# Reserve Accumulation, Growth and Financial Crisis*

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**Abstract**

This paper presents a simple model that reproduces two facts characterizing the international monetary system: i) Developing countries that grow faster accumulate more international reserves and ii) Fast growing developing countries are associated with lower net capital inflows. In our framework the government uses foreign exchange reserves to internalize the growth externalities present in the tradable sector and to provide liquidity to the corporate sector during periods of financial stress. This creates a positive link between reserve accumulation, current account surpluses and growth. Importantly, official reserves and private debt are imperfect substitutes, so that the reserve policy of the government cannot be perfectly offset through borrowing by private agents. We use the model to compare the laissez-faire equilibrium and the optimal reserve policy in an economy that is opening to international capital flows. We find that the optimal reserve management entails a fast rate of reserve accumulation, as well as higher growth and larger current account surpluses compared to the economy with no policy intervention. We also find that the welfare gains of reserve policy are large, in the order of 1 percent of permanent consumption equivalent.

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# 1 Introduction

One of the most spectacular recent trends in the international monetary system is the considerable built up in foreign exchange reserves by emerging countries, in particular East Asian economies and China. As shown by figure 1a, the average reserves-to-GDP ratio in developing countries more than doubled between 1980 and 2010, increasing from 9.5 to 23.3 percent. The increase has been particularly marked in East Asia, where the average reserves-to-GDP ratio passed from 15.5 percent in 1980 to 55.3 percent in 2010.

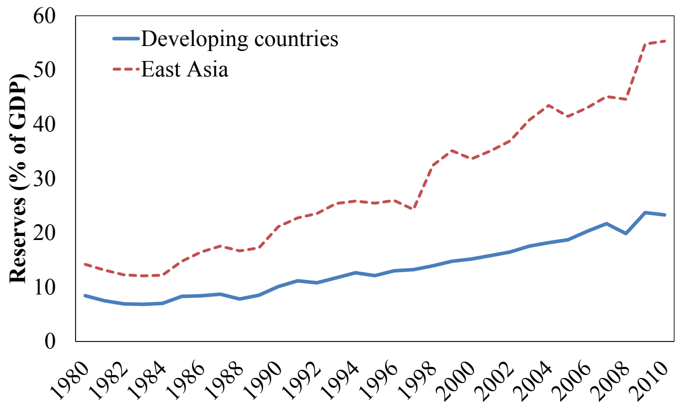
The large accumulation of foreign reserves is not just interesting in itself, but it also represents a key element for understanding the direction and allocation of international capital flows among developing economies. As noticed by Gourinchas and Jeanne (2011), while the neoclassical growth model would suggest that capital should be directed towards those economies that experience faster productivity growth, in the data we observe that faster growing economies are associated with lower net capital inflows (figure 1b). International reserves play a fundamental role in explaining this puzzling pattern of the data. In fact, Gourinchas and Jeanne (2011) show that fast growing countries are net exporters of capital because of their policy of international reserve accumulation (figure 1c).

In this work we propose an analytical framework that is able to replicate the stylized pattern of international capital flows observed in the data:

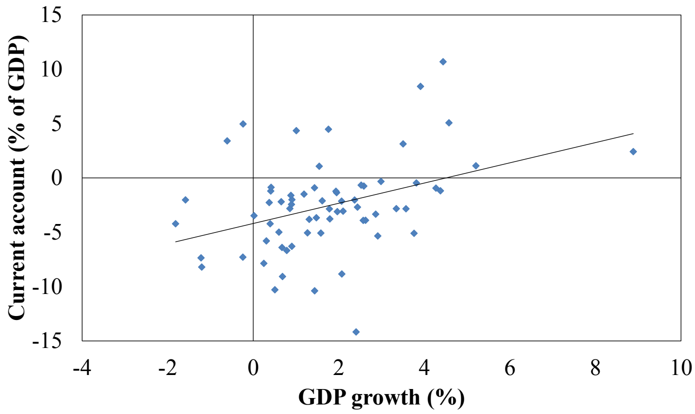
Fact 1) Countries that grow faster accumulate more international reserves.
Fact 2) Fast growing countries are associated with lower net capital inflows.

There are two aspects of our analysis that are crucial. First, the rationale behind reserve accumulation. Second, the interaction between public and private capital flows. In fact, a novelty of our framework is to propose a theory of public intervention in capital flows in which international reserves are distinct from private flows.

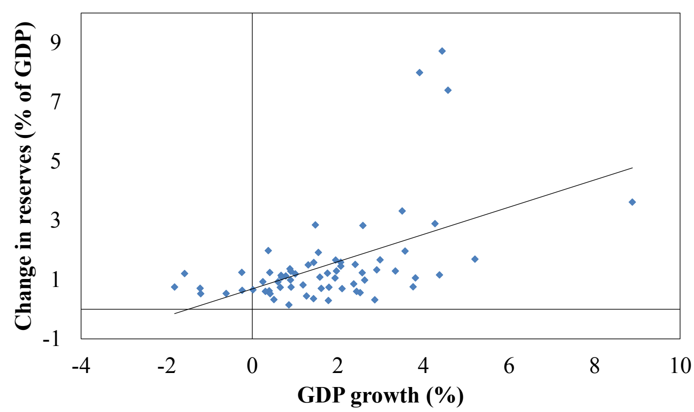
The model is a two-sector small open economy that produces tradable and non-tradable goods. There are two distinctive features of our framework. First, we assume that the tradable sector exhibits a learning by importing externality so that the level of technology in the economy depends on the aggregate amount of intermediate inputs imported. This creates scope for policy intervention, since atomistic firms do not internalize the impact of their decisions on aggregate productivity and use an inefficiently low amount of imported inputs compared to the social optimum. Second, we assume that firms in the tradable sector are subject to financial frictions, in the form of an occasionally binding borrowing constraint. This allows us to capture the role of foreign exchange reserves in providing liquidity to the corporate sector during periods of financial stress.




(a) Reserves in percent of GDP




(b) Average per capita GDP growth and average current account balances between 1980 and 2010




(c) Average per capita GDP growth and average reserve accumulation between 1980 and 2010

Figure 1: Motivating facts. Notes: the sample is composed of 66 developing countries. East Asia refers to the unweighted average of China, Hong Kong, Indonesia, South Korea, Malaysia, Philippines, Singapore and Thailand. Data are from the World Bank Development Indicators.

We show that the presence of growth externalities and financial frictions in the tradable sector can explain the pattern of capital flows observed in the data. During tranquil times, that is when firms are not financially constrained, the government can increase the growth rate of the economy by accumulating international reserves. In fact, to accumulate reserves the government withdraws resources from private agents and induces a depreciation of the real exchange rate (the relative price of non-tradable goods). This generates a shift of labor out of the non-tradable sector, an increase in the production of tradable goods and a rise in the use of imported inputs and in the growth rate of the economy. As a result, faster growing economies experience higher trade surpluses and an undervalued real exchange rate compared to the laissez-faire equilibrium. Moreover, by using reserves to provide liquidity to the corporate sector during financial crises, the government can mitigate the impact of financial frictions on production and further increase the growth rate of the economy compared to the laissez-faire equilibrium. These interventions are welfare improving because under laissez-faire the growth rate of the economy is inefficiently low.

While in the neoclassical growth model the accumulation of international reserves would be offset by capital inflows by the private sector, in our economy the imperfect substitutability among these two assets limits this effect. The imperfect substitutability arises for two main reasons. First, domestic agents are subject to an international borrowing limit so that the possibility that the limit might be binding drives a wedge between the return on international reserves and private bonds. Secondly, international reserves can be used to provide international liquidity when a sudden stop in international capital flows hits the economy, while private assets do not. Hence, in our framework while the economy as a whole runs a current account surplus and accumulates foreign reserves, the private sector accumulates foreign liabilities.

We examine the impact on welfare of reserve accumulation. We first show that a social planner that is unconstrained in terms of policy tools would choose not to accumulate reserves. This implies that foreign exchange reserves are a second-best policy tool. We then compute within a class of simple rules the optimal reserve policy and we find that, despite being a second-best policy tools, the welfare gains from optimal reserve management can be quite large. As an example, we find that the gains from public intervention in capital flows for a country that is opening itself to international capital markets are in the order of 1 percent of permanent consumption equivalents.

This paper is related to several strands of the literature. The goal of our paper is to explain the puzzling negative correlation between productivity growth and capital inflows in developing countries observed by Prasad et al. (2007) and by Gourinchas and Jeanne (2011). Gourinchas and Jeanne (2011) find that the allocation of capital among developing economies is driven by the pattern of reserve accumulation and this motivates our focus on foreign exchange reserves. The central role of government intervention in shaping capital flows to developing countries relates our paper to the so-called "Bretton Woods 2" perspective on the international monetary system of Dooley et al. (2003), according to which the large accumulation of international reserves by the public sector in emerging economies is part of an export-led growth strategy. Our paper is also related to Rodrik (2008), who provides empirical evidence in favor of a causal link from real exchange rate undervaluation to growth.

