Central Bank Digital Currency: When Price and Bank Stability Collide

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Abstract

With the introduction of a central bank digital currency or CBDC, we argue that the central bank needs to confront classic issues of banking, i.e. the tension between providing liquid means of payments and desirable maturity transformation. We analyze these issues in a nominal version of a Diamond and Dybvig (1983) model, when the central bank additionally has a price stability objective. While the central bank can always deliver on its nominal obligations, runs can nonetheless occur, manifesting themselves either as excessive real asset liquidation or as a failure to maintain price stability. We demonstrate an impossibility result that we call the CBDC trilemma: of the three goals of efficiency, financial stability (i.e., absence of runs), and price stability, the central bank can achieve at most two.

Keywords: Central bank digital currency, monetary policy, bank runs, financial intermediation, inflation targeting, CBDC trilemma.

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1 Introduction

Many central banks and policymaking institutions, such as the Bank of Canada, the Bank of England, the Bank for International Settlements, the ECB, the IMF, the People's Bank of China, the Sveriges Riksbank, and the G30, are openly debating the introduction of a central bank digital currency, or CBDC (Barrdear and Kumhof, 2016; Bech and Garratt, 2017; Chapman et al., 2017; Lagarde, 2018; Ingves, 2018; Kahn et al., 2019; Davoodalhosseini et al., 2020; Auer and Böhme, 2020; Auer et al., 2020; Group of 30, 2020).

In this paper, we seek to understand the consequences of introducing CBDCs, by thinking through its consequences to the very end. Indeed, the introduction and adoption of CBDCs have the potential to be a watershed for the monetary and financial systems of advanced economies. With a CBDC, households will have access to an electronic means of payment and thus an attractive alternative to traditional deposit accounts. This remains true, even if retail banks provide the "front end" for CBDC transactions: there may no need to hold traditional deposit accounts on top. Indeed, the "financial inclusion" of households hitherto excluded from banking is often touted as one of the attractive features of a CBDC, and we agree. However, the potential decline of traditional deposit accounts raises the issue of disintermediation: retail banks may no longer be able engage in maturity transformation and fund long-term investments with traditional demand deposits. This decline can be avoided and the old equilibrium restored, if the central bank engages in "pass through", funneling the funds from the households back to the retail banks and the long-term investments, as Brunnermeier and Niepelt, 2019 have argued. With that, though, could the traditional tension between maturity transformation and the possibility for bank runs return, and how? What is the interplay of this tension with the traditional key objective of central banks to maintain price stability?

We show that a CBDC trilemma arises, see figure 1: of the three goals of efficiency, absence of runs and price stability, the central bank can achieve at most two. We build on the classical model by Diamond and Dybvig (1983), which emphasizes a bank's role in maturity transformation. In their model, a bank pools resources and finance long-term projects with demand deposits that can be withdrawn at a short time horizon to meet impatient consumption needs. In our model, households hold CBDC rather than demand deposits. There can no longer be a "withdrawal", since households already hold the most liquid financial instrument. Instead, spending CBDC balances at a short time horizon replaces deposit withdrawals. A "run" then manifests itself as a spending spree in that more than just the fraction of impatient agents spending their CBDC balances at short horizon. Their spending decision versus the available quantity of goods for purchase impacts the price level via market clearing. Given the absence of deposit accounts or other means of payments, real goods can only be traded against money, implicitly setting a form of a cash-in-advance constraint in the tradition of Svensson (1985) and Lucas and Stokey (1987).

The households obtain their CBDC balances in the initial period, by selling their initial endowment of goods to the central bank, which in turn invests these goods, using available technologies. In our model, it is the central bank, which decides on the amount of long-term projects to be liquidated at short horizon, observing the fraction of agents seeking to spend their CBDC balances. That project liquidation policy ultimately is key to understand efficiency, the possibility for runs and price stability and thus the trilemma in figure 1. The obvious alternative is to consider the classic central bank intervention of changing the money supply in response to a run. In section 7, we investigate the possibilities. We show that the central bank would need to reduce the money supply or suspend portions of the money supply as a means of payments. We argue that these are policy responses likely to wreck havoc with the trust that households place in the monetary system.

Our assumption that it is the central bank engaging in maturity transformation and potential liquidation of long-term projects is made entirely for simplicity. In section 10, we show, how one can think of our model as a stripped-down version of a rich financial system, where retail banks finance loans to firms, who conduct long-term projects, where households buy their goods from firms, and where short-term loans between the various parties allow the financial resources to flow.

As an alternative and in order to address the concern that the central bank "takes over" the production side of the economy, we consider an extension of our model in section 8, where we allow for direct competition between retail banks and the central bank to households interested in the risk sharing arrangement offered by traditional deposit accounts. Retail banks then need to obtain CBDC from the central bank to service withdrawals. The central bank then needs to carefully choose the additional instruments of interest rates charged on bank loans as well as a market share tax in order to keep the system competitive.

The rest of the paper is organized as follows. Section 2 reviews the related literature.

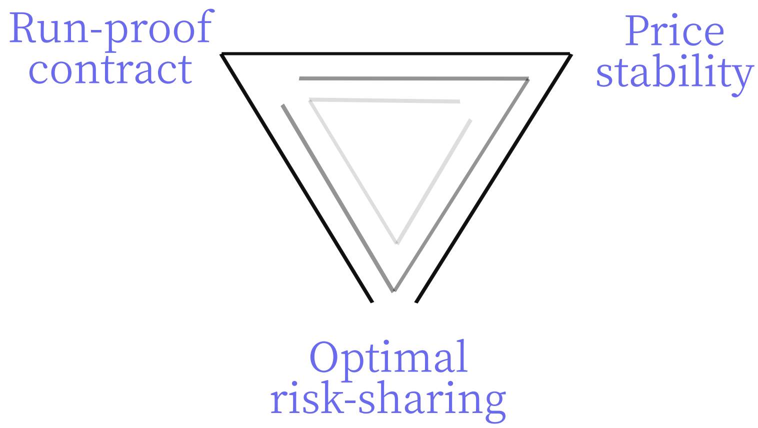

Figure 1: CBDC Trilemma: For the central bank, it is impossible to attain all three objectives at a time. When prioritizing one objective, at least one other objective has to be sacrificed.

Section 3 introduces our model. Section 4 presents the main analysis of the model, defines an equilibrium, and describes some of its fundamental properties. Section 5 discusses how the social optimum can be implemented. Section 6 deals with price stability and how it relates to the implementation of the social optimum. In Section 7, we discuss what may appear to be a natural resolution: the adjustment of the money supply in a state-contingent manner. Issues arising out of private-sector competition such as private investment or a competing private banking sector are taken up in Section 8. In Section 11, we analyze a token-based system and hybrid systems. Section 12 concludes.

2 Related literature

Our paper contributes to several strands of the literature. The three papers closest to ours are Diamond and Dybvig (1983), Skeie (2008), and Allen et al. (2014). First, we contribute to the literature of financial intermediation and bank fragility. Building on the seminal Diamond and Dybvig (1983) model, we stress the central bank's role in liquidity transformation when issuing a CBDC that allows depositors to share idiosyncratic liquidity risk. Similar to Diamond and Dybvig (1983), we study the microincentives of depositors to withdraw ("spend") from the bank. But unlike them, we employ nominal instead of real demand-deposit contracts, giving "the bank" an additional tool –the price level– to prevent runs.

Nominal demand-deposit contracts have previously been considered by Allen and Gale (1998), Skeie (2008), Allen et al. (2014), Leiva and Mendizábal (2019), and Andolfatto et al. (2020), among others. In Skeie (2008), large withdrawals of nominal deposits can lead to an increase in the price level, reducing the real allocation and deterring runs. In a similar model, Allen et al. (2014) show that optimal risk-sharing can be achieved via nominal contracts, but their setting cannot exclude runs. In particular, compare their Section 4.4 to our Lemma 5.2. In their case, the price level reacts passively and cannot be fine-tuned to the agent's spending decisions. As we mentioned above, in both Skeie (2008) and Allen et al. (2014), the "real" side is arising from the interplay between workers and entrepreneurs (and their customers), leaving the nominal side to the banking system and the central bank. Andolfatto et al. (2020) incorporate Diamond-Dybvig financial intermediation into the new monetarist model of Lagos and Wright (2005). Di Tella and Kurlat (2021, forthcoming) ask and answer, why banks are exposed to monetary policy. In our framework, we examine a drastically simplified model, dropping the financial intermediary sector, while these issues would arise in a richer setting.

Unlike in all these papers, in our framework, the central bank is a strategic player that observes withdrawals and, as a response, determines the real goods supply to alter either the depositors' incentives to withdraw or the price level according to its objectives. Therefore, we can show that the central bank can always implement the efficient allocation in dominant strategies, and runs no longer occur. Since implementation in dominant strategies requires giving up price stability, we can also discuss the flip side of this result. We further differ from the literature above by considering a more stylized model, abstracting from private banks and firms. In our framework, the central bank takes over the activity of real investment, financial intermediation, and the management of the money supply.

Second, we contribute to a growing literature on the macroeconomic implications of introducing a CBDC. Berentsen (1998) is perhaps the first analysis of the monetary policy implications of digital money. Chiu et al. (2019) discuss issues regarding the competition with and support of private banks. Keister and Sanches (2019) explore how the presence of a CBDC affects the liquidity premium on bank deposits and, through it, investment. Böser and Gersbach (2019a) gauge the implications of CBDCs for banking panics. Böser and Gersbach (2019b) show that the introduction of a CBDC transfers default risk to the central bank when a CBDC competes with private deposits. Fernández-Villaverde et al.

(2020) demonstrate that competition for deposits between private banks and the central bank can lead to a deposit monopoly at the central bank when commercial banks cannot commit. Skeie (2019) analyzes inflation-driven digital currency runs in a nominal model where a private digital currency competes with a CBDC. In contrast to this strand of the literature, our analysis abstracts from competition between a CBDC and deposits at private banks, respectively a CBDC and private digital currency, by modeling the central bank as the monopolistic provider of demand deposits. Brunnermeier and Niepelt (2019) derive an equivalence result of allocations when introducing a CBDC if the central bank commits to redepositing CBDC funds in private banks. In comparison, we are more explicit about the micro incentives of agents to run on the central bank. Ferrari et al. (2020) discuss monetary policy transmission in a two-country DSGE model when introducing a CBDC. In our model, we focus on one country and do not feature firms, other financial agents, or assets. Instead, we focus on the depositors' microincentives to (not) run on the central bank.

Lastly, we contribute to the growing literature on cryptoeconomics that analyzes the price and exchange rate implications of crypto mining (Choi and Rocheteau, 2020; Garratt and van Oordt, 2019; Huberman et al., 2017; Prat and Walter, 2018), the micro and macroeconomics of blockchain (Amoussou-Guenou et al., 2019; Biais et al., 2019a,b; Ebrahimi et al., 2019; Leshno and Strack, 2020; Saleh, 2020) and token issuance (Cong et al., 2020; Li and Mann, 2020; Prat et al., 2019), and the macroeconomic implications of cryptocurrencies via currency competition (Benigno, 2019; Benigno et al., 2019; Fernández-Villaverde and Sanches, 2019; Schilling and Uhlig, 2019). Our paper abstracts from the existence of competing digital currencies and assumes full functionality of the CBDC account and ledger system.

3 The basic framework

Our framework builds on the classical Diamond and Dybvig (1983) model of banking. Time is discrete with three periods $t = 0, 1, 2$. There is a $[0, 1]$-continuum of agents, each endowed with 1 unit of a real consumption good in period $t = 0$. Agents are symmetric in the initial period, but can be of two types in period 1: patient and impatient. Impatient agents value consumption only in period 1. In contrast, patient agents value consumption in period $t = 2$. An agent is impatient with likelihood $\lambda \in (0, 1)$ and otherwise is patient. The agent's type is randomly drawn at the beginning of period 1 and types are private information. Since we have a continuum of agents, there is no aggregate uncertainty about the measure of patient and impatient types in the economy. Thus, λ also denotes the share of impatient agents. Preferences are represented by a strictly increasing, strictly concave, and continuously differentiable utility function over consumption $u(\cdot) \in \mathbb{R}$. We further assume a relative risk aversion, $-x \cdot u''(x)/u'(x) > 1$, for all consumption levels $x \geq 0$.

There exists a long-term production technology in the economy. For each unit of the good invested in $t = 0$, the technology yields either 1 unit at $t = 1$ or $R > 1$ units at $t = 2$. Additionally, there is a storage technology between periods 1 and 2, yielding 1 unit of the good in $t = 2$ for each unit invested in $t = 1$. All agents can access both technologies. Let $x_1 \geq 0$ denote the agent's real consumption when deciding to spend (or "withdraw") at $t = 1$, and let $x_2 \geq 0$ denote the agent's consumption when spending at $t = 2$.

3.1 Optimal risk-sharing

Following Diamond and Dybvig (1983), we derive, first, the optimal allocation. The social planner collects and invests the aggregate endowment in the long technology. Given that all agents behave according to their type, the social planner maximizes *ex-ante* welfare

$$W = \lambda u(x_1) + (1-\lambda)u(x_2) \tag{1}$$

by choosing (x_1, x_2), subject to the feasibility constraint $\lambda x_1 \leq 1$, and the resource constraint $(1-\lambda)x_2 \leq R(1-\lambda x_1)$. The interior first-order condition for this problem implies that the optimal allocation (x_1^*, x_2^*) satisfies:

$$u'(x_1^*) = Ru'(x_2^*). \tag{2}$$

Given our assumptions, the resource constraint binds in the optimum

$$R(1-\lambda x_1^*) = (1-\lambda)x_2^*. \tag{3}$$

This condition, together with equation (2), uniquely pins down (x_1^*, x_2^*) and delivers the familiar optimal deposit contract in Diamond and Dybvig (1983). Together with $R > 1$ and the concavity of $u(\cdot)$, equation (2) implies that the optimal consumption of patient agents is higher than the consumption of impatient ones: $x_1^* < x_2^*$.

