Does It Matter (For Equilibrium Determinacy) What Price Index The Central Bank Targets?

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Abstract:

What inflation rate should the central bank target? We address determinacy issues related to this question in a two-sector model in which prices can differ in equilibrium. We assume that the degree of nominal price stickiness can vary across the sectors and that labor is immobile. The contribution of this paper is to demonstrate that a modified Taylor Principle holds in this environment. If the central bank elects to target sector one, and if it responds with a coefficient greater than unity to price movements in this sector, then this policy rule will ensure determinacy across all sectors. The results of this paper have at least two implications. First, the equilibrium-determinacy criterion does not imply a preference to any particular measure of inflation. Second, since the Taylor Principle applies at the sectoral level, there is no need for a Taylor Principle at the aggregate level.

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1. Introduction

Since at least Taylor (1993) it has been commonplace to think of monetary policy in terms of directives for the nominal interest rate. The "Taylor Rule" posits that the central bank moves its interest rate instrument in reaction to movements in inflation and output. The recent literature on Taylor rules is voluminous. See Clarida, Galí and Gertler (1999) for a survey.

One branch of this literature is concerned with the issue of equilibrium determinacy: what Taylor Rule coefficients ensure uniqueness of the equilibrium? The problem is that following a rule in which the central bank responds to endogenous variables may introduce real indeterminacy and sunspot equilibria into an otherwise determinate economy.[1] These sunspot fluctuations are welfare-reducing and can potentially be quite large. The policy conclusion of this literature is that a benevolent central banker should only use a Taylor Rule that ensures determinacy of equilibrium.[2] A familiar result is that a necessary condition to ensure determinacy is that the central bank's response to inflation must be at least unity, *i.e.*, a one percentage point increase in the inflation rate should lead to a greater than one percentage point increase in the nominal interest rate.[3] This has been called the "Taylor Principle."

There are numerous operational issues that arise when implementing the Taylor Principle. For example, should the central bank base its policy on forecasts of inflation, or actual realized rates of inflation? That is, should the central bank look forward or backward? In an aggregative

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[1] It should also be recalled that sunspot equilibria are endemic if the interest rate is set to react to exogenous shocks only (Woodford, 1999).
[2] Since studies generally indicate that the welfare advantages of a first-best policy rule are quite small, it is doubly important that a central banker "do no harm" and not follow a policy rule that may introduce sunspot fluctuations into the economy.
[3] Kerr and King (1996) and Clarida, Galí and Gertler (2000) were the first to derive this result in a model similar to that analyzed here. Leeper (1991) has a related discussion.

model with CIA-timing similar to that analyzed here, and including capital, Carlstrom and Fuerst (2000) demonstrate that a backward-looking rule is necessary and sufficient for determinacy.

A second operational issue is what inflation rate should be targeted. The entire consumer price index (CPI)? The CPI stripped of food and energy prices? The median CPI? The fundamental contribution of this paper is to demonstrate that a modified Taylor Principle holds. If the central bank elects to target a subset of goods in the economy, and if it responds with a coefficient greater than unity to past price movements of these good, then this policy rule will ensure price level determinacy across all sectors.[4] The key assumption driving this result is that households purchase goods made in both sectors so that households, and thus both sectors' firms, care about relative prices. Determinacy of the price level in one sector implies price determinacy in the other sector through this relative price mechanism.

This paper therefore confirms and refines an idea that dates back to at least Patinkin (1965). "In brief, a necessary condition for the determinacy of the absolute price level ... is that the central bank concern itself with some money value…" (Chapter 12, Section 6). What is important for determinacy is that the central bank cares enough about, in the sense of being willing to respond forcefully enough to, movements in some nominal anchor. Exactly which nominal price it cares about does not really matter. What does matter is that it cares about *some* nominal price. This price may be anything, from the price of gold to core-CPI.[5]

There are at least two implications of the results of this paper. First, the equilibrium-determinacy criterion does not imply a preference to any particular measure of inflation. The

[4] In this paper, we interpret "targeting" as "reacting to." This is different from Svensson's (2002) definition of targeting, which refers to variables that are included in the central bank's loss function.
[5] We thank Peter Ireland for pointing this out to us. It can also be shown that this nominal anchor need not be a nominal price but may also be nominal money growth.

choice of which inflation rate to target can be made on other grounds.[6] Since the Taylor Principle applies at the sectoral level, there is no need for a Taylor Principle at the aggregate level. For example, suppose that the central bank targets inflation in sector one with a Taylor coefficient of $\tau > 1$, but that the econometrician estimates a Taylor rule using the total CPI. Depending upon the variances and covariances of shocks across the sectors, the estimated Taylor coefficient could be much less than unity. Hence, we cannot conclude that the Taylor Principle is violated by simply looking at aggregate CPI numbers.

The second implication is that in a currency union such as the euro-zone, the ECB will be able to ensure determinacy of the economy even by reacting to inflation in only a subset of countries. For example, P. Benigno (2001) demonstrates that it is optimal to attach more weight to inflation in countries with higher degrees of nominal rigidity. Benigno does not address how such a policy could be operationalized. To the extent that this is achieved through a Taylor-type interest rate rule, our results can be used to analyze whether his optimal policy is determinate. This is important since Benigno does not address this question.