The engine of growth in our model is the accumulation of knowledge trough the imports of foreign intermediate inputs. This is motivated by the empirical findings of Coe et al. (1997), who document the role of imports of capital goods in transferring technological discoveries made in developed countries to developing economies.[1] Our assumption that knowledge is a non-rival and non-excludable good follows a long standing tradition in the literature on economic growth, dating back to Arrow (1962) and Romer (1986).[2]

From a theoretical perspective, our paper is connected to the growing literature providing formal models that reproduce the negative correlation between growth and capital inflows characterizing developing countries. Examples include Aghion et al. (2006), Angeletos and Panousi (2011), Buera and Shin (2009), Broner and Ventura (2010) and Sandri (2010). These papers all focus on private capital flows, while in our model the negative correlation between growth and capital inflows is driven by reserve accumulation by the public sector, consistent with the empirical findings of Gourinchas and Jeanne (2011). Aguiar and Amador (2011) provide a model in which public flows may generate a negative correlation between growth and capital inflows, but the mechanism that they emphasize is different from ours. In fact, in their model the government decreases its stock of foreign debt in order to credibly restrain from expropriating the return from private investment, thus stimulating investment and growth. In contrast, in our framework reserve accumulation by the public sector shifts productive resources toward the tradable sector in order to exploit the knowledge spillovers coming from the imports of foreign capital goods.

Our paper is also related to that fast growing literature that examines the determinants of reserve accumulation in emerging markets. Aizenman and Lee (2007) and Korinek and Servén (2010) emphasize the link between reserve accumulation and growth externalities, while Durdu et al. (2009), Jeanne and Rancière (2011) and Bacchetta et al. (2011) focus on the precautionary motive of holding interna-

[1] See Keller (2004) for a survey of the literature on technology transmission through trade.

[2] Klenow and Rodriguez-Clare (2005) is an excellent survey of the literature on the role of externalities in economic growth.

tional reserves. Our framework encompasses both approaches and differs critically from the existing literature in the modeling of public versus private capital flows.

The rest of the paper is structured as follows. Section 2 introduces the framework. Section 3 presents the social planning allocation and discusses the political barriers that may prevent a government from implementing the first best through trade policies. Section 4 provides intuition about the effect of reserve management. Section 5 presents the results of our policy experiment on financial liberalization and provides estimates of the welfare gains from implementing the optimal reserve policy. Section 6 concludes.

# 2 Model

We consider an infinite-horizon small open economy. Time is discrete and indexed by $t$. The economy is populated by a continuum of mass 1 of households and by a large number of firms. The firms are owned by the households and produce tradable and non-tradable consumption goods. Firms producing the tradable good engage in financial transactions with foreign investors. There is also a government that manages foreign exchange reserves.

## 2.1 Households

The representative household derives utility from consumption and supplies inelastically one unit of labor each period. The household's lifetime expected utility is given by

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma} \right]. \tag{1}$$

In this expression, $E_t[\cdot]$ is the expectation operator conditional on information available at time $t$ , $\beta < 1$ is the subjective discount factor, $\gamma > 0$ is the coefficient of relative risk aversion and $C_t$ denotes a consumption composite good. $C_t$ is defined as a Cobb-Douglas aggregator of tradable $C_t^T$ and non-tradable $C_t^N$ consumption goods

$$C_t = \left(C_t^T\right)^{\omega} \left(C_t^N\right)^{1-\omega}, \tag{2}$$

where $1 > \omega > 0$ denotes the share of expenditure in consumption that the household allocates to the tradable good.

Each period the household faces the following flow budget constraint

$$C_t^T + P_t^N C_t^N = W_t + \Pi_t^T + \Pi_t^N. \tag{3}$$

The budget constraint is expressed in units of the tradable good. The left-hand side represents the household's expenditure. We define $P_t^N$ as the relative price of the non-tradable good in terms of the tradable good, so $C_t^T + P_t^N C_t^N$ is the household's consumption expenditure expressed in units of the tradable good. The right-hand side represents the income of the household. $W_t$ denotes the household's labor income. $\Pi_t^T$ and $\Pi_t^N$ are the dividends that the household receives from firms operating respectively in the tradable and in the non-tradable sector. For simplicity, we have assumed that domestic households do not trade directly with foreign investors. As we will see below, households can access international financial markets indirectly through their ownership of firms.

Each period the representative household chooses $C_t^T$ and $C_t^N$ to maximize expected utility (1) subject to the budget constraint (3). The first order conditions are

$$\frac{\omega C_t^{1-\gamma}}{C_t^T} = \lambda_t \tag{4}$$

$$\frac{(1-\omega) C_t^{1-\gamma}}{C_t^N} = \lambda_t P_t^N, \tag{5}$$

where $\lambda_t$ denotes the Lagrange multiplier on the budget constraint, or the household's marginal utility of wealth. By combining (4) and (5), we obtain the standard intratemporal equilibrium condition that links the relative price of non-tradable goods to the marginal rate of substitution between tradable and non-tradable goods

$$P_t^N = \frac{1-\omega}{\omega} \frac{C_t^T}{C_t^N}. \tag{6}$$

According to this expression, $P_t^N$ is increasing in $C_t^T$ and decreasing in $C_t^N$. In what follows we will use $P_t^N$ as a proxy for the real exchange rate.

## 2.2 Firms in the tradable sector

The tradable sector is meant to capture a modern sector characterized by dynamic productivity gains and open to financial transactions with foreign investors. Firms in the tradable sector produce using labor $L_t^T$, an imported intermediate input $M_t$ and the stock of accumulated knowledge $X_t$, according to the production function

$$Y_t^T = \left(X_t L_t^T\right)^{\alpha_T} M_t^{1-\alpha_T}, \tag{7}$$

where $Y_t^T$ is the amount of tradable goods produced in period $t$ and $0 < \alpha_T < 1$ is the labor share in gross output in the tradable sector. Knowledge is non-rival and can be freely used by firms producing tradable goods.

Firms in the tradable sector have access to international credit markets and can trade in a non-contingent one period bond denominated in units of tradable goods that pays a fixed interest rate $R$. At the end of the period the representative firm distributes to the households the dividends

$$\Pi_t^T = Y_t^T - W_t L_t^T - P^M M_t - B_{t+1} + RB_t - T_t. \tag{8}$$

where $B_t$ denotes the firm's holding of foreign bonds at the start of period $t$, $W_t$ is the wage paid to workers in the tradable sector, $P^M$ is the price of the imported input and $T_t$ are lump-sum taxes paid to the government.[3]

We assume that the tradable sector is subject to a working capital constraint. A fraction $\phi$ of the intermediate inputs has to be paid at the beginning of the period and requires working capital financing. To finance their working capital, firms have access to intraperiod loan contracts. Under these contracts, the funds borrowed by firms at the start of the period have to be repaid at the end of the same period. We assume that the interest rate charged on intraperiod loans is equal to zero. The domestic government provides an amount $D_t$ of working capital loans. The remaining part $\phi P^M M_t - D_t$ has to be covered using intraperiod loans from foreign investors.

In addition, we introduce financial frictions by assuming that at the end of the period each firm can choose to default on its debts toward international investors. In case of default international investors are able to collect an amount of tradable goods equal to $\kappa_t X_t$. To prevent defaults, international investors impose on domestic firms the borrowing constraint

$$\phi P^M M_t - D_t - RB_t \leq \kappa_t X_t, \tag{9}$$

where $\kappa_t$ measures the tightness of the borrowing constraint. On the left-hand side, we have net liabilities of the firms at the beginning of period $t$. Notice that both the intertemporal loans and the loans used to finance the working capital expenses enter the constraint. We introduce credit shocks in the model by assuming that the parameter $\kappa_t$ is stochastic. In what follows we refer to a financial crisis as a period in which the borrowing constraint (9) holds with equality.

Each period the representative firm chooses $L_t^T$, $M_t$ and $B_{t+1}$ to maximize its expected stream of dividends discounted by the households' marginal utility of wealth

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t \lambda_t \Pi_t^T \right], \tag{10}$$

subject to the borrowing constraint (9). The optimality conditions are given by

$$\alpha_T Y_t^T = W_t L_t^T \tag{11}$$

[3]The assumption that lump-sum taxes are paid by firms in the tradable sector, rather than by households, is made to simplify the exposition and it does not affect our results.

$$(1-\alpha_T)Y_t^T = P^M M_t \left(1+\phi\frac{\mu_t}{\lambda_t}\right) \tag{12}$$

$$\lambda_t = \beta R E_t\left[\lambda_{t+1}+\mu_{t+1}\right] \tag{13}$$

$$\mu_t\left(\phi P^M M_t - D_t - RB_t - \kappa_t X_t\right) = 0, \quad \mu_t \geq 0, \tag{14}$$

where $\mu_t$ denotes the multiplier on the borrowing constraint. Equation (11) represents the optimal demand for labor, which implies equality between the marginal product of labor and the wage. The optimal demand for imported inputs is given by equation (12). When the borrowing constraint is not binding ($\mu_t = 0$), the marginal product of the imported input is equated to its price. When the borrowing constraint is binding ($\mu_t > 0$), firms are unable to purchase the optimal amount of imported inputs. This shows up in the equation as an increase in the marginal cost of purchasing one unit of the imported input. Equation (13) is the modified Euler equation for the case in which international borrowing might be constrained. The expectation of a future binding borrowing constraint has an effect similar to an increase in the cost of intertemporal debt that induces agents to decrease their borrowing. Finally, equation (14) is the complementary slackness condition for the borrowing constraint.

