Financial Intermediation, International Risk Sharing, and Reserve Currencies

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Abstract

I provide a framework for understanding the global financial architecture as an equilibrium outcome of the risk sharing between countries with different levels of financial development. The country that has the most developed financial sector takes on a larger proportion of global fundamental and financial risk because its financial intermediaries are better able to deal with funding problems following negative shocks. This asymmetric risk sharing has real consequences. In good times, and in the long run, the more financially developed country consumes more, relative to other countries, and runs a trade deficit financed by the higher financial income that it earns as compensation for taking greater risk. During global crises, it suffers heavier capital losses than other countries, exacerbating its fall in consumption. This country's currency emerges as the world's reserve currency because it appreciates during crises and so provides a good hedge. The model is able to rationalize these facts, which characterize the role of the US as the key country in the global financial architecture.

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The global financial architecture is characterized by the existence of a key country. This role has been fulfilled by the United States of America (US) since the Second World War; prior to the First World War it was fulfilled by the United Kingdom (UK). An important characteristic of the key country is the depth of its financial markets and, in particular, of its funding markets. The empirical literature has highlighted stylized facts that characterize the US international position: its external portfolio is characterized by riskier assets than liabilities; it runs a persistent trade deficit; it transfers wealth to the rest of the world (RoW) during global crises; and its currency is the world's reserve currency and earns a safety premium.

Despite extensive debates on the factors underpinning the global financial architecture, as well as its sustainability, there are few formal models that analyze its economic foundations. I provide a theoretical framework based on financial frictions that rationalizes the role of the key country in the global financial architecture and jointly explains the stylized facts that characterize the US external position.

The key country has the most developed financial sector and takes on a larger proportion of global fundamental and financial risk because its financial intermediaries are better able to deal with funding problems following negative shocks. In good times and in the long run it consumes more, relative to other countries, and runs a trade deficit financed by the higher financial income that it earns as compensation for taking greater risk. During global crises, however, capital losses on its external portfolio lead to a wealth transfer to the RoW. This increases the wealth loss suffered by the key country as a result of the crisis and exacerbates the fall in its consumption.

The key country's currency emerges as the reserve currency because it appreciates during crises, thus representing a global safe asset. This occurs, despite the key country's wealth losses, because of shifts in the relative demand for goods. The increase in the RoW's relative demand for RoW goods, which originates from the wealth transfer from the key country to the RoW, is more than offset by the fall in the key country's relative demand for RoW goods, which originates from increased RoW export costs.

The model not only provides a theoretical framework that jointly makes sense of the empirical stylized facts; its main contribution is to do so by providing the underlying economic foundations through the explicit modeling of financial intermediation and its frictions. The model recognizes the importance of financial intermediation from the key country as both the means of sharing risks globally and a potential source of risk and instability for the global financial architecture.

The model shows that the global financial architecture is affected by endogenous financial instability, with negative fundamental shocks being amplified by the financial system. The amplification occurs because financial intermediaries are levered and invest in similar risky assets; the resulting systemic risk exacerbates the effects of adverse shocks through a fire-sale mechanism.

I summarize the empirical evidence that motivates this paper in four stylized facts:

Fact 1: The US external balance sheet is characterized by risky assets, mainly denominated in foreign currencies, and safer liabilities, mainly denominated in US dollars. Figure 1 shows the US external balance sheet, as of year-end 2007. US residents' holdings of foreign assets were focused on riskier assets, such as equity and foreign direct investment (FDI), which together accounted for 56% of total US assets. By contrast, foreign residents' holdings of US assets were concentrated in safer assets such as debt, which accounted for 69% of total US liabilities.[1] Figure 2 confirms this by plotting the above percentages for 1970-2010. Figure 3 highlights that the majority of US external assets, 64% on average, are denominated in foreign currencies. US external liabilities are instead predominantly denominated in US dollars, 90% on average.[2]

Fact 2: The US runs a persistent trade deficit. The US has run a trade deficit every year since 1976; in 2010, its trade deficit was 3.4% of GDP.[3]

Fact 3: During global crises, the US transfers substantial amounts of wealth to the RoW. The US net foreign asset position deteriorated by $1.4 trillion in 2008. This corresponds to a transfer of 10% of US GDP to the RoW over that year.[4]

Fact 4: The US dollar is the world reserve currency and earns a safety premium.

