Repo Markets, Counterparty Risk,

and the 2007-2009 Liquidity Crisis*

Christian Ewerhart**

Jens Tapking***

October 2, 2009

*) The opinions expressed herein are those of the authors alone and do not necessarily reflect the views of the European Central Bank. First version: October 2007. For useful comments and discussions, we thank Helmut Bester, James Chapman, Darrell Duffie, Marco Galbiati (discussant), Michel Habib, Cornelia Holthausen, David Lando, Marcel Naas (CEO Eurex Repo), Martina Nardon (discussant), Kjell Nyborg, Raphael Repullo, and Pierre-Olivier Weill. We also thank seminar participants at the European Central Bank, the University of Bonn, on the joint Banque de France / ECB conference "Liquidity in interdependent transfer systems," the Econometric Society Meetings in Pittsburgh, the EEA Meetings in Milan, at a panel discussion with the Eurex Repo Forum, and at the Bank of Canada Workshop "Payments, Collateral, and Liquidity." This research has been financially supported by NCCR Finrisk.

**) University of Zurich and NCCR Finrisk; postal address: Chair for Information Economics and Contract Theory, Winterthurerstrasse 30, CH-8006 Zurich, Switzerland; e-mail: christian.ewerhart@iew.uzh.ch; phone: 41-44-6343733; fax: 41-44-6344978.

***) European Central Bank.

Repo Markets, Counterparty Risk,

and the 2007-2009 Liquidity Crisis*

Abstract. A standard repurchase agreement between two counterparties is considered to examine the endogenous choice of collateral, the feasibility of secured lending, and the welfare impact of the central bank's collateral framework. As an important innovation, we allow for two-sided counterparty risk. It is shown that efficient repo contracting typically leads to non-negligible exposures for both sides of the market. As a consequence, there is a joint benefit of using the most liquid and least risky assets as collateral in market transactions first. Moreover, expected utilities of borrower, lender, and central bank may all increase when a broader range of assets is accepted as collateral by the central bank

Keywords. Counterparty risk, repurchase agreements, collateral, liquidity, haircuts, welfare.

JEL-Codes. G21 - Banks; Other Depository Institutions; Micro Finance Institutions; Mortgages; E51 - Money Supply; Credit; Money Multipliers; G11 - Portfolio Choice; Investment Decisions; G32 - Financing Policy; Financial Risk and Risk Management; Capital and Ownership Structure.

Introduction

Standard (sale and) repurchase agreements, or repos (RPs) in short, are used by both private and public counterparties to conveniently swap cash against collateral for a pre-specified period of time. In a typical contract, the lender of cash is compensated by an interest that is calculated from the transaction's nominal value, the term, and the so-called repo rate. Moreover, a haircut is applied to the collateral to limit the lender's exposure to counterparty risk. Indeed, the lender faces the combined risk that the borrower is unable to repay the principal plus interest, and that at the same time the liquidation value of the collateral falls short of the lender's claim. Putting up more collateral keeps this risk contained.

The repo segment has gained considerable importance in international money markets. For instance, daily turnover in the euro repo market has approximately doubled between 2002 and 2007, while the unsecured market segment has been expanding only moderately over the same period.[1] For the U.S. market, Demiralp et al. (2006) write that "the overall repo market is reportedly far larger than the market for federal funds and overnight interbank Eurodollars." The growth of international repo markets can be attributed to a wide range of factors including an increasing reliance on innovative strategies of funding and leveraging, benefits from banking regulation, a high degree of standardization in the contract documentation, and a prominent role of the instrument in central banks' implementation frameworks.[2]

The theoretical analysis of repurchase agreements started with with a seminal contribution by Duffie (1996) who pointed out that when owners of a specific asset incur frictional costs from using the asset as collateral, the repo rate for the asset may fall significantly below the repo rate charged for general collateral. Moreover, through its impact on funding conditions, such specialness is predicted to add a premium to the asset's market price. In a number of recent papers, this theoretical prediction on competitive repo markets has been empirically confirmed from different perspectives.[3]

[1] See ECB (2007a).

[2] There is also an increasing interest in national markets. See, for instance, Baba and Inamura (2004), Fan and Zhang (2007), Jordan and Kugler (2004), and Wetherilt (2003).

[3] See, in particular, Jordan and Jordan (1997), Buraschi and Menini (2002), and Krishnamurthy (2002). Vayanos and Weill (2008) use a repo search market to explain yield differences between on-the-run and off-the-run Treasury bonds.

An assumption underlying this existing theory of the repo market is that there is an investor (the "Short") who seeks to get hold of a well-specified asset through the repo market transaction. However, it has been noted that repo markets are generally open not only to investors interested in a specific security, but also to investors that are interested primarily in the cash side of the transaction. That is, there are also repurchase agreements that are driven mainly by either the funding motive of the cash borrower or by the deposit motive of the cash lender.[4] In these cases, the choice of collateral becomes part of the negotiation. As a practical matter, the difference in the motive for approaching the market not only needs to be revealed early in the negotiation, but is also reflected in differences in the margining, which is done either in cash or in collateral. Moreover, in the case of cash-driven repos, the repo rate for less liquid collateral may also exceed the rate for general collateral.[5] The present paper aims at exploring the determinants of collateral in such cash-driven repurchase agreements. To this end, we introduce counterparty risk into a model of bilaterally negotiated repurchase agreements.

Two empirical regularities have motivated this route of inquiry. One observation is that typically only collateral of the very high quality is accepted in the repo market. The second is that with of the advent of the market turmoil, collateral standards in the market tightened further, while the quality of collateral held with the Eurosystem declined.[6]

To establish the first regularity, we compare the collateral used in the European repo market with the collateral used in repo auctions conducted by the European Central Bank (ECB). Specifically, as shown in Table I, the collateral used during 2006 and 2008 in the private euro repo market has been mostly government bonds. Illiquid and risky assets such as asset-backed securities (ABS) are *not* commonly employed as collateral in the private bilateral repo market. This situation stands in stark contrast with the composition of collateral held with the ECB that accepts a wide range of assets including government bonds (issued either by central or regional authorities), bank bonds (both uncovered and covered), corporate bonds, ABS, other marketable securities, and credit claims. In fact, only 15% of assets deposited for

Table I about here

[4] For evidence, see Buraschi and Menini (2002).
[5] See, for instance, Griffiths and Winters (1997).
[6] The evidence for the U.S. is discussed in Section VI.

use as collateral in Eurosystem credit operations were issued by governments in 2008 (down from 29% in 2006). As Table I indicates, the bulk of central bank collateral in the euro area has been composed of less liquid asset types such as uncovered bank bonds and asset-backed securities.

The second regularity in the data relates to the market turmoil. Following the summer of 2007, requirements on collaterals imposed by cash lenders in the interbank market became even stricter than they usually are. For instance, Frediani et al. (2007, pp. 15-16) report that the share of structured securities used as collateral in so-called tri-party repos had fallen from 35 percent to 25 percent between June 1 and September 14, 2007, with ABS Auto, Card, CDOs, and MBS the most affected through the subprime crisis.[7] This is consistent with observations by Comotto (2008, p. 19) who writes that "Concern over the quality of collateral could explain the reduction in the share of tri-party repos, which has been the preferred way of managing non-government collateral. It definitely explains [...] the unusually high share of government bond collateral in tri-party repos." In contrast, the composition of central bank collateral has shown just the opposite development. Indeed, as Table II summarizes, the share of illiquid and relatively risky assets, here asset-backed securities, has increased significantly since the beginning of the turbulences in August 2007.

Table II about here

To better understand these observations, the present paper takes a closer look at the role of collateral in interbank lending relationships. A scenario is studied in which two counterparties, a borrower of cash and a lender of cash, negotiate simultaneously about (a) the collaterals to be used, (b) the haircut, and (c) the repo rate. In contrast to the existing literature, we allow for two-sided counterparty risk, i.e., we allow for the possibility that not only the borrower, but also the lender may default. This assumption proves to have crucial consequences for the economic determinants of collateral. The analysis will also enable us to formally discuss welfare implications of the central bank's collateral farmework.

With two-sided risk, the bilateral negotiation can be expected to lead to an agreement that balances financial benefits and risks on both sides of the transac-

[7] In a tri-party repo, counterparties sign an additional agreement with a custodian who determines daily valuations for the usually less liquid collateral assets.

tion. Specifically, the lender may be willing to accept a somewhat lower repo rate in exchange for a somewhat increased haircut, as a higher haircut implies better protection. Conversely, the borrower may be willing to provide somewhat more collateral for a somewhat lowered repo rate. Even when collateral bears no opportunity cost, there is a trade-off here for the borrower because of the risk that collateral deposited by the borrower may get lost in the lender's insolvency mass. In fact, this is a critical feature that distinguishes repo contracts from, e.g., mortgage loans.[8] An efficient bargaining outcome is achieved, therefore, by making the marginal rate of substitution between haircut and repo rate congruent between the two counterparties. It turns out that, if collateral is not perfect, i.e., if price fluctuations or a bid-ask spread are possible, then it is typically efficient to expose both counterparties to some credit risk. In fact, as we will show, exceptions from this rule may occur only when collateral is, as we will say, either insured or of junk quality, or else if the borrower's collateral is exploited through the transaction. Moreover, this conclusion does not depend on the counterparties' risk attitudes.

The efficiency of bilateral exposure is what ultimately drives our second main result. This result says that, provided that collaterals can be ranked in a linear way along the riskyness and illiquidity dimension, efficient repo transactions will make use of the most liquid and the least risky assets of the borrower as collateral first. Thus, under this assumption, in a bilateral transaction between two counterparties that may each default with positive probability, good collateral drives bad collateral out of circulation, suggesting an analogy with Gresham's law for commodity money.

We go on and study the economic feasibility of secured contracting under market stress. It is shown that if the most liquid and least risky assets of the borrower are still relatively illiquid or risky then the two counterparties may, even under symmetric information and zero opportunity costs of collateral, not be able to agree on a transaction at all. This outcome occurs in particular if default probabilities are perceived as non-negligible, which might relate our analysis to the developments in money markets following early August 2007. The imperfection of the repo market under two-sided credit risk also adds to existing structural explanations of the microstructure of the money market based on asymmetric information, and suggests a

[8] See, for instance, Stigum (1989), Corrigan and de Terán (2007), or Garbade (2006).

new theoretical rationale for central counterparties. Last but not least, this second result allows us to apply an argument that has been put forward by Kashyap et al. (2002).

The final part of the paper explores the question of how the central bank's collateral framework affects overall welfare. We show first that, with a linear ranking on collaterals, the expansion of the set of eligible collateral is typically accompanied by a replacement of liquid collateral by illiquid collateral. I.e., in contrast to the prediction obtained for market transactions, bad collateral drives out good collateral in lending relationships with the central bank. As we discuss, this observation might have a bearing on fiscal competition between euro area member countries. We then show that less restrictive eligibility may lead to a strict Pareto improvement for lender, borrower, and central bank. Thus, a theoretical rationale for changing the collateral framework during market distress can be given.

