

# Are Central Banks' Projections Meaningful?\*

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## Abstract

Central banks' projections—i.e. forecasts conditional on a given interest rate path—are often criticized on the grounds that their underlying policy assumptions are inconsistent with the existence of a unique equilibrium in many forward-looking models. The present paper describes three alternative approaches to constructing projections that are not subject to that criticism, using a standard New Keynesian model as a reference framework. Yet, and even though they imply an identical path for the interest rate, the three approaches are shown to generate different projections for inflation and output. That result calls into question the meaning and usefulness of such projections.

*Keywords:* interest rate path, inflation targeting, conditional forecasts, interest rate rules, constant interest rate projections, multiple equilibria.

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# 1 Introduction

The periodic construction and publication of macroeconomic forecasts is a common activity among central banks in advanced industrialized economies. Those forecasts get considerable attention from analysts, market participants and the financial press. Among the central banks that have adopted an explicit inflation-forecast targeting strategy those forecasts play a central role in the internal decision process for, in many cases, policy settings are chosen in order to attain a certain inflation target at a specified horizon. Those forecasts are also viewed as an important element in central banks' communication policy. More generally, and using the words of the ECB, "macroeconomic projections play an important role as a tool for aggregating and organizing existing information on current and future economic developments" (ECB (2001)).

The methods and assumptions behind those forecasts vary considerably across central banks. In particular, practices differ regarding the assumption on the path for the short-term nominal interest rate (henceforth, the interest rate) underlying those forecasts. The relative merits of alternative assumptions on which to condition the published forecasts remains the subject of considerable debate, partly sparked by the recent decision by some central banks to change their practice in that regard.<sup>1</sup>

At the risk of oversimplification, one can distinguish three alternative assumptions underlying central banks' forecasting practices. In some cases the interest rate is assumed to remain constant at its current level over the forecasting horizon, giving rise to the so-called *constant interest rate (CIR)* forecasts. Other central banks construct their forecasts under the assumption that the interest rate will follow a path consistent with current market expectations, with the latter being approximated with the forward rates implicit in the yield curve prevailing at the time the projection is made. We refer to those as *market expectations (ME)* forecasts. A third practice found among central banks consists in constructing forecasts based on the assumption that the interest rate will follow whatever path the central bank expects it to follow, i.e. a path consistent with the central bank's "own policy rule," independently of whether the latter had been made explicit or not. We refer to those forecasts as *central bank' expectations (CBE)* forecasts.<sup>2</sup> Since the

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<sup>1</sup>See, e.g. the recent announcements by the Riksbank (2007) and the Federal Reserve Board (2007).

<sup>2</sup>Many central banks that publish CBE-type forecasts also report the own forecast for

latter forecasts are based on the central bank's best assessment of what the interest rate path will be, they can be interpreted as unconditional forecasts. By contrast, CIR or ME forecasts are conditioned on a path of interest rates that does not generally coincide with the central bank's own expectations on that path. As a result, those forecasts are not necessarily the best predictors of future outcomes, as is implicitly reflected in their common labeling as "conditional forecasts" or "projections" For convenience, in the remainder of the paper (as well as in the title) I use the term "projections" to refer to forecasts conditional on a *given* interest rate path.

Each of the forecasting procedures has its own advantages and disadvantages, which may explain the observed diversity of practices.<sup>3</sup> That diversity is illustrated in Table 1, which summarizes the assumptions on the interest rate path underlying the forecast practices of seventeen major central banks.

The present paper's starting motivation is a concern frequently voiced regarding central banks' projections: the inconsistency of their underlying assumptions with the existence of unique equilibrium in a variety of forward-looking models, including the new vintage of optimizing models commonly used for monetary policy analysis.<sup>4</sup> That argument hinges on the notion that, in order to construct such projections, the path of the interest rate is taken *as given* by the central bank, i.e. it is assumed that the interest rate will vary according to a pre-specified path, independently of how inflation or other endogenous variables may end up evolving. In other words, the model is fed with an exogenous interest rate path as a description of monetary policy, which is known to generate an indeterminate equilibrium (and, hence, indeterminate forecasts) in many dynamic models with forward-looking components.<sup>5</sup>

Here I seek to contribute to that debate by clarifying the sense in which macroeconomic projections conditional on a given interest rate path are or are not feasible or well defined. First, I show that it is indeed possible to overcome the curse of indeterminacy and to generate *determinate* macroeconomic projections consistent with an arbitrary path for the interest rate. This is done using a canonical version of the New Keynesian (NK) model as a reference framework. As many as three approaches are proposed to construct the

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the interest rate path (e.g. the RB of New Zealand, Norway's Norges Bank and Sweden's Riksbank). Others do not (the U.S. Federal Reserve).

<sup>3</sup>See, e.g., Svensson (2006) for a detailed discussion.

<sup>4</sup>See, e.g., Leitimo (2003), Honkapohja and Mitra (2005), and Woodford (2005, 2007).

<sup>5</sup>See, e.g. Woodford (2003) or Galí (2008) for a detailed discussion.

desired projections. The first approach, based on the "modest interventions" model of Leeper and Zha (2003), consists of adding unexpected policy shocks to the central bank's rule in order to generate the desired interest rate path.<sup>6</sup> The second and third approaches rely on two alternative interest rate rules designed to generate the desired path of the interest rate as an endogenous equilibrium outcome, while responding systematically to inflation and other macro variables in a way that guarantees the determinacy of the equilibrium (and, hence, uniqueness of the projections).