[1] Source: Balance of Payment Statistics. The percentages are computed as (Equity+FDI)/(Total Assets-Derivatives) and (Debt+Other Investments)/(Total Liabilities-Derivatives).

[2] Source: Lane and Shambaugh (2010). The average is for the period 1990-2004.

[3] Source: IMF.

[4] Source: Balance of Payment Statistics and author's calculations. The deterioration is due in part to changes in the US external portfolio positions and in part to capital losses. I calculate that the capital losses alone constitute a transfer of 7.5% of US GDP. See also Gourinchas, Rey, and Truempler (2011).

Institutions around the world, both private and governmental, hold reserves of US dollars.[5] Figure 4 shows the estimated[6] compensation required by investors for holding a basket of foreign currencies while funding themselves in US dollars: the US dollar safety premium. The annualized premium is, on average, 1%; however, it increases significantly in times of crisis. At the height of the recent financial crisis in October 2008, the US dollar safety premium was as high as 53%.

To make sense of these facts I introduce three successive models. In Section II, I introduce a general-equilibrium model of financial intermediation in a closed economy. This autarky model highlights the mechanisms that play an important role in the open economy case; however, its implications for asset pricing are of independent interest. In Section III, I introduce a simple open economy model with two countries and a single world endowment. This model highlights the core result of the paper: the asymmetric risk sharing between the key country and the RoW, from which *Facts 1-3* emerge. This model cannot account for *Fact 4* because, by design, no exchange rate is present. In Section IV, I allow each country to have an endowment of a differentiated good. In addition to considering how financial frictions affect demand for financial assets, I also consider how they affect demand for goods by introducing trade costs. This final model not only allows me to analyze the exchange rate, but also generalizes the results from the previous section.

In the autarky model in Section II, savings are deposited with financial intermediaries, which in turn invest in risky assets. Since financial intermediaries may choose not to repay their depositors, their funding is potentially credit constrained. When financial intermediaries are well capitalized, the high level of capital acts as a safety buffer against potential investment losses; they can therefore easily raise funding and invest in risky assets. When financial intermediaries are poorly capitalized, concerns over their viability restrict their funding and therefore curtail their ability to invest in risky assets. Financial intermediaries are concerned about two sources of risk: fundamental risk and financial

[5] Eichengreen (2011, page 64) shows that 63% of world official reserves were held in US dollars at year-end 2009, a figure close to the average for the period 1965-2009.

[6] See Maggiori (2010) for details of the estimation. Lustig, Roussanov, and Verdhelan (2010) provide evidence of a counter-cyclical US dollar currency premium.

risk. The former stems from variations in output, while the latter results from variations in the aggregate capital of financial intermediaries. In equilibrium, the presence of financing frictions induces intermediaries to discount risky assets more than in a frictionless model.

In the open economy model in Section III, the greater depth of the US's financial development is represented by the key country's financial intermediaries being better able to raise funding for investment purposes, even when they are poorly capitalized. This, in turn, induces the key country's financial intermediaries to be less concerned about taking leveraged risk: in equilibrium, they take more risk. On the other hand, RoW financial intermediaries accumulate precautionary long positions in safer assets in order to insulate their capital from negative shocks. The asymmetric US balance sheet (*Fact 1*) emerges from this asymmetric risk sharing. The US trade deficit (*Fact 2*) emerges from the higher consumption that it enjoys in good times and in the long run, as compensation for the greater risks that it takes. Similarly, wealth transfers occur in bad times (*Fact 3*) because of the heavier losses suffered by the key country following negative shocks.

The role of the US dollar as a global safe asset is challenging to explain within traditional models. These would predict that a transfer of wealth from the US to the RoW during crises would result in a US dollar depreciation, because the wealth transfer would increase the relative demand for RoW goods, as long as the RoW residents were spending a higher proportion of the wealth that they received on RoW goods than on US goods. If this were the case, the US dollar would represent a risky asset for RoW residents, since it would pay low in bad states of the world.

The tension between the wealth transfer from the US to the RoW and the role of the US dollar as a global safe asset creates a "reserve currency paradox". In Section IV, I rationalize these seemingly contradictory forces by showing that the paradox can be resolved if, in addition to the channel described above, the US relative demand for US goods also increases during crises. I directly model a set-up where RoW export costs increase whenever RoW financial intermediaries lose capital and decrease the availability of credit to RoW exporters. A less literal interpretation of the model also accommodates frameworks where US and RoW exports are differentiated and, in particular, where the demand for US goods is more resilient to global downturns.