The literature on collateralized lending is vast and divers. Only four of the main strands will be mentioned here. The role of *borrowers' hidden characteristics* has been stressed by Stiglitz and Weiss (1981) pointed out that credit rationing may occur as a consequence of asymmetric information either at a pre- or post-contracting stage. Bester (1985), in particular, has argued that in the case of pre-contracting asymmetric information, the self-selection problem may be resolved when commitment to costly collateral is feasible for entrepreneurs with relatively low risks. As noted by Hellwig (1987), it may be hard to decide whether collateral may serve as an effective sorting device in a given credit market, because the nature of the refined equilibrium will typically depend on the way in which competition is modeled under adverse selection. Berger and Udell (1990) even conclude that existing theoretical and empirical approaches to the use of collateral still have to be reconciled. A potential solution has been offered more recently by Coco (1999). As an alternative to this self-selection view on the use of collateral, models have been developed that assign a key role to *borrowers' observable characteristics*. Boot et al. (1991) developed an approach based on the idea that collateral mitigates moral hazard on the side of the borrower, so that borrower risk is positively correlated with collateral usage. Manove and Padilla (1999, 2001) argue that collateral induces banks to do less careful screening of loan applicants. In an empirical study of

Spanish data, Jiménez and Saurina (2004) come to the conclusion that the incentive view explains the use of collateral better than the self-selection view. More recently, Jiménez et al. (2006) have found support for the hypothesis that, for the case of business credit, observable risks matter in decisions about collateral. Booth and Booth (2006) likewise conclude that collateral pledges are correlated with riskier loans. Inderst and Mueller (2007) show that collateral can help to resolve an inefficiency in credit markets with imperfect competition. Another strand of literature is composed of papers that focus on the *risk characteristics of collateral assets*. Barro (1976) performs comparative statics in a model with one-sided strategic default that occurs whenever the value of collateral falls below principal plus interest. There is a divergence between the borrower's expected interest rate, the explicit loan rate, and the lender's expected interest rate. When the lender's expected return is competitive, then the explicit loan rate and the borrower's expected interest rate rise with the loan-value ratio, the competitive rate, and the transaction costs associated with default. Assuming likewise strategic default, Benjamin (1978) shows that the characteristics of the collateral asset, such as marketability and expectations about its future price impact on the payments schedule of the debt. An important paper is Plaut (1985) who clarifies the role of collateral characteristics under one-sided default risk. Specifically, his analysis shows that riskier assets may make better or worse collateral, that assets with higher expected returns may make better or worse collateral, that collateral assets that are 'overvalued' relative to their value under the capital asset pricing theory may be preferred, in which case these assets would only be used for collateral. Cossin and Hricko (2003) study, from an asset-pricing perspective, the impact of risky collateral on credit risk. The role of correlation is stressed. Still another strand of literature, related to the present study through its focus on liquidity provision, is concerned with *rediscounting and payments*. Freeman (1996) considers a model with overlapping generations in which fiat money is used both for consumption and for repayment of loans. It is shown that an elastic provision of liquidity within the period can resolve temporary tensions in liquidity demand without affecting price levels for the consumption good. Mills (2006) considers liquidity provision from a mechanism design perspective, and shows in particular that distortions may occur when the central bank requires collateral that offers alternative benefits for borrowers in the economy. Further references will be given later.

The rest of the paper is structured as follows. Section I introduces the model and discusses efficient risk mitigation in repurchase agreements involving two-sided default risk. In Section II, we describe an important effect that might explain why interbank transactions rely predominantly on relatively liquid and riskless collateral such as government bonds. Section III studies frictions in the repo market. The residual nature of central bank collateral is discusses in Section IV. Section V derives welfare implications of the collateral framework. The case of the U.S. is discussed in Section VI. Section VII concludes. All proofs can be found in the Appendix.

I. The basic model

We consider a money market over three dates, date 0, date 1, and a terminal date 2. There are $1 + m$ assets: cash and $m \geq 1$ collaterals $j = 1, ..., m$. Cash is riskless and does not carry interest, whereas collaterals may be risky and illiquid, as will be made precise later. There are two counterparties $i = 1, 2$, thought of as commercial banks, which draw utility from terminal payoffs. Bank i's utility function $u_i(.)$ is assumed twice continuously differentiable with $u_i'(.) > 0$. Utility in case of own default is normalized to zero.

At date 0, counterparties hold an initial endowment of cash and collaterals. Let $q_j^i \geq 0$ denote bank i's initial endowment of collateral j, for $i = 1, 2$ and $j = 1, ..., m$. Either bank is required to hold a certain amount of cash (potentially zero) at the end of date 1. Cash held at date 1 in excess of these minimum reserve requirements will be of no value.[9] Moreover, bank i's initial endowments $q_0^i \geq 0$ in cash are such that these reserve requirements would be fulfilled without slack in the absence of further transactions.

Between dates 0 and 1, there is an exogenous customer request to transfer an amount $\lambda > 0$ of cash at date 1.[10] With equal probability, the transfer will be from Bank 1 to Bank 2 or vice versa from Bank 2 to Bank 1. The absolute size λ of the liquidity shock will initially be normalized to one. To compensate for the liquidity shock, the bank receiving the transfer will seek to become the *lender* (of cash) in the

[9] I.e., there is no carry-over provision.

[10] Alternatively, an investment opportunity might arise that requires a transfer to another bank.

money market, while the bank sending the funds will seek to become the *borrower* (of cash). We will refer to the former as bank bank i_L, to the latter as bank bank i_B.

At date 2, the state of nature realizes: In the good state G, neither the lender nor the borrower defaults; in state B, only the borrower defaults; and in state L, only the lender defaults. Denote by $\pi_\omega = \pi_\omega(i_B, i_L)$ the probability that state ω realizes, where $\omega \in \{G, B, L\}$. Clearly, $\pi_G + \pi_B + \pi_L = 1$.[11] The following assumption marks the departure from the existing theoretical literature on collateralized lending.[12]

Assumption 1. (Two-sided credit risk) $\pi_B > 0$, $\pi_L > 0$.

To study the determinants of collateral in repurchase agreements, we will focus on a specific contractual form that is motivated by the industry standard.[13] Specifically, it is assumed that counterparties may sign a *standard repurchase agreement (SRA)* $C = (y, h, r)$, which is composed of a collateral composition y, a haircut $h \geq -1$, and a repo rate r; a *composition* is a collection $y = (y_1, ..., y_m)$ of weights $y_j \geq 0$, $j = 1, ..., m$, such that $\sum_{j=1}^{m} y_j = 1$. The agreement foresees that the lender promises to transfer one unit of cash at date 1. The borrower in turn promises to deposit $1+h$ units of collateral of composition y with the lender at date 1. I.e., the common haircut h is applied to all assets.[14] At date 2, in the good state, the borrower will repay the principal plus an interest r. The lender, in turn, redelivers the collateral to the borrower.

The following definition draws a line between a repo contract and either unsecured loans or securities lending.

Definition 1. A contract (y, h, r) is called a *true* SRA if $h > -1$ and $r > -1$.

Indeed, a proper repurchase agreement always involves the transfer of a nonzero amount of collateral to the lender, and always foresees a strictly positive repayment from the borrower to the lender at the end of the term. If, however, $h = -1$,

[11] More generally, default probabilities might depend on the terms of the repo transaction. For a discussion of this possibility, see the working paper version (Ewerhart and Tapking, 2008).

[12] See the references given at the end of the Introduction.

[13] The overwhelming majority of market transactions seems to be based on the so-called Global Master Repurchase Agreement (cf. TBMA, 2000). Counterparties are free to use alternative contracts, but this is rarely done in practice because of significant legal risks. See Garbade (2006), and Comotto (2009). For an analysis of collateralized debt that is not imposing a standardization assumption, see Lacker (2001).

[14] Equivalently, the contract could specify a separate haircut for each collateral asset.

then no collateral is used, and the contract corresponds to an uncollateralized loan. Similarly, if $1+r \leq 0$, then the lender of cash does not receive any repayment, which reduces the agreement to a securities lending transaction.

Collaterals may be illiquid and risky. To capture illiquidity, we will allow for the possibility that the bid-price for selling and the ask-price of buying differ. This is in line both with empirical measures of liquidity and its theoretical foundation in terms of adverse selection (Glosten and Milgrom, 1985). To accomodate risk, we allow for uncertainty about bid and ask prices. In fact, an instructive special case is that of a perfectly liquid collateral, where bid and ask prices are merged into a single market price, which may then be uncertain.

Formally, let $\widetilde{p}_b^j$ denote the ex-ante uncertain liquidation value (or bid price) of asset j, conditional on the borrower's default. Similarly, let $\widetilde{p}_a^j$ denote the ex-ante uncertain replacement cost (or ask price) of asset j at date 2, conditional on the lender's default.[15] The respective distributions of the vectors $(\widetilde{p}_b^1, ..., \widetilde{p}_b^m)$ and $(\widetilde{p}_a^1, ..., \widetilde{p}_a^m)$ on $\mathbb{R}_{\geq 0}^m$ are assumed to be commonly known. For a given composition $y = (y_1, ..., y_m)$, let $\widetilde{p}_b = \sum_{j=1}^m y_j \widetilde{p}_b^j$ and $\widetilde{p}_a = \sum_{j=1}^m y_j \widetilde{p}_a^j$ denote the conditional liquidation value and replacement cost of the collateral portfolio net of haircuts. To avoid uninteresting cases, we assume throughout the paper that distributions of prices have interval supports. Moreover, let $\widetilde{v}_b = (1+h)\widetilde{p}_b$ and $\widetilde{v}_a = (1+h)\widetilde{p}_a$ denote the liquidation value and replacement cost of the collateral portfolio. To refer to the realization of a random variable, the tilde will be dropped. For instance, p_b^j denotes the realization of $\widetilde{p}_b^j$, etc.[16]

We will now specify the payoff consequences of the repurchase agreement in the various states of nature. To see why there is some flexibility, consider the case of borrower default as an example. In this case, as the interbank contract matures, the lender's claim on repayment of principal and interest will meet the borrower's non-monetary claim on the collateral.[17] Consistent with market practice, we will assume

[15] More generally, these prices reflect the respective second-best alternatives.

[16] Our analysis does not presuppose marketability of collateral assets at the time of contracting. However, there is one interpretation of the model in which all collateral assets are perfectly liquid at the time of contracting and possess a market price of 1 at that stage. Clearly, if collateral assets are assumed to be marketable both at the time of contracting and in the good state, outright trading becomes an alternative to the repo, and expected round-trip costs may impose a bound on implicit opportunity rates (cf. Section III).

[17] This might lead easily to a legal dispute. In fact, a "cherry-picking" insolvency agent of the

for this case that borrower's claim for delivery of the collateral is automatically transformed into a monetary claim which can be netted with the lender's claim of repayment. A similar assumption is made in the case that the lender defaults.[18]

Assumption 2. (Netting) *In state B, the borrower's claim on the collateral is replaced by a claim of payment of v_b. In state L, the borrower's claim is replaced by a claim of payment of v_a. Subsequently, the claim of the non-defaulting party vis-à-vis the defaulting party may be used to set off the claim of the defaulting party vis-à-vis the non-defaulting party.*

For instance, in state B, the lender's claim on repayment of principal plus interest is protected by the collateral only if the realized liquidation value v_b of the collateral portfolio at date 2 covers $1+r$. Thus, the lender incurs a potential loss of $\min\{v_b - (1+r); 0\} \leq 0$ compared to state G. Similarly, in state L, the borrower has a potential loss of $\min\{(1+r) - v_a; 0\} \leq 0$, where v_a is the realized replacement cost of the collateral portfolio at date 2. For ease of exposition, we will assume that the potential loss is completely written off.[19]

Assumption 3. (Subordination) *Any positive net claim of the non-defaulting party vis-à-vis the defaulting party will be completely lost.*

As a final matter, the agreement must be specific about what happens when the defaulting party holds a gross claim that exceeds the claim of the non-defaulting party. For instance, in state B, the lender would want to sell the collateral and keep haircuts plus any potential interim increase in the market price. Similarly, in state L, the borrower would want to profit from a decline in the collateral value. Motivated by the details of the standard documentation, such surprise profits will be excluded in the sequel.[20]

Assumption 4. (No windfall profits) *If the defaulting party has a positive*

defaulting borrower would refuse payment, while demanding delivery of the collateral!

[18] Indeed, the Global Master Repurchase Agreement foresees a set-off of mutual claims in case of one-sided insolvency, where the non-defaulting party valuates collateral claims either by actual, quoted, or estimated market prices. Bliss and Kaufman (2006) offer an insightful discussion of netting provisions in the related case of derivatives.

[19] Our results should remain valid if the share of the net claim actually lost is always strictly positive and weakly increasing in the net claim.

[20] Our results should continue to hold if a positive share of windfall profits can be realized, provided that the share is weakly declining in these profits. This includes the case where windfall profits could be fully kept.

net claim vis-à-vis the non-defaulting party then the non-defaulting party has the obligation to pay the net claim (to the insolvency agent of the defaulting party).