## 2.3 Knowledge accumulation

The stock of knowledge available to firms in the tradable sector evolves according to

$$X_{t+1} = \psi X_t + M_t^{\xi} X_t^{1-\xi}, \tag{15}$$

where $\psi \geq 0$ and $0 \leq \xi \leq 1$. This formulation captures the idea that imports of foreign capital goods represent an important transmission channel through which discoveries made in developed economies spill over to developing countries.[4] As mentioned above, we assume that knowledge is a non-rival and non-excludable good. This, combined with the assumption of a large number of firms in the tradable sector, implies that firms do not internalize the impact of their actions on the evolution of the economy's stock of knowledge. This is a typical growth externality.

## 2.4 Firms in the non-tradable sector

The non-tradable sector captures a traditional sector with stagnant productivity, closed to financial transactions with foreign investors. The non-tradable good is produced using labor, according to the production function $Y_t^N = \left(L_t^N\right)^{\alpha_N}$. $Y_t^N$

[4]Coe et al. (1997) provide empirical evidence on the role of imports of capital goods in transferring technological advances from developed to developing countries.

is the output of the non-tradable good, $L_t^N$ is the amount of labor employed and $0 < \alpha_N < 1$ is the labor share in gross output in the non-tradable sector.[5]

The dividends distributed by firms in the non-tradable sector can be written as

$$\Pi_t^N = P_t^N Y_t^N - W_t L_t^N. \quad (16)$$

In this expression we have used the fact that in equilibrium firms in both sectors produce and that this requires equalization between the wages offered in the two sectors. Profit maximization implies

$$\alpha_N P_t^N L_t^{N\,\alpha_N - 1} = W_t. \quad (17)$$

This equation represents the optimal demand for labor from firms in the non-tradable sector. As before, the marginal product of labor is equated to the wage rate.

## 2.5 Credit shocks

The only source of uncertainty in the model concerns $\kappa_t$, the parameter that governs the sum that foreign lenders can recover in case of default. Our aim is to model an economy in which tranquil times alternate with crises. The simplest way to capture this is to assume that $\kappa_t$ can take two values, $\kappa_H$ and $\kappa_L$ with $\kappa_H > \kappa_L$. We will choose values for $\kappa_H$ such that when $\kappa_t = \kappa_H$ the borrowing constraint (9) does not bind, while the value for $\kappa_L$ will be such that when $\kappa_t = \kappa_L$ the borrowing constraint may bind, depending on $B_t$ and on the actions of the government. As mentioned above, we refer to a period in which the borrowing constraint binds as a financial crisis. Moreover, denoting by $\rho_i$ for $i = H, L$ the probability of $\kappa_t = \kappa_i$ knowing that $\kappa_{t-1} = \kappa_i$, we will set $\rho_H > 0.5$ so that crises are rare events and $\rho_L > 1 - \rho_H$ so that crisis events have some persistence.

## 2.6 Government

The government collects taxes from firms in the tradable sector $T_t$, provides working capital loans $D_t$ to firms and trades in foreign exchange reserves $FX_t$. In the spirit of Gertler and Karadi (2011), we assume that lending from the government entails some efficiency losses. Specifically, we assume that in order to lend to firms a sum equal to $D_t$, the government has to employ an amount of tradable goods equal to $D_t/(1-\theta)$, with $0 \leq \theta \leq 1$. Of this amount, $D_t$ is repaid by firms to the

[5] To ensure constant returns to scale in the production of non-tradable goods, we can assume that production is carried out using labor and land according to a constant-returns-to-scale Cobb-Douglas aggregator. The production function in the main text obtains if the supply of land is fixed and normalized to one.

government at the end of the period, while $D_t\theta/(1-\theta)$ is lost during the intervention. Hence, the higher $\theta$ is, the less efficient is the government in providing liquidity to firms.

We can then write the government budget constraint expressed in units of tradable goods as

$$FX_{t+1} = R^{FX} FX_t + T_t - D_t \frac{\theta}{1-\theta}, \tag{18}$$

where $R^{FX}$ is the gross interest rate paid on reserves. To capture some salient features of foreign exchange reserves, we assume that the interest rate paid on reserves is not greater than the interest rate charged on private loans ($R^{FX} \leq R$) and that the government cannot hold negative amounts of foreign reserves

$$FX_t \geq 0. \tag{19}$$

Moreover, the resources employed to provide working capital loans to firms at the start of the period cannot exceed the start of period holdings of foreign reserves

$$\frac{D_t}{1-\theta} \leq R^{FX} FX_t. \tag{20}$$

To simplify the analysis, we restrict our attention to simple forms of intervention. In particular, we assume that to finance reserve accumulation the government levies a tax equal to a fraction $\chi$ of the output of tradable goods during tranquil times, while following a bad credit shock the government sets the tax to zero. More formally, we assume that

$$T_t = \begin{cases} \chi Y_t^T & \text{if } \kappa_t = \kappa_H \\ 0 & \text{if } \kappa_t = \kappa_L \end{cases} \tag{21}$$

where $0 \leq \chi \leq 1$. In addition, we assume that during crises the government provides loans to firms until their borrowing constraint stops binding or until the size of the intervention exceeds a fraction $\chi^{WK}$ of the start-of-period stock of reserves. Formally, we assume that

$$D_t = \max\left(\chi^{WK}(1-\theta)R^{FX}FX_t, \phi P^M M_t^{unc} - RB_t - X_t\kappa_t, 0\right), \tag{22}$$

where $0 \leq \chi^{WK} \leq 1$ and $M_t^{unc}$ is the amount of intermediate inputs that firms would choose in absence of financial frictions, that is if $\phi = 0$.

## 2.7 Market clearing and competitive equilibrium

Market clearing for the non-tradable goods requires that the amount consumed is equal to the amount produced

$$C_t^N = \left(L_t^N\right)^{\alpha_N}. \tag{23}$$

Combining (23), with the households' budget constraint (3), the definitions of firms' profits in the tradable and non-tradable sectors (8) and (16), and the government budget constraint (18), we obtain the market clearing condition for the tradable good

$$C_t^T = Y_t^T - P^M M_t - B_{t+1} + RB_t - FX_{t+1} + R^{FX} FX_t - \frac{\theta}{1-\theta} D_t. \tag{24}$$

Finally, equating the demand and supply of labor gives

$$L_t^T + L_t^N = 1. \tag{25}$$

We are now ready to define a rational expectation equilibrium as a set of stochastic processes $\{C_t, C_t^T, C_t^N, P_t^N, \lambda_t, Y_t^T, L_t^T, L_t^N, M_t, B_{t+1}, \mu_t, W_t, X_{t+1}, FX_{t+1}, T_t, D_t\}_{t=0}^{\infty}$ satisfying (2), (4)-(7), (11)-(14), (17)-(18) and (21)-(25), given the exogenous process $\{\kappa_t\}_{t=0}^{\infty}$, the government policy $\{\chi, \chi^{WK}\}$ and initial conditions $B_0, FX_0$ and $X_0$.

The model has a balanced growth path in which $C_t^T, Y_t^T, M_t, P_t^N, B_{t+1}$ and $W_t$ all grow at the same rate as $X_t$. The real exchange rate grows at a positive rate in the balanced growth path because productivity in the tradable sector exhibits positive trend growth, while productivity in the non-tradable sector is fixed. This is the classic Balassa-Samuelson effect. Since also $GDP_t = Y_t^T - P^M M_t + P_t^N Y_t^N$ grows at the same rate as $X_t$, we will refer to the growth rate of the stock of knowledge as the growth rate of the economy.

## 2.8 Discussion: public and private capital flows

A novel feature of our framework is the distinction between public capital flows in the form of foreign reserves ($FX_t$) and private capital flows ($B_t$). Before we move forward in the analysis, we want to emphasize the differences between the internationally traded private bond and foreign reserves.

The first difference is related to the fact that in our framework, domestic agents have an imperfect access to international private capital markets. In fact, domestic agents are subject to an occasionally binding borrowing constraint that limits their access to foreign credit. From (13) we can see that the possibility that the constraint binds ($\mu_{t+1} > 0$) drives an endogenous wedge (a premium) between the world interest rate, $R$, and the rate at which agents can borrow in international capital market. We also assume that foreign reserves provide a lower return compared to private bonds ($R^{FX} \leq R$). Moreover, similarly to what is also assumed in a first-generation currency crises model, reserves are subject to a lower bound ($FX_t \geq 0$) so that they can only be accumulated. Finally, reserves are more liquid than private bonds and they can be used by the policy authority to provide international liquidity to the private sector during periods of financial turmoil.

These features make the two assets imperfect substitutes. We note here that imperfect substitutability between $B_t$ and $FX_t$ would hold even if $R^{FX} = R$ as long as there is a possibility that the borrowing constraint that private agents face might be binding. This feature of the model creates the key difference with respect to the neoclassical growth model in which the accumulation of foreign reserves would be exactly offset one-for-one by private capital inflows. From our reading of the literature the distinction between the private and public nature of capital flows is novel and differs from existing contributions that identify the international reserves accumulated by the government with the stock of foreign wealth accumulated by domestic agents.

# 3 Social planner

Before considering the foreign reserve policy, we first characterize the social planner allocation. This is useful to build intuition about the source of inefficiency in the competitive equilibrium that creates scope for policy intervention.

