Optimal Macroeconomic Policies in a Heterogeneous World*

James Bullard† **Riccardo DiCecio**‡ **Aarti Singh**§
Jacek Suda¶

This draft: July 19, 2023

Abstract

We study a DSGE model with "massive" heterogeneity, enough to approach Gini coefficients for income, wealth, and consumption in the U.S. data. The economy features three aggregate shocks as well as both permanent and temporary idiosyncratic risk. We introduce policymakers that can mitigate these risks for households, and a welfare theorem outlines how the proposed policies imply an optimal allocation of resources. The proposed policies include achieving the Wicksellian natural real rate of interest, social insurance, and a set of taxes and transfers designed to reduce consumption inequality. We calibrate the model to U.S. data from 1995-2023 assuming the optimal set of policies and argue that the model fit is promising. This suggests that, broadly speaking, U.S. macroeconomic policy has in recent decades been close to optimal. Improvements beyond this set of policies requires design of responses to very large shocks, and we suggest some directions in which such a design may proceed.

Keywords: Optimal monetary and fiscal policy, life cycle economies, heterogeneous households, credit market friction, nominal GDP targeting, non-state contingent nominal contracting, inequality, Gini coefficients, hand-to-mouth households.

JEL codes: E4, E5.

*The authors thank Manuel Amador, Javier Bianchi, Varadarajan Chari, Davide Debortoli, Ben Malin, Fabrizio Perri, Tom Tallerini, Abigail Wosniak, and seminar participants at the FRB-Minneapolis and the Norges Bank, IMF and *IMF Economic Review* "The Future of Macroeconomic Policy" conference for valuable input on an early version of this work. Any errors are the responsibility of the authors.

†Federal Reserve Bank of St. Louis. Any views expressed are those of the authors and do not necessarily reflect the views of others on the Federal Open Market Committee.

‡Federal Reserve Bank of St. Louis.

§University of Sydney.

¶SGH, Warsaw School of Economics and Narodowy Bank Polski (NBP). Any views expressed are those of the authors and do not necessarily reflect the views of the Narodowy Bank Polski.

1 Introduction

1.1 Motivation

Broadly speaking, macroeconomics has been moving toward greater granularity and more explicit levels of heterogeneity in recent decades. This has been due, in part, to a desire among macroeconomists to be able to articulate with greater accuracy the effect of various policies recommended at the macroeconomic level on specific household types at the microeconomic level. This might be labeled as the "Krusell-Smith" approach to macroeconomics, in which the vision is that the economy is buffeted by aggregate shocks at the economy-wide level, but individuals in the economy are also buffeted by shocks at the household level.[1] A macroeconomic stabilization policy based only on an attempt to mitigate aggregate shocks—as traditional macroeconomics has it—may appear to be appropriate or even optimal from a perspective that ignores idiosyncratic risk, but may be much less appropriate if that risk is better taken into account. Further, the conventional wisdom is that the sort of risk that actually impacts households most is at the household level, less so at the aggregate level.

One literature that has explored these issues in a rapidly expanding set of papers analyzes the heterogeneous agent New Keynesian (HANK) model.[2] This literature, reviewed by Galí (2018), emphasizes Aiyagari-Bewley-Huggett-style uninsured idiosyncratic risk and has analyzed the changes in the transmission mechanism of monetary policy due to the more explicit presence of heterogeneous households. This literature is evolving but so far suggests a rethink of traditional macroeconomic policy programs based on models designed only to mitigate shocks at the aggregate level.

In this paper we present a benchmark heterogeneous agent incomplete markets (HAIM) model intended to be complementary to the models in the HANK literature, but with a simpler model structure that permits pencil and paper solutions and an explicit description of the set of macroeconomic policies needed to achieve the first-best allocation of resources. Indeed, this paper includes a welfare theorem whereby the social planner would declare the competitive equilibrium to be a social optimum.

What does this optimal set of macroeconomic policies look like, and does the set resemble suites of actual policies in use today? In this paper we argue that broadly speaking, the optimal set of macroeconomic policies recommended based on the model are similar in nature to actual policies used in many economies. We map the model-recommended policies into the "four horsemen" that we see in operation in actual economies: (1) monetary policy responds to shocks each period in order to achieve the "Wicksellian natural real rate of interest" popularized by Woodford (2003); (2) a Treasury authority maintains a stock of government debt which is rolled over in perpetuity; (3) a labor market authority operates a social (unemployment) insurance

[1] See Krusell and Smith (1998).

[2] See, for instance, Kaplan, Moll, and Violante (2018).

program to mitigate idiosyncratic labor income risk; and (4) a fiscal authority maintains an income redistribution program to drive the consumption Gini coefficient to a socially desirable level. Policy programs with these general features are in use today in many economies. By itself, this would suggest that macroeconomic policy in the U.S. and many other jurisdictions around the globe is essentially on the right track, and that it is the detailed nature of the implementation of these types of policies that should receive the most attention in the literature.

We then turn to extend these results to a calibrated case for the U.S. economy. In the calibrated case, we argue that certain key features of the U.S. panel data, that is, aggregate time series data with both intra- and inter-cohort variability, can be matched by this model assuming that the optimal set of policies is in place. In particular, we provide *prima facie* evidence that the model equilibrium generally provides a good description of U.S. macro- and micro-economic outcomes from 1995-2022. However, for very large shocks—ones we interpret as beyond the ambient stochastic structure in the model—the model equilibrium does not match the data. These cases correspond generally to the global financial crisis of 2007-2009 and the global pandemic of 2020-2021. We conclude that the model-recommended set of macroeconomic policies, to the extent they were implemented in actual U.S. policy from 1995 to 2022, were insufficient to handle these very large shocks, but are sufficient to handle the more ordinary shocks associated with the remainder of the sample period. At the end of the paper we suggest possible extensions to the existing suite of recommended macroeconomic policies to handle these more extreme cases and to continue to improve macroeconomic policy going forward.

1.2 What we do

We study a DSGE life cycle economy with explicit stochastic growth. The model includes three aggregate shocks—to the pace of technological advance, the pace of labor force growth, and, in preferences, the state of aggregate demand. There is idiosyncratic risk of two types, a permanent type realized at the beginning of the life cycle as an agent enters the model as an economic decision-maker, and a temporary type that we will label as unemployment risk. These risks can be mitigated by policy authorities operating different policymaking tools: a monetary authority, a treasury authority, a labor market authority, and a fiscal authority. The direct "redistributional" aspect of the paper comes from the fiscal authority that can operate a tax and transfer scheme differentially across heterogeneous households in order to target the consumption Gini coefficient. However, we stress that really all policies together jointly attain the socially desirable consumption allocations across cohorts and within cohorts by allowing for smoothly functioning credit markets.[3]

[3] Smoothly functioning credit markets are a type of redistributional machine as they allow uneven labor income to be allocated toward consumption at different dates.

We argue that the optimal monetary-fiscal policy mix has the monetary policymaker meeting a targeting criterion analogous to the one outlined by Giannoni and Woodford (2004) for the New Keynesian model. This policy provides a type of insurance against the nominal risk faced by the households with regard to the aggregate shocks. The labor market authority operates a social insurance program—unemployment insurance—funded by linear labor income taxes. The fiscal authority operates a tax-and-transfer scheme which is designed to mitigate the effects of the permanent shock experienced at the moment the agent entered the economy as an economic decision maker. This provides a type of *ex post* insurance against that particular risk. Jointly these policy programs achieve a competitive equilibrium that would be judged first-best by a social planner.

We then turn to a calibration based on U.S. data from 1995-2022.[4] In this comparison, we assume that the model-recommended optimal policies are, in effect, close to the ones actually used by U.S. policymakers during this period. We present comparisons of key predictions from the model to the U.S. data, including consumption-output correlations, nominal interest rate time series versus nominal consumption growth time series, consumption growth by quartile, labor supply elasticities, marginal propensities to consume, and Gini coefficients. We argue that these observations from U.S. data match up well versus the model equilibrium and that this bolsters the case that the model provides a good benchmark for comparison. However, we also show that during certain time periods in the sample, actual U.S. data evolution does not match what would have been predicted by the model on one or more of these dimensions. These discrepancies are mostly or entirely concentrated in the periods associated with the GFC or the global pandemic. We then argue that these are exceptionally large disturbances, above and beyond what is contemplated in the ambient stochastic structure of the model, and that separate and distinct macroeconomic policies would have to be prepared and at the ready should these types of shocks occur again in the future.

1.3 Recent related literature

This paper contains an analysis of monetary policy in an economy with heterogeneous households and a nominal friction. This is a burgeoning field of macroeconomic research, and Galí (2018) provides a survey. Relative to that literature, the heterogeneity here differs in that there is more emphasis on the life cycle structure, less emphasis on idiosyncratic risk, and a different nominal friction, which is nominal contracting here versus sticky prices in most of the existing literature. This paper makes some contact with the HANK literature surveyed by Galí (2018) because we include "hand-to-mouth" households. Also, despite the somewhat different nominal friction, the role of monetary policy is arguably the same as in the New Keynesian

[4] We begin in 1995 as that is when the U.S. achieved and began maintaining a two percent inflation rate. Not all the data we use extends as far back as 1995.

model of Woodford (2003), as the monetary policymaker wishes to take action to maintain the "Wicksellian natural real rate of interest"—the real rate of interest that would characterize the equilibrium if there were no nominal friction at all—in order to attain the first-best allocation of resources. In the present paper, the monetary authority will be able to conduct this first-best monetary policy, and the Wicksellian natural real rate of interest will be equal to the stochastic rate of real output growth.[5]

Non-state contingent nominal contracting (NSCNC) as the nominal friction is supported in part by the evidence presented in Doepke and Schneider (2006). Their analysis documented substantial holdings of nominally-denominated assets in the U.S. and suggested large redistributive effects from unanticipated movements in inflation similar to the effects present in the model of this paper. Optimal monetary policy in the presence of the NSCNC friction has been studied by Koenig (2013) and Sheedy (2014), and the optimal monetary policy suggested in this paper is an extension of their work. Bullard (2014) and Werning (2014) provided commentary on the Sheedy (2014) paper.

The model includes a shock experienced by households as they enter the model. It is a permanent shock to their entire lifetime productivity profile, shifting that profile up or down depending on the draw. We motivate this shock as a proxy for the life experience of the agent from birth to age 20—including schooling, parenting, work experience, and other factors—that has contributed to their human capital before they enter our analysis. Huggett, Ventura, and Yaron (2011) studied the effect of initial conditions at the beginning of the economic life cycle versus shocks experienced during the economic life cycle on lifetime earnings. They found that a substantial fraction (63 percent) of lifetime earnings could be explained by initial conditions as opposed to shocks. The present paper has a more extreme and stylized version of this finding, and we sometimes call this shock the "HVY" shock.

Bhandari, Evans, Golosov, and Sargent (2021) study a HANK-type model with two aggregate shocks, uninsurable idiosyncratic labor income risk, and agents that can trade only non-state contingent nominal government debt. They develop new computational techniques to compute solutions to Ramsey problems in which the Ramsey planner is, in the first instance, restricted to changing nominal interest rates and transfers, and in the second instance is allowed to additionally change tax rates. They argue that recommended policy choices based on this approach are far more active and volatile than those recommended from the representative agent version of the model. Relative to Bhandari, et al., (2021), the present paper finds an exact representation of optimal policy which involves, in effect, insurance for all households against the nominal aggregate risk as well as the permanent and transitory idiosyncratic risk they face. We argue that this latter set of policies bears a close resemblance to actual policies in use in many economies today.

Davila and Schaab (2023) study optimal monetary policy in a one-asset HANK

[5]The optimal monetary policy in this paper is a form of nominal GDP targeting. For a discussion of nominal GDP targeting in a New Keynesian context, see Woodford (2012).

economy, and relate their findings to the burgeoning literature on this topic.[6] They contribute to the technical aspects of problems in this area by using sequence-space perturbation methods. The present paper is meant to be complimentary to this literature but downplays technical optimization aspects in favor of a simplified setting in which the asset distribution is part of the equilibrium but is easy to track. Departures from the exact equilibrium calculation we use would require the computational methods being developed in the literature.

Sargent, Wang, and Yang (2021) have recently suggested that it is likely to be important to incorporate life cycle elements into models within the Bewley-Aiyagari-Huggett tradition in order to match income and wealth inequality observed in the U.S. data. They work with a simple and stylized continuous time model with idiosyncratic risk and stochastic transitions between life cycle phases. They seek, like us, "pencil and paper" solutions. Their model economies do not have aggregate shocks. The model in the present paper has a continuous time counterpart. We are hopeful that starting from the life cycle side of this puzzle with aggregate shocks we will soon be able to meet some of the challenges laid out by Sargent, Wang, and Yang (2021).

McKay and Wolf (2023a) survey the literature on monetary policy and inequality. They discuss both normative and positive aspects of the literature. With respect to positive aspects, they argue that "... our understanding of the effects of monetary policy on macro outcomes has not changed very much" (p. 16). This is a similar conclusion to the one reached in the present paper.

The present paper has no financial intermediation sector, but including such a sector could be quite important. Chiang and Zoch (2023) provide one analysis in a related framework.

2 Model environment

In this section we describe the model environment. The model consists of a list of carefully chosen assumptions such that, when we put all the pieces of the model together, including the assumed set of macroeconomic policies, we will be able to guess and verify the general equilibrium solution. That is, despite its relative complexity, the model will have a "pencil and paper" solution.

2.1 Demographics

The model is built in the life cycle tradition—we wish to track household behavior between age 20 and age 80 based on the idea that such tracking will give us a good read on the behavior of the macroeconomy as a whole. We want to address issues at

[6] See for instance Acharya, Challe, and Dogra (2023), Auclert (2019), Bhandari, et al. (2021), Fahri and Werning (2016), Gonzalez, et al., (2021), Le Grande, et al., (2022), McKay and Wolf (2023b), Nuño and Thomas (2022), and Smirnov (2022).

the quarterly frequency, and so we allow households to make decisions each quarter. Allowing for an exact middle of economic life (age 50) leads us to use a 241-period life cycle for households. We will think of these decision points as the "stage of economic life" of the household, and we will use index $s = 0, ..., 240$ in the notation.

Each cohort entering the economy at each date consists of a continuum of households with identical preferences but differentiated by their life cycle productivity endowment as described below.

Real time t extends from the infinite past to the infinite future.

2.2 Notation and terminology

The model has nominal and real quantities. Most of the model is denoted in real terms. However, household net asset holdings, a, are written in nominal terms in order to put focus on the nominal credit market friction. Aggregate variables, such as total net nominal asset holding A, the real physical capital stock K, and the price level P, are generally denoted in capital letters. We use the terminology "households" and "agents" interchangeably.

2.3 Assets and the credit market friction

There are three *nominally-denominated* assets in the model: privately-issued debt, publicly-issued debt, and claims to physical capital which we will think of as "corporate bonds." These three assets will compete in a perfectly competitive market. Accordingly all assets will pay the same nominal rate of return, and also the same real rate of return.

Inspired by Doepke and Schneider (2006), we include a credit market friction: non-state contingent nominal contracting (NSCNC).[7] The nature of the friction is that debt contracts are agreed to in nominal terms at a stated nominal interest rate, with non-state-contingent repayment. This type of contracting introduces a distortion to the model equilibrium. However, monetary policy will be able to overcome this friction by, in effect, converting the non-state-contingent nominal contracts into the state-contingent real contracts that are optimal given the homothetic preferences we use.

2.4 Household types

There are two types of households: "hand-to-mouth" and "life cycle." There is a continuum of each type, and we will weight the relative size of the two types in the calibration. These two household types have different productivity profiles but identical preferences as we now discuss.

[7]See for instance Sheedy (2014) and Koenig (2013).

2.5 Preferences

Households $i \in (0,1)$ entering the economy at date t (both types) have preferences

$$U_{t,i} = \sum_{s=0}^{T} \eta \ln \tilde{c}_{t,i}(t+s) + (1-\eta)\ln \ell_{t,i}(t+s), \tag{1}$$

where $\eta \in (0,1)$ controls the desirability of real consumption relative to leisure in the consumption-leisure bundle. We use subscripts to denote the date of entry into the model and parentheses to denote real time, so that $\tilde{c}_{t,i}(t+s)$ represents the date $t+s$ consumption of a household i who entered the economy at date t, and $\ell_{t,i}(t+s) \in (0,1)$ represents the date $t+s$ leisure choice of a household i that entered the economy at date t.