In analyzing determinacy, we adopt a framework that shares important features with P. Benigno's (2001) two-country, currency area model. First, we consider two different sectors and allow for the prices in these sectors to differ in equilibrium. We assume that the degree of nominal price stickiness can vary across the sectors. Second, we assume that labor is immobile across the two sectors. This extreme assumption makes it more difficult to generate determinacy as labor flows are not available to mitigate price differences. In short, we set up the model so that it is difficult to generate determinacy under a rule in which the central bank targets inflation in only one sector. This is important given the international implications of our results.

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[6] For example, a trimmed-mean CPI is a better predictor of the future CPI than is the CPI itself.

The paper proceeds as follows. The next section develops the model. Section 3 lays out the basic determinacy results, and Section 4 concludes.

2. The Model

Our model is a two-sector version of the standard New-Keynesian setup used in the recent literature on monetary policy. We limit our discussion to a perfect foresight model as our focus is on equilibrium determinacy. We first describe the behavior of households and firms, respectively, and then turn to the linearized system that will be the focus of our analysis.

2.A. The Representative Household

The economy is populated by a continuum of households between 0 and 1. The representative household consists of two agents. One of these supplies labor to firms in sector 1, the other supplies labor to firms in sector 2. These agents jointly maximize an intertemporal utility function that depends on the household's consumption of a basket of goods C_t, on the household's holdings of real money balances A_t / P_t (where A_t is nominal money holdings and P_t is the CPI), and on the disutility of the two agents from supplying labor in sectors 1 and 2, L_t^1 and L_t^2, respectively:

$$\sum_{t=0}^{\infty} \beta^t U\left(C_t,\ A_t/P_t,\ L_t^1,\ L_t^2\right), \qquad 1 > \beta > 0\,. \tag{1}$$

For simplicity, we assume:

$$U\left(C_t,\ A_t/P_t,\ L_t^1,\ L_t^2\right) = \log C_t + \log\left(A_t/P_t\right) - \left(L_t^1\right)^2/2 - \left(L_t^2\right)^2/2\,. \tag{2}$$

The consumption basket C is a CES aggregate of sub-baskets of individual goods produced in sectors 1 and 2:

$$C_t = \left[b^{\frac{1}{\omega}} \left(C_t^1 \right)^{\frac{\omega-1}{\omega}} + (1-b)^{\frac{1}{\omega}} \left(C_t^2 \right)^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}}, \qquad \omega > 0,\ 1 > b > 0. \tag{3}$$

Sectors 1 and 2 are populated by monopolistically competitive firms, which produce differentiated brands of the sectors' goods. Sector 1 consists of firms in the interval between 0 and b; sector 2 consists of firms between b and 1. The sectoral consumption sub-baskets are:

$$C_t^1 = \left[\left(\frac{1}{b} \right)^{\frac{1}{\theta}} \int_0^b \left(C_t^1(z) \right)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}, \quad C_t^2 = \left[\left(\frac{1}{1-b} \right)^{\frac{1}{\theta}} \int_b^1 \left(C_t^2(z) \right)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}, \quad \theta > 1. \tag{4}$$

Given the consumption index in (3), the CPI equals:

$$P_t = \left[b \left(P_t^1 \right)^{1-\omega} + (1-b) \left(P_t^2 \right)^{1-\omega} \right]^{\frac{1}{1-\omega}}, \tag{5}$$

where P_t^1 and P_t^2 are the price sub-indexes for sectors 1 and 2, respectively:

$$P_t^1 = \left[\frac{1}{b} \int_0^b \left(P_t^1(z) \right)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}, \quad P_t^2 = \left[\frac{1}{1-b} \int_b^1 \left(P_t^2(z) \right)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}, \tag{6}$$

and $P_t^j(z)$ denotes the price of individual brand z produced in sector j, j = 1, 2.

Given these price indexes, the household allocates its consumption to individual brands of the goods in each sector according to the demand schedule:

$$C_t^j(z) = \left(\frac{P_t^j(z)}{P_t^j} \right)^{-\theta} \left(\frac{P_t^j}{P_t} \right)^{-\omega} C_t, \qquad j = 1, 2. \tag{7}$$

Since our focus is on symmetric equilibria within each sector we henceforth drop the firm-specific index z, and instead consider a representative firm in each sector j = 1,2.