Thus, by showing how projections can be constructed using *any* of the above methods, the paper refutes the claim that such forecasts are not feasible due to the indeterminacy underlying the associated equilibrium. The latter finding, however, can hardly provide any consolation to advocates of conditional projections: I show by means of a simple example that the three methods will generally *yield different projections for variables other than the interest rate itself*, despite the fact that, by construction, the latter exhibits an identical path across methods. Thus, the problem is not necessarily one of multiplicity of equilibria resulting from an exogenous interest rate path, but one of *multiplicity of rules* that are consistent with the latter path, each having its own implications for other variables. That observation calls into question the usefulness of projections conditional on a given interest rate path since, in principle, there is no obvious reason to prefer one method over another in order to generate the same path.

That problem does not arise when constructing CBE forecasts, which are based on the rule effectively followed by the central bank when constructing those forecasts. More generally, the problem disappears whenever the projections are conditional on *a rule* (as long as the latter is consistent with a unique equilibrium), and *not* on an *interest rate path* (which may be supported by a multiplicity of rules). It is in that sense that the present paper provides an additional argument against the reporting of central banks' projections conditional on a given interest rate path, thus implicitly making a case for the construction of *rule-based* forecasts, ideally (and naturally) using the central bank's perceived own rule.

The remainder of the paper is organized as follows. Section 2 describes the reference model used in the subsequent analysis. Section 3 revisits the problem of indeterminacy when the interest rate follows an exogenous path,

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<sup>6</sup>As shown below, the "modest interventions" approach is equivalent to assuming a switch right after the forecast horizon to the usual interest rate rule.

and shows how this indeterminacy is inherited by the associated projections. Section 4 describes three approaches that overcome that indeterminacy problem. Section 5 illustrates, using a calibrated version of the reference model, how those three approaches lead to significantly different inflation and output projections conditional on an unchanged interest rate, at any finite horizon. Section 6 concludes.

## 2 A Baseline Model

I use a simple version of the New Keynesian model as a reference framework for the analysis of alternative macro projections.<sup>7</sup> The non-policy block of the model is made up of the following two equations:

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma} (i_t - E_t\{\pi_{t+1}\}) \quad (1)$$

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa y_t + u_t \quad (2)$$

where  $y_t$  denotes the output gap,  $i_t$  is the short-term nominal interest rate,  $\pi_t \equiv p_t - p_{t-1}$  is the rate of inflation between  $t-1$  and  $t$  (with  $p_t$  denoting the log of the price level), and  $u_t$  is an exogenous cost-push shock which follows an AR(1) process with autoregressive coefficient  $\rho_u \in (0, 1]$ . Equation (1) can be obtained by log-linearizing the representative household's Euler equation and a market clearing condition that equates consumption to output.<sup>8</sup> Equation (2) is a version of the so-called New Keynesian Phillips curve, which can be derived by aggregating the price-setting decisions of monopolistically competitive firms subject to Calvo-type constraints on the frequency of price adjustment, combined with standard assumptions on technology and labor markets. In that context, parameter  $\sigma$  corresponds to the coefficient of relative risk aversion,  $\beta$  is the household's discount factor and  $\kappa$  is a coefficient which is inversely related to the degree of price rigidities. All variables are expressed in terms of deviations from their values in a zero inflation steady state.

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<sup>7</sup>This is the model used in the optimal policy analysis of Clarida, Galí and Gertler (1999). The reader is referred to King and Wolman (1996), Woodford (2003) or Galí (2008a) for a detailed description of that model and a derivation of (1) and (2).

<sup>8</sup>We implicitly assume a constant natural rate of interest, to lighten the notation. The analysis below carries over, with suitable modifications, to the case of a time-varying natural rate of interest.

A block describing how monetary policy is conducted completes the model. We assume that the central bank follows a simple interest rate rule of the form

$$i_t = \phi \pi_t \quad (3)$$

where  $\phi > 1$ . As is well known, the latter condition guarantees uniqueness of the equilibrium.

The remainder of the paper examines three alternative approaches to constructing forecasts for inflation and output in the above economy, under a (counterfactual) assumption regarding how interest rates evolve over the forecast horizon. But before turning to that analysis I take a brief detour to describe the basic concern associated with the construction of macroeconomic forecasts conditional on an arbitrary, exogenously given interest rate path.

### 3 Projections Conditional on an Exogenous Interest Rate Path: The Indeterminacy Problem

For the sake of concreteness, let us assume that at the central bank is interested in the  $k$ -period ahead forecasts of inflation and the output gap *conditional* on an arbitrary, exogenous path for the interest rate,  $\{i_t^*\}$ . As mentioned above, both the CIR and ME projections constructed and published by many central banks can be viewed as particular examples of such an exercise since, in both cases, the path of the interest rate is given to the forecaster, with no room allowed for possible adjustments in response to developments in the economy, at least over the horizon for which the interest rate path is defined.

