$ defined in the appendix.

Note that, in the frictionless case, $\nu_\pi = \frac{1}{\bar{\kappa}}\frac{1}{\kappa_\pi}$ and $\nu_e = 0$ so that the optimality condition becomes

$$\widehat{\pi}_t = -\frac{1}{\bar{\kappa}}\frac{1}{\kappa_\pi}\left(\widetilde{Y}_t - y^*\right)$$

which corresponds to standard results in the new-Keynesian case – see, for instance, Woodford (2003, chapter 7, p. 471). This target criterion implies that the central bank will choose to engineer a constant, positive inflation rate, given the output-inflation trade-off implicit in the Phillips curve. Any rise in inflation above that level would be met by a policy response such as to produce a negative output gap.

In our model, the target criterion which would be followed by a central bank under discretion is affected by the existence of financial frictions. While the output gap remains important, both other endogenous variables and shocks limit the ability of the central bank to use the output gap to achieve the desired level of inflation. A surge in inflation could also be countered through actions which affect the spread $\widehat{\Delta}_t$ and the nominal interest rate.

In addition exogenous shocks, including both the financial shock $\widehat{\mu}_t$ and technology shocks (through the efficient level of output $\widehat{Y}_t^e$), affect the target criterion. This implies that the optimal inflation rate varies in the face of these shocks. Differently from the benchmark new-Keynesian case, some temporary deviations from the central bank's objective may occasionally be desirable.

### 5.2 Optimal monetary policy under commitment

We characterize numerically the optimal monetary policy under commitment in the special case characterized above in which linear terms disappear from the quadratic approximation of the welfare function. Under this assumption, steady state inflation is zero and the linear system in equations (23)-(25) is a correct approximation. To characterize more general cases, we need a full second-order approximation of the policy equations, which we perform using Dynare.[2] In all cases, we concentrate on optimal policy under a timeless perspective, as in Woodford (2003).

Figure 4 displays impulse responses to a technology shock when monetary policy is set optimally. Once again, for the technology shock we contrast optimal policy in the credit

[2] We compute the first order conditions of the welfare maximisation problem of the policy maker using Giovanni Lombardo's lq_solution routine available at http://home.arcor.de/calomba/symbsolve4_lnx.zip.

channel model with the optimal policies which would arise in a model with the cost-channel and in the standard new-Keynesian model.[3]

Compared to the results in Figure 2, optimal policy leads to more significant differences in the three models considered here. As is well-known, optimal policy would ensure complete price stability and full stabilization of the output gap in the new-Keynesian model. The policy interest rate would fall on impact and then return slowly to the baseline.

Under the credit channel, near-full inflation stabilization remains optimal in response to technology shocks. However, the path of the policy interest rate which achieves this outcome would be somewhat different. The policy rate is kept constant for one period, before reaching levels roughly consistent with those in the new-Keynesian model. Partly as a result of the slowed monetary easing, output increases less than in the efficient equilibrium and a negative output gap ensues.

The impulse response of the output gap highlights the differences between our model and the cost channel model. In the latter case, the impact reduction in the nominal interest rate is more marked – even if not as aggressive as in the new-Keynesian benchmark – and strongly expansionary, so that the output gap increases after the technology shock. This is not the case in our model because of the increase in the spread, which remains procyclical as in the simple rule benchmark.

Figure 5 displays impulse responses to a positive shock to $\widehat{\mu}_t$. This shock is representative of a broader array of "financial shocks" which could be defined in our model, notably shocks to the $\tau$ subsidy, or to the standard deviation of idiosyncratic shocks. All these shocks would produce similar impulse responses.

The increase in $\widehat{\mu}_t$ generates immediately an increase in the loan-deposit rate spread – the shock is in fact normalized to produce a 1 percentage point increase in the spread. The larger spread acts like a classical cost-push shock. It depresses households' consumption and output while creating inflationary pressures. Under a Taylor rule, the net effect of the shock is a 1 percentage point increase in inflation, in spite of a progressive policy tightening in nominal terms. At the same time, there is a pronounced fall in households' consumption, because of the output loss due to bankruptcies. The amount of credit also falls after the shock.

[3] In all cases, we assume the existence of a steady state subsidy which eliminates first-order terms in output from the second-order expansion of individuals' utility. The subsidy is slightly different in the three cases: it is equal to $\chi/(\chi-1)$ in the new-Keynesian model, $R\chi/(\chi-1)$ in the cost-channel model, and $q\chi/(\chi-1)$ in our model.

Note that, contrary to the case of the technology shock, the spread moves anti-cyclically in response to a financial shock.

Compared to the responses under the Taylor rule, those obtained under optimal policy are striking because they produce an increase in the volatility of many target variables, including the output gap, the spread and the deposit rate. More specifically, in spite of the inflationary pressure, the policy interest rate is cut on impact very aggressively for approximately one year. Consequently, some inflation ensues, even if less high, and less persistent than in the Taylor rule case.

The main reason for this policy response is that the financial shock is inefficient and therefore tends to lead to a fall in households' consumption which is entirely undesirable. The marked expansion in monetary policy is aimed at smoothing households' consumption path after the shock. Compared to the Taylor rule case, households' consumption is almost unchanged on impact and only reaches levels consistent with those attained under the Taylor rule after 3 quarters.

The pronounced fall in the real interest rate creates a small output gap for 1 quarter, which is one of the reasons for the short-lived inflationary outcome.