Preferences for the households entering the economy at date $t-1$ can be defined analogously and in a time-consistent manner as

$$U_{t-1,i} = \sum_{s=0}^{T-1} \eta \ln \tilde{c}_{t-1,i}(t+s) + (1-\eta)\ln \ell_{t-1,i}(t+s) \tag{2}$$

and similarly for all households entering the economy at earlier dates.

The variable $D(t)$ is the stochastic "state of aggregate demand." Households will desire more consumption on dates when the state of aggregate demand is relatively high, and less on dates when the state of aggregate demand is relatively low. We define $\tilde{c}_{t,i}(t+s) \equiv D(t+s)\, c_{t,i}(t+s)$ for all s, and similarly $\tilde{c}_{t-1,i}(t+s) = D(t+s)\, c_{t-1,i}(t+s)$ for all s, and so on for all other households entering the economy at earlier dates $t-2, ..., t-T$, where $D(t+s) > 0$. The state of aggregate demand evolves according to

$$D(t) = \delta(t-1,t)\, D(t-1) \tag{3}$$

where $\delta(t-1,t)$ is the gross growth rate of aggregate demand from date $t-1$ to date t, which follows an appropriate stochastic process with mean value δ that keeps $D(t) > 0\ \forall t$. Households will know today's value of aggregate demand when decisions are made, but will not know future values, making them unsure about exactly how much they may desire to consume at each date in the future. Following Bai, Ríos-Rull, and Storesletten (2017), we allow the state of aggregate demand to influence productive activity in the economy, as detailed in the technology sub-section below. To study the economy without the demand shock, we can set $D(t) = 1\ \forall t$.

2.6 Productivity endowment profiles

Each household $i \in (0,1)$ entering the economy at date t is endowed with a productivity profile. Life cycle households will be endowed with a hump-shaped productivity profile, while hand-to-mouth households will be endowed with a flat profile. We now describe this process.

We begin with a baseline hump-shaped productivity endowment profile denoted as $e = \{e_s\}_{s=0}^{T}$ corresponding to each of the $T + 1 = 241$ stages of economic life s. The hump-shaped nature of the profile means that, for all life cycle households, middle-age productivity is higher than productivity at the beginning or end of the life cycle. The profile is chosen such that households will choose to work the fraction of hours by age that we observe in the U.S. data, as detailed in the calibration section below.

We follow Bullard and DiCecio (2021) and add a stochastic scaling factor to create within-cohort heterogeneity in life cycle productivity profiles. The scaling factor is denoted by x_{lc} and is distributed as a lognormal random variable with standard deviation σ_{lc}. Each incoming life cycle household i is therefore endowed with a productivity profile $e_i = x_{lc,i}e$. The value of the realization $x_{lc,i}$ for a particular household dictates the entire lifetime profile of productivity units. Bullard and DiCecio (2021) used this method to represent an unmodeled human capital accumulation process—parenting, schooling, on-the-job training—that occurs before agents enter the economy. We sometimes call this the HVY shock after Huggett, Ventura, and Yaron (2011).[8] With this type of scaling, within-cohort heterogeneity will allow for arbitrarily rich and arbitrarily poor households.

We also assume that this process of assigning life cycle productivity profiles to agents as they enter the economy is distorted. The main idea is that the processes observed in actual economies regarding development of human capital before age 20 are far from perfect. To account for this, we assume that the assignment process creates more variability in the life cycle productivity than would be necessary if the assignment process was ideal. The ideal assignment process in the economy would assign life cycle productivity profiles with a standard deviation of $\sigma_{lc}^{\star}$. The actual process of assigning life cycle productivity profiles, however, uses a distribution with standard deviation $\sigma_{lc} > \sigma_{lc}^{\star}$. This is a proxy for the idea that the unmodeled human capital accumulation process that takes places before agents enter the economy is imperfect, and ends up assigning life cycle productivity profiles with too much dispersion relative to the ideal distribution in the economy.

2.7 Hand-to-mouth households

The HANK literature has emphasized the importance of households with little or no access to financial markets as a part of the analysis of monetary policy.[9] These households are often described as hand-to-mouth households because they consume their entire labor income—they have a marginal propensity to consume of one. We

[8] Bullard and DiCecio (2021) cited the empirical evidence provided by Huggett, Ventura, and Yaron (2011) suggesting that about 63 percent of the variance of lifetime earnings could be attributed to agent characteristics at age 23. The model here encapsulates a version of that result.

[9] See, for instance, Kaplan, Moll, and Violante (2018).

will include a fixed measure of hand-to-mouth agents to each cohort that enters the model.

The hand-to-mouth (HTM) households have preferences as described above but are endowed with a productivity endowment profile which is perfectly flat, which means that these households are equally productive at all dates, work the same hours at all dates, and do not have peak earning years. We set this baseline equal to $x_{htm}e_{s,htm}$ where $e_{s,htm} = h\bar{e} = e^{htm}$, (a constant), for $s = 0, ..., T$, $h \leq 1$, where $\bar{e}$ is equal to the average endowment in the baseline life cycle productivity profile e (which can itself be normalized to unity), and x_{htm} is a log normal random variable with standard deviation σ_{htm}, the realization of which occurs as the HTM household enters the model. The decision problems for HTM households are solved using the same considerations as for life cycle households, and their net asset holdings will be zero at all dates. We will generally think of $h << 1$ so that the HTM households are poorer on average than the life cycle households. There will be many fewer HTM households than life cycle households in our calibration. As with the life cycle households, we assume $\sigma_{htm} > \sigma^{\star}_{htm}$, where $\sigma^{\star}_{htm}$ corresponds to the ideal distribution for hand-to-mouth households.[10]

2.8 Idiosyncratic risk

All households are subject to idiosyncratic risk at each date in the form of household-specific unemployment shocks u_i. We will simply assume these shocks are *i.i.d.* across the entire distribution of households, taking on a value of 1 with probability p and a value of zero with probability $1 - p$. When a household encounters an unemployment shock $u_i = 0$, it means that there is no work for that household on that date and the household cannot earn labor income.

2.9 Technology

In the spirit of simplicity, we will assume there is a single, representative "stand-in" firm that behaves competitively and operates the technology on behalf of the households. The firm is debt-financed—it issues one-period nominal "corporate" non-state-contingent debt B^c each period according to

$$B^c(t+1) = R^n(t, t+1) B^c(t), \tag{4}$$

meaning that it rolls over the outstanding debt each period at the going contract nominal interest rate given below.

The firm operates a standard Cobb-Douglas technology

$$Y(t) = [D(t) Q(t) N(t)]^{1-\alpha} K(t)^{\alpha} [L(t)]^{1-\alpha} \tag{5}$$

[10] In the calibration we simply set $\sigma_{htm} = 0$, so that all HTM agents receive the same flat productivity profile, as it turns out to not be empirically important for the findings in this paper.

where $K(t)$ is the real value of the physical capital stock at date t, $L(t)$ is aggregate effective human capital supply, that is, the aggregate of hours chosen at date t by the various households multiplied by the productivity of those households at date t, $Q(t)$ is an index of total factor productivity, $N(t)$ is an index of the size of the labor force, $D(t)$ is the state of aggregate demand entering household preferences, and $Y(t)$ is the aggregate level of real output. Total factor productivity and labor force size evolve over time as

$$Q(t) = \lambda(t-1,t)\,Q(t-1), \tag{6}$$
$$N(t) = \nu(t-1,t)\,N(t-1) \tag{7}$$

respectively, where the gross growth rates $\lambda(t-1,t)$ and $\nu(t-1,t)$ follow appropriate stochastic processes with mean values λ and ν, respectively, that keep $Q(t) > 0$ $\forall t$ and $N(t) > 0$ $\forall t$. The value of $Q(t)$ represents the current value of the level of technological prowess, and is viewed as an increasing function over time. Capital depreciates at a constant rate δ^k.

The state of aggregate demand enters the production function because the households may wish to consume somewhat more or less on a particular date, depending on the state of aggregate demand $D(t)$, and the firms (perhaps thought of as "restaurants") need to be ready in case a lot of customers show up at a particular moment.[11] If $D(t)$ is relatively high, then production will be more intensive than otherwise (workers will work harder and produce more "meals"), whereas if $D(t)$ is low workers will not work as hard during that particular period ("evening") and output will be lower than otherwise. Hours worked are the same regardless of the value of $D(t)$ (as hours worked will depend only on the stage of the life cycle) and capital depreciation is assumed to be unaffected. The value of $D(t)$ therefore represents both the desirability of consumption on a particular date as well as the intensity of work effort on that date.

The value of $N(t)$ represents the size of the labor force. The nature of the model is that aggregate effective human capital supply $L(t)$ will be constant over time because households will choose hours based on their stage of the life cycle alone,[12] and these cohort choices will be weighted by constant weights representing the relative size of different cohorts.[13] As $N(t)$ evolves, the size of the labor force is then increasing or decreasing in proportion across all cohorts.[14] Given this discussion the labor input at

[11] This is a simplified version of Bai, Ríos-Rull and Storesletten (2017).

[12] See Bullard and Singh (2019).

[13] One possible set of weights is a geometrically declining sequence as would occur under an assumption of constant labor force growth. However, in the calibration we will use the weights as they are in the U.S. data. Strictly speaking, to use these weights from the U.S. data and maintain consistency with household optimization, we would have to include survival probabilities in the household problem. We have not done this here but we do not think this is empirically important for the issues discussed in this paper.

[14] This exogenous process encapsulates forces like population growth, labor force participation, and net immigration which are beyond the scope of this paper.

date t will be $D(t)\,N(t)\,L(t)$, and total hours worked at date t will be $N(t)\,L(t)$. We will think of $N(t)$ as reflecting the adjustment of aggregate hours on an extensive margin, and $L(t) = L$ as reflecting the adjustment of hours on an intensive margin.

2.10 Macroeconomic policy authorities: Four horsemen

2.10.1 The labor market authority

There is a labor market authority. This authority can levy a linear labor income tax $\tau^u \in (0,1)$ on all agents for all time. The purpose of the tax is to raise the exact amount of revenue necessary to make unemployment insurance payments to those in the economy that have been hit by the *i.i.d.* unemployment shock in a particular period. An agent receiving an unemployment insurance payout receives the after-tax labor income they would have received if they had been able to work in that period.[15] We can think of this tax and its payout as being applied cohort by cohort in the economy, and in fact, because the cohorts themselves consist of a continuum of agents, we can think of the tax and its payout being applied to subsets of agents within a cohort such as the quartiles, deciles, or percentiles of the labor income distribution within the cohort. Taking the limit of this reasoning, the unemployment rate in the economy will be a constant, the probability that $u_i = 0$, which is p; in addition the tax $\tau^u = p$, and the cost to the society of idiosyncratic risk is p times aggregate labor income.

2.10.2 The Treasury authority

The Treasury authority issues nominal debt. We denote the level of nominal federal government debt by $B^g(t)$. The debt issuance process is given by

$$B^g(t) = R^n(t-1,t)\,B^g(t-1). \tag{8}$$

In this expression, $R^n(t-1,t)$ is the contract nominal interest rate between dates $t-1$ and t as described below. Thus the fiscal authority will issue enough debt to pay off existing investors with interest.

The nature of the model is that the life cycle agents will wish to hold assets which are in net positive supply (such as nominal claims to physical capital or nominal government debt) in order to smooth consumption over the life cycle.[16] The hand-to-mouth agents, on the other hand, do not hold assets. If the economy consisted entirely of hand-to-mouth agents, it would be "spartan" and the assets-to-GDP ratio

[15] We also do not allow the agent to enjoy more leisure on the date of unemployment—accordingly we assume the agent has to spend time collecting the unemployment payout that is equivalent to the time they would have otherwise spent working. The agent therefore gets the same utility payoff whether they are employed or unemployed on a particular date.

[16] For a discussion of two-period overlapping generations models with capital and government debt, see Azariadis (1993, section 7.5, pp. 78-84).

would be zero. In the calibration below, we account for observed levels of capital, HTM households, and a fraction of observed government debt in the U.S. data.

2.10.3 The fiscal authority

The fiscal authority operates a fiscal tax and transfer program. The fiscal authority can levy (deliver) an individual-specific linear consumption tax (transfer) τ_i^p on (for) each household at each stage of the life cycle. The purpose of this tax is to redistribute consumption across the economy in order to reduce the consumption Gini coefficient from the one consistent with σ_{lc} and σ_{htm} to a lower value consistent with $\sigma_{lc}^{\star} \geq 0$ and $\sigma_{htm}^{\star} \geq 0$. We will propose this tax and transfer program so that total taxation and transfers from this program sums to zero. The tax-transfer factor $(1-\tau_i^p)$ does not change as the agent moves through the life cycle.

We will show below that $\tilde{c}_{i,t}(t+s)$ is linear in the scaling factor realization $x_{lc,i}$. All other consumption choices are similarly linear in their associated scaling factors. Given this, a natural tax-transfer scheme is to set the tax-transfer factor $(1-\tau_i^p) = x_{lc,i}^{\star}/x_{lc,i}$, where $x_{lc,i}^{\star}$ is the *corresponding* draw from a lognormal distribution with the same mean but lower variance, so that after-tax consumption for this household will be associated with the scaling factor $x_{lc,i}^{\star}$ instead of the original scaling factor $x_{lc,i}$, where $x_{lc}^{\star} \sim LN(\mu, \sigma_{lc}^{\star})$, and where $\sigma_{lc} \geq \sigma_{lc}^{\star} \geq 0$, and similarly for HTM households.

The effect of this tax will be to lower the consumption Gini coefficient. The tax in effect replaces the economy with scaling factors $x_{lc,i}$ and $x_{htm,i}$ with another economy with scaling factors $x_{lc,i}^{\star}$ and $x_{htm,i}^{\star}$. The income and wealth Ginis will also be lowered.

2.10.4 The monetary authority

We assume that the monetary authority controls $P(t)$ directly.[17] The monetary policymaker fully and credibly meets a *targeting criterion* $\forall t$ defined as

$$P(t) = \frac{R^n(t-1,t)}{\delta(t-1,t)\,\lambda(t-1,t)\,\nu(t-1,t)} P(t-1). \tag{9}$$

The term $R^n(t-1,t)$ is the contract nominal interest rate effective between date $t-1$ and date t. The terms $\delta(t-1,t)$, $\lambda(t-1,t)$, and $\nu(t-1,t)$ are the realized rates of growth of aggregate demand, total factor productivity, and the labor force between dates $t-1$ and t, respectively, all of which are known by the policymaker at the moment the price level $P(t)$ is set. Collectively, the denominator represents the aggregate stochastic growth rate of the economy. Because the stochastic growth rate of the economy appears in the denominator, this rule calls for countercyclical price level movements when the actual growth rate differs from the expected real output growth rate embedded in the contract nominal interest rate (10).[18]

[17] For a version that provides a microfoundation for this assumption, see Azariadis, Bullard, Singh, and Suda (2019).

[18] This is a hallmark of nominal GDP targeting as discussed in Koenig (2013) and Sheedy (2014).

2.11 Timing protocol

Nature moves first at the beginning of the period and chooses HVY shocks $x_{lc,i}$ for all life cycle households i entering the economy, HVY shocks $x_{htm,i}$ for all hand-to-mouth households entering the economy, economy-wide unemployment shocks u_i, as well as shocks to the growth rate of aggregate demand, the growth rate of the labor force, and the growth rate of TFP, so that the values of $\delta(t-1,t)$, $\nu(t-1,t)$ and $\lambda(t-1,t)$ are known.

The monetary policymaker moves second and chooses the value of the price level $P(t)$ in reaction to the news on economic growth using (9).

In the third part of the period, all other activities are viewed as occurring simultaneously.

Given this timing protocol, households will be able to make date t decisions without reference to future uncertainty. The monetary policymaker is providing a type of insurance to households.

This completes the description of the model environment.