The representative household enters the period with M_t cash balances and B_{t-1} holdings of nominal bonds. At the beginning of the period, the household visits the financial market, where is carries out bond trading and receives a monetary transfer X_t from the monetary authority. The two agents then split and offer labor in sectors 1 and 2. They meet on the way home from work and go shopping for consumption goods. Before entering the goods market, the household has cash holdings $M_t + X_t + R_{t-1}B_{t-1} - B_t$, where R_{t-1} is the gross nominal interest rate between $t-1$ and t. Agents receive their nominal wage bills $W_t^1 L_t^1$ and $W_t^2 L_t^2$ in the mail at the end of the period. Thus, after engaging in goods trading and opening the mail, the household ends the period with cash balances given by the budget constraint:

$$M_{t+1} = M_t + X_t + R_{t-1}B_{t-1} + W_t^1 L_t^1 + W_t^2 L_t^2 - B_t - P_t C_t . \tag{8}$$

We assume that the money balances that enter the utility function (those that matter for time-t transactions) are those with which the household *enters* the time-t goods market, *i.e.*, cash held in advance of goods market trading. This is the cash-in-advance (CIA) timing of Carlstrom and Fuerst (2001). Hence:[7]

$$A_t = M_t + X_t + R_{t-1}B_{t-1} - B_t . \tag{9}$$

Bond-pricing and money demand equations are as in Carlstrom and Fuerst (2001):

$$\frac{U_m(t) + U_C(t)}{P_t} = R_t \beta \frac{U_m(t+1) + U_C(t+1)}{P_{t+1}}, \tag{10}$$

$$\frac{U_m(t)}{U_C(t)} = R_t - 1, \tag{11}$$

[7] Under the traditional cash-when-I-am-done (CWID) timing of money-in-the-utility-function models, it is: $A_t = M_{t+1} = M_t + X_t + R_{t-1}B_{t-1} + W_t^1 L_t^1 + W_t^2 L_t^2 - B_t - P_t C_t$.

where $U_C(t)$ denotes the marginal utility of consumption at time t and $U_m(t)$ is the marginal utility of time t real money balances. Combining (10) and (11) yields the Fisher equation under CIA timing:[8]

$$\frac{U_C(t)}{P_t} = R_{t+1}\beta \frac{U_C(t+1)}{P_{t+1}}. \tag{12}$$

Labor supplies are determined by:

$$-\frac{U_{L^1}(t)}{U_C(t)} = \frac{W_t^1}{P_t}, \tag{13}$$

$$-\frac{U_{L^2}(t)}{U_C(t)} = \frac{W_t^2}{P_t}, \tag{14}$$

where $-U_{L^1}(t)$ ($-U_{L^2}(t)$) is the marginal disutility of supplying labor to sector 1 (2) firms. We allow for the possibility that real wages in sectors 1 and 2 differ because of labor immobility.

2.B. Firms

Sectors 1 and 2 are populated by monopolistically competitive firms that produce differentiated varieties of the goods in each sector. Price setting in sectors 1 and 2 is subject to Calvo-Yun type nominal rigidity. Given the standard nature of the environment we only sketch a description of firm behavior. Recall that since our focus is on symmetric equilibria we will consider the behavior of a representative firm in each sector.

Firms in each sector produce output according to the linear technology:

$$Y_t^j = L_t^j, \qquad j = 1, 2, \tag{15}$$

[8] The Fisher equation with CWID timing is the familiar:

where Y_t^j and L_t^j are the typical firm's output and labor demand in sector *j*. Firms in each sector *j* = 1, 2, face the downward-sloping demand schedule (7).

Firms choose the amount of labor to be employed and the price of their output to maximize profits in a familiar fashion. Pricing is subject to nominal rigidity. The optimal price in sector *j* satisfies:

$$\frac{P_t^j}{P_t} = \frac{1}{Z_t^j}\frac{W_t^j}{P_t}, \qquad j = 1, 2, \tag{16}$$

where Z^j is marginal cost in sector *j*, so that $(1/Z^j)$ is the markup of price over marginal cost, identical across firms in each sector. Equation (16) follows from either a Calvo-Yun type setup for price stickiness[9] or a quadratic cost of price adjustment as in Rotemberg (1982). Yun (1996) provides the details that link the behavior of marginal cost in each sector to price growth in each sector. For simplicity we omit these details, but simply state the log-linearized version below (equation (23)).

2.C. The Log-Linearized System and Equilibrium

There is a unique steady state to this model. As our focus is on local determinacy questions, we log-linearize the equilibrium conditions around this steady state. Lower-case letters denote percentage deviations from steady-state levels (w_t^j is the log deviation of the *real* wage W_t^j/P_t). When interest and inflation rates are concerned, we consider percentage deviations of *gross* rates from the respective steady-state levels.

$\frac{U_C(t)}{P_t} = R_t\beta\frac{U_C(t+1)}{P_{t+1}}$. Therefore, it is easy to verify that a backward-looking interest rate rule with CIA-timing is equivalent to a current-looking rule with CWID-timing.

[9] Calvo (1983), Yun (1996).

Household behavior is defined by the labor supply equations (13)-(14), the Fisher equation (12), and the demand curves (7). Using the equilibrium condition $l_t^j = c_t^j$, these optimality conditions can be expressed as:

$$w_t^j = c_t + c_t^j, \; j = 1, 2. \tag{17}$$

$$c_{t+1} - c_t = r_{t+1} - \pi_{t+1}, \tag{18}$$

$$c_t^j = -\omega\left(p_t^j - p_t\right) + c_t, \; j = 1, 2. \tag{19}$$

From (5), the CPI is linked to the sectoral prices via

$$p_t = bp_t^1 + (1-b)p_t^2, \tag{20}$$

and prices and inflation are linked by

$$\pi_t = \Delta p_t, \text{ and } \pi_t^j = \Delta p_t^j, \; j = 1, 2, \tag{21}$$

where Δ denotes first differences ($\Delta x_t \equiv x_t - x_{t-1}$ for any variable x).

