

A DSGE model of the Swiss economy with financial frictionsⁱ

Nicolas A. Cuche-Curtiⁱⁱ Jean-Marc Natalⁱⁱⁱ

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Abstract

We extend the small, open economy, DSGE model of Switzerland (Cuche-Curti, Dellas, and Natal, 2009) to include financial frictions and financial shocks. We use the model to study two questions: First, how the pattern of economic activity in Switzerland is affected by financial shocks. And second, how the presence of financial frictions modifies the response of the economy to non-financial disturbances. We find that shocks that reduce entrepreneurial net worth reduce investment and have a pronounced negative effect on economic activity. But we also find that the existence of financial frictions does not alter significantly the response of the economy to non-financial shocks, such as to productivity or monetary policy. This implies that in periods with little financial turbulence, typical DSGE models may be adequate tools for analyzing and forecasting business cycles.

JEL class: E32, E52.

Keyword: DSGE model, financial accelerator, financial shocks, Switzerland.

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ⁱⁱSwiss National Bank and University of St. Gallen

ⁱⁱⁱSwiss National Bank.

Introduction

Recent economic events have brought to center stage the key role played by financial markets in the determination of macroeconomic activity. In particular, shocks that originated in the financial markets (e.g. the sub-prime crisis) seem to have contributed to the deterioration in economic performance in most of the developed world. Developing models that capture such events has become a research priority.

There are –at least– two important questions that these models ought to be able to address. First, how financial shocks impact on real economic activity and prices. And second, how taking into account explicitly financial markets and financial frictions affects the analysis of other macroeconomic shocks (e.g. the effects of a positive productivity shock or of a restrictive monetary policy shock).

The objective of this paper is to address these two questions in the case of Switzerland. We extend the small open economy DSGE model of Switzerland that we developed in previous work (Cuche-Curti, Dellas, and Natal, 2009) by adding a financial accelerator and two financial shocks. The accelerator mechanism is of the type proposed by Bernanke, Gertler, and Gilchrist, 1999 (BGG) and Christiano, Motto, and Rostagno, 2003 and 2007 (CMR). More precisely, we introduce financial intermediaries and a class of entrepreneurs who provide capital services (to intermediate goods producers) using the stock of capital. The entrepreneurs' capital purchases are financed both internally (through own net worth) and externally (through bank loans provided by the financial intermediaries). Entrepreneurs face idiosyncratic productivity shocks and, due to debt contracts, are subject to bankruptcy. The financial shocks we consider are a shock to net worth (or, equivalently, to the value of entrepreneurial assets), and a shock to the dispersion of entrepreneurial performance (or, equivalently, a deterioration in the ability of some firms to pay back their loans).

We calibrate and solve the model according to standard procedures and then examine its dynamic properties following the various shocks. In particular, we compute impulse response functions (IRF) to both financial and non-financial shocks. We reach two main conclusions. First, the effects of financial shocks are broadly consistent with conventional wisdom on this issue. And second, the effects of non-financial shocks are not affected significantly by the

presence or absence of financial frictions. This implies that models that abstract from the financial system and financial frictions (i.e. the typical DSGE models such as DSGE-CH used at the Swiss National Bank) do not suffer from serious mis-specification problems in normal periods, that is, in periods without financial turbulence.

The remaining of the paper is as follows: Section 1 and 2 describe the model and its calibration. Section 3 presents the main results and 4 the conclusions.

1 The Model

As the DSGE model of the Swiss economy is described in great details in Cuche-Curti, Dellas, and Natal, 2009, in this section we only discuss its new features, namely, those associated with the financial accelerator and the financial frictions. The complete model is presented in the appendix.

Our analysis closely follows Collard and Dellas, 2003 who in turn base their analysis on both BGG and CMR. A synoptic view of the model, and in particular how the financial accelerator enter DSGE-CH, is summed up in Figure 1.

<Figure 1 here>

The economy consists of four sectors (sets of firms): The first produces final goods; the second intermediate goods; the third physical capital; and the fourth loans (the ‘banks’). The agents in this economy include consumers, workers, and entrepreneurs. The last group owns the capital stock and provides capital services to the intermediate goods firms. Capital stock purchases are financed both internally (through own net worth) and externally (through bank loans). Entrepreneurs face idiosyncratic productivity shocks and, due to debt contracts, are subject to bankruptcy. The banks borrow from the households and make loans to the entrepreneurs.

We now turn to the description of individual sectors/agents.

1.1 Capital producers

There is a large number of identical capital producers operating under perfect competition. The representative capital producer produces new capital stock in period t , k_{t+1} , by means of investment goods and existing capital stock.¹ The technology is described by the production function

$$k_t = \Theta(i_t, i_{t-1}, k_{t-1}) + (1 - \delta) k_{t-1} \quad (1)$$

with

$$\Theta(i_t, i_{t-1}, k_{t-1}) = i_t (1 - \varpi S(\mathfrak{I}_t)) - (1 - \varpi) \frac{\varphi}{2} \left(\frac{i_t}{k_{t-1}} - \delta \right)^2 k_{t-1} \quad (2)$$

where i_t is investment, $S(\mathfrak{I}_t)$ is equal to $\frac{i_t}{i_{t-1}}$, δ is the constant depreciation rate (between zero and one), $\varphi > 0$ is a technology parameter, and ϖ a parameter allowing to have either investment ($\varpi = 1$) or capital adjustment costs ($\varpi = 0$) in the capital dynamics.

i_t is purchased at price P_t in the goods market, and k_t is the old capital stock purchased at price \tilde{Q}_t in the capital market. The new capital stock is sold at price Q_t .² Thus, profits are given by

$$\Pi_t^k = P_t^c Q_t k_t - P_t^i i_t - P_t^c Q_t k_{t-1} \quad (3)$$

and maximizing profits subject to (1) and (2) leads to the following FOC (with respect to investment)

$$\frac{P_t^i}{P_t^c} = Q_t \Theta_{1t} + \beta \frac{\lambda_{t+1}}{\lambda_t} (1 + \pi_{t+1}^c) Q_{t+1} \Theta_{2t+1}, \quad (4)$$

and (with respect to capital) using in addition the budget constraint of the household

$$\frac{R_t^k Q_{t-1}}{1 + \pi_t^c} = zu - a(u_t) + (\Theta_{3t} + 1 - \delta) Q_t, \quad (5)$$

where $zu - a(u_t)$ comes from the variable capital utilization cost in the budget constraint of the households (z is the rental rate of capital, u is the capital utilization, and $a(u)$ is the cost of changing the utilization rate of capital).

¹Hereafter lowercase letters denote real variables while uppercase letters denote nominal variables.

²Due to the fact that old and new capital are perfect substitutes, there will be a single price for capital $Q_t = \tilde{Q}_t$.

1.2 Entrepreneurs and financial arrangements

1.2.1 Production of capital services

There is a large number of heterogenous entrepreneurs j who buy new capital stock $k_{j,t+1}$ at price Q_t from the capital producers and transform it into capital services $\tilde{x}_{j,t+1}$ according to the linear technology

$$\tilde{x}_{j,t+1} = \omega_{j,t+1} k_{j,t+1}. \quad (6)$$

Capital services are then provided to the firms at the real rental rate z_t . ω denotes the productivity of the transformation technology. Productivity is entrepreneur specific. ω is a random variable drawn from the distribution $F(\omega^*) = \mathbb{P}(\omega \leq \omega^*)$ with $\mathbb{E}\omega = 1$. We will allow for exogenous variation in dispersion of this distribution (holding its mean constant) as a way of capturing changes in the prospects of perspective bank borrowers. A higher dispersion will typically imply a higher expected frequency of bankruptcy. Each entrepreneur draws its type ω after capital $k_{j,t+1}$ has been purchased. The entrepreneur purchases capital by using his net worth $N_{j,t+1}$ and by borrowing from a financial intermediary an amount $B_{j,t+1}$ at the gross rate of interest $R_{j,t+1}^b$. At the end of the period, he resells his capital to the capital producers at the price \tilde{Q}_t . Moreover, by that time, the entrepreneur has either succeeded in his projects in which case he repays his loan to the bank, $R_{j,t+1}^b B_{j,t+1}$; or he has failed. In the later case, the financial intermediary must pay a proportional cost μ to monitor the entrepreneur and confiscate whatever revenues have been generated.

Finally, the entrepreneur has a constant probability γ of surviving into the next period. If not, he shuts his business down. Therefore, $1 - \gamma$ entrepreneurs disappear in each and every period. In order to maintain constant the population of entrepreneur, we assume that $1 - \gamma$ new entrepreneurs are born in the same time. These new entrepreneurs finance their purchases with a transfer, τ_t^e , that they receive from the government.³ As in CMR we allow γ to vary exogenously over time. A positive realization of a shock to γ can be interpreted

³This assumption is made in order to guarantee that all entrepreneurs receive non-zero loans. The debt contract discussed in the next section implies that net worth serves as a collateral. Since the newly born have no pre-existing net worth, the absence of transfer would imply that these entrepreneurs would not receive any loan, and would therefore not be able to purchase capital. The population of entrepreneur would then converge to zero due to the mortality rate $1 - \gamma$. Moreover, the remaining profit, after consumption (propensity χ), of the exiting entrepreneurs is held by the government to finance the transfers τ_t^e

as an exogenous increase in aggregate worth or equivalently as an increase in asset values.