3 Model solution and social optimum

3.1 Nominal contracting

All households meet in a competitive market each period to agree on nominal loans. The fiscal authority is also selling nominally-denominated debt in this market. Nominal claims to capital are also being purchased in this market.. The NSCNC friction means that households must agree on a non-state contingent nominal contract at date t that will pay off at date $t+1$. From the household t, i, Euler equation, the non-state contingent gross nominal interest rate, $R^n(t,t+1)$ in effect from period t to $t+1$ is given by[19]

$$R^n(t,t+1)^{-1} = E_t\left[\frac{\tilde{c}_{t,i}(t)}{\tilde{c}_{t,i}(t+1)}\frac{P(t)}{P(t+1)}\right]. \tag{10}$$

We call this the "contract rate." In the equilibrium we describe, it can be read as the expected rate of nominal GDP growth.

If there was no uncertainty in the economy, this expression would simplify to

$$R^n(t,t+1) = \lambda\nu\delta\pi^{\star}, \tag{11}$$

that is to say, the means of the stochastic growth rates for TFP, labor force, and demand, in the economy, respectively, multiplied by the desired long run gross rate of growth of the price level—the gross inflation rate target $\pi^{\star}$. A credible inflation target is embedded in the contract rate. In order to avoid carrying $\pi^{\star}$ in the notation, we set $\pi^{\star}=1$ in this paper.

[19]For more detail see Chari and Kehoe (1999).

3.2 Model solution

We will guess and verify that there is a "pencil and paper" general equilibrium solution for the proposed economy. The guess is that the general equilibrium is characterized by the real interest rate equal to the stochastic rate of real output growth at each date t. This guess makes sense given the extensive literature on models within this class, along with the carefully chosen assumptions we have made. We provide the details in the Appendix.

The life cycle household i choices in the equilibrium for consumption can be written as

$$\tilde{c}_{t-s,i}(t) = x_{lc,i}\left(1-\tau_i^p\right)\left(1-\tau^u\right) w(t) \frac{\eta}{T+1} \sum_{j=0}^{T} e_j, \tag{12}$$

for $s = 0, ..., T$. Today's consumption choices depend linearly on today's real wage per effective efficiency unit $w(t)$, which itself grows over time at the rate of output growth. It is also scaled by $x_{lc,i}\left(1-\tau_i^p\right)$ as well as the tax factor $\left(1-\tau^u\right)$. The earlier discussion indicated that $\tau^u = p$, so $1-p$ represents the consumption loss due to idiosyncratic risk in the economy. For leisure, each life cycle household i chooses

$$\ell_{t-s,i}(t) = \frac{1-\eta}{T+1} \frac{1}{e_s} \sum_{j=0}^{T} e_j, \tag{13}$$

for $s = 0, ..., T$. These choices do not depend on $w(t)$ or $x_{lc,i}\left(1-\tau_i^p\right)$ or $\left(1-\tau^u\right)$. Agents will work more (take less leisure) when they are more productive (e_s is relatively large) which will be in the middle of the life cycle in our calibration, independently of these other factors. Notably the tax factors do not distort hours worked. For real net asset positions, each life cycle household i chooses

$$\frac{a_{t-s,i}(t)}{P(t)} = x_{lc,i}\left(1-\tau_i^p\right)\left(1-\tau^u\right) w(t) \left\{ \left[\sum_{j=0}^{s} e_j\right] - \left(\frac{s+1}{T+1}\right) \sum_{j=0}^{T} e_j \right\} \tag{14}$$

for $s = 0, ..., T$. These are again linear in today's real wage $w(t)$, the scaling factor $x_{lc,i}\left(1-\tau_i^p\right)$, and the tax factor $\left(1-\tau^u\right)$.

3.3 Social optimum

A monetary policy obeying the price level targeting criterion combined with the debt issuance policy followed by the Treasury authority, the unemployment insurance program administered by the labor market authority, and the tax-transfer scheme followed by the fiscal authority can jointly be viewed as a socially optimal monetary-fiscal policy mix. A theorem similar to the one in Bullard and DiCecio (2021) characterizes this competitive equilibrium as first best provided (i) the social planner places equal weight on all households for all time and for all i, (ii) the planner discounts forward and backward in time at the stochastic real rate of interest, (iii) the planner cannot change an agent's type. In particular, all consumption growth rates for all households of both types are equalized in this equilibrium and furthermore all are equal to the output growth rate. In a limiting case with only life cycle households and a tax-transfer program attaining $\sigma_{lc}^{\star} = 0$, every household for all time would enjoy exactly the same utility modulo the real interest rate adjustment.

4 Mapping to data

4.1 Overview

The model has a relatively limited number of parameters that need to be calibrated in order to compute the equilibrium we wish to study. Most prominently, the hump-shaped life cycle productivity profiles of the life cycle households need to be established, and our strategy is to choose these profiles to match available data on hours worked by age. In conjunction with this, we choose η to match average time devoted to market work across the economy.

For the HTM households, we need to choose the fraction of households that are in this category. Since these households work but do not hold assets, a higher fraction of HTM households will generate a lower ratio of assets to output in the economy. We will calibrate this fraction to the number of unbanked households in the U.S., which is 0.045 in the most recent data.[20] This will mean that the fraction of HTM households will be relatively small in this calibration. We set $h = 0.25$, meaning that a HTM household with the baseline HTM productivity profile will earn income equal to only 25 percent of a life cycle household with the baseline life cycle productivity profile.

All life cycle households as well as all HTM households receive a draw $x_{lc,i}$ or $x_{htm,i}$, respectively, from a lognormal distribution when they enter the model, and these distributions have standard deviations σ_{lc} and σ_{htm} respectively. We choose these values to approach the pre-taxes-and-transfers Gini coefficient for income in the U.S. data.

The tax-transfer scheme in the model is capable of, in effect, changing the standard deviation of the beginning-of-life-cycle scaling distribution from σ_{lc} to $\sigma_{lc}^{\star} < \sigma_{lc}$. This produces a lower consumption Gini coefficient but also lowers the income and financial wealth Gini coefficients. We discuss this below.

4.2 Life cycle productivity profiles

We turn to calibrating the baseline productivity profile of life cycle households. Households receive this profile when they enter the model at age 20. The profile that these

[20]Source: FDIC (2022).

households actually receive is $x_{lc,i}e$ where $x_{lc,i}$ is a draw from a lognormal distribution and $e = \{e_s\}$ with $s = 0, ..., 240$. Accordingly, the shape of the profile is the same for all households, and so we need to specify e.

A property of the model is that household choices for hours worked depend on the stage of the life cycle alone and are independent of $x_{lc,i}$.

These considerations suggest a mapping to available data on household hours worked by age. We use average (2000-2021) hours worked by age data from the U.S. Census ACS for ages 20 to 80.[21] These hours are reported in weekly terms. The model normalizes available hours to unity, and so we divide the hours reported in the ACS by 112 based on 16 possible hours of work per day and a 7-day possible workweek. This yields fractions of available time spent working of below 0.20 in relative youth, close to 0.30 in middle age, and below 0.05 later in the life cycle. We then reverse-engineer a three parameter function that generates a life cycle productivity profile that, in turn, causes households to choose fractions of time spent working out of total available time at each stage of the life cycle close to those observed in the data. This productivity profile e is given by

$$e_s = 1 + p_1 \exp\left[-\left(\frac{s - p_2}{p_3}\right)^4\right] \tag{15}$$

for $s = 0, ..., 240$, with $p_1 = 0.38$, $p_2 = 83.68$, and $p_3 = 91.78$. In conjunction with this profile, we set $\eta = 0.22$. This parameter controls the relative desirability of consumption versus leisure in preferences, and using this value causes the average fraction of time devoted to work over the life cycle to match the data. The productivity profile e is shown in Figure 1. The comparison between hours worked by age in the model versus the data is shown in Figure 2.

4.3 Cohort size and technology

We will allow the cohorts to be of different size in order to match U.S. data, but we will hold the age distribution fixed through time.[22] We use U.S. Census population data from 2000 to 2021 for ages 20-80. Weights are average population shares by age, smoothed by a fourth-order polynominal (Figure 3). These weights are generally declining with age, and the cohort size of older cohorts is less than half that of younger cohorts.[23]

The capital share parameter α is set to 0.33.

[21] Data are available at IPUMS (see Ruggles et al., 2022).

[22] In order for the observed population weights to be more consistent with the model, we would have to include survival probabilities or other factors. We have not done this in this version of the model.

[23] We found that it did not materially affect results whether we used the population rates directly or the smoothed polynomial version of the weights.

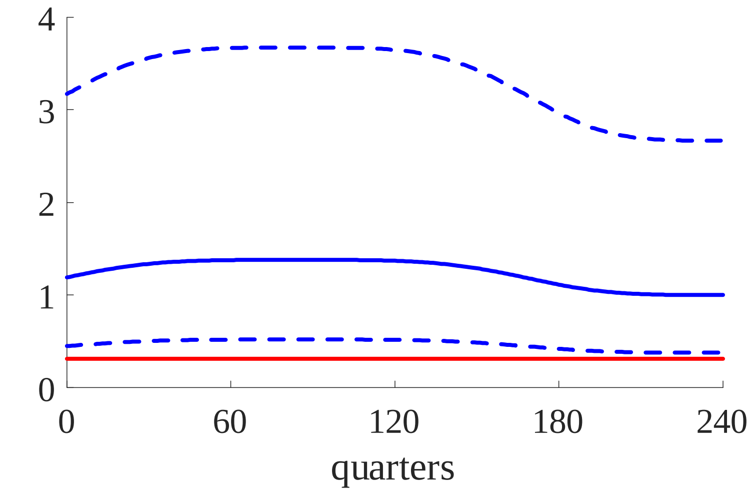

Figure 1: The baseline productivity profile is the blue line. The upper dotted line is the 75th percentile of the scaling factor, and the lower dotted line is the 25th percentile. The red line is the productivity endowment profile for the hand-to-mouth households. We set the variance of the HVY shock for these households to zero.

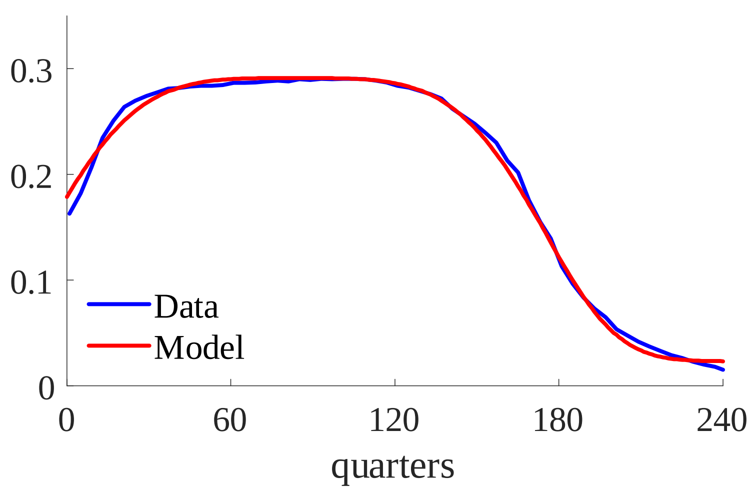

Figure 2: Hours worked by age. Endowment by cohort parameters, p_j, and the utility parameter η are calibrated to minimize the distance between hours by cohort implied by the model (red line) and hours by cohort in the data (blue line).

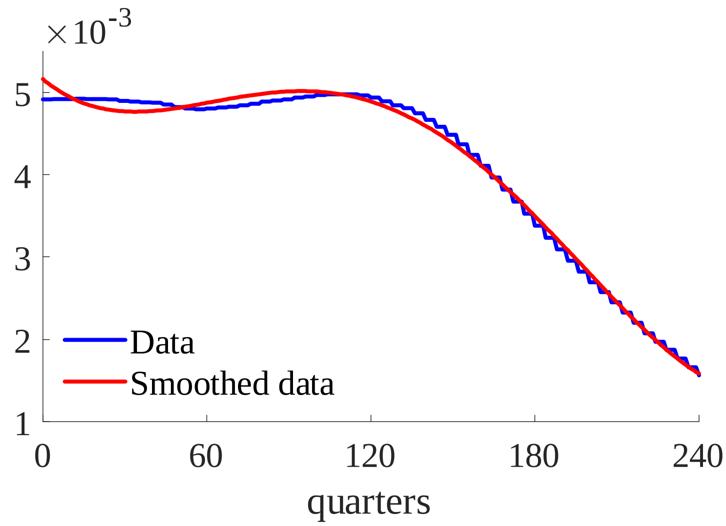

Figure 3: Population weights by age. Population shares by age data have been averaged over 2000-2021 (blue line) and smoothed by fitting a fourth-order polynomial (red line)

4.4 The assets-to-output ratio

The calibrated life cycle productivity profile described above is skewed toward youth compared to the perfectly symmetric profile used in Bullard and DiCecio (2021). After also adjusting for the size of the various cohorts, effective productivity is skewed even more toward the earlier stages of the life cycle. These two changes relative to that paper mean that assets in positive net supply will be held by households as part of the equilibrium, that is, $A(t) > 0$ as opposed to $A(t) = 0$ in Bullard and DiCecio (2021). By itself, these changes would lead $A(t)/[4Y(t)]$ to be 4.18.

The hand-to-mouth households have a flat baseline endowment profile. These households supply labor and increase output but will have zero net assets at all points in the life cycle. Adding a positive measure of HTM households will therefore lower the equilibrium assets-to-output ratio.[24] Taking this into account with the fraction of HTM households in the population set to 0.045 means that $A(t)/[4Y(t)] = 4.13$.

We consider an equilibrium with two assets in positive net supply, federal government debt and capital. The value of the physical capital stock relative to output in the U.S. data is about 3.32 according to Cooley and Prescott (1994). This means the federal government debt to output ratio has to be 0.81 in order for the assets-output ratio to be 4.13. Although the debt-to-GDP ratio is currently much higher (118.6% in 2023Q1), this value is in line with the average level of debt-to-GDP since 1995 (82.5%).

[24] The HTM household work hours choices would have a minor effect on calibrated hours per cohort, but we have not incorporated this feature into the calibration.

4.5 Gini coefficient calibration

The model features stationary distributions for income, financial wealth, and consumption, and we want to target realistic values for the Gini coefficients associated with these distributions. The Gini coefficient for income is the most widely studied in the U.S. data. In the model, there are multiple concepts of income because life cycle households are earning labor income and also capital income. We will focus on a labor earnings Gini. We follow Castañeda, Díaz-Gímenez, and Ríos-Rull (2003) as well as Sargent, Wang, and Yang (2021) and set our target value to $G_y = 0.63$. The Gini coefficient for financial wealth is also widely studied. Castañeda, Díaz-Gímenez, and Ríos-Rull (2003) and Sargent, Wang, and Yang (2021) use a value of $G_w = 0.78$, and we use this value.[25] For consumption, we use the post-taxes-and-transfers value from Heathcote, Perri, and Violante (2010) of $G_c = 0.32$.[26]

We use two remaining parameters, σ_{lc} and σ_{htm}, which control within-cohort HVY shock dispersion for the two types of households, to initially match the target pre-taxes-and-transfers $G_w = 0.78$, and then to match other Gini coefficient values. This is discussed further below.

5 Model fit

5.1 Overview

We compare the calibrated model equilibrium to U.S. data from 1995-2023 on six dimensions: (1) aggregate consumption growth correlations with output; (2) consumption growth by income quartiles; (3) labor supply elasticities; (4) marginal propensities to consume; (5) tax progressivity and the consumption Gini; (6) nominal interest rates versus nominal output growth rates.

5.2 Aggregate consumption growth

The model-suggested optimal monetary-fiscal policy mix in this paper means that credit markets function smoothly. In turn, this means that the model equilibrium is characterized by an aggregate rate of quarterly consumption growth, in both nominal and real terms, that is equal to the quarterly rate of stochastic output growth in both nominal and real terms. The model works directly with growth rates so there are no issues of detrending.

Is this prediction borne out in the data? If so, this would be one piece of evidence that model equilibrium may provide a promising fit to actual data. One immediate issue is that the model does not have some of the objects that are important components of an actual measure of GDP in the data, such as an international sector, a

[25] Davies, Sandstrom, Shorrocks, and Wolf (2011) find a similar value of $G_W = 0.80$.

[26] This is from their Figure 13, using the most recent value in their sample data.