Moreover, the depositors' relative risk-aversion exceeding unity and the resource constraint yield $x_1^* > 1$ and $x_2^* < R$.[1]

Diamond and Dybvig (1983) show that a demand-deposit contract can implement the efficient allocation. A key feature of their analysis is the use of a "real" demand deposit contract (i.e., a contract that promises to pay out goods in future periods). Due to a maturity mismatch between real long-term investment and real deposit liabilities, the Diamond and Dybvig (1983) environment, however, also features a bank run equilibrium under which the social optimum is not implemented. Our main contribution is to show that a nominal contract can lead to the implementation of the efficient allocation in dominant strategies. In other words, runs do not occur along the equilibrium path. The key mechanism is that the central bank can set the price level, thereby controlling the wedge between real long-term investment and nominal deposit liabilities. The implementation in dominant strategies comes at a price, requiring flexibility of the price level.

4 A nominal economy

Consider now an economy with a social planner that uses nominal contracts to implement the efficient allocation.

Nominal contracts. The planner offers contracts in a unit of account for which it is the sole issuer. Because central banks have a monopoly on currency, the planner in our analysis can be equated with the central bank or any other monetary authority with the ability to issue currency. In this paper, we refer to the unit of account as a central bank digital currency (CBDC) or digital euros. Agents who sign a contract with the central bank hand over their real goods endowment and receive CBDC balances in return. The most straightforward interpretation of our environment is to think of a CBDC as an account-based electronic currency in the sense of Barrdear and Kumhof (2016) and Bordo and Levin (2017), i.e., to think of a CBDC as being akin to a deposit account at the central bank. In

[1] Following the proof in Diamond and Dybvig (1983),

$$Ru'(R) = u'(1) + \int_1^R \frac{\partial}{\partial x}(x \cdot u'(x))\,dx = u'(1) + \int_1^R (x \cdot u''(x) + u'(x))\,dx < u'(1) \tag{4}$$

by $-x \cdot u''(x)/u'(x) > 1$ for all x.

Section 11, we show that the results of our paper largely carry over to a token-based system or hybrid systems. Agents can spend their CBDC balances by transferring them to other agents in exchange for goods. As with physical euros, we impose the constraint that agents cannot hold negative amounts of a CBDC. v **Timing.** At $t = 0$, the central bank creates an empty account, i.e., a zero-balance CBDC account, for each agent in the economy. Then, each agent agrees to invest her unit endowment of the good in exchange for $M > 0$ units of digital euros, credited to that agent's account. Next, the central bank invests all goods in the long-term technology.

In $t = 1$, agents learn their type and decide whether to spend their CBDC balances M, that is, either to withdraw them or to roll them over. The central bank contract imposes the constraint that an agent either spends all her balances or no balance at all. Because types are unobservable, the central bank cannot discriminate between patient and impatient agents to deny a patient agent access to her balances. Let $n \in [0, 1]$ denote the share and measure of agents who decide to spend in $t = 1$. The central bank observes n and then decides on the fraction $y = y(n)$ of technology to liquidate, selling that amount in the goods market at the unit price P_1. Notice that through the resource constraint, early liquidation of technology reduces the remaining investment and, hence, the supply of goods in $t = 2$. That is, there is a real payoff externality, and the central bank's liquidation choice in $t = 1$ determines the real supply of goods for both of the periods $t = 1$ and $t = 2$. Given n, the central bank also chooses a nominal interest rate $i = i(n)$ to be paid in period 2 on the remaining CBDC balances. Each digital euro held at the end of $t = 1$ turns into $1 + i(n)$ digital euros at the beginning of $t = 2$. Notice that $i(n) \geq -1$, given that agents cannot hold negative amounts of digital euros.

In $t = 2$, the remaining $1 - n$ depositors each have $(1 + i)M$ digital euros to spend on goods in the market at a price P_2. The remaining investment in the technology matures so that the central bank supplies $R(1 - y(n))$ units of goods in exchange for money balances. Figure 2 summarizes this timing.

Definition 1. *A central bank policy is a triple $(M, y(\cdot), i(\cdot))$, where $y : [0, 1] \to [0, 1]$ specifies the central bank's liquidation policy and $i : [0, 1] \to [-1, \infty)$ is the interest rate policy for every possible spending level $n \in [0, 1]$.*

Notice that M itself is not state-contingent. The logic here is that, traditionally, 1 dollar

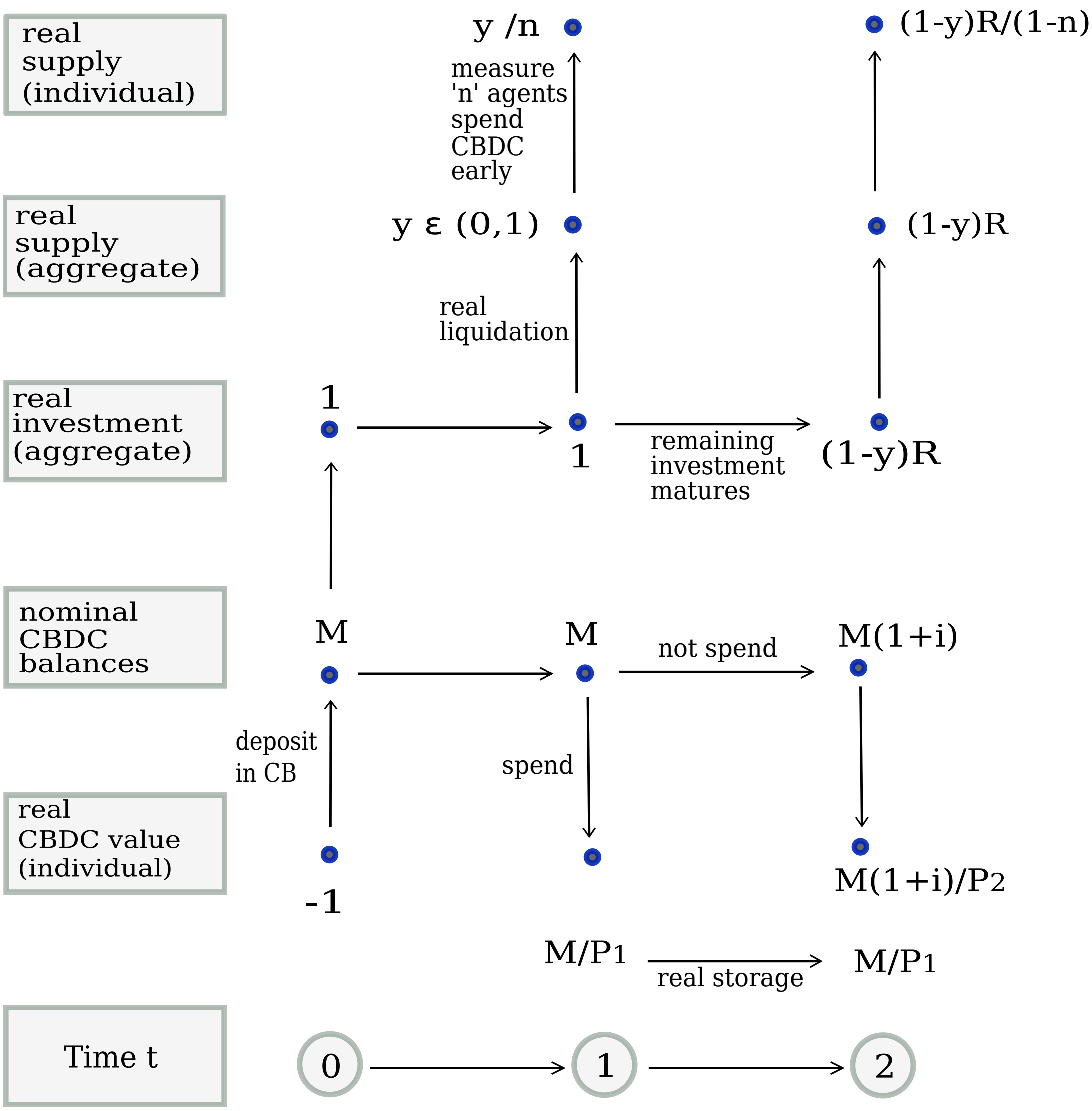

Figure 2: Nominal and real investment and contracts

today is 1 dollar tomorrow: we maintain that tradition with that assumption here. In Section 7, we discuss an extension where we allow M to be state-contingent as well.

Market clearing. In periods 1 and 2, agents spend their money balances for goods in a Walrasian market. The market-clearing conditions are:

$$nM = P_1 y(n) \tag{5}$$

$$(1-n)(1+i(n))M = P_2 R(1-y(n)), \tag{6}$$

which take the form of the quantity theory equation in each period. Given aggregate spending n in $t = 1$ and the central bank's policy, these conditions determine the price level, $P_1 = P_1(n)$ and $P_2 = P_2(n)$, in each period:

$$P_1(n) = \frac{nM}{y(n)} \tag{7}$$

$$P_2(n) = \frac{(1-n)(1+i(n))M}{R(1-y(n))}. \tag{8}$$

The central bank chooses the initial money supply before learning the measure of withdrawals in the intermediate period. The central bank, however, controls the goods supply in the Walrasian market, which is chosen conditional on the measure of withdrawals. As a result, the central bank can control the price level in period 1.[2] The nominal interest rate allows the central bank to control the price level in period 2 independently of the price level in period 1. Because the intermediary is the central bank with a monopoly on the unit of account in which contracts are denominated, the liquidation policy is flexible and becomes a monetary policy tool.

Implied real contract. The real value when spending CBDC balances in $t = 1$ equals

$$x_1 = \frac{M}{P_1}, \tag{9}$$

while the real value when spending balances in $t = 2$ equals

$$x_2 = \frac{(1+i\,(n))M}{P_2}. \tag{10}$$

Aggregate spending n and the liquidation policy $y\,(n)$ jointly determine the allocation of goods via the market-clearing conditions. The real allocations when spending in $t = 1$ versus $t = 2$ can therefore be rewritten as

$$x_1(n) = \frac{y(n)}{n} \tag{11}$$

$$x_2(n) = \frac{1-y(n)}{1-n}R. \tag{12}$$

[2] A private bank, in contrast, would need to take P_1, P_2 as given, which together with the observation n implies a unique liquidation $y(n, P_1)$.

Because all agents that spend CBDC in the same period have the same nominal income, the real goods supply $y(n)$ is equally distributed across all spending agents in period 1, and the supply $R(1 - y(n))$ is equally allocated to all spending agents in period 2.[3]

To summarize: in $t = 0$, the central bank announces and commits to a policy $(M, y(\cdot), i(\cdot))$, pinning down a spending-contingent real goods supply and an offer to a nominal contract $(M, M(1 + i(\cdot)))$ in exchange for 1 unit of the good. All consumers accept the contract and the policy, meaning they have the option to spend either M digital euros in period 1 or $M(1+i(n))$ digital euros in period 2, for every possible level of aggregate spending $n \in [0, 1]$. We discuss voluntary participation in contracts in Section 8.

In $t = 1$, the aggregate spending level n is realized. Finally, the central bank's policy, together with the market-clearing conditions, results in the real consumption amounts $(x_1(n), x_2(n)) = (\frac{M}{P_1}, \frac{M(1+i(n))}{P_2}) = \left(\frac{y(n)}{n}, \frac{1-y(n)}{1-n} R\right)$. Notice that the central bank is fully committed to carry through with its policy (M, y, i), regardless of which n obtains and independently of the implications for the price levels (P_1, P_2). We, therefore, define

Definition 2. *A* **commitment equilibrium** *consists of a central bank policy $(M, y(\cdot), i(\cdot))$, aggregate spending behavior $n \in [0, 1]$ and price levels (P_1, P_2) such that:*

(i) The spending decision of each individual consumer is optimal given aggregate spending decisions n, the announced policy $(M, y(\cdot), i(\cdot))$, and price levels (P_1, P_2).

(ii) Given aggregate spending n, the central bank provides $y(n)$ goods and sets the nominal interest rate $i(n)$.

(iii) Given $(n, y(n), M)$, the price level P_1 clears the market in $t = 1$.
Given $(n, y(n), i(n), M)$, the price level P_2 clears the market in $t = 2$.

As a particular consequence of this equilibrium concept, the price levels (P_1, P_2) flexibly adjust to the aggregate spending realization and the announced central bank policy.

[3] These equations remain intuitive even if $y(n) = 0$ or $y(n) = 1$. Therefore, we assume that they continue to hold, despite one of the price levels being potentially ill-defined or infinite.

5 Implementation of optimal risk sharing

In our model, the implementation of the optimal risk sharing arrangement (x_1^*, x_2^*) is of particular interest to the central bank. Given the preferences and technology that we postulated above, only the real allocation of goods matters to the two types of agents. There is, consequently, no additional motive for the monetary authority to keep prices stable.

However, focusing only on real allocations is a narrow perspective. There is a vast literature arguing in favor of central banks keeping prices stable or setting a goal of low and stable inflation for reasons that are absent from our model. For instance, stable prices minimize the misallocations created by nominal rigidities as in Woodford (2003). Having to hold cash to accomplish transactions, such as in cash-in-advance or money-in-utility models, creates a whole range of distortions that can be minimized by deft management of the price level (think about the logic behind the Friedman rule). It certainly would therefore be reasonable to extend the social planner objective (1) with a term, reflecting a desire to keep prices stable. Rather than extending the model to include these considerations, which would complicate the analysis for an uncertain benefit, we shall proceed by discussing the tradeoffs between achieving the optimal real allocation of consumption and the implications of such an effort for the stability of prices.