Combining the assumed interest rate path with equations (1) and (2) yields the system of difference equations:

$$\begin{bmatrix} \pi_t \\ y_t \end{bmatrix} = \mathbf{A}_0 \begin{bmatrix} E_t\{\pi_{t+1}\} \\ E_t\{y_{t+1}\} \end{bmatrix} + \mathbf{B}_0 \begin{bmatrix} u_t \\ i_t^* \end{bmatrix} \quad (4)$$

where

$$\mathbf{A}_0 \equiv \begin{bmatrix} \beta + \frac{\kappa}{\sigma} & \kappa \\ \frac{1}{\sigma} & 1 \end{bmatrix} \quad ; \quad \mathbf{B}_0 \equiv \begin{bmatrix} 1 & -\frac{\kappa}{\sigma} \\ 0 & -\frac{1}{\sigma} \end{bmatrix}$$

Letting  $\mathbf{x}_t = [\pi_t, y_t]'$  and  $\mathbf{z}_t \equiv [u_t, i_t^*]'$  we can rewrite the previous dynamical system in a more compact way as:

$$\mathbf{x}_t = \mathbf{A}_0 E_t\{\mathbf{x}_{t+1}\} + \mathbf{B}_0 \mathbf{z}_t \quad (5)$$

Iterating forward on (5) we obtain the following expression for  $k$ -horizon projections as of time  $t$ :

$$E_t\{\mathbf{x}_{t+k}\} = \mathbf{A}_0^{-k} (\mathbf{x}_t - \mathbf{f}_t) \quad (6)$$

where  $\mathbf{f}_t \equiv \mathbf{B}_0 \mathbf{z}_t + \mathbf{A}_0 \mathbf{B}_0 E_t\{\mathbf{z}_{t+1}\} + \dots + \mathbf{A}_0^{k-1} \mathbf{B}_0 E_t\{\mathbf{z}_{t+k-1}\}$ . Thus, in order for the projections at any  $k$ -horizon to be well defined, the current values of  $\mathbf{x}_t$  (and, thus, of inflation and the output gap) must be determined uniquely, which in turn requires that the solution to (5) exist and be unique. A necessary and sufficient condition for that to be the case is that the two eigenvalues of  $\mathbf{A}_0$  lie within the unit circle, given that  $\mathbf{x}_t$  consists of two non-predetermined variables.<sup>9</sup> But as is well known, and formally re-stated in the following lemma, that condition is not satisfied under the exogenous interest rate path regime assumed here.<sup>10</sup>

**Lemma.** Let  $\lambda_1$  and  $\lambda_2$  denote the eigenvalues of  $\mathbf{A}_0$ , where  $\lambda_1 \leq \lambda_2$ . Both  $\lambda_1$  and  $\lambda_2$  are real and satisfy  $0 < \lambda_1 < 1 < \lambda_2$ .

*Proof:* see appendix.

The multiplicity of non-explosive solutions to (5) associated with the exogenous interest rate path implies that the equilibrium value for  $\mathbf{x}_t$  is not uniquely determined. In other words there exist a continuum of values for  $\mathbf{x}_t$  that are consistent with a rational expectations equilibrium. It follows from (6), that there is also a continuum of  $k$ -horizon projections for the output gap and inflation,  $E_t\{y_{t+k}\}$  and  $E_t\{\pi_{t+k}\}$ , consistent with the assumed interest rate path.

Next I describe three alternative approaches that allow one to construct inflation and output gap projections that are not subject to "the curse of indeterminacy" described above. A feature common to the three approaches is their reliance on some rule that turns the interest rate into an endogenous

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<sup>9</sup>See, e.g. Blanchard and Kahn (1980). Throughout we restrict the analysis to equilibria that remain in a neighborhood of the zero inflation steady state.

<sup>10</sup>The argument involves a straightforward variation of the analysis in Bullard and Mitra (2002). See also chapter 4 in Galí (2008a).

variable, while guaranteeing that the desired interest rate path  $\{i_t^*\}$  is realized *in equilibrium*. In all cases it is assumed that the interest rate along that path converges to its value consistent with a zero inflation steady state, at least in expectation. Given our normalization this implies that  $\lim_{j \rightarrow \infty} E_t\{i_{t+j}^*\} = 0$ .

## 4 Macroeconomic Projections without the Curse of Indeterminacy

### 4.1 Interest Rate Rule I

The first approach proposed in order to generate unique, well defined projections assumes that the central bank adopts the interest rate rule

$$i_t = \phi \pi_t + v_t \quad (7)$$

where  $\phi > 1$  and  $v_t$  is perceived to be an exogenous *i.i.d.* monetary policy shock. Notice that the latter shock is appended to the "usual" rule (3), in order to capture the deviations ("modest interventions," using Leeper and Zha's terminology) required in order to keep the nominal interest rate on the desired path  $\{i_t^*\}$ .<sup>11</sup>

It is easy to check that there exists a unique stationary solution to the system made up of (1), (2), and (7), given the assumption of an inflation coefficient greater than one.<sup>12</sup> The form of that solution can be guessed to be:

$$\begin{aligned} y_t &= a u_t + c v_t \\ \pi_t &= b u_t + d v_t \end{aligned}$$

Using the method of undetermined coefficients it is straightforward to determine the values of the four coefficients, which are given by:

$$\begin{aligned} a &= -(\phi - \rho_u)\Lambda_u \\ b &= \sigma(1 - \rho_u)\Lambda_u \\ c &= -\frac{1 - \beta\rho_u}{\sigma + \kappa\phi} \\ d &= -\frac{\kappa}{\sigma + \kappa\phi} \end{aligned}$$

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<sup>11</sup>See also the application in Smets and Wouters (2005).