Table 1. Output-Consumption Growth Correlations

	Real (2012 Dollars)		**Nominal**	
	Consumption		*Consumption*	
	y/y % change	*% change AR*	*y/y % change*	*% change AR*
GDP	0.93	0.93	0.97	0.94
FS	0.94	0.95	0.97	0.96
FSD	0.96	0.97	0.97	0.97
FSPD	0.95	0.97	0.96	0.98

Table 1: Output consumption correlations. The model equilibrium under the optimal fiscal-monetary policy mix predicts perfect correlation between output and consumption in both real and nominal terms. The correlations in the U.S. for various measures of output are in excess of 0.9.

government sector that taxes for the purpose of public goods provision, and inventories. Accordingly, we will consider a range of different measures of aggregate output in the data that may be considered alternative relevant measures of the "output" in the model. These measures include GDP itself, final sales (FS, which is GDP less the change in private inventories), final sales to domestic purchasers (FSD, which is GDP less exports plus imports less the change in private inventories), and final sales to private domestic purchasers (FSPDP, which is FSD less government purchases). For consumption, we consider personal consumption expenditures (PCE).

Table 1 uses the quarterly data on real and nominal growth rates of output and consumption from 1995 through 2022. The entries in the table are correlation coefficients—the model prediction is that the correlation is 1.0. The left hand side of the table reports correlations of growth rates measured as percentage change from one year earlier, whereas the right hand side of the table uses growth rates measured as percentage change at an annual rate. The general finding is that the correlations are between 0.93 and 0.98 depending on the measures used, reasonably close to the perfect correlation predicted by the model.

To get a feel for these correlations, Figure 4 plots quarterly nominal growth rates of final sales to domestic purchasers, measured as percent change at an annual rate, and quarterly nominal PCE growth rates measured on the same basis, from 1995 to 2022. The correlation, as reported in Table 1, is 0.98. In the Figure, the shaded regions are recessions as defined by the NBER. The correlation is high even during the global financial crisis 2007-2009, and during the onset of the global pandemic, March-April 2020. The model equilibrium predicts that these two lines should be on top of each other.

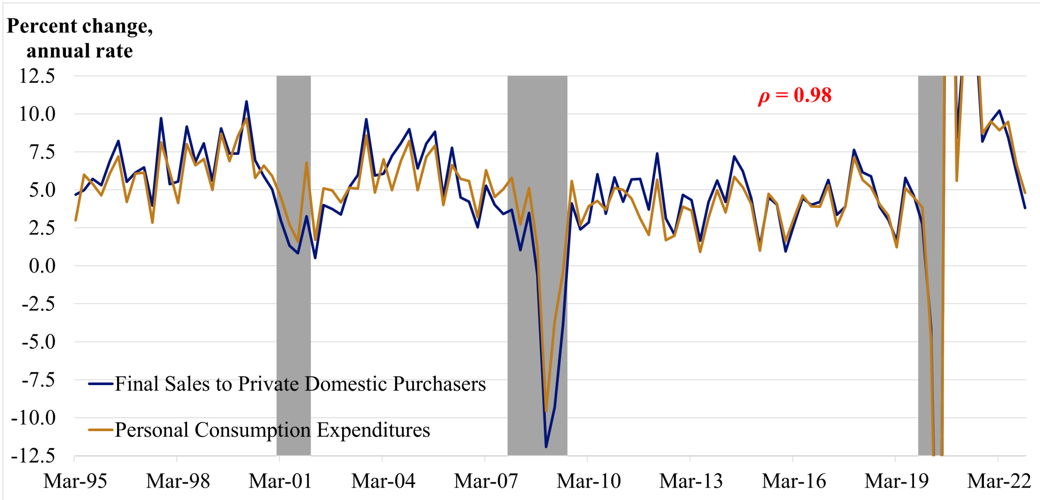

Figure 4: The model equilibrium under the optimal policy mix suggests that the nominal output growth rate and the nominal aggregate consumption growth rate should be equal. This chart shows one measure of nominal output growth and one measure of nominal consumption growth, and the raw correlation is 0.98.

5.3 Consumption growth by quartile within cohort

The model makes more detailed predictions about consumption growth at a more granular level. Under the optimal fiscal-monetary policy mix, the model predicts that both nominal and real consumption growth will be equalized across households at different ages and at different income levels. This is a consequence of a combination of log-linear assumptions used in designing the model—economic growth gets "shared out" appropriately between high and low productivity households who are also using smoothly functioning credit markets.

Here we compare this prediction to available U.S. data at relatively high frequency. The data is credit card expenditures collected weekly[27] between January 2020 and June 2023.[28] The credit card expenditures are sorted by the home ZIP code of the credit card, and the ZIP codes are mapped to data on median income in that particular ZIP code. The data is then sorted into four groups. The first group is spending from ZIP codes with median household income above the 75th percentile in the household-ZIP code income distribution, the second group is spending from ZIP codes with median household income between the 50th and the 75th percentiles in the household-ZIP code income distribution, and so on for the third and fourth groups. The data are measured from a benchmark which is set in the pre-pandemic year 2019.

[27]The data is daily earlier in the sample, and we converted this to weekly to maintain consistency with the latter portion of the sample.

[28]See Chetty et al. (2023). Updates are published regularly on GitHub.

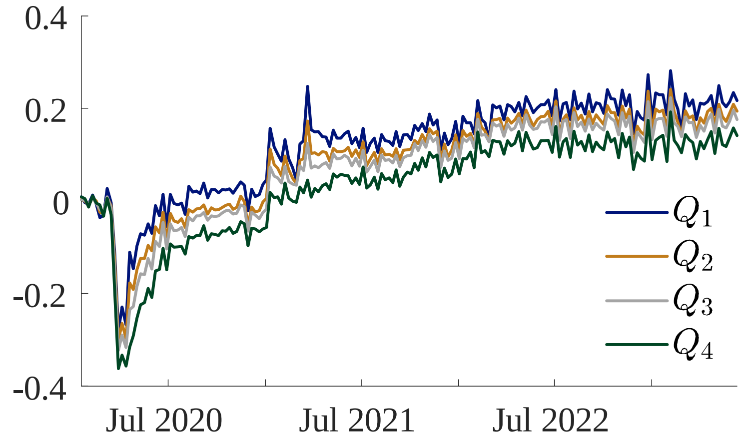

Figure 5: Credit card spending by income group, weekly, January 2020 to June 2023. The model equilibrium predicts that nominal spending growth rates across society should be equalized. The correlation in consumption growth between the groups is indeed very high, as predicted by the model.

Figure 5 plots this data. The model equilibrium predicts that the growth rates of all four series should be perfectly positively correlated. In Figure 5, the four lines are indeed highly correlated. We interpret this as broadly consistent with the equilibrium described under optimal policy in this model.

The correlations associated with this data expressed in levels as in the chart can be found in Table 2. Strictly speaking, the model equilibrium prediction is that consumption *growth rates* are equalized under the optimal monetary-fiscal policy mix. The correlations associated with growth rates are given in Table 3. In either case the correlations tend to be high.

These data cleverly separate individual consumers into income groups, but do not separate the households by age. That is never-the-less consistent from the perspective of the model, because the model equilibrium predicts that households are able to use credit markets in such a way as to make their own personal consumption growth rates equal to the aggregate rate of growth of the economy in both real and nominal terms, regardless of whether they are relatively young, middle-aged, or old, or for that matter whether they are life cycle or hand-to-mouth households. In Figure 5, the lines include households of different ages but the model equilibrium still predicts that the growth rates among the four should be equalized.

These results are confirmed over a longer sample. VISA constructs a momentum index from VISA credit/debit cards spending data.[29] The index ranges between 0 (all credit cards have less spending than the previous year) and 200 (all cards have greater

[29] We are grateful to Dr. Dulguun Batbold (VISA) for providing us with this data.

Table 2. Consumption Expenditures

Correlations in levels

Household ZIP code income distribution

	Q_1	Q_2	Q_3	Q_4
Q_1	1.000	0.994	0.986	0.965
Q_2	–	1.000	0.997	0.985
Q_3	–	–	1.000	0.994
Q_4	–	–	–	1.000

Table 2: Correlation in consumption levels across the household zip code income distribution, January 2020 to March 2023, as measured by credit card expenditure indexed to the home address of the credit card. The correlations between the richest and poorest quartiles are high, consistent with the nature of the model equilibrium.

Table 3. Consumption Expenditures

Correlations in growth rates

Household ZIP code income distribution

	Q_1	Q_2	Q_3	Q_4
Q_1	1.000	0.980	0.956	0.905
Q_2	–	1.000	0.983	0.942
Q_3	–	–	1.000	0.974
Q_4	–	–	–	1.000

Table 3: Correlation in consumption growth rates across the household zip code income distribution, January 2020 to March 2023, as measured by credit card expenditure indexed to the home address of the credit card. The correlations between the richest and poorest quartiles are high, close to the model prediction of 1.0.

Table 4. Consumption Expenditures

Correlations in SMIs

Household ZIP code income distribution

	Q_1	Q_2	Q_3	Q_4
Q_1	1.000	0.996	0.988	0.957
Q_2	–	1.000	0.996	0.967
Q_3	–	–	1.000	0.985
Q_4	–	–	–	1.000

Table 4: Correlation in SMIs across the household ZIP code income distribution.

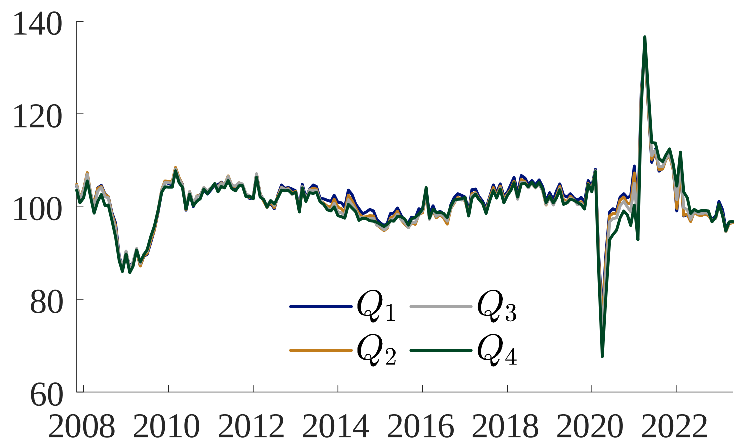

Figure 6: SMI by income quartile.

spending than the previous year). Income quartiles are assigned in the same way as Chetty et al. (2023). The indexes by quartile are monthly and cover the period Nov. 2007-May 2023. The SMIs by quartile are highly correlated (Table 4 and Figure 6).

Recent work by Meyer and Sullivan (2023) on consumption inequality during the postwar era also provides some insightful data that can be used for comparisons to the predictions of the model equilibrium. The model in the present paper predicts that consumption inequality (the consumption Gini as well as the 90:10 ratio) would be constant in the equilibrium with the optimal fiscal-monetary policy mix. Meyer and Sullivan (2023) find that consumption inequality (90:10 ratio) has been about constant or slightly increasing during the 1995 to 2017 period (see their Figure 1). We interpret this aspect of their findings to be consistent with the model equilibrium of the present paper.

Meyer and Sullivan (2023) also report their estimates of consumption growth based on the Consumer Expenditure Surveys indexing on asset holdings by household groups. They report in their Table 6 the consumption growth observed over differing time intervals for various quintiles of the asset-holding distribution. We divide their findings into relatively stable periods, 2000-2006 (their column 7) and 2010-2017 (their column 10), and one unstable period, the one that is centered around the global financial crisis (GFC), 2006-2010 (their column 9). The model equilibrium in the present paper predicts that all of these consumption growth rates should be equal. For the 2000-2006 period, the consumption growth rates across quintiles are reported by Meyer and Sullivan (2023) as $\{16.3, 19.1, 20.0, 23.9, 23.4\}$, and for 2010-2017 they are reported as $\{9.7, 12.5, 10.6, 17.3, 11.8\}$. The ranges are 7.1 and 7.6 percentage points, respectively, for these two periods. These consumption growth rates are clearly not equalized, but might viewed as tolerable given that Meyer and Sullivan (2023) stress that measurement issues are acute in considering household level consumption expenditures. However, for the 2006-2010 period Meyer and Sullivan report $\{11.5, -9.2, -3.4, -12.5, -4.9\}$, and so the range is 24 percentage points. Even acknowledging measurement issues, this suggests that the model equilibrium does not provide a good fit to this data during the 2006 to 2010 period, the time of the GFC. The shock at that time was quite large and likely caused a lot of redistribution. This result will be corroborated when we look at interest rates and consumption growth rates below.

Coibion, Gorodnichenko, and Koustas (2021) show that the rise in expenditure inequality measured at high frequency can be consistent with unchanging consumption inequality due to a decreased shopping frequency for storable goods since 1980.

5.4 Heckman-type labor supply

The model equilibrium is calibrated to match observed hours worked by cohort for life cycle households. A stylized fact emphasized in the empirical labor literature is that an appropriately specified cross-section or panel regression will suggest that hours worked are largely independent of real wage changes. Carneiro and Heckman (2003, p. 196) state that, "In a modern society, in which human capital is a larger component of wealth than is land, a proportional tax on human capital is like a nondistorting Henry George tax as long as labor supply responses are negligible. Estimated intertemporal labor supply elasticities are small, and welfare effects from labor supply adjustment are negligible." Does the model equilibrium here meet the data as characterized by Carneiro and Heckman?

To compare the model equilibrium here to the empirical literature, consider Jantti, Pirttila, and Selin (2015). They have a general framework for an empirical analysis of labor supply elasticities. They suggest the following starting point (their notation):

$$h_{i,t} = \beta \ln \left[\left(1 - \tau_{i,t}\right) w_{i,t} \right] + \gamma R_{i,t} + \epsilon_{i,t}, \tag{16}$$

where i indexes individuals, t indexes time, h is hours worked annually, τ is a marginal tax rate, w is the gross hourly wage rate, R is virtual income which includes other household income including capital income, β and γ are parameters to be estimated and ϵ is an error term. They make further adjustments to this equation for purposes of estimation, but this represents the core idea.

In the present model we know the hours worked choice for all households at all dates, which is given by (see the Appendix):

$$1 - \ell_{t-s,i}(t) = 1 - \frac{1-\eta}{T+1}\frac{1}{e_s}\sum_{j=0}^{T} e_j, \tag{17}$$

for $s = 0, ..., T$ where ℓ is the leisure hours chosen, η is a preference parameter representing the relative desirability of leisure versus consumption and e_s, $s = 0, ..., T$, is the baseline life cycle productivity profile.[30] For life cycle agents hours worked depends on the stage of the life cycle, and portions of the life cycle with higher productivity (the middle) are associated with higher levels of hours worked. And in particular, hours worked in the model's equation (17) does not depend on either the real wage per effective efficiency unit nor on the capital income embedded in $R_{i,t}$ in equation (16). The *prima facie* expectation would be that a regression of the type specified in equation (16) using data produced by an economy where hours worked is being determined by equation (17) would lead to the inference that β and γ are equal to or close to zero depending on the details of how the estimation is carried out.[31]

We interpret these observations as suggesting model equilibrium labor supply elasticities are broadly consistent with those estimated in the empirical labor literature.

These observations also mean that, in the model equilibrium, linear labor income taxes do not distort labor supply so long as the tax is applied equally across all stages of the household's life cycle. To see this, consider a linear tax $(1 - \tau_i)$ on labor earnings applied to each period of an agent's life cycle. In equation (17), such a tax would multiply every element of e_s for agent i. Evidently, the factor $(1 - \tau_i)$ would then cancel out in this expression, leaving hours worked unchanged.[32] The nondistortionary nature of linear labor income taxation in the model equilibrium allows the government to fund an unemployment insurance scheme to mitigate idiosyncratic risks agents face.[33]

[30] For hand-to-mouth agents this formula simplifies to the constant η, as their productivity profiles are perfectly flat.

[31] A possible exception to this statement could be that if the regression was estimated for a panel dataset, then according to the model labor supply would be increasing during certain portions of the life cycle (such as ages 25-54), and at the same time in the model real wages are growing, and hence one may find a positive correlation (positive estimated value for β) in that case.

[32] In fact, agent i has drawn a scale factor $x_{lc,i}$ when entering the economy, but that scale factor also cancels in this expression. This means that both rich and poor agents work the same number of hours at each stage of the life cycle.