Runs on the central bank. The first important property of the equilibrium defined above is that a nominal contract, *per se*, does not rule out the possibility of a run on the central bank. Since impatient agents only care for consumption in $t = 1$, every equilibrium will exhibit aggregate spending behavior of at least λ, implying $n \geq \lambda$.[4] Patient agents, on the other hand, spend their CBDC balances strategically in $t = 1$ or $t = 2$. They spend in $t = 1$ if they believe that the central bank policy implies $x_1 > x_2$. In that case, patient agents will use the storage technology to consume x_1 in period 2. Otherwise, patient agents will find it optimal to wait until the final period. We say,

Definition 3 (Central Bank Run)**.** *A run on the central bank occurs if patient agents also spend in* $t = 1$*,* $n > \lambda$*.*

In a bank run, the central bank is not running out of the item that it can produce freely (i.e., it is not running out of digital money). This feature will distinguish the run equilibrium

[4] When $y(n) = 0$, impatient agents are indifferent between spending and not spending. To break ties, we assume that they spend their CBDC balances in $t = 1$.

here from the bank run equilibrium in Diamond and Dybvig (1983), in which a commercial bank prematurely liquidates all of its assets to satisfy the demand for withdrawals in period 1, therefore, ultimately running out of resources. If $n > \lambda$, the central bank is confronted with a run on deposits. As we will see, the real consequences of a run on the central bank with nominal contracts can be similar to its counterpart in the model with real contracts. However, we shall demonstrate that the central bank's ability to avert a run is necessarily tied to its monopoly on currency and the implementation of a nominal contract. Importantly, by equations (11) and (12), a patient agent's optimal decision whether to run on the central bank, to spend or not, depends on the central bank's choices only through the liquidation policy $y(\cdot)$ and not via the nominal elements M and $i(n)$. By our equilibrium definition, the aggregate spending behavior n has to be consistent with optimal individual choices. These considerations imply the following lemma.

Lemma 5.1. *Given the central bank policy* $(M, y(\cdot), i(\cdot))$*,*

(i) The absence of a run, $n = \lambda$*, is an equilibrium only if* $x_1(\lambda) \leq x_2(\lambda)$*.*

(ii) A central bank run, $n = 1$*, is an equilibrium if and only if* $x_1(1) \geq x_2(1)$*.*

(iii) A partial run, $n \in (\lambda, 1)$*, occurs in equilibrium if and only if patient agents are indifferent between either action, requiring* $x_1(n) = x_2(n)$*.*

This lemma fully characterizes the range of equilibria, given the implied real allocation of a central bank policy. But how can policy attain the first-best allocation?

5.1 Implementation of optimal risk sharing via liquidation policy

By $(x_1^*, x_2^*) = \left(\frac{y^*}{\lambda}, \frac{R(1-y^*)}{1-\lambda}\right)$, the feasibility constraint $y \in [0, 1]$, and the optimality conditions in Section 3.1, the implementation of optimal risk sharing requires a liquidation policy

$$y^*(\lambda) = x_1^* \lambda \in (\lambda, 1] \tag{13}$$

given that only impatient types spend. Similarly to Diamond and Dybvig (1983), the resource constraint $y \in [0, 1]$ and $x_1^* > 1$ imply that optimal risk sharing is not feasible when all agents spend. The implied price level when n agents spend equals $P_1^*(n) = \frac{nM}{\lambda x_1^*}$. These results confirm our assertion at the start of this section that the social optimum is independent of price level stability. Combining the previous derivation with Lemma 5.1, we arrive at the following lemma.

Lemma 5.2. *The central bank policy* $(M, y(\cdot), i(\cdot))$ *implements optimal risk sharing* (x_1^*, x_2^*) *in dominant strategies if the central bank*

(i) sets $y(\lambda) = y^*$ *for any* $n \leq \lambda$*.*

(ii) sets a liquidation policy that implies $x_1(n) < x_2(n)$ *for all* $n > \lambda$*.*

The real allocation to agents and, thus, their incentives to spend or not depend on the central bank policy $(M, y(\cdot), i(\cdot))$ only through the liquidation policy $y(\cdot)$. Given that only impatient agents are spending (i.e., $n = \lambda$), then a policy choice with $y(\lambda) = y^*$ implements the social optimum. That is, there is a multiplicity of monetary policies that implement the first-best since the pair $(M, i(\cdot))$ is not uniquely pinned down. While the pair $(M, i(\cdot))$ does not affect depositors' incentives, it has an impact on prices via equations (7) and (8).

Second, thanks to the existence of the storage technology, patient agents can –but do not have to– spend their CBDC balances at time two. Spending at time two is dominant only if for every possible spending level n the real allocation at time two exceeds the allocation at $t = 1$.

The central bank internalizes depositors' decision making. Since it observes aggregate spending behavior n *before* it liquidates any asset, the central bank is not committed to liquidating y^* if patient agents are also spending. Condition (ii) of this lemma corresponds to the classic incentive-compatibility constraint in the bank run literature: since expectations are rational, in $t = 1$, depositors correctly anticipate the central bank policy that follows spending behavior n. To deter patient agents from spending, the central bank can threaten to implement a liquidation policy $y(\cdot)$ that makes spending non-optimal *ex-post*, i.e., so that $x_1(n) < x_2(n)$ for $n \in (\lambda, 1]$. If the monetary authority can credibly threaten patient agents by setting such a liquidation policy, it deters them from spending ex-ante, and a central bank run does not occur in equilibrium. Therefore, in the unique equilibrium, only impatient agents spend, all patient agents roll over, and the social optimum is always attained.

The central bank implements "spending late" as the dominant equilibrium strategy for patient agents by fine-tuning the real goods supply via its liquidation policy, i.e., by making real asset liquidation spending-contingent.

Definition 4. *We call a liquidation policy $y(\cdot)$ "run-deterring" if it satisfies*

$$y^d(n) < \frac{nR}{1 + n(R-1)}, \quad \text{for all } n \in (\lambda, 1]. \tag{14}$$

Such a liquidation policy implies that "roll over" is ex-post optimal $x_1(n) < x_2(n)$ even though patient agents are withdrawing $n \in (\lambda, 1]$.

The implementation of a run-deterring policy is only possible because the contracts between the central bank and the agents are nominal. The liquidation of investments in the real technology is at the central bank's discretion, thereby controlling the real goods supply and, for a given spending level, the real allocation in either time period. A spending-contingent liquidation policy implies a spending-contingent price level. In the case of real contracts between a private bank and depositors such as in Diamond and Dybvig (1983), in contrast, the real claims of the agents are fixed already in $t = 0$, thus pinning down a liquidation policy for every measure of aggregate spending n. In the case of large withdrawals, rationing must occur. Similarly, in the case of nominal contracts between a private bank and depositors, the private bank has to take the price level as given, which then again pins down the liquidation policy. Alternatively, the price level adjusts via market clearing to high aggregate nominal spending (Skeie, 2008), while here it can serve as a strategic control variable.

As the main result of this paper,

Corollary 5 (Trilemma part I - No price stability)**.** *Every policy choice $(M, y(\cdot), i(\cdot))$, $n \in [0,1]$ with*

$$y(\lambda) = y^* \text{ and } y^d(n) < \frac{nR}{1 + n(R-1)}, \quad \text{for all } n \in (\lambda, 1], \tag{15}$$

deters central bank runs and implements the social optimum in dominant strategies. Such a deterence policy choice requires the interim price level $P_1(n)$ to exceed the withdrawal-dependent bound:

$$P_1(n) > \frac{M}{R}(1 + n(R-1)), \quad \text{for all } n \in (\lambda, 1]. \tag{16}$$

Under a credible liquidation policy (15) all agents have a dominant strategy to spend if and only if the agent is impatient; otherwise they wait. Thus, under rational behavior, runs do not occur, and by $y(\lambda) = y^*$ the social optimum always obtains. That is, a strategic real supply shock enforced by the central bank *causes* a demand shock to CBDC spending that deters runs.

The implementation, however, comes at a price. To attain feasibility of a run-deterring policy $y(\cdot)$, the central bank has to sacrifice price stability. By condition (16), the more agents spend, the larger the required price level threat to deter runs. The threat has to be credible to deter runs *ex-ante*. Agents have to believe that *ex-post* the central bank will give up price stability if realized spending behavior is excessive. Only then do runs and inflation not occur on the equilibrium path.

In Diamond and Dybvig (1983), we learned the dilemma that offering the optimal amount of risk-sharing via demand-deposit contracts requires private banks to be prone to runs. Thus, a bad bank run equilibrium also exists. Our result brings this dilemma to the next level. If the bank is a central bank equipped with the power to set price levels and control the real goods supply, then optimal risk-sharing can be implemented in dominant strategies such that a bank run never occurs, but only at the expense of price stability.

Observe that by the optimality conditions and the resource constraint, $y^* < \frac{\lambda R}{1+\lambda(R-1)}$ holds and that the upper bound for $y^d(n)$ is increasing in n. Therefore, the constant liquidation policy

$$y(n) \equiv y^* \tag{17}$$

implements optimal risk sharing in dominant strategies. However, there exist other liquidation policies that can accomplish the same result. The policy (17) *is equivalent* to the run-proof dividend policy in **Jacklin 198X**, which is there implemented not via demand-deposit contracts but via real equity shares that can be traded at the interim stage among patient and impatient depositors. In Jacklin, the bank announces real dividend payments in $t = 0$ which pins down a supply of goods in $t = 1$ and $t = 2$. The agents can trade claims on dividends at the interim period but trade does not affects the overall goods supply. Therefore, Jacklin provides a special case of a run-deterring policy, implemented via predetermined real dividends. For the same reasons as above, since the real supply in Jacklin is predetermined in $t = 1$ and $t = 2$, the dividend policy in Jacklin does not allow constant price levels in general, see section 8.2 for details. The policy (17) also delivers the same result as does the classic suspension-of-convertibility option, which is known to exclude bank runs in the Diamond-Dybvig world.

There is a subtle but essential difference, though, between suspension and our liquidation policy. Suspension of convertibility requires the bank to stop paying customers who arrive after the fraction λ of agents have withdrawn. By contrast, in our environment, there is no restriction on agents to spend their digital euros in period 1, and there is no suspension of accounts. Instead, it is the supply of goods offered for trade against those digital euros and the resulting change in the price level that generate the incentives for patient agents to prefer to wait. This reasoning also implies that, in our model, (nominal) deposit insurance will not deter agents from running on the central bank.

More concretely, low liquidation and thus supply implies that the price level P_1 is pushed above an upper bound that is increasing in the aggregate spending.[5] The low liquidation policy, on the other hand, deters large spending *ex-ante*, such that the high price level (16) is a threat that is realized only off-equilibrium. But each time we have an off-equilibrium threat, we should worry about the possibility of time inconsistency. In comparison with the classical treatment of time inconsistency in Kydland and Prescott (1977), the concern here is not that the central bank will be tempted to inflate too much, but that it would be tempted to inflate too little. The central bank can avoid suboptimal allocations by committing to let inflation grow whenever necessary. A similar concern appears in models with a zero lower bound on nominal interest rates. Eggertsson and Woodford (2003) have shown that a central bank then wants to commit to keeping interest rates sufficiently low for sufficiently long, even after the economy is out of recession, to get the economy off the zero lower bound (see also Krugman, 1998, for an early version of this idea). But once the economy is away from the zero lower bound, there is an incentive to renege on the commitment to lower interest rates and avoid an increase in the price level.

In our model, we assume that the central bank fully commits such that the threat is credible. But what if the central bank is concerned with price stability and, therefore, refuses to induce a high price level?

6 The classic policy goal: Price level targeting

There are many possible reasons why central banks view the stabilization of price levels or, more generally, inflation rates as one of their prime objectives. The model here should be viewed as part of a larger macroeconomic environment, where the objective of price stability

[5] Our result resembles Theorem 4 in Allen and Gale (1998) and has a similar intuition. In Allen and Gale (1998), a central bank lends to a representative bank an interest-free line of credit to dilute the claims of the early consumers so that they bear a share of the low returns to the risky asset. In their environment, private bank runs are required to achieve the first-best risk allocation.

must be taken into account. That objective could arise out of concerns regarding nominal rigidities or legal mandates, and they may be socially optimal, requiring an appropriate modification of (1). The task at hand, then, is to examine how price stability imposes constraints on central bank policy. In particular, we will document the existence of deep tensions between the three objectives of attaining the first-best outcome, deterring central bank runs, and maintaining price stability.

Addressing the time-inconsistency problem above requires the introduction of an objective function for the central bank. Given an objective function for the central bank, a time-consistent equilibrium is a commitment equilibrium such that the central bank policy $(M, y(n), i(n))$ and the resulting price levels $(P_1(n), P_2(n))$ maximize the central bank's objective function for every value $n \in [0, 1]$. A particular objective is that the central bank pursues price stability above everything else. We shall distinguish between two versions of the objective of price stability: full price stability and partial price stability. Let us start by analyzing the former.

6.1 Full price stability

Definition 6. *We call a central bank policy*

(i) P_1***-stable at level*** $\overline{P}$*, if it achieves* $P_1(n) \equiv \overline{P}$ *for the* ***price level target*** $\overline{P}$*, for all spending behavior* $n \in [\lambda, 1]$.

(ii) ***price-stable at level*** $\overline{P}$*, if it achieves* $P_1(n) = P_2(n) \equiv \overline{P}$ *for the* ***price level target*** $\overline{P}$*, for all spending behavior* $n \in [\lambda, 1]$.

In our definition, price stability here is treated as a mandate and commitment to the price level $\overline{P}$ even for off-equilibrium realizations of n. From the definition, price stability at some level $\overline{P}$ implies P_1 stability at $\overline{P}$. Hence, the second price stability criterion is stronger.

Definition 7. *Given a price goal* $\overline{P}$*, we call a commitment equilibrium a*

- P_1**-price-commitment equilibrium***, if the central bank policy is* P_1*-stable at level* $\overline{P}$
- **price-commitment equilibrium***, if the central bank policy is price-stable at level* $\overline{P}$

What constraints does the price stability objective impose on central bank policy?