<sup>12</sup>See, e.g., Bullard and Mitra (2002).

where  $\Lambda_u \equiv \frac{1}{\sigma(1-\rho_u)(1-\beta\rho_u)+\kappa(\phi-\rho_u)}$ .

Combining (1) with the above solution we obtain the following expression for  $t+k$  output gap and inflation

$$\begin{aligned} y_{t+k} &= E_t\{y_{t+k+1}\} + \frac{1}{\sigma} E_t\{\pi_{t+k+1}\} - \frac{1}{\sigma} i_{t+k}^* \\ &= -(\phi-1)\Lambda_u\rho_u u_{t+k} - \frac{1}{\sigma} i_{t+k}^* \end{aligned} \quad (8)$$

and

$$\begin{aligned} \pi_{t+k} &= \beta E_t\{\pi_{t+k+1}\} + \kappa y_{t+k} + u_{t+k} \\ &= (\beta b\rho_u - \kappa(\phi-1)\Lambda_u\rho_u + 1) u_{t+k} - \frac{\kappa}{\sigma} i_{t+k}^* \\ &= (1-\rho_u)(\sigma + \kappa\phi)\Lambda_u u_{t+k} - \frac{\kappa}{\sigma} i_{t+k}^* \end{aligned} \quad (9)$$

Thus, the central bank's  $k$ -horizon forecasts are given by

$$\begin{aligned} E_t\{y_{t+k}\} &= -(\phi-1)\Lambda_u\rho_u^{k+1} u_t - \frac{1}{\sigma} i_{t+k}^* \\ E_t\{\pi_{t+k}\} &= (1-\rho_u)(\sigma + \kappa\phi)\Lambda_u\rho_u^k u_t - \frac{\kappa}{\sigma} i_{t+k}^* \end{aligned}$$

Note that, ex-post, the central bank imposes a sequence of realizations for  $v_t$  that guarantee that the desired interest rate path  $\{i_t^*\}$  is attained. Formally, this requires

$$v_t = i_t^* - \phi \pi_t$$

for all  $t$ . Combining the previous condition with the expression for equilibrium inflation above we obtain an expression for  $v_t$  in closed form

$$\begin{aligned} v_t &= \left(1 + \frac{\kappa\phi}{\sigma}\right) i_t^* - \phi(\sigma + \kappa\phi)(1-\rho_u)\Lambda_u u_t \\ &= \left(1 + \frac{\kappa\phi}{\sigma}\right) (i_t^* - i_t^r) \end{aligned}$$

where  $i_t^r \equiv \phi\sigma(1-\rho_u)\Lambda_u u_t$  is the interest rate that would prevail under the baseline or "usual" rule. Thus, it is clear that in general  $\{v_t\}$  will not satisfy the *i.i.d.* assumption ex-post, thus violating the rationality of expectations.

This should be recognized by agents if the "intervention" were to last long enough.<sup>13</sup>

The second and third methods described below are not subject to the previous shortcoming, being fully consistent with the assumption of rational expectations. Before we turn to them I take a brief detour to describe an equivalence result.

#### 4.1.1 Modest Interventions vs. Switching Rules: An Equivalence Result

If the interest rate path on which projections must be conditioned is assumed to revert back after the forecast horizon to a level determined by the usual rule, the problem of indeterminacy can be shown to go away, allowing the central bank to construct well defined conditional projections.

To see this assume that the central bank sets the interest rate according to the rule

$$i_{t+j} = i_{t+j}^*$$

for  $j = 0, 1, 2, \dots, k$  and

$$i_{t+j} = \phi \pi_{t+j}$$

for  $j = k + 1, k + 2, \dots$ , i.e. it sets the interest rate at the level determined by the given exogenous path up to the desired forecast horizon, and switches to its regular interest rate rule after that.<sup>14</sup> Such an approach is used by Laséen, Lindé and Svensson (2008), as a way to simulate arbitrary time-varying interest rate rules in the context of the Riksbank's estimated DSGE model (Ramses).

Note that from time  $t + k + 1$  onward the equilibrium dynamics are described by (1), (2), and (3), which are associated with a unique equilibrium. We can solve for that equilibrium using the method of undetermined coefficients, after guessing that both the output gap and inflation will be proportional to the cost-push shock. This yields the following expressions

$$y_{t+k+1} = a u_{t+k+1} \quad ; \quad \pi_{t+k+1} = b u_{t+k+1}$$

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<sup>13</sup>This is acknowledged by Leeper and Zha (2003) who argue that the deviations from the rule should be small enough not to induce a change in agents' expectations about the regime in place.

<sup>14</sup>In that sense, the switching rule approach is more restrictive than the modest interventions approach (or the second and third approaches described below), since it cannot generate finite horizon projections conditional on an arbitrary infinite horizon interest rate path.

where  $a$  and  $b$  are given by the same expressions as above.