# 6 Conclusion

Using a small, microfounded model with nominal rigidities and credit frictions, we have analyzed the implications of financial market conditions on macroeconomic dynamics and on optimal monetary policy. Given the simplicity of our set-up, we have been able to characterize analytically the linearized aggregate relations of the model and to obtain an approximate welfare criterion consistent with the microfoundations of the model.

Our results show that, in general, monetary policy ought to pay attention to the evolution of financial market conditions, as captured for example by changes in credit spreads. On the one hand, these changes matter because they affect firms' marginal costs and have therefore an impact on output and inflation. On the other hand, spreads matter because they affect the economy's bankruptcy rate and the resource costs of loan monitoring.

In quantitative terms, we show that price stability should remain the primary objective of monetary policy. Nevertheless, our results suggest that there might be good reasons for a central bank to react with an aggressive easing to an adverse financial shock. Those types of shocks create an inefficient recession, whose negative consequences on consumption could be partly reduced. At the same time, any aggressive policy easing should not be accompanied by persistent deviations from price stability.

# Appendix

## A The financial contract

The informational structure corresponds to a costly state verification (CSV) problem. The solution is a standard debt contract (see e.g. Gale and Hellwig, 1985) such that: i) the repayment to the financial intermediary is constant in states when monitoring does not occur; ii) the firm is declared bankrupt when the fixed repayment cannot be honoured; iii) in case of bankruptcy, the financial intermediary monitors and completely seizes the firm's output.

Recall that the presence of agency costs implies that $y_{i,t} = \omega_{i,t}\chi_t q_t x_{i,t}$. Define

$$f\left(\overline{\omega}\right) \equiv \int_{\overline{\omega}}^{\infty} \omega\Phi\left(d\omega\right) - \overline{\omega}\left[1 - \Phi\left(\overline{\omega}\right)\right]$$

$$g\left(\overline{\omega};\mu\right) \equiv \int_{0}^{\overline{\omega}} \omega\Phi\left(d\omega\right) - \mu\Phi\left(\overline{\omega}\right) + \overline{\omega}\left[1 - \Phi\left(\overline{\omega}\right)\right]$$

as the expected shares of output accruing respectively to an entrepreneur and to a lender, after stipulating a contract that sets the fixed repayment at $\overline{P}_t\chi_t q_t\overline{\omega}_{it}x_{i,t}$ units of money. In case of default, a stochastic fraction $\mu_t$ of the input costs $x_{i,t}$, measured in units of money, is used in monitoring. We assume that $\mu_t$ follows a AR1 process given by $\mu_t = \left(1-\rho_\mu\right)\mu + \rho_\mu\mu_{t-1} + \epsilon_{\mu,t}$. At the individual firm level, total output is split between the entrepreneur, the lender, and monitoring costs so that

$$f\left(\overline{\omega}_t\right) + g(\overline{\omega}_t) = 1 - \mu_t\Phi\left(\overline{\omega}_t\right).$$

The optimal contract is the pair $(x_{i,t}, \overline{\omega}_{i,t})$ that solves the following CSV problem:

$$\max\ \overline{P}_t\chi_t q_t f(\overline{\omega}_{i,t})x_{i,t}$$

subject to

$$
\begin{aligned}
\overline{P}_t \chi_t q_t g(\overline{\omega}_{i,t}) x_{i,t} &\geq R_t^d P_t \left(x_{i,t} - \tau\right) && (32) \\
\overline{P}_t \left[f\left(\overline{\omega}_{i,t}\right) + g\left(\overline{\omega}_{i,t}; \mu_t\right) - 1 + \mu_t \Phi\left(\overline{\omega}\right)\right] &\leq 0 && (33) \\
\overline{P}_t \chi_t q_t f(\overline{\omega}_{i,t}) x_{i,t} &\geq P_t \tau && (34)
\end{aligned}
$$

The optimal contract maximizes the entrepreneur's expected profits subject to the lender being willing to lend out funds, (32), the feasibility condition, (33), and the entrepreneur being willing to sign the contract, (34). Notice that the intermediary needs to pay back to the household a gross return equal to the safe interest on deposits, $R_t^d$. Since in equilibrium $R_t = R_t^d$, the financial intermediary's expected return on each unit of loans cannot be lower than $R_t$.

The optimality conditions can be written as

$$
\begin{aligned}
q_t &= \frac{R_t}{1 - \mu_t \Phi\left(\overline{\omega}_{i,t}\right) + \frac{\mu_t f(\overline{\omega}_{i,t}) \phi(\overline{\omega}_{i,t})}{f_{\overline{\omega}}(\overline{\omega}_{i,t})}}, && (35) \\
x_{i,t} &= \left\{ \frac{R_t}{R_t - q_t g\left(\overline{\omega}_{i,t}; \mu_t\right)} \right\} \tau. && (36)
\end{aligned}
$$

From equation (35), it follows that the terms of the contract depend on the state of the economy only through the aggregate mark-ups $\chi_t$ and $q_t$ and the return $R_t$. Hence, they are the same for all firms, $\overline{\omega}_{i,t} = \overline{\omega}_t$. Since initial wealth is also the same across firms, it follows from equation (36) that the size of the project is the same across firms.

