

The Term Structure of Interest Rates across Frequencies

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Abstract

This paper tests the expectations hypothesis (EH) of the term structure of interest rates in US data, using spectral regression techniques that allow us to consider different frequency bands. We find a positive relation between the term spread and the change in the long-term interest rate in a frequency band of 6 months to 4 years, whereas the relation is negative at higher and lower frequencies. We confirm that the variance of risk premia relative to expected changes in long-term interest rates varies across frequency bands, leading the EH to be rejected in some bands but not in others.

Keywords: Expectations theory of the term structure, interest rates, spectral regression, frequency domain.

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1. Introduction

The term structure of interest rates, or the yield curve, is a central element in much of modern monetary and financial economics. It is also an important concept for financial institutions. For instance, a whole range of financial assets is priced off the yield curve for government securities. The term structure also plays a central role in monetary policy making. Thus, the spread between long nominal and real yields is used by many central banks to gauge inflation expectations and the credibility of the monetary policy regime; the slope of the term structure is used to assess the probability of recessions; and the entire yield curve is used to assess market expectations about the future course of monetary policy.

Given its central role in both theory and practice, it is unfortunate that the main body of theory developed to understand the term structure – the expectations hypothesis (EH), which holds that long interest rates are determined by the expected future path of short-term interest rates plus a constant, but potentially maturity-dependent, term premium – has been resoundingly rejected by large number of studies using data from different countries, time periods and maturity segments. Despite this, the theory continues to be used to interpret interest-rate movements by analysts in the academic, financial and central-banking sectors alike. One reason for that may be that while time-varying risk premia appear to be present and lead to a statistical rejection of the theory, these are small enough for the EH to be seen as a useful tool for understanding movements in the term structure (Campbell and Shiller 1987, 1991).

Mankiw and Miron (1986) argue that in the presence of time-varying term premia, the empirical performance of the EH will depend on how much they vary over time in comparison to expectations of future changes in short-term interest rates. Thus, the EH will fare poorly in episodes in which the central bank conducts policy in such a way as to make it difficult for financial market participants to forecast future interest rates, but will be more useful for understanding the behaviour of longer interest rates when future short-term rates are more predictable. Mankiw and Miron (1986) study

data around the establishment of Federal Reserve System, which led to a pronounced dampening of the highly predictable seasonal component of interest rates, and find support for this hypothesis.

Gerlach and Smets (1997) provide more evidence about how the predictability of short-term rates affects the results from tests of the EH. Using data from 17 economies on short-term euro-currency interest rates, they find that the EH is not rejected in those economies in which one-month interest rates are relatively easily predictable. Gerlach and Smets (1998), in a closely related study, note that the EH tends not to be rejected in economies with fixed exchange rates, essentially because occasional episodes of exchange-market pressures have led to spikes in short-term interest rates. Since these increases were temporary, market participants expected them to be undone over time, which led to considerable predictability of interest rates that dominated any variability of the term premium. In economies with floating exchange rates, by contrast, future changes in short-term interest rates were more difficult to predict, so that movements in the term-structure reflected largely variations in term-premia. As a consequence, the EH was rejected.

In this paper we use US data to explore the related hypothesis that the predictability of future changes in short-term interest rates varies across frequency bands as a consequence of the conduct of monetary policy. At high frequencies corresponding to periodicities of less than six months, the variance of changes in expected future interest rates may be small, so that movements in the term structure are dominated by term premia and the EH is rejected. By contrast, in the intermediate, or business cycle, frequency band corresponding to periodicities of between six months and four years, short-term rates may contain a relatively large predictable component as a consequence of the conduct of monetary policy. If so, it may be difficult to reject the EH although the presence of a time-varying risk premium renders it, strictly speaking, false. Finally, at low frequencies changes in short-term rates may again be difficult to forecast, for instance because there is in fact little variation in the expected

future path of short-term interest rates in this frequency band. If, however, risk premia do exhibit variations at long frequencies, the EH will again be rejected.

The paper is structured as follows. In the next section we provide a review of some of the relevant literature testing the expectations hypothesis. One method to explore the empirical validity of the EH is to regress, essentially, the realised future path of the short-term interest rate on the term spread and test whether the coefficient on the term spread is unity. This approach is only feasible when testing the short end of the term structure, that is, when the maturity of the long rate is 3, 6 or 12 months. Interestingly, tests using this approach frequently do not reject the EH.

The alternative approach is to regress the change in long yields between t and $t+1$ on the current spread. While theory suggests that, suitably normalised, the slope parameter should be unity, empirically it is generally estimated to be negative and significant. Since this test of the EH focuses on the behaviour of the long end of the yield curve, which may be relatively severely affected by the term premium, it is particularly interesting to focus on this implication of the EH when applying frequency domain techniques.

2. Brief review of the literature

[To be completed later.]

3. Empirical work

The EH has a number of implications for the joint behaviour of interest rates of different maturities. Testing typically exploits the fact that the spread between a long and a short interest rate should predict future changes in interest rates. Denoting the yield of a N -period coupon-paying bond by R_t^N and the one-period interest rate by r_t , Hardouvelis (1994) obtains the following expression for the change in the long rate:

$$(1) \quad R_{t+1}^{N-1} - R_t^N = (D_N - 1)^{-1} \{ (R_t^N - r_t) - E_t \phi_{N,t} + \varepsilon_{t+1} \}$$

where D_N denotes the duration of a N -period bond (which is given by $D_N \approx (1 - g^N)/(1 - g)$, $g = (1 + \text{Rbar})^{-1}$), Rbar denotes the sample mean of R_t^N , $\phi_{N,t}$ is a risk premium and ε_{t+1} is a set of revisions of agents' expectations of future short rates and risk premia (and is thus uncorrelated over time). Hardouvelis considers the regression:

$$(2) \quad R_{t+1}^{N-1} - R_t^N = \alpha + \beta(D_N - 1)^{-1}(R_t^N - r_t) + u_{t+1}$$

and shows that the estimator, $\hat{\beta}$, can be written as

$$(3) \quad \hat{\beta} = \frac{\text{Var}(E_t(R_{t+1}^{N-1} - R_t^N)) + \rho \text{Var}(E_t(R_{t+1}^{N-1} - R_t^N))^{1/2} \text{Var}(\phi_{N,t})^{1/2}}{\text{Var}(E_t(R_{t+1}^{N-1} - R_t^N)) + \text{Var}(\phi_{N,t}) + 2\rho \text{Var}(E_t(R_{t+1}^{N-1} - R_t^N))^{1/2} \text{Var}(\phi_{N,t})^{1/2}}$$