We now discuss the behavior of the entrepreneur and of the financial intermediary and characterize their financial arrangement.

1.2.2 The behavior of entrepreneurs

During period t entrepreneur j accumulates nominal net worth $N_{j,t+1}$ which he can use to purchase capital $k_{j,t+1}$ at price Q_t .⁴ The capital stock undergoes a transformation according to the linear technology (6), and then leased out to the intermediate goods producers at the real price z_t . At the end of the period, the transformed capital stock is sold back to the capital producers at the price \tilde{Q}_{t+1} . Hence, expected revenues are

$$\mathbb{E}_t \left[P_{t+1} z_{t+1} \omega_{j,t+1} k_{j,t+1} + \tilde{Q}_{t+1} \omega_{j,t+1} k_{j,t+1} \right] \quad (7)$$

or, denoting the rate of return on capital as

$$R_{t+1}^k \equiv \frac{P_{t+1} z_{t+1} + \tilde{Q}_{t+1}}{Q_t}, \quad (8)$$

expected revenues can be rewritten as

$$\mathbb{E}_t \left[Q_t R_{t+1}^k \omega_{j,t+1} k_{j,t+1} \right]. \quad (9)$$

Capital purchases are financed either through net worth $N_{j,t+1}$ or through a bank loan in the amount of $B_{j,t+1}$. This implies the following budget constraint

$$Q_t k_{j,t+1} = N_{j,t+1} + B_{j,t+1}. \quad (10)$$

The debt contract then specifies the loan amount $B_{j,t+1}$ and the nominal gross rate $R_{j,t+1}^b$.⁵ Entrepreneurs who can fully repay their loans –those with a good enough productivity type– pay back $R_{j,t+1}^b B_{j,t+1}$ at the end of the period. Those who fail because they drew a bad

⁴It should be clear that the only relevant state variable for an entrepreneur j is his net worth; that is, $k_{j,t+1}$ stands for $k(N_{j,t+1})$.

⁵The amount of the loan and the interest rate are contingent on the level of net worth of the entrepreneur, so that $B_{j,t+1} = B(N_{j,t+1})$ and $R_{j,t+1}^b = R^b(N_{j,t+1})$.

productivity type must deliver whatever revenues they have earned to the financial intermediary. By assumption there exists a cutoff value $\omega_{j,t+1}^*$ (contingent on the level of net worth) defined as

$$R_{j,t+1}^b B_{j,t+1} = Q_t R_{t+1}^k \omega_{j,t+1}^* k_{j,t+1} \quad (11)$$

below which the entrepreneur fails to fully repay.

The expected profit of an entrepreneur is given by

$$\mathbb{E}_t \left[\int_{\omega_{j,t+1}^*}^{\infty} (Q_t R_{t+1}^k \omega_{j,t+1} k_{j,t+1} - R_{j,t+1}^b B_{j,t+1}) dF(\omega) \right] \quad (12)$$

or, using the definition of the cutoff value $\omega_{j,t+1}^*$, and the fact that $k_{j,t+1}$ is decided in period t ,

$$\mathbb{E}_t \left[\int_{\omega_{j,t+1}^*}^{\infty} (\omega_{j,t+1} - \omega_{j,t+1}^*) R_{t+1}^k dF(\omega) \right] Q_t k_{j,t+1}. \quad (13)$$

Equation (13) defines the objective function of an entrepreneur to be maximized.

1.2.3 The behavior of the financial intermediary

Financial intermediaries take deposits from the domestic agents and make loans to the domestic entrepreneurs. In case of a successful entrepreneur ($\omega_{j,t+1} > \omega_{j,t+1}^*$), the bank receives $R_{j,t+1}^b B_{j,t+1}$. Otherwise, it receives $Q_t R_{t+1}^k \omega_{j,t+1} k_{j,t+1}$. But, in the latter case, the bank must first monitor the entrepreneur at the cost –per unit– of $\mu > 0$. Hence, financial intermediary revenues are

$$\int_{\omega_{j,t+1}^*}^{\infty} R_{j,t+1}^b B_{j,t+1} dF(\omega) + (1 - \mu) \int_0^{\omega_{j,t+1}^*} Q_t R_{t+1}^k \omega_{j,t+1} k_{j,t+1} dF(\omega) \quad (14)$$

or

$$(1 - F(\omega_{j,t+1}^*)) R_{j,t+1}^b B_{j,t+1} + (1 - \mu) \int_0^{\omega_{j,t+1}^*} Q_t R_{t+1}^k \omega_{j,t+1} k_{j,t+1} dF(\omega). \quad (15)$$

The financial intermediary draws its funds from depositors at the risk free rate of R_t (which is not contingent on the values of the shocks realized in $t + 1$). The zero profit condition is

$$(1 - F(\omega_{j,t+1}^*)) R_{j,t+1}^b B_{j,t+1} + (1 - \mu) \int_0^{\omega_{j,t+1}^*} Q_t R_{t+1}^k \omega_{j,t+1} k_{j,t+1} dF(\omega) = R_t B_{j,t+1} \quad (16)$$

which also corresponds to the participation constraint of the financial intermediary.

1.2.4 The contract

The debt contract specifies a level of loans, $B_{j,t+1}$ and a gross interest rate $R_{j,t+1}^b$ that maximizes the expected profit of the entrepreneur, equation (13), subject to the participation constraint of the financial intermediary, equation (16). The problem can be equivalently restated as choosing the capital stock and the cutoff point ω^* .

The solution can be recovered from the following equations

$$\mathbb{E}_t \left[[1 - \Gamma(\omega_{t+1}^*)] R_{t+1}^k + \frac{[\Gamma(\omega_{t+1}^*) - \mu G(\omega_{t+1}^*)] R_{t+1}^k - R_t^b}{1 - \mu \omega_{t+1}^* h(\omega_{t+1}^*)} \right] = 0, \quad (17)$$

$$[\Gamma(\omega_{t+1}^*) - \mu G(\omega_{t+1}^*)] Q_t R_{t+1}^k k_{j,t+1} - R_t^b (Q_t k_{j,t+1} - N_{j,t+1}) = 0, \quad (18)$$

where Γ , G , and h are various probability and cumulative distribution functions of the cutoff point ω^* .⁶

1.2.5 Caveats

While the model contains the most popular specification of financial frictions it abstracts from many aspects of the financial system that may be important for business cycles and may have played a prominent role in the current global recession.

For instance, banks are totally safe in this model but may not be so in the real world. There is no interbank market and there is no distinction between the various forms of bank borrowing (deposits, bonds, etc.). There is no role for government bonds, no distinction between classes of risk for various assets. Moreover, the banking sector is perfectly competitive. An important challenge for future research is to identify the features that are important for understanding the theoretical and quantitative properties of the financial market-macroeconomic performance nexus.

⁶See the separate technical appendix for the details of the derivation; available on request.

2 Calibration

The calibration of the model is based on Cuche-Curti, Dellas, and Natal, 2009. For the additional parameters associated with the financial markets/frictions we rely on Collard and Dellas, 2003. Table 1 gives the latter set of parameters.

<Table 1 here>

The properties of the financial contract are determined by the distribution of the productivity parameter ω . We assume that ω is log-normally distributed, such that $\log \omega \sim \mathcal{N}(m_\omega, \sigma_\omega)$. We assume that $\mathbb{E}\omega = 1$. Standard results for log-normal distributions imply that $\mathbb{E}\omega = \exp\left(m_\omega + \frac{\sigma_\omega^2}{2}\right)$ such that $m_\omega = -\sigma_\omega^2/2$. σ_ω^2 is then computed such that $F(\bar{\omega}) = 0.0076$, a 3% annualized business failure rate (the approximate rate in the data), leading to a value of $\sigma_\omega = 0.1149$. This implies a standard deviation for ω of 0.1153 (square root of $\exp(2m_\omega + \sigma_\omega^2)(\exp(\sigma_\omega^2) - 1)$). μ is set such that the model generates a net worth to capital ratio of approximately 2 given that the risk spread, $R^k - R^b$, in annualized values, equals 200 basis point. This yields a value for μ of about 0.11. The firm mortality rate is borrowed from BGG and is set at $\gamma = 0.0272$, implying a survival rate of 0.9728.

Table 1 also reports the persistence parameters of the stochastic processes and the standard deviations of the main shocks. These values come from DSGE-CH and from BGG.

3 The results

3.1 Financial shocks

The model is log-linearized around the deterministic steady state and then solved using Dynare. We first study the effects of financial shocks.