## B The log-linearized system of equilibrium conditions

The equilibrium can then be characterized as the solution to the following system of log-linearized equations in the variables $\left\{\widehat{c}_t, \widehat{Y}_t, \widehat{h}_t, \widehat{q}_t, \widehat{\chi}_t, \widehat{\pi}_t, \widehat{R}_t, \widehat{\Delta}_t\right\}$,

$$
\begin{aligned}
\alpha_1 \widehat{\Delta}_t &= -\widehat{Y}_t - \frac{(y-c)}{c}\widehat{\chi}_t + \widehat{c}_t - \alpha_1 \frac{g_\mu \mu}{g}\widehat{\mu}_t \\
\widehat{q}_t &= a_t - \sigma^{-1}\widehat{c}_t - \varphi \widehat{h}_t - \widehat{\chi}_t \\
\widehat{h}_t &= \widehat{Y}_t - a_t \\
\widehat{R}_t &= \widehat{q}_t - \alpha_2 \widehat{\Delta}_t + \alpha_3 \widehat{\mu}_t \\
\widehat{Y}_t &= \widehat{\chi}_t + \widehat{R}_t + \alpha_4 \widehat{\Delta}_t + \alpha_5 \widehat{\mu}_t \\
\widehat{R}_t &= E_t \widehat{\pi}_{t+1} + \sigma^{-1}\left(E_t \widehat{c}_{t+1} - \widehat{c}_t\right) \\
\widehat{\pi}_t &= -\overline{\kappa}\left(1 + \sigma^{-1}\frac{e}{c}\right)\widehat{\chi}_t + \beta E_t \widehat{\pi}_{t+1}
\end{aligned}
$$

where $\overline{\kappa} \equiv \frac{(1-\theta)(1-\theta\beta)}{\theta}\frac{1}{1+\sigma^{-1}\frac{e}{c}}$, plus a monetary policy rule.

## C The coefficients of the log-linearized equations

The coefficients of the system (19)-(22) are given by

$$
\begin{aligned}
\alpha_1 &= -\frac{\frac{f_{\overline{\omega}}\overline{\omega}}{\chi}\frac{y}{c}}{1 - g_{\overline{\omega}}\Delta} > 0 \\
\alpha_2 &= -\frac{\mu \frac{f\overline{\omega}}{f_{\overline{\omega}}}\left(\phi_{\overline{\omega}} - \frac{\phi^2}{f_{\overline{\omega}}}\right)\frac{q}{R}}{(1 - g_{\overline{\omega}}\Delta)} > 0 \\
\alpha_3 &= \left[\frac{\frac{g_\mu \mu}{g}\frac{f\overline{\omega}}{f_{\overline{\omega}}}\left(\phi_{\overline{\omega}} - \frac{\phi^2}{f_{\overline{\omega}}}\right)}{(1 - g_{\overline{\omega}}\Delta)} + \frac{f\phi}{f_{\overline{\omega}}} - \Phi\right]\mu \frac{q}{R} \\
\alpha_4 &= -\frac{\mu \frac{f\overline{\omega}}{f_{\overline{\omega}}}\left(\phi_{\overline{\omega}} - \frac{\phi^2}{f_{\overline{\omega}}}\right) + \overline{\omega}\left(f_{\overline{\omega}} + \mu\phi\right)}{\left(f + \frac{\mu f \phi}{f_{\overline{\omega}}}\right)(1 - g_{\overline{\omega}}\Delta)} > 0 \\
\alpha_5 &= \left(\frac{\alpha_4 g_\mu}{g} - \frac{\phi}{f_{\overline{\omega}} + \mu\phi}\right)\mu \\
\delta_1 &\equiv \left(1 + \frac{\sigma^{-1}e}{c}\right)\alpha_4 - \sigma^{-1}\alpha_1 - \alpha_2 \\
\delta_2 &\equiv \alpha_3 - \sigma^{-1}\alpha_1 \frac{g_\mu \mu}{g}.
\end{aligned}
$$

## D Case with frictionless financial markets

When $\mu_t = 0$, for all $t$, $f(\overline{\omega}_t) + g(\overline{\omega}_t; \mu_t) = 1$. Also, since there are no monitoring costs, banks set $\overline{\omega}_t$ as high as possible subject to the constraint that the firm is willing to sign the contract, i.e.

$$f(\overline{\omega}_t) = \frac{\tau}{q_t x_t}$$

This maximizes banks' profits, as they can size the production of all defaulting firms at no cost. In such equilibrium,

$$\begin{aligned} g(\overline{\omega}_t; \mu_t) &= 1 - \frac{\tau \chi_t}{y_t} \\ e_t &= \tau \\ Y_t &= c_t + \tau. \end{aligned}$$

Moreover, from the bank's zero profit condition, we have

$$x_t = \frac{R_t \tau}{R_t - q_t \left(1 - \frac{\tau \chi_t}{y_t}\right)}.$$

The log-linearized system can then be written as

$$\widehat{\chi}_t = -\left[\frac{(R-1)\chi\tau}{y}\left(1+\sigma^{-1}+\varphi\right) + \left(\sigma^{-1}+\varphi\right)\right]\widehat{Y}_t + \left(\frac{\chi\tau}{y} - \frac{1}{q}\right) R\widehat{R}_t + \left[\frac{(R-1)\chi\tau}{y} + 1\right](1+\varphi)\, a_t$$

$$\widehat{Y}_t = E_t \widehat{Y}_{t+1} - \sigma\left(\widehat{R}_t - E_t \widehat{\pi}_{t+1}\right)$$

$$\widehat{\pi}_t = -\overline{\kappa}\left(1 + \sigma^{-1}\frac{e}{c}\right)\widehat{\chi}_t + \beta E_t \widehat{\pi}_{t+1}$$

In the limiting case where $\tau \approx 0$, $q_t = R_t$ and the system boils down to the equations reported in the text.