(see Gerlach and Smets, 1997). From this it follows that:

$$(4) \quad \text{plim}\beta = 1 - \frac{q(\rho + q)}{1 + 2q\rho + q^2}$$

where $q = (\text{Var}(E_t \phi_{N,t}) / \text{Var}(E_t(R_{t+1}^{N-1} - R_t^N)))^{1/2}$ and $\rho = \text{Corr}(E_t \phi_{N,t}, E_t(R_{t+1}^{N-1} - R_t^N))$.

Thus, if the term premium is constant over time, as is assumed by the EH, β should be estimated to be unity. However, equation (4) indicates that in the general case in which the term premium varies over time (so that the EH is false), $\text{plim } \beta$ can take any value as is clear from Figure 1 (which is copied from Hardouvelis, 1994, p. 271). Of course, depending on the precision of the estimates, the hypothesis that $\beta = 0$ may not be rejected.

In this paper we test the EH at the long end of the term structure by estimating equation (2). In contrast to the existing literature, we perform the estimation in different frequency bands, using the band spectral regression (BSR) procedure of Engle (1974). The reason for doing so is straightforward. Following Hardouvelis (1994, p. 274), we can obtain an expression equivalent to equation (1) for the spread between long and short interest rates as function of the future path of short rates (relative to the current short rate) and a term premium:

$$(5) \quad \left(\sum_{i=0}^{N-1} w_i E_t r_{t+i} - r_t \right) + E_t \theta_t \equiv R_t^N - r_t$$

where $w_i = g^i / D_N$ and $E_t \theta_t = \sum_{i=0}^{N-1} w_i E_t \phi_{N-1,t+i}$.

The rationale for testing the EH using BSR arises from the fact that the relative importance of the two terms on the left hand side of equation (5) (or, equivalently, equation (1)) may vary across frequency bands. To see this, suppose that central banks do not react to very short-term or high-frequency phenomena – perhaps because the lags by which monetary policy affects the economy are too long for policy to be able to stabilise the economy – but only to shocks that have more persistent economic effects. If so, interest rate expectations would contribute little to movements in the term structure at high frequencies, leaving term premia to be the dominant driving factor and the EH to be rejected. At somewhat lower frequencies in which interest rate expectations vary more as monetary policy reacts to shifts in economic conditions, it is possible that the EH fits the data better. At a still lower frequency band, variation in the interest rate expectations may again be weaker, leaving the EH to be rejected.

4. The Data

We use monthly interest rate data for the US, expressed in percentages, and consider three different maturities for the short-term interest rate, the 1-month London interbank bid (LIBID) rate for US dollars, the 3-month and the 6-month US treasury-bill rates. For the long-term interest rates we use US government bond rates for six different maturities of 1, 2, 3, 5, 7 and 10 years. Interest rates are monthly averages and, except for the 1-month LIBID rate that is from Bloombergs, are obtained from the website of the Federal Reserve Bank of St. Louis. Following Hardouvelis (1994), the short rates are transformed to bond equivalent yields, r , from discount yields, y , according to the formula $r = (365y/100)/(360 - dy/100)$, where d denotes days to

maturity, which are 30 for the 1-month rate, 91 for the 3-month rate and 182 for the 6 month rate.

Since we do not want to deal with the disinflation period in the early eighties when interest rates were unusually volatile, we start our sample in September 1987, that is, at the time Alan Greenspan was appointed Chairman of the Board of Governors of the Federal Reserve System. The sample period ends in February 2007. Figure 2 shows the data. It is apparent that all interest rates share the same movements over time, though the short rates tend to be more volatile than the longer maturities. For the largest part of the sample the slope of the yield curve was positive, with some short exceptions around 2000 and at the end of the sample.

Next we consider the left-hand and the right-hand side variables in the regression equation (2). Figure 3 show changes in the long rates of six different maturities (1, 2, 3, 5, 7 and 10 years), together with the spreads between these long rates and short rates of 1, 3 or 6 months maturity. While the change in the long rates is clearly stationary, this is much less clear whether the term spreads are. In fact, term spreads involving the longest maturities show much less mean reversion and formal (unreported) tests of stationarity often reject.

This pattern carries over to the frequency domain properties of the data. Figure 4 shows the spectra for the changes in the long rate in the first panel and the three different term spreads in the other three panels. While differencing the long rate removes most of the power around the zero frequency and flattens the spectrum, the term spreads are persistent as evidenced by the high spectral density near the origin, and thus display what Granger and Hatanaka (1964) refer to as the typical spectral shape of a nonstationary time series. Figure 4 shows that the change in the one-year interest rate has the most power at the lowest frequencies. Furthermore, the spread between the 1-month and 1-year rate appears to have a greater spectral density at the origin than the other term spreads. This suggests that the EH may fare better in the case in which the short rate is the 1-month rate, and the long rate is the 1 year rate.

Importantly, Figure 4 shows that the spectral densities of the series are quite similar irrespectively of which change in the long rate, or which term spread, is considered. However, the spectral densities of the changes in the long rates do not match closely with the spectral densities of the term spreads. Moreover, the closeness of the match appears to vary between frequencies. This suggests that estimates of equation (2) will vary across frequency bands. Furthermore, they are likely to be better in the frequency band between 6 months and four years, which corresponds to frequencies between 0.33π and 0.04π , where both the spreads and changes of interest rates for at least some maturities have considerable power.¹

We next present a brief exposition of the empirical method used before we discuss the results.

