The Phillips curve under state-dependent pricing*

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Abstract

This article is related to the large recent literature on Phillips curves in sticky-price equilibrium models. It differs in allowing for the degree of price stickiness to be determined endogenously. A closed-form solution for short-term inflation is derived from the dynamic stochastic general equilibrium (DSGE) model with state-dependent pricing developed by Dotsey, King and Wolman. This generalized Phillips curve encompasses the New Keynesian Phillips curve (NKPC) based on Calvo-type price-setting as a special case. It describes current inflation as a function of lagged inflation, expected future inflation, current and expected future real marginal costs, and current and past variations in the distribution of price vintages. We find that current inflation depends positively on its own lagged values giving rise to intrinsic persistence as a source of inflation persistence. Also, we find that the state-dependent terms (that is, the variations in the distribution of price vintages) tend to counteract the contribution of lagged inflation to inflation persistence.

Keywords: State-dependent pricing, inflation dynamics, Phillips curve.

JEL Classification: E31, E32

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1 Introduction

In recent years, dynamic general equilibrium models with nominal rigidities have become the standard tool to analyze the effects of monetary policy on output and prices. These models typically assume that firms choose their prices optimally, while the timing of their price changes is exogenous (time-dependent pricing). The assumption of time-dependent pricing is often useful because it makes a model easier to work with. It implies however that firms do not adjust the time pattern of their price adjustments in response to changes in macroeconomic conditions. This is hardly plausible if we think of an environment with shifts in trend inflation, for example, and therefore it may limit the value of these models for monetary policy analysis. As an alternative, Dotsey et al. (1999) have developed a dynamic general equilibrium model with endogenous timing of price changes. Building on earlier contributions by Sheshinski and Weiss (1983) and Caplin and Leahy (1991), they describe an economy where firms' time pattern of price changes responds to the state of the economy (state-dependent pricing). Recent contributions to this strand of the literature include models emphasizing the role of sticky pricing plans (Burstein (2005)) and idiosyncratic marginal cost shocks (Golosov and Lucas (2003).

The analysis of monetary policy in dynamic general equilibrium models is usually performed by numerical methods. Nevertheless, it is useful for many purposes to have a closed-form solution for short-term inflation. In the case of time-dependent pricing, a structural equation relating inflation dynamics to the level of real marginal costs (or another measure of real activity) has been derived from the Calvo (1983) model. Under zero trend inflation, it relates inflation to real marginal costs and the expectation of next period's inflation. This is known as the New Keynesian Phillips curve (NKPC).[1]

In this paper, we derive a closed-form solution for short-term inflation in the Dotsey et al. (1999) model. The resulting equation is more general than the NKPC, and it nests the latter as a special case. It relates inflation to lagged inflation, expected future inflation, current and expected future real marginal costs, and current and past variations in the distribution of price vintages. The number of leads and the size of the coefficients are endogenous and depend on

[1]See Woodford (2003) for a detailed exposition.

the level of steady-state inflation and on firms' beliefs about future adjustment costs associated with price changes. This structural equation is referred to in this paper as the state-dependent Phillips curve (SDPC).

In contrast to the NKPC, lagged inflation terms affect current inflation in the SDPC. This is an interesting feature since estimates by Galí and Gertler (1999) and many others suggest that the NKPC extended by a lagged inflation term provides a better description of inflation dynamics than the purely forward-looking NKPC. There are various ways to derive a specification with lagged inflation beyond the SDPC. Three approaches have been considered in the recent literature. First, Galí and Gertler (1999) extend the Calvo (1983) model to allow for a subset of price-adjusting sellers that resort to a backward-looking rule of thumb to set prices. Second, Christiano et al. (2005), within the same framework, assume that some firms index their price to past inflation. Third, Fuhrer and Moore (1995), Wolman (1999), Dotsey (2002), Kozicki and Tinsley (2002), Mash (2003), and Guerrieri (2006) use other forms of time-dependent pricing that build on the staggered contract model of Taylor (1980). While all these approaches provide lagged inflation terms, the structure of these Phillips curves is conditional on the assumption about exogenous nominal rigidities. So a key advantage of the SDPC over this class of Phillips curves is its endogenous structure.

We evaluate the properties of the SDPC under a variety of assumptions. The paper first focuses on the coefficients which are functions of the parameters describing the equilibrium outcome of the model. Based on steady-state comparisons, we examine how the coefficients respond to changes in the model calibrations of adjustment costs under both low and high trend inflation environments. The paper then turns to the model's implications for policy analysis. By policy implications, we mean the effect of different monetary policy rules for inflation and output dynamics. We explore how the state-dependency influences the persistence in inflation under various assumptions for policy inertia. This is done by way of examining the dynamic response of the economy to a monetary policy shock, when state-dependent pricing is either present or absent. Also, the paper uses the SDPC framework to examine whether a hybrid NKPC (NKPC extended by a lagged inflation term) can adequately describe inflation dynamics generated in a calibrated state-dependent pricing economy. To explore that issue, artificial data sets for a state-dependent pricing economy are generated under both low and high trend inflation environments. These data are used to estimate the hybrid NKPC and to assess the specification by comparing the estimated coefficients with those derived from the calibrated model and with those typically found in the empirical literature. Finally, the estimated hybrid NKPC is added to a small standard macro model, and the response of this model economy to a monetary policy shock is compared with the corresponding results derived from the full model with state-dependent pricing.

The reminder of this paper is organized as follows. Section 2 reviews the main features of the state-dependent model by Dotsey et al. (1999). In Section 3, we derive the SDPC and show that this generalized Phillips curve nests the NKPC. Section 4 discusses the steady-state comparisons. Section 5 analyzes the dynamic effects of monetary policy shocks and characterizes the performance of the hybrid NKPC against the backdrop of a model economy featuring state-dependent pricing. Section 6 concludes.

2 The state-dependent pricing model

The framework we use in this paper is the dynamic stochastic general equilibrium model with state-dependent pricing of Dotsey et al. (1999). The economy is characterized by monopolistic competition between firms selling final goods. With a common technology and common factor markets real marginal costs are the same for all firms. The novel feature of Dotsey et al. (1999)'s model is the way price adjustment costs are introduced. It is assumed that firms face stochastic costs of price adjustment which are i.i.d. across firms and across time. Firms evaluating their prices weigh the expected benefit from price adjustment against the price adjustment cost they have drawn in the current period. Conditional on the current adjustment costs, some firms do adjust while others do not. All adjusting firms set the same price.

In this section, we focus on the key equations describing the optimal nominal price and the aggregate price level, respectively. Formal details of the rest of the model can be found in Dotsey et al. (1999). To simplify the presentation, we split the price-setting problem into two parts. For a given realization of the adjustment cost, each firm has to decide whether to adjust the price of the final good it produces and, if so, to what level. The former decision problem can be described as

$$V_t = \max\left(v_{0,t} - \xi_t w_t, v_{j,t}\right), \tag{2.1}$$

where $v_{0,t}$ gives the current value of the firm if it adjusts the price in the current period, and $v_{j,t}$ is the value of a firm that last adjusted its price j periods ago. The price adjustment cost is denoted by $\xi_t w_t$, where ξ_t is the realization of the stochastic adjustment cost expressed in labor units, and w_t is the real wage.

The value of the firm in case of a price adjustment in t is determined by

$$v_{0,t} = \max_{P_{0,t}} \left\{z_{0,t} + E_t \beta Q_{t,t+1}\left[(1-\alpha_{1,t+1})v_{1,t+1} + \alpha_{1,t+1}v_{0,t+1} - w_{t+1}\Xi_{1,t+1}\right]\right\} \tag{2.2}$$

with

$$\Xi_{1,t+1} = \int_0^{G^{-1}(\alpha_{1,t+1})} \xi g(x)dx.$$

The corresponding value of a firm that last adjusted its price j periods ago in case of no price adjustment in t is

$$v_{j,t} = z_{j,t} + E_t \beta Q_{t,t+1}\left[(1-\alpha_{j+1,t+1})v_{j+1,t+1} + \alpha_{j+1,t+1}v_{0,t+1} - w_{t+1}\Xi_{j+1,t+1}\right] \tag{2.3}$$

with

$$\Xi_{j+1,t+1} = \int_0^{G^{-1}(\alpha_{j+1,t+1})} \xi g(x)dx,$$

where $z_{j,t}$ denotes the current real profit based on the optimal price set j periods ago, $P_{j,t}$, and the term in square brackets reflects the two possibilities of adjustment and non-adjustment next period. With probability $1 - \alpha_{j+1,t+1}$, the firm will not adjust its price next period; in this case, we have the discounted expected value of a non-adjusting firm, $E_t[\beta Q_{t,t+1} v_{j+1,t+1}]$, where $\beta Q_{t,t+1}$ is the stochastic discount factor which varies with the ratio of future to current marginal utility. With probability $\alpha_{j+1,t+1}$, the firm will adjust its price next period; in this case, we have the discounted expected value of an adjusting firm, $E_t[\beta Q_{t,t+1} v_{0,t+1}]$, less the expected adjustment cost the firm will have to pay, amounting to $E_t[w_{t+1}\alpha^{-1}_{j+1,t+1}\Xi_{j+1,t+1}]$. The average cost in labor units paid conditional on adjustment, $\alpha^{-1}_{j+1,t+1}\Xi_{j+1,t+1}$, depends on $G^{-1}(\alpha_{j+1,t+1})$, where $G(\cdot)$ denotes the distribution of the fixed price adjustment cost. Equation (2.2) refers to a firm which does adjust its price in the current period, so that $j = 0$; otherwise the interpretation is the same as in (2.3).

Now a firm will change its price only if the benefit of a price adjustment exceeds the realization of the random adjustment cost. Formally,

$$\upsilon_{0,t} - \upsilon_{j,t} \geq w_t \xi_t, \quad \forall j = 1, 2, \ldots, J. \tag{2.4}$$

If both sides of (2.4) are equal, the firm is indifferent between adjusting its price and keeping it unchanged. This borderline case can be used to derive the price adjustment probability $\alpha_{j,t}$ of a firm that adjusted its price j periods ago. It is the likelihood of drawing an adjustment cost that is smaller than the benefit expressed in labor units, $(\upsilon_{0,t} - \upsilon_{j,t})/w_t$. This can be written as

$$\alpha_{j,t} = G(\frac{\upsilon_{0,t} - \upsilon_{j,t}}{w_t}), \quad \forall j = 1, 2 \ldots, J. \tag{2.5}$$

Equation (2.5) describes how the adjustment probabilities depend on the state of the economy. As the value functions evolve stochastically with the state of the economy, the adjustment probabilities $\alpha_{j,t}$, $\forall j = 1, 2, \ldots, J-1$, also change. Our notation here reflects the fact that J is the maximum number of periods the firm is willing to go without a price adjustment, i.e. $\alpha_{J,t} = 1$. The number of price vintages is finite because, with adjustment costs bounded from above and positive trend inflation, the net benefit of a price adjustment becomes arbitrarily large over time. The state-dependent behavior of the adjustment probabilities is a key feature of the model. It captures the intuitive notion that adjustment behavior responds to shocks, and that, with positive inflation, a firm which last changed its price a long time ago is more likely to readjust than a firm which changed its price more recently.

The adjustment probabilities $\alpha_{j,t}$, $\forall j = 1, 2, \ldots, J$, can then be used to describe the distribution of price vintages in the economy and the evolution of this distribution through time. Let the firms at the beginning of period t be ordered according to the time that has elapsed since their most recent price adjustment $\tau_{j,t}, \forall j = 1, 2, \ldots, J$, where $\sum_{i=1}^{J} \tau_{j,t} = 1$. In period t, a fraction $\alpha_{j,t}$ of vintage-j firms decides to adjust in accordance with (2.4), and a fraction $(1 - \alpha_{j,t})$ decides to stick to the old price $P_{j,t}$. The total fraction of firms adjusting in period t, $\omega_{0,t}$, is therefore

$$\omega_{0,t} = \sum_{j=1}^{J} \alpha_{j,t} \tau_{j,t} \tag{2.6}$$

and the fractions of the other firms, i.e., the firms that last adjusted their prices j periods ago, are

$$\omega_{j,t} = (1 - \alpha_{jt}) \tau_{j,t}, \qquad \forall j = 1, 2, \ldots, J-1. \tag{2.7}$$

The end-of-period fractions define the distribution of the price vintages at the beginning of period $t+1$: $\tau_{j+1,t+1} = \omega_{j,t}, \forall j = 0,1,\ldots,J-1$. Note that the distribution of prices as well as the adjustment probabilities are conditional on the exogenous adjustment cost distribution function $G(\xi)$. In Section 4.1 below, we will examine the sensitivity of the optimal price-setting behavior with respect to different assumptions for $G(\cdot)$.