# E Welfare approximation

Our monetary policy objective is derived as the second order approximation to a weighted average of the utilities of the household and of the entrepreneur, i.e.

$$E_o \left\{ \sum_{0}^{\infty} \beta^t \left[ \varsigma U_t + (1-\varsigma) U_t^e \right] \right\}$$

where $\varsigma$ is the weight of the utility of households in the policy objective. Households' temporary utility can then be approximated as

$$U_t \simeq U + u_c c \left( \widehat{c}_t + \frac{1}{2} \left( 1 + \frac{u_{cc} c}{u_c} \right) \widehat{c}_t^2 \right) - v_h h \left( \widehat{h}_t + \frac{1}{2} \left( 1 + \frac{v_{hh} h}{v_h} \right) \widehat{h}_t^2 \right)$$

where hats denote log-deviations from the deterministic steady state and $c$ and $h$ denote steady state levels. Similarly, entrepreneurial temporary utility $U_t^e$ can be expanded as

$$U_t^e \simeq e \left( 1 + \widehat{e}_t + \frac{1}{2} \widehat{e}_t^2 \right)$$

where $e$ is the steady state level of entrepreneurial consumption.

Under the functional form $U_t = \frac{c_t^{1-\sigma^{-1}}}{1-\sigma^{-1}} - \psi \frac{h_t^{1+\varphi}}{1+\varphi}$, households' temporary utility can be rewritten as

$$U_t \simeq \frac{c^{1-\sigma^{-1}}}{1-\sigma^{-1}} - \psi \frac{h^{1+\varphi}}{1+\varphi} + c^{1-\sigma^{-1}} \widehat{c}_t - \psi h^{1+\varphi} \widehat{h}_t + \frac{1}{2} \left( c^{1-\sigma^{-1}} \left( 1-\sigma^{-1} \right) \widehat{c}_t^2 - \psi h^{1+\varphi} \left( 1+\varphi \right) \widehat{h}_t^2 \right)$$

We can express hours, households' consumption and entrepreneurial consumption as

$$h_t = \frac{s_t Y_t}{A_t}$$

$$c_t = Y_t - e_t$$

$$e_t = \tau R_t H \left( \mu_t, \overline{\omega}_t \right)$$

where $H(\mu_t, \overline{\omega}_t) \equiv \left[ 1 + \frac{\mu_t \phi(\overline{\omega}_t)}{f_{\overline{\omega}}(\overline{\omega}_t)} \right]^{-1}$.

The period aggregate utility can be approximated as

$$
\begin{aligned}
\varsigma U_t + (1-\varsigma) U_t^e \simeq\; & \varsigma c^{1-\sigma^{-1}} \left( \frac{1}{1-\sigma^{-1}} - \frac{\psi}{1+\varphi} \frac{h^{1+\varphi}}{c^{1-\sigma^{-1}}} \right) + (1-\varsigma) e \\
& + \varsigma c^{1-\sigma^{-1}} \widehat{c}_t + (1-\varsigma) e \widehat{e}_t - \varsigma c^{1-\sigma^{-1}} \frac{\psi h^{1+\varphi}}{c^{1-\sigma^{-1}}} \widehat{h}_t \\
& + \frac{1}{2} \varsigma c^{1-\sigma^{-1}} \left(1-\sigma^{-1}\right) \widehat{c}_t^2 - \frac{1}{2} \varsigma c^{1-\sigma^{-1}} \frac{\psi h^{1+\varphi}}{c^{1-\sigma^{-1}}} (1+\varphi) \widehat{h}_t^2 + \frac{1}{2} (1-\varsigma) e \widehat{e}_t^2
\end{aligned}
$$

Now note that the resource constraint $c_t = Y_t - e_t$ can be approximated to second order as

$$
\widehat{c}_t = \frac{Y}{c} \widehat{Y}_t - \frac{e}{c} \widehat{e}_t + \frac{1}{2} \frac{Y}{c} \widehat{Y}_t^2 - \frac{1}{2} \frac{e}{c} \widehat{e}_t^2 - \frac{1}{2} \widehat{c}_t^2
$$

while the production function implies simply

$$
\widehat{h}_t = -a_t + \widehat{s}_t + \widehat{Y}_t.
$$

It follows that utility can be rewritten as

$$
\begin{aligned}
\frac{\varsigma U_t + (1-\varsigma) U_t^e - \varkappa}{c^{1-\sigma^{-1}}} \simeq\; & -\varsigma \frac{\psi h^{1+\varphi}}{c^{1-\sigma^{-1}}} \widehat{s}_t - \varsigma \left( \frac{\psi h^{1+\varphi}}{c^{1-\sigma^{-1}}} - \frac{Y}{c} \right) \widehat{Y}_t \\
& - \frac{e}{c} \left( \varsigma - (1-\varsigma) c^{\sigma^{-1}} \right) \left( \widehat{e}_t + \frac{1}{2} \widehat{e}_t^2 \right) \\
& - \frac{1}{2} \varsigma \sigma^{-1} \widehat{c}_t^2 - \frac{1}{2} \varsigma \left( \frac{\psi h^{1+\varphi}}{c^{1-\sigma^{-1}}} (1+\varphi) - \frac{Y}{c} \right) \widehat{Y}_t^2 \\
& + \varsigma \frac{\psi h^{1+\varphi}}{c^{1-\sigma^{-1}}} (1+\varphi) a_t \widehat{Y}_t + tips
\end{aligned}
$$