<Figure 2 here>

Figure 2 gives the response of the key macroeconomic variables to an increase in the firms' survival rate, $1 - \gamma$. One can think of this shock as representing a positive shock to asset

values. Because exiting entrepreneurs have, on average, more net worth than entering ones, this translates into an increase in the value of entrepreneurial assets. As can be seen the result is a broad and persistent expansion in economic activity: Output, investment, employment exports all go up, and so do the nominal interest rate and inflation. The only exception to this general expansion is to be found in consumption which experiences initially a decline before it starts to grow. This type of response is standard. Typically, on impact, consumption and investment move in opposite directions in response to shocks that improve investment prospects (the same result is also obtained by CMR).

The second shock we consider is a mean preserving spread in the distribution of the productivity of the entrepreneurs (an increase in σ_ω holding $\mathbb{E}\omega = 1$ constant). The response of the economy is depicted in **Figure 3**.

<Figure 3 here>

The mean preserving spread in the distribution of productivity redistributes mass towards the tails of the distribution and also makes it more skewed. But in general, one can not know how it redistributes mass across the particular cutoff point, ω^* . In the specification we use, the spread leads to a reduction in ω^* , and has a pronounced negative effect on bank lending, net worth, investment and inflation. The effects on employment and output are more intricate. While the *impact* effect is positive (as it is on consumption), the economy shortly thereafter slips into a recession.

The intuition for this is as follows. The reduction in investment implies that the future productivity of labor will be lower, that is, the future wage will be low relative to the present one. The intertemporal substitution effect thus works in making employment expand in the present. This effect is also reinforced by the negative income effect on leisure, so employment (and output) increase on impact. But as time goes on, this intertemporal substitution effect disappears, and the static substitution effect makes employment decline.

3.2 Presence of financial frictions

We now turn to the question of how the presence of financial frictions affects the response of the economy to non financial shocks. **Figures 4-6** show the IRF to a productivity,

foreign output, and monetary policy shock, respectively, for both the model with financial accelerator and a version of DSGE-CH.⁷

<Figures 4-6 here>

The main pattern in these graphs –which echoes similar findings in CMR– is that financial frictions are of little consequence for studying the macroeconomic implications of non-financial shocks.

4 Conclusions

In this paper we have added credit market imperfections to the small, open economy DSGE model of Switzerland in order to study two questions: First, how financial shocks impact on macroeconomic performance. And second, how financial frictions influence the response of the Swiss economy to non-financial disturbances.

We have seen that financial shocks that reduce net worth and discourage investment prove recessionary. And the more immediate the reduction in net worth, the more immediate the negative response of the economy.

Do DSGE models need to incorporate financial frictions in order to produce an accurate picture of the macroeconomic implications of non-financial shocks? Echoing similar findings in the literature, for instance CMR, we find that commonly used imperfections of the financial system do not materially influence how *other* shocks impact on the economy. This finding seems encouraging as the explicit modeling of the financial markets may add realism but at the same time it adds another considerable layer of complexity to the model. To the extent that substantial financial disturbances are infrequent, the standard DSGE model would suffice.

The present analysis, while useful in studying particular episodes of turbulence in financial markets, has remained silent on the issue of the relative importance of financial and non-financial disturbances. That is, on how much of recent business cycles can be attributed to

⁷This version has been calibrated to make the comparison exercise meaningful –DSGE-CH having no entrepreneurs– which implies that it slightly differs from the version used for forecasting at the SNB.

the former vs. the latter. We plan to address this issue in future work, in the context of an estimated version of this model.

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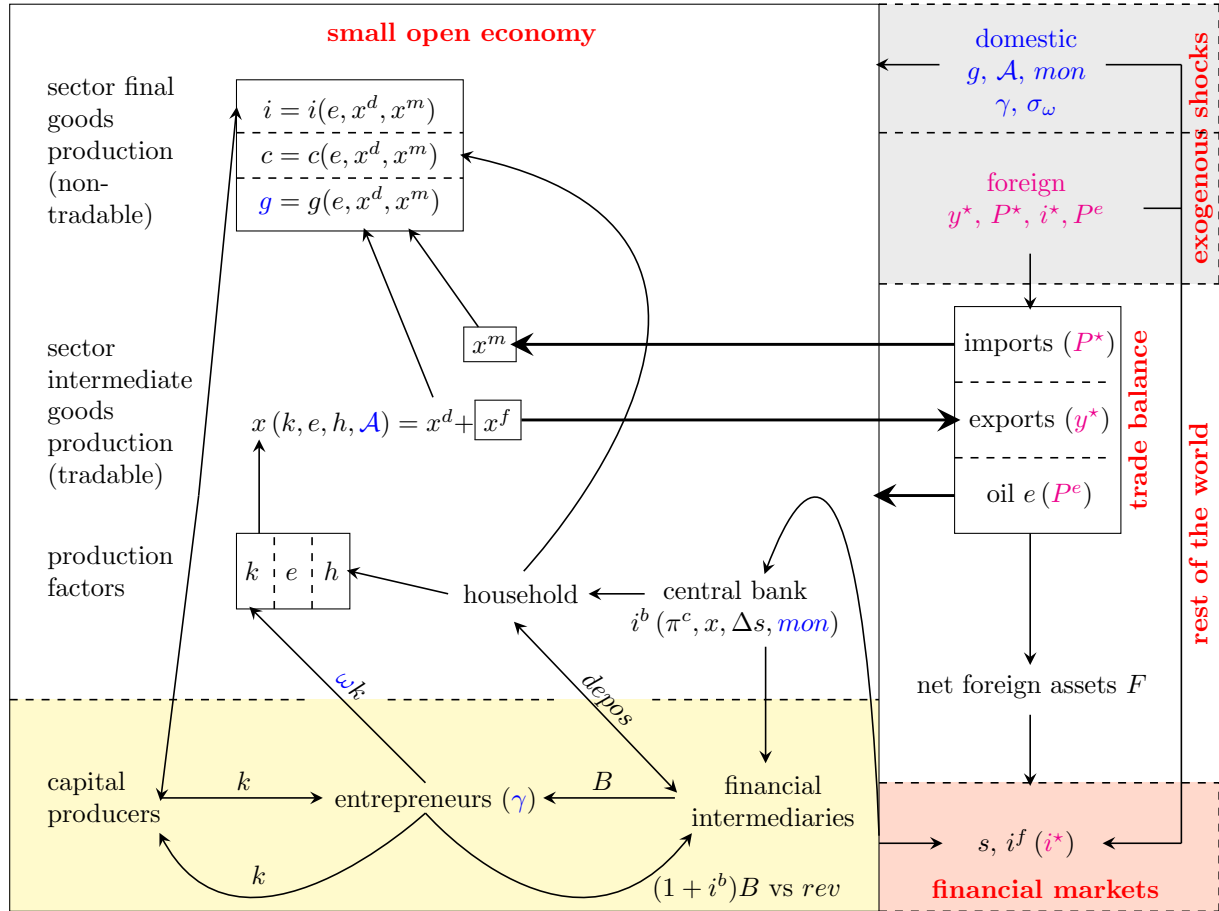
Table

Table 1 – Calibration

Financial parameters		
Mean of ω	$\mathbb{E}\omega$	1.0000
	m_ω	-0.0066
Standard deviation of ω		0.1153
	σ_ω	0.1149
Monitoring cost	μ	0.1100
Failure rate	$F(\bar{\omega})$	0.0076
Mortality rate	γ	0.0272
Survival rate	$1 - \gamma$	0.9728
Propensity to consume	χ	0.0100
Shocks		
Persistence of productivity shock	ρ_A	0.9000
Standard deviation	σ_A	0.0530
Persistence of foreign demand shock	ρ_{y^*}	0.9000
Standard deviation	σ_{y^*}	0.0082
Persistence of monetary policy shock	ρ_{mon}	0.0000
Standard deviation	σ_{mon}	0.0014
Persistence of survival shock	$\rho_{1-\gamma}$	0.7000
Standard deviation	$\sigma_{1-\gamma}$	1.0000
Persistence of asset shock	ρ_ω	0.7000
Standard deviation	σ_ω	1.0000