Collectively, Assumptions 2 through 4 make the contract comprehensive and thereby determine conditional payoffs. Utility is derived from the instrument's isolated return, with variations in the valuations of collaterals being of a temporary nature. In particular, the borrower experiences no changes in utility from (interim) increases to the value of collaterals unless the lender defaults during the term of the repo, in which case the collateral needs to be replaced.[21]

Write $u_L(.) = u_{i_L}(.)$ and $u_B(.) = u_{i_B}(.)$. Let $\widetilde{u}_L$ and $\widetilde{u}_B$, respectively, denote the lender's and borrower's uncertain utility at the time of contracting. We assume that expected utilities are well-defined. Then the lender's expected utility at the time of contracting is given by

$$E[\widetilde{u}_L] = \pi_G u_L(r) + \pi_B E[u_L(\min\{\widetilde{v}_b - 1; r\})], \tag{1}$$

where $E[.]$ denotes the unconditional expectation operator, and the minimum takes care of Assumption 4. Thus, conditional on the borrower's default, the lender is basically (i.e., ignoring the bid-ask spread) exposed to a short European put option on the collateral, where the strike price is determined by the degree of overcollateralization, i.e., by the haircut. Similarly, the borrower's expected utility at the time of contracting, reads

$$E[\widetilde{u}_B] = \pi_G u_B(-r) + \pi_L E[u_B(\min\{1 - \widetilde{v}_a; -r\})], \tag{2}$$

which amounts to being exposed to a short European call option on the collateral, conditional on the lender's default. Note that in contrast to a vulnerable option (see Johnson and Stulz, 1987) that loses the option character in the default case, a repurchase agreement transforms into an option-like exposure *with* the default of either counterparty.

A scenario will be considered now in which lender and borrower negotiate over the parameters of the repurchase agreement. Apparently, the bargaining set for borrower and lender will consist of all standard repurchase agreements (y, h, r) that

[21] It is this focus on liquidity risk that will lead to different expressions for expected utilities compared to what one would obtain in Merton's (1974) option pricing approach to collateralized lending.

satisfy

$$y_j(1+h) \leq q_j^{i_B} \qquad (j = 1, ..., m). \tag{3}$$

A standard repurchase agreement (y, h, r) that satisfies (3) will be called *valid*. Thus, validity captures the fact that the collaterals promised in the contract must be available for transfer from the borrower's balance sheet at the beginning of the term. A valid SRA (y^*, h^*, r^*) is *efficient* when the pair of counterparties' expected utilities resulting from the contract is not dominated, in the Pareto sense, by expected utilities resulting from any other valid SRA.

It turns out that an efficient repurchase agreement will typically expose both lender and borrower to non-trivial counterparty risk. To make this statement precise, the following definitions will be useful. For a given collateral composition y, let $\underline{p}_b = \underline{p}_b(y)$ denote the minimum of the support of $\widetilde{p}_b$, and let $\overline{p}_a = \overline{p}_a(y)$ denote the supremum of the support of $\widetilde{p}_a$.[22]

Definition 2. Collateral is *imperfect* if $\overline{p}_a > \underline{p}_b$ for any composition y.

Definition 3. Collateral is *insured* if there is some y such that either $\underline{p}_b > 0$ and $\underline{p}_b$ is a mass point of $\widetilde{p}_b$, or if $\overline{p}_a < \infty$ and $\overline{p}_a$ is a mass point of $\widetilde{p}_a$.

To understand the latter definition, consider the example that the borrower offers as collateral a number of stocks which are held together with an identical number of put options. Should the value of collateral drop below the strike price, the option goes into the money, so that with positive probability, the lender could realize the strike price in the market. Thus, there would be a mass point in the distribution of $\widetilde{p}_b$. This type of mass point is excluded when collateral is not insured.

We will see below that imperfect and uninsured collateral necessitates an exposure for the lender in any efficient repurchase agreement. However, the borrower need not have an exposure under these conditions. Intuitively, this will be the case if a borrower does not have sufficient collateral. Two further definitions capture the additional conditions needed, namely that collateral can be liquidated with some positive value, and that collateral is not used up through the transaction.

Definition 4. Collateral is *of junk quality* if there is some composition y such that $\widetilde{p}_b \equiv 0$ with probability one.

[22] Clearly, $\overline{p}_a$ may be infinite.

Definition 5. The borrower's collateral is *not exploited* in the valid SRA (y, h, r) if there is an $h' > h$ such that (y, h', r) is valid.

We can state our first main result.

Theorem 1 (Exposure). *Impose that Assumptions 1 through 4 hold, and that collateral is imperfect and not insured. Consider any efficient true SRA* (y, h, r)*. Then* $\text{pr}\{\widetilde{v}_b < 1 + r\} > 0$*, where* $\text{pr}\{.\}$ *denotes the unconditional probability. If, in addition, the borrower's collateral is neither of junk quality nor exploited, then* $\text{pr}\{\widetilde{v}_a > 1 + r\} > 0$.

Thus, under two-sided credit risk, a negotiated standard repo will typically expose both lender and borrower to some counterparty risk. To see why this is true, assume that thepo transaction,e lender ird repurchase as fully protected against any losses from the repo transaction. Then a marginal decrease of the haircut applied to the collateral portfolio may introduce the risk of a small loss for the lender, but this loss occurs, if the collateral is uninsured, only with a small probability. As a consequence, the expected utility of a fully protected lender is not really lowered by a marginal concession in the haircut. However, for the borrower who is not fully protected, a marginal decrease in the haircut reduces losses that occur with strictly positive probability. Therefore, when the lender is fully protected, the lender's reservation price (in terms of the repo rate) for a small concession in the haircut is nil, while the borrower's willingness to pay is strictly positive. Hence, full protection of the lender cannot be efficient.

A similar argument shows that full protection of the borrower cannot be efficient under additional conditions. Indeed, when the borrower is fully protected, the lender would marginally benefit from a compensated increase in the haircut, at least if collateral is not of junk quality, while the borrower would be indifferent at the margin. Moreover, such a compensated increase in the haircut is feasible provided that the borrower's collateral is not fully exploited. Combining both insights, we find that optimal risk sharing using non-junk, yet imperfect and uninsured collateral either exploits the borrower's collateral or results in exposures for both sides of the market.

While it is clear that the only outcome consistent with economic rationality is a Pareto efficient agreement, the question might arise whether such a contract always exists. The answer is positive provided expected utilities depend continuously on the parameters of the contract. To understand why, assume that a given SRA promises expected utilities $E[\widetilde{u}_B]$ and $E[\widetilde{u}_L]$. Since expected utilities depend continuously on the contract (y, h, r), there is a closed set of contracts that yield a weak Pareto improvement. In fact, this set is compact, because y is bounded by definition, $h \geq -1$ is bounded from above by validity, and r is bounded through the fact that the borrower's (lender's) expected utility is downwards-sloping (upwards-sloping) in r. Hence, the set of contracts that allow a weak Pareto improvement is compact and (trivially) nonempty. Thus, under the continuity assumption made, any SRA is indeed weakly dominated by some Pareto efficient SRA.

II. Optimal collateral

A perspective that is sometimes taken is that there is a conflict of interests between lender and borrower insofar that the lender is interested to obtain the best collateral from the borrower, while the borrower is interested to forward only the worst collateral. As we will see in the present section, this perspective is not completely accurate because it neglects that counterparties negotiate, together with the composition of collateral, also about haircut and interest rate. More specifically, we show now that Theorem 1 has testable implications for the use of collateral in repurchase agreements provided that collaterals can be *ordered linearly* along the liquidity and risk dimension. To our knowledge, this is the first result of this type in the literature. We start with an example.

Example 1. Lender and borrower negotiate over the terms of a standard repurchase agreement. Two assets can be used as collateral. Asset 1 has an expected conditional liquidation value of $E[\widetilde{p}_b^1] = 0.98$, and an expected conditional replacement cost of $E[\widetilde{p}_a^1] = 1.02$. Asset 2 has an expected conditional liquidation value of $E[\widetilde{p}_b^2] = 0.97$, and an expected conditional replacement cost of $E[\widetilde{p}_a^2] = 1.05$. We wish to formalize the notion that asset 2 is more risky and less liquid than asset 1. To this end, we compare prices normalized with respect to the respective *mid price*

$$\mu_j = E[\frac{\widetilde{p}_a^j + \widetilde{p}_b^j}{2}] \tag{4}$$

for asset $j = 1, 2$. We assume that the distribution of normalized liquidation values for Asset 2 is dominated by that of normalized liquidation values for Asset 1 in (unconditional) *mean-decreasing* second-order stochastics, while conversely, the distribution of normalized replacement costs for Asset 2 is dominated by that of normalized replacement costs for Asset 1 in *mean-increasing* second-order stochastics.[23] This assumption captures the notion that in expectation, the bid-ask spread for Asset 1 is strictly smaller than that of Asset 2, which is a usual measure for higher liquidity of Asset 1. Moreover, the assumption also captures the notion that realized prices are more difficult to predict for Asset 2 that for Asset 1, which reflects some form of riskyness.

The counterparties consider first a collateral composition $(y_1, y_2) = (80\%, 20\%)$, combined with a haircut of $h = 4\%$, and a repo rate of 2%. Expected conditional prices of the collateral portfolio are then given by

$$E[\widetilde{v}_b] = (1+4\%)(80\% \cdot 0.98 + 20\% \cdot 0.97) = 1.017, \quad (5)$$

$$E[\widetilde{v}_a] = (1+4\%)(80\% \cdot 1.02 + 20\% \cdot 1.05) = 1.067. \quad (6)$$

It can now be shown that, provided that the borrower has unused quantities of collateral 1, there is scope for a Pareto improvement. As an illustration, consider the adjusted collateral composition $(y_1', y_2') = (100\%, 0\%)$. In this situation, there are several combinations of haircut and repo rate that achieve a Pareto improvement. For instance, counterparties might want to combine a haircut of $h = 4.208\%$ with an unchanged repo rate.[24] Conditional prices of the adjusted collateral portfolio are then given by

$$E[\widetilde{v}_b'] = (1+4.208\%)(100\% \cdot 0.98 + 0\% \cdot 0.97) = 1.021, \quad (7)$$

$$E[\widetilde{v}_a'] = (1+4.208\%)(100\% \cdot 1.02 + 0\% \cdot 1.05) = 1.063. \quad (8)$$

Through the adjustment, the expected conditional liquidation value of the collateral portfolio has increased and the expected conditional replacement cost has declined, which is individually beneficial for both counterparties. Moreover, as Asset 2 is more risky and less liquid than Asset 1, the adjustment strictly reduces the volatility of conditional prices. Thus, the improvement of the collateral allows lowers risk exposures for both counterparties.

[23] See Assumption 5 below for a precise definition.

[24] This haircut has been constructed as in the proof of Theorem 2, for $\delta = 0.1996$. The haircut is increasing here because collateral 2 has a higher expected appreciation than collateral 1.

Example 1 illustrates the possibility that, provided that rational counterparties reach an efficient outcome, and credit risk is two-sided, good collateral is used up first in the interbank transaction. The relatively illiquid and risky collateral is not used because it would not allow counterparties to balance their risk exposures as efficiently as the relatively more liquid and less risky collateral. Example 1 thereby captures an effect that might explain the first observation discussed in the Introduction, viz. that interbank repos rely predominantly on relatively liquid and riskless collateral. This effect depends critically on the two-sidedness of default risk, i.e., the effect does not occur with one-sided risk.[25]

The rest of this section extends Example 1 into a general statement. To obtain a clear-cut result also in the case of more than two assets, the following assumption will be imposed.

Assumption 5. (Commonality) *Fix $\widetilde{p}_b^0 \equiv \widetilde{p}_a^0 \equiv 1$ and $\mu_0 = 1$. Then, for $j = 1, ..., m$,*

$$\frac{\widetilde{p}_b^j}{\mu_j} \equiv \frac{\widetilde{p}_b^{j-1}}{\mu_{j-1}} - \widetilde{\varepsilon}_b^j \text{ and } \frac{\widetilde{p}_a^j}{\mu_j} \equiv \frac{\widetilde{p}_a^{j-1}}{\mu_{j-1}} + \widetilde{\varepsilon}_a^j, \tag{9}$$

where $\mu_1, ..., \mu_m > 0$ are constants, and $\{\widetilde{\varepsilon}_b^1, ..., \widetilde{\varepsilon}_b^m\}, \{\widetilde{\varepsilon}_a^1, ..., \widetilde{\varepsilon}_a^m\}$ are collections of independent random variables satisfying $E[\widetilde{\varepsilon}_b^j] > 0, E[\widetilde{\varepsilon}_a^j] > 0$ for $j = 1, ...m$.

For three or more collateral assets, Assumption 5 is more restrictive then the second-order stochastic dominance assumption made in the example above because of the required independence of the error terms across pairs of consecutive collaterals. Without independence, the possibility of risk diversification may make a portfolio consisting of several risky assets less risky than a third collateral, even though it dominates the others individually in terms of risklessness. This possibility is excluded by Assumption 5.

With these preparations, the following result is obtained.