We then turn to the second aspect of the firm's price-setting problem, that is the determination of the optimal nominal price $P_{0,t}$. The adjusting firm will choose $P_{0,t}$ such that $\upsilon_{0,t}$ is maximised. Differentiating (2.2) with respect to $P_{0,t}$ and removing $\upsilon_{1,t+1}$ by recursive forward substitution leads to the optimality condition

$$0 = E_t \sum_{j=0}^{J-1} \beta^j Q_{t,t+j} \frac{\omega_{j,t+j}}{\omega_{0,t}} \frac{\partial z_{j,t+j}}{\partial P_{0,t}}, \tag{2.8}$$

where

$$\omega_{j,t+j}/\omega_{0,t} = \prod_{i=1}^{j} (1-\alpha_{i,t+i}),$$

$$\frac{\partial z_{j,t+j}}{\partial P_{0,t}} = \frac{1-\theta}{P_{t+j}} \left[\frac{P_{0,t}}{P_{t+j}}\right]^{-\theta} C_{t+j} + \frac{\theta}{P_{t+j}} \left[\frac{P_{0,t}}{P_{t+j}}\right]^{-\theta-1} MC_{t+j} C_{t+j}.$$

Here, MC_{t+j}, C_{t+j}, and P_{t+j} denote aggregate real marginal costs, aggregate demand and aggregate prices, and θ is the elasticity of substitution between goods (or equally, the elasticity of demand for any single good). Equation (2.8) is the dynamic counterpart to the static optimality condition for the monopolistic firm's price-setting problem. It requires the sum of the discounted marginal profits due to a price adjustment to be zero, or, since the profits are defined as revenues minus costs, the sum of discounted expected marginal revenues to equal the sum of expected real marginal costs. With common factor markets, as in King and Wolman (1996), the firm's real marginal costs in turn can be expressed as a function of aggregate real marginal costs and aggregate prices.

Solving (2.8) for the optimal price $P_{0,t}$, yields

$$P_{0,t} = \frac{\theta}{\theta-1} \frac{E_t \sum_{j=0}^{J-1} \beta^j Q_{t,t+j} \frac{\omega_{j,t+j}}{\omega_{0,t}} MC_{t+j} P_{t+j}^{\theta} C_{t+j}}{E_t \sum_{j=0}^{J-1} \beta^j Q_{t,t+j} \frac{\omega_{j,t+j}}{\omega_{0,t}} P_{t+j}^{\theta-1} C_{t+j}}. \tag{2.9}$$

This is the central pricing equation and corresponds to that in Dotsey et al. (1999). The optimal price depends on current and expected future aggregate real marginal costs, aggregate demand

and aggregate prices. The weights, $E_t\omega_{j,t+j}/\omega_{0,t}$, reflect the expected probabilities to be stuck with the currently set price for j periods, $E_t\prod_{i=1}^{j}(1-\alpha_{i,t+i})$. These conditional probabilities are endogenous and vary in response to changes in the state variables. They would be neither endogenous nor time varying in a purely time-dependent model.

As all firms have identical real marginal costs and identical expectations of future adjustment costs, $P_{0,t}$ is the same for all adjusting firms.[2] The aggregate price index P_t is therefore given by

$$P_t \equiv \left[\sum_{j=0}^{J-1}\omega_{j,t}(P_{0,t-j})^{1-\theta}\right]^{\frac{1}{1-\theta}}, \tag{2.10}$$

where $\omega_{j,t}$ is the fraction of firms charging the price $P_{0,t-j}$ at time t. A revision of the price adjustment probabilities induced by a monetary shock, for example, thus affects the persistence of the aggregate price level through the re-weighting of individual prices in (2.10).

3 The state-dependent Phillips curve (SDPC)

3.1 Derivation

This section discusses the derivation of a Phillips curve from the model outlined in Section 2. The key equations are (2.9) describing the optimal nominal price set by adjusting firms, $P_{0,t}$, and (2.10) describing the aggregate price level, P_t. Starting from (2.9), we can divide both sides of the equation by P_t to get relative prices. By log-linearising around the steady state and solving for the optimal relative price $x_{0,t}$, we get

$$x_{0,t} = E_t\sum_{j=1}^{J-1}\sum_{i=j}^{J-1}[\theta\rho_i + (1-\theta)\delta_i]\pi_{t+j} + E_t\sum_{j=0}^{J-1}\{\psi_j mc_{t+j} + (\rho_j-\delta_j)[\hat{\omega}_{j,t+j}-\hat{\omega}_{0,t}]\} \tag{3.1}$$

with

$$\rho_j = \frac{\beta^j\omega_j\Pi^{j\theta}}{\sum_{i=0}^{J-1}\beta^i\omega_i\Pi^{i\theta}}, \qquad \delta_j = \frac{\beta^j\omega_j\Pi^{j(\theta-1)}}{\sum_{i=0}^{J-1}\beta^i\omega_i\Pi^{i(\theta-1)}}, \qquad \psi_j = \rho_j + \kappa(\rho_j-\delta_j),$$

where the $\hat{\omega}$-terms denote absolute deviations and the other time-varying lower-case letters denote percentage deviations from their respective steady-state values. Appendix A summarises

[2] Golosov and Lucas (2003) present a menu-cost model in which firms set prices optimally in response to both aggregate and idiosyncratic shocks. In this set-up price adjusting firms may charge different prices.

the main steps of this derivation. Equation (3.1) describes the variations of the optimal relative price around its steady state, $x_{0,t}$, as a function of the deviations of future inflation, π_{t+j}, of current and future real marginal costs, mc_{t+j}, and of future probabilities of non-adjustment, $\hat{\omega}_{j,t+j} - \hat{\omega}_{0,t}$, from their steady-state values. The coefficients depend on steady-state inflation, Π, the steady-state distribution of price vintages, ω_j, the number of price vintages, J, the real discount factor, β, the price elasticity of demand, θ, and the elasticity of aggregate demand with respect to real marginal costs, κ. With an increase in steady-state inflation for instance, the benefit of adjusting relative prices rises for all firms. Hence, the adjustment probabilities increases (according to (2.4)), and the structure and number of the ω_j-terms move (according to (2.6) and (2.7)), thereby affecting the magnitude of the coefficients in (3.1) endogenously.

Starting from (2.10), we then derive the log-linearized version of the aggregate price level in terms of $x_{0,t}$. As shown in Appendix A, this yields

$$x_{0,t} = \mu_0 \pi_t + \sum_{j=1}^{J-2} \mu_j \pi_{t-j} - \sum_{j=1}^{J-1} \omega_j \nu_j x_{0,t-j} - \frac{1}{1-\theta} \sum_{j=0}^{J-1} \nu_j \hat{\omega}_{j,t}, \tag{3.2}$$

$$\mu_j = \frac{1}{\omega_0} \sum_{i=j+1}^{J-1} \omega_i \Pi^{i(\theta-1)}, \qquad \nu_j = \frac{1}{\omega_0} \Pi^{j(\theta-1)}.$$

With $\hat{\Omega}_t = \sum_{j=0}^{J-1} \nu_j \hat{\omega}_{j,t}$, (3.2) can be rewritten as

$$x_{0,t} = \mu_0 \pi_t + \sum_{j=1}^{J-2} \mu_j \pi_{t-j} - \sum_{j=1}^{J-1} \omega_j \nu_j x_{0,t-j} - \frac{1}{1-\theta} \hat{\Omega}_t. \tag{3.3}$$

According to (3.3), $x_{0,t}$ is related to deviations of current and lagged inflation, π_{t-j}, and of lagged optimal relative prices, $x_{0,t-j}$, from their steady-state values. Further it is related to the deviation of the distribution of price vintages from the steady state, $\hat{\Omega}_t$. The coefficients, in turn, depend on steady-state inflation, Π, the steady-state distribution of price vintages, ω_j, the number of price vintages, J, and the price elasticity of demand, θ. With an increase in steady-state inflation, the steady-state adjustment probabilities and the distribution of price vintages change endogenously. Since the aggregate price level depends on the distribution of price vintages, the shifting pattern of the distribution caused by the increase in steady-state inflation affects the dynamics of the aggregate price level expressed in terms of $x_{0,t}$.

To obtain an equation for the dynamics of inflation, we combine (3.1) and (3.3) and solve for π_t:

$$\pi_t = \frac{1}{\mu_0}\left[\sum_{j=1}^{J-1}\sum_{i=j}^{J-1}[\theta\rho_i + (1-\theta)\delta_i]\pi_{t+j} + E_t\sum_{j=0}^{J-1}\psi_j mc_{t+j} + E_t\sum_{j=0}^{J-1}(\rho_j - \delta_j)[\hat{\omega}_{j,t+j} - \hat{\omega}_{0,t}] \right.$$
$$\left. - \sum_{j=1}^{J-2}\mu_j\pi_{t-j} + \sum_{j=1}^{J-1}\omega_j\nu_j x_{0,t-j} + \frac{1}{1-\theta}\hat{\Omega}_t\right]. \quad (3.4)$$

Applying iterative backward substitution to (3.3) allows us to eliminate all optimal relative price terms in (3.4). The procedure is outlined in Appendix B. The resulting equation for the inflation dynamics is given by

$$\pi_t = E_t\sum_{j=1}^{J-1}\delta'_j\pi_{t+j} + E_t\sum_{j=0}^{J-1}\psi'_j mc_{t+j} + E_t\sum_{j=0}^{J-1}\gamma_j[\hat{\omega}_{j,t+j} - \hat{\omega}_{0,t}] + \eta_0\hat{\Omega}_t + \sum_{j=1}^{\infty}\eta_j\hat{\Omega}_{t-j} + \sum_{j=1}^{\infty}\mu'_j\pi_{t-j}, \quad (3.5)$$

where

$$\delta'_j = \frac{1}{\mu_0}\sum_{i=j}^{J-1}[\theta\rho_i + (1-\theta)\delta_i], \qquad \psi'_j = \frac{1}{\mu_0}\psi_j, \qquad \gamma_j = \frac{1}{\mu_0}(\rho_j - \delta_j),$$

$$\mu'_j = \frac{1}{\mu_0}\Big(\sum_{i=1}^{j}\vec{e}[H(-B)^{i-1}A]_{[.,j-(i-1)]} - \mu_j\Big), \qquad \mu_j = 0, \forall j \geq J-1,$$

$$\eta_0 = \frac{1}{\mu_0}\frac{1}{1-\theta}, \qquad \eta_j = -\frac{1}{\mu_0}\sum_{i=1}^{j}\vec{e}[H(-B)^{i-1}C]_{[.,j-(i-1)]}, \forall j \geq 1.$$

The details about the matrices H, A, B and C are given in Appendix B. It is sufficient to note here that $\vec{e}$ is a unity row vector with $[(j+1)(J-1)-1]$ elements and that the matrices H, A, B and C are square matrices of order $[(j+1)(J-1)-1]$. The subscript $[.,j-(i-1)]$ denotes the columns of matrix $[H(-B)^{(i-1)}A]$ and $[H(-B)^{(i-1)}C]$ which are premultiplied by $\vec{e}$.

We refer to (3.5) as the *state-dependent Phillips curve* (SDPC). According to the SDPC, the deviation of current inflation from the steady state, π_t, depends on the deviations from their respective steady-state values of lagged inflation, π_{t-j}, expected future inflation, π_{t+j}, current and expected future real marginal costs, mc_{t+j}, and expected future probabilities of non-adjustment, $\hat{\omega}_{j,t+j} - \hat{\omega}_{0,t}$, and of the lagged distributions of price vintages, $\hat{\Omega}_{t-j}$.

The number of leads for π_{t+j}, mc_{t+j}, and $\hat{\omega}_{j,t+j} - \hat{\omega}_{0,t}$ are finite, while the number of lags for π_{t-j} and $\hat{\Omega}_{t-j}$ are infinite. The infinite lag structure results from the elimination of the relative prices. The coefficients on these lags can be shown to converge to zero. How fast this comes about depends again on the assumption made about the adjustment cost distribution and on the state of the economy. The convergence occurs since the price adjustment cost, and therefore the price-setting behavior, is stochastic implying that $\omega_{0,t} > \omega_{j,t}$, $\forall j = 1, 2, \ldots, J-1$.

The coefficients in the SDPC depend on steady-state inflation, the steady-state distribution of price vintages, the number of price vintages, and the price elasticity of demand. Those on the expected variables also depend on the discount factor; those on the marginal cost terms further depend on the elasticity of aggregate demand with respect to real marginal costs. The price adjustment costs are not made explicit in (3.5), but they are lingering in the background. By affecting the number and the distribution of price vintages, they are indirectly linked to the coefficients of the SDPC. Thus we conclude that with a change in the distribution of adjustment costs or a change in steady-state inflation, the structure of the SDPC will change as well. In this section, we have suggested how an increase in steady-state inflation influences the optimal pricing behavior in the state-dependent model. In Section 4, we shall give a more detailed account based on numerical methods and figures.

3.2 Nesting the New Keynesian Phillips curve

A substantial amount of recent research in monetary economics has focused on theoretical and empirical issues related to the NKPC. The NKPC states that current inflation depends on next period's expected inflation and on real marginal costs (or another measure of economic activity):

$$\pi_t = \beta E_t \pi_{t+1} + \frac{\alpha(1-\beta(1-\alpha))}{(1-\alpha)} mc_t. \tag{3.6}$$

This specification can be derived from a dynamic general equilibrium model with monopolistic competition and Calvo-type price stickiness.[3] Calvo (1983) assumes that the price-setter adjusts his or her price whenever a random signal occurs. The signals are i.i.d. across firms and across time. Thus, there is a constant probability α that a given price-setter will be able to reset his or her price in a given period. The adjustment probability is independent of the time that

[3] See Yun (1996) or Galí and Gertler (1999).

has elapsed since the previous price adjustment, and the adjustment frequency does not depend on the state of the economy.