Turning to firm behavior, the pricing equation (16) has the form

$$z_t^j = w_t^j + p_t - p_t^j, \; j = 1, 2. \tag{22}$$

From Yun (1996), we have the familiar New-Keynesian Phillips curve:[10]

$$\pi_t^j = \lambda^j z_t^j + \beta \pi_{t+1}^j, \; j = 1, 2, \tag{23}$$

where $\lambda^j > 0$ measures the degree of nominal rigidity in sector j. We allow sectors to differ in the extent to which prices are sticky.

To close the model we need to define monetary policy. We specify monetary policy as a Taylor rule in which the nominal interest rate is a function of lagged inflation. We choose this backward-looking rule because current and forecast-based rules are typically prone to sunspot

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[10] See Yun (1996) for the Calvo-Yun setup, or Roberts (1995) for the quadratic cost adjustment scenario.

equilibria in this modeling environment.[11] We consider two alternatives for the rate of inflation to which the central bank is reacting. In the first case, the central bank reacts to CPI inflation:

$$r_t = \tau\pi_{t-1}, \quad \tau > 0. \tag{24}$$

In the second case, the central bank reacts to inflation in sector 1 only:

$$r_t = \tau\pi_{t-1}^1, \quad \tau > 0. \tag{25}$$

In what follows we will call (24) the "CPI Taylor Rule" and (25) the "Sectoral Taylor Rule."

To summarize, the equilibrium of the model consists of the ten sectoral variables $z_t^j, w_t^j, \pi_t^j, c_t^j, p_t^j$, for j = 1, 2, and the three aggregate variables p_t, π_t, and c_t, that satisfy the thirteen restrictions in (17)-(23) and (24) or (25).

3. Equilibrium Determinacy

We now proceed to the issue of determinacy. We proceed in two steps. First, we examine the case of perfect labor mobility. This case is easily dealt with. Second, we turn to the more interesting case of no labor mobility. A key conclusion is that even in this environment, with such an extreme real rigidity, targeting inflation in one sector is sufficient for price level determinacy across all sectors.

3.A. Determinacy with Labor Mobility

Let us begin the analysis with a special case of the model in which labor is instantaneously mobile across sectors so that $w_t^1 = w_t^2$ for all t. In this case, equations (17)-(19)

[11] Carlstrom and Fuerst (2000) demonstrate that an aggressive backward-looking rule ($\tau > 1$) is necessary and sufficient for determinacy in a model that includes capital accumulation.

imply that $c_t^1 = c_t^2 = c_t$ and $p_t^1 = p_t^2 = p_t$, and equations (17) and (22) imply that $z_t = w_t = 2c_t$. The two sectors collapse into one, and we are left with a system solely in aggregates:

$$c_{t+1} - c_t = r_{t+1} - \pi_{t+1}, \tag{26}$$

$$\pi_t = 2\lambda c_t + \beta \pi_{t+1}, \tag{27}$$

where $\lambda = b\lambda^1 + (1-b)\lambda^2$ is the weighted average of λ^j across sectors. Since $\pi_t^1 = \pi_t^2 = \pi_t$, it is irrelevant whether the central bank targets price inflation in sector 1 (equation (25)) or aggregate inflation (equation (24)). The determinacy conditions for this model are identical to the aggregate model studied in, for example, Carlstrom and Fuerst (2001). We have determinacy if and only if $\tau > 1$.

Proposition 1. Assume that the two sectors are characterized by perfect labor mobility but potentially different degrees of price rigidity. Then $\tau > 1$ is a necessary and sufficient condition for determinacy under the CPI Taylor Rule $r_t = \tau\pi_{t-1}$ and the Sectoral Taylor Rule $r_t = \tau\pi_{t-1}^1$.

3.B. Determinacy without Labor Mobility

Suppose that real wages need never equal because labor cannot flow across sectors. One would anticipate that this extreme real rigidity would make it difficult to achieve equilibrium determinacy if the central bank only targets one sector. We begin by assuming that the two sectors are characterized by identical degrees of nominal rigidity ($\lambda^1 = \lambda^2 = \lambda$). We then conclude with the more general case.

To analyze determinacy, it is convenient to define aggregate variables and differences as follows. Given sectoral levels of variables x^1 and x^2, the aggregate level is $x \equiv bx^1 + (1-b)x^2$.

We let x^D denote the difference between sectors 1 and 2: $x^D \equiv x^1 - x^2$. Determinacy of aggregates and differences implies determinacy at the individual sector level since $x^1 = x + (1-b)x^D$ and $x^2 = x - bx^D$. We will exploit this fact in what follows.