Theorem 2 (Gresham's law for collateral, market version). *Assume that collateral is uninsured. Then, under Assumptions 1 through 5, any efficient true SRA (y^*, h^*, r^*) entails the collateral composition*

$$y^* = (\frac{q_1^{i_B}}{1+h^*}, ..., \frac{q_{j^*-1}^{i_B}}{1+h^*}, 1 - \frac{\sum_{j=1}^{j^*-1} q_j^{i_B}}{1+h^*}, \underbrace{0, ..., 0}_{m-j^* \text{ times}}), \tag{10}$$

[25] See, in particular, Plaut (1985).

where j^* *is the smallest index such that* $\sum_{j=1}^{j^*} q_j^{i_B} \geq 1 + h^*$.

Thus, when the scope for diversification across collaterals is limited, then it is of mutual interest of borrower and lender to use up the most liquid and least risky collateral first.[26]

III. Feasibility of the secured market transaction

While the unsecured credit market is known to break down under stress (Flannery, 1995), it was typically understood that collateral ensures access to money markets also when credit risk is non-negligible (see, e.g., Allen et al., 1989). In the present section, it is shown for a fixed composition of collateral that interbank lending may not be feasible even if collateral causes no opportunity costs, information is symmetrically distributed, and physical transaction costs are zero. Sufficient conditions for market frictions are two-sided default risk and imperfect collateral.

Indeed, counterparties will approve a contract only when it is individually rational to do so. We will assume here exogenous outside options granting utility levels of $\underline{u}_L = (\pi_G + \pi_B)u_L(r^D)$ to the lender and $\underline{u}_B = (\pi_G + \pi_L)u_B(-r^B)$ to the borrower, respectively, where r^D is the lender's implicit opportunity rate for deposits, and r^B is the borrower's implicit opportunity rate for borrowings. Figure 1 illustrates indifference curves resulting from outside options. Shown are, as a function of the haircut h, the highest acceptable repo rate $\rho^B(h)$ for the borrower and the lowest acceptable repo rate $\rho^D(h)$ for the lender. A repurchase agreement will be signed by both borrower and lender if and only if $\rho^D(h) \leq \rho^B(h)$ for some h. Clearly, the use of better collateral would lower $\rho^D(h)$ and increase $\rho^B(h)$, making the market transaction more likely.

Figure 1 about here

Before we discuss the general nature of the market friction, it is instructive to look at an example. Explicit conditions for the feasibility of a repo transaction can be given in the case of risk-neutrality and pure illiquidity.

Example 2. Assume that lender and borrower are risk-neutral. Assume also that the liquidation value and the replacement cost of collateral at date 2 are known to be P_b and P_a at date 1, where $P_a > P_b > 0$. Then the lender's expected utility from

[26] Also security-driven repurchase agreements tend to concentrate on liquid assets. This is because of dynamic shorting strategies that depend on the trader's ability to close the position at short notice. We are grateful to Darrell Duffie for pointing this out to us.

contracting is given by

$$E[\widetilde{u}_L] = \pi_G r + \pi_B \min\{(1+h)P_b - 1; r\}. \tag{11}$$

For convenience, impose parameter restrictions $\pi_G > 0$, $r^B > -1$, and $r^D > -1$. When the lender is exposed to credit risk, i.e., if $(1+h)P_b < 1+r$, then a comparison of the resulting expression in (11) with the available outside option shows that for a deposit rate r of at least

$$r \geq r^D + \frac{\pi_B}{\pi_G}(1 + r^D - (1+h)P_b), \tag{12}$$

the lender would be willing to contract against a haircut of h. On the other hand, a lender that is not exposed to credit risk would require at least a deposit rate of $r \geq r^D$. Thus, in either case,

$$\rho^D(h) = r^D + \frac{\pi_B}{\pi_G} \max\{1 + r^D - (1+h)P_b; 0\}. \tag{13}$$

Analogously, one can determine the borrower's break-even rate as

$$\rho^B(h) = r^B + \frac{\pi_L}{\pi_G} \min\{1 + r^B - (1+h)P_a; 0\}. \tag{14}$$

Since the expressions (13) and (14) are piecewise linear, a straightforward graphical argument shows that an agreement will be signed if and only if conditions

$$\rho^D(\frac{1+r^B}{P_a} - 1) > r^B \text{ and } \rho^B(\frac{1+r^D}{P_b} - 1) < r^D \tag{15}$$

are simultaneously satisfied. Thus, an agreement will be signed if and only if

$$\frac{\pi_B}{\pi_G + \pi_B} \cdot \frac{P_a - P_b}{P_a} \leq \frac{r^B - r^D}{1 + r^B} \tag{16}$$

and

$$\frac{\pi_L}{\pi_G + \pi_L} \cdot \frac{P_a - P_b}{P_b} \leq \frac{r^B - r^D}{1 + r^D} \tag{17}$$

hold. It is instructive to compare these conditions with the fact that when $\pi_L = 0$ and collateral is ample, it is always efficient to have a repo transaction.

The following result confirms that the possibility of a friction is not driven by the simplifying assumptions of the example.

Theorem 3. (Market imperfection) *Impose Assumptions 1 through 4, and assume that collateral is imperfect. Then, for any interest rate level* $r_0 > 0$ *and for* *any collateral composition y, there are opportunity rates r^B and r^D for borrower and lender, respectively, such that $r^B > r_0 > r^D$, and such that no market transaction is individually rational for both lender and borrower.*

Theorem 3 captures a type of friction in interbank markets that had gone unnoticed so far. The critical issue to note is that again, with one-sided default risk, the market will always work provided that the borrower still holds ample collateral, even if of very low quality. However, when default risk is two-sided, capital mobility is imperfect since not only the lender, but also the borrower cares about potential losses.[27]

Illustrations. Theorem 3 might relate to three distinct developments during the recent liquidity crunch. First, a market failure might have been a motivation for the ECB and the Swiss National Bank to offer U.S. Dollar funds to euro-area and Swiss counterparties since December 2007 through a participation in the U.S. Fed's term auction facility (TAF). To understand why, note that a euro-area counterparty in search of dollar funding would in principle have been able to access euro funding through the Eurosystem's open market operations. Apparently, however, there has been a problem with turning this euro funding into dollars, which normally could be done by a forex swap transaction with some U.S. bank. One explanation might be differences in time zones. Our model suggests an alternative explanation. Specifically, while forex swaps differ from repos in many aspects, the underlying economic structure is similar when one currency is interpreted as the collateral security. Our theory would then suggest that if there is two-sided counterparty risk, and if exchange rates are volatile, it may be difficult for euro-area counterparties to obtain dollar funding.

Another visible market disruption that could be mentioned here is the repo run on dealers in the U.S. in March 2008, related to the near-fall of Bear Stearns. The Bank of England (2008, p. 9) writes that "Bear Stearns was not only unable to obtain funding in unsecured markets, but also could not secure funds against high-quality collateral. That led to a rapid fall in its sizable reserves of liquid assets

[27] The friction should be stronger when the value of collateral is positively correlated with the borrower's equity. Conversely, the analysis suggests that a borrower may find it easier to transact in the interbank market by offering collateral whose market value is positively correlated with the lender's equity, such as the lender's own uncovered bonds.

... and the firm was forced to seek support from JPMorgan Chase & Co. and the Federal Reserve Bank of New York." Other sources have been less enthusiastic about the quality of collateral that Bear Stearns was able to offer. Apparently also other primary dealers had difficulties in obtaining short-term funding. In a quite unconventional move, the Federal Reserve decided, effective on Tuesday, March 11, 2008, to offer primary dealers an amount of $200 bn in Treasury bonds and bills in exchange for mortgage-backed securities. The new element of that so-called Term Securities Lending Facility (TSLF) has been that individual transactions have a term of 28 days (rather than overnight).[28] The Fed went further by implementing, effective March 16, the so-called Primary Dealer Credit Facility (PDCF) that offers penalty-free access to overnight repo loans against a range of collaterals that strictly includes securities accepted in open market operations. In contrast to the discount window, this facility is open to primary dealers of the Federal Reserve Bank of New York, yet not to depository institutions. In the specific case of Bear Stearns, funding problems might have resulted either from insufficiency of collateral (which would preclude lending even in the case of one-sided credit risk), fears by potential lenders of getting involved in complicated default procedures, and even outright predatory behavior. Still, Theorem 3 captures an effect that generally might have increased frictions in the repo market, and which could have contributed to the debacle.

Finally, it will be recalled that on August 9, 2007, problems with subprime loans in the U.S. led, among other things, to a sudden dry-out of the market for asset-backed commercial paper (ABCP), which had served as a source of funding for so-called structured investment vehicles (SIVs). Banks with credit commitments vis-à-vis such vehicles had an unexpected increase in liquidity needs. Illiquid assets held by the vehicles, such as collateralized debt obligations (CDOs), could no longer serve as collateral. At the same time, those investors that had refused to roll over commercial paper have received significant cash transfers to their bank accounts. Kashyap et al. (2002) have put forward the argument that commercial banks have the unique ability to pool imperfectly correlated liquidity risks resulting from loan

[28]More recently, the Bank of England has implemented a similar measure, the so-called Special Liquidity Scheme, which offers terms of one year, renewable to up to three years.

commitments and deposit contracts. Gatev and Strahan (2006) find empirical support for a similar mechanism in the context of the commercial paper market. The stylized facts mentioned above might relate our analysis to the pooling argument. Specifically, one could argue that before the turbulences, numerous banks might have decided to specialize and to exploit the synergies identified by Kashyap et al. across the money market, assuming that liquidity risks can be shared effectively with other banks. Then, during the turbulence, some of those banks (e.g., investment banks) would have to satisfy a loan commitment, while others would receive a liquidity inflow in the form of additional deposits. However, in view of Theorem 3, a market transaction that matches supply and demand may not be guaranteed. Thus, using the terminology introduced by Kashyap et al., with specialized banks, synergies across banks may become a prerequisite to synergies across the two sides of the balance sheet.

Implications for market structure. Under normal market conditions, the analysis suggests that also in the secured segment of the money market, a counterparty may be constrained to trading with a counterparty that has a relatively good credit standing. Under normal market conditions, this effect should be reflected in the topology of the interbank network. Two types of regularities are predicted. First, counterparties with an excellent rating may be able to intermediate in the repo market. In practice, this should lead to a *two-tiered structure* of the repo market, just as predicted for the unsecured market by Freixas and Holthausen (2004). The second regularity should be the emergence of *central counterparty* trading, where a clearing house with good standing intermediates the transaction by becoming a counterparty to both the lender and the borrower.

IV. Central bank collateral and haircuts

In Section II, it has been shown that with two-sided credit risk, and under additional assumptions on collateral characteristics, counterparties seek to use the most liquid and least risky assets as collateral first. What is the impact of this effect on central bank collateral? Probably none if the central bank determines the collateral. However, in the Eurosystem's open market operations, and more recently also in certain facilities offered by the Federal Reserve, the counterparties have a significant discretion concerning the collaterals to be forwarded.[29] This suggests that assets that are less liquid and more risky are more likely to end up on the balance sheet of central banks that offer this type of flexibility to market participants. In the present section, we outline a simple extension of the basic model that captures this point. Specifically, we will examine stable compositions of central bank collateral under a given collateral framework. As a byproduct, insights on the role of haircuts for the composition of central bank collateral are obtained.

Thus, extending the set-up considered so far, there is now a central bank, and it is assumed that Bank 1 and Bank 2 have debt positions $D_1 > 0$ and $D_2 > 0$, respectively, outstanding vis-à-vis the central bank from date 0 onwards. The central bank's *collateral framework* will be formalized as a pair (J, η), where $J \subseteq \{1, ..., m\}$ is the *set of eligible assets* accepted as collateral, and $\eta = \{\eta_j\}_{j \in J}$ is a *haircut rule* that determines a haircut $\eta_j \geq -1$ for each eligible asset $j \in J$. For simplicity, we will consider only policies where $J = \{1, ..., m_{\text{CB}}\}$ for some $1 \leq m_{\text{CB}} \leq m$. Asset m_{CB} may then be interpreted as the lowest-quality collateral accepted by the central bank.

Denote by $\theta^i = (\theta^i_1, ..., \theta^i_m)$ the composition of bank i's collateral deposits, net of haircuts, with the central bank at date 0. By definition of the central bank's collateral framework, $\theta^i_j = 0$ for $j \notin J$. Against this backdrop, the liquidity shock occurs. In contrast to the discussion so far, the size of the liquidity shock λ is now the realization of a random variable $\widetilde{\lambda} > 0$ with support $\mathbb{R}_{\geq 0}$.