If we consider the Dotsey et al. (1999) model under the assumption that price-setting follows Calvo (1983) and that the level of steady-state inflation is constant at zero (i.e. $\Pi = 1$ in gross terms), we can show that the SDPC representation of inflation dynamics collapses to the NKPC. Since the Calvo pricing assumption implies that the adjustment probability is constant for all firms, $\alpha_{j,t} = \alpha$, the number of price vintages becomes infinite and the weights of the price vintages can be written as a function of α and j:

$$\omega_{j,t} = \alpha(1-\alpha)^j, \qquad \forall j = 0, 1, \ldots \infty.$$

With these modifications, (3.5) takes the form

$$\pi_t = E_t \sum_{j=1}^{\infty} \delta_j' \pi_{t+j} + E_t \sum_{j=0}^{\infty} \psi_j' mc_{t+j} + E_t \sum_{j=0}^{\infty} \gamma_j [\hat{\omega}_{j,t+j} - \hat{\omega}_{0,t}] + \sum_{j=1}^{\infty} \mu_j' \pi_{t-j} + \sum_{j=0}^{\infty} \eta_j \hat{\Omega}_{t-j}, \quad (3.7)$$

where

$$\delta_j' = \frac{\alpha}{(1-\alpha)} \beta^j (1-\alpha)^j, \quad \psi_j' = \frac{\alpha(1-\beta(1-\alpha))}{(1-\alpha)} \beta^j (1-\alpha)^j, \quad \gamma_j = 0, \quad \mu_j' = 0, \quad \eta_j = 0.$$

There are three points to note here. First, under the assumption of Calvo-type price-setting and zero trend inflation, the SDPC does not include any lagged terms. This is the consequence of the definition of the aggregate price level in (2.10). The infinite geometric lag structure allows us to abstract from the weights of the previously set optimal prices and to summarise the whole pricing history in terms of the previous period's aggregate price level. This result holds regardless of the level of steady-state inflation. Second, the effect of the state-dependent pricing behavior (reflected in (3.5) by $\hat{\omega}_{j,t+j} - \hat{\omega}_{0,t}$, $\hat{\Omega}_{t-j}$) disappears. Third, equation (3.7) includes an infinite number of leads for expected inflation and expected real marginal costs. As shown in Figure 1, the coefficients on these leaded variables take a geometrically falling and infinite form.[4]

After isolating expected next period's inflation and current real marginal costs in (3.7), the

[4] Although the actual number of leads is infinite in the Calvo model, there are only 15 leads displayed in Figure 1.

SDPC representation of the Calvo model takes the form

$$\pi_t = \alpha\beta E_t\pi_{t+1} + \frac{\alpha(1-\beta(1-\alpha))}{(1-\alpha)}mc_t \\ + \frac{\alpha}{(1-\alpha)}\sum_{j=2}^{\infty}\beta^j(1-\alpha)^j E_t\pi_{t+j} + \frac{\alpha(1-\beta(1-\alpha))}{(1-\alpha)}\sum_{j=1}^{\infty}\beta^j(1-\alpha)^j E_t mc_{t+j}. \quad (3.8)$$

The geometrically falling and infinite coefficient structure allows us to express the whole lead structure in (3.8) in terms of $E_t\pi_{t+1}$:

$$\beta(1-\alpha)E_t\pi_{t+1} = \frac{\alpha}{(1-\alpha)}\sum_{j=2}^{\infty}\beta^j(1-\alpha)^j E_t\pi_{t+j} + \frac{\alpha(1-\beta(1-\alpha))}{(1-\alpha)}\sum_{j=1}^{\infty}\beta^j(1-\alpha)^j E_t mc_{t+j}. \quad (3.9)$$

Making use of (3.9), the SDPC representation in equation (3.8) reduces to the NKPC in (3.6).[5] The quantitative effect of this simplification on the coefficient of $E_t\pi_{t+1}$ is shown in Figure 1. The coefficient on expected next period's inflation is $\alpha\beta$ in the SDPC representation of the Calvo model and β in the NKPC. Since $0 < \alpha < 1$, the coefficient is larger in the NKPC. Note that the coefficient on current real marginal costs is the same in both representations.

4 Evaluation of the SDPC

In this section we evaluate the SDPC with respect to different rates of trend inflation and different types of price-setting. The analysis is conducted for the steady state. We calibrate the model and solve for the equilibrium. The calibrations are chosen such that the average duration of price rigidity turns out to be three quarters for an annualized rate of steady-state inflation of 6%.

One way of describing price-setting behavior is by the sequence of price adjustment probabilities $[\alpha_1, \ldots, \alpha_j, \ldots, \alpha_{J-1}]'$ considered by the firm. We compare three such sequences which are based on three different distributions of price adjustment costs. Following Dotsey et al. (1999), the distribution functions are assumed to have the form $G(\xi) = c_1 + c_2 \tan[c_3\xi - c_4]$. Figure 2 illustrates the three cases of price-setting. The first distribution function, labelled 'flat cdf',

[5] Similarly, it can be shown that the SDPC nests the NKPC specifications derived under positive trend inflation by Ascari (2004) and Bakhshi et al. (2003).

indicates that a firm is likely to draw either a very small or a very large adjustment cost over the interval $[0, 0.0178]$; the likelihood of drawing an intermediate adjustment cost is very small. The second function, labelled 'S-shaped cdf', implies that a firm again is likely to draw either a small or a large adjustment cost; but the interval now is $[0, 0.014]$ and the likelihood of drawing an adjustment cost in the middle range is higher than under the first distribution function. This second function is qualitatively similar to the one adopted by Dotsey et al. (1999). The third distribution function, labelled 'linear cdf', approximates a uniform distribution of adjustment costs over the interval $[0, 0.008]$.

The rest of the model calibration is the same for all three cases of price-setting. For the price elasticity of demand, we set $\theta = 10$ implying a flexible price markup of 11%. In addition, we set $\beta = 0.984$ for the quarterly real discount rate, and $\Pi = 1.03$ (in gross terms) for the annual rate of steady-state inflation. The alternative steady-state inflation rate that will be used for comparison is set at $\Pi = 1.06$ (in gross terms). Table 1 summarizes the calibrations.[6]

4.1 Steady-state comparisons of adjustment probabilities and fractions of firms in price vintages

We start our evaluation by looking at the steady-state price adjustment probabilities, α_j, and the corresponding distribution of price vintages, ω_j. Figure 3 summarizes the results for the three types of price-setting behavior (flat cdf, S-shaped cdf and linear cdf) and the two levels of steady-state inflation (3% and 6%). The horizontal axis indicates the vintages ordered by the number of quarters j since the price has been set. The panels in the first column of Figure 3 display the adjustment probabilities, whereas the distributions of price vintages are in the second column.

As we would expect, the adjustment probability α_j is not constant across price vintages. It rises with j in all three models. The reason is that in an inflationary environment the expected benefit of adjusting prices is larger for firms of vintage $j+1$ than for firms of vintage j, resulting in a higher adjustment probability. With the S-shaped cdf and $\pi_{ss} = 3\%$, for example, there

[6] With few exceptions, the parameter values are as in Dotsey (2002). The parameter value of θ is taken form Chari et al. (2000). The calibrations of the distribution of adjustment costs and of the level of steady-state inflation are our own.

is a probability of just 5% that firms which adjusted their price in the previous period ($j = 1$) do adjust again in the current period. By contrast, firms which set their price two years ago ($j = 8$) will expect a sizable profit gain from readjusting. Hence, the probability of adjusting in the current period is considerably higher (59%).

With the level of steady-state inflation rising from 3% to 6%, the relative prices of the various firms erode more rapidly. As a result, the firms adjust their prices more frequently. This is reflected in higher adjustment probabilities α_j. At the same time, the number of price vintages increases with higher levels of steady-state inflation. Consider again the model with the S-shaped cdf for adjustment costs. There are 11 price vintages when steady-state inflation is 3%. As the rate of steady-state inflation rises to 6%, the number of vintages declines to 6, and the average duration of price rigidity falls from a good four quarters to three quarters.

Turning to the fractions of firms in the different price vintages, we note that the fractions decline as j rises. Also, the number of price vintages is smaller with higher steady-state inflation, and the number of firms in vintages with low j is larger. Under the S-shaped cdf for example, ω_0 increases from 0.24 at 3% inflation to 0.33 at 6% inflation.

Finally, we observe that the shape of the adjustment probabilities displayed in Figure 3 differs depending on the adjustment cost distribution function. This difference is not transmitted to the distribution of price vintages, however. At least for low rates of steady-state inflation, the distributions of price vintages are strikingly similar across the three price-setting assumptions.

4.2 Adjustment cost distributions and the SDPC

The reminder of Section 4 focuses on the SDPC coefficients. As the distributions of price adjustment costs (flat cdf, S-shaped cdf, linear cdf) cause substantial differences between sequences of adjustment probabilities, we may wonder how the three distributions influence the reduced-form coefficients in the SDPC. Figure 4 displays the SDPC coefficients computed under the assumption of 3% steady-state inflation. The leads (+) and lags (−) of the variables are given on the horizontal axis, the coefficients (δ'_j, μ'_j, ψ'_j, γ_j, η_j) on the vertical axis.

The coefficients on expected future inflation, δ'_j, and on current and expected future real marginal costs, ψ'_j, take their highest values at low leads and fall off smoothly with higher leads in a slightly convex pattern. This pattern is similar to the one we observed in Figure 1 for the SDPC representation of the Calvo model implied by (3.8). The coefficients on lagged inflation, μ'_j, are quantitatively important at low lags but fall off rapidly at higher lags and converge to zero. The coefficients on the state-dependent behavioral terms, γ_j and η_j, are converging to zero in an oscillating pattern.

The comparison across the different types of price-setting behavior indicates that the differences between the three adjustment cost distribution functions have little effect on the reduced-form coefficients of the SDPC. The explanation is based on the definition of the coefficients. According to (3.5), the coefficients depend on Π, J, ω_j, β, θ, and κ. In our calibration, it is assumed that $\Pi = 3\%$, $\beta = 0.984$, $\theta = 10$ and $\kappa = 0.66$ in all three models. Also, as shown in Section 4.1, the number of price vintages (flat cdf: $J = 10$, S-shaped cdf: $J = 10$, linear cdf: $J = 8$) and the distribution of price vintages, ω_j, vary little across the three models despite marked differences in the sequences of adjustment probabilities. Thus, the similarity can be traced back to the parameters and steady-state values going into the reduced-form coefficients of the SDPC, which are all either equal or similar across the three types of price-setting behavior.

4.3 Trend inflation and the SDPC

Next, we explore the effect of the steady-state inflation rate on the reduced-form coefficients of the SDPC. We have seen in Figure 3 that a higher level of steady-state inflation leads to an upward revision of the optimal adjustment probabilities. As a consequence, prices are adjusted more frequently, the number of price vintages, J, declines and the distribution of price vintages, ω_j, is modified. Since the other factors determining the reduced-form coefficients of the SDPC (β, θ and κ) remain unchanged, any effect of the steady-state inflation rate on the SDPC coefficients must be attributed to this mechanism.

We consider two steady-state inflation rates: 3% and 6%. The results displayed in Figure 5 show that with steady-state inflation rising from 3% to 6%, the number of future inflation terms falls. The coefficients on expected future inflation, δ'_j, increase with inflation at low leads while falling off more rapidly at higher leads. The same pattern holds for the coefficients on current and future real marginal costs, ψ'_j, on the expected variations in state-dependent price-setting behavior, γ_j, and – with lags instead of leads – on lagged inflation. At the same time, the oscillating pattern gets more distinct. Note, finally, that the coefficients on expectations about future state-dependent deviations from steady-state adjustment behavior are small for all three types of adjustment costs.

4.4 Lagged inflation terms

With few exceptions, the coefficients on lagged inflation displayed in Figures 4 and 5 are positive. The largest value is on the first lag. This coefficient is positive in all six cases considered. With higher lags, the coefficients fall off rapidly. Table 2 presents the coefficient on the first lag, μ_1, and the sum of the coefficients on all lagged inflation terms, approximated by $\sum_{j=1}^{3J} \mu_j$, for the three price-setting assumptions (flat cdf, S-shape cdf, linear cdf) and the two steady-state rates of inflation (3% and 6%). The results confirm that the sum of the coefficients on lagged inflation is positive in all cases considered.

This pattern is interesting for two reasons. First, it is consistent with a large amount of empirical evidence which suggests that inflation depends positively on its own lagged values even after controlling for fundamental factors like current and anticipated real marginal costs (or other measures of economic activity). Second, it indicates that models with state-dependent pricing offer, at least in principle, an explanation of intrinsic persistence. Standard models with micro-foundations do not produce an independent positive effect of lagged inflation on current inflation. In the NKPC which is based on Calvo-style pricing, lagged inflation is irrelevant in determining inflation, and while models based on Taylor-style staggered contracts do allow current inflation to depend on past inflation, the coefficients on lagged inflation will be negative (see Dotsey (2002), Whelan (2004) and Guerrieri (2006)). Such problems have been addressed by introducing rule-of-thumb price setters or indexation into otherwise purely forward-looking models (as in Galí and Gertler (1999) and Christiano et al. (2005)). Yet these adjustments are open to criticism, in that they are not micro founded.