[33] Prescott (2004) argued that the tax differences on labor income across countries account for observed differences in hours worked—broadly speaking, across Europe versus the U.S. The present model would put more emphasis on differences in life cycle productivity profiles across economies.in driving differences in hours worked.

Business cycle employment changes in the model come from $N(t)$ which can be interpreted as an extensive margin, and which is not modeled in more detail in this paper.[34]

5.5 Marginal propensities to consume

The model equilibrium naturally predicts that some life cycle agents will be hand-to-mouth consumers at some points in time, in the sense that they will consume the entirety of their labor income in a particular period. Of course, the hand-to-mouth households are, by design, a household type that supplies labor each period inelastically and does not use asset markets to smooth consumption, so that they consume their entire labor income each period.

We take the derivative of consumption with respect to a change in labor income as a measure of the MPC in our model equilibrium. This MPC is easy to derive and will differ over the life cycle. In particular, life cycle households will, at times, have MPCs of this type equal to unity, in particular near the beginning of the life cycle and again toward the end of the life cycle. The first case represents (along with HTM households) asset-poor agents with MPCs of unity, while the second case represents relatively asset-rich households with MPCs equal to one. These ideas are illustrated in Figure 7 for our calibrated case.

Kaplan and Violante (2022) survey a range of models and their implications for the marginal propensity to consume, which they define as the propensity to consume out of a relatively small windfall to income. They emphasize that many models predict MPCs that appear to be too low (lower than the MPCs in Figure 7) compared to empirical estimates. In this sense the MPCs in Figure 7 might be viewed as more realistic than what would be obtained from many other models. Kaplan and Violante (2022) also emphasize that actual MPCs are quite heterogeneous, and Figure 7 also suggests heterogeneous MPCs, which might be considered a step in the right direction.

The optimal monetary-fiscal policy mix suggested in this paper does not, however, hinge on the heterogeneity of MPCs nor on the specific values of the MPCs. Instead, these MPCs are an outcome that would be observed when the optimal monetary-fiscal monetary policy mix is in place. In particular, the life cycle households need to use credit markets to smooth consumption over the life cycle, and the optimal monetary policy fully mitigates the credit market friction so that the credit market works smoothly for this purpose.

[34]See Peterman (2012) for a discussion of "reconciling micro and macro frisch elasticities." Peterman (2012) emphasizes including what might be called extensive margin workers in empirical analysis of labor supply elasticities, and that these types of workers account for most of the difference in estimates available in the literature.

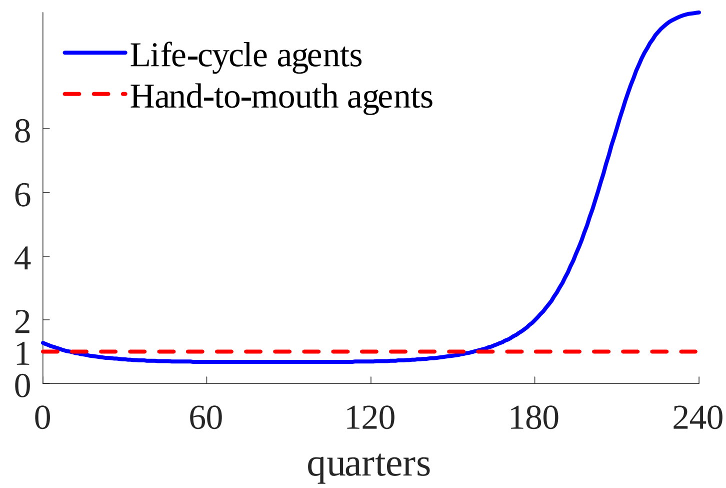

Figure 7: A cross-section diagram of marginal propensities to consume out of labor earnings at each date in the model equilibrium. Relatively young (asset poor) and older (asset rich) life cycle agents have a MPC larger than one. The MPC of life cycle agents during the middle of life is 0.69.

5.6 Tax progressivity and the consumption Gini

Life cycle agents in this model receive a scaled version of the standard life cycle productivity profile shown in Figure 1. The scaled version is denoted $x_{lc,i}e_s$, $s = 0, ..., T$, and the HVY shock $x_{lc,i}$ is a draw from a lognormal distribution with standard deviation σ_{lc}. Agents that receive a high draw will earn a correspondingly high level of before-tax labor income over their life cycle, whereas agents that receive a low draw will earn a correspondingly low level of before-tax labor income over their life cycle. This is a summary method of representing an unmodeled process of human capital accumulation through pre-age 20 youth that would include parenting, schooling, on-the-job training and other factors. We assume this process is distorted. If the process was ideal, the resulting scaling distribution would have a standard deviation of $\sigma^{\star}_{lc} < \sigma_{lc}$, that is, these processes would develop pre-age 20 human capital exactly appropriately and in a non-distortionary manner. Because σ_{lc} is "too large," the consumption Gini will be larger than desired in the model equilibrium. The goal of an optimal tax-transfer scheme is to lower the variance of beginning-of-the-life-cycle scaling for life cycle households to $\sigma^{\star}_{lc}$. In a limiting case, we could think of $\sigma^{\star}_{lc} = 0$, that is, every life cycle agent receives the same baseline profile e as they enter the model.

We have assumed a progressive labor income tax scheme to reduce the consumption Gini to the level consistent with $\sigma^{\star}_{lc}$. The effect is to undo some of the variance in the initial productivity profile allocation process. The government is insuring against beginning of economic life HVY shocks.

Table 5. Assets and Gini Coefficients

Parameters	h	0.25
	$\sigma_{lc,pre-tax}$	1.45
	$\sigma_{lc,post-tax}$	0.55
	σ_{htm}	0
U.S. data	$A/(4Y)$	4.14
	$G_{w,pre-tax}$	0.78
	$G_{y,pre-tax}$	0.63
	$G_{c,pre-tax}$	–
	$G_{c,post-tax}$	0.32
Model	$A/(4Y)$	4.13
	$G_{w,pre-tax}$	0.78
	$G_{w,post-tax}$	0.62
	$G_{y,pre-tax}$	0.72
	$G_{y,post-tax}$	0.39
	$G_{c,pre-tax}$	0.70
	$G_{c,post-tax}$	0.32

Table 5: Assets-to-output ratio and Gini coefficients in the model versus the U.S. data. "Pre-tax" denotes the values that would occur if the progressive labor income tax was zero. "Post-tax" denotes the values that occur in the presence of a tax-transfer program sufficient to lower the consumption Gini to the observed U.S. data value of 0.32.

The tax-transfer itself for agent i can be represented by $(1-\tau_i^p)\, x_{lc,i}\tilde{c}_{s,i}$, and we set $(1-\tau_i^p) = \frac{x_{lc,j}}{x_{lc,i}}$, so that we are using the tax-transfer to simply replace $x_{lc,i}$ with $x_{lc,j}$, where $x_{lc,j}$ is the *corresponding* draw to $x_{lc,i}$ but from a distribution with standard deviation $\sigma_{lc}^{\star}$ instead of σ_{lc}. Agents far above the median are taxed the most, agents far below the median receive the largest transfer in proportional terms, net revenue is zero, and there is no distortion to real output. Other Gini coefficients are also affected. We apply this tax to attain the consumption Gini in the U.S. data, as outlined in Table 5.

Table 5 shows that the model can be calibrated to match the financial wealth Gini of 0.78 in the U.S. data by choosing $\sigma_{lc} = 1.45$ and $\sigma_{htm} = 0$ (the fraction of HTM agents is low, so variance on this dimension is not empirically important). This choice is associated with an income Gini $G_y = 0.72$ versus the U.S. data value of 0.63. We could alternatively choose a somewhat lower value of $\sigma_{lc} = 0.55$ in order to match the income Gini exactly, but at the cost of missing the wealth Gini to the low side. But regardless of which way one proceeds on this question, the model equilibrium suggests a pre-tax and transfer consumption Gini that is not too far below the income Gini—in Table 5 this value is 0.70. We interpret this to mean that progressive taxation and other measures in actual U.S. policy are sufficient to reduce the consumption Gini substantially. Accordingly, we apply our redistributive consumption tax to match the post-tax-and-transfers consumption Gini observed in the data of 0.32. The effects of this are shown in Table 5 as "post-tax" values.

These ideas are further illustrated in Figure 8. The Figure plots values for the consumption, income, and wealth Gini coefficients in the model equilibrium for alternative values of $\sigma_{lc} \geq 0$. The model equilibrium maintains an ordering with the wealth Gini highest, the income Gini second highest, and the consumption Gini lowest, as in the U.S. data. High values of σ_{lc} drive all three Gini coefficients toward one. A particular value of σ_{lc} can be chosen to match either the wealth, the income, or the consumption Gini, but not all three. The calibrated value in Table 5 matches the pre-tax (that is, zero progressive taxation) wealth Gini but leaves the pre-tax income Gini too high, as the vertical black line in Figure 8 indicates. The Figure indicates that, generally speaking, any calibration that matches either the wealth or the income Gini on a pre-tax basis will have a relatively high associated pre-tax-and-transfer consumption Gini.

This leaves room for a redistributive consumption tax to lower the consumption Gini in order to match the post-tax-and-transfer consumption Gini observed in the U.S. data. The redistributive tax is doing a lot of work: According to our calibration, it is reducing the consumption Gini from 0.70 to 0.32.[35] However, as Figure 8 makes clear, if the redistributive scheme we outline was doing this much work, the associated post-tax income and wealth Gini coefficients would also be substantially lower. We conclude that while the model can match certain Gini coefficients in the U.S. data, it does not match all three simultaneously, and that tax-transfer programs interact with observed Gini coefficients in quantitatively important ways.

Figure 8 also illustrates that, even at $\sigma_{lc} = 0$ (which is the case where all life cycle agents receive the same beginning of economic life profile) the wealth and income Ginis would be positive, as shown on the vertical y-axis in the Figure. This is due to life cycle effects alone—even completely equal households earn more income in the middle of economic life than they do at the beginning or the end, and they hold assets over the life cycle in order to smooth lifetime consumption, driving positive income and wealth Gini coefficient values.[36]

5.7 Nominal returns

The model equilibrium predicts equalized nominal and real returns for three assets under optimal monetary policy: capital, MBS and Treasuries. These assets are not further differentiated inside the model. To compare to the U.S. data, we need an asset

[35] In recent work using German data, Haan, Kempster, and Prowse (2019) use a life cycle model to estimate that the tax-and-transfer system is sufficient to offset 54 percent of the inequality in lifetime earnings. This would be the same order of magnitude as what the model in the present paper suggests for the U.S.

[36] Also see the discussion in Heathcote, Storesletten, and Violante (2020).

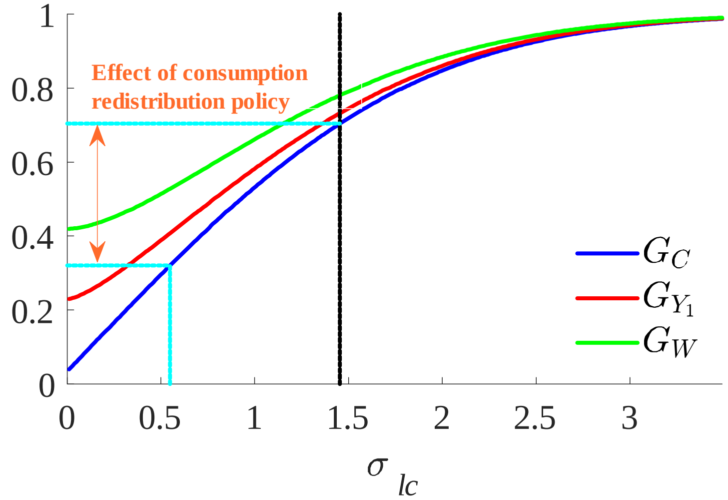

Figure 8: The consumption, income, and financial wealth Gini coefficients in the model equilibrium for values of $\sigma_{lc} \geq 0$.The redistributive consumption tax is lowering the consumption Gini from 0.69 to 0.32, but at the expense of missing on other Gini coefficients. The social optimum at $\sigma_{lc} = 0$ still leaves positive income and wealth Ginis due to life cycle effects.

representing a return to capital in a format with risk characteristics similar to MBS and Treasuries. One candidate is a high-quality corporate bond. We use a seven-year nominal investment grade corporate bond metric. In the model and the data, this type of bond has a seven-year horizon but can be refinanced each period.

The model equilibrium predicts that the nominal return on the assets should be equal to the nominal consumption growth rate, or, equivalently in the model, the nominal output growth rate. This prediction might be expected to hold in periods of relative stability with optimal monetary, fiscal, and labor market policy. In these circumstances the private sector is able to set nominal debt contracts relying on the monetary authority to set the price level that ratifies those debt contracts ex-post.

Arguably, however, the U.S. economy has been disturbed by two large unanticipated shocks since 2005: (1) the global financial crisis, and (2) the global pandemic. For the purposes of this paper, these events are simply “large disturbances” outside the scope of this model. The interim period, $2011 - 2019$, fits the model assumptions better and we may expect the model provide a better model fit to the data during this time frame.

Figure 9 shows that measures of U.S. nominal consumption growth and nominal GDP growth[37] on a 12-month basis are approximately equal to the nominal return

[37] Lewis-Mertens-Stock is an index using weekly data meant to track the percent change in real GDP growth from one year earlier. Adding 12-month core PCE inflation monthly gives a monthly measure of nominal GDP growth from one year earlier.

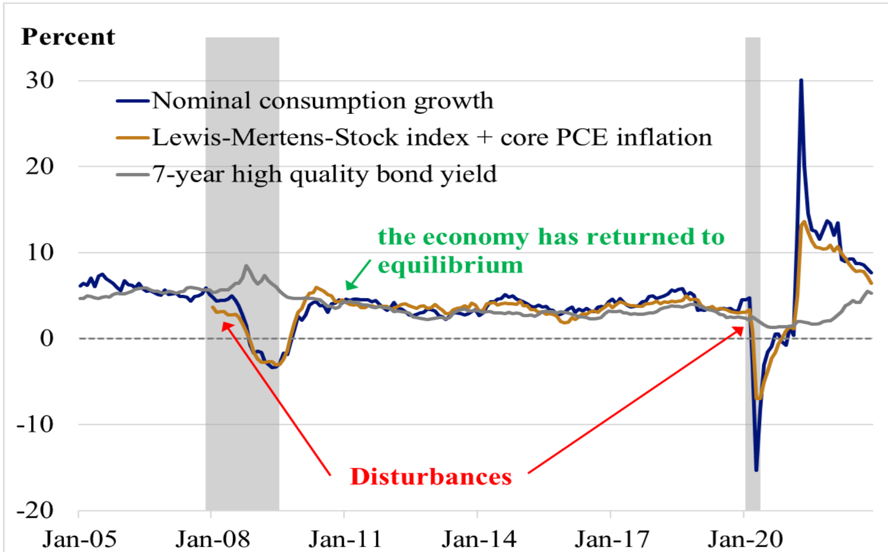

Figure 9: The model with the optimal monetary-fiscal policy mix predicts the gray line representing the nominal interest rate on newly-issued corporate debt will coincide with nominal consumption growth and nominal GDP growth represented by the Lewis-Mertens-Stock index plus monthly PCE inflation. This prediction broadly holds in the Figure outside of the two large disturbances.

on a 7-year high quality corporate bond between 2011 and 2019, as predicted by the model equilibrium under optimal policy. However, nominal growth rates and interest rates are considerably different during large, unanticipated shocks like the GFC and the pandemic. The chart suggests that the conditions of macroeconomic equilibrium were re-established relatively quickly after the GFC, and also appear to be close to being re-established following the pandemic.

5.8 Summary and future extensions

This calibration and discussion has highlighted several findings. First, the model predicts consumption behavior and labor supply behavior which appears to be broadly consistent with observed U.S. data. Consumption growth by income group does not, however, match the data presented by Meyer and Sullivan (2023) for the global financial crisis. Similarly, the time series data on nominal returns and nominal consumption growth broadly matches the predictions of the model, but not during the global financial crisis or during the pandemic. A natural conclusion is that the model works well during "normal times" but does not provide reliable results for exceptionally large shocks. The core assumption in the calibration that the four macroeconomic policies represent an optimal package falls short in these cases.