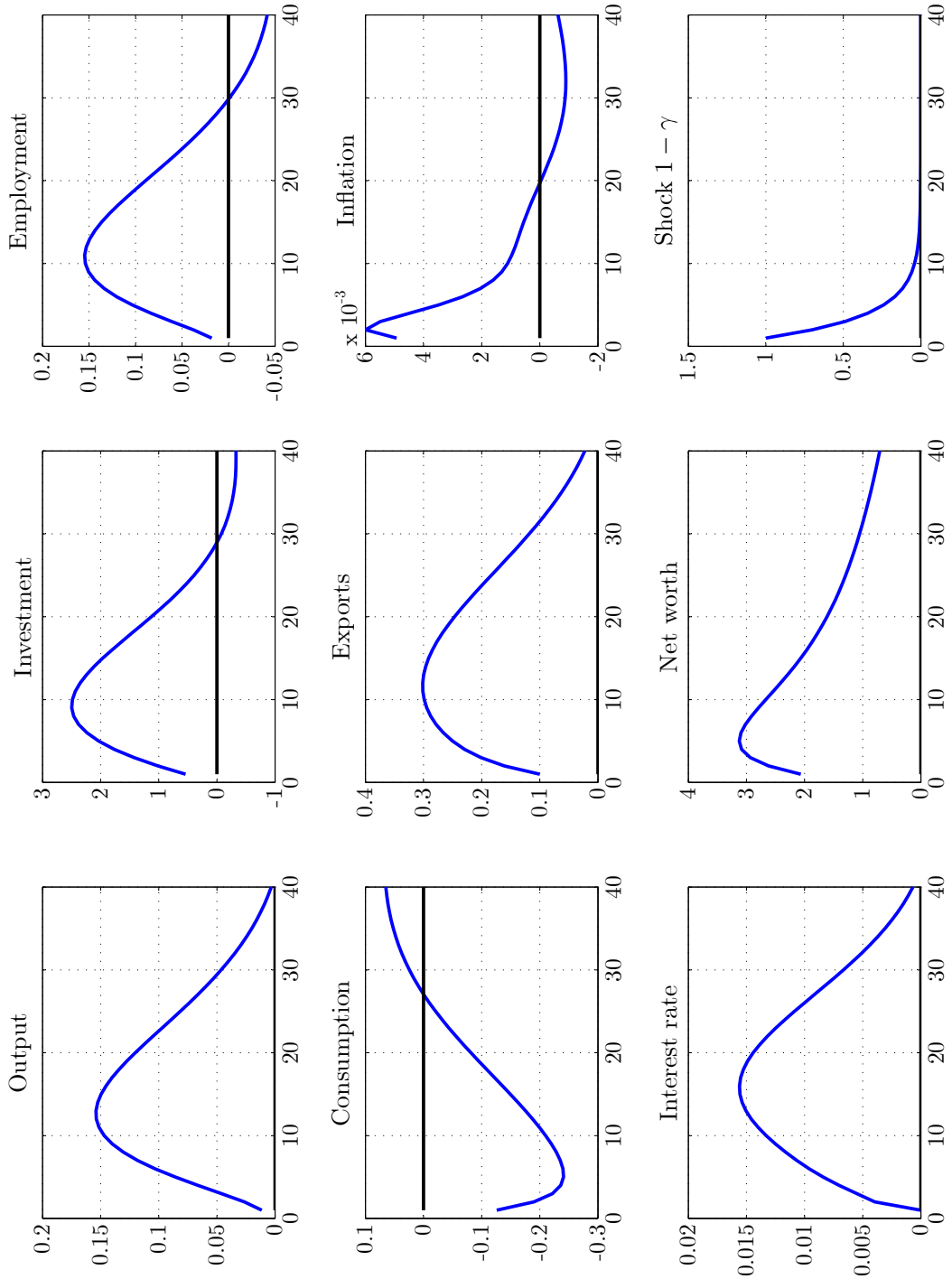
Figures

Figure 1 – Main agents and mechanisms in DSGE-CH with financial accelerator



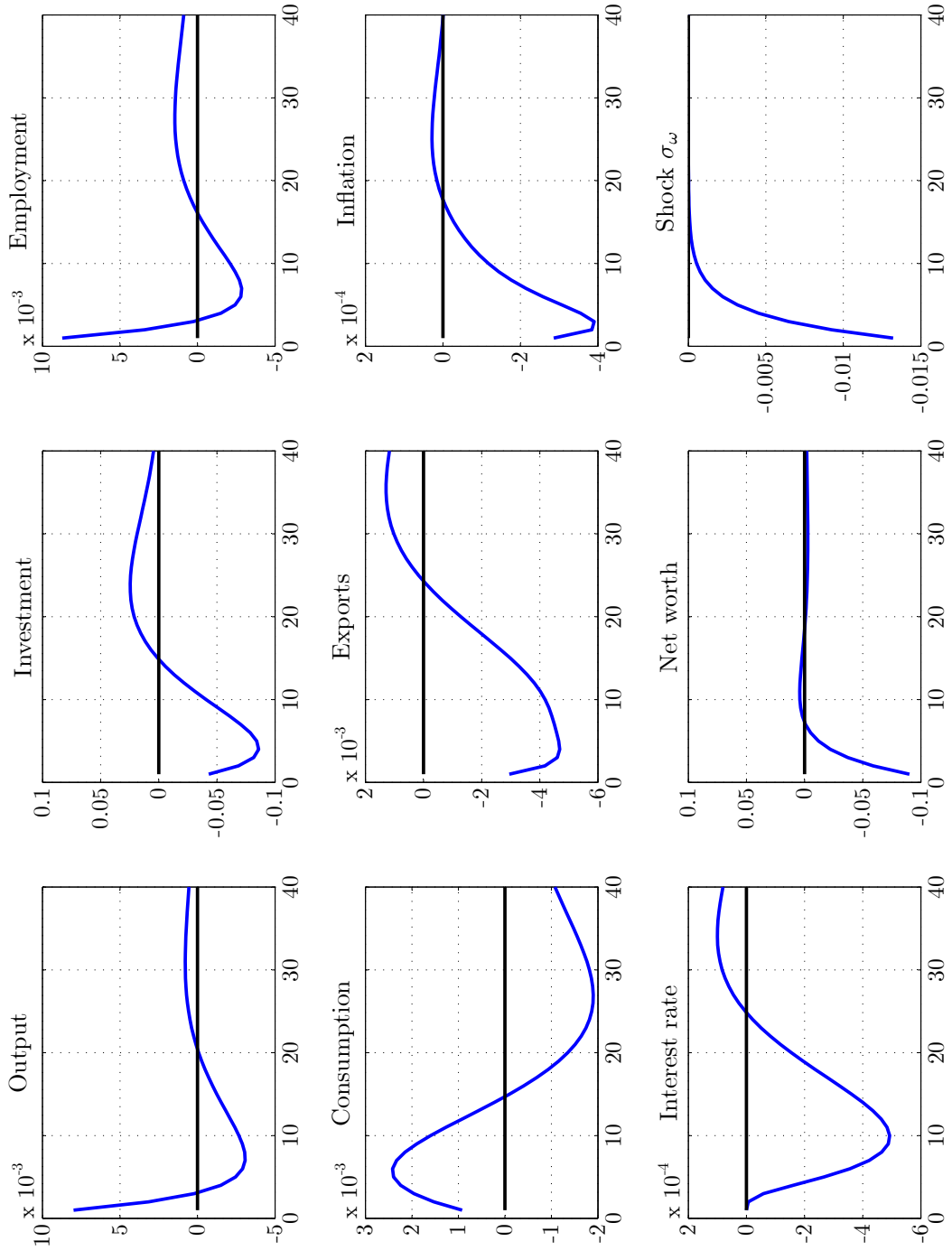
Note: i = investment, c = consumption, g = public expenditures, e = oil, x = intermediate goods, x^d = intermediate goods used domestically, x^f = exported intermediate goods, x^m = imported intermediate goods, k = capital, h = labor, ω = productivity in capital services production, B = loans, rev = revenues, i^b = domestic interest rate, π^c = consumer price inflation, Δs = appreciation of foreign currency, mon = monetary policy shock, A = productivity, γ = survival rate of entrepreneurs, σ_ω = variance of productivity in capital services production, y^* = foreign output, P^* = foreign inflation, i^* = foreign interest rate, P^e = oil price, F = net foreign assets, i^f = world interest rate, s = exchange rate (Swiss francs for a unit of foreign currency), $depos$ = deposits.