To examine the SDPC coefficients on lagged inflation, it is useful to consider their determinants. Since the generalized structure given in (3.5) is somewhat cumbersome, it is more convenient to consider individual coefficients. The coefficient on the first lag of inflation is given by

$$\mu_1' = \frac{\omega_1 \Pi^{(\theta-1)}}{\omega_0} - \frac{\sum_{i=2}^{J-1} \frac{\omega_i}{\omega_0} \Pi^{i(\theta-1)}}{\sum_{i=1}^{J-1} \frac{\omega_i}{\omega_0} \Pi^{i(\theta-1)}}. \tag{4.1}$$

The first term on the right-hand-side of (4.1) is the product of the probability of non-adjustment one period after the price has been reset, $\frac{\omega_1}{\omega_0} = (1 - \alpha_1)$, and the level of steady-state inflation. The second term is a ratio of two sums, where the numerator is the sum of inflation weighted probabilities of non-adjustment, $\frac{\omega_j}{\omega_0} = (1 - \alpha_1)(1 - \alpha_2)(1 - \alpha_3)...(1 - \alpha_j)$, from t through $t + j$ starting at $j = 2$, and the denominator is the corresponding sum starting at $j = 1$. For the first term, it can be shown that it is greater than one for plausible values of Π, θ and α_1. The second term on the other hand is always less than one. Therefore we can conclude that the SDPC coefficient on the first lag of inflation is positive.[7]

5 The SDPC and inflation dynamics

In this section we consider various issues related to inflation persistence, i.e., the tendency of inflation to move gradually towards its long-term value. We show how the response of inflation to a monetary policy shock changes with variations in policy inertia and shifts in trend inflation. And closer to the main focus of this paper, we examine how the price setting assumption underlying the SDPC modifies inflation dynamics. We set the SDPC against the popular hybrid NKPC proposed by Galí and Gertler (1999) to consider the implications of using the hybrid NKPC, when the true price-setting model is in fact state dependent, so the correct inflation equation is the SDPC.

5.1 The consequences of a monetary policy shock

To close the model described in Section 2, we adopt a standard monetary policy rule of the form

$$i_t = \rho_1 i_{t-1} + (1 - \rho_1)(\phi_\pi \pi_t + \phi_y y_t) + \epsilon_t. \tag{5.1}$$

[7] When price-setting behavior is according to Calvo (1983) and the level of steady-state inflation is constant at zero (i.e., $\Pi = 1$), μ_1' and all coefficients on higher lags of inflation fall to zero since the two terms in (4.1) (and in the corresponding definitions for higher lags) exactly add up to zero.

Here, i_t is the nominal interest rate, y_t is the output gap, and ϵ_t is an i.i.d. monetary policy shock.[8] The parameters ϕ_π and ϕ_y are set to 1.5 and 0.5.

We refer to the complete model with (5.1) as the SDP model. The model is linearized around the equilibrium outcome based on the S-shaped cdf for adjustment costs. The steady-state inflation rate is set at 3% and the parameter defining the degree of policy inertia, ρ_1, is set to 0.8. The responses of inflation and other variables to a (negative) interest rate shock of 100 basis points are summarized in Figure 6 (dotted lines). We can see that inflation and output rise by 1.4% and 1.6% on impact. Thereafter, the effects gradually decline. The fraction of price adjusting firms jumps by 35 percentage points on impact before declining gradually as well. In addition, we can see from the panel at the bottom of the table that the positive contribution of lagged inflation terms to inflation is partly offset by negative contributions from lagged state-dependent variations in the distribution of price setters ($\hat{\Omega}$ terms). The presence of state-dependent adjustment behavior, therefore, tends to counteract the intrinsic persistence in inflation.

The persistence in inflation displayed in Figure 6 is strongly affected by the assumption on policy inertia. This can be seen by comparing the impulse responses based on $\rho_1 = 0.8$ with the case of no policy inertia defined as $\rho_1 = 0$ (solid lines). We find that the impact effects on inflation (0.25%) and output (0.4%) are considerably smaller without policy inertia. The same holds for the fraction of extra adjusters. Forward-looking price setters, by anticipating output and inflation to return rapidly to the steady-state values, will expect a smaller benefit from price adjustment than in the presence of policy inertia. As a result, the fraction of extra adjusters is smaller. One period after the shock, the output effect is basically zero, while a small inflation effect persists. Again, we observe that the presence of state-dependent adjustment behavior tends to counteract the effect coming from the lagged inflation terms.

To highlight the role of the state-dependent terms in greater detail, we analyze a parsimonious model from the SDP model. In this alternative model, the adjustment probabilities are held fixed at their steady-state values. Essentially, this is the time-dependent pricing counterpart of the SDP model and is therefore referred to, in what follows, as the TDP model. In the

[8]Dotsey et al. (1999) by contrast consider a money supply rule, not an interest rate rule.

TDP model, the SDPC defined by (3.5) reduces to the TDPC:

$$\pi_t = E_t \sum_{j=1}^{J-1} \delta'_j \pi_{t+j} + E_t \sum_{j=0}^{J-1} \psi'_j mc_{t+j} + \sum_{j=1}^{\infty} \mu'_j \pi_{t-j}, \tag{5.2}$$

where it should be noted that the coefficients on π_{t+j}, mc_{t+j} and π_{t-j} are identical with those in (3.5).[9]

The consequences of a monetary policy shock in the TDP model are shown in Figure 7. We assume a steady-state inflation rate of 3%, and $\rho_1 = 0.8$ for the degree of policy inertia. The corresponding results for the SDP model are given for convenience. The comparison illustrates the effect of state-dependent price-setting on the dynamics of inflation. We find that by suppressing state-dependent pricing the impact effect on inflation declines. The reason is that the fraction of adjusting firms does not respond to the state of the economy so that there are no extra adjusters in the aftermath of a monetary policy shock. Prices are stickier on impact, and the effect on output is amplified. In addition, we can see that the persistence in inflation and in output is larger than in the SDP model. Three quarters after the shock, the contribution of the lagged inflation terms accounts for half of the response of inflation to the monetary policy shock. In the SDP model, on the other hand, as noted above, the intrinsic persistence is counteracted by the lagged $\hat{\Omega}$ terms.

We have assumed so far that the rate of steady-state inflation is set at 3%. By raising steady-state inflation to 6%, we examine how the dynamic response of the economy to a monetary policy shock is affected by changes in trend inflation. Results for the SDP model under high policy inertia ($\rho_1 = 0.8$) are reported in Figure 8. The impact effect of a monetary policy shock on inflation is larger when steady-state inflation is higher. But, the persistence in inflation declines. As pointed out in Section 4.1, the adjustment probabilities in each price vintage and the average frequencies of price adjustment increase with steady-state inflation. As a consequence, the total number of price vintages, and therefore inflation persistence, is reduced. For output, this mechanism gives the opposite pattern. The impact effect of a monetary policy shock on output

[9]It can be shown that the TDPC representation of inflation dynamics collapses to the NKPC if we assume a constant price adjustment probability for all firms (implying an infinite number of price vintages) and zero trend inflation.

is smaller when steady-state inflation is shifted from 3% to 6%, and the effect of the shock on output persistence is larger. In quantitative terms, all these differences are fairly modest. At least for the calibration used here, the response of inflation and output to a monetary shock does not seem to be very sensitive to whether steady-state inflation is 3% or 6%.

5.2 Performance of the hybrid NKPC in an SDPC economy

We now move to the implications of assuming a simplified description of inflation dynamics when the true inflation process is given by the more general SDPC. In particular, we assess the performance of the popular hybrid NKPC which is extensively used in the theoretical and empirical literature on inflation determination. Galí and Gertler (1999) derived the hybrid NKPC based on the assumption that some firms set their prices in a forward-looking optimizing way à la Calvo (1983), while other firms apply a backward-looking rule of thumb when setting prices. The resulting equation is

$$\pi_t = \gamma_b \pi_{t-1} + \gamma_f E_t \pi_{t+1} + \lambda mc_t. \tag{5.3}$$

Unlike the purely forward looking NKPC described by (3.6), the hybrid NKPC features a lagged inflation term, π_{t-1}, which can capture intrinsic persistence in inflation. Empirical evidence for several countries suggests that the hybrid NKPC provides a better description of inflation dynamics than the purely forward-looking NKPC. Examples include Galí and Gertler (1999), Galí et al. (2001), Leith and Malley (2002), and Smets and Wouters (2003), Gagnon and Khan (2005).[10] Estimates of (5.3) typically show that the coefficient on expected future inflation, $\hat{\gamma}_f$, exceeds the coefficient on lagged inflation, $\hat{\gamma}_b$, and the coefficient on measured real marginal cost, $\hat{\lambda}$, is positive though not always statistically significant.

[10] In response to criticisms by Rudd and Whelan (2005) and Linde (2005), Galí et al. (2005) present empirical evidence for the robustness of their estimates of the hybrid NKPC. For evidence in support of the purely forward-looking NKPC, see Sbordone (2002) and Lubik and Schorfheide (2004).

5.2.1 Estimates of the hybrid NKPC

To investigate whether the hybrid NKPC is a good approximation of the SDPC, we start by estimating (5.3) based on artificial data generated by the SDP.[11] These estimates are then compared with the typical pattern provided by estimates of (5.3) based on real data. In addition, they are compared with the true coefficients of the SDPC displayed in Figures 4 and 5, which we have derived from the calibrated model. To gain further insight, the exercise is repeated by simulating data from the TDP model.

For the generation of the data sets, we assume three types of shocks: to preferences, to technology, and to the interest rate (monetary policy). The monetary policy shocks are assumed to be i.i.d., whereas the shocks to preferences and technology are assumed to follow an AR(1) process with a persistence parameter of 0.5. The standard deviation of the innovation to a shock is 1% (monetary policy, preferences) and 0.7% (technology), respectively. The two models (SDP model and TDP model) are log linearized around the steady state based on the assumption of the S-shaped distribution of adjustment costs and two different rates of steady-state inflation (3% and 6%). A high degree of policy inertia is assumed throughout ($\rho_1 = 0.8$). The average duration of price stickiness turns out as roughly four quarters at 3% and three quarters at 6% steady-state inflation. For each of the four cases considered, 1,000 samples of 150 observations are generated.

Based on these data sets, we estimate (5.3) using the Generalized Method of Moments (GMM) approach. The instrument set we use comprises of four lags each of inflation, real marginal costs and the output gap. This lag length corresponds to that typically used in the empirical literature (see, for example, Galí and Gertler (1999)). Table 3 presents the mean estimates of the coefficients λ, γ_f and γ_b over the respective 1,000 data sets. The interval in square brackets is given by the 10% and the 90% quantiles of the distributions of the coefficient estimates. The share of the 1,000 data sets with a significant t-value for λ is given in brackets. J^* indicates the fraction of the 1,000 data sets where the Sargan-Hansen instrument validity test is passed.

[11] Dotsey (2002) conducts a similar experiment. He estimates (5.3) based on data generated by a three-period forward-looking truncated Calvo model under zero steady-state inflation.

We find that the results presented in Table 3 are broadly consistent with the pattern typically found in real data. That is, the estimated coefficient on expected future inflation, $\hat{\gamma}_f$, is larger than the estimated coefficient on lagged inflation, $\hat{\gamma}_b$, and the point estimate of the real marginal cost coefficient, $\hat{\lambda}$, is positive, though not always significant. This holds for all four cases considered, implying that the correspondence appears to be independent of steady-state inflation and the assumption of state-dependent pricing.

Next, we compare the estimated coefficients of the hybrid NKPC with the coefficients of the SDPC and the TDPC displayed in Figure 4 (for a steady-state inflation rate of 3%) and Figure 5 (for a steady-state inflation rate of 6%). As noted earlier, the coefficients on π_{t-j}, π_{t+j} and mc_{t+j} derived from the calibrated model do not differ between SDPC and TDPC. The comparison with the estimated coefficients of the hybrid NKPC provides mixed results. The estimate of $\hat{\gamma}_b$ is in the ballpark of our metric of intrinsic persistence, that is the sum of the coefficients on lagged inflation reported in Table 2. However, the estimated coefficient on current marginal costs, $\hat{\lambda}$, is some way below the corresponding coefficients derived from the calibrated model. This is all the more striking as expected future marginal costs are completely ignored in the hybrid NKPC, whereas the coefficients on these terms are all positive in Figure 4 and Figure 5. Finally, we note that the estimates of the duration of price stickiness, $\hat{D}$ - calculated using the estimates of $\hat{\gamma}_f$, $\hat{\lambda}$ and the calibrated β - are all slightly below the average durations derived from the calibrated model, reported in Table 2.

5.2.2 Responses to monetary shocks

Based on the results presented in Section 5.2.1, we then examine whether the misspecification of the hybrid NKPC really matters. Specifically, we perform a simulation exercise that should give us some idea about how much we are misled by mistakenly using the estimated hybrid NKPC when quantifying the effects of a monetary policy impulse. That is, we compare the dynamic responses of inflation, output and the interest rate to a monetary policy shock in the SDP model and the TDP model with those generated in a small New Keynesian macro model that includes the hybrid NKPC. The latter is a standard three equation model consisting of a log-linearized

Euler equation,

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1}) + \epsilon_{y,t}, \tag{5.4}$$

the hybrid NKPC (5.3) parameterized with the mean estimates reported in Table 3, and the monetary policy rule (5.1) with $\rho_1 = 0.8$.

As in Section 5.1, the monetary policy shock is a (negative) shock to the interest rate of 100 basis points. We compute the dynamic responses of the model with the hybrid NKPC and compare the results with those obtained from the SDP model and the TDP model. Figure 9 reports the results of this comparison based on a steady-state inflation rate of 3%. The corresponding results for steady-state inflation set at 6% are in Figure 10.[12]

We find that the model with the hybrid NKPC tends to understate the impact effect of the monetary policy shock on inflation. The impact effect on output in turn is overstated. This result is consistent with our finding that the estimate of the coefficient on current real marginal costs in the hybrid NKPC is biased downwards. It also reflects the fact that the expected future real marginal costs do not show up in the hybrid NKPC, whereas the coefficients on these terms are all positive in the SDPC and TDPC. Also, we note that the differences in impact effects are considerably larger under state-dependent pricing than under time-dependent pricing. This is due to the fact that under state-dependent pricing the fraction of extra adjusters will jump after a monetary policy shock. As a result, the impact effect on inflation is amplified and the impact effect on output is muted relative to the TDP model.