This suggests, in turn, that there is room for improvement to enlist additional macroeconomic policies that would be used on the relatively rare occasions when there is a major financial crisis, a pandemic, or some type of other relatively rare but large shock. The existing policies in the model can be viewed as forms of insurance for an uncertain world: when households are confronted by some aspect of the world which is considered risky, a natural response is to insure. A more elaborate model would include additional insurance for the relatively rare but large shocks.

Can the macroeconomic policies suggested in this paper substitute for one another? The prospects for substitutability appear to be quite limited. Instead, the model suggests we should look to develop multiple tools for multiple problems. In particular, the model does not suggest stretching monetary policy to attempt to solve the idiosyncratic risk problem (better addressed by social insurance, such as unemployment insurance), provide an additional asset for the economy (better addressed by the Treasury policy), or provide needed redistribution (better provided by the fiscal authority).

6 Conclusion

This paper has provided a benchmark model to evaluate macroeconomic policy in the U.S. since 1995 and suggest directions for further improvement. The model recommends a monetary policy characterized by achievement of the Wicksellian natural real rate of interest, a social insurance policy funded by linear labor income taxes, a fixed stock of national debt relative to aggregate output, and an income redistribution program designed to lower the level of the consumption Gini coefficient. A welfare theorem characterizes the sense in which the implied allocations would be viewed by a planner as first-best.

Broadly speaking, this type of macroeconomic policy package is the one that is actually followed in many countries in recent decades.

We have calibrated this model to U.S. data assuming that these optimal model policies are close to the ones actually being used. We presented *prima facie* evidence that the model equilibrium fits the data reasonably well in normal times across a variety of metrics. In more extreme times, when shocks are considerably larger and arise from sources not considered in the ambient stochastic structure of the model, model fit deteriorates and the model predicts considerable reallocation. We conclude that better macroeconomic policy design for the future will leave the current macroeconomic policy package in place but will have specialized and possibly very different policies that would be called upon in reaction to very large and unusual shocks like the GFC and the global pandemic.

References

[1] Acharya, S., E. Challe, and K. Dogra. 2023. "Optimal Monetary Policy According to HANK." *American Economic Review* 113(7): 1741-1782.

[2] Auclert, A. 2019. "Monetary Policy and the Redistribution Channel." *American Economic Review* 109(6): 2333-2367.

[3] Azariadis, C. 1993. *Intertemporal Macroeconomics*. Blackwell.

[4] Azariadis, C., J. Bullard, A. Singh, and J. Suda. 2019. "Incomplete Markets and Monetary Policy." *Journal of Economic Dynamics and Control* 103: 83-101.

[5] Bhandari, A., D. Evans, M. Golosov, and T. Sargent. 2021. "Inequality, Business Cycles, and Monetary-Fiscal Policy." *Econometrica* 89(6): 2559-2599.

[6] Bai, Y., J-V. Ríos-Rull, and K. Storesletten. 2017. "Demand Shocks as Productivity Shocks." Unpublished manuscript, University of Pennsylvania.

[7] Board of Governors of the Federal Reserve System. 2019. *Report on the Economic Well-Being of U.S. Households in 2018*. May.

[8] Bullard, J. 2014. "Comment on K. Sheedy, 'Debt and Incomplete Financial Markets.' " *Brookings Papers on Economic Activity*, Spring, pp. 362-368.

[9] Bullard, J., and R. DiCecio. 2021. "Optimal Monetary Policy for the Masses." Federal Reserve Bank of St. Louis *Working Paper 2019-009D.*

[10] Bullard, J., and J. Feigenbaum. 2007. "A Leisurely Reading of the Life Cycle Consumption Data." *Journal of Monetary Economics* 54(8): 2305-2320.

[11] Carneiro, P., and J. Heckman. 2003. "Human Capital Policy." In: *Inequality in America: What Role for Human Capital Policies?*, edited by Benjamin Friedman, 77–239. Cambridge, Mass.: MIT Press.

[12] Castañeda, A., J. Díaz-Gímenez, and J-V. Ríos-Rull. 2003. "Accounting for U.S. Earnings and Wealth Inequality." *Journal of Political Economy* 111(4): 818-857.

[13] Chari, V., and P. Kehoe. 1999. "Optimal Fiscal and Monetary Policy." *Handbook of Macroeconomics* Volume 1, Part C, pp. 1671-1745.

[14] Chetty, R., J. Friedman, N. Hendren, M. Stepner and the Opportunity Insights Team. 2023. "The Economic Impacts of COVID-19: Evidence from a New Public Database Built Using Private Sector Data." *Quarterly Journal of Economics,* forthcoming.

[15] Chiang, Y-T, and P. Zoch. 2022. "Asset Supply and Liquidity Transformation in HANK." Federal Reserve Bank of St. Louis *Working Paper 2022-038A.*

[16] Coibion, O., Y. Gorodnichenko and D. Koustas. 2021. "Consumption Inequality and the Frequency of Purchases." *American Economic Journal: Macroeconomics* , 13(4): 449-482.

[17] Cooley, T., and E. Prescott. 1995. "Economic Growth and Business Cycles." In T. Cooley, ed., *Frontiers of Business Cycle Research*, Princeton University Press, pp. 1-38.

[18] Davila, E., and A. Schaab. 2023. "Optimal Monetary Policy with Heterogeneous Agents: Discretion, Commitment, and Timeless Policy." NBER Working Paper #30961. February.

[19] Doepke, M., and M. Schneider. 2006. "Inflation and the Redistribution of Nominal Wealth." *Journal of Political Economy* 114(6): 1069-1097.

[20] Fahri, E., and I. Werning. 2016. "A Theory of Macroprudential Policies in the Presence of Nominal Rigidities." *Econometrica* 84(5): 1645-1704.

[21] Federal Deposit Insurance Corporation (FDIC). 2022. *2021 FDIC National Survey of Unbanked and Underbanked Households*. Available at: **??** .

[22] Feng, Z., and M. Hoelle. 2017. "Indeterminacy in Stochastic Overlapping Generations Models: Real Effects in the Long Run." *Economic Theory* 63: 559-585.

[23] Galí, J. 2018. "The State of New Keynesian Economics: A Partial Assessment." *Journal of Economic Perspectives* 32(3): 87-112.

[24] Giannoni, M., and M. Woodford. 2004. "Optimal Inflation Targeting Rules." In B. Bernanke and M. Woodford, eds., *The Inflation Targeting Debate*, University of Chicago Press, Chapter 3, pp. 93-171.

[25] Gonzalez, B., G. Nuno, D. Thaler, and S. Albrizio. 2021. "Optimal Monetary Policy with Heterogeneous Firms." Working paper, Banco de Espana.

[26] Greenspan, A. 2007. *The Age of Turbulence*. Penguin Press.

[27] Haan, P., D. Kemptner, and V. Prowse. 2019. "Insurance, Redistribution, and the Inequality of Lifetime Income." Working paper, DIW Berlin. September 30.

[28] Heathcote, J., Perri, F., and G. Violante. 2010. "Unequal We Stand: An Empirical Analysis of Economic Inequality in the U.S., 1967-2006." *Review of Economic Dynamics* 13(1): 15-51.

[29] Heathcote, J., K. Storesletten, and G. Violante. 2020. "How Should Tax Progressivity Respond to Rising Income Inequality?" Working paper, Princeton.

[30] Huggett, M., G. Ventura, and A. Yaron. 2011. "Sources of Lifetime Inequality." *American Economic Review* 101(7): 2923-2954.

[31] Jantti, M., J. Pirttila, and H. Selin. 2015. "Estimating Labor Supply Elasticities Based on Cross-Country Micro Data: A Bridge Between Micro and Macro Estimates?" *Journal of Public Economics* 127: 87-99.

[32] Kaplan, G., B. Moll, and G. Violante. 2018. "Monetary Policy According to HANK." *American Economic Review* 108(3): 697-743.

[33] Kaplan, G., and G. Violante. 2022. "The Marginal Propensity to Consume in Heterogeneous Agent Models." NBER Working Paper 30013.

[34] Koenig, E. 2013. "Like a Good Neighbor: Monetary Policy, Financial Stability, and the Distribution of Risk." *International Journal of Central Banking*, June, pp. 57-82.

[35] Krusell, P., and T. Smith. 1998. "Income and Wealth Heterogeneity in the Macroeconomy." *Journal of Political Economy* 106(5): 867-896.

[36] Le Grand, F., A. Martin-Baillon, and X. Ragot. 2022. "Should Monetary Policy Care About Redistribution? Optimal Fiscal and Monetary Policy with Heterogeneous Agents." Unpublished manuscript, University of Copenhagen.

[37] McClelland, R., and S. Mok. 2012. "A Review of Recent Research on Labor Supply Elasticities." Congressional Budget Office *Working paper 2012-12.*

[38] McKay, A., and C. Wolf. 2023a. "Monetary Policy and Inequality." *Journal of Economic Perspectives* 37(1): 121-144.

[39] McKay, A., and C. Wolf. 2023b. "Optimal Monetary Policy Rules in HANK." Unpublished manscript, MIT.

[40] Meyer, B., and J. Sullivan. 2023. "Consumption and Income Inequality in the U.S. Since the 1960s." *Journal of Political Economy* 131(2): 247-284 February.

[41] Nuño, G., and C. Thomas. 2022. "Optimal Redistributive Inflation." *Annals of Economic and Statistics* 146: 3-64.

[42] Peterman, W. 2012. "Reconciling Micro and Macro Estimates of the Frisch Labor Supply Elasticity." Board of Governors of the Federal Reserve System *Working paper 2012-75.*

[43] Prescott, E. 2004. "Why Do Americans Work So Much More Than Europeans?" Federal Reserve Bank of Minneapolis *Quarterly Review* 28(1): 2-13.

[44] Ríos-Rull, J-V. 1996. "Life Cycle Economies and Aggregate Fluctuations." *Review of Economic Studies* 63(3): 465-489.

[45] Ruggles, S., S. Flood, R. Goeken, J. Grover, E. Meyere, J. Pacas, and M. Sobek. 2022. *IPUMS USA: Version 12.0 [dataset].* Minneapolis, MN: IPUMS.

[46] Sargent, T., N. Wang, and J. Yang. 2021. "Stochastic Earnings Growth and Equilibrium Wealth Distributions." NBER Working Paper 28473.

[47] Sheedy, K. 2014. "Debt and Incomplete Financial Markets: A Case for Nominal GDP Targeting." *Brookings Papers on Economic Activity*, Spring, pp. 301-361.

[48] Smirnov, D. 2022. "Optimal Monetary Policy in HANK." Unpublished manuscript, Universitat Pompeu Fabra.

[49] U.S. Census Bureau, *American Community Survey*, 2000-2021.

[50] Werning, I. 2014. "Comment on K. Sheedy, 'Debt and Incomplete Financial Markets.'" *Brookings Papers on Economic Activity*, Spring, pp. 368-372.

[51] Woodford, M. 2003. *Interest and Prices.* Princeton University Press.

[52] Woodford, M. 2012. "Methods of Policy Accommodation at the Interest-Rate Lower Bound." Paper presented at the Jackson Hole Symposium, Federal Reserve Bank of Kansas City, September 16.

A Appendix: Solution details

A.1 Overview

The model features heterogeneous households and an aggregate shock, so that the evolution of the asset-holding distribution in the economy is part of the description of the equilibrium. This would normally require numerical computation. However, symmetric scaling, log preferences, and other simplifying assumptions allow solution by "pencil and paper" methods. In this appendix we outline this solution in some detail. A key feature of the solution is that the asset-holding distribution will be linear in the date t real wage $w(t)$, and so will simply shift up and down with changes in

$w(t)$. We do not claim uniqueness of this equilibrium, but we regard the equilibrium we isolate as a natural focal point for this analysis.[38]

The heart of the matter is that the model solution comes down to a sequence of real interest rates for doubly-infinite real time $t \in (-\infty, +\infty)$. The guess is that, thanks to many simplifying assumptions, this sequence will be exactly equal to the stochastic growth rate of the economy at each date, that is, "real interest rate equals output growth rate," a familiar finding from parts of the theoretical OG literature. To verify that this guess is correct, we will show that households solve their optimization problems, the representative firm optimizes profits, and the general equilibrium condition that aggregate asset supply and aggregate asset demand are equalized at the proposed interest rate is met.

The model includes four macroeconomic policies. Two of these are simple taxation policies (the linear labor income tax of the labor market authority, and the redistributional linear labor income tax of the fiscal authority). The monetary authority has a state-contingent policy which is to set the price level after observing the stochastic growth rate each period according to (9). We can substitute these three policies into the households' problems. (The fourth policy, the Treasury authority debt issuance policy, shows up only in the general equilibrium portion of the analysis as it provides a second asset in positive net supply).

With these considerations, we can proceed to solve household problems with the three policies substituted in. The main result is that under the guess that the real interest rate is equal to the stochastic growth rate, the household problems can be solved without reference to future uncertainty, that is, that date t consumption, asset holding, and labor supply can be represented in closed form for all households i of both types in the economy. The monetary authority is providing a type of insurance to households by adjusting the price level to ratify the nominal contracts committed to by households in the previous period.[39]

It can then be shown that the general equilibrium condition is also met, namely, that aggregate asset supply is equal to aggregate asset demand at the proposed interest rate.

This completes the description of the equilibrium. We now turn to show these results in detail.

(1) We first state household problems with the two macroeconomic tax policies included, but with nominal assets so that we can see where the monetary policy will enter these problems.

(2) We then substitute in the monetary policy and restate the household problems.

[38] See Feng and Hoelle (2017) for a recent discussion and analysis. Typical quantitative-theoretic applications in the area of stochastic OLG would be unable to address the issues brought out by the Feng and Hoelle (2017) analysis.

[39] Another way to say this is that the monetary authority ensures that households have exactly the nominal income necessary to pay off the nominal contract at the nominal contract rate R^n.This is the same result discussed in Koenig (2013) and Sheedy (2014).

(3) We then state the conjecture that the real interest rate will equal the output growth rate and substitute this into the household problems.

(2) We solve household problems for the cohort entering the economy at an arbitrary date t under the proposed policies and the conjecture for the real interest rate, and show that households can choose date t consumption, net assets, and hours worked without reference to future uncertainty.

(3) We solve household problems for other households entering the economy at earlier dates with shorter horizons and asset holdings from the previous period and show that, similarly, these households can choose date t consumption, net assets, and hours worked without reference to future uncertainty.

(4) We provide related solutions for hand-to-mouth households. These households hold no net assets.

(5) We verify that the aggregate asset market clearing condition is satisfied. This establishes the equilibrium.

We assume interior solutions, which implies a set of joint restrictions on the value of η and the baseline endowment sequence $e = \{e_s\}_{s=0}^{T}$. This set of joint restrictions is sufficient to guarantee interior solutions for all households i. Our calibration meets these interiority conditions.