Figure 2 – IRF to a positive asset value shock



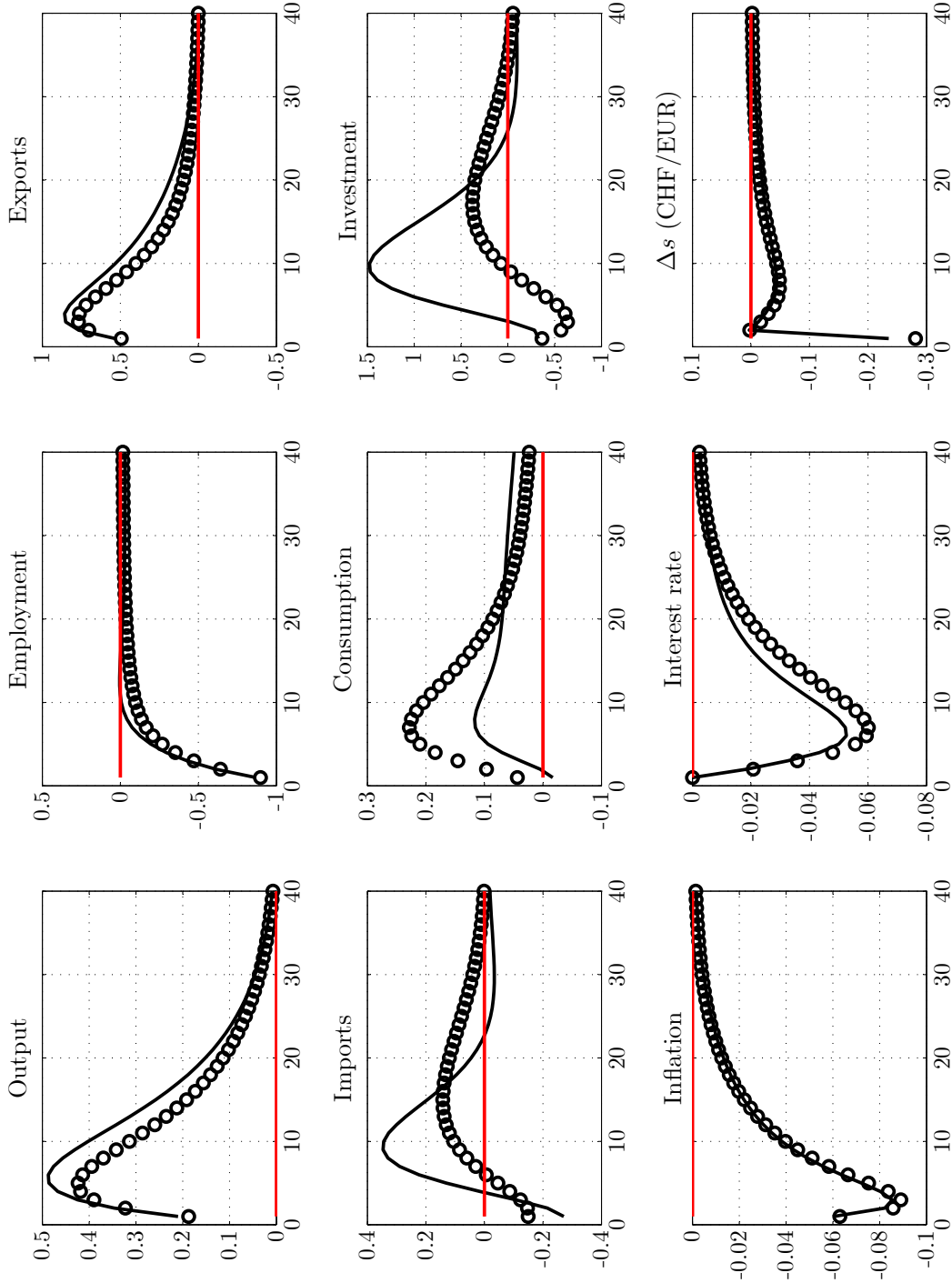
Note: The shock happens to the survival rate $1 - \gamma$ and is interpreted as a positive shock to net worth or, equivalently, to the value of entrepreneurial assets.

Figure 3 – IRF to a mean preserving spread in the productivity of entrepreneurs



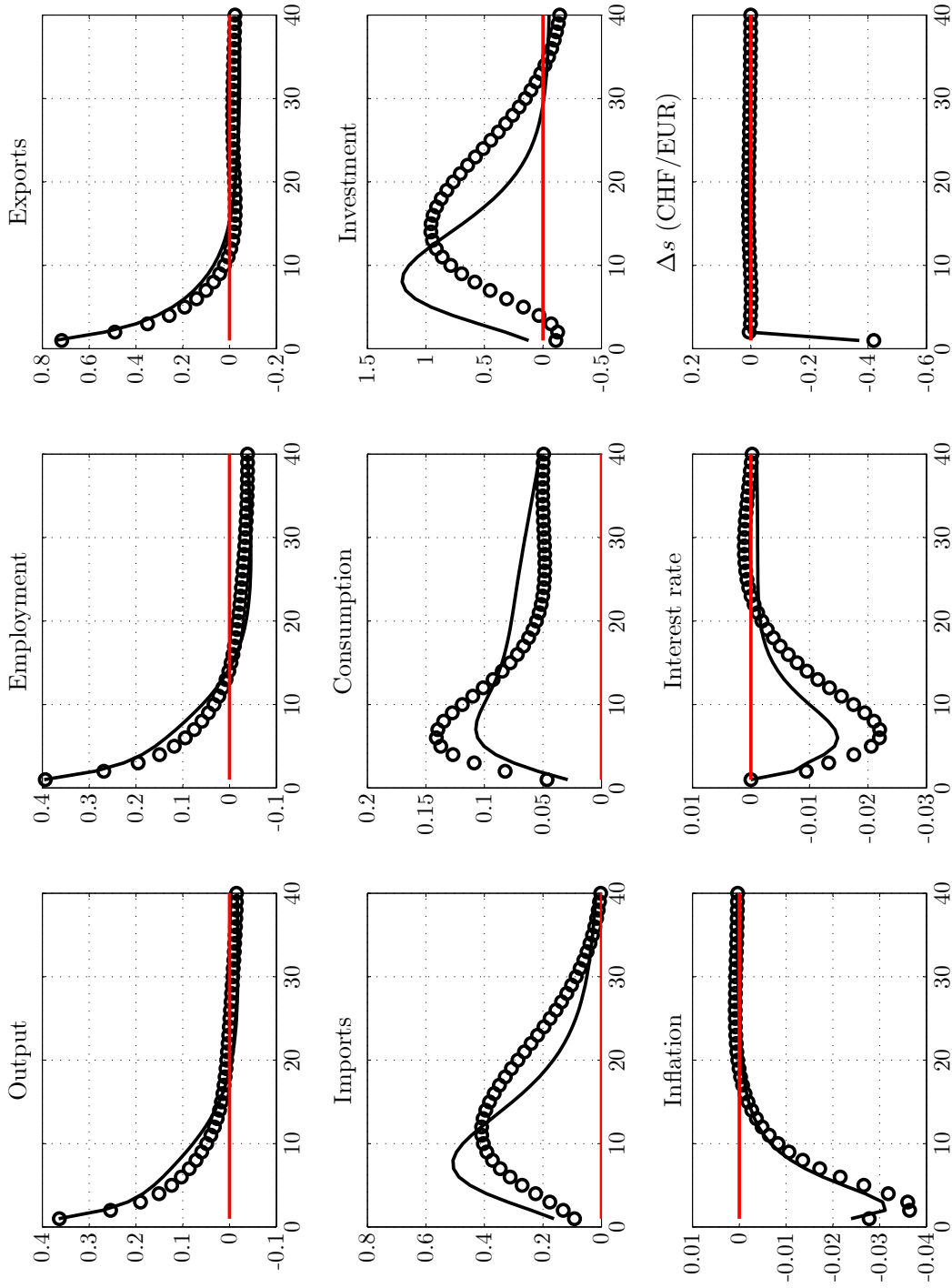
Note: The positive shock is to the variance of the productivity of entrepreneurs σ_ω and is interpreted as an increase in the riskiness of bank lending or, equivalently, as a worsening of the ability to repay loans.

Figure 4 – IRF to domestic productivity shock



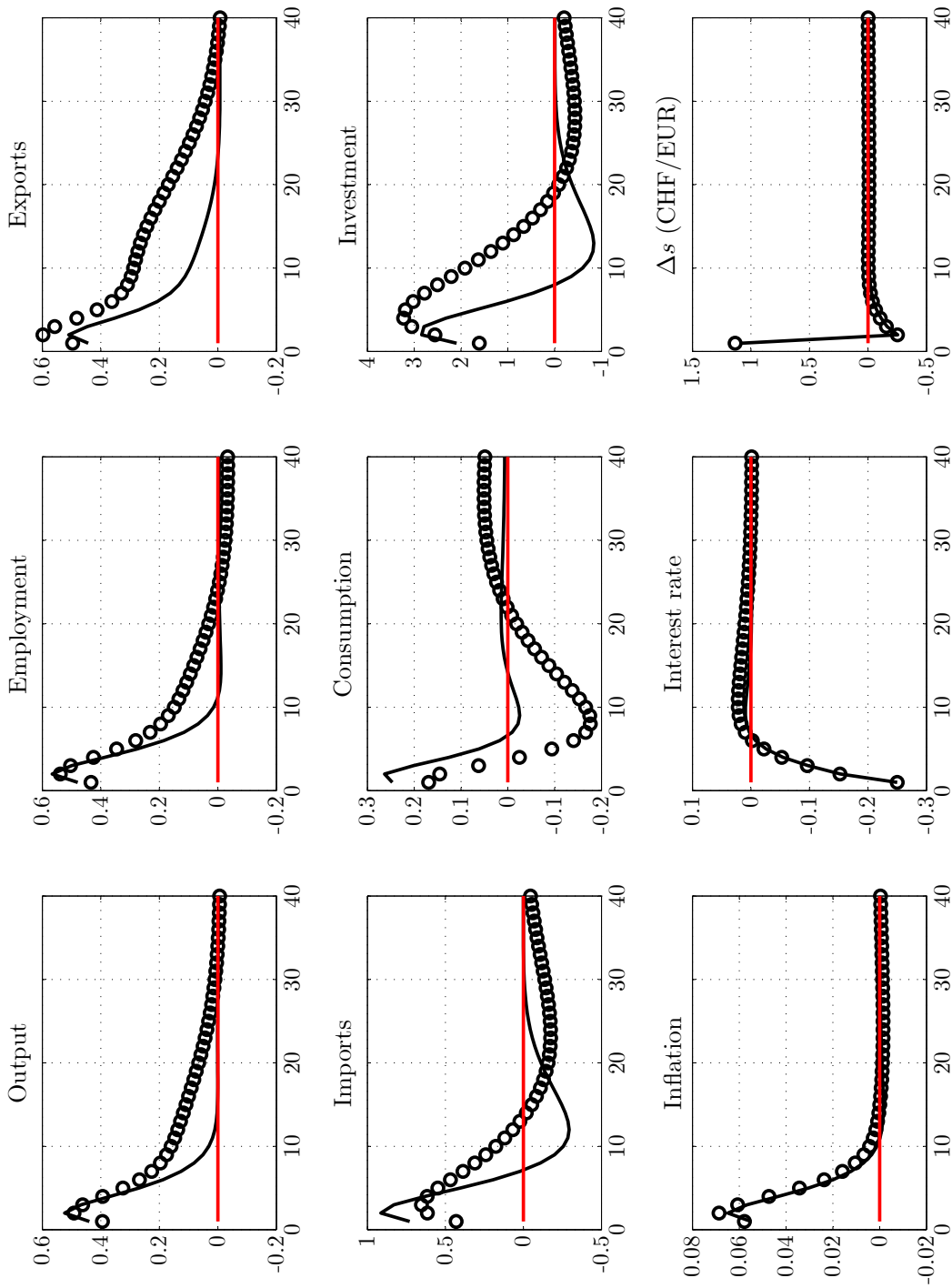
Note: The positive shock happens to the TFP in the intermediate goods sector (\mathcal{A}).