I Related Literature

The closed economy model contributes to the study of the implications of the financial sector for macroeconomics and finance, in the tradition[7] of Bernanke and Gertler (1989) and Kiyotaki and Moore (1997). In particular, it adapts the modeling of financial intermediation of Gertler and Kiyotaki (2010)[8] to a continuous-time endowment-economy framework. These modifications allow me to provide global solutions,[9] analyze risk premia, and characterize both the stochastic steady state and the stationary distribution of wealth. The global solutions, which are analytical up to the solution of a system of two ordinary differential equations (ODEs), show that the equilibrium has substantial non-linearities that cannot be readily analyzed by log-linearizing around the deterministic steady state. This solution method also allows me to exactly characterize the international portfolios in the open economy model.

The key assumption of greater financial development[10] of the US compared to the RoW is in the spirit of Caballero, Farhi, and Gourinchas (2008) and Mendoza, Quadrini, and Ríos Rull (2009). Kindleberger (1965) and Despres, Kindleberger, and Salant (1966) were among the first to argue that the asymmetric external balance sheet of the US, and previously of the UK, could be due to differences in financial development. Caballero et al. (2008) analyze a deterministic model where the US's greater ability to supply tradable assets rationalizes the emergence of global imbalances, the US trade deficit, and low long-term interest rates. Mendoza et al. (2009) analyze a production economy with idiosyncratic risk and limited contract enforceability, where the US's greater ability to enforce contracts leads to a lower US interest rate and an asymmetric US balance

[7]For a sample of this literature see: Bernanke, Gertler, and Gilchrist (1999), Fostel and Geanakoplos (2008), Simsek (2009), Kurlat (2009), He and Krishnamurthy (2010), Brunnermeier and Sannikov (2010), Gârleanu and Pedersen (2011).

[8]This paper builds on the work of Gertler and Karadi (2011) and Kiyotaki and Moore (2008).

[9]"Global" refers to solving the equations that characterize the equilibrium of the model for the entire range of the state variables, rather than solving them by a log-linear approximation of the model around the non-stochastic steady state.

[10]I am not suggesting that financial development is the only characteristic. Recent literature, for example, has emphasized the importance of country size for currency returns (Hassan (2010), Martin (2011)). My goal is to isolate the role of one important characteristic, financial development, and to analyze its equilibrium implications.

sheet. The most closely related work is that of Gourinchas, Govillot, and Rey (2010),[11] who study the role of the US as an insurance provider to the RoW in a representative agent framework with complete markets, where agents differ in the coefficient of relative risk aversion.

I add to this literature not only by providing a risk-based view of the role of the key country, which differs from the traditional macroeconomic view; more importantly, I do so by providing the underlying economic foundations through the explicit modeling of financial sector frictions and aggregate risk. The former is important to understanding the characteristics that distinguish the key country and its role, while the latter allows me to analyze the benefits and the costs of asymmetric risk sharing, especially during financial crises.

I also analyze exchange rate dynamics, which are important to understanding why the RoW considers US-dollar-denominated short-term debt to be safe. Previous papers do not model the role of the US dollar as a reserve currency or its safety premium. In addition, the risk-based view of the key currency that I provide is in contrast to Krugman (1980) and Matsuyama, Kiyotaki, and Matsui (1993), who instead stress the vehicle role of the key currency for international trade.

II Autarky: the Banking Economy

The output of the economy is determined by a tree with stochastic dividend process

$$\frac{dY(t)}{Y(t)} = \mu\ dt + \sigma\ dz(t), \tag{1}$$

where $z(t)$ is a standard Brownian motion,[12] and μ and σ are constant.

The set-up of financial intermediation is a continuous time adaptation of Gertler and Kiyotaki (2010). The economy is populated by a continuum of measure one of households.

[11] The paper also provides evidence that the US earns positive excess returns on its external portfolio. See Curcuru, Dvorak, and Warnock (2008) for a contrarian view.

[12] The Brownian motion is defined on a complete probability space and generates a filtration $\mathscr{F}$. Throughout the paper, "adapted process" means $\mathscr{F}(t)$ adapted. For brevity, I state all results without explicitly referencing the regularity conditions necessary for the applications of stochastic calculus in this paper. Where necessary, some regularity conditions are explicitly verified in the proofs in Appendix A.