Combining the previous result with (1), (2) and the fact that  $i_{t+k} = i_{t+k}^*$ , we can write the equilibrium conditions corresponding to period  $t+k$  as

$$\begin{aligned} y_{t+k} &= E_t\{y_{t+k+1}\} + \frac{1}{\sigma} E_t\{\pi_{t+k+1}\} - \frac{1}{\sigma} i_{t+k}^* \\ &= -(\phi - 1)\rho_u \Lambda_u u_{t+k} - \frac{1}{\sigma} i_{t+k}^* \end{aligned}$$

$$\begin{aligned} \pi_{t+k} &= \beta E_t\{\pi_{t+k+1}\} + \kappa y_{t+k} + u_{t+k} \\ &= (1 - \rho_u)(\sigma + \kappa\phi)\Lambda_u u_{t+k} - \frac{\kappa}{\sigma} i_{t+k}^* \end{aligned}$$

which are expressions identical to (8) and (9) above. The corresponding projections as of time  $t$  are thus uniquely determined and correspond to those generated by the "modest interventions" method.<sup>15</sup>

## 4.2 Interest Rate Rule II

Consider next an interest rate rule of the form

$$i_t = i_t^* - \gamma i_{t-1}^* + \gamma(\pi_t + \sigma \Delta y_t) \quad (10)$$

where  $\gamma$  is a constant coefficient satisfying  $\gamma > 1$ .<sup>16</sup> Combining (10) with (1) we obtain the difference equation:

$$i_t - i_t^* = \frac{1}{\gamma} E_t\{i_{t+1} - i_{t+1}^*\} \quad (11)$$

Note that under the assumption that  $\gamma > 1$  the only non-explosive solution to (11) is  $i_t = i_t^*$  for all  $t$ . In other words, by following rule (10) the central bank can support any desired interest rate path  $\{i_t^*\}$ .

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<sup>15</sup>One should note, however, that the previous equivalence result hinges critically on  $E_t\{y_{t+k+1}\}$  and  $E_t\{\pi_{t+k+1}\}$  corresponding in both cases to the expected value conditional on the baseline rule, and hence on being history-independent. The previous observation suggests that the equivalence result presented here is unlikely to carry over to a more general setting with endogenous state variables.

<sup>16</sup>The present rule generalizes the one considered in Galí (2003, 2008b) to the case of an arbitrary interest rate path.

The equilibrium dynamics under rule (10) are described by (2) and

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma} (i_t^* - \gamma i_{t-1}^* + \gamma (\pi_t + \sigma \Delta y_t) - E_t\{\pi_{t+1}\}) \quad (12)$$

where the latter equation can be obtained by using (10) to eliminate the interest rate in (1). Equivalently, and more compactly, we can write (2) and (12) as:

$$\begin{bmatrix} \pi_t \\ y_t \\ y_{t-1} \end{bmatrix} = \mathbf{A}_1 \begin{bmatrix} E_t\{\pi_{t+1}\} \\ E_t\{y_{t+1}\} \\ y_t \end{bmatrix} + \mathbf{B}_1 \begin{bmatrix} u_t \\ i_t^* - \gamma i_{t-1}^* \end{bmatrix} \quad (13)$$

where

$$\mathbf{A}_1 \equiv \begin{bmatrix} \beta & 0 & \kappa \\ 0 & 0 & 1 \\ \frac{\beta\gamma-1}{\sigma\gamma} & -\frac{1}{\gamma} & 1 + \frac{1}{\gamma} + \frac{\kappa}{\sigma} \end{bmatrix} \quad ; \quad \mathbf{B}_1 \equiv \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ \frac{1}{\sigma} & \frac{1}{\sigma\gamma} \end{bmatrix}$$

Note that the system (13) involves one predetermined and two non-predetermined variables. Thus, it has a unique non-explosive solution if and only if two eigenvalues of  $\mathbf{A}_1$  lie inside, and one outside, the unit circle. The following proposition establishes a necessary and sufficient condition for that property to obtain.

**Proposition 1.** A necessary and sufficient condition for (13) to have a unique non-explosive solution is given by  $\gamma > 1$

*Proof.* see appendix.

More generally, one can show that an interest rate rule of the form

$$i_t = \gamma \pi_t + \gamma \Delta y_t + v_t$$

generates a unique equilibrium, where  $\gamma > 1$  and  $\{v_t\}$  is an arbitrary exogenous process. By setting  $\gamma \equiv \sigma\gamma$  and  $v_t = i_t^* - \gamma i_{t-1}^*$  we guarantee that such a unique equilibrium is associated with the desired interest rate path  $\{i_t^*\}$ .

Letting  $\mathbf{x}_t = [\pi_t, y_t, y_{t-1}]'$  and  $\mathbf{z}_t \equiv [u_t, i_t^* - \gamma i_{t-1}^*]'$ , we can compute projections  $E_t\{\pi_{t+k}\}$  and  $E_t\{y_{t+k}\}$  using

$$E_t\{\mathbf{x}_{t+k}\} = \mathbf{A}_1^{-k} (\mathbf{x}_t - \mathbf{g}_t)$$

where  $\mathbf{g}_t \equiv \mathbf{B}_1 \mathbf{z}_t + \mathbf{A}_1 \mathbf{B}_1 E_t\{\mathbf{z}_{t+1}\} + \dots + \mathbf{A}_1^{k-1} \mathbf{B}_1 E_t\{\mathbf{z}_{t+k-1}\}$ , and where  $\mathbf{x}_t$  is the unique solution to (13), which can be obtained using standard formulae (see, e.g. Blanchard and Kahn (1980)).