The planner maximizes domestic households' utility (1), subject to the economy-wide resource constraints (23), (24) and (25), the borrowing constraint (9) and the two constraints on reserve management (19) and (20). Importantly, the social planner takes into account the effect that imported inputs have on the accumulation of knowledge, and so also the equation describing the evolution of the stock of knowledge (15) enters as a constraint in the planner's problem.

Appendix A provides a formal characterization of the social planning allocation. Here we notice that, as long as $R^{FX} < R$, the social planner chooses not to hold reserves, that is she sets $FX_{t+1} = 0$ for every $t$.[6] Intuitively, the social planner chooses not to hold reserves because they represent an inefficient saving vehicle compared to foreign bonds, as they pay a lower interest rate. This happens notwithstanding the fact that reserves can be used to provide liquidity during crises. To understand this result, notice that the working capital constraint is affected by the private net foreign asset position at the beginning of period $t$. Due to the lower interest rate paid on reserves compared to private bonds, the most efficient way from the social planner perspective to relax the constraint in period $t$ is by reducing the net debt position in period $t-1$ (i.e. increasing $B_t$), rather than accumulating reserves and using them in the event of a crisis.

As showed in appendix A, the social planner allocation is characterized by the same equations as the competitive equilibrium in which $FX_{t+1} = D_t = 0$

[6] If $R^{FX} = R$ the planner may hold foreign reserves, but imposing $FX_{t+1} = 0$ for every $t$ on her allocation does not prevent the planner from reaching the first best. See the appendix for the details.

is imposed in every period.[7] The only difference is given by equation (12), the optimality condition that determines the choice of imported inputs. In fact, in the social planner allocation equation (12) is replaced by

$$P^M \left(1 + \phi \frac{\mu_t^{SP}}{\lambda_t^{SP}}\right) = (1-\alpha_T)\frac{Y_t^T}{M_t} + \underbrace{\beta\xi \left(\frac{X_t}{M_t}\right)^{1-\xi} E_t \left(\frac{\lambda_{t+1}^{SP}}{\lambda_t^{SP}} \left(\alpha_T \frac{Y_{t+1}^T}{X_{t+1}} + \kappa_{t+1}\frac{\mu_{t+1}^{SP}}{\lambda_{t+1}^{SP}}\right)\right)}_{\text{growth externality}},$$

where $\mu_t^{SP}$ is the Lagrange multiplier on the borrowing constraint (9) and $\lambda_t^{SP}$ is the Lagrange multiplier on the resource constraint for tradable goods (24). The left-hand side of this expression represents the marginal cost of increasing the use of imported inputs, taking into account the impact of the borrowing constraint, captured by the term $\mu_t^{SP}$. The first term on the right-hand side is the benefit from the increase in the output of tradable goods generated by an increase in the use of imported inputs. These two terms are equivalent to the ones that would arise in the competitive equilibrium allocation (12). The second term on the right-hand side is specific to the social planner problem and captures the benefits derived from the increase in the stock of knowledge implied by an increase in the use of imported inputs. Increasing the stock of knowledge is beneficial for two reasons. First, the social planner internalizes the fact that a higher usage of imported inputs today leads to higher knowledge and higher productivity tomorrow and thus to a higher amount of tradable goods produced in the future. Second, the social planner internalizes the fact that an increase in productivity tomorrow relaxes the borrowing constraint by increasing the sum that foreign investors can recover in case of default. These two effects imply that in every period the amount of imported inputs used is higher in the social planner allocation than in the competitive equilibrium without policy intervention. Because of this, the economy grows at a faster rate under the social planner allocation compared to the competitive equilibrium with no policy intervention.

It is possible to decentralize the social planner allocation in the competitive equilibrium by subsidizing the purchase of imported inputs at rate

$$\tau_t = \frac{\beta\xi}{P^M} \left(\frac{X_t}{M_t}\right)^{1-\xi} E_t \left(\frac{\lambda_{t+1}^{SP}}{\lambda_t^{SP}} \left(\alpha_T \frac{Y_{t+1}^T}{X_{t+1}} + \kappa_{t+1}\frac{\mu_{t+1}^{SP}}{\lambda_{t+1}^{SP}}\right)\right),$$

while financing the subsidy using lump-sum taxes. This subsidy scheme is able to restore the first best, but in practice this form of intervention might be politically

[7] To be precise, if the economy starts with a positive amount of reserves ($FX_0 > 0$) and it is hit by a bad credit shock during the first period ($\kappa_0 = \kappa_L$) the planner may use the initial stock of reserves to finance working capital and $D_0$ may be positive. Even in this case, $FX_{t+1} = 0$ for any $t$ and so $D_t = 0$ for any $t > 0$.

hard to implement. For instance, a government might not be able to openly subsidize firms in the export sector due to the existence of trade agreements such as the WTO rules. In the next section we show how an appropriate management of foreign exchange reserves can serve as a second best policy to internalize the growth externalities in the tradable sector, without breaking the rules dictated by free trade agreements.

# 4 Reserve policy and growth

In this section we discuss the mechanisms through which a policy of reserve accumulation during tranquil times and liquidity provision during crisis times works. In particular we are interested in providing intuition on how foreign reserves can be used as a second best policy tool aimed at internalizing the growth externalities in the tradable sector.

We start by examining the impact of foreign reserve accumulation in states in which the borrowing constraint is not binding. Combining equations (11), (12) and (17) and using the fact that when the borrowing constraint does not bind $\mu_t = 0$, we obtain the demand for imported inputs, $M_t$, as a function of the real exchange rate, $P_t^N$

$$M_t = \left(\frac{1-\alpha_T}{P^M}\right)^{\frac{1}{\alpha_T}} X_t \left[1 - \left(\frac{\alpha_N}{\alpha_T}\frac{P_t^N}{X_t}\left(\frac{P^M}{1-\alpha_T}\right)^{\frac{1-\alpha_T}{\alpha_T}}\right)^{\frac{1}{1-\alpha_N}}\right].$$

When the real exchange rate appreciates ($P_t^N$ rises) the demand for imported inputs decreases. Intuitively, an increase in $P_t^N$, the relative price of non-tradable goods, increases the marginal product of labor in the non-tradable sector. This causes a shift of labor out of the tradable sector that decreases the productivity of the imported intermediate inputs and induces firms to reduce $M_t$. This suggests that in order to increase the use of imported inputs and the growth rate of the economy above their competitive equilibrium values, the government can implement policies that reduce $P_t^N$, that is to engineer a real exchange rate undervaluation.[8]

To understand the link between reserve accumulation and real exchange rate determination in tranquil times, we combine equations (6), (18) and (24) and use the fact that during tranquil times $D_t = 0$ to obtain

$$P_t^N = \frac{1-\omega}{\omega}\frac{Y_t^T - P^M M_t - B_{t+1} + RB_t - FX_{t+1} + R^{FX} FX_t}{C_t^N}.$$

[8]We refer to a policy-induced real exchange rate undervaluation when the real exchange rate, net of the Balassa-Samuelson effect, is undervalued in the competitive equilibrium allocation with policy intervention compared to its value in the laissez-faire equilibrium.

Holding everything else constant, this equation implies a negative relationship between $P_t^N$ and $FX_{t+1}$. The intuition is simple: In order to accumulate foreign reserves the government needs to withdraw resources from the private sector. Since only tradable goods can be sold to foreigners in exchange for reserves, the government must appropriate tradable goods from the private sector.[9] Private agents are then forced to reduce their consumption of tradable goods. This leads to a real exchange rate depreciation which in turns stimulates production in the tradable sector and imports of the intermediate good. Through this channel, a policy of accumulating reserves during tranquil times has the potential to increase the growth rate of the economy and to internalize, at least partly, the growth externalities present in the tradable sector.

Clearly, in general equilibrium a change in $FX_{t+1}$ affects all the other endogenous variables. In particular private agents tend to offset the impact of the increase in foreign reserves on consumption by borrowing from abroad. Indeed, in a model in which private borrowing and reserves are perfect substitutes, the accumulation of $FX_{t+1}$ would be counterbalanced by a corresponding decline in $B_{t+1}$. In our framework the imperfect substitutability between the two assets prevents private agents from completely offsetting the actions of the government.

We now illustrate the general equilibrium implications of a policy of reserve accumulation during tranquil times by examining how the stochastic steady state of our economy varies when we change the value of $\chi$, our proxy for the resources employed to accumulate reserves during tranquil times.[10]

The six panels of figure 2 show the long-run mean values of the following variables: The growth rate of GDP, the percentage deviations of the real exchange rate from its value in the equilibrium with no policy intervention, the trade balance-to-GDP ratio, the private net foreign assets-to-GDP ratio, consumption of tradable goods and aggregate consumption as a function of $\chi$, the fraction of tradable output devoted to reserve accumulation during tranquil times. The real exchange rate is normalized by the stock of knowledge to control for the Balassa-Samuelson effect. The same normalization is applied to consumption of tradable goods and to aggregate consumption.

As suggested by the partial equilibrium analysis, the growth rate of the economy is increasing in the amount of resources devoted to reserves accumulation during tranquil times. Stronger accumulation of foreign exchange reserves also produces a depreciation of the real exchange rate and an increase in the trade balance-to-GDP ratio. Both of these effects are driven by the fall in the consump-

[9] In our model, we can think of tradable goods as a proxy for the international currency.