Figure 5 IRF to a foreign output shock



Note: The positive shock happens to the foreign demand (y^*).

Figure 6 – IRF to a monetary policy shock



Note: The positive monetary policy shock happens to the domestic interest rate (*mon*).

Appendix

Final goods production, consumption

Eq.1:	$x_t^{d,c} = \omega_c \mathbf{g}(x_t^c) \left(\omega_{e,c} c_t \right)^{\frac{1-\rho_{e,c}}{1-\rho_c}} \left(p_t^{d,c} \right)^{\frac{1}{\rho_c-1}}$
Eq.2:	$x_t^{m,c} = (1 - \omega_c) \mathbf{g}(x_t^c) \left(\omega_{e,c} c_t \right)^{\frac{\rho_{e,c}-\rho_c}{1-\rho_c}} \left(p_t^{m,c} \right)^{\frac{1}{\rho_c-1}}$
Eq.3:	$\mathbf{g}(x_t^c) = \omega_{e,c} c_t \left[\omega_c \left(p_t^{d,c} \right)^{\frac{\rho_c}{\rho_c-1}} + (1 - \omega_c) \left(p_t^{m,c} \right)^{\frac{\rho_c}{\rho_c-1}} \right]^{\frac{1}{\rho_c} \frac{1-\rho_{e,c}}{1-\rho_c}}$

Final goods production, investment

Eq.4:	$x_t^{d,i} = \omega_i \mathbf{g}(x_t^i) \left(\omega_{e,i} i_t \right)^{\frac{\rho_{e,i}-\rho_i}{1-\rho_i}} \left(\omega_{e,i} i_t \right)^{\frac{1-\rho_{e,i}}{1-\rho_i}} \left(\frac{P_t^i}{P_t^c} \right)^{\frac{1}{1-\rho_i}} \left(p_t^{d,c} \right)^{\frac{1}{\rho_i-1}}$
Eq.5:	$x_t^{m,i} = (1 - \omega_i) \mathbf{g}(x_t^i) \left(\omega_{e,i} i_t \right)^{\frac{\rho_{e,i}-\rho_i}{1-\rho_i}} \left(\omega_{e,i} i_t \right)^{\frac{1-\rho_{e,i}}{1-\rho_i}} \left(\frac{P_t^i}{P_t^c} \right)^{\frac{1}{1-\rho_i}} \left(p_t^{m,c} \right)^{\frac{1}{\rho_i-1}}$
Eq.6:	$\mathbf{g}(x_t^i) = \omega_{e,i} i_t \left(\frac{P_t^i}{P_t^c} \right)^{\frac{1}{1-\rho_{e,i}}} \left[\omega_i \left(p_t^{d,c} \right)^{\frac{\rho_i}{\rho_i-1}} + (1 - \omega_i) \left(p_t^{m,c} \right)^{\frac{\rho_i}{\rho_i-1}} \right]^{\frac{1}{\rho_i} \frac{1-\rho_{e,i}}{1-\rho_i}}$
Eq.7:	$\frac{P_t^i}{P_t^c} = \left\{ \omega_{e,i} \left[\omega_i \left(p_t^{d,c} \right)^{\frac{\rho_i}{\rho_i-1}} + (1 - \omega_i) \left(p_t^{m,c} \right)^{\frac{\rho_i}{\rho_i-1}} \right]^{\frac{\rho_{e,i}(1-\rho_i)}{1-\rho_{e,i}}} \left(1 - \omega_{e,i} \right)^{\rho_i} + (1 - \omega_{e,i}) \left(p_t^{e,c} \right)^{\frac{\rho_{e,i}}{\rho_{e,i}-1}} \right\}^{\frac{\rho_{e,i}-1}{\rho_{e,i}}}$

Final goods production, government expenditures

Eq.8:	$x_t^{d,g} = \omega_g \mathbf{g} (x_t^g) \frac{\rho_{e,g} - \rho_g}{1 - \rho_g} (\omega_{e,g} g_t) \frac{P_t^g}{P_t^c} \frac{1 - \rho_{e,g}}{1 - \rho_g} \left(p_t^{d,c} \right)^{\frac{1}{\rho_g - 1}}$
Eq.9:	$x_t^{m,g} = (1 - \omega_g) \mathbf{g} (x_t^g) \frac{\rho_{e,g} - \rho_g}{1 - \rho_g} (\omega_{e,g} g_t) \frac{1 - \rho_{e,g}}{1 - \rho_g} \left(\frac{P_t^g}{P_t^c} \right)^{\frac{1}{1 - \rho_g}} (p_t^{m,c})^{\frac{1}{\rho_g - 1}}$
Eq.10:	$\mathbf{g} (x_t^g) = \omega_{e,g} g_t \left(\frac{P_t^g}{P_t^c} \right)^{\frac{1}{1 - \rho_{e,g}}} \left[\omega_g \left(p_t^{d,c} \right)^{\frac{\rho_g}{\rho_g - 1}} + (1 - \omega_g) \left(p_t^{m,c} \right)^{\frac{\rho_g}{\rho_g - 1}} \right]^{\frac{1}{\rho_g} \frac{1 - \rho_g}{1 - \rho_{e,g}}}$
Eq.11:	$\frac{P_t^g}{P_t^c} = \left\{ \omega_{e,g} \left[\omega_g \left(p_t^{d,c} \right)^{\frac{\rho_g}{\rho_g - 1}} + (1 - \omega_g) \left(p_t^{m,c} \right)^{\frac{\rho_g}{\rho_g - 1}} \right]^{\frac{\rho_{e,g}(1 - \rho_g)}{(1 - \rho_{e,g})\rho_g}} + (1 - \omega_{e,g}) \left(p_t^{e,c} \right)^{\frac{\rho_{e,g}}{\rho_{e,g} - 1}} \right\}^{\frac{\rho_{e,g} - 1}{\rho_{e,g}}}$
Eq.12:	$x_t^{d,o} = \omega_o \mathbf{g} (x_t^o) \frac{\rho_{e,o} - \rho_o}{1 - \rho_o} (\omega_{e,o} O_t) \frac{1 - \rho_{e,o}}{1 - \rho_o} \left(\frac{P_t^o}{P_t^c} \right)^{\frac{1}{1 - \rho_o}} \left(p_t^{d,c} \right)^{\frac{1}{\rho_o - 1}}$