Each household consists of a continuum of measure one of family members, or agents, of which a fraction $\beta \in (0,1)$ are savers and a fraction $1-\beta$ are financiers. All agents, both savers and financiers, have logarithmic utility and identical rate of time preferences. Each financier within a household manages a financial intermediary; these are all, in turn, owned by the household. Savers deposit funds with these financial intermediaries.

By assumption, there is perfect consumption insurance within each household because all agents pay out their earnings to be shared equally across the entire household. This assumption, combined with an application of the law of large numbers across households, allows for the construction of the representative agent.

In order to create a meaningful role for financial intermediation, I assume that only financiers, through their financial intermediaries, can hold shares in the output tree.[13] Savers can only deposit funds with financial intermediaries and they receive a pre-determined return $r_d(t)$.

The saver's problem, therefore, is to choose how much to consume and how much to deposit with the financial intermediaries:

$$\begin{aligned} &\max_{\{C(u)\}_{u=t}^{\infty}} \; E_t\left[\int_t^{\infty} e^{-\rho(u-t)} log(C(u))du\right] \\ &s.t. \quad dD(t) = [r_d(t)D(t) - C(t)]dt + \Pi(t)dt, \end{aligned} \tag{2}$$

where D is the aggregate savers' deposits and Π is the aggregate net transfers from the financiers, described later. Because the economy has a representative agent, I directly write the saver's optimization problem in terms of aggregate quantities. Throughout the paper, upper-case letters denote aggregate quantities, while lower-case letters denote individual agents' quantities. In addition, I use the equilibrium outcome of no default on deposits to directly write the dynamics of the deposit account as being risk free.

Financiers can use their own capital and the deposits that they have raised to invest in the risky asset. The balance sheet of a financial intermediary is $Q(t)s(t) = n(t) + d(t)$, where $s(t)$ is the number of shares of the output tree owned by the financial intermediary,

[13] A number of papers motivate this assumption by developing micro-foundations where monitoring problems make it inefficient for savers to directly hold assets. These papers delegate the asset management problem to financial intermediaries in equilibrium. I follow Gertler and Kiyotaki (2010) in directly assuming that an unmodelled monitoring problem prevents savers from directly holding assets.

$Q(t)$ is the price of the output tree, and $n(t)$ is the financial intermediary's net worth. The stock price dynamics follow the continuous diffusion process

$$\frac{dQ(t)}{Q(t)} + \frac{Y(t)}{Q(t)}dt = \mu_Q(t)dt + \sigma_Q(t)dZ(t).$$

The drift and volatility terms need to be solved for in equilibrium.

Financiers face a credit constraint, which requires that the value of the financial intermediary that they manage remains positive. To motivate this constraint, I introduce an incentive compatibility problem. Financiers can walk away from their financial intermediary; if this occurs, the financial intermediary is wound down and its depositors recover the value of the financial intermediary's assets: $s(t)Q(t)$.[14]

Savers only deposit funds with financial intermediaries owned by other households. In particular, they spread their deposits sufficiently across the financial intermediaries owned by the various households to allow the law of large numbers to hold.[15] This allows a simple aggregation of the model, while still maintaining a meaningful incentive for financiers to walk away from negative net worth financial intermediaries. In short, the incentive compatibility problem provides the micro foundations for a credit constraint.

Since financiers and savers have identical utility functions, there are no incentives for financiers to pay dividends from their financial intermediaries. Instead, financiers would choose to accumulate capital and their financial intermediaries would "grow out" of the credit constraint. To prevent this outcome, I assume that financiers and savers switch roles based on exponential probability functions with intensity λ and $\lambda\frac{1-\beta}{\beta}$, respectively.[16] When a financier switches role, she pays all her accumulated net worth to her household.

[14]More precisely, savers receive $min\{(s(t)Q(t), d(t)\}$, with excess funds, if any, being returned to the financier's household. In equilibrium, however, the financier has no incentive to walk away from the financial intermediary if its deposits can be fully recovered, so the simplified formulation is adopted in the main text.

[15]To motivate this assumption one can think of a set-up with idiosyncratic risk in each intermediary, such that savers want to diversify their deposits across intermediaries, and then let this risk shrink to zero.

[16]The different intensities maintain the populations of savers and financiers constant.