[10] More precisely, for each value of $\chi$ we solved the model numerically. Then we drew a 10000 periods-long simulation, discarded the first 100 periods, and computed the long run average values of the variables of interest. In all the simulations we set $\chi^{WK} = 0$, details on the value of the other parameters are provided in section 5.1.

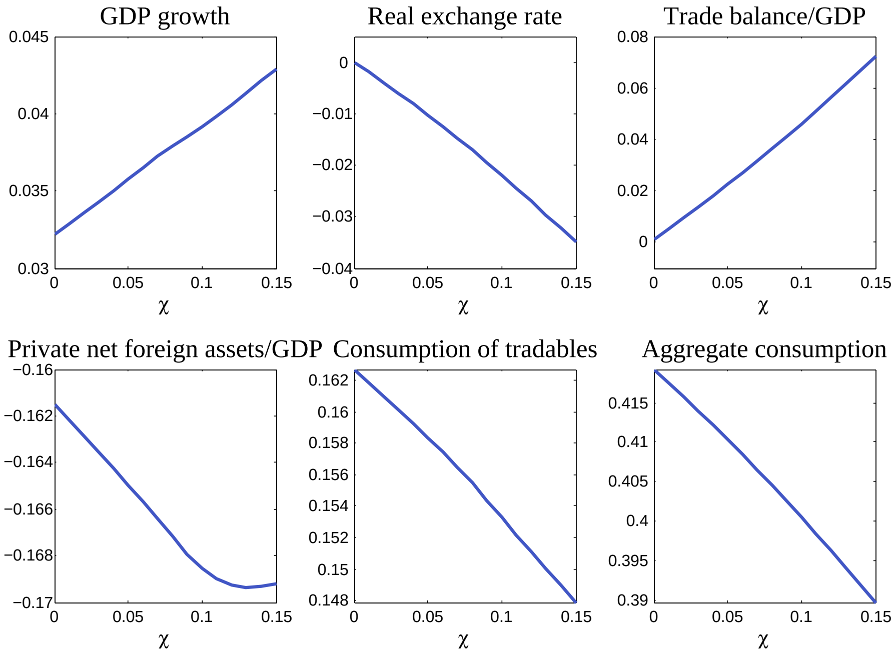



Figure 2: Impact of reserve accumulation. Notes: $\chi$ is the fraction of tradable output devoted to reserve accumulation during tranquil times. The real exchange rate, net of the Balassa-Samuelson effect, refers to the percentage change of $P_t^N/X_t$ with respect to its value in absence of government intervention ($\chi = 0$). The trade balance is defined as $Y_t^T - P^M M_t - C_t^T$. The private net foreign assets-to-GDP ratio is defined as $B_{t+1}/GDP_t$. Consumption is normalized by the stock of knowledge. In all the simulations we set $\chi^{WK} = 0$, details on the value of the other parameters are provided in section 5.1.

tion of tradable goods caused by the withdrawal of resources from private agents. The increase in the production of tradable goods implied by the real exchange rate depreciation also contributes to the improvement in the trade balance-to-GDP ratio.

Figure 2 shows that as the government increases the pace at which it accumulates foreign exchange reserves the private foreign debt-to-GDP ratio rises. As we mentioned above, this occurs as private agents partially offset the increase in public savings implied by faster reserve accumulation by decreasing private savings and hence by accumulating more foreign debt.

De-trended consumption of tradable goods and aggregate consumption are both decreasing in the rate of reserve accumulation. This highlights a key trade-off that determines the impact on welfare of government intervention. On the one hand, faster reserve accumulation induces higher growth and this has a positive effect on welfare. On the other hand, in order to accumulate foreign exchange reserves the government has to subtract resources that would otherwise be consumed, and this affects welfare negatively. The balance between these two effects determines whether reserve accumulation during tranquil times has a positive or negative impact on welfare, as we will document later.

We now turn to the impact of crisis-times interventions. During crisis times, the borrowing constraint binds and the amount of imported inputs used in production is given by

$$M_t = \frac{X_t \kappa_L + RB_t + D_t}{\phi P^M}.$$

This equation makes clear that in order to increase the amount of imported inputs used by firms above its value in the equilibrium without intervention, the government has to provide working capital loans during crisis events (i.e. set $D_t > 0$). Hence, in the model the existence of growth externalities in the tradable sector, coupled with financial frictions, provides a justification for the use of reserves during crises.

Figure 3 compares the response to a negative credit shock for two different economies.[11] The solid lines refer to an economy in which the government does not intervene during the crisis ($\chi^{WK} = 0$). When the bad credit shock hits the economy in period 3, firms become borrowing constrained, they are forced to cut their imports of intermediate inputs and this negatively affects production of tradable goods and GDP. The real exchange rate depreciates because households have to cut their consumption of tradable goods and because labor flows toward the non-tradable sector, thus increasing the supply of non-tradable goods. Moreover, since credit shocks are persistent, households decrease their stock of inter-temporal foreign debt in order to self-insure against the increased risk of a future bad credit shock.

The dashed lines refer to the case in which the government uses its stock of reserves to provide working capital loans to firms in the tradable sector ($\chi^{WK} > 0$). When the bad credit shock hits the economy, the government starts drawing down its stock of reserves to finance the purchase of imported inputs. This mutes the impact of the credit shock on GDP and on the real exchange rate. In addition, the bad credit shock does not translate into an increase in private savings, i.e. a decrease in foreign debt, because households anticipate that the government will intervene in case of a future bad credit shock.

[11] To construct this figure, we simulated the economy with $\chi = 0.09$ and $\chi^{WK} = 1$ for 10000 periods, discarded the first 100 periods and then collected all the periods with a negative credit shock ($\kappa_t = \kappa_L$). We then constructed windows around each period $t$ with a bad credit shock going from $t-2$ years before the shock to $t+12$ years after. We then collected the median path for $\kappa_t$ and the median initial values for the state variables $B_{t-2}$ and $FX_{t-2}$ across all the windows. Finally, we fed this path for the credit shock and these initial conditions to the model without intervention during crises ($\chi^{WK} = 0$) and to the model with intervention ($\chi^{WK} = 1$).

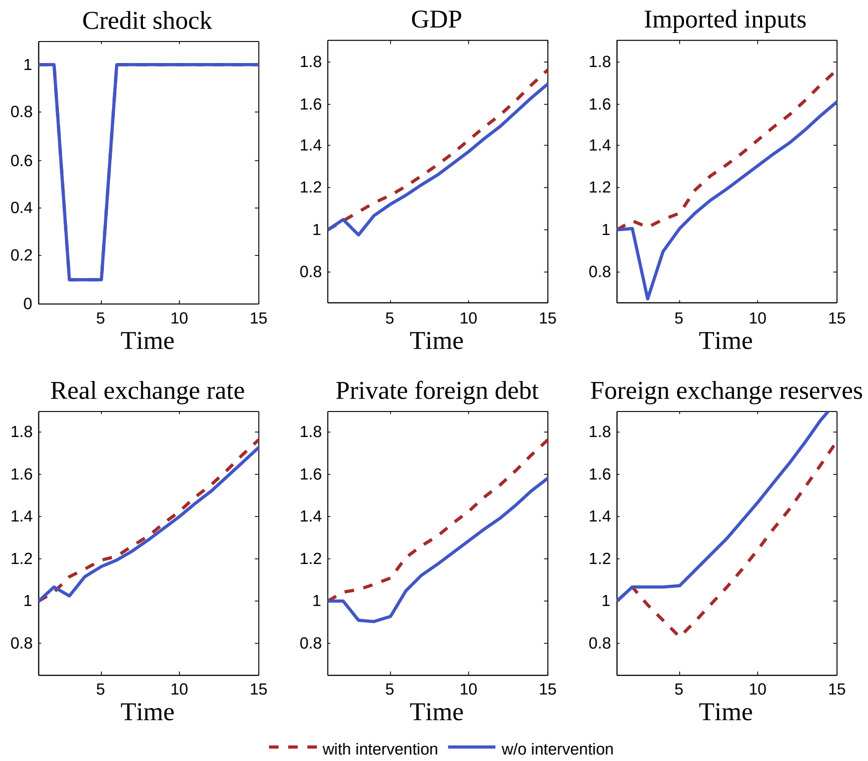



Figure 3: Intervention during crises. Notes: All the variables are normalized by their first-period value. Private net foreign debt is defined as $-B_{t+1}$. Foreign exchange reserves refer to $FX_{t+1}$. In all the simulations we set $\chi = 0.09$. In the model with intervention $\chi^{WK}$ is set equal to 1, while in the model without intervention $\chi^{WK}$ is set equal to 0. Details on the value of the other parameters are provided in section 5.1.

Notice that the crisis entails a permanent difference in the level of GDP between the two economies. This stems from the fact that in our model an economy hit by a crisis never fully recovers to its pre-crisis growth path.[12] Because of this reason, intervening during crises has a positive impact on the average growth rate of the economy.

One interesting feature of the model is that the relationship between growth and the real exchange rate depends on whether the economy is borrowing constrained or not. In fact the binding borrowing constraint reverses the negative relationship between growth and real exchange rate observed during tranquil times. This happens because to stimulate growth during crises the government has to provide loans to firms in the tradable sector. This shifts productive resources toward the

[12] Cerra and Saxena (2008) provide empirical evidence showing that countries that are hit by a crisis hardly get back to their pre-crisis growth path.

tradable sector, allowing households to consume more tradable goods. At the same time, the production of non-tradable goods decreases and so the real exchange rate rises, creating a positive relationship between real exchange rate, use of imported inputs and growth.