where

$$
\varkappa \equiv \varsigma c^{1-\sigma^{-1}} \left( \frac{1}{1-\sigma^{-1}} - \frac{1}{1+\varphi} \frac{\psi h^{1+\varphi}}{c^{1-\sigma^{-1}}} \right) + (1-\varsigma) e
$$

Now consider the FOC

$$
\frac{\psi h^{\varphi}}{c_t^{-\sigma^{-1}}} = \frac{A_t}{q_t \chi_t}
$$

Under perfect competition and frictionless credit markets, firms set the real wage at the marginal product of labor, $w_t = A_t$. In our model, equation (11) implies that $w_t = \frac{A_t}{q_t \chi_t}$. We provide households with a subsidy $\Omega_1$ such that $\frac{\psi h^{\varphi}}{c^{-\sigma^{-1}}} = w_t (1-\Omega_1)$, and

$$
1 - \Omega_1 = q\chi.
$$

It follows that in such a steady state

$$\frac{\psi h^{1+\varphi}}{c^{1-\sigma^{-1}}} = \frac{Ah}{c} = \frac{Y}{c}.$$

This subsidy will allow us to ignore first order terms in output from welfare. Note, however, that the same subsidy will not bring the steady state of the economy back to the efficient (steady state) level. The actual steady state can in fact be written as

$$Y = \psi^{-\left(\varphi+\sigma^{-1}\right)^{-1}} \left[ \frac{1-\Omega_1}{q\chi\left(1-\frac{f(\overline{\omega})}{\chi}\right)^{\sigma^{-1}}} \right]^{\left(\varphi+\sigma^{-1}\right)^{-1}}$$

while the steady state level of efficient output is $Y^e = \psi^{-\left(\varphi+\sigma^{-1}\right)^{-1}}$. The subsidy which would make the actual steady state efficient is $1-\Omega_1 = q\chi\left(1-\frac{f(\overline{\omega})}{\chi}\right)^{\sigma^{-1}} > q\chi$. Following Woodford (2003), for small values of the distortions, $\Omega_1$ can be treated as an expansion parameter. The steady state gap $y^* \equiv \log Y - \log Y^e$ can then be loglinearized to yield $y^* \simeq -\left(\varphi+\sigma^{-1}\right)^{-1}\Omega_1$.

In addition, we focus on the case of a special Pareto weight $\varsigma = \frac{c^{\sigma^{-1}}}{1+c^{\sigma^{-1}}}$, which allows us to ignore first order terms in entrepreneurial consumption. It follows that the loss can be written as

$$\frac{\varsigma U_t + (1-\varsigma)\, U_t^e - \varkappa}{\varsigma c} \simeq -\frac{Y}{c}\widehat{s}_t - \frac{1}{2}\frac{Y}{c}\varphi\widehat{Y}_t^2 - \frac{1}{2}\sigma^{-1}\left(\frac{Y}{c}\widehat{Y}_t - \frac{e}{c}\widehat{e}_t\right)^2 + \frac{Y}{c}\,(1+\varphi)\,a_t\widehat{Y}_t + t.i.s.p.$$

Now note that in the fully-efficient steady state we would have

$$(1+\varphi)\,a_t = \sigma^{-1}\widehat{c}_t^e + \varphi\widehat{Y}_t^e + \left(\varphi+\sigma^{-1}\right)(y-y^e)$$

which can be used to substitute out the technology shock $a_t$ from the loss function.

In addition, a first order approximation to the equation for price dispersion, of first-order in $\widehat{s}_t$ and second-order in $\widehat{\pi}_t$ takes the form

$$\widehat{s}_t \simeq \frac{\theta}{1-\theta}\varepsilon\frac{\widehat{\pi}_t^2}{2} + \theta\widehat{s}_{t-1}.$$

This latter can be integrated forward to obtain

$$\widehat{s}_t \simeq \frac{\theta}{1-\theta}\varepsilon \sum_{s=t_0}^{t} \theta^{t-s}\frac{\widehat{\pi}_s^2}{2} + \theta^{t-t_0+1}\widehat{s}_{t_0-1}.$$

Multiplying this by $\beta^{t-t_0}$ and realizing that multiples of $\widehat{s}_{t_0-1}$ are independent of policy, we obtain

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0}\widehat{s}_t \simeq \frac{\theta}{(1-\theta)(1-\beta\theta)}\varepsilon \sum_{t=t_0}^{\infty} \beta^{t-t_0}\frac{\widehat{\pi}_t^2}{2} + t.i.p.$$