Proposition 8 (Policy under Full Price Stability)**.** *A central bank policy is:*

(i) P_1*-stable at level* $\overline{P}$*, if and only if its liquidation policy satisfies:*

$$y(n) = \frac{M}{\overline{P}}n, \textit{ for all } n \in [0,1] \tag{18}$$

implying a real interim allocation:

$$x_1(n) \equiv \overline{x}_1 = \frac{M}{\overline{P}} \leq 1. \tag{19}$$

(ii) A central bank policy is price-stable at level $\overline{P}$*, if and only if its liquidation policy satisfies equation* (18)*, its price level satisfies (19), and its interest policy satisfies:*

$$i(n) = \frac{\frac{\overline{P}}{M} - n}{1-n}R - 1. \tag{20}$$

A price-stable liquidation policy (18) requires asset liquidation in constant proportion to aggregate spending for all $n \in [0,1]$; see the green line in Figure 3, where we plot $y(n)$ for partial versus full price-stable liquidation policies. As a consequence, individual real consumption x_1 is constant regardless of aggregate spending behavior, and cuts below 1 since, due to the resource constraint, the central bank cannot liquidate more than the entire investment. Hence, a price-stable liquidation policy excludes rationing or all kinds of suspension policies. By equation (19) and again due to the resource constraint, for a given money supply M, only price levels $\bar{P} \geq M$ can be P_1- stable or price-stable. The slope of the liquidation policy is, thus, equal or below 1. In other words, the rationing problem shows up indirectly through a lower bound on all possible price-stable central bank policies.

There is a caveat here. Should agents be able to operate the savings technology on their own, then they can always assure themselves a payoff of 1 in period $t = 1$ for every good stored in period $t = 0$. Thus, the only CBDC contract acceptable to these agents would be a "green line" coinciding with the 45-degree line and a slope of 1. Slopes below 1 are agreeable, however, if the central bank is the only entity capable of operating this technology or the only entity capable of intermediation with operators of that technology.

Recall from Section 5, that optimal risk sharing satisfies $x_1^* > 1$, while from Proposition

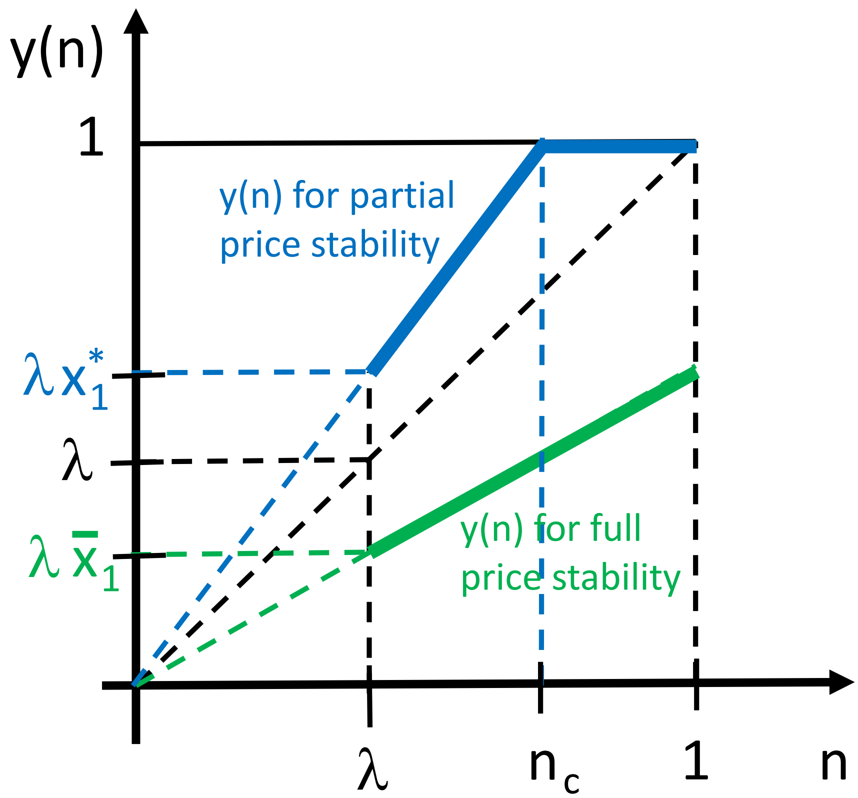

Figure 3: Partial vs. full price-stable liquidation policies

8 a P_1 price-stable policy requires $x_1 \leq 1$. Therefore, we can show the second part of our trilemma:

Corollary 9 (Trilemma part II - No optimal Risk-sharing)**.** *If the central bank commits to a P_1-stable policy, then:*

(i) Optimal risk sharing is never implemented.

(ii) The no-run equilibrium is implemented in dominant strategies, i.e., there is a unique equilibrium in which only impatient agents spend, $n^* = \lambda$, and there are no central bank run equilibria.

(iii) If the central bank commits to a price-stable central bank policy, then the nominal interest rate is increasing in n and non-negative $i(n) \geq 0$ for all $n \in [\lambda, 1]$.

Intuitively, no runs take place under a P_1-stable policy since the real allocation in $t = 1$ is too low, causing all patient agents to prefer to spend late.

6.2 Partial price stability

While price stability and the absence of central bank runs may be desirable, the constraint (19), i.e., the failure to implement optimal risk sharing, is not. In particular, the implementation of the social optimum is impossible under complete price stability. Recall that optimal risk-sharing at $x_1^* > 1$ triggers potential bank runs in models of the Diamond-Dybvig variety: thus, part (ii) of the proposition above should not be a surprise. Demanding price stability for all possible spending realizations of n is thus too stringent: for sufficiently high spending levels of n, equation (18) exhausts the liquidation possibilities available to a central bank, as $y(n)$ cannot exceed 1. We therefore examine a more modest goal: a central bank may still wish to assure price stability, but may deviate from its goal in times of crises. We capture this with the following definition.

Definition 10. *A central bank policy is*

(i) ***partially*** P_1***-stable at level*** $\overline{P}$*, if for all spending behavior* $n \in [\lambda, 1]$*, either the policy achieves* $P_1(n) = \overline{P}$ *for some* ***price level target*** $\overline{P}$*, or aggregate spending satisfies* $n > \bar{P}/M$*. In the latter case, we require full liquidation,* $y(n) = 1$*.*

(ii) ***partially price-stable at level*** $\overline{P}$*, if for all spending behavior* $n \in [\lambda, 1]$*, either the policy achieves* $P_1(n) = P_2(n) = \overline{P}$ *for some* ***price level target*** $\overline{P}$*, or aggregate spending satisfies* $n > \bar{P}/M$*. In the latter case, we require* $y(n) = 1$*.*

For a graphical illustration, see the blue line in Figure 3. Obviously, P_1-stable central bank policies are also partially P_1-stable, and price-stable central bank policies are also partially price-stable.

Definition 11. *Given a price goal* $\overline{P}$*, we call a commitment equilibrium a*

- **partial** P_1**-price-commitment equilibrium***, if the central bank policy is partially* P_1*-stable at level* $\overline{P}$
- **partial price-commitment equilibrium***, if the central bank policy is partially price-stable at level* $\overline{P}$

Recall that only price levels above the money supply $\overline{P} \geq M$ can attain full price stability. We therefore now concentrate on lower price levels $M > \overline{P}$, since attaining optimality requires $1 < x_1^* = M/\bar{P}$. Nevertheless, we also encounter a (weaker) feasibility constraint for partially price-stable policies. Since the central bank cannot liquidate more than the entire asset, $y(n) = x_1 n \in [0, 1]$ for all $n \in [\lambda, 1]$, it faces the constraint $\lambda x_1 \leq 1$. Feasibility, therefore, implies a lower bound on all possible partially stable price levels, $\overline{P} \geq \lambda M$. Furthermore, partial price stability restricts central bank policies:

Proposition 12 (Policy under Partial Price-Stability)**.** *Suppose that $M > \overline{P} \geq \lambda M$.*

(i) A central bank policy is partially P_1-stable at level $\overline{P}$, if and only if its liquidation policy satisfies:

$$y(n) = \min\left\{\frac{M}{\overline{P}}n, 1\right\}. \tag{21}$$

(ii) For every partially P_1-stable central bank policy at level $\overline{P}$, there exists a critical aggregate spending level $n_c \equiv \frac{\overline{P}}{M} \in (0, 1)$ such that

(ii.a) For all $n \leq n_c$, the price level is stable at $P_1(n) = \overline{P}$ and the real goods purchased per agent in period $t = 1$ equal $x_1(n) = \overline{x}_1 = \frac{M}{\overline{P}} > 1$ while real goods purchased per agent in period $t = 2$ equal $x_2(n) = R(1 - \overline{x}_1 n)/(1 - n)$.

(ii.b) For spending $n > n_c$, the real goods purchased per agent in period $t = 1$ equal $x_1(n) = 1/n$ while $x_2(n) = 0$ and the price level $P_1(n)$ proportionally increases with total spending n: $P_1(n) = Mn$

(iii) A central bank policy is partially price-stable at $\overline{P}$, if and only if its liquidation policy satisfies equation (21) and its interest policy satisfies:

$$i(n) = \frac{\frac{\overline{P}}{M} - n}{1 - n}R - 1, \qquad \textit{for all } n \leq n_c. \tag{22}$$

For $n > n_c$, there is no supply of real goods in $t = 2$. Thus, P_2 and $i(n)$ are irrelevant.

(iv) For a partially price-stable central bank policy at $\overline{P}$, there exists a spending level

$$n_0 = \frac{R\frac{\overline{P}}{M} - 1}{R - 1} = \frac{Rn_c - 1}{R - 1} \in [0, n_c), \tag{23}$$

such that the nominal interest rate turns negative for all $n \in (n_0, n_c)$. For $R < M/\overline{P}$, the nominal interest rate is negative for all $n \in [0, n_c)$.

Proposition 12 reflects the central bank's capacity to keep the price level and the real interim allocation x_1 stable as long as spending remains below the critical level n_c. The stabilization of the price level requires liquidation of real investment proportionally to aggregate spending by factor $M/\overline{P}$. At the critical spending level n_c, the central bank is forced to liquidate the entire asset to maintain the price level P_1. Since the central bank cannot liquidate more than its entire investment, as spending exceeds the critical level n_c, price level stabilization via liquidation of real assets becomes impossible. For all spending behavior $n > n_c$, the real allocation to late spending agents is thus zero. The rationing of real goods implies that the price level has to rise and the real allocation declines in aggregate spending.

The spending level $n_0 < n_c$ is the level at which the real allocation to early and late spenders is just equal

$$x_1(n_0) = x_2(n_0) = \bar{x}_1. \tag{24}$$

Notice that $x_2(n)$ declines in n for $n \in [0, n_c]$. Thus, if fewer than measure n_0 of agents spend, not spending is optimal for patient agents. But for all spending realizations $n > n_0$, the allocation at $t = 2$ undercuts the allocation at $t = 1$: $x_2(n) < x_1(n)$, turning the real interest rate on the CBDC negative, and causing "spend early" to be a patient agent's optimal response to an aggregate spending behavior in excess of n_0. Consequently, self-fulfilling runs are possible as in Diamond and Dybvig (1983), and we obtain the following result as a corollary of Proposition 12:

Corollary 13 (Trilemma part III- Runs on the Central Bank (Fragility))**.** *Under every partially P_1-stable central bank policy with $M > \overline{P} \geq \lambda M$, there is a multiplicity of equilibria:*

(i) There exists a good equilibrium in which only impatient agents spend, $n^* = \lambda$. In that case, there is no run on the central bank, the social optimum is attained and the price level is stable at level $\overline{P}$.

(ii) There also exists a bad equilibrium in which a central bank run occurs, $n^* = 1$, the social optimum is not attained, and the price level is unstable.

Proposition 12 is in marked contrast to Proposition 8. One could argue that when banking is interesting, i.e., $x_1^* > 1$, then the goal of price stability induces the possibility of runs on the central bank, the necessity for negative nominal interest rates, and the abolishment of the price stability goal, if a run indeed occurs.

7 Money supply policy or suspension of spending

It is natural to ask why the central bank cannot resort to a much more classical monetary policy to resolve the trilemma and attain price stability: expansion or reduction of the money supply. In this section, let us then allow for the possibility that M is state-contingent, i.e., M is chosen as a function of aggregate spending $M = M(n)$ at $t = 1$. Therefore, a central bank policy consists of three functions $(M(\cdot), y(\cdot), i(\cdot))$.

The analysis is now straightforward and easiest to explain for the case where the liquidation policy is not state-contingent, $y(n) \equiv y^*$. To maintain price stability at some level $\overline{P}$, market clearing demands

$$nM(n) = \overline{P}y^*. \tag{25}$$

As a result, the total money balances spent in $t = 1$ stay constant in n, implying

$$n\,M(n) \equiv \lambda\,M(\lambda), \quad \text{for all } n \in [\lambda, 1]. \tag{26}$$

But spending per agent alters, as does the total money supply $M(n)$. That is, the central bank would have to commit itself to **reduce** the quantity of money in circulation in response to a demand shock encapsulated in n: the more people go shopping, the lower are individual money balances. With the policy (25), $y(n) \equiv y^*$ and $i(n) \equiv i^*$ chosen so that $P_2 = \overline{P}$, the central bank can now achieve full price stability, efficiency and financial stability. The CBDC trilemma appears to be resolved. There are several ways of thinking about this.

State-contingent money supply. A first approach is to make the amount of CBDC balances available for shopping in $t = 1$ state-contingent. Having such CBDC accounts with random balances is an intriguing possibility: it is quite impossible with paper money but fairly straightforward with electronic forms of currency. A different interpretation of this approach is to think in terms of a state-contingent nominal interest rate paid on CBDC accounts between $t = 0$ and $t = 1$. One should recognize that both of these routes are a bit odd, and contrary to how we usually treat money and interest rates. As for money, a dollar today is a dollar tomorrow: changing that amount in a state-contingent fashion probably risks severely undermining the trust in the monetary system, and trust is key for maintaining a fiat currency. As for interest rates, it is commonly understood that interest rates are agreed upon before events are realized in the future. A state-contingent interest rate turns accounts into risky and equity-like contracts, likewise undermining trust in the safety of the system.