Definition 6. For a given collateral framework (J, η), a pair (θ^1, θ^2) of collateral compositions will be called *stable* if there is, for any $i_B \in \{1, 2\}$ and for any $\lambda > 0$, either a market imperfection or an efficient true SRA that does not require the replacement of central bank collateral.

Under the Assumptions of Theorem 2, banks have an interest to liberate good collateral held with the central bank to the extent that it will serve a purpose in the repo market. It is clear, therefore, that the only remaining reason for a bank to *not* optimize collateral held with the central bank is that collateral is correctly expected not to be used. This possibility is excluded by the following assumption.

Assumption 6. (Market usage of eligible collateral) *For any $j \in J$, and*

[29] See also Section VI.

any $(q_1^{i_B}, ..., q_m^{i_B}) \in \mathbb{R}^m_{\geq 0}$, *there is a* $\lambda > 0$ *such that counterparties conclude an SRA* (y, h, r) *with* $h > -1$ *and* $y_j > 0$.

We are ready to formally capture the residual nature of central bank collateral.

Theorem 4. (Gresham's law for collateral, central bank version) *Fix a collateral framework* (J, η), *and impose that collateral is uninsured. Then, under Assumptions 1 through 6, the unique stable pair* $(\theta^1(J,\eta), \theta^2(J,\eta))$ *of collateral compositions is given by*

$$\begin{aligned} \theta^i(J,\eta) \quad = \quad (& \underbrace{0,...,0}_{j_*(i)-1 \text{ times}}, 1 - \sum_{j=j_*(i)+1}^{m_{\text{CB}}} \frac{q_j^i}{(1+\eta_j)D_i}, \frac{q_{j_*(i)+1}^i}{(1+\eta_{j_*(i)+1})D_i}, ... \\ & ..., \frac{q_{m_{\text{CB}}}^i}{(1+\eta_{m_{\text{CB}}})D_i}, \underbrace{0,...,0}_{m-m_{\text{CB}} \text{ times}}), \end{aligned} \tag{18}$$

where $j_*(i)$ *denotes the largest index such that* $\sum_{j=j_*(i)}^{m_{\text{CB}}} q_j^i/(1+\eta_j) \geq D_i$, *and* $i = 1, 2$.

Theorem 4 captures the observations discussed in the Introduction by suggesting that commercial banks have an incentive to forward first the least liquid and most risky asset accepted by the central bank. Indeed, as more liquid and less risky assets allow a more efficient mitigation of exposure risk in interbank repo transactions, there is an endogenous opportunity cost of taking high-quality collateral to the central bank. The crucial point to note is that this effect takes place essentially irrespective of haircuts. Moreover, the residual nature of central bank collateral should become more evident in times of increasing perceptions of default risks.

Haircuts and fiscal competition. The result above might also help to clarify the role of haircuts applied by central banks. Haircuts have always been an instrument of risk management, both for commercial banks and for central banks. However, as Theorem 4 suggests, haircuts cannot properly be used to steer the composition of central bank collateral. Indeed, the opportunity costs of using the least liquid and most risky assets accepted by the central bank will remain negligible as long as the borrower's holdings of such assets are ample enough. Changing haircuts should therefore not be sufficient to induce counterparties to take more liquid and less risky collateral to the central bank. In particular, Theorem 4 suggests that haircuts are not suitable as an instrument for fine-tuning the composition of central bank collateral along, say, issuing fiscal authorities. This provides a clear-cut answer to a question of significant practical interest (cf. Buiter and Sibert, 2005).

V. Welfare implications

To the extent that the availability of relatively liquid and riskless collateral is a prerequisite for interbank lending, a policy issue arises when collateral standards set by the central bank have the potential to withhold such high-quality collateral from the market. The example below captures the point that a change in the collateral framework may be socially desirable irrespective of distributional concerns.

Example 3. This example continues Example 2. Consider a set-up with two assets. One asset has a certain liquidation value of P_b and a certain replacement cost of P_a, as before. The other asset has a liquidiation value $0 < P_b' < P_b$ and, to keep things simple, the same replacement cost P_a as the first asset. Assume that the central bank initially insists on the liquid asset, and that, in fact, all of the borrower's liquid assets are deposited with the central bank. Assume also that default risks and illiquidity of the less liquid asset are so severe that, while amply available on the borrower's balance sheet, no market transaction comes about. I.e.,

$$\frac{\pi_B}{\pi_G + \pi_B} \cdot \frac{P_a - P_b'}{P_a} > \frac{r^B - r^D}{1 + r^B}, \tag{19}$$

$$\frac{\pi_L}{\pi_G + \pi_L} \cdot \frac{P_a - P_b'}{P_b'} > \frac{r^B - r^D}{1 + r^D}. \tag{20}$$

To see the impact of a relaxed collateral policy, assume that the central bank accepts the less liquid asset, but with a haircut $\eta_2 = (1 + r_{CB})(D_2/P_b') - 1$ that takes away any default risk from the central bank. Then, the borrower will substitute the liquid assets held with the central bank by illiquid assets. Moreover, provided the liquid asset is liquid enough such that at least one of conditions (19), (20) holds with reversed inequality when P_b' is replaced by P_b, the borrower will offer it to the lender as collateral who will accept. If this happens, then a strict Pareto improvement is obtained because at least one of the market participants has a strict gain in expected utility, while the central bank at least keeps its expected utility level.

Thus, in the example, expected utilities for both lender and borrower can be increased, while the exposure for the central bank remains unchanged. The increase in welfare is possible since the central bank is more likely to return the collateral than any market player. As a consequence, ample resources of illiquid collateral can be used vis-à-vis the central bank, but not in the market.

The example provides a rationale for the decisions of several central banks to broaden the range of assets accepted as collateral during the financial turmoil. Our framework thereby captures an *informal* argument that has been used quite frequently by policy makers during the 2007-2009 liquidity crisis. It should be noted that the formal argument, however, relies crucially on the two-sided default risk assumption which distinguishes our paper from the existing literature.[30]

As an illustration, we mention the cases of the U.S. Fed, the Bank of England, the Bank of Canada, and the Bank of Australia. Before August 2007, these central banks generally accepted only a quite narrow range of assets, mainly government bonds, as collateral. During the turbulences, however, all of these institutions significantly broadened the range of eligible collateral. Also the Eurosystem, which already had a policy of accepting a relatively broad list of assets as collateral before the outset of the crisis, decided to accept additional classes of collaterals in late 2008. The example provides a rationale for such policy adjustment.

VI. The case of the U.S.

Our analysis has been motivated by Table I: the European Central Bank accepts largely lower-quality collateral in its refinancing operations, while repo transactions between private parties largely take place using more liquid collateral like central government bonds. Interestingly, the situation in the U.S. looks exactly reversed. Prior to the onset of the crisis, the Federal Reserve accepted only very liquid assets (mostly U.S. Treasury securities) in its open market operations. In contrast, repo transactions involving other assets such as mortgage-backed securities (MBS) took place entirely between private parties. While this pattern has changed somewhat since the Federal Reserve began expanding its lending operations, it is not clear from what has been said so far that the current situation in the U.S. resembles that presented in Table I for Europe. To understand why the markets look so different and to see that central bank policies are, in fact, moving closer together as a result of the crisis, we will recall a number of important institutional differences.

It should be noted first that the ECB and the Fed follow quite different approaches with respect to collateral (cf., e.g., ECB, 2007b). In case of the ECB, each

[30] The example above focuses on a revival of disrupted market. Alternatively, one can consider the availability of better collateral in a functioning market. This scenario is explored in the working paper version.

counterparty decides which assets it will use as collateral for credit from the Eurosystem, provided these assets are *eligible*.[31] If a counterparty wants to use mainly ABS rather than government bonds, it can do so. Furthermore, the counterparty does not have to pay a higher interest rate on the credit because of this decision. In the U.S., however, it is predominantly the Fed that defines which types of assets it will receive. For example, in its open market operations the Fed accepts only domestic treasury securities, agency bonds, and agency-backed MBS. The Fed decides how much of these three asset types are respectively accepted. Typically, agency-backed MBS are accepted only at higher rates compared to treasury and agency securities. This is illustrated by the following quote (cf. Federal Reserve Bank of New York, 2008): "In recent years the distribution by collateral tranche of outstanding RPs has been weighted heavily toward the Treasury tranche...until financial market strains appeared in short-term funding markets. At that point dealers' propositions against agency and MBS collateral tranches that it accepts on its RPs became more attractive on a relative basis."

However, there is one important exception. Depository institutions can get credit from the Fed via the Discount Window and the Term Auction Facility (TAF). In either case, the respective depository institution decides which eligible assets it puts forward as collateral. Moreover, the rate to be paid on the credit does not depend on the collateral used. Table III illustrates this point. Specifically, it can be seen that domestic depository institutions mainly put forward highly illiquid assets (non-marketable loans and ABS) as collateral in this context, confirming that banks use less liquid assets as collateral with the central bank not only in the euro area, but also in U.S.[32]

Table III about here

Also the differences between the euro area and the U.S. concerning the usage of collateral in interbank repo transactions have an institutional background. Generally, the euro area repo market appears to be characterized by a higher number of institutional players, and by a somewhat more fragmented clearing and settlement landscape. Moreover, the U.S. market is concentrated on the overnight segment,

[31] Whether a specific asset is eligible or not can be seen immediately from a web document maintained and published by the ECB.

[32] The first descriptive studies available confirm our perspective. See, e.g., Hördahl and King (2008).

while in the euro area, terms between overnight and one year are all common. Furthermore, tri-party repos are more common in the U.S. than in the euro area. Specifically, in the U.S., the tri-party segment corresponds to very roughly one quarter of the repo market, in the euro area, the share has typically been about one tenth of total market turnover. To explain why collaterals not issued by the U.S. government have played a major role in interbank repo markets before the outset of the crisis, one must observe that, before the turmoil, markets for U.S. bonds were typically much deeper than markets for EU bonds. This was the case in particular for the much-used mortgage-backed securities, because projected cash flows tended to be backed by agencies such as Freddie Mac and Fanny Mae, considered safe at the time. Thus, having a prevalent use of non-governmental collateral in the private market is consistent with our theory, provided such collateral is backed by a suitable third party.

VII. Conclusion

Modern funding strategies increasingly rely on repurchase agreements through which cash is exchanged short-term against collaterals of longer maturities. Interestingly, the bulk of such refinancing is based on securities that are very stable in value and actively traded. Minimum standards concerning collateral quality tend to become even more restrictive when interbank market conditions tighten, as during the credit crunch following August 2007. On the other hand, there has been a tendency to deposit more and more illiquid assets for use in central banks' liquidity-providing operations, whenever flexibility has been granted by policy makers.

The present study has derived a number of theoretical predictions that might help to clarify these and related observations. Our key results says it is typically in the best interest of both lender and borrower to be exposed to some credit risk in the repurchase agreement. This result has four main implications. First, if collateral can be ranked linearly along the riskyness and illiquidity dimension, then the most liquid and least risky asset will be preferably used in the interbank market. Second, if the best collateral available is still relatively illiquid or risky, and if there is non-negligible bilateral counterparty risk, then no market transaction may come about at all. This captures a potentially important friction in the repo market. The third implication concerns the use of collateral in liquidity-providing market operations. Imposing again the linear ordering of assets, we have shown that essentially unaffected by the haircut policy, the least liquid and most risky assets will be deposited with the central bank. As a final implication, it was shown that a less restrictive collateral framework may lead to a Pareto improvement for market participants and central bank.

In particular, the analysis provides a theoretical rationale for the decisions of several central banks, including the Federal Reserve, to broaden the range of assets accepted as collateral during the financial turmoil.