5. Empirical Methods and Results

Since we expect the variability of the term premium to differ across frequency bands, or, loosely speaking, across time horizons, it seems natural to investigate the EH using spectral methods. Band spectrum regression (BSR) was first proposed by Engle (1974), who shows that if $y = x\beta + \varepsilon$ is a valid regression model in the time domain, it can be transformed into the frequency domain by applying a Fourier transformation to both the dependent and the independent variables. Denoting the transformed variables as \tilde{x} and \tilde{y} , the regression in the frequency domain is $\tilde{y} = \tilde{x}\beta + \tilde{\varepsilon}$.² The estimator, $\hat{\beta}$, then can be written as:

$$(6) \quad \hat{\beta} = \left[\sum_{k=0}^{T-1} \hat{f}_{xx}(\omega_k) \right]^{-1} \sum_{k=0}^{T-1} \hat{f}_{xy}(\omega_k),$$

¹ Measuring frequency, ω , in fractions of π , periodicity in quarters is given by $2\pi/\omega$. Thus, a frequency of $\omega=0.1\pi$ corresponds to a periodicity of 20 months.

² The transformation to the frequency domain does not affect the standard regression results.

where T is the sample size, $\hat{f}_{xx}(\omega)$ is the periodogram of the series in x at each frequency ω and $\hat{f}_{xy}(\omega)$ is a vector of cross periodograms.³ The benefit of transferring the regression model into the frequency domain is that it permits a test of the hypothesis that a specific model applies to some but not to all frequencies. In this case we premultiply the regression model by a $T \times T$ matrix A with unity on the diagonal for each included frequency and zero elsewhere,

$$(7) \quad A\tilde{y} = A\tilde{x}\beta + A\tilde{\varepsilon}, \text{ where } E(A\tilde{\varepsilon})(A\tilde{\varepsilon})^* = \sigma^2 A$$

with an asterisk, “*”, denoting the complex conjugate of the transposed matrix. Thus, to compute $\hat{\beta}$ we sum over a frequency band instead of the full range of frequencies as in equation (6).⁴ If equation (6) is estimated only for a subset of frequencies, but is true for all frequencies, the estimator is consistent but inefficient as it does not use all available information. By contrast, if the model applies only to a specific frequency band, using information from all frequencies might obscure the relationship between the variables.

5.1 Regression results

Next we estimate equation (2), focusing on the intermediate (6 to 48 month) frequency band in which we believe that the EH may be accepted by the data. Before interpreting the results, we explain how the table is constructed. The third column shows the estimate of β and, between parentheses, its standard error, while the fourth columns presents the t -statistics from tests of the hypotheses that β is significantly different from zero and from unity, respectively. If we are unable to reject that β equals unity but at the same time β is significantly different from zero, we interpret this as evidence in favour of the EH. For comparison, columns five and

³ Since the estimator of β averages over periodograms, there is no need to smooth these as is necessary when estimating the spectrum.

⁴ Though the cross-periodograms in equation (5) are complex, $\hat{\beta}$ will be real if the k^{th} frequency component is included along with the $T - k^{\text{th}}$ component.

six present β and the t -statistics from the same tests when all frequencies are used, which corresponds to estimating equation (2) using OLS.

The third column of Table 1 shows that the point estimate of β exceeds unity for all regressions. By contrast, the OLS estimates are close to unity only for the regression involving maturities of up to two years and confirm the results in the literature, in which the long rate is frequently taken to be a 10-year rate, that generally obtains a negative point estimate. Moreover, the coefficient estimates in the third column increase when longer maturities are used as the dependent variable. In only one of 18 regressions – when the short rate is 3 months and the long rate is 3 years – do we not reject (at the 5% level of significance) the hypothesis that β equals zero, though the t -value of 1.94 is marginally below the critical value. By contrast, on only two occasions do we obtain a significant slope parameter in the OLS regressions.

Furthermore, in ten cases are we unable to reject the hypothesis that β equals unity when we restrict attention to the 6-to-48-months frequency band. By contrast, we only do so on two occasions when the OLS estimates are used. In sum, the regressions are much more supportive of the EH when we focus attention on the 6-month to 4-year frequency band than when all frequencies are used.

To better understand the common rejections of the EH when all frequencies are included (that is, when OLS is used), Figure 5 shows the coefficient estimates for three different frequency bands – a low-frequency band comprising fluctuations with a periodicity of more than 4 years, the intermediate-frequency band comprising fluctuations of between 6 months and 4 years we discussed above, and a high-frequency band containing fluctuations with a periodicity of less than 6 months. (The results are also available in Table A1 in the appendix.)

Figure 5 shows the coefficients for different maturities of the long rate using the term spreads relative to the 1-month rate (defining the term spreads relative to the 3-month and 6-month rates yield very similar results and we therefore do not show them in the figure). The different behaviour of β across frequency bands is

immediately apparent. First, the coefficient is small at high frequencies and becomes increasingly negative when the dependent variable or the term spread is computed using longer maturities. This illustrates why conventional methods often find negative point estimates for β . Second, the estimate of β is positive in the intermediate band, though it generally exceeds unity. Third, in the low-frequency band β is positive for short maturities of the dependent variable such as 1 to 3 years, if a term spread relative to the 1-month and 3-month rate is used, and 1 year if the term spread is defined using the 6-month rate. This confirms that it is more likely to find evidence in favour of the EH with conventional methods when using short-term interest rates.

Engle (1974) shows that a conventional F -test can be used to test for equality of the parameters across frequency bands. Performing such a test, we find, not surprisingly, that the equality of β across frequency bands is strongly rejected for all 18 pairs of long rates and term spreads with test statistics between $F_{1,230} = 20.47$ to $F_{1,230} = 43.38$, which all far exceed the critical value at the 5% significance level of 3.89.

5.2 How important is time-varying term premium?

Our findings support the notion that term premia is the dominant factor driving movements in the term structure at high and very low frequencies, whereas interest rate expectations play a more important role in the intermediate frequency band. Next we explore this hypothesis more directly. Of course, neither term premia nor expected changes in the interest rate are observable so that we have to take an indirect approach to do so.