Finally, note that entrepreneurial consumption can be written as

$$\widehat{e}_t = \widehat{R}_t + \delta_3\widehat{\Delta}_t - \delta_4\widehat{\mu}_t$$

where

$$\begin{aligned} \delta_3 &= -\frac{\mu\overline{\omega}\left(\phi_{\overline{\omega}} - \frac{\phi^2}{f_{\overline{\omega}}}\right)}{(f_{\overline{\omega}} + \mu\phi)(1-\Delta g_{\overline{\omega}})} > 0 \\ \delta_4 &= \left(\frac{\phi}{\mu\phi + f_{\overline{\omega}}} + \frac{\delta_3 g_\mu}{g}\right)\mu \end{aligned}$$

It follows that the approximated welfare function can be written as in the main text, for

$$\kappa_\pi \equiv \frac{1}{2}\frac{Y}{c}\frac{\varepsilon\theta}{(1-\theta)(1-\beta\theta)} > 0$$

# F Optimal policy under discretion

Under discretion, the central bank tries to minimize the loss function (31) subject to the three-equation system (23)-(25). Denote as $\eta_{\Delta,t}$, $\eta_{Y,t}$ and $\eta_{\pi,t}$ the Lagrangean multipliers associated to the constraints. The first order conditions can be written as

$$\eta_{\pi,t} = 2\kappa_\pi\widehat{\pi}_t$$

and

$$
\begin{aligned}
\eta_{Y,t} &= \frac{Y}{c}\varphi\left(\widetilde{Y}_t - y^*\right) + \sigma^{-1}\frac{Y}{c}z_t + \sigma^{-1}\frac{Y-c}{c}\frac{Y}{c}\left(\widehat{Y}_t^e + y^*\right) + \frac{1+\varphi+\sigma^{-1}\frac{Y}{c}}{\delta_1}\eta_{\Delta,t} + 2\overline{\kappa}\kappa_\pi\left(\sigma^{-1}+\varphi\right)\widehat{\pi}_t \\
\eta_{\Delta,t} &= -\frac{Y-c}{c}\delta_3\sigma^{-1}z_t - \sigma^{-1}\frac{Y-c}{c}\frac{Y}{c}\delta_3\left(\widehat{Y}_t^e + y^*\right) - \frac{\alpha_1 - \alpha_2\frac{e}{c}}{1-\varphi\frac{e}{c}}\eta_{Y,t} + 2\overline{\kappa}\kappa_\pi\left(\sigma^{-1}\alpha_1 + \alpha_2\right)\widehat{\pi}_t \\
0 &= -\frac{Y-c}{c}\sigma^{-1}z_t - \sigma^{-1}\frac{Y-c}{c}\frac{Y}{c}\left(\widehat{Y}_t^e + y^*\right) - \frac{\sigma^{-1}\frac{e}{c}}{\delta_1}\eta_{\Delta,t} - \sigma\frac{1}{1-\varphi\frac{e}{c}}\eta_{Y,t} + 2\overline{\kappa}\kappa_\pi\widehat{\pi}_t
\end{aligned}
$$

where

$$
z_t \equiv \frac{Y}{c}\left(\widetilde{Y}_t - y^*\right) - \frac{Y-c}{c}\left(\widehat{R}_t + \delta_3\widehat{\Delta}_t - \delta_4\widehat{\mu}_t\right)
$$

These equations can be solved for the three Lagrange multipliers and yield an additional optimality condition. In the case $\varphi = 0$ and $\sigma^{-1} = 1$, the latter condition can be written simply as

$$
\widehat{\pi}_t = \nu_e\left(\widehat{Y}_t^e + y^*\right) - \nu_\pi\left[\left(\widetilde{Y}_t - y^*\right) - \frac{Y-c}{Y}\left(\widehat{R}_t + \delta_3\widehat{\Delta}_t - \delta_4\widehat{\mu}_t\right)\right]
$$

where

$$
\begin{aligned}
\nu_e &\equiv \frac{1}{\overline{\kappa}}\frac{1}{\kappa_\pi}\frac{Y}{c}\frac{e}{c}\frac{2\,\frac{Y}{c}\,\delta_3 + \frac{e}{c}\,\alpha_2\,\left(1+\frac{Y}{c}-\frac{e}{c}\,\delta_3\right) - \delta_1\,\left(1+\delta_3\right) - \alpha_1\,\left(1+\frac{Y}{c}-\frac{e}{c}\,\delta_3\right)}{\left(1+\frac{Y}{c}\right)\,\alpha_1 + \left(2+3\,\frac{e}{c}+2\,\frac{e^2}{c^2}+2\,\frac{Y}{c}+\frac{Y-c}{c}\frac{Y}{c}\right)\,\alpha_2 + \delta_1} \\
\nu_\pi &\equiv \frac{1}{\overline{\kappa}}\frac{1}{\kappa_\pi}\frac{\frac{e}{Y}\,\alpha_1 - \frac{e}{Y}\frac{e}{c}\,\alpha_2 + \left(1+\frac{e}{Y}\right)\,\delta_1 - 2\,\frac{e}{c}\,\delta_3}{\left(1+\frac{y}{c}\right)\,\alpha_1 + \left(2+3\,\frac{e}{c}+2\,\frac{e^2}{c^2}+2\,\frac{Y}{c}+\frac{e}{c}\frac{Y}{c}\right)\,\alpha_2 + \delta_1}
\end{aligned}
$$