The financier's optimization problem is, therefore, to maximize the value of the financial intermediary that she manages, subject to the credit constraint:

$$\max_{\{d(u),s(u)\}_{u=t}^{\infty}} \Lambda_\lambda(t)V(t) = E_t\left[\int_t^\infty \Lambda_\lambda(u)\ \lambda\ n(u)du\right] \tag{3}$$
$$s.t. \quad dn(t) = s(t)(dQ(t) + Y(t)dt) - r_d(t)d(t)dt$$
$$V(t) \geq 0,$$

where $\Lambda_\lambda(t) \equiv e^{-(\rho+\lambda)t}\frac{1}{C(t)}$ is the agents' marginal utility modified for the intensity with which financiers change roles, and $V(t)$ is the value of the financial intermediary. Intuitively, the value of the intermediary is the expected discounted value of its dividends.[17] The first constraint is the evolution of the financial intermediary's net worth, while the second is the credit constraint.

When a saver becomes a financier, she needs capital with which to operate. I assume that this start-up capital is received from the household. In particular, I assume that each new financier is endowed with a fraction $\frac{\delta}{\lambda(1-\beta)}$ of the existing financiers' assets. Therefore, the aggregate net worth of the financial sector evolves according to:

$$dN(t) = (r_d(t) - \lambda)N(t)dt + S(t)Q(t)[(\mu_Q(t) + \delta - r_d(t))dt + \sigma_Q(t)dz(t)].$$

Similarly, the sum of net transfers from financiers to households is

$$\Pi(t) = \lambda N(t) - \delta S(t)Q(t).$$

The market clearing conditions are

$$C(t) = Y(t); \qquad S(t) = 1.$$

The number of shares in the output tree is normalized to one.

[17]The appropriate discount factor is the marginal value of consumption of the agent receiving the dividends. The financier pays a dividend only once, when she is selected to switch role. The term $e^{-\lambda u}\lambda$ is the probability density function for this exponentially distributed event.

The micro-foundations of the model are intended to capture an array of financial intermediaries, spanning from traditional retail banks to investment banks and the shadow banking system. Despite the heterogeneity of these players, I emphasize their common characteristic: a balance-sheet transmission channel. They are funded by a combination of equity capital and short-term borrowing, while their assets are long term and risky. Financial intermediaries' risky assets are represented in the model by shares in the output tree. The paper focuses on the debt funding of financial intermediaries, with the savers' deposits intended to capture not only retail deposits but also other common debt-funding sources. In particular, interbank debt contracts are formally introduced in Sections III and IV when discussing the model of an open economy. He and Krishnamurthy (2010) study an endowment economy with a financial sector under equity funding.

A Optimal Consumption and Investment

Throughout the paper, I scale variables by the value of current output, with a tilde denoting the scaled version of the corresponding variable.[18] I restrict my attention to the class of Markovian equilibria. The concept of equilibrium is the standard Walrasian one.[19] I suppress the time notation of stochastic processes throughout the rest of the paper, except where necessary to clarify formulas.

A.1 The Saver's Problem

Savers choose how much to consume and how much to deposit with financial intermediaries, as a fraction of the economy's current output. I conjecture that the saver's value function, denoted U, only depends on deposits and the financial sector's net transfers, both scaled by output: $(\tilde{D}, \tilde{\Pi})$. The marginal saver is atomistic and therefore does not take into account the effect of her saving decision on the financial sector's net transfers.

[18]In the current autarky setting, the consumption good is the numeraire and scaling by the value of output is achieved by dividing variables by Y.

[19]Consumption and investment decisions are adapted processes such that the financier's and saver's optimization problems are satisfied and markets clear.

Lemma 1. *The optimality conditions for the saver's optimization in equation (2) imply that the saver prices risk-free deposits according to*

$$-r_d\; dt = E_t\left[\frac{d\Lambda}{\Lambda}\right], \qquad where \quad \Lambda \equiv e^{-\rho t}\frac{1}{C}. \tag{4}$$

This and all other proofs are reported in Appendix A. The saver's Euler equation is unaffected by frictions and has the standard intuition of the optimal trade-off between consumption and savings, given the interest rate.

A.2 The Financier's Problem

Since each financier is atomistic and, therefore, does not affect expected returns in equilibrium, the value of a financial intermediary is scale invariant: an intermediary with ten times more net worth has a value that is ten times higher. Consequently, I conjecture that the financier's value function is linear in the individual financial intermediary's net worth: $V(\tilde{N}, n) = \Omega(\tilde{N})\; n$.