# 5 Financial liberalization and optimal management of foreign exchange reserves

In this section we use our framework to describe the impact of international reserve management on the transition from financial autarky to a regime in which foreign borrowing is allowed, but limited by the borrowing constraint (9). This experiment demonstrates the model's ability to reproduce the pattern of growth, capital flows and reserve accumulation observed in the data. Moreover, we use this exercise to evaluate the significance of the welfare gains that can be obtained through an appropriate management of foreign exchange reserves.

## 5.1 Parameterization

The model cannot be solved analytically and so we must resort to numerical simulations. In order to preserve the non-linearities present in our framework we solve the model using a a global solution method.[13] The model is too simple to lend itself to a careful calibration exercise, hence we choose reasonable values for the parameters in order to illustrate the model's properties.

Some parameters are standard in the literature. The risk aversion parameter is set at $\gamma = 2$. The interest rate at which domestic agents can borrow from foreign investors is assumed equal to $R = 1.04$, while the discount factor is set to $\beta = 1/R$. We choose identical labor shares in the two sectors $\alpha_T = \alpha_N = 0.65$. The share of tradable goods in consumption is set to $\omega = 0.341$ as in Durdu et al. (2009). The price of imported inputs $P^M$ is normalized to 1 without loss of generality.

The parameters governing the financial frictions are set so that the version of the model without government intervention reproduces salient characteristics of developing countries. We set the borrowing limit $\kappa_L$ equal to 0.1. This gives an average net foreign assets-to-GDP ratio of $-16$ percent, in the range of the values commonly observed in developing countries.[14] The probability of experiencing a bad credit shock is set to $1 - \rho_H = 0.1$ as in Jeanne and Rancière (2011), while the probability of exiting an episode of financial turbulence is set to $1 - \rho_L = 0.5$,

[13]More precisely, we solve the model by iterating on the equilibrium conditions as proposed by Coleman (1990).

[14]The precise value of $\kappa_H$ does not affect the simulations, as long as it is sufficiently high so that the borrowing constraint does not bind when $\kappa_t = \kappa_H$.

**Table 1: Parameters**

| Parameter | Symbol | Value |
|---|---|---|
| Risk aversion | $\gamma$ | 2 |
| Interest rate on private borrowing | $R$ | 1.04 |
| Discount factor | $\beta$ | $1/R$ |
| Labor share in output in tradable sector | $\alpha_T$ | 0.65 |
| Labor share in output in non-tradable sector | $\alpha_N$ | 0.65 |
| Share of tradable goods in consumption | $\omega$ | 0.341 |
| Price of imported inputs | $P^M$ | 1 |
| Borrowing limit | $\kappa_L$ | 0.1 |
| Probability of bad credit shock | $1-\rho_H$ | 0.1 |
| Probability of exiting bad credit shock | $1-\rho_L$ | 0.5 |
| Working capital coefficient | $\phi$ | 0.33 |
| Elasticity of TFP w.r.t. imported inputs | $\xi$ | 0.15 |
| Constant in knowledge accumulation process | $\psi$ | 0.34 |
| Interest rate on reserves | $R^{FX}$ | 1 |
| Efficiency of government intervention during crises | $\theta$ | 0.5 |

following Alfaro and Kanczuk (2009). The fraction of imported inputs that has to be paid in advance $\phi$ is set to 0.33 to match an average working capital-to-GDP ratio of 6 percent. This is the same target as in Mendoza and Yue (2011).

To parameterize the process for the accumulation of knowledge we use the estimates provided by Coe et al. (1997). They find that the elasticity of TFP with respect to imports of machinery and equipment in developing countries is close to 0.3. They do not estimate which part of the effect can be attributed to spillovers that are not internalized by firms, so 0.3 is likely to be an upper bound for our parameter $\xi$. We take a pragmatic approach and set $\xi = 0.15$ in our baseline parameterization. We then conduct sensitivity analysis in order to check how our results vary with this parameter. The constant in the knowledge accumulation process $\psi$ is set to 0.34, in order to match an average growth rate of 3 percent in the competitive equilibrium without government intervention.

The gross interest rate paid on reserves $R^{FX}$ is equal to 1. This gives a spread between private borrowing cost and the interest rate paid on reserves of 4 percent, in the range of the values considered by Rodrik (2006). We could not find good estimates for $\theta$, the parameter that determines the efficiency of government intervention during crises. Hence, we set it to 0.5 in our baseline parameterization and then we check how our results are affected by changes in this parameter.

## 5.2 Results

We start by exploring how the foreign reserve policy affects an economy that opens up to international capital flows. To capture the opening to international credit markets, we look at economies that start with no foreign debt ($B_0 = 0$) and with no reserves ($FX_0 = 0$) and we follow them during the transition to a steady state in which foreign borrowing is allowed, but constrained by condition (9). We also assume that the economy starts in tranquil times ($\kappa_0 = \kappa_H$).

We compare two different economies. First, we look at an economy in which the government does not intervene, that is in which $\chi = \chi^{WK} = 0$. Second, we consider an economy in which the government optimally chooses the parameters governing the foreign reserve policy, $\chi$ and $\chi^{WK}$. To compute the optimal policy we constructed grids for $\chi$ and $\chi^{WK}$ and then we searched for the combination of these two parameters that maximizes the expected lifetime utility of the representative household. In our baseline parameterization the optimal policy is characterized by $\chi = 0.09$ and $\chi^{WK} = 1$.

We derived forecast functions that describe the transition from financial autarky to the steady state with financial liberalization using the following procedure. For each model economy we performed 100000 stochastic simulations lasting for 15 periods each, taking as initial conditions $B_0 = FX_0 = 0$ and $\kappa_0 = \kappa_H$. For each period we then averaged across all the simulations to obtain our forecast functions. Figure 4 shows the results of the experiment. To facilitate comparison, GDP, consumption of tradable goods, consumption of non-tradable goods and the real exchange rate are all expressed in percentage deviations from their first-period value in the equilibrium without government intervention.

Start by considering the solid lines, which describe the economy without government intervention. Upon opening to the international credit markets, the economy embarks in a period of accumulation of foreign debt that lasts for around five years, when the private net foreign assets-to-GDP ratio reaches its steady state value of $-16$ percent. The accumulation of foreign debt is the result of two forces. On the one hand, households living in an economy that is growing faster than the rest of the world, as we are implicitly assuming, have the desire to frontload their consumption stream and this pushes domestic agents to accumulate foreign debt. On the other hand, a high stock of foreign debt increases the negative impact of a bad credit shock on production of tradable goods. Because of this, domestic agents accumulate precautionary savings to self-insure against the risk of a bad credit shock and this puts a brake to the buildup of foreign debt. The counterpart to the process of debt accumulation are the high initial current account deficits, that progressively decrease until the current account-to-GDP ratio reaches its steady state value of $-1$ percent.

The first years following financial liberalization also see a progressive increase in

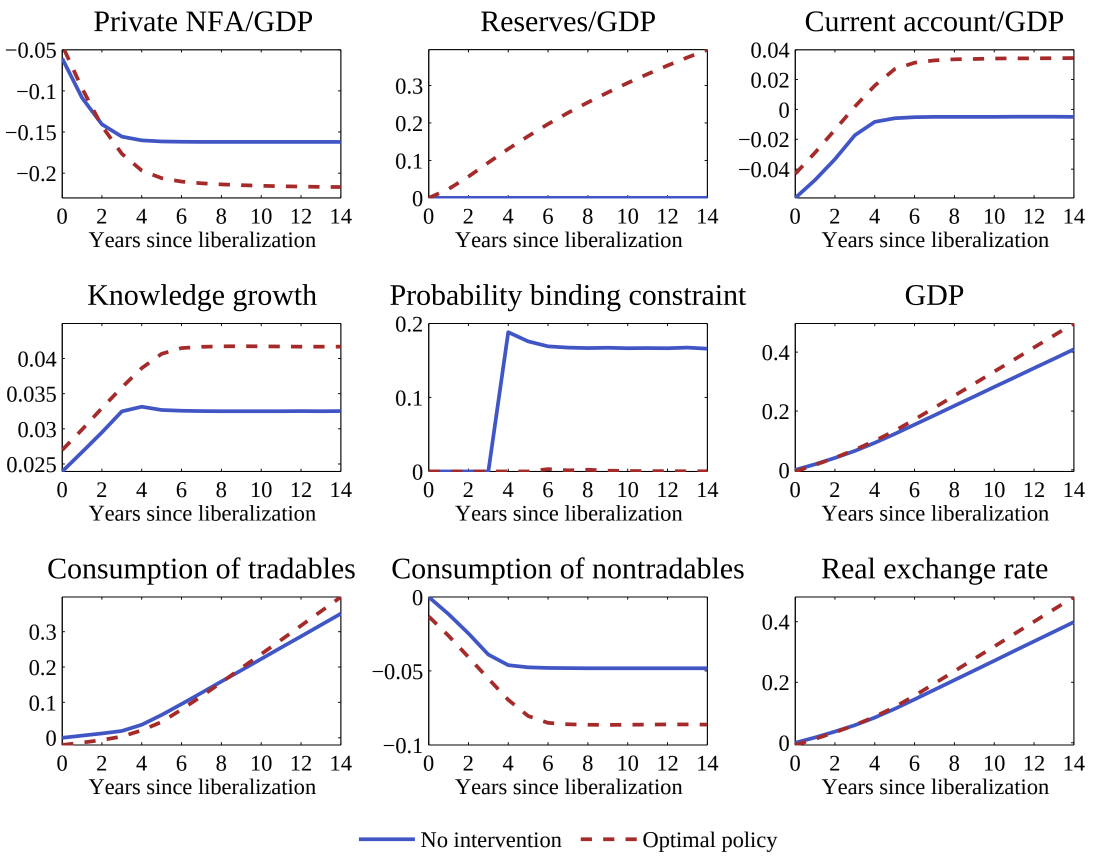



Figure 4: Effects of financial liberalization. Notes: GDP, consumption of tradables, consumption of non-tradables and the real exchange rate are all expressed in percentage deviations from their first-period value in the equilibrium without government intervention. NFA refers to net foreign assets.