Helicopter drops. A third way to think about the state-contingent nature of M corresponds to a classic monetary injection in the form of state-contingent lump-sum payments ("helicopter drops") $M(n) - \bar{M}$ (or taxes, if negative), compared to some original baseline $\bar{M}$. If one wishes to insist that $M(n) - \bar{M} \geq 0$, i.e., only allowing helicopter drops, then the central bank would choose $\bar{M} \leq M(1)$ as payment for goods in period $t = 0$ and thus always distribute additional helicopter money in the "normal" case $n = \lambda$ in period 1. Notice that distributional issues would arise in richer models, where agents are not coordinating on the same action, thereby distorting savings incentives.

Suspension of spending. With an account-based CBDC, there is an additional and rather drastic policy tool at the disposal of the central bank: the central bank can simply disallow agents to spend (i.e., transfer to others) more than a certain amount on their account. In other words, the bank can impose a "corralito" and suspend spending. This policy is different from the standard suspension of liquidation, as the amount of goods to-be-made available is a policy-induced choice that still exists separately from the suspension of spending policy. Notice also that "suspension of spending" should perhaps not be called "suspension of withdrawal." Since there are only CBDC accounts and they cannot be converted into something else, the amounts can only be transferred to another account. With the suspension of spending policy, the central bank could arrange matters in such a way that not more than the initially intended amount of money $\lambda M(\lambda)$ will be spent in period 1; see equation (26). In practice, the central bank would then either take all spending requests at once and, if the total spending requests exceeded the overall threshold, impose a pro-rata spending limit. Alternatively, it could arrange and work through the spending requests in some sequence (first-come-first-served), thereby possibly imposing different limits depending on the position of a request in that queue.

Monetary neutrality. Last but not least, a state-contingent money supply cannot replace the central bank's liquidation policy as the active policy variable. Not only price-targeting but also the deterrence of runs is an objective of the central bank for attaining optimal risk sharing.

A state-contingent money supply, however, does not impact the agent's spending behavior: the individual agents exclusively care for their individual real allocation at $t = 1$, y/n, versus $t = 2$, $R(1 - y)/(1 - n)$. These allocations are independent of nominal quantities (M, P_1). That is, money is neutral. Given a realization of an individual real allocation y/n, the identity:

$$\frac{y}{n} = \frac{M(n)}{P_1} \tag{27}$$

pins down a relationship that needs to hold between the money supply and the price level that clears the market. The central bank can implement all money supplies and price level pairs (M, P_1) that satisfy equation (27). And as soon as the price level goal P_1 is pinned down, contingent on the realization $\frac{y}{n}$, the money supply that solves equation (27) is unique. But in equation (27) the classic dichotomy holds, and the choice of the right-hand side (M, P_1) cannot alter the left-hand side, i.e., cannot alter incentives to run. Consequently, if the central bank wants to impact consumers' behavior to run on the central bank to implement the social optimum, it can only do so by altering the real goods supply y through adjustment of its liquidation policy.

In summary. Given the previous discussion, a state-contingent money supply strikes us as odd monetary policy. First, the usual inclination for central banks is to accommodate an increase in demand with a rise, rather than a decline in the money supply. A central bank that reacts to an increase in demand by making money scarce may undermine trust in the monetary system. In particular, and needless to say, a spending suspension might create considerable havoc; the experience in Argentina at the end of 2001 provides ample proof. Even if this was not the case, monetary neutrality implies that adjusting the money supply does not affect the run decisions of agents. Therefore, we think that this particular escape route from the CBDC trilemma needs to be treated with considerable caution.

8 Voluntary participation in CBDC and competition by private banks

The main model assumes that all consumers invest in a CBDC. It remains to clarify whether agents may be better off using the investment technology on their own, rather than relying on the central bank. This is an important question: if agents were to decide to stay in autarky and invest in the investment technology directly, they may have incentives to supply goods at the interim stage, thus, potentially undermining the central bank's liquidation policy. Similarly, if the outside option is not autarky but investing in deposits with a different, private bank, then the liquidation policy of that private bank has implications for the aggregate real goods supply at the interim stage, again impairing the effectiveness of the central bank's policy. We now discuss both.

8.1 Autarky and voluntary participation in a CBDC

Assume all but one agent invest in a CBDC. Assume that this single agent invests in the real technology at $t = 0$, yielding storage between $t = 0$ and $t = 1$, and yielding $R > 1$ when held between t$t = 0$ and $t = 2$. Then, at $t = 1$, she would learn her type. If she is impatient, she will liquidate the technology, yielding 1 unit of the real good, and she would consume her good. She would not sell the good against nominal CBDC deposits, since she only cares about consumption at $t = 1$. In the case where she is impatient, she is worse off in comparison to an agent who invested in CBDCs with the central bank if the central bank offers optimal risk sharing and manages to implement a run-deterring policy. This is so, since under the latter, an individual impatient agent gets $x_1^* > 1$ real goods.

If the individual agent is patient, she will stay invested in the technology until time two. There, the technology yields $R > 1$ units of the good. The agent will, thus, be better off than under investment in a CBDC since $x_2^* < R$; see Section 3.1. But, in particular, also in the patient case, the individual agent will not offer goods for sale in the interim period, since liquidation and selling against a CBDC will only yield x_2^* in $t = 2$. Thus, in any case, patient or impatient, the agent who invests in autarky will not have an incentive to undermine the central bank's policy by increasing the goods supply in the interim period.

Does the agent prefer to remain in autarky rather than participating in the CBDC? *Ex-ante*, the risk-averse agent cannot know whether she will turn out to be patient or impatient. Diamond and Dybvig (1983) show that pooling of resources via banking can attain the social optimum under an absence of runs, while investment under autarky cannot. That is, the single agent is always better off investing in the CBDC account if the central bank offers optimal risk-sharing and implements a run-deterring policy. Thus, participation in the CBDC account is individually rational.

What if the central bank runs a policy of full price stability at goal $\bar{P}$? In that case, our second main result, Corollary 9, shows that runs on the central bank do not occur but $x_1 \leq 1$. Thus, for all $x_1 < 1$, investing in a CBDC is dominated by investing in autarky. Voluntary participation thus requires $x_1 = 1$ or $M = \bar{P}$, implying $x_2 = R$. The agent is then indifferent between investing in a CBDC and staying in autarky. Yet, if she stayed in autarky, she will not undermine the central bank's liquidation policy for the reasons above.

In the case of a partial price-stable policy, the situation is as in Diamond and Dybvig (1983). *Ex-ante*, the agent cannot know whether a run occurs or not. Conditional on the no-run equilibrium, we implement the social optimum and the agent is better off investing in a CBDC. But conditional on the run equilibrium, she was better off in autarky. From within the model, it is not possible to attach likelihoods for each equilibrium.

8.2 Can private banks undermine the central bank's policy?

The question of under what circumstances consumers prefer investing in a CBDC account with the central bank rather than investing in demand deposits with private banks, with implications for how both types of banks can coexist is addressed in Fernández-Villaverde et al. (2020). In this section, we will analyze the private banks' incentives to provide goods at the interim stage, *conditional on the coexistence of private banks with the central bank.*

Goods supply. If the central bank coexists with private banks, it controls the market of goods only partially, with the remainder of the real goods being supplied by commercial banks. As before, the measure of agents is normalized to one, divided between a share $\alpha \in (0, 1)$ of agents who are CBDC customers at the central bank and a share $1 - \alpha$ who are customers at private banks. Assume that all agents invest their 1 unit endowment in their corresponding bank and that the private banks invest in the same asset as the central bank does. Then, at $t = 1$, the central bank can supply up to α goods via liquidation, while private banks can supply up to $1 - \alpha$ goods.

Assume that there is one centralized goods market to which customers and banks have access. That is, CBDC depositors can spend CBDC balances on goods supplied by private banks and private bank customers can spend their private deposit balances on goods supplied by the central bank. Let n denote the total measure of spending agents across both customer groups at the central bank and private banks, given by

$$n = \alpha\, n_{CB} + (1-\alpha)\, n_P, \tag{28}$$

where n_{CB} is the total share of consumers at the central bank who spend, while n_P is the total share of consumers at the private bank who spend. Given total spending n in period $t = 1$, let $y_P(n)$ be the share of assets liquidated by private banks. In contrast, let $y_{CB}(n)$ be the central bank's liquidation policy, i.e., the share of assets liquidated by the central bank. The total goods supply y in the centralized market at the interim stage is then:

$$y(n) = \alpha\, y_{CB}(n) + (1-\alpha)\, y_P(n). \tag{29}$$

Private deposit making. To collect investment in $t = 0$, the private banks offer a nominal demand-deposit account in return for 1 unit of the real good. The private nominal accounts are denominated in units of the CBDC. Due to competition with the central bank, the private contract also offers M units of the CBDC in $t = 1$ or $M(1 + i(n))$ units in $t = 2$.

To service withdrawals in terms of CBDC, private banks first observe their customers' CBDC withdrawal needs n_P, and borrow the required amount $(1-\alpha)n_P M$ of the CBDC from the central bank at the beginning of period $t = 1$. The central bank creates the CBDC quantity $(1-\alpha)n_P M$ on demand for the private banks. Private banks observe CBDC spending at the central bank n_{CB}, yielding aggregate spending n. During period one, the private banks sell the share $y_P(n)$ of their real goods investment at price P_1 at the centralized market to all consumers, thus receiving proceeds of $P_1 y_P(n)(1-\alpha)$ units of the CBDC in return, where P_1 satisfies market clearing:

$$M\Big((1-\alpha)n_P + \alpha n_{CB}\Big) = P_1\Big(y_P(n)(1-\alpha) + y_{CB}(n)\,\alpha\Big). \tag{30}$$

The private banks use these CBDC proceeds to (partially) repay their loan to the central bank at zero interest within period one. Since the central bank retains only partial control over the goods market, it generically no longer holds $n_{CB}M = P_1 y_{CB}(n)$. As a consequence, the private banks can hold positive or negative CBDC balances $(1-\alpha)(P_1 y_P(n) - n_P M)$ with the central bank between $t = 1$ and $t = 2$.

We seek to examine a range of possibilities for the private bank withdrawals n_P as well as liquidation choices y_P. Thus, it is useful to impose the condition that private banks make zero profits, regardless of the "circumstances" n_P or their choice for y_P. This requires some careful calculation, which we provide in Appendix 14, and only summarized here.

We assume that the central bank charges or pays the nominal interest rate $z = (RP_2/P_1) - 1$ on the excess or deficit CBDC balances of private banks, to be settled at the end of $t = 2$. Note that $z > i$, if $x_1 > 1$ and equals the internal nominal shadow interest rate regarding private bank liquidation choices. Moreover, we impose a market share tax at the end of period $t = 2$ in order to compensate for profits or losses due to circumstances n_P.

At $t = 2$, the remaining private customers spend the quantity $(1-\alpha)(1-n_P)M(1+i(n))$ of private CBDC accounts that the private banks borrow from the central bank at the beginning of $t = 2$. The private banks sell their returns on the remaining investment $R(1-y_P(n))(1-\alpha)$ at price P_2, where P_2 satisfies market clearing

$$M(1 + i(n)) \Big((1 - \alpha)(1 - n_P) + \alpha(1 - n_{CB})\Big) = P_2 R \Big((1 - y_P(n))(1 - \alpha) + (1 - y_{CB}(n))\, \alpha\Big). \quad (31)$$

At the end of $t = 2$, the private banks settle their accounts with the central bank, taking into account the remaining balances at $t = 1$ adjusted for interest, the end-of-period tax compensating for circumstances n_P, the loan at the beginning of $t = 2$ and the sales proceeds at $t = 2$.

Joint liquidation policies. The actions of private banks and the central bank may not be perfectly aligned when it comes to the liquidation of assets and the supply of goods at the interim stage. Private banks can have various objectives depending on their ownership structure and may be subject to regulation of their liquidation policy, both shaping y_P. Independently of whether private banks maximize depositor welfare as in Diamond and Dybvig (1983), or pursue some other objective, the prevention of runs is key. We have seen above that runs occur if the provision of real goods at the interim stage is high. Since the market is centralized, for the spending incentives of bank customers it is irrelevant whether these goods are provided by the central bank's or the private bank's liquidation of assets.

Hence, as before, a run-deterring liquidation policy $y(\cdot)$ is a function of aggregate spending

n such that the real allocation at $t = 1$ undercuts the real allocation at $t = 2$:

$$\frac{y(n)}{n} < R\frac{(1 - y(n))}{1 - n}, \quad \text{for all } n \in [\lambda, 1]. \tag{32}$$

Thus, again, a run-deterring policy satisfies

$$y(n) < \frac{nR}{1 + n(R - 1)}, \quad \text{for all } n \in [\lambda, 1]. \tag{33}$$

Perfect coordination. If the central bank and the private banks coordinate perfectly, i.e., act as one entity, and have full control over the asset liquidation, then all run-deterring policies are possible, as in the case where the central bank is a monopolist. But why would they coordinate perfectly? By the market's centralization, the destiny of the central bank is intertwined with the destiny of the private banks and both types of banks have an interest in deterring runs. In particular, the private bank will, therefore, not undermine a central bank's run-deterring policy by supplying additional goods when, for instance, prices are high, since this might cause a run not only on the central bank but also on the private bank. Coordination is therefore among the equilibrium outcomes.

Runs under imperfect coordination. For general liquidation policies y_P of private banks, runs can occur, as the following example shows. Assume that the private bank for some reason follows a liquidation rule $y_P(n) \in [0, 1]$ where $y_P(n_b) = 1$ for all $n \geq n_b$ where $n_b \in (0, 1)$. For instance, $n_b = 1 - \alpha$, i.e., the private bank is subject to regulation and has to liquidate all assets if a fraction of its customers equal to its market share spends. In that case, as we show next, the central bank's capacity to deter runs depends on the size of the private banking sector, i.e., its market power α. Since the central bank can only control the liquidation of its own investment y_{CP}, via (32) and (29), a run-deterring policy y_{CB} needs to satisfy

$$y_{CB}(n) < \frac{Rn - (1 - \alpha)y_P(n)(Rn + 1 - n)}{\alpha(Rn + 1 - n)}, \quad \text{for all } n \in [\lambda, 1]. \tag{34}$$

Now assume $n > n_b$, such that $y_P(n) = 1$. If in addition the central bank has a small market share $\alpha \to 0$, then the numerator converges to $-(1 - n)$, while the denominator goes to zero, $\alpha(1 + (R - 1)n) \to 0$. That is, for $n_b < n < 1$, the right-hand side in (34) goes to minus infinity such that (34) cannot hold. This implies that the run equilibrium exists.