A.2 Solutions for the date t life cycle cohort

First consider a life cycle household i entering the economy at date t with the preferences we have assumed, where we have substituted in the progressive labor income tax factor $(1 - \tau_i^p)$, the labor income tax imposed by the labor authority running the unemployment insurance program $(1 - \tau^u)$, and we have left asset holding in nominal terms so that we can see where the price level enters the household problem:

$$\max_{\{\tilde{c}_{t,i}(t+s),\ell_{t,i}(t+s)\}_{s=0}^{T}} E_t \left[\sum_{s=0}^{T} \eta \ln \tilde{c}_{t,i}(t+s) + (1-\eta) \ln \ell_{t,i}(t+s) \right], \tag{18}$$

with $\tilde{c}_{t,i}(t+s) \equiv D(t) c_{t,i}(t+s)$ as defined in the text, subject to the consolidated[40] lifetime budget constraint

$$\begin{aligned} \tilde{c}_{t,i}(t) + \sum_{s=1}^{T} \left(\frac{P(t+s)}{P(t)} \frac{\tilde{c}_{t,i}(t+s)}{\mathcal{R}^n} \right) &\leq x_{lc,i}(1-\tau_i^p)(1-\tau^u) e_0 w(t) [1 - \ell_{t,i}(t)] \\ &+ \sum_{s=1}^{T} \left(\frac{P(t+s)}{P(t)} \frac{x_{lc,i}(1-\tau_i^p)(1-\tau^u) e_s w(t+s) [1-\ell_{t,i}(t+s)]}{\mathcal{R}^n} \right) \end{aligned} \tag{19}$$

[40]Consolidation, that is, of the sequence of period-by-period budget constraints through the remainder of the agent's economic life cycle.

where

$$\mathcal{R}^n = \prod_{j=1}^{s} R^n \left(t+j-1, t+j\right). \tag{20}$$

The proposed state-contingent price level path used by the monetary authority is given by

$$P\left(t+1\right) = \frac{R^n \left(t, t+1\right)}{\nu \left(t, t+1\right) \delta \left(t, t+1\right) \lambda \left(t, t+1\right)} P\left(t\right) \tag{21}$$

for all t, with $P\left(0\right) > 0$. Substitution into the budget constraint, and with some rearranging, yields

$$\tilde{c}_{t,i}\left(t\right) + \sum_{s=1}^{T} \left(\frac{\tilde{c}_{t,i}\left(t+s\right)}{\mathcal{R}}\right)$$
$$\leq x_{lc,i}\left(1-\tau_i^p\right)\left(1-\tau^u\right)\left[e_0 w\left(t\right)\left[1-\ell_{t,i}\left(t\right)\right] + \sum_{s=1}^{T} \frac{e_s w\left(t+s\right)\left[1-\ell_{t,i}\left(t+s\right)\right]}{\mathcal{R}}\right]. \tag{22}$$

where

$$\mathcal{R} = \prod_{j=1}^{s} R\left(t+j-1, t+j\right), \tag{23}$$

and where $R\left(t+j-1, t+j\right)$ is the gross real rate of interest between date $t+j-1$ and date $t+j$.

We now turn to the representative firm's problem in order to obtain values for $w\left(t+s\right)$ in (22). The marginal product of labor is the real wage per effective unit of human capital supply, given by

$$w\left(t\right) = \left[D\left(t\right) Q\left(t\right) N\left(t\right)\right]^{1-\alpha}\left(1-\alpha\right) K\left(t\right)^{\alpha} L\left(t\right)^{-\alpha}. \tag{24}$$

We denote by $\left[1-\ell_{t-s,i}\left(t\right)\right] \in \left(0,1\right)$ the fraction of time spent working by household i of cohort $t-s$, $s = 0, ..., T$, where $0 < \ell_{t-s,i}\left(t\right) < 1$.[41] The aggegate effective human capital supply $L\left(t\right)$ can be written as $L^{lc}\left(t\right) + L^{htm}\left(t\right)$, where

$$L^{lc}\left(t\right) = n_0^{lc}\int x_{lc,i} e_0\left[1-\ell_{t,i}\left(t\right)\right] dF_{x_{lc}} + n_1^{lc}\int x_{lc,i} e_1\left[1-\ell_{t-1,i}\left(t\right)\right] dF_{x_{lc}}$$
$$+ \cdots + n_T^{lc}\int x_{lc,i} e_T\left[1-\ell_{t-T,i}\left(t\right)\right] dF_{x_{lc}}, \tag{25}$$

and similarly for hand-to-mouth households, where $n_0^{lc}, n_1^{lc}, ..., n_T^{lc}$ are fixed weights representing the relative size of the various cohorts.[42] In the equilibrium we are

[41] Our assumptions are sufficient to ensure only interior solutions for leisure choices. Households will work very few hours in some periods, but not zero.

[42] These weights can be set to unity for this discussion if desired; the weights are intended to be used for the calibration portion of the paper.

constructing, the leisure choices in this expression will depend on fixed parameters alone, and not on $w(t)$, and consequently $L(t) = L$, a constant. We sometimes call this "core hours," and we will verify the result below. Using this, we can calculate the gross rate of growth in the real wage per efficiency unit as

$$\frac{w(t+1)}{w(t)} = \left[\frac{D(t+1)\,Q(t+1)\,N(t+1)}{D(t)\,Q(t)\,N(t)}\right]^{1-\alpha}\left[\frac{K(t+1)}{K(t)}\right]^{\alpha}. \tag{26}$$

In this expression, $K(t)$ is the real value of the physical capital stock, but it could also be viewed as the capital-core hours ratio since core hours is constant. The representative firm is going to have to invest enough in the capital stock to replace depreciating capital and also to account for growth in the economy in order to keep the capital-core hours ratio constant along a balanced growth path, so that investment $X(t)$ is given by

$$\begin{aligned} X(t) &= K(t+1) - \left(1-\delta^{k}\right)K(t) \\ &= \left[\delta(t,t+1)\,\lambda(t,t+1)\,\nu(t,t+1) - 1 + \delta^{k}\right]K(t). \end{aligned}$$

These considerations mean that, using (26),

$$w(t+1) = \delta(t,t+1)\,\lambda(t,t+1)\,\nu(t,t+1)\,w(t). \tag{27}$$

With the law of motion for the real wage per effective efficiency unit (27) in hand, we can use it in (22) to obtain another version of the budget constraint,

$$\begin{aligned} &\tilde{c}_{t,i}(t) + \sum_{s=1}^{T}\left(\frac{\tilde{c}_{t,i}(t+s)}{\mathcal{R}}\right) \\ &\quad \leq x_{lc,i}\left(1-\tau_i^{p}\right)\left(1-\tau^{u}\right)w(t)\left[e_0\left[1-\ell_{t,i}(t)\right] + \sum_{s=1}^{T}\frac{e_s\mathcal{G}\left[1-\ell_{t,i}(t+s)\right]}{\mathcal{R}}\right]. \end{aligned} \tag{28}$$

where

$$\mathcal{G} = \prod_{j=1}^{s}\delta(t+j-1,t+j)\,\nu(t+j-1,t+j)\,\lambda(t+j-1,t+j), \tag{29}$$

and

$$\mathcal{R} = \prod_{j=1}^{s}R(t+j-1,t+j). \tag{30}$$

Just to fix ideas, when $s = 1$, $\mathcal{G}$ is equal to the growth rate from date t to date $t+1$, while $\mathcal{R}$ is equal to the real interest rate from date t to date $t+1$; and similarly in the instance when $s = 2$, $\mathcal{G}$ is equal to the compound growth rate from date t through date $t+2$, and $\mathcal{R}$ is equal to the compound interest rate date t through date $t+2$, and so on until $s = T$.

We are now ready to use the conjecture that the real interest rate is equal to the stochastic output growth rate at each date. The budget constraint now simplifies to,

$$\tilde{c}_{t,i}(t)+\sum_{s=1}^{T}\left(\frac{\tilde{c}_{t,i}(t+s)}{\mathcal{R}}\right)\leq x_{lc,i}\left(1-\tau_i^p\right)\left(1-\tau^u\right)w(t)\sum_{s=0}^{T}e_s\left[1-\ell_{t,i}(t+s)\right]. \quad (31)$$

Since $w(t)$ is known at the time this problem is solved, and since $x_{lc,i}$ is a shock realization that was drawn as this household entered the economy, we have reduced the budget constraint to one that contains no uncertainty about future income on the right hand side. The household then solves this problem where μ is the multiplier on the life-time budget constraint.

The sequence of first order conditions for $s=0,1,...,T$ with respect to consumption are

$$\frac{\eta}{\tilde{c}_{t,i}(t)}=\mu, \quad (32)$$

and

$$\frac{\eta}{\tilde{c}_{t,i}(t+s)}=\frac{\mu}{\mathcal{R}} \quad (33)$$

for $s=1,...,T$, which implies

$$\tilde{c}_{t,i}(t+s)=\mathcal{R}\tilde{c}_{t,i}(t) \quad (34)$$

for $s=1,...,T$. The conjectured state-contingent plan for consumption is that the household will increase consumption at the stochastic rate of output growth.

The first order conditions with respect to leisure ℓ are

$$\frac{1-\eta}{\ell_{t,i}(t+s)}=\mu x_{lc,i}\left(1-\tau_i^p\right)\left(1-\tau^u\right)w(t)e_s \quad (35)$$

for $s=0,...,T$. Using the FOC for $\tilde{c}_{t,i}(t)$ gives choices for leisure

$$\ell_{t,i}(t+s)=\frac{1-\eta}{\eta}\frac{\tilde{c}_{t,i}(t)}{w(t)x_{lc,i}\left(1-\tau_i^p\right)\left(1-\tau^u\right)e_s} \quad (36)$$

for $s=0,...,T$.

We can now substitute consumption choices back into the budget constraint, which yields

$$(T+1)\tilde{c}_{t,i}(t)\leq w(t)x_{lc,i}\left(1-\tau_i^p\right)\left(1-\tau^u\right)\sum_{s=0}^{T}e_s\left[1-\ell_{t,i}(t+s)\right]. \quad (37)$$

Substituting leisure choices into this expression gives

$$(T+1)\tilde{c}_{t,i}(t)\leq w(t)x_{lc,i}\left(1-\tau_i^p\right)\left(1-\tau^u\right)\sum_{s=0}^{T}e_s\left(1-\frac{1-\eta}{\eta}\frac{\tilde{c}_{t,i}(t)}{w(t)x_{lc,i}\left(1-\tau^u\right)e_s}\right). \quad (38)$$

This is

$$(T+1)\frac{\eta}{\eta}\tilde{c}_{t,i}(t)+(T+1)\frac{1-\eta}{\eta}\tilde{c}_{t,i}(t)=w(t)\,x_{lc,i}\left(1-\tau_i^p\right)(1-\tau^u)\sum_{s=0}^{T}e_s \tag{39}$$

or

$$\tilde{c}_{t,i}(t)=w(t)\,x_{lc,i}\left(1-\tau_i^p\right)(1-\tau^u)\frac{\eta}{T+1}\sum_{s=0}^{T}e_s \tag{40}$$

We conclude that the choice for $\tilde{c}_{t,i}(t)$ —the current consumption choice—is linear in the date t wage and does not depend on future wages. Consumption (40) is scaled by $x_{lc,i}\left(1-\tau_i^p\right)$. The current consumption choice contains the current state of aggregate demand via $\tilde{c}_{t,i}(t)=D(t)\,c_t(t)$, but $D(t)$ is also known at date t. Other quantities below will depend only on $\tilde{c}_{t,i}(t)$ and so only on the current state of aggregate demand and not future states.

We can substitute first period consumption (40) into (36) to obtain

$$\begin{aligned}\ell_{t,i}(t+s) &= \left(\frac{1-\eta}{T+1}\right)\\ &\quad\times\left(\frac{1}{x_{lc,i}\left(1-\tau_i^p\right)(1-\tau^u)\,e_s}\right)x_{lc,i}\left(1-\tau_i^p\right)(1-\tau^u)\sum_{s=0}^{T}e_s\\ &= \left(\frac{1-\eta}{T+1}\right)\left(\frac{1}{e_s}\right)\sum_{j=0}^{T}e_j \end{aligned}\tag{41}$$

for $s=0,...,T$. The amount of leisure chosen at date $t+s$ depends on how much productivity the agent has at that stage s of the life cycle, e_s, relative to the entire sum over the life cycle.[43] If $\eta\to 1$ the household will choose almost no leisure. If $\eta\to 0$ and the value of a particular e_s is small enough, then $\ell_{t,i}(t+s)$ could be larger than one, meaning the household would supply no labor on those dates. This would violate our interior solution assumption. Accordingly, in our calibration we use an endowment profile along with parameter choices sufficient to maintain interior leisure choices. Also, the household's choices of $\ell_{t,i}(t+s)$ are independent of the scaling realization $x_{lc,i}$. That is, all life cycle households i work the same number of hours if they are the same age.

This household will also choose a nominal net asset position to carry into the next

[43] See Bullard and Singh (2019).

period. The date t real value of this position is given by

$$\frac{a_{t,i}(t)}{P(t)} = x_{lc,i}(1-\tau_i^p)(1-\tau^u)e_0[1-\ell_{t,i}(t)]w(t) - \tilde{c}_{t,i}(t)$$

$$= x_{lc,i}(1-\tau_i^p)(1-\tau^u)e_0\left[1-\left(\frac{1-\eta}{T+1}\right)\frac{1}{e_0}\sum_{s=0}^{T}e_s\right]w(t)$$

$$-\frac{\eta}{T+1}w(t)x_{lc,i}(1-\tau_i^p)(1-\tau^u)\sum_{s=0}^{T}e_s$$

$$= w(t)x_{lc,i}(1-\tau_i^p)(1-\tau^u)\left[e_0-\left(\frac{1-\eta}{T+1}\right)\sum_{s=0}^{T}e_s-\frac{\eta}{T+1}\sum_{s=0}^{T}e_s\right]$$

$$= w(t)x_{lc,i}(1-\tau_i^p)(1-\tau^u)\left[e_0-\frac{1}{T+1}\sum_{s=0}^{T}e_s\right]. \quad (42)$$

We conclude that net asset positions are linear in $w(t)$ and scaled by $x_{lc,i}(1-\tau_i^p)$ for each life cycle household i.

A.3 Solutions for older life cycle cohorts

There are also life cycle households that entered the economy at date $t-1$, $t-2$, $\cdots$, $t-T$ that solve a similar problems at date t. These households have existing net nominal asset positions $a_{t-1,i}(t-1)$, $a_{t-2,i}(t-1)$, ..., $a_{t-T,i}(t-1)$, respectively, and have a shorter remaining horizon in their life cycle. Here we show the solution to a household i problem that entered the economy at date $t-1$. We will then infer solutions for all of the other household problems for households that entered the economy at dates $t-2,\cdots,t-T$.

Household i that entered the economy at date $t-1$ solves, at date t,

$$\max_{\{\tilde{c}_{t-1,i}(t+s),\ell_{t-1,i}(t+s)\}_{s=0}^{T-1}} E_t\left[\sum_{s=0}^{T-1}\eta\ln\tilde{c}_{t-1,i}(t+s)+(1-\eta)\ln\ell_{t-1,i}(t+s)\right] \quad (43)$$

subject to the consolidated lifetime budget constraint

$$\tilde{c}_{t-1,i}(t)+\sum_{s=1}^{T-1}\left(\frac{P(t+s)}{P(t)}\frac{\tilde{c}_{t-1,i}(t+s)}{\mathcal{R}^n}\right) \leq x_{lc,i}(1-\tau_i^p)(1-\tau^u)e_1w(t)[1-\ell_{t-1,i}(t)]$$

$$+\sum_{s=2}^{T}\left(\frac{P(t+s-1)}{P(t)}\frac{x_{lc,i}(1-\tau_i^p)(1-\tau^u)e_sw(t+s-1)[1-\ell_{t-1,i}(t+s-1)]}{\mathcal{R}^n}\right)$$

$$+\frac{R^n(t-1,t)a_{t-1,i}(t-1)}{P(t)}, \quad (44)$$

where

$$\mathcal{R}^n = \prod_{j=1}^{s} R^n \left(t + j - 1, t + j\right). \tag{45}$$

In this "remaining lifetime" budget constraint, we can see from (42) what the nominal value of $a_{t-1,i}\left(t-1\right)$ must have been from last period, namely

$$a_{t-1,i}\left(t-1\right) = P\left(t-1\right) w\left(t-1\right) x_{lc,i}\left(1-\tau_i^p\right)\left(1-\tau^u\right)\left[e_0 - \frac{1}{T+1}\sum_{s=0}^{T} e_s\right]. \tag{46}$$

We can therefore write the value of $R^n\left(t-1,t\right) a_{t-1,i}\left(t-1\right)$ as

$$\begin{aligned} &R^n\left(t-1,t\right) a_{t-1,i}\left(t-1\right) \\ &\quad = R^n\left(t-1,t\right) P\left(t-1\right) w\left(t-1\right) x_{lc,i}\left(1-\tau_i^p\right)\left(1-\tau^u\right)\left[e_0 - \frac{1}{T+1}\sum_{s=0}^{T} e_s\right]. \end{aligned} \tag{47}$$

We can use the price level targeting criterion $P\left(t\right) = \frac{R^n(t-1,t)}{\delta(t-1,t)\nu(t-1,t)\lambda(t-1,t)} P\left(t-1\right)$ and the law of motion for the real wage per effective efficiency unit

$$w\left(t\right) = \delta\left(t-1,t\right)\nu\left(t-1,t\right)\lambda\left(t-1,t\right) w\left(t-1\right) \tag{48}$$

to simplify the right hand side as

$$\begin{aligned} &R^n\left(t-1,t\right) a_{t-1,i}\left(t-1\right) = \\ &\qquad R^n\left(t-1,t\right)\frac{P\left(t\right)\delta\left(t-1,t\right)\nu\left(t-1,t\right)\lambda\left(t-1,t\right)}{R^n\left(t-1,t\right)} \\ &\quad \times \frac{w\left(t\right)}{\delta\left(t-1,t\right)\nu\left(t-1,t\right)\lambda\left(t-1,t\right)} x_{lc,i}\left(1-\tau_i^p\right)\left(1-\tau^u\right)\left[e_0 - \frac{1}{T+1}\sum_{s=0}^{T} e_s\right] \\ &\qquad = P\left(t\right) w\left(t\right) x_{lc,i}\left(1-\tau_i^p\right)\left(1-\tau^u\right)\left[e_0 - \frac{1}{T+1}\sum_{s=0}^{T} e_s\right]. \end{aligned} \tag{49}$$

Therefore the entire real-valued term is given by

$$\frac{R^n\left(t-1,t\right) a_{t-1,i}\left(t-1\right)}{P\left(t\right)} = w\left(t\right)\left[e_{0,i} - \frac{1}{T+1}\sum_{s=0}^{T} e_{s,i}\right]. \tag{50}$$

We can also write this as

$$\frac{R^n\left(t-1,t\right) a_{t-1,i}\left(t-1\right)}{P\left(t\right)} = w\left(t\right) x_{lc,i}\left(1-\tau_i^p\right)\left(1-\tau^u\right)\left[\frac{T}{T+1} e_0 - \frac{1}{T+1}\sum_{s=1}^{T} e_s\right]. \tag{51}$$

Since $w(t)$ and $P(t)$ are known at date t when the consumption-saving decision is made, this is a nonstochastic object. It is linear in $w(t)$. It enters in lump-sum fashion in the budget constraint and so does not affect the first order conditions.