the growth rate of the economy. This happens because the accumulation of foreign debt props up the consumption of tradable goods for a given amount of tradable goods produced. This gives an incentive to shift labor toward the production of non-tradable goods, which is higher during the first years after liberalization compared to its steady state value. As the economy approaches its steady state, progressively more labor is allocated to the production of tradable goods, more intermediate inputs are imported and the growth rate of the economy increases until it reaches its steady state value.

Finally, during the first years after the opening to international credit markets the probability of experiencing a binding borrowing constraint is zero, because of the low stock of initial debt. As the stock of foreign debt increases, so does the probability of entering a financial crisis.

The dashed lines refer to the economy in which the government implements the optimal policy. After the opening to the international credit markets the government starts to accumulate foreign reserves at a fast pace. In fact, in the first fifteen years after financial liberalization the reserves-to-GDP ratio passes from 0 to almost 40 percent. Afterward, the reserves-to-GDP ratio keeps growing until it reaches its steady state value of 84 percent. Because of this policy, net capital inflows are lower compared to the laissez-faire equilibrium. Indeed, in steady state the current account-to-GDP ratio in the economy with policy intervention is 5 percentage points higher than in the economy without intervention.

Interestingly, the economy with government intervention posts higher current account surpluses despite higher accumulation of foreign debt from the private sector. The large buildup of private debt is driven by two effects. First, as discussed in section 4, private agents take on foreign debt to partly offset the impact of reserve accumulation on consumption. Second, in the economy with government intervention the incentives for private agents to build a stock of precautionary savings are weaker, because firms in the tradable sector anticipate that the government will supply working capital financing during crisis events. The result is that in steady state the private net foreign assets-to-GDP ratio is 5 percentage points lower compared to the economy without policy intervention.

Despite the reaction of private agents and because of the imperfect substitutability between private and public capital flows, the government policy succeeds in engineering a real exchange rate undervaluation that shifts productive resources out of the non-tradable sector and into the production of tradable goods.[15] Moreover, the government intervention during crises reduces to almost zero the probability of facing a binding borrowing constraint. These two effects lead to a higher use of imported inputs and to a faster growth rate of the economy compared to the equilibrium with no policy intervention. In fact, in steady state the growth rate of the stock of knowledge is 1 percent higher than under laissez-faire.

The model is thus able to replicate the negative correlation between growth and capital inflows observed in the data. Moreover, consistent with empirical evidence, the correlation is driven by the accumulation of foreign reserves from the public sector.

Figure 4 can also be used to illustrate the intuition underlying the impact on welfare of government interventions. During the first years after financial liberalization, consumption of tradable goods is lower in the economy with policy

[15]Notice that the undervaluation refers to the real exchange rate purged from the Balassa-Samuelson effect. In absolute terms, the real exchange rate in the economy with policy intervention is undervalued compared to the laissez-faire equilibrium only during the first years after liberalization. Due to faster productivity growth in the tradable sector induced by reserve accumulation, the real exchange rate in the economy with government intervention eventually becomes more appreciated than in the economy with no intervention.

intervention compared to the laissez-faire equilibrium. This happens because the government appropriates tradable goods from the private sector to finance the accumulation of reserves. However, the government policy also leads to faster growth and this explains why from year 9 on the consumption of tradable goods becomes higher in the equilibrium with policy intervention compared to the one without intervention. Hence, the government faces a trade-off between lower consumption of tradable goods in the present, in exchange for faster growth and thus higher consumption of tradable goods in the future.

To describe the impact on welfare of different reserve management policies, we report the welfare gains that can be obtained from government intervention for an economy that undergoes financial liberalization. We compute the welfare gains of moving from the equilibrium with no intervention to a generic policy regime $i$ as the proportional increase in consumption for all possible future histories that households living in the economy with no policy intervention must receive in order to be indifferent between remaining the no-intervention economy and switching to policy regime $i$. Formally, the welfare gain $\eta$ is defined as

$$E_0\left[\sum_{t=0}^{\infty}\beta^t\frac{\left((1+\eta)\,C_t^n\right)^{1-\gamma}}{1-\gamma}\right]=E_0\left[\sum_{t=0}^{\infty}\beta^t\frac{C_t^{i1-\gamma}}{1-\gamma}\right],$$

where the superscripts $n$ and $i$ denote allocations respectively in the economy with no policy intervention and under policy regime $i$. Since we want to look at economies that start from financial autarky we set the initial states to $B_0=0$, $FX_0=0$ and $\kappa_0=\kappa_H$.

Figure 5 presents the results of our welfare analysis by plotting the welfare gains as a function of the resources employed to accumulate reserves during tranquil times $\chi$, for different intensities of the intervention during crises $\chi^{WK}$.

The first thing to notice is that the welfare gains from policy intervention are quantitatively significant. For instance, the optimal policy delivers welfare gains above 1 percent of permanent consumption equivalent. Moreover, the bulk of the welfare gains seems to come from the ability to provide liquidity to firms during crises. This can be seen from the large welfare differences between the economy with no intervention during crises ($\chi^{WK}=0$) and those in which the government does intervene to provide liquidity during periods of financial turbulence ($\chi^{WK}>0$). In addition, under the welfare maximizing rule reserves are accumulated at a fast pace, since 9 percent of the output of tradable goods is devoted to the accumulation of reserves each tranquil period.

Interestingly, some welfare gains, albeit small, can be obtained through the accumulation of foreign exchange reserves also when they cannot be used to intervene during crises. This can be seen by looking at the $\chi^{WK}=0$ line, which reaches its maximum corresponding to a consumption equivalent of 0.02 percent

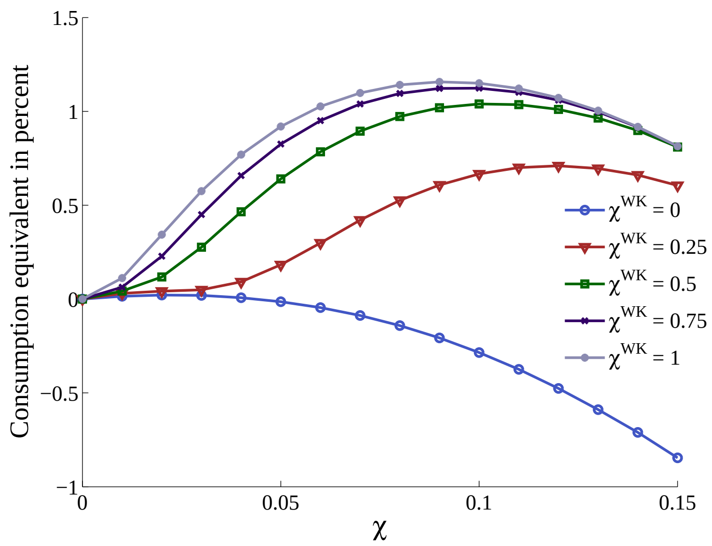



Figure 5: Welfare impact of policy interventions. Notes: $\chi$ is the fraction of tradable output devoted to reserve accumulation during tranquil times. $\chi^{WK}$ is the maximum amount of resources that the government is willing to use during crisis events, expressed as a fraction of the start-of-period stock of foreign exchange reserves.

when $\chi = 0.02$. Thus reserve accumulation can be a welfare enhancing policy also when reserves cannot perform their traditional role of liquidity provider during financial crises.

### 5.3 Sensitivity analysis

TO BE WRITTEN

## 6 Conclusion

This paper presents a simple framework that it is able to reproduce two facts characterizing the international monetary system: i) Developing countries that grow faster accumulate more international reserves and ii) Fast growing developing countries are associated with lower net capital inflows. In our framework the government uses foreign exchange reserves to internalize the growth externalities present in the tradable sector and to provide liquidity to the corporate sector during periods of financial stress. This creates a positive link between reserve accumulation, current account surpluses and growth. Importantly, in our framework official reserves and private debt are imperfect substitutes, so that the reserve policy of the government cannot be perfectly offset through borrowing by private agents.

We use the model to compare the laissez-faire equilibrium and the optimal reserve policy in an economy that is opening to international capital flows. We find that the optimal reserve management entails a fast rate of reserve accumulation, as well as higher growth and larger current account surpluses compared to the economy with no policy intervention. We also find that the welfare gains of reserve policy are large, in the order of 1 percent of permanent consumption equivalent.