I also conjecture that the marginal value of net worth, Ω, only depends on the aggregate financial sector net worth, scaled by output. Aggregate net worth affects the incentives for financiers to walk away from their financial intermediaries; consequently, it intuitively also determines the tightness of the credit constraint and, in turn, expected returns to financial capital.

Lemma 2. *The optimality conditions for the financier's optimization in equation (3) imply that the financier prices risk-free deposits and shares in the tree according to:*

$$0 = \lambda\Lambda Q(1-\Omega)dt + \Lambda\Omega Y dt + E_t\left[d(\Lambda\Omega Q)\right] \tag{5}$$

$$0 = \lambda\Lambda D_a(1-\Omega)dt + E_t\left[d(\Lambda\Omega D_a)\right], \tag{6}$$

where D_a is the deposit asset with dynamics $\frac{dD_a}{D_a} = r_d\; dt$.

The financier is concerned about two risks: consumption risk and financial risk.[20]

[20] In Appendix A, I show that the financier's Euler equations imply that assets are priced according to a multi-factor asset pricing model, where the two factors are consumption and aggregate scaled net worth. This model extends the Consumption Capital Asset Pricing Model (CCAPM) to account for financial risk.

The financier dislikes assets with low returns when aggregate consumption is low and when her financial intermediary has low net worth. The former, which is consistent with standard consumption-based asset pricing models, is captured by the term Λ. The latter, which would result in a tightening of the credit constraint, is captured by the multiplicative term Ω. If financial risk and consumption risk are positively correlated, as they are in equilibrium, financiers discount the risky asset more than an investor with equal consumption but logarithmic utility, hereafter referred to as the log investor.

Ω can be interpreted as the "q price" of installed financial capital. Capital outside the financial sector is worth its purchase value of 1, since the consumption good is the numeraire. However, installed capital inside the financial sector is worth more than 1 because financial intermediaries earn, from the perspective of a log investor, abnormal risk-adjusted returns. Intuitively, the term $\lambda(1-\Omega)$ in the above Euler equations accounts for the probability λdt with which a financier switches role in the next dt units of time and the fact that, upon switching, capital is only worth 1 rather than Ω.

B Equilibrium

B.1 The Lucas Economy

Assume that there are no frictions, so that financiers always have to repay all deposits. In this case, the equilibrium is equivalent[21] to that of a standard Lucas endowment economy (Lucas (1978)), where the endowment is given by equation (1) and there exists a representative agent with logarithmic preferences who can trade both shares in the output tree and a risk-free bond. I refer to this economy in short as the Lucas Economy.[22]

Intuitively, the distribution of wealth between deposits and financial capital does not affect the equilibrium; this is because financiers can always raise sufficient deposits to achieve the desired investment in the risky asset.[23] It follows that the marginal value of

[21]See Appendix A.

[22]It is well known that the equilibrium of this economy features a constant risk-free rate and a constant and low risk premium. Assets are priced according to the CCAPM, with consumption as the only risk factor.

[23]Following investment losses, and even in the case where net worth becomes negative, depositors are always repaid in full because financiers can roll over deposits. Furthermore, if a financier with negative net worth is selected to switch roles, she pays negative net worth out to her household; that is, the household repays in full the selected financier's depositors.

net worth, Ω, is constant at 1. Consequently, the pricing equations in equations (5-6) simplify to the classic Lucas equations.

B.2 The Banking Economy

The equilibrium of the economy with frictions is affected by the wealth distribution, that is, the amount of capital inside the financial sector. When financial intermediaries have low capital, financiers are concerned about losing further capital; consequently, financial intermediation becomes disrupted and wealth cannot readily be invested in the risky asset. By contrast, when financial intermediaries are better capitalized there is a buffer against investment losses, leading to an investment allocation closer to the one in the Lucas Economy.

Proposition 1. *The financier's and saver's optimization problems can be written in terms of a single state variable: the aggregate financial sector net worth scaled by output $\tilde{N}$. Furthermore, the state variable is a strong Markov process with dynamics*

$$\begin{aligned} \frac{d\tilde{N}}{\tilde{N}} &= \left[\rho - \lambda + \phi(\mu_Q - r_d + \delta - \sigma\sigma_Q)\right] dt + (\phi\sigma_Q - \sigma)dz \\ &\equiv \mu_{\tilde{N}} dt + \sigma_{\tilde{N}} dz, \end{aligned} \tag{7}$$

where $\phi \equiv \frac{Q}{N}$ is the financial sector leverage. The equilibrium is characterized by a system of two coupled second-order ODEs for the price-dividend ratio, $\tilde{Q}(\tilde{N})$, and the marginal value of net worth, $\Omega(\tilde{N})$:

$$0 = \mu_Q - r_d - \sigma\sigma_Q + \sigma_\Omega\sigma_Q \tag{8}$$

$$0 = \lambda\frac{1-\Omega}{\Omega} + \mu_\Omega - \sigma\sigma_\Omega, \tag{9}$$

where $\frac{d\Omega}{\Omega} = \mu_\Omega dt + \sigma_\Omega dz$.