Appendix: Proofs

Proof of Theorem 1. Fix some efficient true SRA (y, h, r). As shown in Section I, under Assumptions 2 through 4, the lender's expected utility at the time of contracting is given by

$$E[\widetilde{u}_L] = \pi_G u_L(r) + \pi_B \int u_L(\min\{(1+h)p_b - 1; r\}) dF_b(p_b), \tag{21}$$

where $F_b(p_b) = \text{pr}\{\widetilde{p}_b \leq p_b\}$ denotes the cumulative distribution function of $\widetilde{p}_b$. The integrand in (21) will be $u_L(r)$ for all $p_b > p^* = (1+r)/(1+h)$, and $u_L((1+h)p_b - 1)$ otherwise. Consequently,

$$\begin{aligned} E[\widetilde{u}_L] &= (\pi_G + \pi_B(1 - F_b(p^*)))u_L(r) + \pi_B \int_{p_b \leq p^*} u_L((1+h)p_b - 1) dF_b(p_b) \\ &= (\pi_G + \pi_B)u_L(r) - \pi_B(1+h) \int_{p_b \leq p^*} F_b(p_b) u_L'((1+h)p_b - 1) dp_b, \end{aligned} \tag{22}$$

where we applied integration by parts on the Stieltjes integral. Using Leibniz' rule and $\partial p^*/\partial r = 1/(1+h)$, one obtains

$$\frac{\partial E[\widetilde{u}_L]}{\partial r} = (\pi_G + \pi_B(1 - F_b(p^*)))u_L'(r). \tag{23}$$

A similar calculation starting from (22) and involving $\partial p^*/\partial h = -p^*/(1+h)$ yields

$$\begin{aligned} \frac{\partial E[\widetilde{u}_L]}{\partial h} &= -\pi_B \int_{p_b \leq p^*} F_b(p_b) u_L'((1+h)p_b - 1) dp_b \qquad (24) \\ &\quad + \pi_B p^* F_b(p^*) u_L'(r) - \pi_B(1+h) \int_{p_b \leq p^*} F_b(p_b) p_b u_L''((1+h)p_b - 1)) dp_b \\ &= \pi_B p^* F_b(p^*) u_L'(r) - \pi_B \int_{p_b \leq p^*} F_b(p_b) d(p_b u_L'((1+h)p_b - 1)) \qquad (25) \end{aligned}$$

Integrating again by parts,

$$\frac{\partial E[\widetilde{u}_L]}{\partial h} = \pi_B \int_{p_b \leq p^*} p_b u_L'((1+h)p_b - 1) dF_b(p_b). \tag{26}$$

From (23) and (26), the lender's marginal rate of substitution between haircut and repo rate is given by

$$\text{MRS}_{h,r}^L = \frac{\partial E[\widetilde{u}_L]/\partial r}{\partial E[\widetilde{u}_L]/\partial h} = \frac{(\pi_G + \pi_B \text{pr}\{\widetilde{p}_b > p^*\}) u_L'(r)}{\pi_B \int_{p_b \leq p^*} p_b u_L'((1+h)p_b - 1) dF_b(p_b)}. \tag{27}$$

A completely analogous derivation yields the borrower's marginal rate of substitution

$$\text{MRS}_{h,r}^B = \frac{\partial E[\widetilde{u}_B]/\partial r}{\partial E[\widetilde{u}_B]/\partial h} = \frac{(\pi_G + \pi_L \text{pr}\{\widetilde{p}_a < p^*\}) u_B'(-r)}{\pi_L \int_{p_a \geq p^*} p_a u_B'(1 - (1+h)p_a) dF_a(p_a)}, \tag{28}$$

where $F_a(.)$ denotes the distribution function of $\widetilde{p}_a$. To provoke a contradiction, assume that the lender is fully protected under (y, h, r), i.e., assume $\text{pr}\{\widetilde{v}_b < 1+r\} = 0$. Since $h > -1$, this implies $\text{pr}\{\widetilde{p}_b < p^*\} = 0$. Hence, $p^* \leq \underline{p}_b$. Moreover, since $r > -1$, also $p^* > 0$. Thus, given that collateral is not insured, p^* is not a mass point of $\widetilde{p}_b$, so that even $\text{pr}\{\widetilde{p}_b \leq p^*\} = 0$. From Assumption 1, we must have $\pi_G + \pi_B \text{pr}\{\widetilde{p}_b > p^*\} > 0$. Thus $\text{MRS}_{h,r}^L = \infty$. On the other hand, $\text{pr}\{\widetilde{p}_b \geq p^*\} = 1$, and so, as collateral is imperfect, $\text{pr}\{\widetilde{p}_a \leq p^*\} < 1$ or, equivalently, $\text{pr}\{\widetilde{p}_a > p^*\} > 0$. Using (28) and again Assumption 1 yields $\text{MRS}_{h,r}^B < \infty$, which implies that counterparties would jointly prefer to use a lower haircut. Contradiction. Hence, $\text{pr}\{\widetilde{v}_b < 1+r\} > 0$. Assume now that in addition to being imperfect and uninsured, the borrower's collateral is not of junk quality, and not exploited. If, under these assumptions, the borrower were fully protected under (y, h, r), i.e., if $\text{pr}\{\widetilde{v}_a > 1+r\} = 0$, then from $h > -1$, we would have $\text{pr}\{\widetilde{p}_a > p^*\} = 0$. Then, clearly, $p^* \geq \overline{p}_a$. As collateral is not insured, p^* cannot be a mass point of $\widetilde{p}_a$, hence even $\text{pr}\{\widetilde{p}_a \geq p^*\} = 0$ and the denominator in (28) vanishes. Moreover, using Assumption 1, one finds $\text{MRS}_{h,r}^B = \infty$, i.e., the borrower would be willing to accept a small increase in the haircut essentially without any compensation in the repo rate. On the other hand, collateral is imperfect, hence $\overline{p}_a > \underline{p}_b$. Using $p^* \geq \overline{p}_a$, this yields $p^* > \underline{p}_b$. But collateral is not of junk quality, so that $\overline{p}_b > 0$. Invoking the assumption of interval supports, we have therefore that $\text{pr}\{0 < \widetilde{p}_b \leq p^*\} > 0$. Hence, using Assumption 1 again, the denominator in (27) does not vanish, and so $\text{MRS}_{h,r}^L < \infty$, i.e., the lender would be willing to compensate the borrower for a non-marginal increase in the haircut by a non-marginal reduction in the repo rate. Since not all collateral is exploited, an increase in the haircut is indeed feasible, and a discrepancy in marginal rates of substitution cannot be efficient. Contradiction. Thus, $\mathrm{pr}\{\widetilde{v}_a > 1 + r\} > 0$, which proves also the second claim. □

Proof of Theorem 2. Consider an efficient true SRA $C = (y, h, r)$ with collateral composition $y = (y_1, ..., y_m)$. By definition, C is valid. It suffices to show that it is Pareto dominated for lender and borrower to simultaneously use one collateral and not fully use up another collateral with a lower index. To provoke a contradiction, assume that $y_{k+1} > 0$ and $(1 + h)y_k < q_k^{i_B}$ for some $k \in \{0, ..., m - 1\}$. From Assumption 5, we know that there are mid prices $\mu_1 > 0, ..., \mu_m > 0$, as well as collections of independent random variables $\{\widetilde{\varepsilon}_b^1, ..., \widetilde{\varepsilon}_b^m\}, \{\widetilde{\varepsilon}_a^1, ..., \widetilde{\varepsilon}_a^m\}$ with strictly positive means, such that for any $j = 1, ..., m$,

$$\frac{\widetilde{p}_b^j}{\mu_j} \equiv \frac{\widetilde{p}_b^{j-1}}{\mu_{j-1}} - \widetilde{\varepsilon}_b^j \text{ and } \frac{\widetilde{p}_a^j}{\mu_j} \equiv \frac{\widetilde{p}_a^{j-1}}{\mu_{j-1}} + \widetilde{\varepsilon}_a^j, \tag{29}$$

where $\widetilde{p}_b^0 \equiv \widetilde{p}_a^0 \equiv 1$ and $\mu_0 = 1$. To achieve a Pareto improvement, we seek a new SRA (y', h', r') with $y' = (y_1', ..., y_m')$ such that notional amounts in each asset class satisfy $y_k'(1 + h') > y_k(1 + h)$, yet also $y_{k+1}'(1 + h') < y_{k+1}(1 + h)$, and finally $y_j'(1+h') = y_j(1+h)$ for all $j \neq k, k+1$. This can be achieved as follows. Let $\delta \geq 0$ be small. Define the new SRA $C'(\delta) = (y', h', r')$ by

$$h' = \frac{1 + h}{1 - (\mu_{k+1}/\mu_k - 1)\delta} - 1, \tag{30}$$

$$y_k' = (1 - (\mu_{k+1}/\mu_k - 1)\delta)y_k + \delta\mu_{k+1}/\mu_k, \tag{31}$$

$$y_{k+1}' = (1 - (\mu_{k+1}/\mu_k - 1)\delta)y_{k+1} - \delta, \tag{32}$$

$$y_j' = (1 - (\mu_{k+1}/\mu_k - 1)\delta)y_j \qquad (j \neq k, k + 1), \tag{33}$$

and $r' = r$. Clearly, $C'(0) = C$, and for $\delta > 0$ small enough, the haircut h' is well-defined. Moreover, using (3), (30), $y_{k+1} > 0$, and $(1 + h)y_k < q_k^{i_B}$, it is straightforward to check that for δ small enough, we have $0 \leq (1 + h')y_j' \leq q_j^{i_B}$ for $j = 1, ..., m$. Another straightforward calculation exploiting (31) through (33) as well as $\sum_{j=1}^{m} y_j = 1$ shows that $\sum_{j=1}^{m} y_j' = 1$. Hence, for $\delta > 0$ small enough, the contract $C'(\delta)$ is well-defined and valid. It is claimed now that for $\delta > 0$ small enough, the SRA $C'(\delta)$ achieves a strict Pareto improvement over C. To see why, consider the conditional liquidation value $\widetilde{v}_b' = (1+h')\sum_{j=1}^m \widetilde{p}_b^j y_j'$ of the collateral portfolio deposited under the new agreement. Using (31) through (33), one obtains

$$\widetilde{v}_b' = (1+h)\sum_{j=1}^m \widetilde{p}_b^j y_j + (1+h')\frac{\mu_{k+1}}{\mu_k}\delta\widetilde{p}_b^k - (1+h')\delta\widetilde{p}_b^{k+1}. \tag{34}$$

Recall that $\widetilde{v}_b = (1+h)\widetilde{p}_b$. Then, using (29) for $j = k+1$ delivers

$$\widetilde{v}_b' \equiv \widetilde{v}_b + (1+h')\delta\mu_{k+1}\widetilde{\varepsilon}_b^{k+1}. \tag{35}$$

An induction argument involving Assumption 5 shows that

$$\widetilde{v}_b \equiv (1+h)\sum_{j=1}^m y_j \widetilde{p}_b^j \equiv (1+h)\sum_{j=1}^m y_j\mu_j - \sum_{j=1}^m \gamma_j\widetilde{\varepsilon}_b^j, \tag{36}$$

with parameters $\gamma_j \geq 0$ for $j = 1, ..., m$. Combining (35) and (36), one finds

$$\widetilde{v}_b' = \widetilde{z} - \delta'\widetilde{\varepsilon}_b^{k+1}, \tag{37}$$

where $\widetilde{z}$ is a random variable independent from $\widetilde{\varepsilon}_b^{k+1}$, and

$$\delta' = \gamma_{k+1} - (1+h')\mu_{k+1}\delta = \gamma_{k+1} - \frac{(1+h)\mu_{k+1}\delta}{1-(\mu_{k+1}/\mu_k - 1)\delta}. \tag{38}$$