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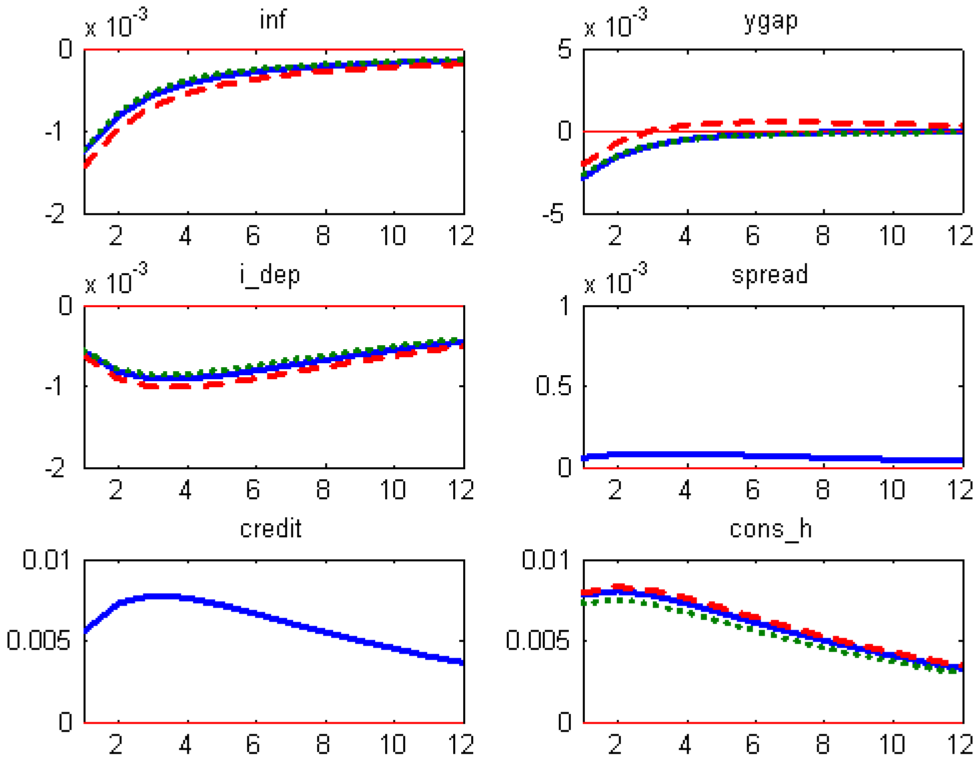
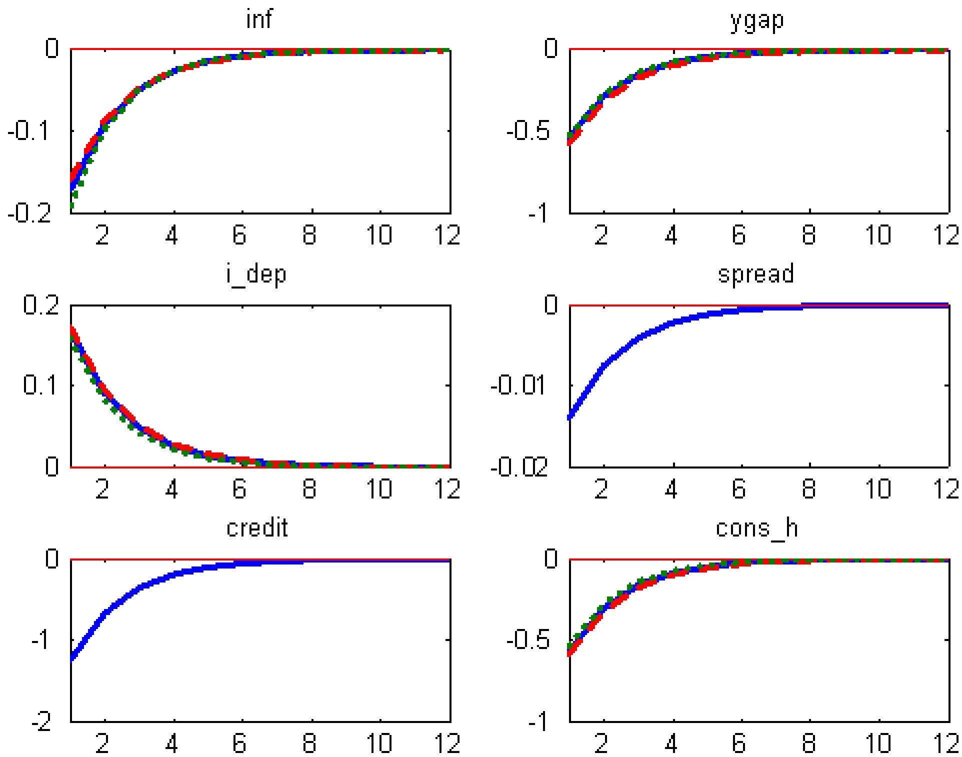
Figure 2: Impulse responses to a technology shock under a Taylor rule within different models