A sufficient condition: Run-deterrence under imperfect coordination. The example above makes clear that the central bank's share in the deposit market needs to be large enough in order to prevent runs. The following proposition provides the appropriate bound under which the central bank can ensure the absence of a run, regardless of the private bank's liquidation schedule $y_P : [\lambda, 1] \rightarrow [0, 1]$.

Proposition 14. *Suppose that the central bank's share in the deposit market satisfies*

$$\alpha > \frac{1-\lambda}{(1-\lambda+R\lambda)}. \tag{35}$$

Then the central bank can always find a run-deterring liquidation policy $y_{CB} : [\lambda, 1] \rightarrow [0, 1]$, regardless of the private bank's liquidation policy $y_P : [\lambda, 1] \rightarrow [0, 1]$.

Such an $\alpha \in (0, 1)$ exists since $\frac{1-\lambda}{(1-\lambda+R\lambda)} \in (0, 1)$. Thus, the right-hand side $\frac{1-\lambda}{(1-\lambda+R\lambda)}$ of equation (35) imposes a lower bound on the balance-sheet size of the central bank as a percentage of the total demand deposit market, such that run-deterring policies remain possible despite coexisting private banks that are subject to liquidation restrictions.

Proof. [Proposition 14] We need to show that for any private bank liquidation policy $y_P : [\lambda, 1] \rightarrow [0, 1]$, there is a central bank liquidation policy $y_{CB} : [\lambda, 1] \rightarrow [0, 1]$ so that (34) is satisfied. To derive a sufficient condition on the central bank's market share α under which it can nevertheless implement a run-deterring policy, note that by $R > 1$, the right-hand side in (34) declines in the value y_p for all $\alpha \in (0, 1)$. Thus, if a central bank policy y_{CP} is run-deterring for $y_P = 1$ for all $n \in [0, 1]$, then y_{CP} is also run-deterring for a private bank policy $y_P(n) \leq 1$ for all $n \in [0, 1]$. Thus, assume $y_P = 1$ for all $n \in [0, 1]$. Then, a sufficient condition for a run-deterring policy against all private bank policies y_P is:

$$y_{CB}(n) < \frac{Rn - (1-\alpha)(Rn + (1-n))}{\alpha(1 + (R-1)n)} = 1 - \frac{1-n}{\alpha(1 + (R-1)n)}, \quad \text{for all } n \in [\lambda, 1]. \tag{36}$$

The right-hand side is increasing in n and $y_{CB}(n)$ cannot undercut zero. Thus, an n α such that:

$$0 < 1 - \frac{1-\lambda}{\alpha(1 + (R-1)\lambda)} \tag{37}$$

□

is a sufficient condition for the existence of a policy $y_{CB} \in [0, 1]$ that satisfies (36).

9 Trade in Equity Shares (Jacklin, 1987)

New section

In a real banking model, Diamond and Dybvig (1983) show that banks can offer the socially optimal risk-sharing allocation via demand-deposit contracts but are, as a consequence, prone to runs. Jacklin (1987) demonstrates that optimal risk sharing can be implemented in a run-proof way when the bank does not offer demand-deposits but rather shares in equity if (i) real dividend payments $D = \lambda c_1^*$ in $t = 1$ and $R(1 - D)$ in $t = 2$ are predetermined in $t = 0$ and (ii) there exists a market to trade claims on dividends in $t = 1$. The dividends accrue to all investors, patient and impatient. If equity markets exist and open in $t = 1$, then Jacklin demonstrates that patient investors have an incentive to puchase the impatient agent's late dividend payments in return for the lower, early dividend payments. Likewise, impatient agents have an incentive to trade their claims on a late dividend payment in return for early dividend payments. This trade happens in an incentive compatible way so that all agents, before learning their types in $t = 0$, are willing to agree to the predetermined dividend payments. In $t = 1$, impatient types cannot revolt or run on the bank to demand an additional share of their late dividend payment. Therefore, runs that enforce excess asset liquidation can no longer occur.

The question arises, would the Jacklin (1987) environment also work in our nominal banking model to prevent runs on the central bank. The answer is not only yes, but in fact, the dividend policy proposed in Jacklin (1987) is a special case of a run-deterring liquidation policy with a dividend payment equal to

$$D = \lambda\, c_1^* = y^*, \qquad \text{for all } n \in [0, 1] \tag{38}$$

That is, the liquidation policy discussed around equation (17) which implements the social optimum in dominant strategies via CBDC demand-deposits *is* the real allocation that is implemented in Jacklin (1987) via equity shares and trade in dividends. Similarly to the central bank's liquidation policy in our main model, in Jacklin (1987), the real dividends to be paid in $t = 1$ and $t = 2$ are predetermined in $t = 0$. Moreover, in Jacklin, all agents, patient and impatient, receive dividends. Therefore, also in Jacklin, the total real liquidation in $t = 1$, and thus, the total goods supply in $t = 1$ and $t = 2$ is predetermined in $t = 0$, independently of the individual agents' trading behavior at the interim stage. Like in our setting, the fixed supply of goods in $t = 1$ enforces incentive-compatible self-selection, thereby, implementing optimal risk sharing. In our setting, the fixed goods supply deters patient types from spending, while in Jacklin (1987) it enforces trade in real dividends between the patient and impatient agent group at a market clearing price.

Note, however, that our banking model features nominal contracts while in Jacklin (1987), dividends are denominated in real terms. By our main result (5), a run-deterring policy requires an inflation threat (16), otherwise, patient types would not self-select but may be tempted to shop early.

What if the dividend payments is Jacklin (1987) were nominal? Does inflation necessarily arise there too? And what is a run on a bank under trade in equity shares, i.e. in the Jacklin (1987) setting?

Assume the extreme case where agents can hold equity shares in the central bank. The total measure of all agents remains at one. The central bank offers nominal equity shares and invests in the real technology. Assume, all agents receive 1 unit of CBDC shares when investing their real goods in $t = 0$ and all agents, irrespective of their type, are paid a nominal dividend D_1 in $= 1$ and another nominal dividend D_2 by the central bank in $t = 2$.

The central bank follows a liquidation policy $y(n)$ and call (D_1, D_2) the central bank's dividend policy, where as before, $n \in [0, 1]$ denotes the measure of agents who go shopping with CBDC in $t = 1$.

Since dividends are paid to all shareholders, the total nominal CBDC supply equals D_1 in $t = 1$ and equals D_2 in $t = 2$. The central bank sets a price level P_1 at time $t = 1$ and P_2 in $t = 2$ that clears the goods market.

In $t = 1$, types realize and impatient types want to consume as much as possible in $t = 1$. Similar to Jacklin (1987), impatient types can can sell their claims on a nominal dividend D_2 in $t = 2$ in return for nominal dividends D_1 in $t = 1$ to purchase consumption goods provided by the central bank.

In Jacklin (1987), since dividends are real, they promise consumption in a one-to-one relation. With nominal dividends, this is no longer true. Crucially, the central bank sees shareholders and shoppers as two different agent groups.

Let $n \in [0, 1]$, the measure of agents who go shopping with CBDC in $t = 1$ to spend dividends $\tilde{D} \geq D$, after trade in nominal dividends has take place. Recall that there is no storage technology for nominal dividends. That is, either an agent trades D for consumption goods with the central bank directly or sells D in return for a claim on a larger nominal dividend D_2 in $t = 2$. Consequently, in the aggregate, the total measure D of nominal CBDC is supplied in the market by D/n agents who are demanding $y(n)$ goods at a market clearing price P_1. We can define a run on nominal equity shares as the incidence where patient types are not willing to trade their early dividends for late dividends with impatient types. That is, more than just impatient types $n > \lambda$ go shopping for real goods by spending their nominal dividends D_1, and the trade in dividends between the agent groups partially collapses.

After observing the total measure of shoppers n who jointly supply dividends D, the central bank supplies $y(n)$ goods according to her policy. The market clearing price P_1 satisfies

$$D = P_1 \, y(n) \tag{39}$$

Likewise in $t = 2$

$$D_2 = P_2 R(1 - y(n)) \tag{40}$$

The real allocations per agent equal

$$\frac{y(n)}{n} = \frac{D}{P_1 n}$$

in $t = 1$ and

$$\frac{R(1-y)}{1-n} = \frac{D_2}{P_2(1-n)}$$

in $t = 2$.

One difference to the nominal CBDC demand-deposit contract discussed above is, under nominal equity shares, and for a liquidation policy that is fixed in n, $y(n) = const$ for all $n \in [0, 1]$, the price level is stable in both $t = 1$ and $t = 2$. Nevertheless, in this nominal version of Jacklin (1987) runs can occur, in the sense that patient types are not willing to trade their early nominal dividends for late nominal dividends. That is, the key mechanism in Jacklin (1987) is not the fixation of equity shares and dividends but rather that the dividend payments and thus the real goods supply in $t = 1$ is predetermined in $t = 0$:

Set $y(n) = 1$ for all $n \in [0, 1]$. That is, the central bank liquidates all real technology at the interim stage so that the goods supply in $t = 2$ is zero, $R(1-y(n)) = 0$. Consequently, late dividend payments D_2 have zero real value, $P_2 \to \infty$, and all agents patient and impatient go shopping for goods in $t = 1$, implying $n = 1$ and trade in nominal equity shares collapses.

The central bank can however implement the social optimum by setting a liquidation policy $y(n) = y^*$ for $n = \lambda$. Moreover, the central bank has to deter patient types from shopping early so that nominal equity shares are traded, which happens when the liquidation policy is run-deterring $\frac{y(n)}{n} < \frac{R(1-y(n))}{1-n}$, for all $n \in (\lambda, 1]$, yielding again the familiar constraint $y(n) < \frac{nR}{1+n(R-1)}$ as in equation (14). This implies a particular design on the real value of the aggregate dividends via (39)

$$\frac{D}{P_1} < \frac{nR}{1 + n(R-1)}, \qquad \text{for all } n \in (\lambda, 1] \tag{41}$$

Since the nominal dividend payments are predetermined in $t = 0$, they cannot depend on n. The right hand side of (41) is increasing in n. Therefore, if the central bank wants to follow a fixed price level path, $P_1 = \bar{P}$, then the dividends have to satisfy

$$D < \bar{P} \frac{\lambda R}{1 + \lambda(R-1)} \tag{42}$$

so that patient types have no incentive to shop early. By $\hat{y} := \frac{\lambda R}{1+\lambda(R-1)} \in (0, 1)$, the constant liquidation policy $\hat{y}$ is feasible, run-proof and implements the price level $\bar{P}$. For a spending-flexible liquidation policy $y(n)$ that varies in n, the price level will have to adjust, as in the case of the nominal CBDC demand-deposit contract. Thus, generically, also with nominal equity shares the trade-off between implementing the social optimum in a run-proof way and keeping prices stable exists. Moreover, recall that under a run-deterring liquidation policy, the nominal CBDC demand-deposit contract has flexible prices only off equilibrium.

10 The financial system

Our model abstracts away from a number of features of the financial system, which may generally be important, but appear to be rather tangential for the key issues at hand. In our baseline setting, we only have households and the central bank interacting with each other. This can appear as rather different from from the institutional framework seen in practice and the risk-sharing framework in place[6]. In our framework, we examine a drastically simplified model, dropping the financial intermediary sector entirely.

The purpose of this section is to link the two, and to motivate the stripped-down setup in section 3 and beyond.

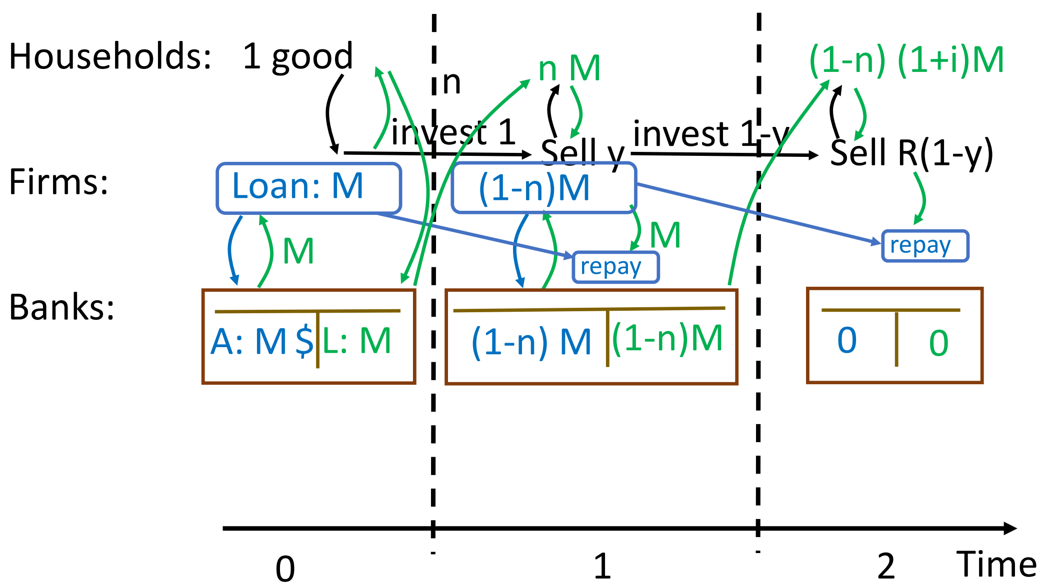

Figure 4: The Financial System 1: households, firms and banks.