Our empirical approach follows Gerlach and Smets (1997, 1998) who test the EH hypothesis by regressing the realised future path of short rates on the current term spread, using data for a sample of 17 countries and three definitions of the term spread. Next they use an ancillary regression to estimate the predictability of future short rates and ask whether it can explain the observed cross-currency differences in the estimates of β . (Of course, in our case we compare across frequency bands rather

than across currencies.) Dividing the numerator and the denominator of the right-hand side in equation (2) by $\text{Var}(R_{t+1}^{N-1} - R_t^N)$, the estimated slope coefficient $\hat{\beta}$ can be written as

$$(8) \quad \hat{\beta} = \frac{R^2 + \rho R \theta}{R^2 + 2\rho R \theta + \theta^2},$$

where ρ is defined as in equation (4), R^2 denotes the ratio of the expected variance of the change in the long-term interest rate to its actual variance, $R^2 = \text{Var}(E_t(R_{t+1}^{N-1} - R_t^N)) / \text{Var}(R_{t+1}^{N-1} - R_t^N)$, and θ^2 the ratio of the variance of the term premium to the variance of changes in the long rate, $\theta^2 = \text{Var}(\phi_{N,t}) / \text{Var}(R_{t+1}^{N-1} - R_t^N)$. R^2 thus can be interpreted as a measure of the predictability of the change in the long interest rate, whereas θ^2 measures the relative importance of the term premium.

We measure the predictability of the change in the long interest rate by regressing the change in the long rate for each frequency band on a constant, the lagged short interest rate and the lagged term spread.

$$(9) \quad R_{t+1}^{N-1} - R_t^N = \gamma_0 + \gamma_1 r_t + \gamma_2 (R_t^N - r_t) + \varepsilon_t$$

We then use the R^2 from this regression, together with the estimated β coefficients from equation (4) to obtain estimates for ρ and θ . Assuming that θ and ρ are constant across maturities, we have 18 observations for $\hat{\beta}$ and R^2 for each of the three frequency bands, which we use to estimate the following regression:

$$(10) \quad \hat{\beta}_i = \frac{R_i^2 + \alpha_1 R_i}{R_i^2 + 2\alpha_1 R_i + \alpha_2} + \xi_i, \quad i = 1, 2, 3,$$

where $\alpha_1 = \rho\theta$ and $\alpha_2 = \theta^2$. The resulting coefficient estimates are shown in Table 2. All coefficients are of plausible sign and magnitude. The variance of the term premium ranges from 18 to 41 percent of the variance of the change in the long rate across frequency bands. Moreover, the relative variability of the term premium is lowest in the intermediate frequency band though the difference to the low-frequency band is marginal. At high frequencies, however, substantial variation in

the term premium seems to be present. The estimate of α_1 is negative for all frequency bands, which corresponds to results found in the literature.⁵ The correlation of the term premium and the expected change in the long rate is close to minus unity, but it decreases in the lower frequency band. Of course, these results may be subject to generated regressor bias, since both the dependent and the independent variable stem from first-step regressions, and thus have to be treated with caution.

We finally compare our estimates of β with $\text{plim } \beta$ in equation (3) using our estimates of ρ and θ . Figure 5 shows the observations from all three frequency bands.⁶ Results are broadly consistent with the theoretical relation. The symbols represent the individual estimates of β , while the solid line shows the theoretical relation. The estimates from the high frequency band are shown as solid squares. They all lie in the negative part of the graph around $q = 1$ and are much smaller than the theoretical relation. One reason for this can be that the use of monthly data biases results in the high-frequency band because the smallest cycle we can distinguish is a cycle of two months (the so-called Nyquist frequency) whereas interest rates change and expectations are formed on a daily basis. The estimates for the intermediate band are marked by a circle and scatter around a q of slightly below unity in the positive part of the graph, whereas the estimates for the low-frequency band, marked by a triangle, concentrate at large values of q .

6. Conclusions

In this paper we test the EH of the term structure of interest rates in US data, using spectral techniques that allow us to consider different frequency bands. Strikingly, our estimates show that the slope coefficient on the term spread changes sign across frequency bands. At high frequencies, defined as fluctuations with a periodicity of

⁵ See, e.g., Mankiw and Miron (1986).

⁶ We can obtain q by dividing θ by the R^2 from the forecasting regression.

less than 6 months, we find a negative relation between the term spread and the change in the long-term interest rate. In an intermediate frequency band, corresponding to a time-horizon of 6 months to 4 years, we find a positive reaction of the future change in the long rate to the term spread, whereas at low frequencies beyond 4 years the relation is positive for short maturities and negative for longer maturities. Our conjecture that the variance of the term premium relative to expected changes in the long rate varies across frequency band is corroborated by the data.

Moreover, we try to relate the findings to time-varying term premia and find some support in the data. We find that the variance of risk premia relative to expected changes in long-term interest rates varies across frequency bands, leading the EH to be rejected in some bands but not in others.

Appendix

Table A1. Estimates of equation (2) in the 2-to-6 months and 48-to-∞ months frequency band.

<i>Change in</i>	<i>Term spread</i>	β (2-to-6 month band)	<i>Standard error</i>	β (48-to-∞ month band)	<i>Standard error</i>
1-year rate	1 month	-3.991*	0.730	1.110*	0.341
1-year rate	3 months	-11.529*	1.440	4.321*	0.490
1-year rate	6 months	-13.929*	3.402	3.928*	1.059
2-year rate	1 month	-9.302*	1.581	0.502	0.426
2-year rate	3 months	-21.181*	2.040	1.233	0.705
2-year rate	6 months	-23.399*	3.537	-0.173	0.749
3-year rate	1 month	-14.022*	2.284	0.115	0.416
3-year rate	3 months	-29.077*	2.938	0.221	0.594
3-year rate	6 months	-31.194*	4.554	-0.578	0.590
5-year rate	1 month	-20.735*	3.604	-0.341	0.331
5-year rate	3 months	-42.349*	4.369	-0.509	0.415
5-year rate	6 months	-43.513*	6.079	-0.827*	0.403
7-year rate	1 month	-26.579*	4.565	-0.490	0.313
7-year rate	3 months	-50.631*	5.977	-0.696	0.378
7-year rate	6 months	-49.789*	7.961	-0.913*	0.366
10-year rate	1 month	-33.529*	5.956	-0.455	0.285
10-year rate	3 months	-62.868*	7.314	-0.643	0.335
10-year rate	6 months	-62.138*	9.508	-0.785*	0.326

Note: The dependent variable is indicated in column 1. The sample period is 1987M9 to 2007M2. Standard errors are given in parentheses. An asterisk indicates significance at the 5% level.