Turning to the persistence in inflation generated by the various models, we find fairly modest differences between the model with the hybrid NKPC and the TDP model, and considerable differences between the former and the SDP model. The inflation persistence generated by the TDP model is tracked well by the model with the hybrid NKPC, indicating that the persistence generated by the sequence of lagged inflation terms in the TDPC, $\sum_{j=1}^{3J} \mu_j$, is well approximated by the coefficient on lagged inflation in the estimated hybrid NKPC. The comparison with the SDP model in turn suggests that the inflation response generated by the model with the hybrid NKPC is not only biased downward on impact but also significantly more persistent than the

[12] Some of the impulse responses based on the SDP model and on the TDP model are repeated from Figures 6 to 8.

effects predicted by the SDP model. Again, the considerably larger impact effect reflects again the large swings in the fraction of adjusting firms allowed by the SDP model. Overall, these results suggest that the hybrid NKPC is a good approximation of the inflation dynamics provided by more general time-dependent models or by state-dependent models when the variations in the distribution of price vintages are minor.[13]

6 Conclusions

We have used the state-dependent pricing model of Dotsey et al. (1999) to derive a general specification for the Phillips curve which allows for positive steady-state inflation and state-dependent price-setting behavior. In the state-dependent Phillips curve (SDPC) inflation depends on current and expected future real marginal costs, past and expected future inflation, and past and expected future fluctuations in the price adjustment pattern. As it turns out, the specific nature of firms' price-setting behavior and the structural parameters of the model, such as the level of steady-state inflation, have important implications for the coefficients and the lead-lag structure of the SDPC.

An interesting property of the SDPC is that it offers an explanation of intrinsic persistence. That is, it implies positive coefficients on lagged inflation for a wide range of price-setting behavior. We have illustrated how the lagged inflation terms contribute to the overall persistence in inflation. As might be expected, the effect of lagged inflation gives rise to a considerable amount of inflation persistence as long as there are no or only little state-dependent variations in price vintages. But if a considerable number of price setters feel compelled to reset prices after an event, the persistence in inflation is reduced. This reflects the fact that the state-dependent pricing mechanism adds more price flexibility to economic dynamics. In extreme, prices could become fully flexible and inflation persistence would disappear.

Also, we have illustrated that the monetary policy rule has an influence on price-setting

[13]This is consistent with the findings of Klenow and Kryvtsov (2005). They present empirical evidence based on U.S. inflation data suggesting that the dominant contribution to the variance of inflation comes from the average size of price changes (as opposed to the fraction of items with price changes). When they calibrate the model of Dotsey et al. to match this variance decomposition, they find that the model's impulse responses are very close to those in a simple time-dependent model.

and thereby on inflation persistence. In the absence of monetary policy inertia, there is little incentive to deviate from the equilibrium price adjustment pattern after a shock and therefore only little effect from the variation in the distribution of price vintages on inflation. But with high monetary policy inertia, a larger number of price-setters will decide to reset prices after a shock such that inflation persistence is reduced significantly relative to a model without state-dependent pricing.

Finally we assessed the performance of the popular hybrid NKPC (with one lag of inflation) in an economy with state-dependent price-setting. Data are simulated from plausibly calibrated model economies and are used to estimate the hybrid NKPC. We find that the hybrid NKPC does well in tracking the intrinsic persistence found in the data-generating model, but fails to replicate the sensitivity of inflation with respect to real marginal costs. Specifically, using the estimated hybrid NKPC specifications to generate dynamic responses to a monetary shock in a small New Keynesian model, we find that the estimated hybrid NKPC captures the macroeconomic dynamics fairly well as long as there is little or no state-dependent price-setting. But, if sizeable state-dependent variations in the distribution of price vintages are present, the hybrid NKPC generates too much persistence in the macroeconomic variables, and understates the impact response of inflation to shocks.

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A Log-linearization of the main pricing equations

Consider the first-order condition for an optimal nominal price, as given by equation (2.8) in the text. After some rearrangement, we obtain

$$E_t \sum_{j=0}^{J-1} \beta^j Q_{t,t+j} \frac{\omega_{j,t+j}}{\omega_{0,t}} \left[\frac{P_{0,t}}{P_t} \frac{P_t}{P_{t+j}} \right]^{1-\theta} C_{t+j} = \frac{\theta}{\theta-1} E_t \sum_{j=0}^{J-1} \beta^j Q_{t,t+j} \frac{\omega_{j,t+j}}{\omega_{0,t}} MC_{t+j} \left[\frac{P_{0,t}}{P_t} \frac{P_t}{P_{t+j}} \right]^{-\theta} C_{t+j}. \quad \text{(A.1)}$$

Replacing $P_{0,t}/P_t$ by $X_{0,t}$ and using $P_t/P_{t+j} = 1/\prod_{i=1}^{j} \Pi_{t+i}$, A.1 can be rewritten as

$$E_t \sum_{j=0}^{J-1} \beta^j Q_{t,t+j} \frac{\omega_{j,t+j}}{\omega_{0,t}} \left[\frac{1}{\prod_{i=1}^{j} \Pi_{t+i}} \right]^{1-\theta} C_{t+j} X_{0,t} = \frac{\theta}{\theta-1} E_t \sum_{j=0}^{J-1} \beta^j Q_{t,t+j} \frac{\omega_{j,t+j}}{\omega_{0,t}} MC_{t+j} \left[\frac{1}{\prod_{i=1}^{j} \pi_{t+i}} \right]^{-\theta} C_{t+j}. \quad \text{(A.2)}$$

Log-linearizing A.2 around the steady-state values $\omega_j = \omega_j, \forall j = 0, \ldots, J-1$, $C = C$, $\Pi = \Pi$, $X = \frac{P_0}{P}$, $Q = 1$, and $MC = MC$ yields

$$E_t \sum_{j=0}^{J-1} \left[q_{t,t+j} + \hat{\omega}_{j,t+j} - \hat{\omega}_{0,t} + (\theta-1) \sum_{i=1}^{j} \pi_{t+i} + c_{t+j} + x_{0,t} \right] \beta^j \frac{\omega_j}{\omega_0} \Pi^{j(\theta-1)} CX = \frac{\theta}{\theta-1} E_t \sum_{j=0}^{J-1} \left[q_{t,t+j} + \hat{\omega}_{j,t+j} - \hat{\omega}_{0,t} + mc_{t+j} + \theta \sum_{i=1}^{j} \pi_{t+i} + c_{t+j} \right] \beta^j \frac{\omega_j}{\omega_0} \Pi^{j\theta} CMC, \quad \text{(A.3)}$$

where $\hat{\omega}$-terms denote absolute deviations and the other time-varying lower-case letters denote percentage deviations of variables from their respective steady-states values. Using

$$X = \frac{\theta}{\theta-1} \frac{\sum_{j=0}^{J-1} \beta^j \omega_j \Pi^{j\theta}}{\sum_{j=0}^{J-1} \beta^j \omega_j \Pi^{j(\theta-1)}} MC, \quad \text{(A.4)}$$

we can solve for the optimal relative price:

$$x_{0,t} = E_t \sum_{j=0}^{J-1} \left[q_{t,t+j} + \hat{\omega}_{j,t+j} - \hat{\omega}_{0,t} + mc_{t+j} + \theta \sum_{i=1}^{j} \pi_{t+i} + c_{t+j} \right] \frac{\beta^j \omega_j \Pi^{j\theta}}{\sum_{i=0}^{J-1} \beta^i \omega_i \Pi^{i\theta}} - E_t \sum_{j=0}^{J-1} \left[q_{t,t+j} + \hat{\omega}_{j,t+j} - \hat{\omega}_{0,t} + (\theta-1) \sum_{i=1}^{j} \pi_{t+i} + c_{t+j} \right] \frac{\beta^j \omega_j \Pi^{j(\theta-1)}}{\sum_{i=0}^{J-1} \beta^i \omega_i \Pi^{i(\theta-1)}}. \quad \text{(A.5)}$$

Relating real marginal cost fluctuations to the fluctuations of output, we can write:

$$c_t = \kappa mc_t, \tag{A.6}$$

where κ denotes the elasticity of aggregate demand with respect to real marginal costs. If the expected future fluctuations of the stochastic discount factors, q_{t+j}, are ignored, substituting A.6 into A.5 and rearranging yields

$$x_{0,t} = E_t \sum_{j=1}^{J-1} \sum_{i=j}^{J-1} [\theta\rho_i + (1-\theta)\delta_i]\pi_{t+j} + E_t \sum_{j=0}^{J-1} \{\psi_j mc_{t+j} + (\rho_j - \delta_j)[\hat{\omega}_{j,t+j} - \hat{\omega}_{0,t}]\}, \tag{A.7}$$

where

$$\rho_j = \frac{\beta^j \omega_j \Pi^{j\theta}}{\sum_{i=0}^{J-1} \beta^i \omega_i \Pi^{i\theta}}, \qquad \delta_j = \frac{\beta^j \omega_j \Pi^{j(\theta-1)}}{\sum_{i=0}^{J-1} \beta^i \omega_i \Pi^{i(\theta-1)}}, \qquad \psi_j = \rho_j + \kappa(\rho_j - \delta_j).$$

Equation A.7 corresponds to (3.1) in the text.

Consider next the aggregate price level described by (2.10) in the text:

$$P_t = \left[\sum_{j=0}^{J-1} \omega_{j,t} (P_{0,t-j})^{1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{A.8}$$

Equation A.8 can be rewritten such that all elements are constant along the inflationary steady state:

$$1 = \sum_{j=0}^{J-1} \left[\omega_{j,t} \left(\frac{P_{0,t-j}}{P_t}\right)^{1-\theta}\right] = \sum_{j=0}^{J-1} \left[\omega_{j,t} \left(\frac{P_{0,t-j}}{P_{t-j}} \frac{P_{t-j}}{P_t}\right)^{1-\theta}\right]. \tag{A.9}$$

Replacing $P_{0,t-j}/P_{t-j}$ by $X_{0,t-j}$ and P_{t-j}/P_t by $1/\prod_{i=0}^{j-1} \Pi_{t-i}$, we obtain

$$1 = \sum_{j=0}^{J-1} \omega_{j,t} \frac{X_{0,t-j}^{1-\theta}}{\prod_{i=0}^{j-1} \Pi_{t-i}^{\;1-\theta}}. \tag{A.10}$$

Log-linearizing A.10 around the steady-state values $X_{0,t-j} = X$, $\omega_j = \omega_j$, and $\Pi = \Pi$ gives

$$0 = \sum_{j=0}^{J-1} \left[\frac{X^{1-\theta}}{\Pi^{j(1-\theta)}} \hat{\omega}_{j,t} + (1-\theta)\omega_j \frac{X^{1-\theta}}{\Pi^{j(1-\theta)}} x_{0,t-j} - (1-\theta)\omega_j \frac{X^{1-\theta}}{\Pi^{j(1-\theta)}} \sum_{i=0}^{j-1} \pi_{t-i}\right]. \tag{A.11}$$

Solving for current inflation gives

$$(1-\theta) \sum_{j=1}^{J-1} \omega_j \Pi^{j(\theta-1)} \pi_t = \sum_{j=0}^{J-1} \left[\frac{1}{\Pi^{j(1-\theta)}} \hat{\omega}_{j,t} + (1-\theta)\omega_j \frac{1}{\Pi^{j(1-\theta)}} x_{0,t-j} - (1-\theta)\omega_j \frac{1}{\Pi^{j(1-\theta)}} \sum_{i=1}^{j-1} \pi_{t-i}\right] \tag{A.12}$$

and

$$\pi_t = \frac{1}{\sum_{j=1}^{J-1}\omega_j \Pi^{j(\theta-1)}} \sum_{j=0}^{J-1} \left[\Pi^{j(\theta-1)}\hat{\omega}_{j,t} + \omega_j \frac{1}{\Pi^{j(1-\theta)}} x_{0,t-j} - \omega_j \Pi^{j(\theta-1)} \sum_{i=1}^{j-1} \pi_{t-i} \right], \tag{A.13}$$

where again $\hat{\omega}$-terms denote absolute deviations and the other time-varying lower-case letters denote percentage deviations of variables from their respective steady-states values. Since $X = P_0/P$, we have

$$0 = \left(\frac{P_0}{P}\right)^{1-\theta} \sum_{j=0}^{J-1} \left[\Pi^{j(\theta-1)} \left(\frac{1}{1-\theta}\hat{\omega}_{j,t} + \omega_j (x_{0,t-j} - \sum_{i=0}^{j-1} \pi_{t-i}) \right) \right]. \tag{A.14}$$

Solving for the optimal relative price, $x_{0,t}$, yields, after some rearrangement,

$$x_{0,t} = \frac{1}{\omega_0} \left[\sum_{j=0}^{J-2} \sum_{i=j+1}^{J-1} \omega_i \Pi^{i(\theta-1)} \pi_{t-j} - \sum_{j=1}^{J-1} \omega_j \Pi^{j(\theta-1)} x_{0,t-j} - \frac{1}{1-\theta} \sum_{j=0}^{J-1} \Pi^{j(\theta-1)} \hat{\omega}_{j,t} \right]. \tag{A.15}$$

Equation A.15 corresponds to (3.2) in the text.