Consider figure 4, which shows a three-period model with households, firms and banks. Households start with 1 unit of some good each in period 0. In period 0, banks provide firms with one-period loans totalling M units of money. The firms provide banks with an IOU or loan agreement, adding to the asset side of the bank. Firms use their M units of money

[6]For example, Di Tella and Kurlat (2021, forthcoming) ask and answer, why banks are exposed to monetary policy

to purchase the goods from households, and invest them. Households in turn deposit the money received with banks, creating bank liabilities of M in the form of deposit accounts. Total assets are equal to total liabilities of the bank, indicated by the T-account. In period 1, a fraction n of households withdraws their deposits and use them to buy goods from firms. Firms sell y units of the goods to the households, keeping $1-y$ invested for sale in period 2. The firm uses its receipts of nM units of money for the partial repayment of the one period loan obtained in period 0. The firm thus needs to obtain a new loan, totalling $(1-n)M$, and use that loan as well as the money received in the goods market in order to repay its period-0 loan completey. In period 2, the remaining investment of the firm generates $R(1-y)$ goods, which are sold to the household. We allow the bank to pay a nominal interest rate of i on deposits held until period 2. Households withdraw their entire deposits. With the cash receipts at hand, the firm repays the original loan. The bank finishes the period 2 with a balance sheet of length zero.

A few remarks are in order. We have assumed that there is no interest between period 0 and period 1. They can be introduced, but would clutter notation at this point. We have assumed that loans are one-period loans[7]. The degree to which banks are willing to roll over these loans determines the number of goods the firms have to sell in period 1, in order to be able to repay the original loan in full. [HARALDS COMMENT: This needs a bit more thought ... it is the crucial step later on, though, through which the central bank may force firms into selling more of their goods. Seems the firms always get nM units of money ... so, how are they are forced into liquidating goods? How do we think about that in a usual DD model? May be something about individual incentives vs market clearing.] In period 2 and in order for the balance sheet of the bank to end up with zero on both sides as well as with the firms selling all output against the all remaining money on the deposit accounts of households, prices and interest rates have to appropriately clear the markets. These issues will be sorted out in our model: for the purpose of the motivational description here, we shall simply assume that this is so.

Note that"money" in figure 4 is inside money, created by the banking system. In principle, all the transactions could take place per appropriate bank-to-bank and account-to-account transfers within the banking system. In practive, however, "withdrawal" of deposit accounts is understood to allow the conversion of deposit accounts into cash and then paying with

[7] Alternatively, we could have assumed that loans are long-term, but callable in period 1.

cash in turn. There is no "cash" in figure 4. There is no central bank.

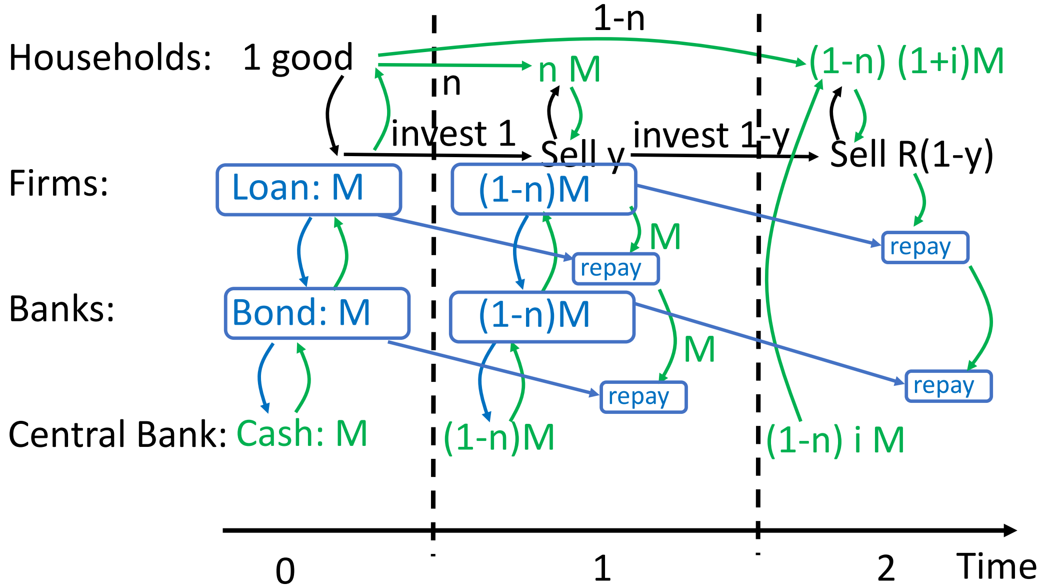

Figure 5: The Financial System 2: introducing a central bank and open market operations.

Figure 5 thus introduces a central bank and a role for cash transactions. Indeed, we now assume that all transactions are for cash only, and that households hold cash across periods rather than using deposit accounts at banks. While it is not hard to enrich matters further and allow for a hybrid deposit-cash-based system, it would seem to unnecessarily complicate matters further. What is important here is that cash is the most liquid means of payment. Households no longer "withdraw" some accounts, turning their withdrawal into cash: it is cash that they have at hand.

In figure 5 therefore, the firms seek to obtain cash, when obtaining a loan from a bank. Banks cannot create cash: cash is outside money. The banks therefore first need to obtain cash from the central bank. They do so by selling one-period bonds to the central banks for cash, as a first step in period 0. This is a familiar and rather standard open market operation: the central bank purchases bonds, using central bank money. Note that the bonds sold by the banks in figure 5 are bonds underwritten by the bank. In practice and outside financial distress episodes, central banks insist on only purchasing government bonds in open market operations. It would not be hard to introduce another layer into the structure in figure 5, where a government issues bonds to originally be held by banks, who in turn may sell them to the central bank, but that would not create a substantive difference for our analysis. In practice, central banks furthermore typically pay for OMT bond purchases using reserves rather than cash, i.e. crediting the central bank accounts of selling banks. The system shown in figure 5 could be enriched by allowing for the distinction between central bank reserves and cash. In practice, however, banks can turn these reserve accounts into cash, as needed: the distinction would not make a substantive difference. We allow the central bank to pay a nominal interest on cash held by households between period 1 and period 2. While this may appear to be rather futuristic, and perhaps hard to do with cash, it will be rather easily feasible when cash is replaced with CBDC: we thus include this feature here.

In sum, figure 5 shows the complete financial system, including banks, firms and a central bank, and it shows a central bank interacting with banks via standard open market operations. It is now the central bank, which determines via its open market operations in period 1 the degree to which banks can extend loans to firms in period 1, in turn influencing real activity per the sales y by firms. This influence of the central bank on the volume of loans extended by private banks and on real activity should be familiar from standard textbooks on money and banking. While matters are more complicated in practice, and involve additional detail and steps, the important point is that the financial system in figure 5 is standard and conventional.

For the purpose of our benchmark analysis, the banks in figure 5 turn out to be an unnecessary layer: one might as well have firms sell their loans directly to the central bank. This is the system envisioned in figure 6. One might even see this system as compatibe with some of the measures undertaken during recent financial crisis or pandemic episodes, where central banks indeed purchased bonds issued by the private sector rather than issued by banks or the governments. The key for us, however, is simply that there is little substantive distinction between the financial structure in figure 5 and figure 6.

Finally, one can strip out the firms too, and simply assume that the central bank undertakes the real investment and sales of goods itself, as shown in figure 7. This picture of the financial system surely looks odd: central banks do not engage in real production nor sell goods directly to households! We agree. The point is simply that the financial structure shown in figure 7 can be understood as the conventional financial structure shown in figure 5, stripping out layers of little relevance to our analysis. However, it is good to keep the system shown in figure 5 in mind in order to understand, how households interact with the

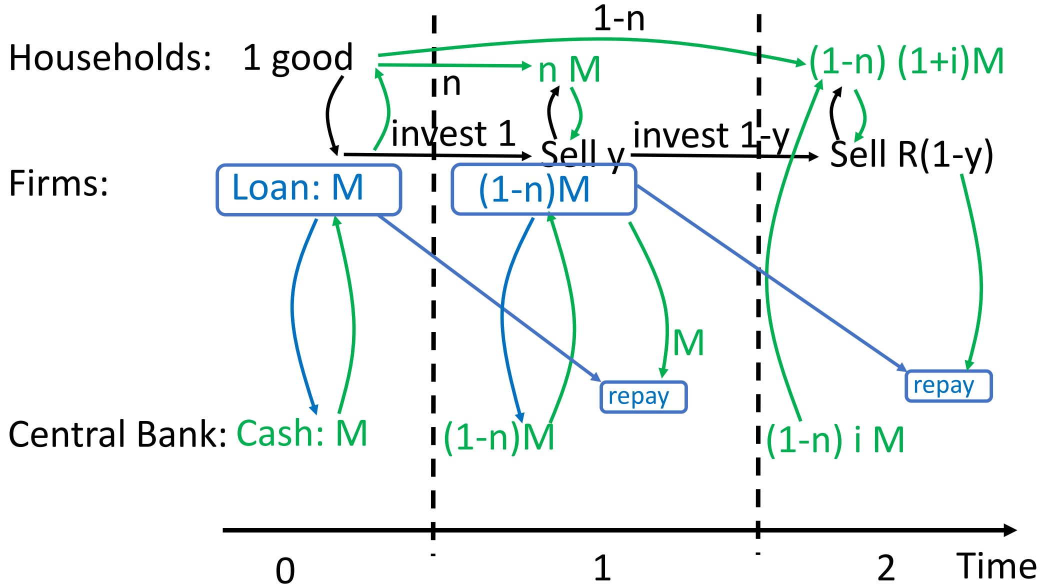

Figure 6: The Financial System 3: stripping out banks.

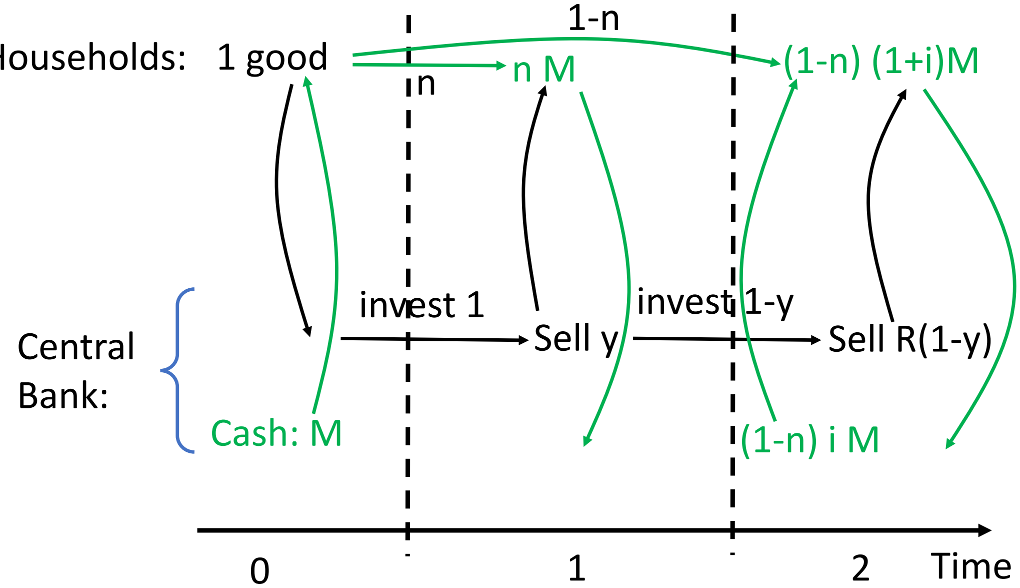

Figure 7: The Financial System 4: only households and a central bank.

central bank. Households are not "entitled" to real goods, using their cash, as may seem the case in figure 7: rather households just spend cash, as was made clear in the description of figure 5. Moreover, households do not "withdraw" balances from some central bank account in figure 7. They receive cash in period 0, and there is nothing more to be withdrawn: cash is the most liquid form of payment. Finally and given that we allow an interest on remaining cash balances to be paid in period 2, we could allow for interest payments on the overall cash balance between period 0 and 1. What matters below in the analysis is the amount of cash in the hands of households in period 1 and not, how much cash they originally received in period 0.

11 Extensions

11.1 Token-based CBDC

With a token-based CBDC, a central bank issues anonymous electronic tokens to agents in period 1, rather than accounts.[8] These electronic tokens are more akin to traditional banknotes than to deposit accounts. Trading with tokens only requires trust in the authenticity of the token rather than knowledge of the identity of the token holder. Thus, token-based transactions can be made without the knowlegde of the central bank.

Technically, and with appropriate software, digital tokens can be designed in such a way that each unit of a token in $t = 1$ turns into a quantity $1 + i$ of tokens in $t = 2$, with i to be determined by the central bank at the beginning of period $t = 2$: even a negative nominal interest rate is possible.[9]

With that, the analysis in the previous sections still holds, since nothing of essence depends on the identity of the spending agents other than total CBDC tokens spent in the goods market. With a token-based CBDC, agents obtain M tokens in period $t = 0$, and decide how much to spend in periods $t = 1$ and $t = 2$. Thus, the same allocations can

[8] This can be done with or without a blockchain. In the second case, a centralized ledger to record transactions can be kept by a third-party that is separate from the central bank. That third party could also potentially pay interest or impose a suspension of spending. For the purpose of this paper, we do not need to worry about the operational details of such a third party or to specify which walls should exist between it and the central bank to guarantee the anonymity of tokens.

[9] Historically, we have examples of banknotes bearing positive interest (for instance, during the U.S. Civil War, the U.S. Treasury issued notes with coupons that could be clipped at regular intervals) and negative interest (demurrage-charged currency, such as the prosperity certificates in Alberta, Canada, during 1936). Thus, an interest-bearing electronic token is only novel in its incarnation, but not in its essence.

be implemented except for those that require the suspension of spending, as discussed in Subsection 7.

For the latter, the degree of implementability depends on technical details outside the scope of this paper. Note that even with a token-based system, the transfer of tokens usually needs to be registered somewhere, e.g., on a blockchain. It is technically feasible to limit the total quantity of tokens that can be transferred on-chain in any given period. A pro-rata arrangement can be imposed by taking all the pending transactions waiting to be encoded in the blockchain, taking the sum of all the spending requests, and accordingly dividing each token into a portion that can be transferred and a portion that cannot. It may be that off-chain solutions arise circumventing some of these measures, but their availability depends on the precise technical protocol of the CBDC token-based system. In the case where the token-based CBDC is operated by a centralized third party, such an implementation is even easier.