We now substitute the price level targeting criterion into the rest of the budget constraint to obtain

$$\begin{aligned} &\tilde{c}_{t-1,i}(t) + \sum_{s=1}^{T-1} \left(\frac{\tilde{c}_{t-1,i}(t+s)}{\mathcal{R}} \right) \\ &\leq w(t) x_{lc,i} \left(1-\tau_i^p\right)\left(1-\tau^u\right) \left(\left[\sum_{s=1}^{T} e_s \left[1-\ell_{t-1,i}(t+s-1)\right] \right] + \left[e_0 - \frac{1}{T+1} \sum_{s=0}^{T} e_s \right] \right). \end{aligned} \tag{52}$$

where

$$\mathcal{R} = \prod_{j=1}^{s} \delta(t+j-1, t+j)\, \nu(t+j-1, t+j)\, \lambda(t+j-1, t+j) \tag{53}$$

As previously there is no income uncertainty on the right hand side for this household.

The FOC with respect to consumption are given by

$$\frac{\eta}{\tilde{c}_{t-1,i}(t)} = \mu$$

along with

$$\frac{\eta}{\tilde{c}_{t-1,i}(t+s)} = \frac{\mu}{\mathcal{R}} \tag{54}$$

for $s = 1, ..., T-1$, which implies

$$\tilde{c}_{t-1,i}(t+s) = \mathcal{R}\tilde{c}_{t-1,i}(t) \tag{55}$$

for $s = 1, ..., T-1$. This household has a state-contingent consumption plan which is to increase consumption in future periods at the stochastic growth rate of the economy. The first order conditions with respect to leisure are

$$\frac{1-\eta}{\ell_{t-1,i}(t+s)} = \mu w(t)\, x_{lc,i}\left(1-\tau_i^p\right)\left(1-\tau^u\right) e_s \tag{56}$$

for $s = 0, ..., T-1$. We combine each of these with the corresponding FOC for date t consumption to give the choices for leisure

$$\ell_{t-1,i}(t+s-1) = \frac{1-\eta}{\eta} \frac{\tilde{c}_{t-1,i}(t)}{w(t)\, e_{s,i}} \tag{57}$$

for $s = 1, ..., T$.

We can now substitute consumption and leisure choices back into the "remaining life" budget constraint. This yields, using (51),

$$T\tilde{c}_{t-1,i}\left(t\right) = \frac{w(t)x_{lc,i}\left(1-\tau_i^p\right)\left(1-\tau^u\right)}{} \times\left(\left[\sum_{s=1}^{T} e_s\left(1-\frac{1-\eta}{\eta}\frac{\tilde{c}_{t-1,i}\left(t\right)}{w\left(t\right)x_{lc,i}\left(1-\tau^u\right)e_s}\right)\right]+\left[\frac{T}{T+1}e_0-\frac{1}{T+1}\sum_{s=1}^{T}e_s\right]\right) \quad (58)$$

which is

$$T\frac{\eta}{\eta}\tilde{c}_{t-1,i}\left(t\right)+T\frac{1-\eta}{\eta}\tilde{c}_{t-1,i}\left(t\right) = w\left(t\right)x_{lc,i}\left(1-\tau_i^p\right)\left(1-\tau^u\right)\sum_{s=1}^{T}e_s+w\left(t\right)x_{lc,i}\left(1-\tau_i^p\right)\left(1-\tau^u\right)\left[\frac{T}{T+1}e_0-\frac{1}{T+1}\sum_{s=1}^{T}e_s\right],$$

or

$$\frac{T}{\eta}\tilde{c}_{t-1}\left(t\right)=\frac{T+1}{T+1}w\left(t\right)x_{lc,i}\left(1-\tau_i^p\right)\left(1-\tau^u\right)\left[\sum_{s=1}^{T}e_s\right] -\frac{1}{T+1}w\left(t\right)x_{lc,i}\left(1-\tau_i^p\right)\left(1-\tau^u\right)\left[\sum_{s=1}^{T}e_s\right]+\frac{T}{T+1}w\left(t\right)x_{lc,i}\left(1-\tau_i^p\right)\left(1-\tau^u\right)e_0 \quad (59)$$

or

$$\frac{T}{\eta}\tilde{c}_{t-1}\left(t\right)=\frac{T}{T+1}w\left(t\right)x_{lc,i}\left(1-\tau_i^p\right)\left(1-\tau^u\right)\left[\sum_{s=1}^{T}e_s\right]+\frac{T}{T+1}w\left(t\right)x_{lc,i}\left(1-\tau_i^p\right)\left(1-\tau^u\right)e_0 \quad (60)$$

which is

$$\tilde{c}_{t-1,i}\left(t\right)=w\left(t\right)\frac{\eta}{T+1}x_{lc,i}\left(1-\tau_i^p\right)\left(1-\tau^u\right)\left[e_0+\sum_{s=1}^{T}e_s\right], \quad (61)$$

and finally

$$\tilde{c}_{t-1,i}\left(t\right)=w\left(t\right)\frac{\eta}{T+1}x_{lc,i}\left(1-\tau_i^p\right)\left(1-\tau^u\right)\sum_{s=0}^{T}e_s. \quad (62)$$

This is linear in $w\left(t\right)$ and $x_{lc,i}\left(1-\tau_i^p\right)$ as well as the tax factor $\left(1-\tau^u\right)$. This formula is the same as was derived for $\tilde{c}_{t,i}\left(t\right)$ above. This indicates that two agents i, j that share the same productivity profile (they have the same value of $x_{lc,i}\left(1-\tau_i^p\right)$) will consume the same amount at each stage of the life cycle. Also, we can infer from the solution for $\tilde{c}_{t,i}\left(t\right)$ above that $\tilde{c}_{t-1,i}\left(t-1\right)$ must have been $w\left(t-1\right)\frac{\eta}{T+1}x_{lc,i}\left(1-\tau_i^p\right)\left(1-\tau^u\right)\sum_{s=0}^{T}e_s$, and therefore that consumption growth for this household is $w(t)/w(t-1)$ which is the stochastic rate of output growth consistent with the conjecture.

The leisure choice at date t would then be, setting $s=1$ in (57),

$$\begin{aligned} \ell_{t-1,i}(t) &= \frac{1-\eta}{\eta}\frac{\tilde{c}_{t-1,i}(t)}{w(t)\,x_{lc,i}(1-\tau_i^p)(1-\tau^u)\,e_1} \\ &= \frac{1-\eta}{\eta}\frac{1}{w(t)\,x_{lc,i}(1-\tau_i^p)(1-\tau^u)\,e_1}w(t)\frac{\eta}{T+1}x_{lc,i}(1-\tau_i^p)(1-\tau^u)\sum_{s=0}^{T}e_s \\ &= \frac{1-\eta}{T+1}\frac{1}{e_1}\sum_{s=0}^{T}e_s. \end{aligned}$$

This choice is independent of $w(t)$ and $x_{lc,i}(1-\tau_i^p)$.

This household would then have a net asset position at date t to be carried into date $t+1$:

$$\frac{a_{t-1,i}(t)}{P(t)} = x_{lc,i}(1-\tau_i^p)(1-\tau^u)\,e_1 w(t)\left[1-\ell_{t-1,i}(t)\right]-\tilde{c}_{t-1,i}(t)+\frac{R^n(t-1,t)\,a_{t-1,i}(t-1)}{P(t)}. \tag{63}$$

Using (50), we have

$$\frac{a_{t-1,i}(t)}{P(t)} = x_{lc,i}\left(1-\tau_i^p\right)\left(1-\tau^u\right)e_1 w(t)\left[1-\ell_{t-1,i}(t)\right]$$
$$-\tilde{c}_{t-1,i}(t) + \frac{R^n(t-1,t)\,a_{t-1,i}(t-1)}{P(t)}$$
$$= x_{lc,i}\left(1-\tau_i^p\right)\left(1-\tau^u\right)e_1 w(t)\left[1-\frac{1-\eta}{\eta}\frac{\tilde{c}_{t-1,i}(t)}{w(t)\,x_{lc,i}\left(1-\tau_i^p\right)\left(1-\tau^u\right)e_1}\right]$$
$$-\frac{\eta}{\eta}\tilde{c}_{t-1,i}(t) + w(t)\,x_{lc,i}\left(1-\tau_i^p\right)\left(1-\tau^u\right)\left[e_0-\frac{1}{T+1}\sum_{s=0}^{T}e_s\right],$$
$$= x_{lc,i}\left(1-\tau_i^p\right)\left(1-\tau^u\right)e_1 w(t)$$
$$-\frac{1}{\eta}\tilde{c}_{t-1,i}(t) + w(t)\,x_{lc,i}\left(1-\tau_i^p\right)\left(1-\tau^u\right)\left[e_0-\frac{1}{T+1}\sum_{s=0}^{T}e_s\right],$$
$$= x_{lc,i}\left(1-\tau_i^p\right)\left(1-\tau^u\right)e_1 w(t)$$
$$-\frac{1}{\eta}w(t)\frac{\eta}{T+1}x_{lc,i}\left(1-\tau_i^p\right)\left(1-\tau^u\right)\left[\sum_{s=0}^{T}e_s\right]$$
$$+w(t)\,x_{lc,i}\left(1-\tau_i^p\right)\left(1-\tau^u\right)\left[e_0-\frac{1}{T+1}\sum_{s=0}^{T}e_s\right],$$
$$= w(t)\,x_{lc,i}\left(1-\tau_i^p\right)\left(1-\tau^u\right)\left[e_1-\frac{1}{T+1}\left[\sum_{s=0}^{T}e_s\right]+\left[e_0-\frac{1}{T+1}\sum_{s=0}^{T}e_s\right]\right]$$
$$= w(t)\,x_{lc,i}\left(1-\tau_i^p\right)\left(1-\tau^u\right)\left[e_0+e_1-\frac{2}{T+1}\sum_{s=0}^{T}e_s.\right] \quad (64)$$

For all other life cycle households i at date t who entered the economy at date $t-2, t-3, ..t-T$, consumption and net assets at date t will also be linear in $w(t)$ and $x_{lc,i}\left(1-\tau_i^p\right)$.

A.4 General formulas

The considerations above indicate that the following formulas apply for date t consumption, date t leisure choices, and date t net asset positions for each life cycle household i that entered the economy at date $t-s$, for $s=0,...,T$. For consumption, each life cycle household i chooses

$$\tilde{c}_{t-s,i}(t) = x_{lc,i}\left(1-\tau_i^p\right)\left(1-\tau^u\right)w(t)\frac{\eta}{T+1}\sum_{j=0}^{T}e_j, \quad (65)$$

for $s=0,...,T$. For leisure, each life cycle household i chooses

$$\ell_{t-s,i}(t)=\frac{1-\eta}{T+1}\frac{1}{e_s}\sum_{j=0}^{T}e_j, \tag{66}$$

for $s=0,...,T$. And for net asset positions, each life cycle household i chooses

$$\frac{a_{t-s,i}(t)}{P(t)}=x_{lc,i}\left(1-\tau_i^p\right)\left(1-\tau^u\right)w(t)\left\{\left[\sum_{j=0}^{s}e_j\right]-\left(\frac{s+1}{T+1}\right)\sum_{j=0}^{T}e_j\right\} \tag{67}$$

for $s=0,...,T$.

A.5 Hand-to-mouth households

Hand-to-mouth households are identical to life-cycle households except that they are assigned a baseline productivity profile which is perfectly flat instead of hump-shaped. We can set this baseline equal to $x_{htm}e_s$ where $e_s=e^{htm}$ for $s=0,...,T$, and e^{htm} is a constant, and x_{htm} is a log normal random variable the value of which is realized as the household enters the economy. It follows from the above general formulas that

$$\tilde{c}_{t-s,i}^{htm}(t)=x_{htm,i}\left(1-\tau_i^p\right)w(t)\,\eta e^{htm}, \tag{68}$$

for $s=0,...,T$; that

$$\ell_{t-s,i}^{htm}(t)=1-\eta, \tag{69}$$

for $s=0,...,T$; and that

$$\frac{a_{t-s,i}^{htm}(t)}{P(t)}=0 \tag{70}$$

for $s=0,...,T$. That is, hand-to-mouth households will always work the same number of hours and will always have a net asset position of zero, but their consumption will be scaled by $x_{htm,i}\left(1-\tau_i^p\right)$, meaning that more productive HTM households will consume more than less productive HTM households.

A.6 General equilibrium

The only role of the Treasury authority is to issue nominal-interest-bearing debt according to

$$B^g(t+1)=R^n(t,t+1)\,B^g(t), \tag{71}$$

rolling it over in perpetuity at the going contract nominal interest rate, and to collect no taxes of any kind nor to embark on any government spending of any kind. The amount of this asset held on net in equilibrium is the difference between total nominal assets $A(t)$ and nominal claims to capital (corporate debt) $B^c(t)$:

$$B^g(t)=A(t)-B^c(t). \tag{72}$$

Combining these two equations gives the general equilibrium condition as

$$A(t+1) - B^c(t+1) = R^n(t, t+1)\left[A(t) - B^c(t)\right]. \qquad (73)$$

This can be written as

$$\left[A(t+1) - B^c(t+1)\right]\frac{P(t+1)}{P(t+1)} = R^n(t, t+1)\frac{P(t)}{P(t)}\left[A(t) - B^c(t)\right] \qquad (74)$$

which is

$$\frac{\left[A(t+1) - B^c(t+1)\right]}{P(t+1)} = R^n(t, t+1)\frac{P(t)}{P(t+1)}\frac{\left[A(t) - B^c(t)\right]}{P(t)} \qquad (75)$$

$$\frac{\left[A(t+1) - B^c(t+1)\right]}{P(t+1)} = R(t, t+1)\frac{\left[A(t) - B^c(t)\right]}{P(t)}, \qquad (76)$$

where $R(t, t+1)$ is the gross real rate of interest between dates t and $t+1$. We have conjectured that $R(t, t+1)$ is equal to the gross stochastic real output growth rate given by $\lambda(t, t+1)\,\nu(t, t+1)\,\delta(t, t+1)$. Net asset holdings have been shown to be linear in the real wage and therefore grow at the output growth rate. The capital stock also grows at the output growth rate as do real claims to capital, and so this general equilibrium condition is met. Whether $B^g(t)$ is positive in this equilibrium is a quantitative question which is discussed in the calibration section; it is positive in our calibration.