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Table 1. Estimates of equation (2) in the 6 to 48 month frequency band and with OLS.

<i>Change in</i>	<i>Term spread</i>	β	$Tstat (\beta= 0/1)$	βOLS	$Tstat (\beta= 0/1)$
1-year rate	1 month	1.845	3.890*	0.723	1.315
		(0.474)	1.781	(0.550)	-0.504
1-year rate	3 months	1.862	2.517*	1.846	2.262*
		(0.740)	1.165	(0.816)	1.037
1-year rate	6 months	3.689	3.341*	2.726	2.165*
		(1.104)	2.435*	(1.259)	1.371
2-year rate	1 month	2.218	2.982*	0.506	0.628
		(0.744)	1.637	(0.806)	-0.613
2-year rate	3 months	2.043	2.171*	0.925	0.952
		(0.941)	1.108	(0.917)	-0.078
2-year rate	6 months	3.227	2.812*	0.905	0.775
		(1.147)	1.941	(1.168)	-0.081
3-year rate	1 month	2.721	2.641*	0.284	0.300
		(1.031)	1.670	(0.949)	-0.754
3-year rate	3 months	2.424	1.937	0.482	0.429
		(1.251)	1.138	(1.123)	-0.461
3-year rate	6 months	3.733	2.525*	0.442	0.353
		(1.478)	1.849	(1.251)	-0.446
5-year rate	1 month	3.811	2.784*	-0.001	-0.001
		(1.369)	2.053*	(1.026)	-0.976
5-year rate	3 months	3.374	2.080*	0.004	0.004
		(1.622)	1.464	(1.179)	-0.844
5-year rate	6 months	5.091	2.783*	0.038	0.031
		(1.829)	2.236*	(1.233)	-0.780
7-year rate	1 month	5.124	3.166*	-0.087	-0.081
		(1.618)	2.548*	(1.063)	-1.022
7-year rate	3 months	4.359	2.279*	-0.144	-0.119
		(1.913)	1.756	(1.204)	-0.950
7-year rate	6 months	6.534	3.091*	-0.032	-0.026
		(2.114)	2.618*	(1.231)	-0.838
10-year rate	1 month	6.575	3.332*	-0.080	-0.072
		(1.973)	2.825*	(1.119)	-0.965
10-year rate	3 months	5.444	2.348*	-0.195	-0.155
		(2.319)	1.917	(1.257)	-0.951
10-year rate	6 months	8.023	3.166*	-0.042	-0.033
		(2.534)	2.772*	(1.270)	-0.821

Note: The dependent variable is indicated in column 1. The first entry in columns 3 and 5 tests the hypothesis $\beta = 0$, the second $\beta = 1$. The sample period is 1987M9 to 2007M2. Standard errors are given in parentheses. An asterisk indicates significance at the 5% level.

Table 2. Estimates of predictability

Coefficient	<i>Low-frequency band</i>	<i>Intermediate-frequency band</i>	<i>High-frequency band</i>
α_1	-0.636 (0.003)	-0.417 (0.010)	-0.386 (0.049)
α_2	0.405 (0.004)	0.177 (0.008)	0.183 (0.059)
Uncentered R ²	0.462	0.859	0.231
Durbin Watson	0.437	1.084	0.615
θ	0.636	0.421	0.428
ρ	-0.999	-0.991	-0.901

Note: Estimate of equation (8) with 18 observations. Estimated by nonlinear least squares (Gauss-Newton) with robust standard errors.

Figure 1. The regression slope as a function of the relative variability of the risk premium.