Final goods production, other expenditures

Eq.13:	$x_t^{m,o} = (1 - \omega_o) \mathbf{g} (x_t^o) \frac{\rho_{e,o} - \rho_o}{1 - \rho_o} (\omega_{e,o} O_t) \frac{1 - \rho_{e,o}}{1 - \rho_o} \left(\frac{P_t^o}{P_t^c} \right)^{\frac{1}{1 - \rho_o}} (p_t^{m,c})^{\frac{1}{\rho_o - 1}}$
Eq.14:	$\mathbf{g} (x_t^o) = \omega_{e,o} O_t \left(\frac{P_t^o}{P_t^c} \right)^{\frac{1}{1 - \rho_{e,o}}} \left[\omega_o \left(p_t^{d,c} \right)^{\frac{\rho_o}{\rho_o - 1}} + (1 - \omega_o) \left(p_t^{m,c} \right)^{\frac{\rho_o}{\rho_o - 1}} \right]^{\frac{1}{\rho_o} \frac{1 - \rho_o}{1 - \rho_{e,o}}}$
Eq.15:	$\frac{P_t^o}{P_t^c} = \left\{ \omega_{e,o} \left[\omega_o \left(p_t^{d,c} \right)^{\frac{\rho_o}{\rho_o - 1}} + (1 - \omega_o) \left(p_t^{m,c} \right)^{\frac{\rho_o}{\rho_o - 1}} \right]^{\frac{\rho_{e,o}(1 - \rho_o)}{(1 - \rho_{e,o})\rho_o}} + (1 - \omega_{e,o}) \left(p_t^{e,c} \right)^{\frac{\rho_{e,o}}{\rho_{e,o} - 1}} \right\}^{\frac{\rho_{e,o} - 1}{\rho_{e,o}}}$
Eq.16:	$O_t = k_{t-1} a (u_t) + \mu G (\omega_t^*) \frac{R_t^k Q_{t-1} k_{t-1}}{1 + \pi_t^c}$

Oil demand

Eq.17:	$e_t^c = (p_t^{e,c})^{\frac{1}{\rho_{e,c}-1}} (1 - \omega_{e,c}) c_t$
Eq.18:	$e_t^i = (p_t^{e,i})^{\frac{1}{\rho_{e,i}-1}} (1 - \omega_{e,i}) i_t \left(\frac{P_t^i}{P_t} \right)^{\frac{1}{1-\rho_{e,i}}}$
Eq.19:	$e_t^g = (p_t^{e,g})^{\frac{1}{\rho_{e,g}-1}} (1 - \omega_{e,g}) g_t \left(\frac{P_t^g}{P_t} \right)^{\frac{1}{1-\rho_{e,g}}}$
Eq.20:	$e_t^o = (p_t^{e,o})^{\frac{1}{\rho_{e,o}-1}} (1 - \omega_{e,o}) o_t \left(\frac{P_t^o}{P_t} \right)^{\frac{1}{1-\rho_{e,o}}}$

Intermediate goods production

Eq.21:	$x_t = \mathcal{A}_t \left\{ \alpha_c^{\frac{1}{\sigma_e}} \left(x_t^{kl} (h_t, \check{k}_t) \right)^{\frac{\sigma_e-1}{\sigma_e}} + (1 - \alpha_c)^{\frac{1}{\sigma_e}} (e_t^x)^{\frac{\sigma_e-1}{\sigma_e}} \right\}^{\frac{\sigma_e}{\sigma_e-1}}$
Eq.22:	$x_t^{kl} (h_t, \check{k}_t) = \left(\alpha_l^{\frac{1}{\sigma_{kl}}} h_t^{\frac{\sigma_{kl}-1}{\sigma_{kl}}} + (1 - \alpha_l)^{\frac{1}{\sigma_{kl}}} \check{k}_t^{\frac{\sigma_{kl}-1}{\sigma_{kl}}} \right)^{\frac{\sigma_{kl}}{\sigma_{kl}-1}}$
Eq.23:	$\check{k}_t = \left(\frac{\Psi_t}{z_t} \right)^{\sigma_{kl}} \mathcal{A}_t^{\sigma_{kl} \left(\frac{\sigma_e-1}{\sigma_e} \right)} x_t^{kl} (h_t, \check{k}_t)^{\frac{\sigma_e - \sigma_{kl}}{\sigma_e}} (1 - \alpha_l) (\alpha_c x_t)^{\frac{\sigma_{kl}}{\sigma_e}}$
Eq.24:	$h_t = \left(\frac{\Psi_t}{w_t} \right)^{\sigma_{kl}} \mathcal{A}_t^{\sigma_{kl} \left(\frac{\sigma_e-1}{\sigma_e} \right)} x_t^{kl} (h_t, \check{k}_t)^{\frac{\sigma_e - \sigma_{kl}}{\sigma_e}} \alpha_l (\alpha_c x_t)^{\frac{\sigma_{kl}}{\sigma_e}}$
Eq.25:	$e_t^x = \left(\frac{\Psi_t}{p_t^{e,x}} \right)^{\sigma_e} \mathcal{A}_t^{\sigma_e-1} (1 - \alpha_c) x_t$

Assets and yields

Eq.26: $\lambda_{1,t} = (c_t^{PI} - \varrho c_{t-1}^{PI})^{-\sigma} - \varrho \beta \mathbb{E}_t (c_{t+1}^{PI} - \varrho c_t^{PI})^{-\sigma}$

Eq.27: $\lambda_{1,t} = R_t^b \beta \mathbb{E}_t \left(\frac{\lambda_{1,t+1}}{1 + \pi_t^c} \right)$

Eq.28: $\lambda_{1,t} = R_t^f \beta \mathbb{E}_t \left(\Delta s_{t+1} \frac{\lambda_{1,t+1}}{1 + \pi_{t+1}^c} \right)$

Eq.29: $R_t^f = R_t^* + port_t - \iota \frac{\pi_t^f}{\Delta s_t p_t^*} b_t^f \quad (\text{for } b_t^f = s_t F_t / P_t^c)$

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Monetary policy

Eq.30: $\hat{i}_t^b = \rho_R \hat{i}_{t-1}^b + (1 - \rho_R) \left(k_n \hat{\pi}_t^c + k_x \hat{x}_t + k_s \widehat{\Delta s}_t \right) + mon$

Capital producers

Eq.31: $k_t = \Theta(i_t, i_{t-1}, k_{t-1}) + (1 - \delta) k_{t-1}$

Eq.32: $\frac{P_t^i}{P_t^c} = Q_t \Theta_{1,t} + \frac{\beta \lambda_{1,t+1}}{\lambda_{1,t}} (1 + \pi_{t+1}^c) Q_{t+1} \Theta_{2,t+1}$

Eq.33: $c_t^c = \chi (1 - \gamma_t) (1 - \Gamma(\omega_t^*)) \frac{R_t^k Q_{t-1} k_{t-1}}{1 + \pi_t^c}$

Entrepreneurs

Eq.34: $\frac{R_t^k Q_{t-1}}{1+\pi_t^e} = zu - a(u_t) + (\Theta_{3t} + 1 - \delta) Q_t$

Eq.35: $z_t = a'(u_t)$

Eq.36: $N_{t+1} = \gamma_t (1 - \Gamma(\omega_t^*)) \frac{R_t^k Q_{t-1} k_{t-1}}{1+\pi_t^e} + \tau_t^e$

Eq.37: $\mathbb{E}_t \left[(1 - \Gamma(\omega_{t+1}^*)) R_{t+1}^k \right] + \frac{[\Gamma(\omega_{t+1}^*) - \mu G(\omega_{t+1}^*)] R_{t+1}^k - R_t}{1 - \mu \omega_{t+1}^* h(\omega_{t+1}^*)} = 0$

Eq.38: $[\Gamma(\omega_t^*) - \mu G(\omega_t^*)] Q_{t-1} R_t^k k_{t-1} = R_{t-1} (Q_{t-1} k_{t-1} - N_{t-1})$

Price setting in the intermediate goods sector

$$\text{Eq.39: } \hat{\pi}_t^d = \hat{p}_t^d - \hat{p}_{t-1}^d + \hat{\pi}_t^c$$

$$\text{Eq.40: } p_t^* = \frac{s_{t-1}P_t^*}{P_{t-1}^*} \rightarrow \hat{P}_t^* = \hat{p}_{t-1}^* + \widehat{\Delta s_{t-1}} + \hat{\pi}_t^* - \hat{\pi}_{t-1}^c$$

$$\text{Eq.41: } rer_t = \frac{s_t P_t^*}{P_t^c} = \frac{\Delta s_t p_t^*}{\pi_t^c} \rightarrow \widehat{rer}_t = \widehat{\Delta s_t} + \hat{p}_t^* - \hat{\pi}_t^c$$

$$\text{Eq.42: } \hat{\pi}_t^m = \hat{p}_t^m - \hat{p}_{t-1}^m + \hat{\pi}_t^c$$

$$\text{Eq.43: } \hat{\pi}_t^e = \hat{p}_t^e - \hat{p}_{t-1}^e + \hat{\pi}_t^c$$

$$\text{Eq.44: } \hat{\pi}_t^c = \omega_{e,c} \left(\omega_c \hat{\pi}_t^{d,c} + (1 - \omega_c) \hat{\pi}_t^{m,c} \right) + (1 - \omega_{e,c}) \hat{\pi}_t^{e,c}$$

$$\text{Eq.45: } \hat{p}_t^d = \frac{\tau}{1+\tau^2\beta} \left(\beta \mathbb{E}_t \hat{p}_{t+1}^d + \hat{p}_{t-1}^d + \beta \mathbb{E}_t \hat{\pi}_{t+1}^c + \Gamma \hat{\pi}_{t-1}^c - (1 + \Gamma\beta) \hat{\pi}_t^c + \frac{(1-\tau)(1-\tau\beta)}{\tau} \left(\hat{\psi}_t - \text{Mup} \right) \right)$$