It should be noted here that the role of parameter  $\gamma$  is restricted to guaranteeing that the target path for the interest rate is attained, which is the case for *any* value of  $\gamma$  larger than one. The particular choice of  $\gamma$  has no influence, however, on the resulting equilibrium path of inflation and the output gap and, hence, on the corresponding projections. To see this, note that under rule (10) the linear combination  $\pi_t + \sigma \Delta y_t$  will be equal to  $i_{t-1}^*$ , for all  $t$ . The latter condition can in turn be combined with inflation equation (2) to obtain a difference equation for the output gap, which can be solved independently of  $\gamma$ .

### 4.3 Interest Rate Rule III

Consider finally an interest rate rule of the form

$$i_t = i_t^* - \gamma i_{t-1}^* + \gamma (\pi_t + r_{t-1}) \quad (14)$$

where  $r_t \equiv i_t - E_t\{\pi_{t+1}\}$  is the (ex-ante) real interest rate and  $\gamma > 1$ . Combining (14) with the definition of the real interest rate, yields again a difference equation of the form

$$i_t - i_t^* = \frac{1}{\gamma} E_t\{i_{t+1} - i_{t+1}^*\}$$

whose only stationary solution is  $i_t = i_t^*$  for all  $t$  given our assumption that  $\gamma > 1$ .

The equilibrium dynamics under rule (14) are described by three equations: inflation equation (2), the dynamic IS equation (1) and the interest rate rule (14), with the latter two rewritten in terms of the real interest rate, that is,

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma} r_t \quad (15)$$

$$r_t + E_t\{\pi_{t+1}\} = i_t^* - \gamma i_{t-1}^* + \gamma (\pi_t + r_{t-1}) \quad (16)$$

The previous equilibrium conditions can be written in compact form as:

$$\begin{bmatrix} \pi_t \\ y_t \\ r_{t-1} \end{bmatrix} = \mathbf{A}_2 \begin{bmatrix} E_t\{\pi_{t+1}\} \\ E_t\{y_{t+1}\} \\ r_t \end{bmatrix} + \mathbf{B}_2 \begin{bmatrix} u_t \\ i_t^* - \gamma i_{t-1}^* \end{bmatrix} \quad (17)$$

$$\mathbf{A}_2 \equiv \begin{bmatrix} \beta & \kappa & -\frac{\kappa}{\sigma} \\ 0 & 1 & -\frac{1}{\sigma} \\ \frac{1}{\gamma} - \beta & -\kappa & \frac{1}{\gamma} + \frac{\kappa}{\sigma} \end{bmatrix} \quad ; \quad \mathbf{B}_2 \equiv \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ -1 & -\frac{1}{\gamma} \end{bmatrix}$$

Once again, the system of difference equations (17) involves two non-predetermined and one predetermined variables. Thus, it has a unique non-explosive solution if and only if two eigenvalues of  $\mathbf{A}_2$  lie inside, and one outside, the unit circle. The following proposition establishes that the condition  $\gamma > 1$  is both necessary and sufficient condition for uniqueness to obtain.

**Proposition 2.** A necessary and sufficient condition for (17) to have a unique non-explosive solution is given by  $\gamma > 1$

*Proof:* see appendix.

Letting now  $\mathbf{x}_t = [\pi_t, y_t, r_{t-1}]'$  and  $\mathbf{z}_t \equiv [u_t, i_t^* - \gamma i_{t-1}^*]'$ , we can compute conditional forecasts  $E_t\{\pi_{t+k}\}$  and  $E_t\{y_{t+k}\}$  using

$$E_t\{\mathbf{x}_{t+k}\} = \mathbf{A}_2^{-k} (\mathbf{x}_t - \mathbf{g}_t)$$

where  $\mathbf{g}_t \equiv \mathbf{B}_2 \mathbf{z}_t + \mathbf{A}_2 \mathbf{B}_2 E_t\{\mathbf{z}_{t+1}\} + \dots + \mathbf{A}_2^{k-1} \mathbf{B}_2 E_t\{\mathbf{z}_{t+k-1}\}$ , and where  $\mathbf{x}_t$  is the unique solution to (17).

Note finally that under rule (14) the linear combination  $\pi_t + r_{t-1}$  is (ex-post) equal to  $i_{t-1}^*$ , for all  $t$ . That condition can be combined with (2) and (15) to obtain a system of three difference equations with three endogenous variables ( $\pi_t$ ,  $y_t$ , and  $r_t$ ), which is independent of  $\gamma$ . Hence, and as it was the case for interest rate rule II, the specific choice of  $\gamma$  has no influence on the resulting projections.

## 5 Multiple Determinate Projections: An Example with a Constant Interest Rate Path

The previous section has described three alternative approaches to the construction of projections conditional on a given path for the nominal interest rate which are not subject to the problem of indeterminacy. Such a multiplicity of approaches raises a natural question: Are the inflation and output gap projections generated by the different approaches identical, if they are

conditioned on the same interest rate path? The answer to that question is negative, as the exercise described next makes clear.