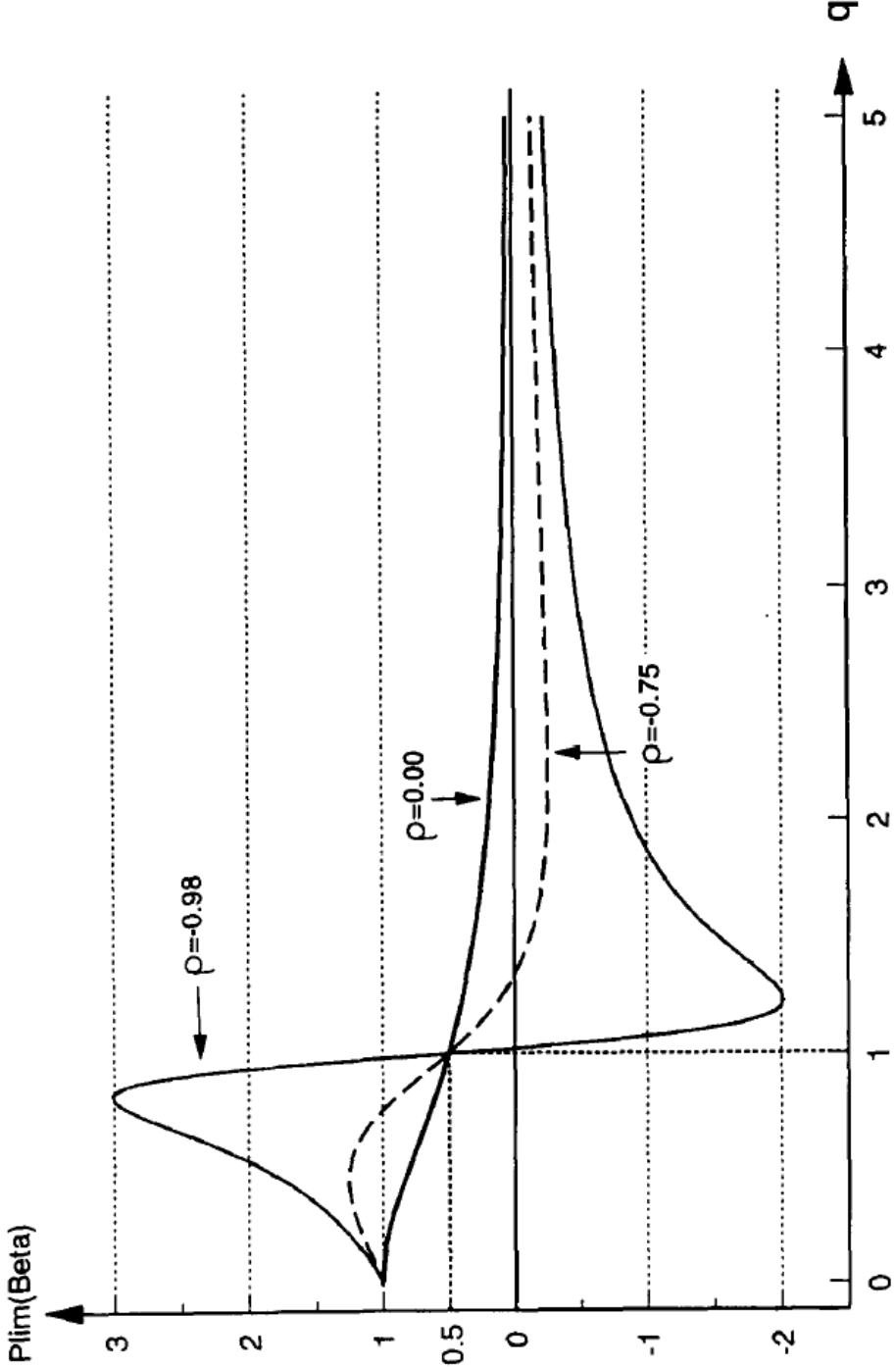


Fig. 1. The regression slope as a function of the relative variability of the risk premium.

$plim(\beta) = 1 - q(\rho + q)/(1 + 2\rho q + q^2)$, where β is the slope coefficient of the regression equation: $R_{t+1} - R_t = \alpha + \beta[1/(D_N - 1)](R_t - r_t) + u_{t+1}$; q denotes the relative variability of the risk premium and equals the ratio of the standard deviation of the expected risk premium to the standard deviation of the expected change in the long rate; ρ is the correlation between the expected risk premium and the expected change in the long rate.