B Derivation of the SDPC coefficients

Consider (3.1) and (3.3) in the text. Combining these two equations and solving for π_t, one obtains

$$\begin{aligned} \pi_t = \frac{1}{\mu_0} \Bigg[& E_t \sum_{j=1}^{J-1} \sum_{i=j}^{J-1} [\theta\rho_i + (1-\theta)\delta_i] \pi_{t+j} + E_t \sum_{j=0}^{J-1} \psi_j mc_{t+j} + E_t \sum_{j=0}^{J-1} (\rho_j - \delta_j)[\hat{\omega}_{j,t+j} - \hat{\omega}_{0,t}] \\ & - \sum_{j=1}^{J-2} \mu_j \pi_{t-j} + \sum_{j=1}^{J-1} \omega_j \nu_j x_{0,t-j} + \frac{1}{1-\theta} \hat{\Omega}_t \Bigg], \end{aligned} \tag{B.1}$$

where

$$\rho_j = \frac{\beta^j \omega_j \Pi^{j\theta}}{\sum_{i=0}^{J-1} \beta^i \omega_i \Pi^{i\theta}}, \qquad \delta_j = \frac{\beta^j \omega_j \Pi^{j(\theta-1)}}{\sum_{i=0}^{J-1} \beta^i \omega_i \Pi^{i(\theta-1)}}, \qquad \psi_j = \rho_j + \kappa(\rho_j - \delta_j),$$

$$\mu_j = \frac{1}{\omega_0} \sum_{i=j+1}^{J-1} \omega_i \Pi^{i(\theta-1)}, \qquad \nu_j = \frac{1}{\omega_0} \Pi^{j(\theta-1)}, \qquad \hat{\Omega}_t = \sum_{j=0}^{J-1} \nu_j \hat{\omega}_{j,t}.$$

Using (3.2) and applying matrix notation, the weighted lagged relative price terms in B.1, can be written as

$$H\vec{x}_t = HA\vec{\pi}_t - HBx_{t-1} - HC\vec{\Omega}_t, \tag{B.2}$$

where

$$\vec{x}_t = \begin{bmatrix} x_{0,t-1} \\ x_{0,t-2} \\ \vdots \\ x_{0,t-(J-1)} \\ \vdots \\ x_{0,t-2J+2} \\ \vdots \\ x_{0,t-T} \end{bmatrix} \quad \vec{\pi}_t = \begin{bmatrix} \pi_{t-1} \\ \pi_{t-2} \\ \vdots \\ \pi_{t-(J-1)} \\ \vdots \\ \pi_{t-2J+2} \\ \vdots \\ \pi_{t-T} \end{bmatrix} \quad \vec{\Omega}_t = \begin{bmatrix} \hat{\Omega}_{t-1} \\ \hat{\Omega}_{t-2} \\ \vdots \\ \hat{\Omega}_{t-(J-1)} \\ \vdots \\ \hat{\Omega}_{t-2J+2} \\ \vdots \\ \hat{\Omega}_{t-T} \end{bmatrix}$$

$$H = \begin{bmatrix} \omega_1\nu_1 & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & \omega_2\nu_2 & 0 & \cdots & \cdots & \cdots & \cdots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \omega_{J-1}\nu_{J-1} & 0 & \cdots & \cdots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} \mu_0 & \mu_1 & \cdots & \mu_{J-2} & 0 & \cdots & \cdots & 0 \\ 0 & \mu_0 & \mu_1 & \cdots & \mu_{J-2} & 0 & \cdots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & 0 & \mu_0 & \mu_1 & \cdots & \mu_{J-2} \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} \omega_1\nu_1 & \omega_2\nu_2 & \cdots & \omega_{J-1}\nu_{J-1} & 0 & \cdots & \cdots & 0 \\ 0 & \omega_1\nu_1 & \omega_2\nu_2 & \cdots & \omega_{J-1}\nu_{J-1} & 0 & \cdots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & 0 & \omega_1\nu_1 & \omega_2\nu_2 & \cdots & \omega_{J-1}\nu_{J-1} \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} \frac{1}{1-\theta} & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & \frac{1}{1-\theta} & 0 & \cdots & \cdots & \cdots & \cdots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \cdots & 0 & \frac{1}{1-\theta} & 0 & \cdots & \cdots & \vdots \\ \vdots & \ddots & \ddots & 0 & \frac{1}{1-\theta} & 0 & \cdots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 & \cdots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \end{bmatrix}$$

By iterative backward substitution, the lagged relative price terms in B.2 can be expressed in terms of lagged inflation rates and lagged deviations of the distributions of price vintages from their steady-state distribution:

$$\begin{aligned} H\vec{x}_t &= HA\vec{\pi}_t - HBx_{t-1} - HC\vec{\Omega}_t \\ &= HA\vec{\pi}_t - HB[A\pi_{t-1} - Bx_{t-2} - C\Omega_{t-1}] - HC\vec{\Omega}_t \\ &\vdots \\ &= \sum_{j=0}^{k} H(-B)^j[A\vec{\pi}_{t-j} - C\vec{\Omega}_{t-j}] + H(-B)^{k+1}x_{t-(k+1)} \\ &\vdots \\ &= \lim_{k\to\infty} \sum_{j=0}^{k} H(-B)^j[A\vec{\pi}_{t-j} - C\vec{\Omega}_{t-j}]. \end{aligned} \tag{B.3}$$

Thus, if we unwind the lagged relative price terms in B.2 to the infinite past, B.1 can be expressed as

$$\pi_t = E_t\sum_{j=1}^{J-1}\delta'_j\pi_{t+j} + E_t\sum_{j=0}^{J-1}\psi'_j mc_{t+j} + E_t\sum_{j=0}^{J-1}\gamma_j[\hat{\omega}_{j,t+j} - \hat{\omega}_{0,t}] + \sum_{j=1}^{\infty}\mu'_j\pi_{t-j} + \sum_{j=0}^{\infty}\eta_j\hat{\Omega}_{t-j}, \tag{B.4}$$

where

$$\delta'_j = \frac{1}{\mu_0}\sum_{i=j}^{J-1}[\theta\rho_i + (1-\theta)\delta_i] = \frac{\omega_0}{\sum_{k=1}^{J-1}\omega_k\Pi^{k(\theta-1)}}\sum_{i=j}^{J-1}[\theta\frac{\beta^i\omega_i\Pi^{i\theta}}{\sum_{k=0}^{J-1}\beta^k\omega_k\Pi^{k\theta}} + (1-\theta)\frac{\beta^i\omega_i\Pi^{i(\theta-1)}}{\sum_{k=0}^{J-1}\beta^k\omega_k\Pi^{k(\theta-1)}}],$$

$$\psi'_j = \frac{1}{\mu_0}\psi_j = \frac{\omega_0}{\sum_{i=1}^{J-1}\omega_i\Pi^{i(\theta-1)}}\frac{\beta^j\omega_j\Pi^{j\theta}}{\sum_{i=0}^{J-1}\beta^i\omega_i\Pi^{i\theta}} + \kappa(\frac{\beta^j\omega_j\Pi^{j\theta}}{\sum_{i=0}^{J-1}\beta^i\omega_i\Pi^{i\theta}} - \frac{\beta^j\omega_j\Pi^{j(\theta-1)}}{\sum_{i=0}^{J-1}\beta^i\omega_i\Pi^{i(\theta-1)}}),$$

$$\gamma_j = \frac{1}{\mu_0}(\rho_j - \delta_j) = \frac{\omega_0}{\sum_{i=1}^{J-1}\omega_i\Pi^{i(\theta-1)}}(\frac{\beta^j\omega_j\Pi^{j\theta}}{\sum_{i=0}^{J-1}\beta^i\omega_i\Pi^{i\theta}} - \frac{\beta^j\omega_j\Pi^{j(\theta-1)}}{\sum_{i=0}^{J-1}\beta^i\omega_i\Pi^{i(\theta-1)}}),$$

$$\mu'_j = \frac{1}{\mu_0}\Big(\sum_{i=1}^{j} \vec{e}[H(-B)^{i-1}A]_{[.,j-(i-1)]} - \mu_j\Big), \qquad \mu_j = 0, \forall j \geq J-1,$$

$$\eta_0 = \frac{1}{\mu_0}\frac{1}{1-\theta}, \qquad \eta_j = -\frac{1}{\mu_0}\sum_{i=1}^{j} \vec{e}[H(-B)^{i-1}C]_{[.,j-(i-1)]}, \qquad \forall j \geq 1.$$

Note that $\vec{e}$ is a unity row vector with $[(j+1)(J-1)-1]$ elements and that the matrices H, A, B and C are square matrices of order $[(j+1)(J-1)-1]$. The subscript $[., j-(i-1)]$ then denotes the column of matrix $[H(-B)^{(i-1)}A]$ and $[H(-B)^{(i-1)}C]$ which are pre-multiplied by $\vec{e}$.

Table 1: Model calibrations

Quarterly discount factor	$\beta = 0.984$				
Risk aversion	$\sigma = 1$				
Labor supply elasticity	∞				
Labor share	$\alpha_L = 0.667$				
Demand elasticity	$\theta = 10$				
Steady-state inflation	$\pi_{ss} = 3\%, 6\%$				
Adjustment costs:					
Flat cdf	$c_1 = 0.34$	$c_2 = 0.02$	$c_3 = 171.6$	$c_4 = 1.513$	B= 0.018
S-shaped cdf	$c_1 = 0.52$	$c_2 = 0.17$	$c_3 = 178.4$	$c_4 = 1.26$	B= 0.014
Linear cdf	$c_1 = 1.26$	$c_2 = 1.00$	$c_3 = 80.93$	$c_4 = 0.90$	B= 0.008

Notes: B = upper bound of price adjustment costs.

Table 2: SDPC and intrinsic persistence

		$\pi_{ss} = 3\%$			$\pi_{ss} = 6\%$	
cdf	μ'_1	$\sum_{j=1}^{3J} \mu'_j$	D	μ'_1	$\sum_{j=1}^{3J} \mu'_j$	D
flat	0.198	0.297	3.9	0.190	0.341	3.0
S-shaped	0.280	0.409	4.2	0.370	0.421	3.1
linear	0.303	0.453	4.2	0.459	0.465	3.0

Notes: D = average duration of price stickiness in quarters.

Table 3: GMM estimates of the hybrid NKPC, data generated based on S-shaped cdf

π_{ss}	Model	$\hat{\lambda}$	$\hat{\gamma}_f$	$\hat{\gamma}_b$	$\hat{D}$	J*
3%	tdp	0.027 (0.534)	0.574	0.426	3.8	0.970
		[0.000 , 0.063]	[0.464 , 0.710]			
3%	sdp	0.041 (0.304)	0.610	0.390	3.5	0.909
		[-0.009 , 0.103]	[0.514 , 0.728]			
6%	tdp	0.069 (0.794)	0.566	0.434	2.7	0.981
		[0.018 , 0.129]	[0.479 , 0.692]			
6%	sdp	0.088 (0.395)	0.614	0.386	2.7	0.778
		[-0.017 , 0.198]	[0.519 , 0.738]			

Notes: $\gamma_f + \gamma_b = 1$,
$\hat{D}$ = estimated average duration of price stickiness,
J^* = proportion of 1000 simulations passing the J-test.