Let us consider the case of a central bank which, as of time  $t$ , wants to produce conditional forecasts of inflation and the output gap for period  $t+k$ . As above, the non-policy block of the economy is described by equations (1) and (2). For simplicity, we assume that the economy was at its steady state position in period  $t-1$ , i.e.  $y_{t-1} = \pi_{t-1} = i_{t-1} = 0$ . A cost-push shock of unit size is assumed to hit the economy in period  $t$ , vanishing over time in proportion to  $\rho_u^k$ , for  $k = 1, 2, 3, \dots$

What are the model-based projections for inflation and the output gap in period  $t+k$  conditional on the central bank keeping the interest rate unchanged? Next I compute those projections under each of the three approaches discussed above for a calibrated version of the baseline model. For the purposes of this exercise, I assume the following parameter values, which are similar to those often used in the literature:  $\beta = 0.99$ ,  $\sigma = 1$ ,  $\kappa = 0.1$ ,  $\phi = 1.5$  and  $\rho_u = 0.5$ .<sup>17</sup> It should be clear that the main finding of this section is a qualitative one, and does not hinge on the details of the calibration.

Figures 1 and 2 display the inflation and output gap projections at horizons up to 12 quarters associated with the three rules described in the previous section. The figures also display the (unconditional) forecast associated with the "true" rule (3). Note that, even though the three rules support an unchanged interest rate through the forecast horizon (and beyond, in the case of rules II and III), their associated projections for inflation and output are very different. The differences among them involve both the size of the projected changes in inflation and the output gap as a result of the assumed cost-push shock, but also in the patterns of those responses and, in one case, even the sign of those responses. We also see that none of them tracks the unconditional forecast well, but this was to be expected since the latter implies a different interest rate path. Interestingly, the constant interest rate projections differ even in terms of the sign of their deviation from the unconditional forecasts.

Why do the three rules considered above generate different projections despite being associated with an identical nominal interest rate path? Put simply, the intuition behind that result is that any given nominal interest rate path is consistent with different paths of the real rate and expected inflation.

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<sup>17</sup>As argued above, the specific choice of  $\gamma$  has no influence on the projections, as long as it is larger than one, which we assume here.

In the New Keynesian model, the presence of sticky prices makes it possible for the central bank to influence real variables, including the real interest rate. The three rules considered achieve the same nominal rate path through different combinations of real rates and expected inflation. Not surprisingly, each of those combinations is associated with different paths for the output gap and, as a result, for inflation as well. This leads, in turn, to different projections.

The previous finding clearly calls into question the usefulness of projections conditional on a given interest rate path, since there is no obvious reason to prefer one method over another in order to generate the same path. Put differently, the information required in order to compute well defined macroeconomic projections goes beyond the specification of the interest rate path and the horizon of the projections. A complete description would need to include the nature and specification of the *policy rule* that will be followed in order to support the desired interest rate path.

## 6 Concluding Remarks

There appears to be a growing tendency among central banks to construct and report macroeconomic projections consistent with their own views about the future evolution of nominal interest rates. Many economists have welcome that development on different grounds, including its likely benefits from the viewpoint of transparency and ease of communication of the monetary authority's decisions and overall strategy to the public.

In the present paper I have also tried to make a case (at least implicitly) for the adoption of projections based on the central bank's own interest rate forecasts. But I have done so by arguing that the alternative, i.e. conditioning projections on an exogenously given interest rate path, rests on shaky theoretical grounds. The latter assessment does not follow from the often heard argument that such projections will typically be indeterminate in forward-looking models. On the contrary, I have described as many as three different approaches to construct determinate projections conditional on an arbitrary interest rate path. Instead, I have argued that the main shortcoming of those projections lies precisely in the multiplicity of methods (each associated with a different rule) that are available to generate them, together with the fact that the different methods generally yield divergent projections for variables other than the interest rate itself. That observation

calls into question the usefulness of projections conditional on a given interest rate path since, in principle, there is no obvious reason to prefer one method over another in order to generate that path.

## Appendix

### *Proof of Lemma*

From the properties of matrices and their eigenvalues, we have  $tr(\mathbf{A}_0) = 1 + \beta + \frac{\kappa}{\sigma} = \lambda_1 + \lambda_2$  and  $\det(\mathbf{A}_0) = \beta = \lambda_1\lambda_2$ . Thus we have  $\lambda_1 = \frac{1+\beta+\frac{\kappa}{\sigma}-\sqrt{(1+\beta+\frac{\kappa}{\sigma})^2-4\beta}}{2}$  and  $\lambda_2 = \frac{1+\beta+\frac{\kappa}{\sigma}+\sqrt{(1+\beta+\frac{\kappa}{\sigma})^2-4\beta}}{2}$ . Note that

$$\begin{aligned}\Delta &\equiv \left(1 + \beta + \frac{\kappa}{\sigma}\right)^2 - 4\beta \\ &> (1 + \beta)^2 - 4\beta \\ &= (1 - \beta)^2\end{aligned}\tag{18}$$

from which it follows that both eigenvalues are real. Also, (18) implies  $\lambda_2 > \frac{1+\beta+\frac{\kappa}{\sigma}+\sqrt{(1-\beta)^2}}{2} > 1$ .

Note also that  $\sqrt{(1 + \beta + \frac{\kappa}{\sigma})^2 - 4\beta} < 1 + \beta + \frac{\kappa}{\sigma}$ , from which we have  $\lambda_1 > 0$ . Finally,  $\lambda_1 < 1$  follows from the fact that  $\lim_{\frac{\kappa}{\sigma} \rightarrow 0} \lambda_1 = \beta < 1$ , and  $\frac{\partial \lambda_1}{\partial (\kappa/\sigma)} < 0$ . *QED*.