Figure 2. Data

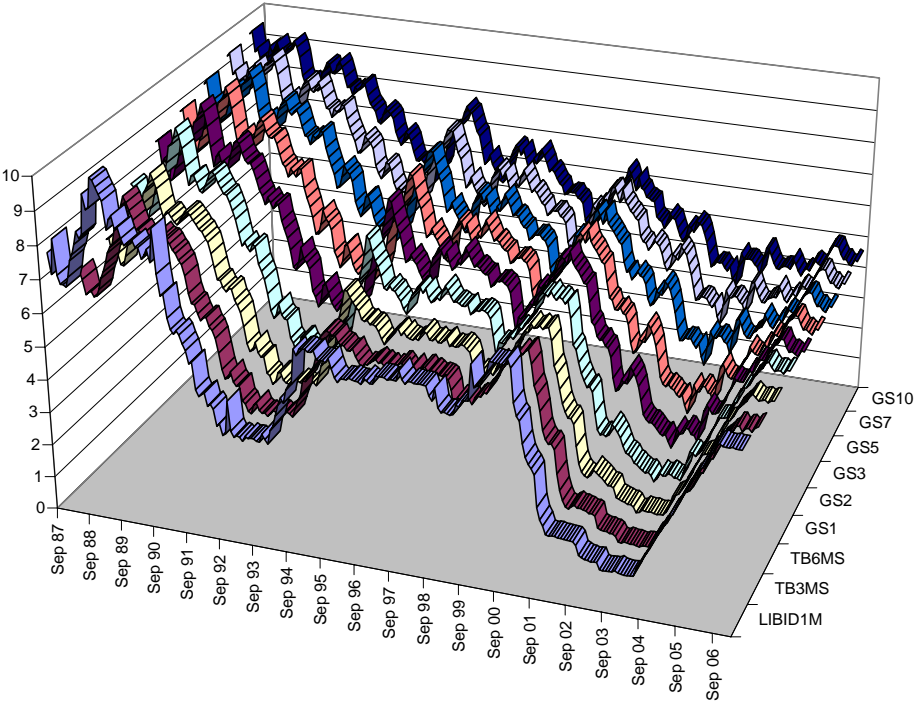


Figure 3. Change in the long rate and term spreads

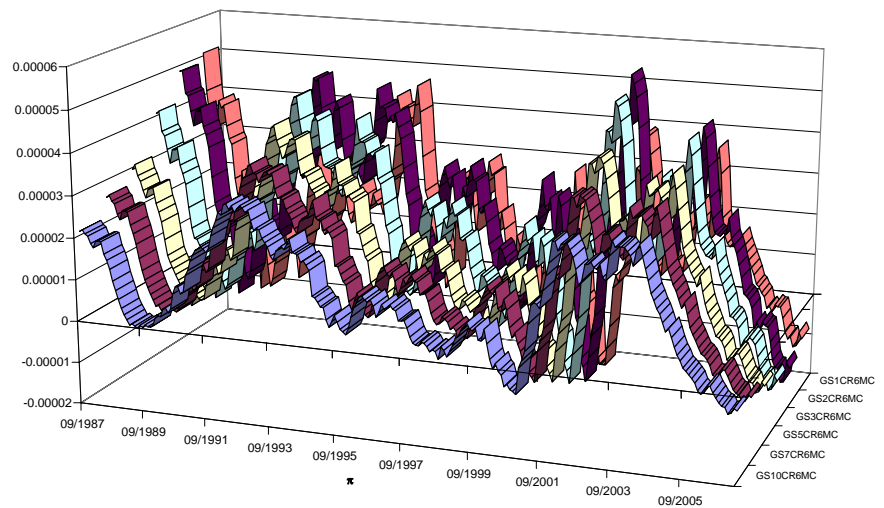
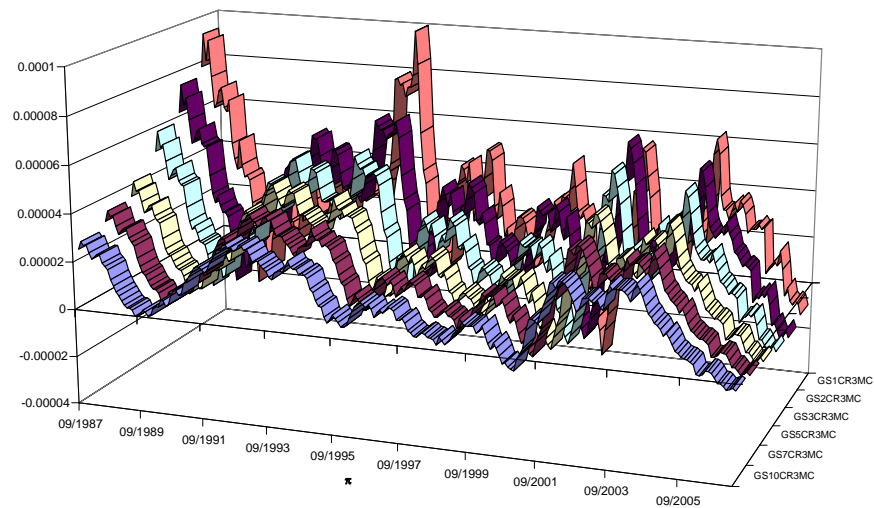
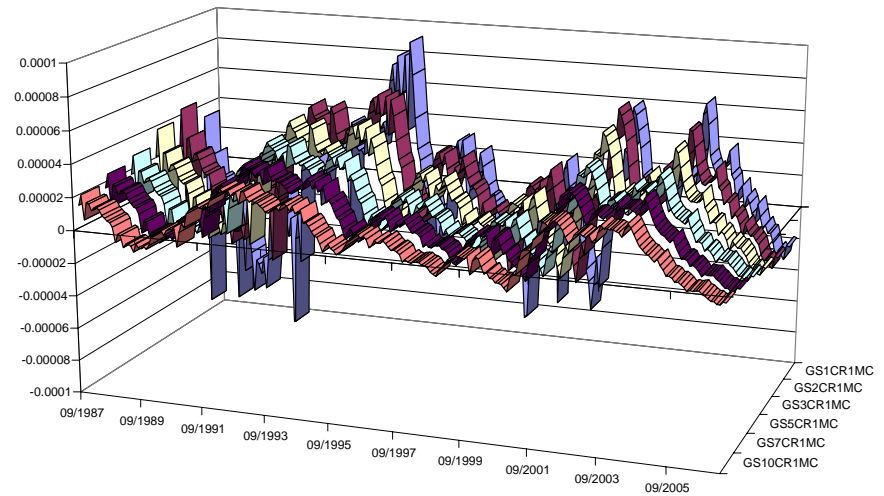
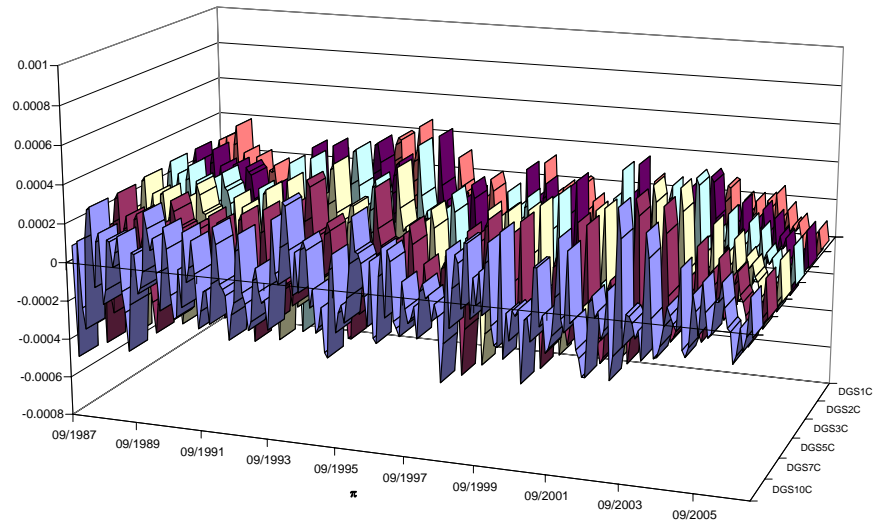