With perfect labor mobility the key finding was that the prices between the two sectors had to be equal $p_t^1 = p_t^2 = p_t$ irrespective of the policy rule. This immediately implied that it made no difference whether the central bank targeted one or both sectors. Without labor mobility these two prices may no longer be equal. Despite this, however, we demonstrate that there is determinacy of relative prices regardless of the policy rule. This will then enable us to turn to the issue of aggregate determinacy and show that we have determinacy for either the CPI Taylor Rule or the Sectoral Taylor Rule as long as $\tau > 1$.

3.B.1. Determinacy of Relative Prices

Given identical degrees of nominal rigidity ($\lambda^1 = \lambda^2 = \lambda$) the dynamics of the cross-sector inflation differential are described by:

$$\pi_t^D = \lambda z_t^D + \beta \pi_{t+1}^D . \tag{28}$$

Using (17), (19) and (22), we have the following cross-sector link between marginal cost and relative prices:

$$z_t^D = -(\omega + 1) p_t^D . \tag{29}$$

Relationship (29) is key. In fact, without sticky prices $z_t = z_t^D = 0$ so that prices in the two sectors are again equal and thus the CPI- and Sectoral-Taylor Rules are the same.

With sticky prices, since households purchase goods in both sectors there is a link between relative prices, and thus marginal costs, in each sector. Because this cross-sector link is *negative*, relative prices are always pinned. A high price in sector 1 implies a low demand for sector 1's good. This in turn leads to: a low demand for sector 1 labor; a low wage in sector 1; and thus a low marginal cost in sector 1. This negative cross-sector link is opposite the positive link implied by the Phillips curve (28). This incompatibility eliminates the possibility of self-fulfilling behavior in relative prices and thus generates determinacy of relative prices.

We will now demonstrate this formally by exploiting the link in (29). Since the Phillips curve is in terms of π_t^D, but (29) need not hold at time $t-1$, we first consider determinacy from the vantage point of time $t+1$.[12] We then use the restrictions implied by this dynamic equation to see whether relative marginal cost, and hence relative prices, are determined once we take into account the extra time-*t* restriction implied by (28) and (29). Scrolling (29) forward and writing it in difference form we have

$$\Delta z_{t+1}^D = -(\omega+1)\pi_{t+1}^D . \tag{30}$$

Substituting this into the time $t+1$ Phillips curve (28):

$$-\Delta z_{t+1}^D = \lambda(\omega+1) z_{t+1}^D - \beta \Delta z_{t+2}^D , \tag{31}$$

or

$$\beta z_{t+2}^D - \left[\lambda(\omega+1) + 1 + \beta\right] z_{t+1}^D + z_t^D = 0 . \tag{32}$$

The characteristic polynomial of equation (32) has one root inside and one root outside the unit circle. This implies that z_{t+1}^D is a unique function of z_t^D, so that Δz_{t+1}^D and, from equation (30), π_{t+1}^D are also unique functions of z_t^D. Using this knowledge, we return to the time-*t* restrictions. In particular plugging (29) into (28), and using $\pi_t^D = p_t^D - p_{t-1}^D$ and $\pi_{t+1}^D = \pi_{t+1}^D\left(z_t^D\right)$, we are left with:

$$-(1+\omega)z_t^D - p_{t-1}^D = \lambda z_t^D + \beta\pi_{t+1}^D(z_t^D).$$

Thus, z_t^D is determined from above and, from (29), p_t^D is also determined. Hence, we have determinacy of price level differences (relative prices) across sectors.

3.B.2. Determinacy of Aggregates

We will now turn to the behavior of the aggregates. There are two cases here depending upon the form of the Taylor rule.

Case 1. The CPI Taylor Rule

Suppose the central bank reacts to CPI inflation as in (24). Aggregating inflation across sectors yields:

$$\pi_t = \lambda z_t + \beta\pi_{t+1}. \tag{33}$$

$$w_t = z_t. \tag{34}$$

$$w_t = 2c_t. \tag{35}$$

Hence, our system is the familiar

$$c_{t+1} - c_t = r_{t+1} - \pi_{t+1}, \tag{36}$$

$$\pi_t = 2\lambda c_t + \beta\pi_{t+1}. \tag{37}$$

This is identical to (26)-(27) so that we have determinacy of aggregates if and only if $\tau > 1$.

Case 2. The Sectoral Taylor Rule

[12] If the economy starts at time *t*, there is no condition (29) for time *t* - 1.

Suppose that the central bank sets the interest rate according to rule (25). Recall that the nature of monetary policy (in particular, the inflation rate to which the central bank reacts) was irrelevant for the argument surrounding determinacy of differences across sectors. This will be key in what follows.

Equations (36)-(37) hold also when the central bank reacts to sector 1 inflation only. Note that inflation in sector 1 can be written as $\pi_t^1 = \pi_t + (1-b)\pi_t^D$, where we already know that the inflation differential is determinate. Hence, equation (36) becomes:

$$c_{t+1} - c_t = \tau\pi_t + \tau(1-b)\pi_t^D - \pi_{t+1}. \tag{38}$$

Our system is thus (37)-(38). But since π_t^D is determinate regardless of the policy rule, we are left with the familiar determinacy condition $\tau > 1$.