$$\text{Eq.46: } \hat{p}_t^m = \frac{\tau_m}{1+\tau_m^2\beta} \left(\beta \mathbb{E}_t \hat{p}_{t+1}^m + \hat{p}_{t-1}^m + \beta \mathbb{E}_t \hat{\pi}_{t+1}^c + \gamma_m \hat{\pi}_{t-1}^c - (1 + \gamma_m\beta) \hat{\pi}_t^c + \frac{(1-\tau_m)(1-\tau_m\beta)}{\tau_m} \left(\widehat{rer}_t - \text{Mup} \right) \right)$$

$$\text{Eq.47: } \hat{p}_t^f = \frac{\tau_f}{1+\tau_f^2\beta} \left(\beta \mathbb{E}_t \hat{p}_{t+1}^f + \hat{p}_{t-1}^f + \beta \mathbb{E}_t \hat{\pi}_{t+1}^c + \gamma_f \hat{\pi}_{t-1}^c - (1 + \gamma_f\beta) \hat{\pi}_t^c + \frac{(1-\tau_f)(1-\tau_f\beta)}{\tau_f} \left(\hat{\psi} - \widehat{rer}_t - \text{Mup} \right) \right)$$

Wage setting

Eq.48:
$$\hat{w}_t = \frac{1}{\Upsilon(1+\beta)}\hat{w}_{t-1} + \frac{\beta}{\Upsilon(1+\beta)}\mathbb{E}_t\hat{w}_{t+1} + \frac{1}{\Upsilon(1+\beta)}\left(\beta\left(\mathbb{E}_t\hat{\pi}_{t+1}^c - \gamma_w\hat{\pi}_t^c\right) - \left(\hat{\pi}_t^c - \gamma_w\hat{\pi}_{t-1}^c\right)\right) + \frac{(1-\tau_w)(1-\tau_w\beta)}{(1-\tau_w)(1-\tau_w\beta)+\tau_w(1+\beta)}\left(1-\nu\frac{h}{1-h}\frac{1}{\vartheta-1}\right)\widehat{MRS}_t$$

Equilibrium conditions

Eq.49:
$$x_t^d = x_t^{d,c} + x_t^{d,i} + x_t^{d,g} + x_t^{d,o}$$

Eq.50:
$$x_t^m = x_t^{m,c} + x_t^{m,i} + x_t^{m,g} + x_t^{m,o}$$

Eq.51:
$$e_t = \int_0^1 e_t^x(i) di + e_t^c + e_t^i + e_t^g + e_t^o$$

Eq.52:
$$x_t = x_t^d + x_t^f$$

Eq.53:
$$x_t^f = \left(p_t^f/P_t^*\right)^{\frac{1}{\sigma_f-1}}(1-\omega^*)y_t^*$$

Eq.54:
$$b_t^f = \frac{\Delta s_t b_t^f}{\pi_t^c} R_{t-1}^F + \frac{\Delta s_t p_t^f}{\pi_t^c} p_t^f x_t^f - \frac{\Delta s_t p_t^f}{\pi_t^c} x_t^m - p_t^f e_t$$

Exogenous shocks

$$\text{Eq.55: } \mathcal{A}_t = \rho_{A1}\mathcal{A}_{t-1} + \rho_{A2}\mathcal{A}_{t-2} + \rho_{A3}\mathcal{A}_{t-3} + \rho_{A4}\mathcal{A}_{t-4} + (1 - \rho_{A1} - \rho_{A2} - \rho_{A3} - \rho_{A4})\mathcal{A} + \varepsilon_t^A$$

$$\text{Eq.56: } g_t = \rho_{g1}g_{t-1} + \rho_{g2}g_{t-2} + (1 - \rho_{g1} - \rho_{g2})g + \varepsilon_t^g$$

$$\text{Eq.57: } R_t^* = \rho_{R^*1}R_{t-1}^* + \rho_{R^*2}R_{t-2}^* + (1 - \rho_{R^*1} - \rho_{R^*2})R^* + \varepsilon_t^{R^*}$$

$$\text{Eq.58: } \hat{\pi}_t^* = \rho_{\hat{\pi}^*}\hat{\pi}_{t-1}^* + (1 - \rho_{\hat{\pi}^*})\hat{\pi}^* + \varepsilon_t^{\hat{\pi}^*}$$

$$\text{Eq.59: } y_t^* = \rho_{y^*}y_{t-1}^* + (1 - \rho_{y^*})y^* + \varepsilon_t^{y^*}$$

$$\text{Eq.60: } p_t^e = \rho_{p^e}p_{t-1}^e + (1 - \rho_{p^e})p^e + \varepsilon_t^{p^e}$$

$$\text{Eq.61: } port_t = \rho_{port}port_{t-1} + (1 - \rho_{port})port + \varepsilon_t^{port}$$

$$\text{Eq.62: } \dots$$

$$\text{Eq.63: } \dots$$

$$\text{Eq.64: } \dots$$

$$\text{Eq.65: } mon_t = \rho_{mon}mon_{t-1} + (1 - \rho_{mon})mon + \varepsilon_t^{mon}$$

$$\text{Eq.66: } \gamma_t = \rho_\gamma\gamma_{t-1} + (1 - \rho_\gamma)\gamma + \varepsilon_t^\gamma$$

$$\text{Eq.67: } m_{\omega t} = \rho_{m_\omega}m_{\omega t-1} + (1 - \rho_{m_\omega})m_\omega + \varepsilon_t^{m_\omega}$$

Definitions

Eq.68:	$S(\mathfrak{J}_t) = g_3 \left(e^{g_1(\mathfrak{J}_t-1)} + \frac{g_1}{g_2} e^{-g_2(\mathfrak{J}_t-1)} - \left(1 + \frac{g_1}{g_2} \right) \right)$
Eq.69:	$S'(\mathfrak{J}_t) = g_1 g_3 (e^{g_1(\mathfrak{J}_t-1)} - e^{-g_2(\mathfrak{J}_t-1)})$
Eq.70:	$S_{12} = -\mathfrak{J}_t S(\mathfrak{J}_t)$
Eq.71:	$S_2 = \frac{\varphi}{2} \left(\frac{i_t}{k_{t-1}} - \delta \right)^2 k_{t-1}$
Eq.72:	$S_{21} = \varphi \left(\frac{i_t}{k_{t-1}} - \delta \right)$
Eq.73:	$S_{23} = -\varphi \left(\frac{i_t}{k_{t-1}} - \delta \right) \frac{i_t}{k_{t-1}}$
Eq.74:	$\Theta(i_t, i_{t-1}, k_{t-1}) = i_t (1 - \varpi S(\mathfrak{J}_t)) - (1 - \varpi) \frac{\varphi}{2} \left(\frac{i_t}{k_{t-1}} - \delta \right)^2 k_{t-1}$
Eq.75:	$\Theta_{1_t} = 1 - \varpi S(\mathfrak{J}_t) - \varpi \mathfrak{J}_t S'(\mathfrak{J}_t) - (1 - \varpi) \varphi \left(\frac{i_t}{k_{t-1}} - \delta \right)$
Eq.76:	$\Theta_{2_t} = \varpi \mathfrak{J}_t^2 S'(\mathfrak{J}_t)$
Eq.77:	$\Theta_{3_t} = (1 - \varpi) \frac{\varphi}{2} \left[\left(\frac{i_t}{k_{t-1}} \right)^2 - \delta^2 \right]$

Definitions (continued)

Eq.78: $a(u_t) = a_1 e^{a_2(u_t-1)} - a_1$

Eq.79: $a'(u_t) = a_1 a_2 e^{a_2(u_t-1)}$

Eq.80: $G(\omega_t^*) = \Phi\left(\frac{\ln \omega_t^* - m_\omega}{\sigma_\omega} - \sigma_\omega\right)$

Eq.81: $\Gamma(\omega_t^*) = \omega_t^* \left(1 - \Phi\left(\frac{\ln \omega_t^* - m_\omega}{\sigma_\omega}\right)\right) + G(\omega_t^*)$

Eq.82: $h(\omega_t^*) = \frac{F'(\omega_t^*)}{1-F(\omega_t^*)} = \frac{f(\omega_t^*)}{\Gamma(\omega_t^*) - G(\omega_t^*)} = \frac{\omega_t^* f(\omega_t^*)}{\Gamma(\omega_t^*) - G(\omega_t^*)} = \frac{1}{1-\Phi\left(\frac{\ln \omega_t^* - m_\omega}{\sigma_\omega}\right)} \frac{1}{\sigma_\omega \sqrt{2\pi}} e^{-\frac{(\ln \omega_t^* - m_\omega)^2}{2\sigma_\omega^2}}$

Eq.83: $m_t = \left(\lambda_{1,t} \frac{i_t^b}{1+i_t^b}\right)^{-\frac{1}{\eta}}$