Figure 4. Spectra

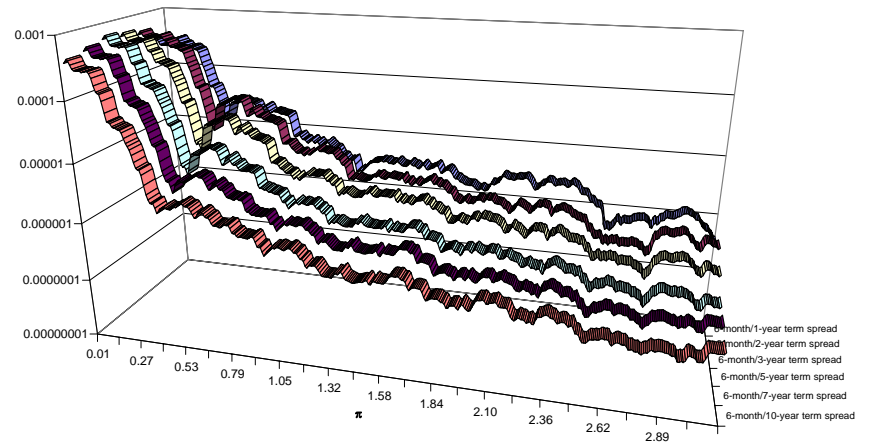
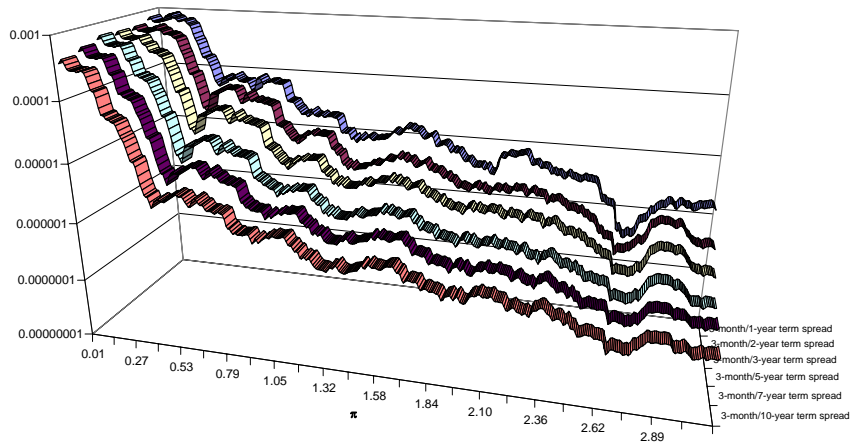
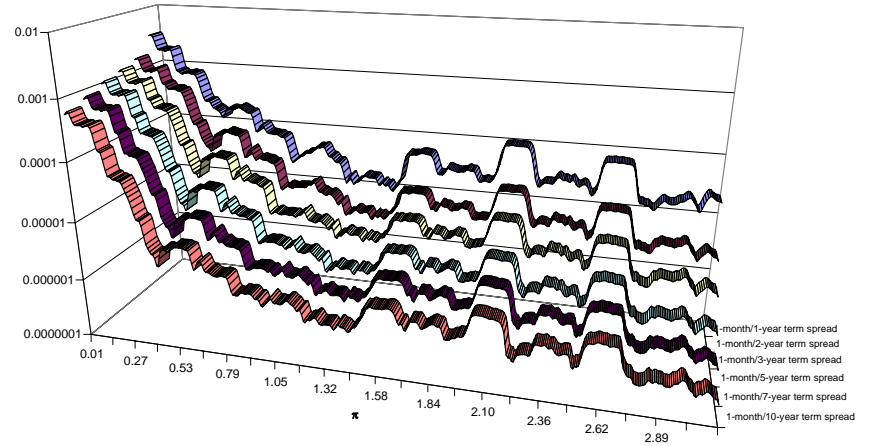
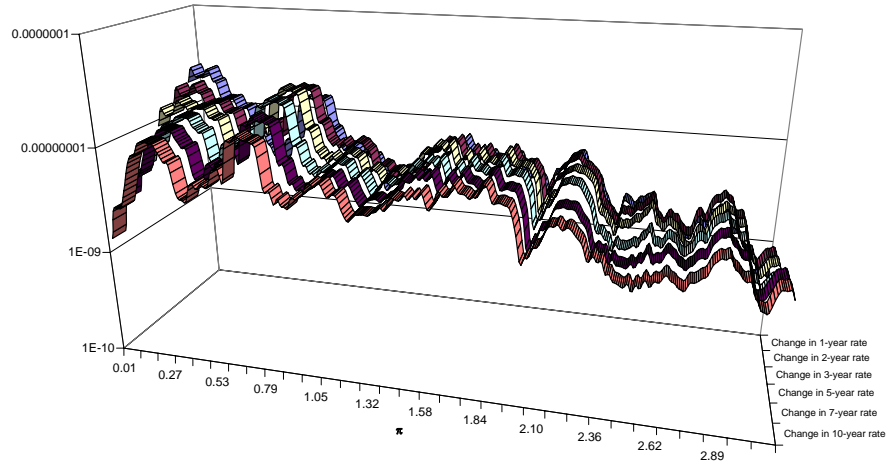


Figure 5. Estimated β parameters across frequency bands

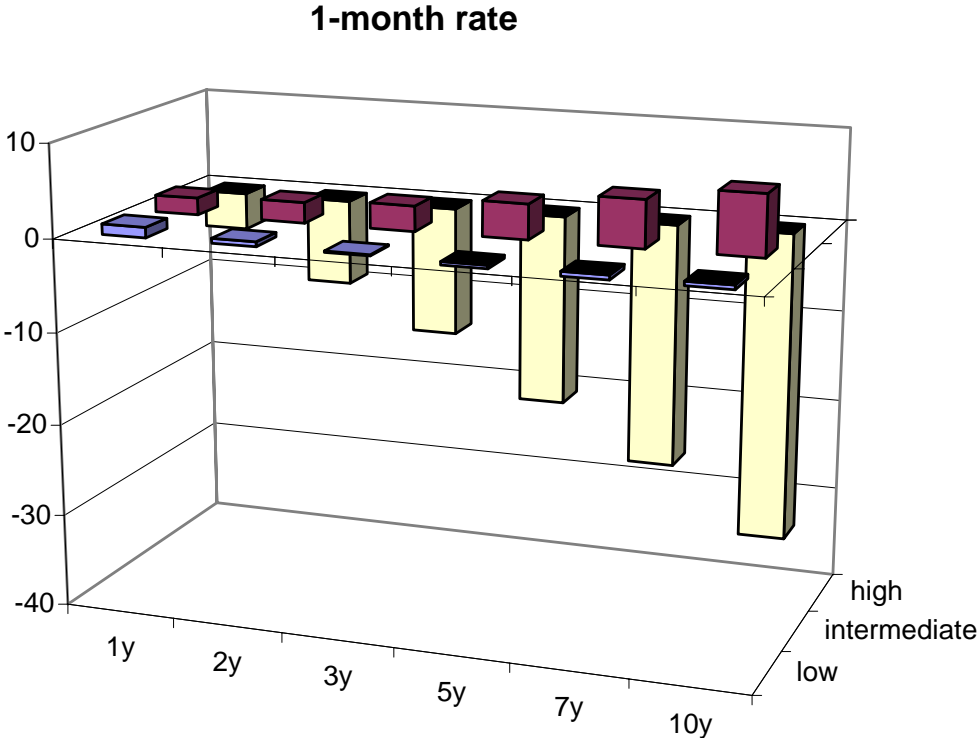


Figure 6. Estimated β and plim β