I conjecture, and it is the case in equilibrium, that the state variable is pro-cyclical: $\sigma_{\tilde{N}} \geq 0$. This occurs because financiers are levered and raise risk-free funding while investing in the risky asset; consequently, a positive dividend shock increases net worth on more than a one-for-one basis.

The system of ODEs[24] has an intuitive interpretation, though a formal analysis of the boundary conditions and the numerical solution method are also included in Appendix B. The ODE (8) implies that the Sharpe ratio is higher than in the Lucas Economy; this occurs because financiers are worried about losses of capital that could restrict their investment opportunity set. To see this, re-write equation (8) as

$$\frac{\mu_Q - r_d}{\sigma_Q} = \sigma - \sigma_\Omega.$$

The Sharpe ratio has two components. The first, the volatility of consumption, which in equilibrium is equal to σ, is the same as in the Lucas Economy. The second, σ_Ω, accounts for financiers' required compensation, measured per unit of risk, to take on risk that is correlated with their net worth. In equilibrium, $\sigma_\Omega < 0$ because the marginal value of net worth increases when financiers lose capital. The ODE (9) is a restriction on the dynamics of Ω; it ensures that financiers and savers agree on the pricing of risk-free deposits.[25]

Endogenously, financiers cut their risky investments sufficiently quickly following losses that a default never occurs and the credit constraint never binds. Therefore, other than fundamental risk σ, all risk in the model is liquidity risk. This arises because the financial sector engages in maturity-risk-liquidity transformation[26] by borrowing in instantaneous fixed-rate deposits and investing in long-term risky assets.

The equilibrium dynamics are illustrated in Figure 5. A quantitative analysis is beyond the scope of this paper; the equilibria described in this and the following sections are numerical examples rather than calibrations.

[24]Here and in subsequent propositions the ODEs (8-9) are expressed implicitly since the drifts and volatilities are themselves only functions of $\tilde{N}$ and the level and first two derivatives of the functions Ω and $\tilde{Q}$. The explicit form of the ODEs is provided in Appendix A.

[25]The saver's Euler equation (4) and the fact that, in equilibrium, consumption equals output together imply that the risk-free deposit rate equals the risk-free rate in the Lucas Economy. For financiers to agree on the pricing of the risk-free rate, the ODE (9) requires that the intertemporal (elasticity of substitution) and intratemporal (precautionary) effects of financial risk (Ω) on the risk-free rate exactly offset each other. See Appendix A for details.

[26]The concepts of maturity, risk, and liquidity transformation have been defined in various ways in the literature. I follow here the definitions in Brunnermeier, Eisenbach, and Sannikov (2010), according to whom: the maturity transformation occurs because debt funding is instantaneous while the asset is infinitely lived; the risk transformation occurs because debt funding is risk-free while the asset is risky; and the liquidity transformation occurs because debt, being instantaneous, is continuously regenerated in the liquid consumption good, while equity sales have different price impacts depending on the level of the state variable.

Figure 5 shows that the effects of bank capitalization on the equilibrium are non-linear. In particular, there are two regions with markedly different equilibrium dynamics. In the first region, which covers most of the state space, the decreasing capitalization of financial intermediaries leads to a fall in the stock market and an increase in volatility. In the second region, the extremely low net worth of financial intermediaries leads to an increase in the stock market and a decrease in volatility. I describe the dynamics of each of the regions in turn.

In the first region, a negative output shock not only results in financiers losing capital; their concern about further potential losses also induces them to decrease investments in the risky asset, as a precautionary measure. The resulting fall in demand for the risky asset can be observed in its declining price-dividend ratio. As all financiers have similar balance sheets, the initial small *iid* fundamental shock is amplified by systemic risk. With all financiers trying to sell the risky asset, a vicious cycle of fire sales[27] commences: each individual financier trying to sell depresses the stock price, inducing further capital losses and triggering a requirement to sell even more shares. The model therefore endogenously generates a flight-to-safety effect.