This result is quite general and powerful. Since relative prices were pinned independent of the policy rule chosen, the determinacy conditions for sectoral and Taylor rules will be identical if the monetary authority reacts to any aggregate variable in addition to sectoral/aggregate-inflation.

Proposition 2. Suppose that the two sectors are characterized by zero labor mobility but identical degrees of nominal rigidity. Then $\tau > 1$ is a necessary and sufficient condition for determinacy under the CPI Taylor Rule $r_t = \tau\pi_{t-1}$ and the Sectoral Taylor Rule $r_t = \tau\pi_{t-1}^1$.

We have shown that the Taylor principle ($\tau > 1$) is a necessary and sufficient condition for determinacy in a two-sector economy with identical degrees of nominal rigidity across sectors regardless of labor mobility and, more importantly, regardless of whether the central bank is reacting to aggregate CPI inflation or inflation in one sector only. Note that the latter result holds irrespective of the value of *b*, the share of sector 1 in the consumption basket. Even if *b* were extremely small, a more than proportional reaction of the nominal interest rate to past inflation in sector 1 would be sufficient to ensure determinacy, regardless of labor mobility.

As we mentioned above, the intuition for this result revolves around equation (29). Households purchase goods in both sectors so that they respond to relative prices. Households also supply labor to both sectors. A low relative price implies a high relative demand for the product, and thus a high wage and marginal cost in that sector. This negative link between prices and marginal cost across sectors is opposite the firm's desire to have prices increasing in marginal cost. This general equilibrium tension in relative pricing results in relative price determinacy. But once relative prices are determined we need only consider aggregate behavior and we are quickly led to the familiar Taylor Principle of $\tau > 1$. This conclusion is quite robust: In the next proposition, we establish the general result that $\tau > 1$ ensures determinacy of the two-sector economy even when the degrees of nominal rigidity differ across sectors.

Proposition 3. Suppose that the two sectors are characterized by zero labor mobility and different degrees of nominal rigidity. Then $\tau > 1$ is a necessary and sufficient condition for determinacy under the CPI Taylor Rule $r_t = \tau\pi_{t-1}$ and the Sectoral Taylor Rule $r_t = \tau\pi^1_{t-1}$.

Proof: See the Appendix.

4. Conclusion

A well-known result in the recent work on central bank interest rate policies is the Taylor Principle: to ensure equilibrium determinacy, the central bank must respond aggressively ($\tau > 1$) to movements in inflation. This result comes from an aggregative sticky-price model. The contribution of this paper is to demonstrate that a modified Taylor Principle holds in a multi-sector economy in which the sectors differ by the degree of price stickiness irrespective of whether labor is mobile between the two sectors. In particular, it does not matter what price index the central bank targets—the median CPI, core CPI, or the entire CPI—an aggressive response to any one of these price indexes is sufficient for determinacy.

Another interesting question on which this paper may help shed light is whether it matters in an open-economy set up if central banks target tradable goods, non-tradable goods, or the entire CPI inflation. Benigno and Benigno (2001) show that the Taylor principle holds in a model with flexible exchange rates, purchasing power parity, and Taylor-type policy rules where the central banks react to the inflation rate of domestic products only. This paper suggests that it does not matter which price level the central bank of an open economy targets. Relative price adjustments should ensure determinacy given a properly aggressive reaction to any of the inflation rates above even if labor is completely immobile between countries. Work in progress is presently trying to verify this hunch.

We conclude with an example that will illustrate the empirical relevance of our theoretical result. Kozicki (1999) provides estimates of backward-looking Taylor rules over the period 1983-97.[13] Using CPI inflation as the measure of inflation she estimates $\tau = .88$. This is a

[13] Kozicki also includes a measure of the output gap in her estimation, but this is irrelevant for the issue at hand because the central bank's reaction to the output gap has a negligible effect on the determinacy conditions (the corresponding determinacy condition is $2\lambda(\tau - 1) + (1 - \beta)\gamma > 0$, where γ is the coefficient on the output gap). The numbers we report are from her Table 3 with the Taylor measure of the output gap. A similar result arises for the IMF and DRI measures of the output gap.

violation of the Taylor Principle suggesting that the economy over that period could be subject to sunspots. However, Kozicki also estimates a Taylor rule for this same period, where the central bank responds to core CPI inflation instead—a narrower measure of inflation. This estimate was $\tau = 1.28$, indicating that sunspots would not be a problem over this time period. In general, her results suggest that the U.S. central bank responds to core CPI inflation and not total CPI. This may have important implications for papers such as that by Clarida, Galí, and Gertler (2000), who estimate whether sunspots are a potential problem for certain sub-periods in U.S. history.

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6. Appendix

Proposition 3. Suppose that the two sectors are characterized by zero labor mobility and different degrees of nominal rigidity. Then $\tau > 1$ is a necessary and sufficient condition for determinacy under the CPI Taylor Rule $r_t = \tau\pi_{t-1}$ and the Sectoral Taylor Rule $r_t = \tau\pi_{t-1}^1$.