### *Proof of Proposition 1*

The characteristic polynomial of  $\mathbf{A}_1$  is given by

$$\begin{aligned}p_{A_1}(z) &= z^3 - \left(1 + \beta + \frac{\kappa}{\sigma} + \frac{1}{\gamma}\right) z^2 + \left(\frac{1}{\gamma} \left(1 + \beta + \frac{\kappa}{\sigma}\right) + \beta\right) z - \frac{\beta}{\gamma} \\ &= (z - \lambda_1) (z - \lambda_2) (z - \gamma^{-1})\end{aligned}$$

where  $\lambda_1 \equiv \frac{1+\beta+\frac{\kappa}{\sigma}-\sqrt{(1+\beta+\frac{\kappa}{\sigma})^2-4\beta}}{2}$  and  $\lambda_2 \equiv \frac{1+\beta+\frac{\kappa}{\sigma}+\sqrt{(1+\beta+\frac{\kappa}{\sigma})^2-4\beta}}{2}$ . Using the same logic as in the proof of the Lemma above we conclude that both  $\lambda_1$  and  $\lambda_2$  are real, and satisfy the inequality  $0 < \lambda_1 < 1 < \lambda_2$ . Thus, and as long as  $\gamma > 1$ , two eigenvalues of  $\mathbf{A}$  lie inside and one outside, the unit circle. *QED*.

### *Proof of Proposition 2.*

The characteristic polynomial of  $\mathbf{A}_2$  is given by

$$p_{A_2}(z) = z^3 - \left(1 + \beta + \frac{\kappa}{\sigma} + \frac{1}{\gamma}\right) z^2 + \left(\frac{1}{\gamma} \left(1 + \beta + \frac{\kappa}{\sigma}\right) + \beta\right) z - \frac{\beta}{\gamma}$$

which is identical to that of  $\mathbf{A}_1$ . Hence, the same condition for uniqueness of the equilibrium applies.

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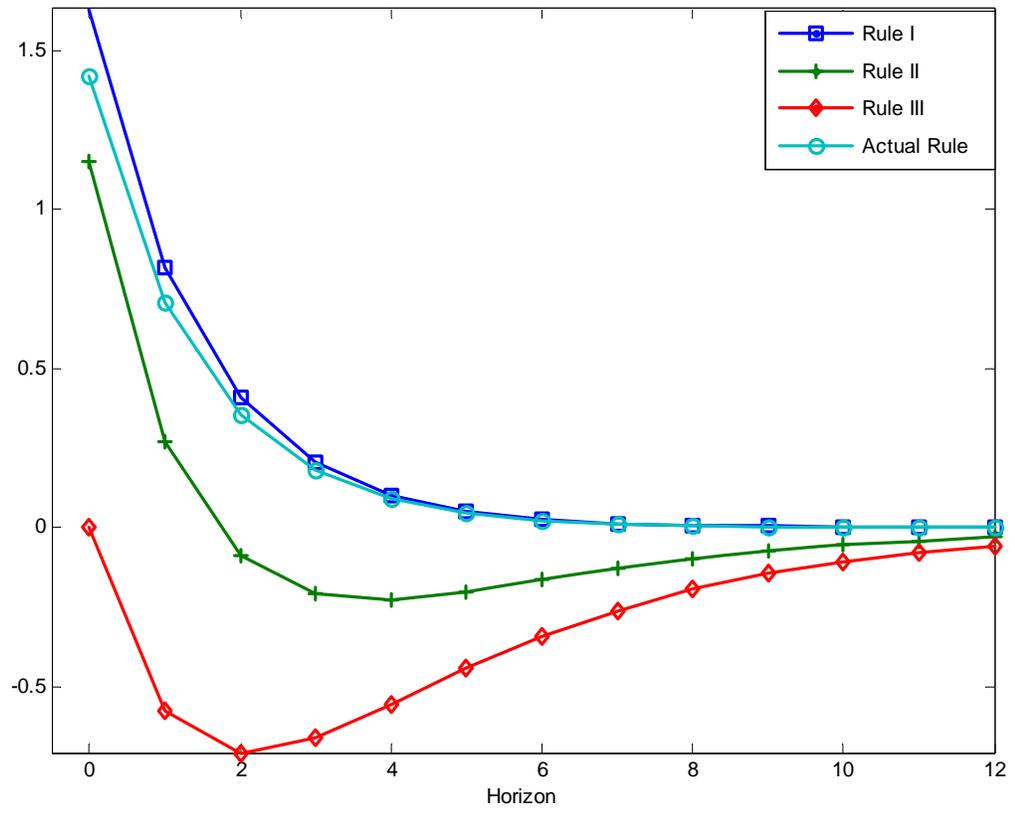
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Table 1 Central Banks' Forecasting Practices	
Reserve Bank of Australia	CIR
Central Bank of Brazil	CIR and ME
European Central Bank	ME
Bank of Japan	ME
Reserve Bank of New Zealand	CBE
Norges Bank	CBE
Monetary Authority of Singapore	Global rates
Riksbank	CBE
Swiss National Bank	CIR
Bank of England	ME
Federal Reserve System	CBE (individual FOMC members)

Note. CIR: constant interest rate; ME: market expectations ; CBE: central bank expectations. Source: Bank for International Settlements (2007) and national sources.

**Figure 1**

**Constant Interest Rate Projections: Inflation**



**Figure 2**

**Constant Interest Rate Projections: Output Gap**