As $\partial\delta'/\partial\delta < 0$, it suffices to show that $\partial E[\widetilde{u}_L]/\partial\delta' < 0$, where the derivative is evaluated at $\delta' = \gamma_{k+1}$. Let $G(.)$ and $H(.)$ denote the distribution functions of random variables $\widetilde{z}$ and $\widetilde{\varepsilon}_b^{k+1}$, respectively. Then, using (37), expected utility (1) for the lender at the time of contracting reads

$$E[\widetilde{u}_L] = \pi_G u_L(r) + \pi_B \iint u_L(\min\{z - \delta'\varepsilon_b^{k+1} - 1; r\})dH(\varepsilon_b^{k+1})dG(z), \tag{39}$$

where z and ε_b^{k+1} denote the realizations of random variables $\widetilde{z}$ and $\widetilde{\varepsilon}_b^{k+1}$, respectively. The weak inequality $\partial E[\widetilde{u}_L]/\partial\delta' \leq 0$ follows from standard arguments (cf., e.g., Tesfatsion, 1976), but the strict inequality requires a proof. For this, note that the interior integral in (39) reads

$$E[\widetilde{u}_L|B,z] = \int u_L(\min\{z - \delta'\varepsilon_b^{k+1} - 1; r\})dH(\varepsilon_b^{k+1}) \tag{40}$$

$$= u_L(r)H(\frac{r-z+1}{\delta'}) + \int_{\frac{r-z+1}{\delta'}}^{\infty} u_L(z - \delta'\varepsilon_b^{k+1} - 1)dH(\varepsilon_b^{k+1}), \tag{41}$$

and can be differentiated with respect to δ' at $\delta' = \gamma_{k+1}$. We obtain

$$\frac{\partial}{\partial \delta'} E[\widetilde{u}_L | B, z] = -\mu_{k+1} \int_{\frac{r-z+1}{\delta'}}^{\infty} \varepsilon_b^{k+1} u_L'(z - \delta' \varepsilon_b^{k+1} - 1) dH(\varepsilon_b^{k+1}) \tag{42}$$

$$\leq -u_L'(r) \mu_{k+1} \int_{\frac{r-z+1}{\delta'}}^{\infty} \varepsilon_b^{k+1} dH(\varepsilon_b^{k+1}), \tag{43}$$

where the inequality follows from the fact that $u_L'(.)$ is weakly declining. Now, by Assumption 5,

$$E[\widetilde{\varepsilon}_b^{k+1}] = \int_{-\infty}^{\infty} \varepsilon_b^{k+1} dH(\varepsilon_b^{k+1}) > 0, \tag{44}$$

so that $\partial E[\widetilde{u}_L | B, z]/\partial \delta' \leq 0$. It suffices to show that $\partial E[\widetilde{u}_L | B, z]/\partial \delta' < 0$ is strict for "sufficiently many" z. Note that Assumption 5 implies that collateral is imperfect. From Theorem 1 and efficiency, $\mathrm{pr}\{\widetilde{v}_b < 1+r\} > 0$. Thus, by (37) and independence,

$$\mathrm{pr}\{\widetilde{v}_b \geq 1 + r\} = \int H(\frac{1+r-z}{\gamma^{k+1}}) dG(z) < 1. \tag{45}$$

Therefore, there must be a compact interval Z satisfying $\int_Z dG(z) > 0$ such that for any $z \in Z$, we have $H(\frac{1+r-z}{\gamma^{k+1}}) < 1$. Fix $z \in Z$. From (44) and $H(\frac{1+r-z}{\gamma^{k+1}}) < 1$, clearly

$$\int_{\frac{1+r-z}{\gamma^{k+1}}}^{\infty} \varepsilon_b^{k+1} dH(\varepsilon_b^{k+1}) = (1 - H(\frac{1+r-z}{\gamma^{k+1}})) E[\widetilde{\varepsilon}_b^{k+1} | \widetilde{\varepsilon}_b^{k+1} \geq \frac{1+r-z}{\gamma^{k+1}}] \tag{46}$$

$$\geq (1 - H(\frac{1+r-z}{\gamma^{k+1}})) E[\widetilde{\varepsilon}_b^{k+1}] > 0, \tag{47}$$

so that by (43), we find indeed that $\partial E[\widetilde{u}_L | B, z]/\partial \delta' \leq 0$ for all $z \in Z$. Hence, $\partial E[\widetilde{u}_L]/\partial \delta' < 0$. Thus, for small enough $\delta > 0$, the lender's expected utility at the time of contracting is strictly increasing in δ. Clearly, the borrower's expected utility at the time of contracting is weakly increasing with a change from C to $C'(\delta)$. Thus, the initial SRA C cannot be efficient. □

Proof of Theorem 3. Define r^B, r^D, h_0 as in Lemma A.1 below. Then $r^B > r_0 > r^D$. Moreover, for any haircut $h \geq -1$, either $h < h_0$ or $h \geq h_0$. If $h < h_0$, then $\rho^D(h) \geq \rho^D(h_0) > r^B \geq \rho^B(h)$, so there is no repo rate for which the market transaction is individually rational for lender and borrower at the same time. If $h \geq h_0$, then $\rho^B(h) \leq \rho^B(h_0) < r^D \leq \rho^D(h)$, and again no market transaction is feasible. □

Lemma A.1. *There is a haircut* $h_0 \geq -1$ *and interest rates* r^B, r^D *satisfying* $r^B > r_0 > r^D$ *such that* $\rho^D(h_0) > r^B$ *and* $\rho^B(h_0) < r^D$.

Proof. As collateral is imperfect, there is a cut-off price p^* such that $F_b(p^*) > 0$ and $F_a(p^*) < 1$. Define the haircut h_0 by $p^* = (1+r_0)/(1+h_0)$. Let $r^D = r_0 - \varepsilon$ and $r^B = r_0 + \varepsilon$ for $\varepsilon > 0$ small. It will be shown that for ε small enough, $\rho^D(h_0) > r^B$ and $\rho^B(h_0) < r^D$. By the definition of $\rho^D(h_0)$,

$$\begin{aligned}(\pi_G + \pi_B)u_L(r^D) &= (\pi_G + (1 - F_b(p_b^*))\pi_B)u_L(\rho^D(h_0)) \\ &\quad + \pi_B \int_{p_b \leq p_b^*} u_L((1+h_0)p_b - 1)dF_b(p_b),\end{aligned} \tag{48}$$

where $p_b^* = (1 + \rho^D(h_0))/(1 + h_0)$. Re-arranging (48) yields

$$\begin{aligned}u_L(\rho^D(h_0)) &= u_L(r^D) \\ &\quad + \frac{\pi_B}{\pi_G + F_b(p_b^*)\pi_B} \int_{p_b \leq p_b^*} (u_L(r^D) - u_L((1+h_0)p_b - 1))dF_b(p_b),\end{aligned} \tag{49}$$

where the integral is either positive or zero. To provoke a contradiction, assume that $\rho^D(h_0) \leq r^B$ for all small $\varepsilon > 0$. Then $p_b^* \leq \widehat{p}_b = (1+r^B)/(1+h_0)$, and consequently,

$$\begin{aligned}&u_L(\rho^D(h_0)) \\ \geq\ & u_L(r^D) + \frac{\pi_B}{\pi_G + F_b(\widehat{p}_b)\pi_B} \int_{p_b \leq p_b^*} (u_L(r^D) - u_L((1+h_0)p_b - 1))dF_b(p_b).\end{aligned} \tag{50}$$

For $p_b < \widehat{p}_a = (1 + r^D)/(1 + h_0)$, the expression integrated in (50) is positive, while for $p_b \geq \widehat{p}_a$, the expression is negative or zero. Hence, splitting the integral yields

$$\begin{aligned}&u_L(\rho^D(h_0)) \\ \geq\ & u_L(r^D) + \frac{\pi_B}{\pi_G + F_b(\widehat{p}_b)\pi_B} \int_{p_b < \widehat{p}_a} (u_L(r^D) - u_L((1+h_0)p_b - 1))dF_b(p_b) \\ & - \frac{\pi_B}{\pi_G + F_b(\widehat{p}_b)\pi_B} \int_{\widehat{p}_a \leq p_b \leq p_b^*} (u_L((1+h_0)p_b - 1) - u_L(r^D))dF_b(p_b) \qquad (51) \\ \geq\ & u_L(r^D) + \frac{\pi_B}{\pi_G + F_b(\widehat{p}_b)\pi_B} \int_{p_b < \widehat{p}_a} (u_L(r^D) - u_L((1+h_0)p_b - 1))dF_b(p_b) \\ & - \frac{\pi_B}{\pi_G + F_b(\widehat{p}_b)\pi_B} \int_{\widehat{p}_a \leq p_b \leq \widehat{p}_b} (u_L((1+h_0)p_b - 1) - u_L(r^D))dF_b(p_b) \qquad (52) \\ \geq\ & u_L(r^D) + \frac{\pi_B}{\pi_G + F_b(\widehat{p}_b)\pi_B} \int_{p_b < \widehat{p}_a} (u_L(r^D) - u_L((1+h_0)p_b - 1))dF_b(p_b) \\ & - \frac{\pi_B}{\pi_G + F_b(\widehat{p}_b)\pi_B} \int_{\widehat{p}_a \leq p_b \leq \widehat{p}_b} (u_L(r^B) - u_L(r^D))dF_b(p_b). \qquad (53)\end{aligned}$$

For $\varepsilon \to 0$, we would have $\rho^D(h_0) \to r_0$, and therefore in the limit

$$u_L(\rho^D(h_0)) \geq \\ u_L(r^D) + \frac{\pi_B}{\pi_G + F_b(p^*)\pi_B} \int_{p_b<p^*} (u_L(r^D) - u_L((1+h_0)p_b - 1))dF_b(p_b). \tag{54}$$

For any values $\widehat{p}_b, \widehat{p}_a$ sufficiently close to p^* it is still true that $F_a(\widehat{p}_b) < 1$ and $F_b(\widehat{p}_a) > 0$. In particular, the integral in (54) is strictly positive. Using Assumption 1, we find a contradiction to the assumption that $\rho^D(h_0) \leq r^B$ for all small $\varepsilon > 0$. Thus, $\rho^D(h_0) > r^B$ for *some* sufficiently small ε. But for decreasing ε, the interest rate r^B is decreasing, while r^D is increasing so that $\rho^D(h_0)$ is non-decreasing. Hence, $\rho^D(h_0) > r^B$ for *any* sufficiently small ε. An analogous argument can be used to show that also $\rho^B(h_0) < r^D$ for all sufficiently small ε. □

Proof of Theorem 4. It is immediate from Theorem 2 that the pair of compositions (18) is stable. For the uniqueness part, assume a pair of compositions (θ^1, θ^2) such that for two eligible collaterals $k' > k$, the borrower has forwarded to the central bank a positive amount of collateral k, and at the same time kept a positive amount of collateral k'. Without loss of generality, $k' = k+1$, i.e., there exists a k such that $\theta_k^{i_B} > 0$ and $(1+\eta_{k+1})\theta_{k+1}^{i_B} D_{i_B} < q_{k+1}^{i_B}$. To show the instability of (θ^1, θ^2), note that, by Assumption 6, borrower and lender find it in their joint interest to conclude an SRA such that the eligible collateral $k+1$ will be used in this agreement. Following now the lines of the proof of Theorem 2, using that collateral is uninsured, it can be seen that a strict Pareto improvement for lender and borrower is feasible if the borrower substitutes a small amount of collateral k deposited with the central bank by the corresponding amount of collateral $k+1$. Hence, (θ^1, θ^2) cannot be stable unless it is of the form (18). □

References

Allen, L., Peristiani, S., Saunders, A. (1989), Bank size, collateral, and net purchase behavior in the federal funds market: Empirical evidence, *Journal of Business* **62(4)**, 501-515.

Baba, N., and Inamura, Y. (2004), The Japanese repo market: Theory and evidence, *Bank of Japan Monetary and Economic Studies*, March, 65-90.

Bank of England (2008), *Financial Stability Report,* Issue 23 (April).

Barro, R. (1976), The loan market, collateral, and rates of interest, *Journal of Money, Credit and Banking* **8(4)**, 439-456.

Benjamin, D. K. (1978), The use of collateral to enforce debt contracts, *Economic Inquiry* **16(3)**, 333-359.

Berger, A. N., and Udell, G. F. (1990), Collateral, loan quality, and bank risk, *Journal of Monetary Economics* **25**, 21-42.

Bester, H. (1985), Screening vs rationing in credit markets with imperfect information, *American Economic Review* **75(4)**, 850-855.

Booth, J. R., and Booth, L. C. (2006), Loan collateral decisions and corporate borrowing costs, *Journal of Money, Credit, and Banking* **38(1)**, 67-90.

Bliss, R. R., and Kaufman, G. G. (2006), Derivatives and systemic risk: Netting, collateral, and closeout, *Journal of Financial Stability* **2**, 55-70.

Buiter, W., and Sibert, A. (2005), How the Eurosystem's treatment of collateral in its open market operations weakens fiscal discipline in the eurozone (and what to do about it), *CEPR Discussion Paper* No. 5387.

Buraschi, A., and Menini, D. (2002), Liquidity risk and specialness, *Journal of Financial Economics* **64**, 243-284.

Chan, Y.-S., and Thakor, A. (1987), Collateral and competitive equilibria with moral hazard and private information, *Journal of Finance* **42(2)**, 345-363.

Coco, G. (1999), Collateral, heterogeneity in risk attitude and the credit market equilibrium, *European Economic Review* **43,** 559-574.