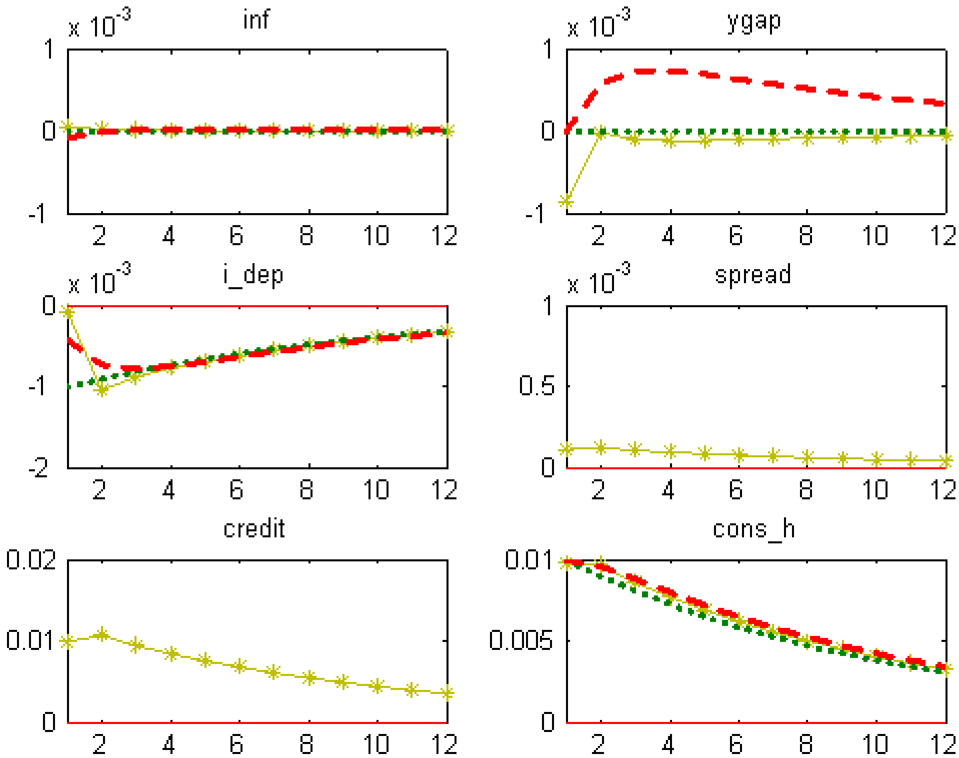

Legend: blue solid line: credit channel model; dashed red line: cost channel model; dotted green line: new-Keynesian model.

Figure 3: Impulse responses to a policy shock under a Taylor rule within different models



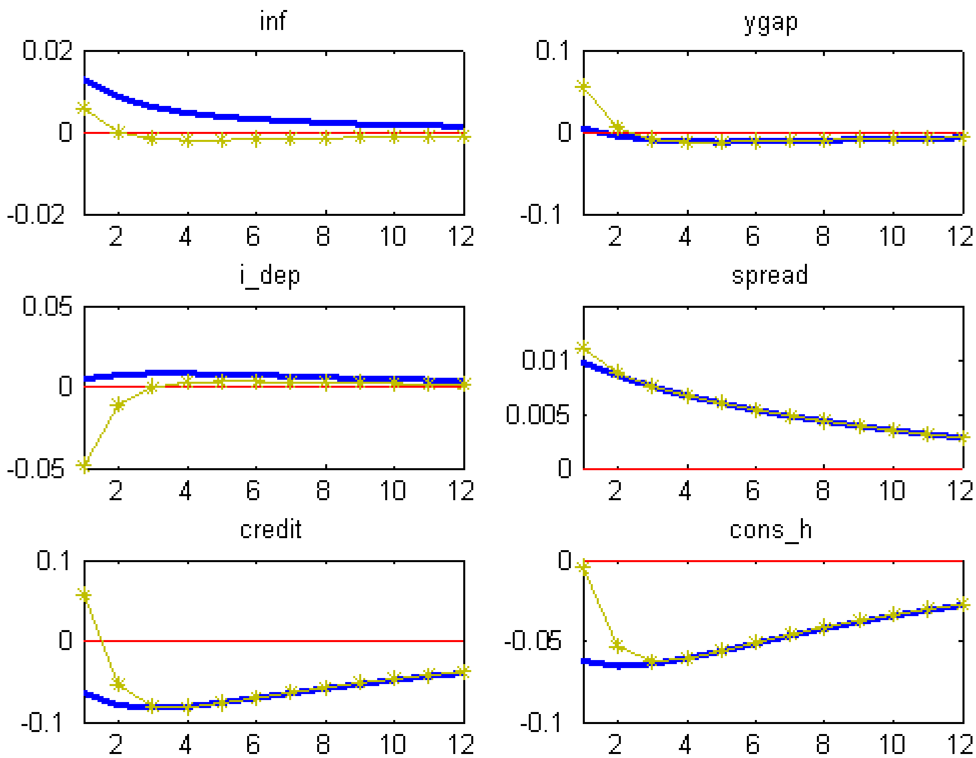

Legend: blue solid line: credit channel model; dashed red line: cost channel model; dotted green line: new-Keynesian model.

Figure 4: Impulse responses to a technology shock under optimal policy within different models




Legend: brown stars: optimal policy in the credit channel model; dashed red line: optimal policy in the cost channel model; dotted green line: optimal policy in the new-Keynesian model.
Note: in all cases, linear terms in the second order expansion of utility are set to zero through an appropriate steady state subsidy and, for the credit channel model, through a particular Pareto weight.

Figure 5: Impulse responses to a $\widehat{\mu}_t$ shock in the credit channel model: Taylor rule vs optimal policy




Legend: brown stars: optimal policy; blue solid line: simple rule.
Note: linear terms in the second order expansion of utility are set to zero through an appropriate steady state subsidy and a particular Pareto weight.