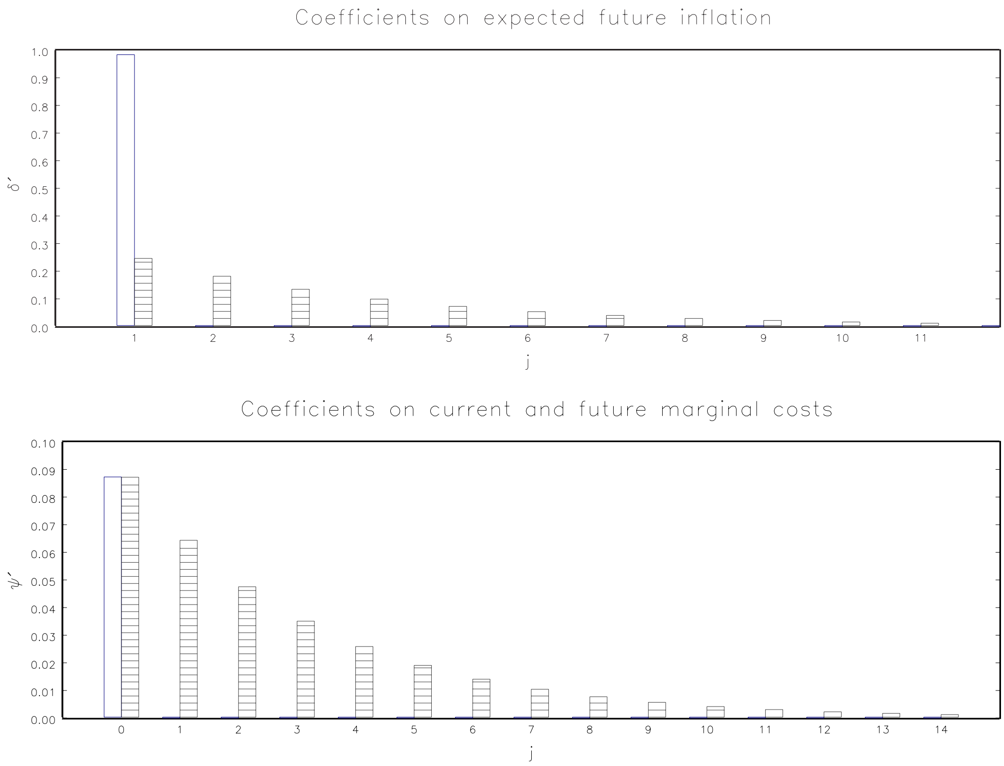

Figure 1: NKPC curve in its SDPC representation, $\pi_{ss} = 0\%$

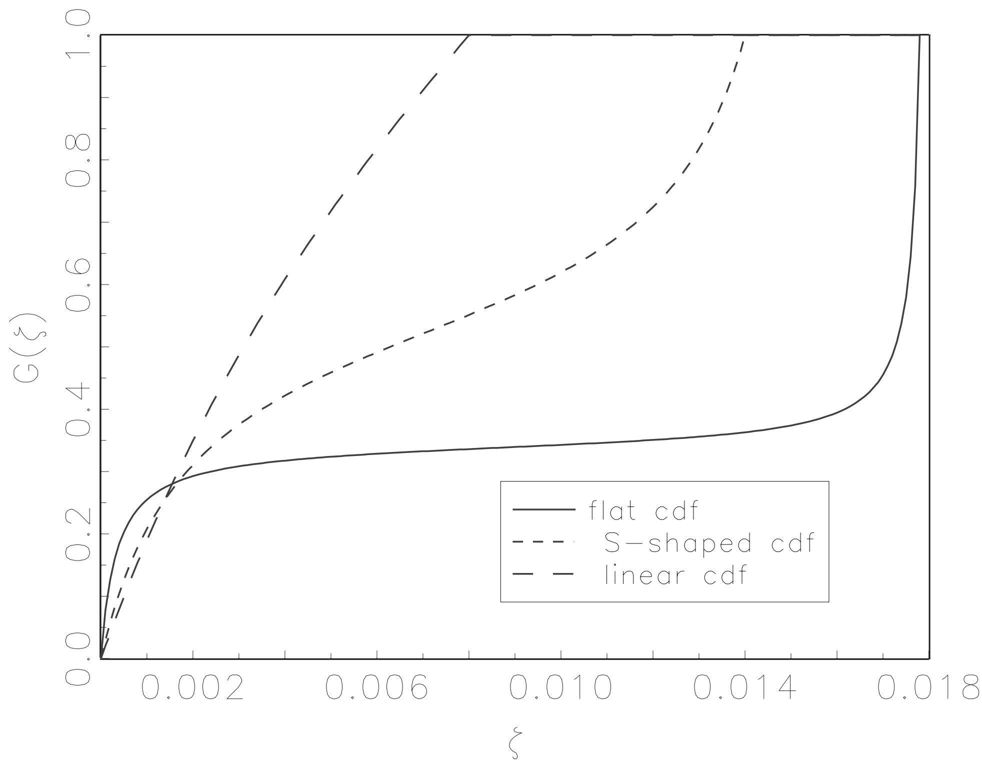

Figure 2: Cumulative distribution functions (cdf) of fixed adjustment costs.

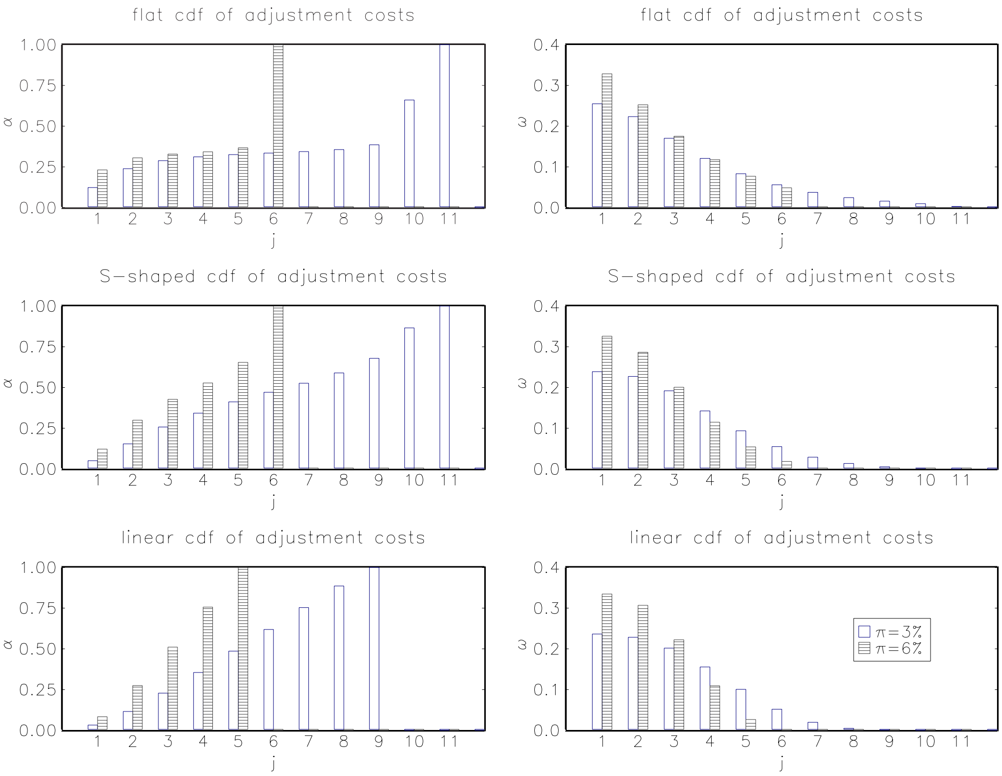

Figure 3: Characterization of price-setting behavior along steady state.

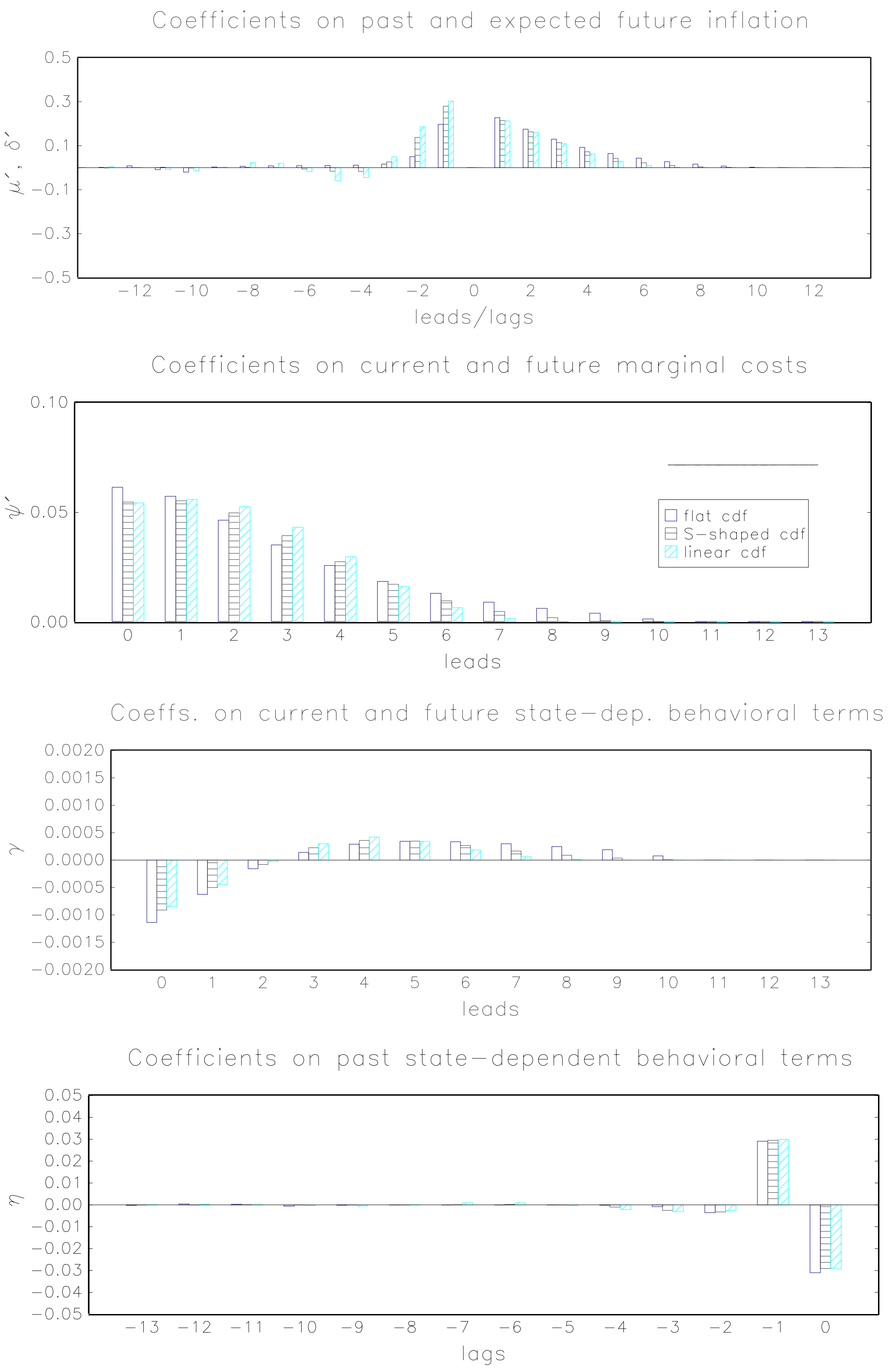

Figure 4: SDPC coefficients across three adjustment cost cdf, $\pi_{ss} = 3\%$.

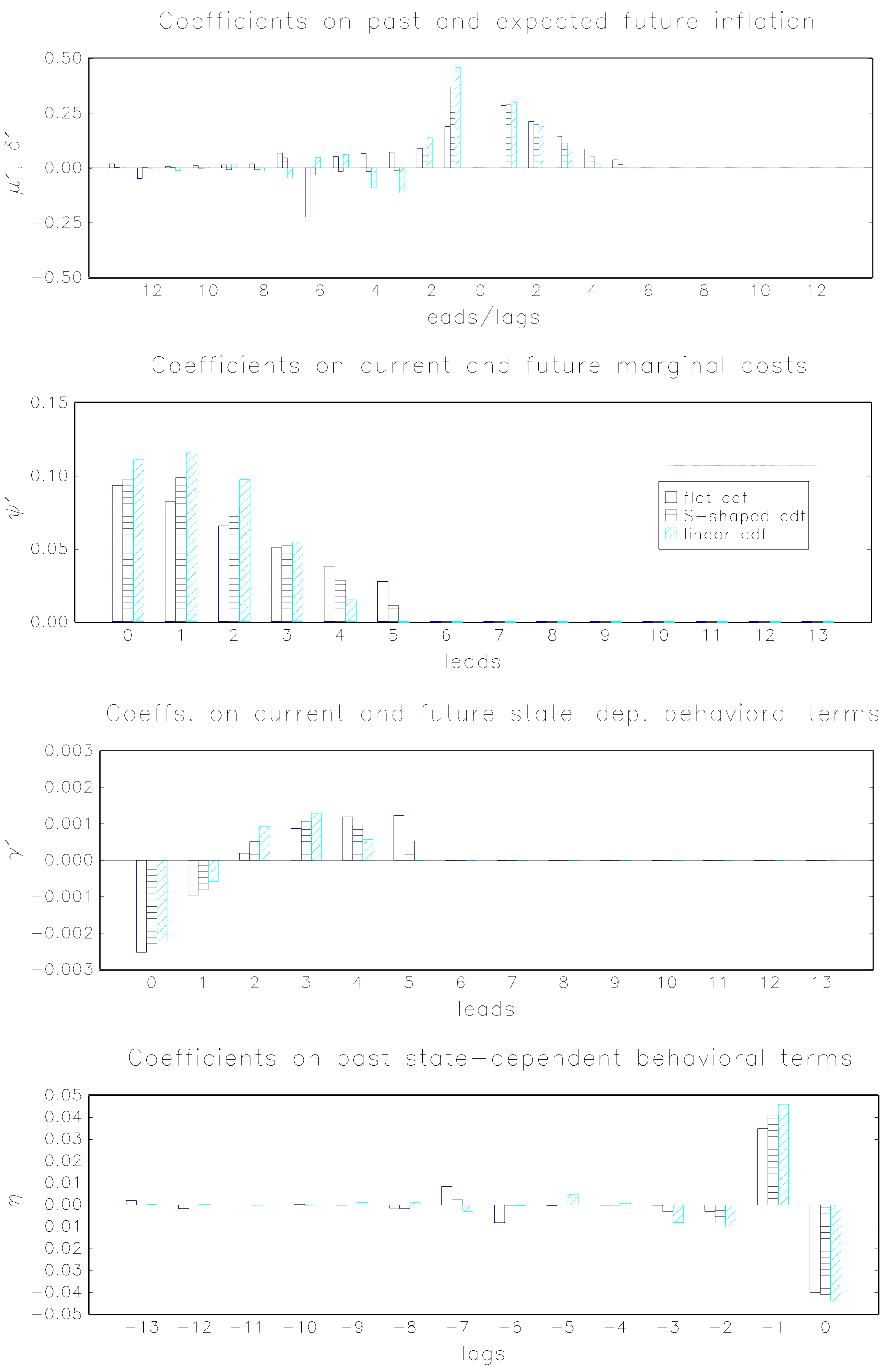

Figure 5: SDPC coefficients across three adjustment cost cdf, $\pi_{ss} = 6\%$.