11.2 Synthetic CBDC and retail banking

With a synthetic CBDC, agents do not hold the central bank's digital money directly. Rather, agents hold accounts at their own retail bank, which in turn holds a CBDC not much different from current central bank reserves. This may be due to tight regulation by the monetary authority. The retail banks undertake the real investments envisioned for the central bank in our analysis above. A synthetic CBDC, therefore, corresponds to the model sketched in Section 8.2 with $\alpha = 0$.

The key difference from the current cash-and-deposit-banking system is that cash does not exist as a separate central bank currency or means of payment. That is, in a synthetic CBDC system, agents can transfer amounts from one account to another, but these transactions are always observable to the banking system and, thereby, the central bank. Likewise, agents (and banks) cannot circumvent negative nominal interest, while they could do so in a classic cash-and-deposit banking system by withdrawing cash and storing it.

For the purpose of our analysis, observability is key. Our analysis is relevant in the case of a systemic bank run, i.e., if the economy-wide fraction of spending agents exceeds the equilibrium outcome. Much then depends on the interplay between the central bank and the system of private banks. For example, if the liquidation of long-term real projects is up to the retail banks, and these retail banks decide to make the same quantity of real goods available in each period, regardless of the nominal spending requests by their depositors, then the aggregate price level will have to adjust. The central bank may seek to prevent this either by imposing a suspension of spending at retail banks or by forcing banks into higher liquidation of real projects: both would require considerable authority for the central bank. Proposition 14, for instance, says that with $\alpha = 0$, the central bank alone cannot implement a run-deterring policy when offering a synthetic CBDC. Run deterrence then requires retail banks to control liquidation in a particular way.

11.3 Cash

The key difference to a fully cash-based system is that spending decisions can only be observed in the goods market, rather than by also tracing accounts or transactions on the blockchain. In principle, the payment of nominal interest rates on cash is feasible, but is demanding in practice. Excluding nominal interest rates on cash, due to these practical considerations, implies the cash-and-deposit banking system discussed in Section 11.2 and the restrictions discussed there. The tools available to a central bank are now considerably more limited. These limitations may be a good thing, as they may impose a commitment technology and may thus lead to the prevention of an equilibrium systemic bank run in the first place, but the restricted tool set may be viewed as a burden *ex-post*, should such a bank run occur.

12 Conclusion

Diamond and Dybvig (1983) have taught us that the implementation of the social optimum via the financial intermediation of banks comes at the cost of making these banks prone to runs. We have argued that this dilemma becomes a trilemma when the central bank acts as the intermediary offering a CBDC because central banks are additionally concerned about price stability. As summarized in Figure 1, a central bank that wishes to simultaneously achieve a socially efficient solution, price stability, and financial stability (i.e., absence of runs) will see its desires frustrated. We have shown that a central bank can only realize two of these three goals at a time.

In its role as the intermediary, the central bank collects and invests the real goods endowments of the agents in a real production technology, offering a nominal CBDC contract in return. At an interim period, the agents learn whether they enjoy late (patient agent) or early (impatient agent) consumption and, then, make their nominal spending decisions. A central bank run occurs if patient agents also decide to spend their CBDC balances early. Patient agents do spend early when the real value from early spending exceeds the real value from late spending. But real values depend on the central bank's liquidation policy of real investment. The central bank observes aggregate nominal spending and then decides how much of its real investment to liquidate in order to supply goods to the agents spending their balances. The price level for real goods then adjusts such that nominal CBDC spending clears the real goods market. In contrast, a private intermediary would need to take the price level as given such that the price level jointly with aggregate nominal spending pins down the necessary liquidation of its real investment.

As our main result, we have demonstrated that the central bank can always implement optimal risk sharing in dominant strategies and deter central bank runs at the price of threatening inflation off-equilibrium. If price-stability objectives for the central bank imply that the central bank would not follow through with that threat, then allocations either have to be suboptimal or prone to runs.

We hope to extend our analysis in several important directions. For instance, we can have a richer "real" side of the economy, including analyzing how a CBDC can affect heterogeneous agents. We can also study how a CBDC will affect a wider range of financial assets beyond demand deposits. These are vital considerations to judge the desirability of moving toward a CBDC world.

13 Appendix A: Proofs

Proof. [Proposition 8] Proof (i): Via the market clearing condition (7), setting $P_1(n) \equiv \bar{P}$ for all n requires $y(n) = \frac{M}{\bar{P}}n$, for all $n \in [0,1]$. Thus, via (11), $x_1(n) = y(n)/n = \frac{M}{\bar{P}}$ is constant for all n. Last, since the central bank cannot liquidate more than the entire investment in the real technology, $y(n) \in [0,1]$ for all n, together with x_1 constant requires, in particular, $\frac{M}{\bar{P}} = x_1 = x_1(1) = y(1) \leq 1$. Thus, $M \leq \bar{P}$. Proof (ii): When additionally requiring price stability, $P_1(n) = P_2(n) \equiv \bar{P}$, the market clearing condition (8) together with (18) yields (20). □

Proof. [Corollary 9] Proof (i): We know that price stability demands $x_1 \leq 1$ but the social optimum satisfies $x_1^* > 1$. Proof (ii): $\overline{x}_1 \leq 1$ implies $x_2(n) = \frac{1-y(n)}{1-n}R = \frac{1-n\overline{x}_1}{1-n}R \geq R > 1 \geq \overline{x}$. Since the real value of the allocation at $t = 2$ always exceeds the real value of the time one allocation at $t = 1$, patient agents never spend at $t = 1$; thus, there are no runs. Proof (iii): By equation (19), $\frac{\overline{P}}{M} \geq 1$, implies $i(n) = \frac{\frac{\overline{P}}{M}-n}{1-n}R - 1 \geq R - 1 > 0$ for all $n \in [\lambda, 1]$ by $R > 1$. Further, $\frac{\overline{P}}{M} \geq 1$ implies that $i(n)$ increases in n. □

Proof. [Proposition 12] Proof (i): Equation (21) follows immediately from (7) and the constraint $y(n) \leq 1$. Proof (ii): In $n = n_c$, we have $\frac{M}{\bar{P}}n = 1$. Therefore, $n_c > 0$. By assumption $\bar{P} < M$, thus $n_c < 1$, with $n_c \in (0,1)$. Equation (21) implies that $x_1(n) = y(n)/n$ is constant at the level $\overline{x} = M/\overline{P}$, as long as $y(n) < 1$: this is the case for $n < n_c$. For $n \geq n_c$, $y(n) \equiv 1$. All goods are liquidated, so $x_1(n) = 1/n$. Equation $P_1(n) = Mn$ follows from equation (7). Proof (iii): Equation (22) follows from (8) combined with (21). Proof (iv): This is straightforward, when plugging in (21) into $P_2(n)$ and observing that n_0 is positive only for $R > M/\overline{P}$. □

14 Appendix B: Private bank accounting

Consider the collective of private banks with market share $(1-\alpha) \in (0,1)$. For the sake of brevity, we refer to the collective as "the private bank." A fraction n_P of the private bank's customers spend in $t = 1$, while a fraction n_{CB} of the central bank's customers do so, for a total fraction n of all agents $n = (1-\alpha)n_P + \alpha n_{CB}$. Agents are promised M units of the CBDC, when spending in $t = 1$, or $M(1+i)$ units, when spending in $t = 2$. The central bank liquidates y_{CB} goods in period $t = 1$, while the private bank liquidates y_P, for total liquidation $y = (1-\alpha)y_P + \alpha y_{CB}$. For accounting, we introduce some notation. The private bank borrows CBDC L_1 from the central bank to meet withdrawals at the beginning of each period, repaying the loan at the end of the period with the sales proceeds S_1 from selling real goods. No interest is charged for the within-period loan.

The difference D_1 at the end of period $t = 1$ is kept on account at the central bank, earning or paying the nominal interest rate z, to be settled at the end of period $t = 2$. Further, the bank has to pay a tax $\tau(1-\alpha)$ denoted in CBDC at the end of period 2 (or receive this as a subsidy, if $\tau < 0$). The interest rate z and the tax τ are chosen by the central bank (CB in the accounting below), and may depend on n_P and choices y_P of the private bank. We seek to calculate x and τ so that the private bank makes zero profits, i.e., is left with zero CBDC balances D_2 at the end of period 2, after having liquidated and sold all its remaining goods at the end of period 2. Accounting requires

Accounting in period $t = 1$:

$$\begin{aligned}
\text{Loan from CB:} \quad L_1 &= (1-\alpha)n_P M \\
\text{Sales proceeds:} \quad S_1 &= (1-\alpha)P_1 y_P \\
\text{Difference:} \quad D_1 &= S_1 - L_1 = (1-\alpha)(P_1 y_P - n_P M)
\end{aligned}$$

Accounting in period $t = 2$:

$$\begin{aligned}
\text{Loan from CB:} \quad L_2 &= (1-\alpha)(1-n_P)(1+i)M \\
\text{Sales proceeds:} \quad S_2 &= (1-\alpha)P_2 R(1-y_P) \\
\text{CB account:} \quad A_2 &= (1+z)D_1 - \tau(1-\alpha) \\
\text{Difference:} \quad D_2 &= A_2 + S_2 - L_2 \\
&= (1-\alpha)\Big(P_2 R + ((1+z)P_1 - P_2 R)y_P - (1+i)M - (z-i)n_P M - \tau\Big)
\end{aligned}$$

Market clearing:

$$\begin{aligned}
\text{In } t = 1: \quad P_1 y &= nM \\
\text{In } t = 2: \quad P_2 R(1-y) &= (1-n)(1+i)M
\end{aligned}$$

Sum $(1+i)$ times the market clearing equation for P_1 with the equation for P_2 to obtain $P_2 R + ((1+i)P_1 - P_2 R)y = (1+i)M$. Use the latter equation to replace $(1+i)M$ in the last expression for D_2 to find

$$\frac{D_2}{P_1(1-\alpha)} = (i-s)(y_P - y) + (z-i)(y_P - n_P x_1) - \frac{\tau}{P_1} \tag{43}$$

where, as usual, $x_1 = \frac{M}{P_1}$ is the amount of real goods acquired by agents in period $t = 1$ and where we introduce:

$$s = \frac{P_2}{P_1} R - 1 \tag{44}$$

to denote the "shadow" nominal interest rate for private banks, equating liquidating a unit of the good in $t = 1$, selling at P_1 and investing at the shadow nominal return $1 + s$ to keeping the unit of good and thus selling R units at price P_2. Notice that $y = n x_1$ and the market clearing equations imply

$$1 + s = (1+i)\frac{1-n}{1-x_1 n} x_1 \tag{45}$$

and, thus, $s > i$, whenever $x_1 > 1$. In particular, this is the case at the efficient outcome. We note that $s = i$, if and only if $x_1 = 1$, which is the maximal full price-stable solution as well as the market allocation, when agents engage in self-storage.

Suppose now that the private bank sells exactly as many goods as purchased by their withdrawing customers, i.e., $y_P = n_P x_1$. Absent τ, equation (43) reveals that the private bank will make a loss or profit, if $x_1 \neq 1$ and if $y_P \neq y$, i.e., $n_P \neq n$. For example, if the share of private-bank customers who go shopping in $t = 1$ is larger than the average share of customers who shops economy-wide, $n_P > n$, and if the allocation achieves $x_1 > 1$ and thus $s > i$, then the private bank incurs a loss $D_2 < 0$, absent τ, as the opportunity costs for servicing agents in $t = 1$ are high. We shall use these observations to fix the tax τ to compensate for these losses or profits, and assume that

$$\tau = P_1(i-s)(n_P - n)x_1 \tag{46}$$

from here onward. This τ depends on the specifics of the bank only via the "circumstances" n_P and does not depend on the choice y_P. To take care of the case where $y_P \neq n_P x_1$, we use the central bank-account interest rate z. Solving for z per setting $D_2 = 0$ in (43) and imposing (46) yields the following result, which we formulate as a proposition.

Proposition 15. *Suppose τ satisfies (46). Then*

$$\{D_2 = 0\} \Leftrightarrow \Big(\{y_P = n_P x_1\} \ or \ \{z = s\}\Big). \tag{47}$$

In sum, taxing the "circumstance" profits per (46) and paying an internal interest rate z on central bank balances equal to the shadow nominal interest rate s achieves the objective that private banks make zero profits, regardless of their circumstances n_P and regardless of their liquidation choice y_P.

Lemma 16. *If the private bank sets $y_P \equiv y_{CB}$, then the interest rate for which the private bank's balances with the central bank are zero equals $z = i$ and $\tau = 0$.*

That is, if the private bank liquidates the same share of assets as does the central bank, then the interest rate on CBDC balances $z = i$ sets bank profits to zero.

Proof. [Lemma 16] With $\tau = 0$, the CBDC balance at the end of $t = 2$ equals

$$\begin{aligned} D_2 &= (1-\alpha)\left(P_2 R(1-y_p) - (1-n_p)(1+i)M + (1+z)(P_1 y_p - n_p M)\right) \\ &= (1-\alpha)M * \begin{pmatrix} (1+i)\left(\frac{(1-y_p)(1-n)}{1-y} - (1-n_p)\right) \\ +(1+z)\left(\frac{n y_p}{y} - n_p\right) \end{pmatrix} \end{aligned} \tag{48}$$

where, at the last equality, we have plugged in P_1 and P_2. Then,

$$\frac{(1-y_p)(1-n)}{1-y} - (1-n_p) = -\left(\frac{n y_p}{y} - n_p\right) \tag{49}$$

if and only if

$$\frac{y(1-y_p) - n(y-y_p)}{y(1-y)} = 1 \tag{50}$$

For $\alpha \in (0,1)$, $y_P \equiv y_{CB}$ implies $y_p = y$. If $y = y_p$, then equations (50) and (49) are true. Thus, for $y = y_p$ the choice $z = i$ puts $D_2 = 0$. □

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