Proof. (We consider the case of the Sectoral Taylor Rule. The proof in the case of the CPI Taylor Rule is symmetric.) To begin, we collapse the system (17)-(23) into a system solely in terms of the sectoral prices. First, eliminate w_t^j and c_t^j by substituting (17) and (19) into (22):

$$2c_t = z_t^j + (1+\omega)(p_t^j - p_t)\,,\ j = 1, 2. \tag{39}$$

Using one of these two expressions we can eliminate c_t from the system, and then use the two Phillips curves (23) to eliminate z_t^j. The aggregate price level can be eliminated with the use of (20), and we use (21) to turn the inflation rates into price level differences. This gives us the following dynamic system:

$$A_1 \begin{pmatrix} p_{t+2}^2 \\ p_{t+1}^2 \\ p_t^2 \\ p_{t+1}^1 \\ p_t^1 \end{pmatrix} = A_0 \begin{pmatrix} p_{t+1}^2 \\ p_t^2 \\ p_{t-1}^2 \\ p_t^1 \\ p_{t-1}^1 \end{pmatrix},$$

where:

$$A_1 \equiv \begin{pmatrix} -\beta\lambda_2 & \lambda_1\lambda_2(1-\omega)(1-b) & \lambda_1\lambda_2\begin{bmatrix}(1-\omega)(1-b)\\ +2\tau\end{bmatrix} & \lambda_2\begin{bmatrix}\lambda_1(1+\omega)(1-b)\\ +(1+2\beta)\end{bmatrix} & \lambda_2\begin{bmatrix}\lambda_1(1+\omega)(1-b)\\ -(2+\beta)\end{bmatrix} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & \dfrac{-\beta\lambda_2}{1+\omega} & -\dfrac{\lambda_1[(1+\omega)\lambda_2+(1+\beta)]}{1+\omega} & \dfrac{\beta\lambda_1}{1+\omega} & \dfrac{\lambda_2[(1+\omega)\lambda_1+(1+\beta)]}{1+\omega} \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

and

$$A_0 \equiv \begin{pmatrix} 0 & 0 & -\lambda_2 & 0 & -2\tau\lambda_1\lambda_2 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \dfrac{\lambda_2}{1+\omega} & 0 & \dfrac{-\lambda_1}{1+\omega} \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}.$$

Inverting A_1 we have:

$$\begin{pmatrix} p_{t+2}^2 \\ p_{t+1}^2 \\ p_t^2 \\ p_{t+1}^1 \\ p_t^1 \end{pmatrix} = A \begin{pmatrix} p_{t+1}^2 \\ p_t^2 \\ p_{t-1}^2 \\ p_t^1 \\ p_{t-1}^1 \end{pmatrix}$$

where $A = A_1^{-1} A_0$.

Our interest is in the roots of A. One root is always unity as we are writing the system in price levels (in difference form we have inflation rates). The characteristic equation of the remaining system is proportional to a fourth-order polynomial, $J(q)$. For determinacy, three roots of J must be outside the unit circle and one root must be within the unit circle. $J(q)$ has the form

$$J(q) = J_4 q^4 + J_3 q^3 + J_2 q^2 + J_1 q + J_0,$$

where

$$J_4 = -\lambda_1\lambda_2\beta^2,$$

$$J_3 = \left[(1+\omega)\lambda_1 + b(\omega-1)(\lambda_2-\lambda_1) + 2(1+\beta+\lambda_2)\right]\beta\lambda_1\lambda_2,$$

$$J_2 = -\begin{bmatrix}(1+4\beta+\beta^2)+\lambda_1(1+\beta)(\omega(1-b)+(1+b)) \\ +\lambda_2(2(1+\beta(1+\tau)+\lambda_1)+(1+\beta)b(\omega-1))+2\lambda_1\lambda_2\omega\end{bmatrix}\lambda_1\lambda_2,$$

$$J_1 = 2(1+\omega)\lambda_1^2\lambda_2^2(\tau-1),$$

$$J_0 = -\lambda_1\lambda_2(1+2\lambda_2\tau),$$

with $J_4 < 0$, $J_3 > 0$, $J_2 < 0$, $J_1 > (<) 0$ if $\tau > (<) 1$, and $J_0 < 0$. Note that J_4 and J_3 do not depend upon the key policy parameter τ. Its easy to show that $J(0) < 0$, $J'(0) > 0$, $J''(0) < 0$, and $J'''(0) > 0$. Hence, all the roots of J have positive real parts. The product of the four roots is equal to $J_0/J_4 > 1$. Furthermore $J(1)$ has the sign of $(\tau-1)$. Therefore, if $\tau < 1$, there are either 0 or 2 roots in (0, 1), so that we can never have determinacy. Hence, $\tau > 1$ is necessary for determinacy. We now turn to sufficiency.