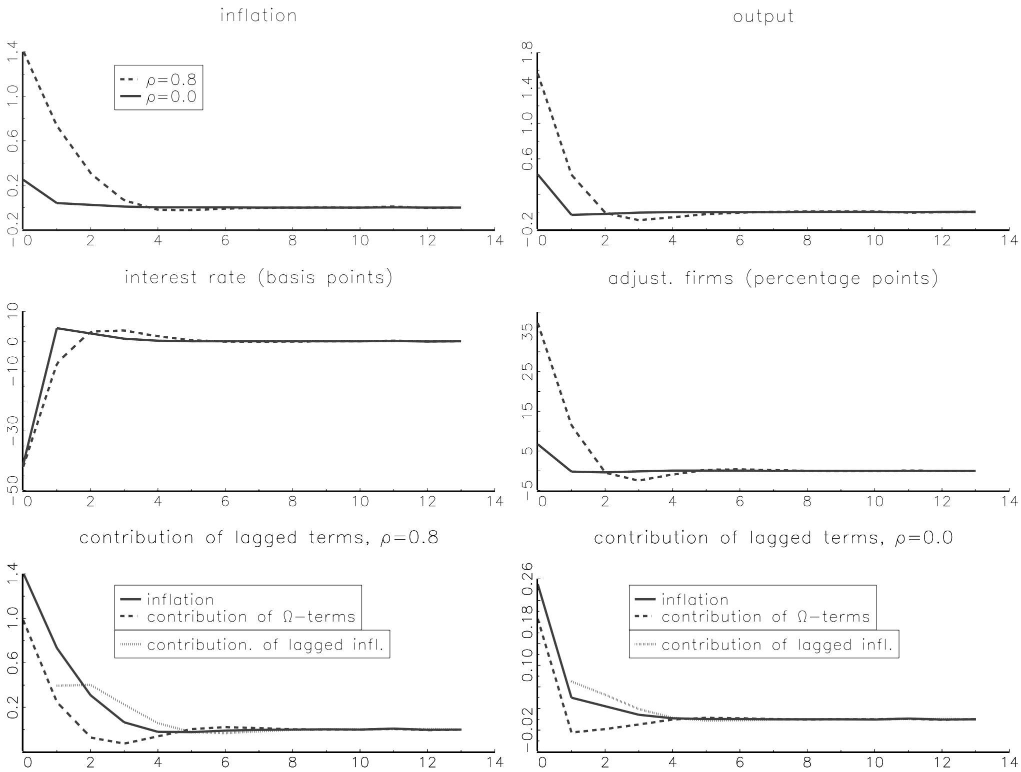

Figure 6: Responses to an expansionary interest rate shock (100 basis points): high policy inertia ($\rho = 0.8$) vs. no policy inertia ($\rho = 0$). SDP model with S-shaped distribution of adjustment costs, $\pi_{SS} = 3\%$.

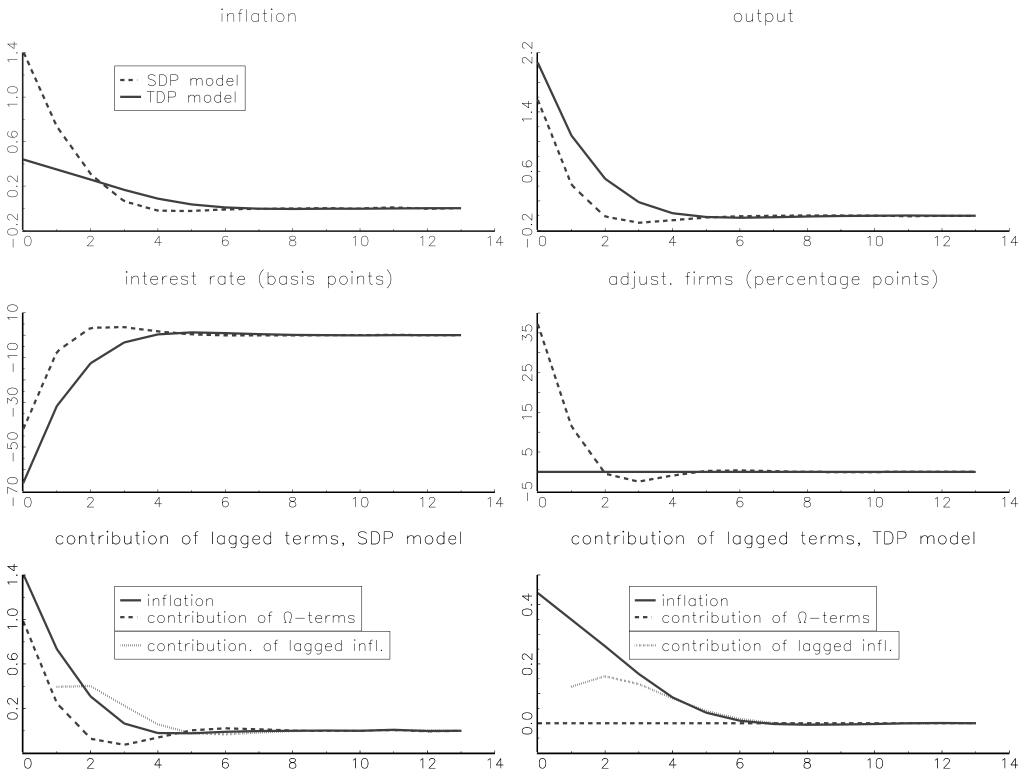

Figure 7: Responses to an expansionary interest rate shock (100 basis points): SDP model vs. TDP model. S-shaped distribution of adjustment costs, high policy inertia, ($\rho = 0.8$) $\pi_{SS} = 3\%$.

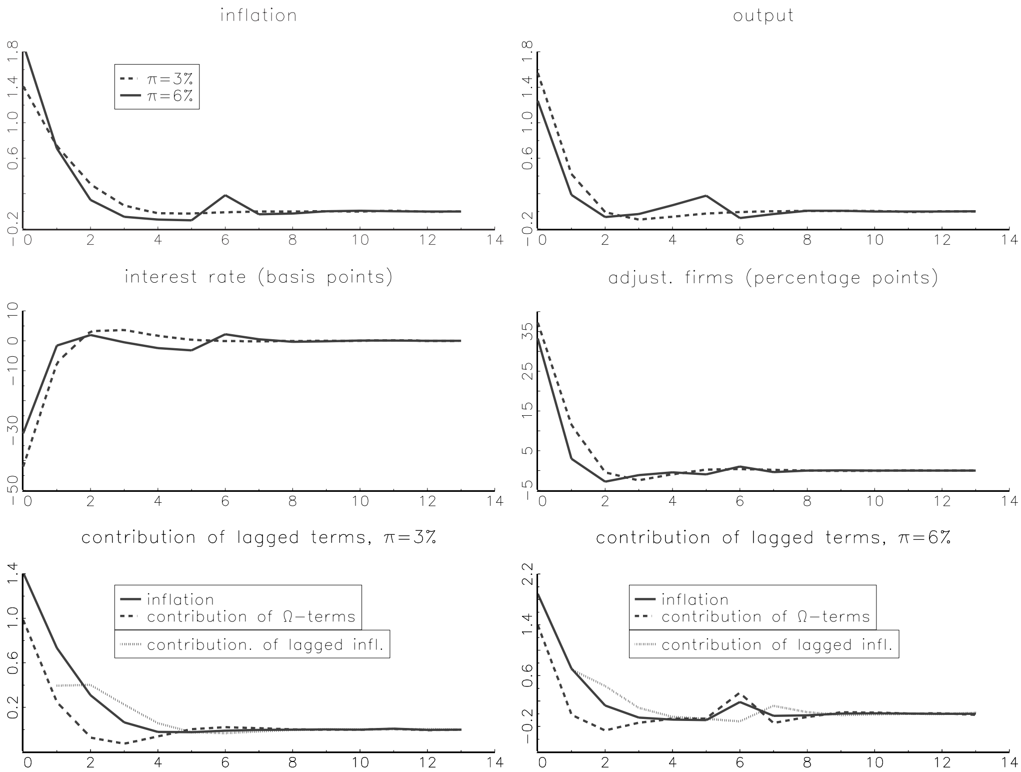

Figure 8: Responses to an expansionary interest rate shock (100 basis points): $\pi_{SS} = 3\%$ vs. $\pi_{SS} = 6\%$. SDP model with S-shaped distribution of adjustment costs, high policy inertia ($\rho = 0.8$).

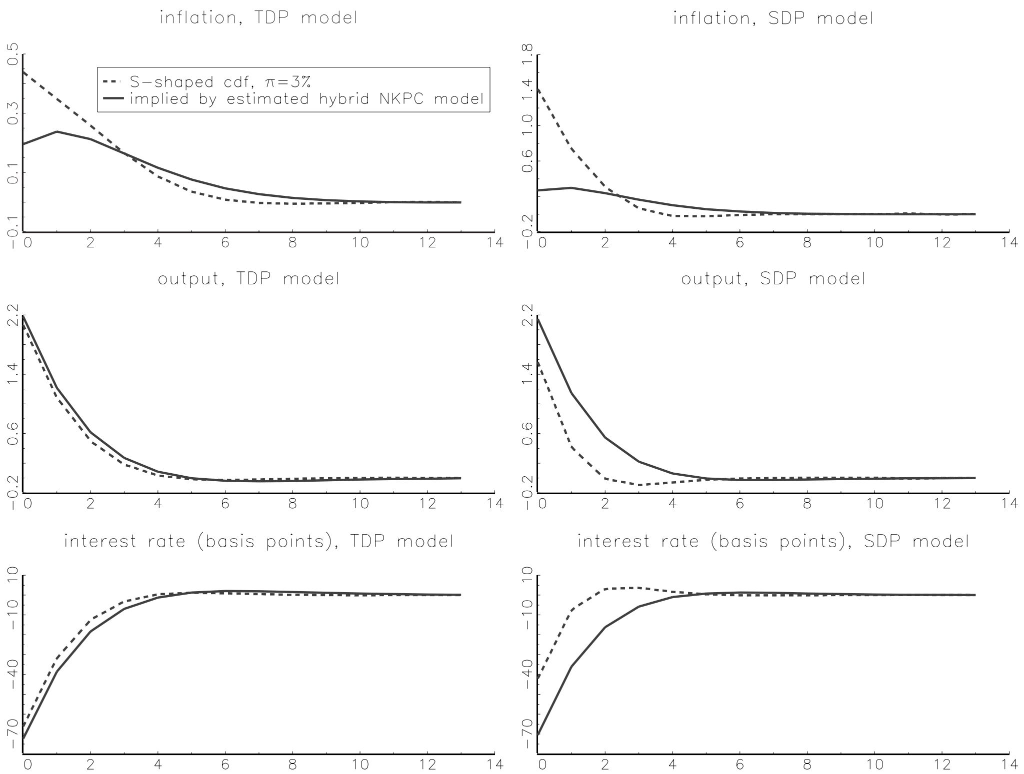

Figure 9: Responses to an expansionary interest rate shock (100 basis points). SDP model with S-shaped distribution of adjustment costs and $\pi_{SS} = 3\%$ vs. model with estimated hybrid NKPC: high policy inertia ($\rho = 0.8$).

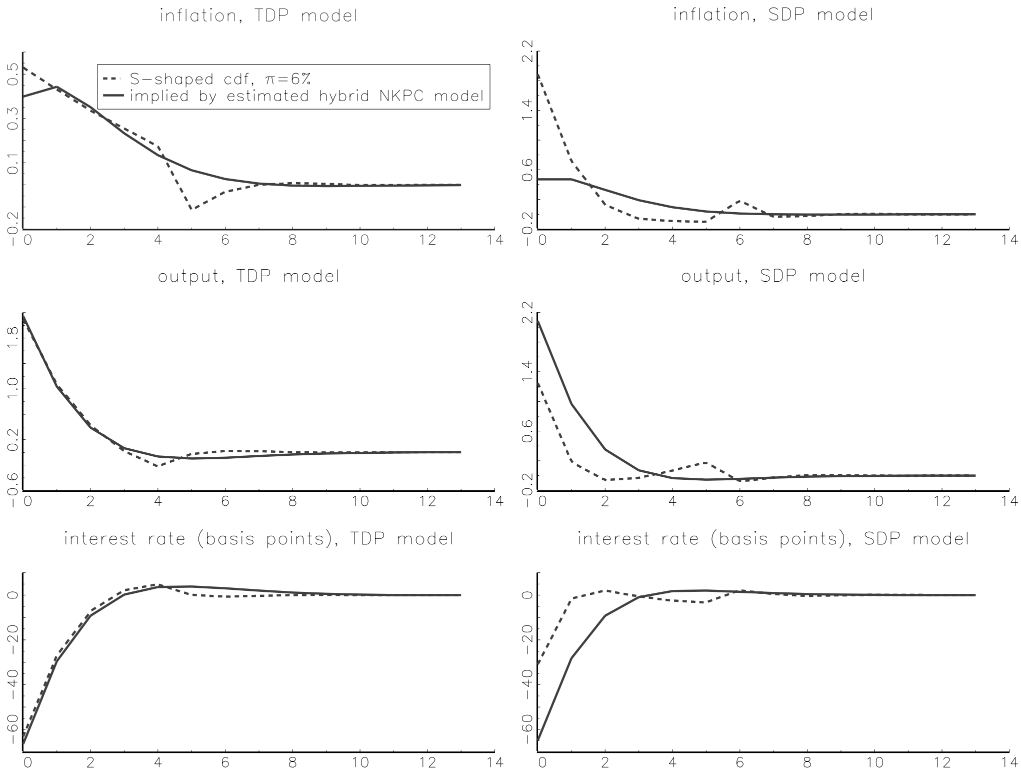

Figure 10: Responses to an expansionary interest rate shock (100 basis points). SDP model with S-shaped distribution of adjustment costs and $\pi_{SS} = 6\%$ vs. model with estimated hybrid NKPC: high policy inertia ($\rho = 0.8$).