The simple framework that we propose can be extended in a number of directions to study several issues related to the international monetary system. For example, extending the model to a two country framework sheds light on the impact of reserve accumulation from developing countries on global interest rates and on the country issuing the reserve currency (Benigno and Fornaro (2012)). It would also be interesting to introduce into the model the possibility for the government to implement controls on private capital flows. We conjecture that the imposition of barriers to private borrowing would make the impact of reserve accumulation on growth more effective. In light of this, the model could provide an explanation for the practice of imposing tight controls on capital flows characterizing many developing economies. We are planning to address these topics in future research.

# Appendices

## A Social planner allocation

In this appendix we formally characterize the social planner allocation. The social planner chooses $\left\{C_t^N, C_t^T, L_t^T, L_t^N, M_t, B_{t+1}, FX_{t+1}, D_t\right\}_{t=0}^{\infty}$ to maximize households' expected utility (1), subject to the economy-wide resource constraints (23), (24) and (25), the borrowing constraint (9), the two constraints on reserve management (19) and (20) and the law of motion for the stock of knowledge (15). The first order conditions of the planner's problem can be written as

$$(1-\omega)\frac{C_t^{1-\gamma}}{C_t^N} = \lambda_t^N,$$

$$\omega\frac{C_t^{1-\gamma}}{C_t^T} = \lambda_t^{SP}$$

$$\alpha_N \left(1-L_t^T\right)^{\alpha_N-1} \lambda_t^N = \alpha_T A_t X_t^{\alpha_T} L_t^{\alpha_T-1} M_t^{1-\alpha_T} \lambda_t^{SP}$$

$$\begin{aligned} P^M \left(1+\phi\frac{\mu_t^{SP}}{\lambda_t^{SP}}\right) &= (1-\alpha_T)\frac{Y_t^T}{M_t} \\ &+ \beta\xi\left(\frac{X_t}{M_t}\right)^{1-\xi} E_t\left(\frac{\lambda_{t+1}^{SP}}{\lambda_t^{SP}}\left(\alpha_T\frac{Y_{t+1}^T}{X_{t+1}}+\kappa_{t+1}\frac{\mu_{t+1}^{SP}}{\lambda_{t+1}^{SP}}\right)\right) \end{aligned} \tag{A.1}$$

$$\lambda_t^{SP} = \beta R\left(\lambda_{t+1}^{SP}+\mu_{t+1}^{SP}\right) \tag{A.2}$$

$$\lambda_t^{SP} = \beta R^{FX}\left(\lambda_{t+1}^{SP}+\mu_{t+1}^{FX}\right)+\nu_t \tag{A.3}$$

$$\mu_t^{SP} = \frac{\mu_t^{FX}}{1-\theta}+\frac{\theta}{1-\theta}\lambda_t^{SP}, \tag{A.4}$$

plus the complementary slackness conditions for the inequality constraints. $\lambda_t^N$, $\lambda_t^{SP}$, $\mu_t^{SP}$, $\nu_t$ and $\mu_t^{FX}$ are the Lagrange multipliers respectively on constraints (23), (24), (9), (19) and (20).

Combining equations $(A.2)$, $(A.3)$ and $(A.4)$ gives

$$\beta R\left(\lambda_{t+1}^{SP}+\mu_{t+1}^{SP}\right) = \beta R^{FX}(1-\theta)\left(\lambda_{t+1}^{SP}+\mu_{t+1}^{SP}\right)+\nu_t.$$

This expression has strong implications for the social planner's management of foreign reserves. Start by assuming that $R^{FX} < R$. Then the equation above implies that $FX_t = 0$ for each $t > 0$. This means that if the return on foreign reserves is less than the return on foreign bonds the social planner chooses to hold a zero amount of reserves during each period. If the social planner starts with a positive amount of reserves she may use them to finance the purchase of imported inputs during the initial period, but she will choose to hold no reserves from period 1 on.

Now consider the case $R^{FX} = R$, so that the return on the two assets is equalized. If $\theta = 0$, then it is easy to see that $B_t$ and $FX_t$ become perfect substitutes and that the planner cares only about the economy's net foreign asset position $B_t + FX_t$ and not about its composition between private bonds and reserves. If $\theta > 0$, that is if using reserves during crises is costly, the two assets cease to be perfect substitutes, but the planner is again indifferent about the composition of foreign assets as long as the foreign assets position allows her to set $D_t = 0$ for each $t > 0$. In any case, also when $R^{FX} = R$, setting $FX_{t+1} = 0$ in every period does not prevent the planner from reaching the first best allocation.[16]

Indeed, the social planner allocation is characterized by the same equations as the competitive equilibrium in which $FX_t = D_t = 0$ is imposed in every $t > 0$. The only exception concerns the optimality condition for imported inputs, which

[16] Again, the social planner might use reserves to provide working capital loans during period 0 if it starts with a positive amount of reserves.

is replaced by equation ($A.1$). This happens because the social planner internalizes the impact of imported inputs on the stock of knowledge, while atomistic agents don't.

# References


Aghion, P., D. Comin, and P. Howitt (2006) "When Does Domestic Saving Matter For Economic Growth?," NBER Working Paper No. 12275.

Aguiar, M. and M. Amador (2011) "Growth in the Shadow of Expropriation," *The Quarterly Journal of Economics*, Vol. 126, No. 2, pp. 651–697.

Aizenman, J. and J. Lee (2007) "International reserves: precautionary versus mercantilist views, theory and evidence," *Open Economies Review*, Vol. 18, No. 2, pp. 191–214.

Alfaro, L. and F. Kanczuk (2009) "Optimal reserve management and sovereign debt," *Journal of International Economics*, Vol. 77, No. 1, pp. 23–36.

Angeletos, G.M. and V. Panousi (2011) "Financial integration, entrepreneurial risk and global dynamics," *Journal of Economic Theory*, Vol. 146, No. 3, pp. 863–896.

Arrow, K.J. (1962) "The economic implications of learning by doing," *The review of economic studies*, Vol. 29, No. 3, pp. 155–173.

Bacchetta, P., K. Benhima, and Y. Kalantzis (2011) "Capital Controls with International Reserve Accumulation: Can this Be Optimal?," Unpublished Manuscript.

Benigno, G. and L. Fornaro (2012) "Reserve Accumulation and Global Imbalances," Work in progress.

Broner, F.A. and J. Ventura (2010) "Rethinking the effects of financial liberalization," NBER Working Paper No. 16640.

Buera, F.J. and Y. Shin (2009) "Productivity growth and capital flows: The dynamics of reforms," NBER Working Paper No. 15268.

Cerra, V. and S.C. Saxena (2008) "Growth dynamics: the myth of economic recovery," *The American Economic Review*, Vol. 98, No. 1, pp. 439–457.

Coe, D.T., E. Helpman, and A.W. Hoffmaister (1997) "North-South R & D Spillovers," *The Economic Journal*, Vol. 107, pp. 134–149.

Coleman, W. J. (1990) "Solving the stochastic growth model by policy-function iteration," *Journal of Business Economic Statistics*, Vol. 8, No. 1, pp. 27–29.

Dooley, M.P., D. Folkerts-Landau, and P. Garber (2003) "An essay on the revived Bretton Woods system," NBER Working Paper No. 9971.

Durdu, C.B., E.G. Mendoza, and M.E. Terrones (2009) "Precautionary demand for foreign assets in Sudden Stop economies: An assessment of the New Mercantilism," *Journal of Development Economics*, Vol. 89, No. 2, pp. 194–209.

Gertler, M. and P. Karadi (2011) "A model of unconventional monetary policy," *Journal of Monetary Economics*, Vol. 58, No. 1, pp. 17–34.

Gourinchas, P.O. and O. Jeanne (2011) "Capital Flows to Developing Countries: The Allocation Puzzle," NBER Working Paper No. 13602.

Jeanne, O. and R. Rancière (2011) "The Optimal Level of International Reserves For Emerging Market Countries: A New Formula and Some Applications," *The Economic Journal*, Vol. 121, pp. 905–930.

Keller, W. (2004) "International Technology Diffusion," *Journal of Economic Literature*, Vol. 42, No. 3, pp. 752–782.

Klenow, P.J. and A. Rodriguez-Clare (2005) "Externalities and growth," *Handbook of economic growth*, Vol. 1, pp. 817–861.

Korinek, A. and L. Servén (2010) "Real Exchange Rate Undervaluation: Static Losses, Dynamic Gains," World Bank Policy Research Working Paper No. 5250.

Mendoza, E.G. and V.Z. Yue (2011) "A General Equilibrium Model of Sovereign Default and Business Cycles," NBER Working Paper No. 17151.

Prasad, E.S., R.G. Rajan, and A. Subramanian (2007) "Foreign capital and economic growth," NBER Working Paper No. 13619.

Rodrik, D. (2006) "The social cost of foreign exchange reserves," *International Economic Journal*, Vol. 20, No. 3, pp. 253–266.

——— (2008) "The real exchange rate and economic growth," *Brookings Papers on Economic Activity*, pp. 365–412.

Romer, P.M. (1986) "Increasing returns and long-run growth," *The Journal of Political Economy*, pp. 1002–1037.

Sandri, D. (2010) "Growth and Capital Flows with Risky Entrepreneurship," IMF Working Paper.