Since $J(1) > 0$ for $\tau > 1$, we know that J has (at least) two real roots, one in the unit circle and one outside. Let us refer to these two real roots as $e_1 < 1$ and $e_2 > 1$. Our task is to examine the remaining two roots of J and demonstrate that they are outside the unit circle if $\tau > 1$.
The proof proceeds in three steps. First, we show that these two remaining roots are real and outside the unit circle for $\tau = 1 + \varepsilon$, where $\varepsilon > 0$ and ε is arbitrarily small. Therefore, we have determinacy for τ slightly greater than one. The second step, which is part of the case when the two roots are real, demonstrates that, as long as these two roots stay real, they must remain outside the unit circle for larger values of τ. Finally, we turn to the case when these two roots are complex and show that, once their norm is outside the unit circle, it must stay there. The first two steps of the proof ensure that, once the roots become complex, they are outside the unit

circle. The last step guarantees that if they ever become real again they will once again be outside the unit circle.

Case 1. The two remaining roots are real

We first demonstrate that, if these two remaining roots are real, then J must have three roots outside the unit circle.

Define the function $h(y) \equiv J(q)$ where $y = q - 1$. The function h is also a quartic with coefficients h_0, h_1, h_2, h_3, and h_4. Note that $h_0 = J(1)$, $h_1 = J'(1)$, $h_2 = J''(1)/2$, $h_3 = J'''(1)/3!$, etc. Inspection of the J function implies that $h_0 > 0$, $h_3 > 0$ and $h_4 < 0$. Descartes' Rule of Signs implies that there is indeterminacy if and only if $h_1>0$ and $h_2>0$. In the neighborhood of $\tau = 1$, both $J'(1)$ and $J''(1)$, however, cannot be greater than or equal to zero (assuming $\beta > 1/2$) since $(2\beta - 1)J'(1 \mid \tau = 1) + (1 - \beta)J''(1 \mid \tau = 1) < 0$. This then implies that $h(y)$ ($J(q)$) has three roots greater than zero (unity). Hence, we have determinacy for τ just slightly greater than unity.

As long as these two roots remain real, they must remain outside the unit circle for larger values of τ. This is true because $J(0) < 0$ and $J(1) > 0$ for all $\tau > 0$, so that the only way for there to be indeterminacy is to have three roots within the unit circle. This can never be the case without the roots first becoming complex. Therefore, as we increase τ out of the neighborhood 1 + ε, we must continue to have exactly one root in the unit circle.

We next show that the two remaining roots must be real for τ just slightly greater than unity. Recall that $J(0) < 0$ and $J(1) = 0$ for $\tau = 1$. If the two remaining roots are complex, then J must have a double real root at $q = 1$ when $\tau = 1$. Furthermore, since $J(1) > 0$ for $\tau > 1$, J must be complex at $q = 1$. But we demonstrated earlier that both $J'(1)$ and $J''(1)$ cannot be non-negative. Hence, the two remaining roots must be real.

Once the roots become complex, they are determinate. We show below that, once these two roots become complex, they must stay outside the unit circle. This then implies that, even if they become real again, they will be determinate.

Case 2. The remaining roots are complex

The problem with the above proof is that, as τ increases, the remaining two roots may become complex. Define

$G(q) \equiv (q - e_1)(q - e_2)(q - a - bi)(q - a + bi)$,

where the two complex roots are $(a + bi)$ and $(a - bi)$. Let $x \equiv (a^2 + b^2)$ denote the norm of these two roots. We calculate $dx/d\tau = a(db/d\tau) + b(da/d\tau)$ as follows. Expanding G, we end up with coefficients G_3, G_2, G_1, and G_0, with $G_3 = J_3/J_4$, $G_2 = J_2/J_4$, $G_1 = J_1/J_4$, and $G_0 = J_0/J_4$. We then construct the following matrix:

$$\begin{bmatrix} \frac{dG_0}{da} & \frac{dG_0}{db} & \frac{dG_0}{de_1} & \frac{dG_0}{de_2} \\ \frac{dG_1}{da} & \frac{dG_1}{db} & \frac{dG_1}{de_1} & \frac{dG_1}{de_2} \\ \frac{dG_2}{da} & \frac{dG_2}{db} & \frac{dG_2}{de_1} & \frac{dG_2}{de_2} \\ \frac{dG_3}{da} & \frac{dG_3}{db} & \frac{dG_3}{de_1} & \frac{dG_3}{de_2} \end{bmatrix} \begin{bmatrix} \frac{da}{d\tau} \\ \frac{db}{d\tau} \\ \frac{de_1}{d\tau} \\ \frac{de_2}{d\tau} \end{bmatrix} = \begin{bmatrix} \frac{d(J_0/J_4)}{d\tau} \\ \frac{d(J_1/J_4)}{d\tau} \\ \frac{d(J_2/J_4)}{d\tau} \\ \frac{d(J_3/J_4)}{d\tau} \end{bmatrix}.$$

Evaluating $dx/d\tau$ at $x = 1$, it can be shown that $dx/d\tau > 0$, *i.e.*, if the complex roots get to the border of the unit circle, they are pushed back out. **QED**