Comotto, R. (2007-2008), *European Repo Market Survey,* International Capital Market Association, Zurich, issues 12 and 14, URL: http://www.icma-group.org/market_info/surveys/repo/latest.html.

Corrigan, D., and de Terán, N. (2007), *Collateral: Securities Lending, Repo, OTC Derivatives and the Future of Finance,* Global Custodian, London.

Cossin, D., and Hricko, T. (2003), A structural analysis of credit risk with risky collateral: A methodology for haircut determination, *Economic Notes* **22 (3)**, 243-282.

Demiralp, S., Preslopsky, B., and Whitesell, W. (2006), Overnight interbank loan markets, *Journal of Economics and Business* **58**, 67-83.

Duffie, D. (1996), Special repo rates, *Journal of Finance* **51(2)**, 493-526.

ECB (2005), *The Implementation of Monetary Policy in the Euro Area*, February.

ECB (2007a), *Euro Money Market Survey*, November.

ECB (2007b), The collateral frameworks of the Federal Reserve System, the Bank of Japan and the Eurosystem, *Monthly Bulletin*, October, 85-100.

Ewerhart, C., and Tapking, J. (2008), Repo Markets, Counterparty Risk, and the 2007/2008 Liquidity Crisis, ECB Working Paper No. 909.

Fan, L., and Zhang, C. (2007), Beyond segmentation: The case of China's repo markets, *Journal of Banking and Finance* **31**, 939-954.

Federal Reserve Bank of New York (2008), *Domestic Open Market Operations during 2007*, Report prepared for the Federal Open Market Committee by the Markets Group of the Federal Reserve Bank of New York, February, URL: http://www.new yorkfed.org/markets/omo/omo2007.pdf.

Flannery, M. (1995), Financial crises, payment system problems, and discount window lending, *Journal of Money, Credit, and Banking* **28(4)**, 804-824.

Frediani, G., Morosini, P., and Wilkin, J.-R. (2007), *Enhancing your liquidity through a collateral agent*, The 12th Global Securities Financing Summit, Documentation, September 19-20, URL: http://www.clearstream.com/ci/dispatch/en/binary/ci_ content_pool/attachements/ 03_gsf/010_gsf_summit/2007_summit/presentat ions/DBAG_Workshop_-Frediani-Morosini-Wilkin.pdf.

Freeman, S. (1996), The payments system, liquidity, and rediscounting, *American Economic Review* **86(5)**, 1126-1138.

Freixas, X., and Holthausen, C. (2004), Interbank market integration under asymmetric information, *Review of Financial Studies* **18(2)**, 459-490.

Garbade, K. D. (2006), The evolution of repo contracting conventions in the 80s, *FRBNY Economic Policy Review*, May.

Gatev, E., and Strahan, P. (2006), Banks' advantage in hedging liquidity risk: Theory and evidence from the commercial paper market, *Journal of Finance* **61(2)**, 867-892.

Glosten, L., and Milgrom, R. (1985). Bid, ask, and transaction prices in a specialist market with heterogeneously informed traders, Journal of Financial Economics **14**, 71-100.

Griffiths, M., and Winters, D. (1997), The effect of Federal Reserve accounting rules on the equilibrium level of overnight repo rates, *Journal of Business Finance & Accounting* **24(6)**, 815-832.

Hellwig, M. (1987), Theory of competition in markets with adverse selection, *European Economic Review* **31**, 319-325.

Hördahl, P., and King, M. (2008), Developments in repo markets during the financial turmoil, *BIS Quarterly Review*, December, 37-53.

Inderst, R., and Mueller, H. (2007), A lender-based theory of collateral, *Journal of Financial Economics* **84**, 826-859.

Jiménez, G., Saurina, J. (2004), Collateral, type of lender and relationship banking as determinants of credit risk, *Journal of Banking and Finance* **28**, 2191-2212.

Jiménez, G., Salas, V., and Saurina, J. (2006), Determinants of collateral, *Journal of Financial Economics* **81**, 255-281.

Jordan, B., and Jordan, S. (1997), Special repo rates: An empirical analysis, *Journal of Finance* **52(5)**, 2051-2072.

Johnson, H., and Stulz, R., 1987, The pricing of options with default risk, *Journal of Finance* **42(2)**, 267-280.

Jordan, T., and Kugler, P. (2004), Implementing Swiss monetary policy: Steering the 3M-Libor with repo transactions, *Swiss Journal of Economics and Statistics* **140(3)**, 381-393.

Kashyap, A. K., Rajan, R., and Stein, J. C. (2002), Banks as liquidity providers: An explanation for the coexistence of lending and deposit-taking, *Journal of Finance* **57(1)**, 33-73.

Krishnamurthy, A. (2002), The bond/old-bond spread, *Journal of Financial Economics* **66**, 463–506.

Lacker, J. (2001), Collateralized debt as the optimal contract, *Review of Economic Dynamics* **4(4)**, 842-859.

Manove, M., Padilla, A.J. (1999), Banking (conservatively) with optimists. *RAND Journal of Economics* **30**, 324–350.

Manove, M., Padilla, A.J. (2001), Collateral versus project screening: a model of lazy banks. *RAND Journal of Economics* **32**, 726–744.

Merton, R. C. (1974), On the pricing of corporate debt: The risk structure of interest rates, *Journal of Finance* **29**, 449-470**.**

Mills, D. C. (2006), Alternative central bank credit policies for liquidity provision in a model of payments, *Journal of Monetary Economics* **53**, 1593-1611.

Plaut, S. E. (1985), The theory of collateral, *Journal of Banking and Finance* **9**, 401-419.

Stiglitz, J., and Weiss, A. (1981), Credit rationing in markets with imperfect information, *American Economic Review* **71(3)**, 393-410.

Stigum, M. (1989), *The Repo and Reverse Markets*, Irwin, New York.

Tesfatsion, L. (1976), Stochastic dominance and maximization of expected utility, *Review of Economic Studies* **43**, 301-315.

The Bond Market Association (2000), *TBMA/ISMA Global Master Repurchase Agreement*, URL: http://archives1.sifma.org/agrees/global_master_repurchase_agreement.pdf.

Vayanos, D., and Weill, P.-O. (2008), A search-based theory of the on-the-run phenomenon, Journal of Finance **63**, 1361-1398.

Wetherilt, A. (2003), Money market operations and short-term interest rate volatility in the United Kingdom, *Applied Financial Economics* **13**, 701-719.

Eurosystem (Source: ECB data)	2006	2008
Central Gov	23%	11%
Regional Gov	6%	4%
Uncov Bank Bonds	32%	28%
Cov Bank Bonds	13%	10%
Corporates	6%	5%
ABS	12%	30%
Other marketable	4%	1%
Credit claims	4%	11%
Total	100%	100%

Repo market (Source: ICMA)	2006	2008
EU Central Gov	84.2%	82.3%
Other EU	15.8%	17.7%
Total	100.0%	100.0%

Table I. Average Collateral Usage during 2006 and 2008 in Primary and Secondary Funding

The entries on the left-hand side refer to market values of assets, net of haircuts, held as collateral by counterparties with the Eurosystem as an average of monthly data. Shown are the percentage shares of different types of assets eligible as collateral with the Eurosystem. The entries on the right-hand side represent percentage shares of different types of EU collateral used in the euro repo market. Reported are averages over values reported by financial units as outstanding mid June and mid December.

Eurosystem (Source: ECB data)		2004	2005	2006	2007	2008
	Central Gov	31%	28%	23%	15%	11%
	ABS	6%	10%	12%	16%	30%

	2007				2008			
	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
Central Gov	na	na	na	na	12%	12%	10%	9%
ABS	12%	13%	16%	23%	30%	32%	29%	28%

Table II. Development of Collateral Usage in Commercial Bank Refinancing

The table refers to average market values of assets, net of haircuts, held as collateral by counterparties with the Eurosystem. Shown are the percentage shares of different types of collateral. The upper part of the table exhibits annual data, the lower part quarterly figures. No quarterly data were available for the usage of government bonds during 2007.

Federal Reserve System (Source: www.federalreserve.gov)	
Loans	
Commercial	378
Residual Mortgage	152
Commercial Real Estate	146
Consumer	137
Securities	
US Treasury/Agency	27
Municipal	41
Corporate Market Instruments	50
MBS/CMO: Agency-backed	71
MBS/CMO: Other	33
Asset-backed	142
International (Sovereign, Agency, Municipal, and Corporate)	49
Total	1,226

Table III. Lending to U.S. Depository Institutions: Collateral Pledged by Borrowers
Shown are lendable values in $ billions of collateral pledged by borrowers of primary, secondary, seasonal, and Term Auction Facility (TAF) credit in April 2009. This table is updated by the Federal Reserve approximately every 60 days.

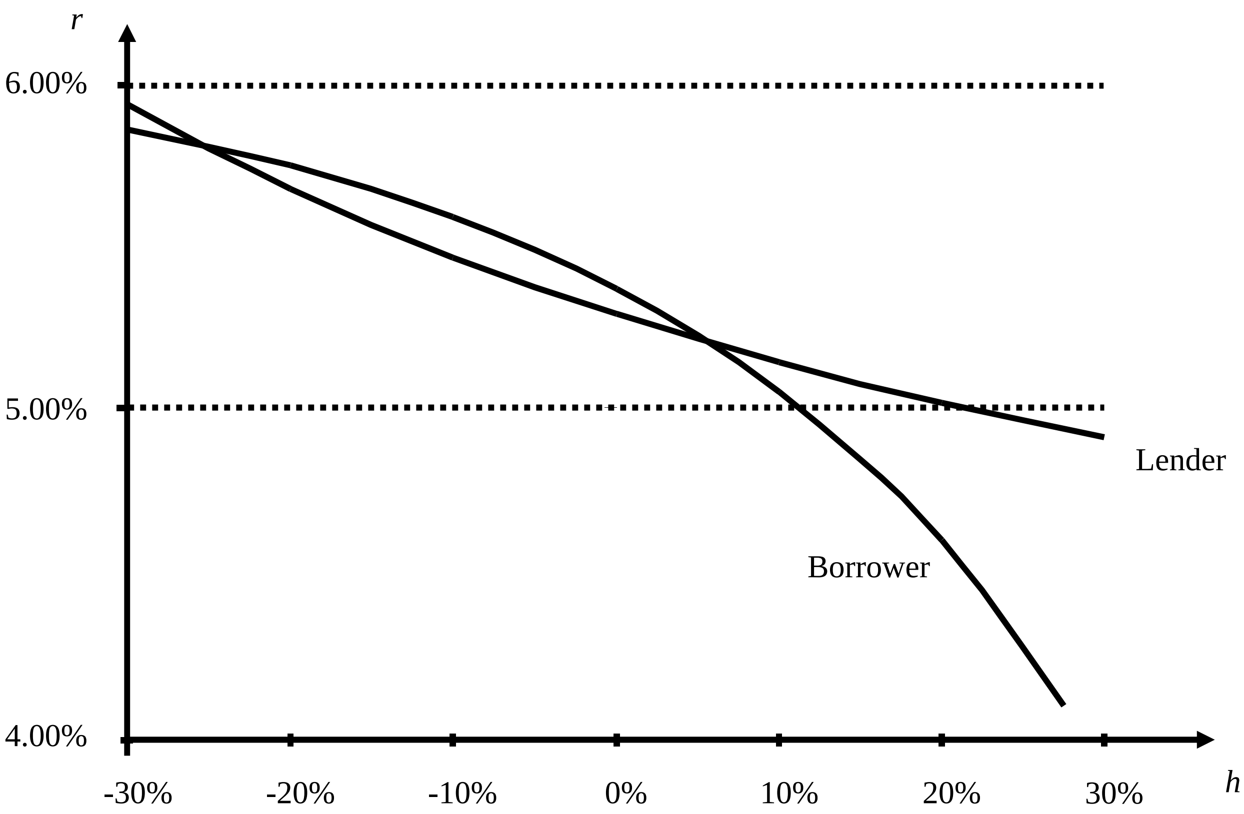

Figure 1. Indifference Curves of Counterparties

Shown are the break-even repo rates for lender and borrower as a function of the haircut. The numerical example captures the theoretical possibility of negative haircuts even when the lender has the higher default probability than the cash borrower. The effect is caused by a right-skewness of the assumed price distributions, which exposes the borrower to significant counterparty risk.