The amplification also generates an increase in the volatility of asset prices. The diffusion terms of the stock and of scaled net worth can be written as

$$\sigma_Q = \frac{\phi - \tilde{Q}'}{\phi(1 - \tilde{Q}')}\sigma; \qquad\qquad \sigma_{\tilde{N}} = \phi\ \sigma_Q - \sigma, \tag{10}$$

where the superscript $'$ denotes the first derivative of a function. Endogenously, $\phi \geq 1$ and $\tilde{Q}' < 1$. Asset prices are more volatile than dividends whenever $\tilde{Q}'(\phi - 1) > 0$, with the extent of the amplification depending on financial intermediaries' leverage and on the

[27]As in Shleifer and Vishny (1992), the financiers attempting to sell the asset depress its price because the "natural buyers", the other financiers, have also suffered capital losses and are also attempting to sell. The risky asset is non redeployable since savers, by assumption, value it at zero (cannot hold it). Fire-sale transactions never occur in equilibrium; financiers' attempts to sell the asset reduce its price sufficiently to induce them to hold it. As in Kiyotaki and Moore (1997), a dynamic feedback effect amplifies this static effect. In my set-up, however, the dynamic effect arises from endogenous movements in the discount factor rather than in cash flows. Capital losses heighten intermediaries' concerns about further losses and increase their discount factor for the risky asset. Since capital cannot be immediately replenished, the increase in the discount factor is persistent. The higher discount factor for future cash flows dynamically feeds back into lower present asset prices, thus further lowering intermediaries' present net worth.

reaction of the price-dividend ratio to changes in net worth.[28] There is no amplification only if financial intermediaries are not levered ($\phi = 1$) or if the price-dividend ratio does not react to changes in intermediary capital ($\tilde{Q}' = 0$). In the first region, amplification is positive since intermediaries are levered ($\phi > 1$) and the price-dividend ratio falls whenever intermediaries lose capital ($\tilde{Q}' > 0$).

The equilibrium dynamics in this first region illustrate common characteristics of financial crises. These dynamics change as further negative shocks push financial intermediaries into the second region, where their capital is close to zero. Recall that, in aggregate, the credit constraint takes the form $\Omega\tilde{N} \geq 0$. The tightness of the constraint is determined by the balance of two opposing effects: losses of capital, reflected in a lower $\tilde{N}$, induce increases in the value of capital, represented by a higher Ω.

In the first region, losses of capital outweigh the effect of increases in the value of capital and tighten the constraint almost linearly. As financial intermediaries' capital decreases further and we enter the second region, the increase in the value of capital alleviates the losses of capital and the constraint tightens more slowly. Intuitively, the higher Sharpe ratio mitigates the incentives of financiers to walk away from poorly capitalized financial intermediaries. This causes the price-dividend ratio to increase whenever there are intermediary capital losses ($\tilde{Q}' < 0$). In this case, equation (10) shows that capital gains have a stabilizing effect on losses of net worth and dampen the volatility of asset prices. The risky asset begins to mimic the risk-free one and, in the limit as net worth approaches zero, the risky asset is locally risk less.[29]

[28] This emphasizes, as in Brunnermeier and Pedersen (2008), the interaction of market liquidity, i.e. the price impact of transactions in the risky asset ($\tilde{Q}'$), and funding liquidity, i.e. the ability of financial intermediaries to raise capital for investment (ϕ).

[29] This second region of the state space provides an endowment economy equivalent to financial depressions, such as the one experienced in Japan starting in the early 1990s. Following the most acute phase of a crisis, where the stock market crashes and volatility increases, further losses of capital lead to a depression region. Here, stock prices are so high compared to dividends that risky investment returns are low. Consequently, financiers are not able to quickly escape this region by accumulating net worth through positive returns on investments. Figure 7 confirms the intuition by showing a fall in the drift and volatility of aggregate financial net worth. In the limit, as $\tilde{N} \downarrow 0$, the drift approaches $\delta\tilde{Q}$ and can be set arbitrarily close to zero, and the volatility goes to zero. Brunnermeier and Sannikov (2010) provide a similar "area of attraction" in the low region of the state space. In my model, the main difference is that the depression is caused solely by endogenous changes in the discount factor, while cash flows are exogenous.

Under the restriction $\delta = \lambda - \rho$, the economy eventually converges to the Lucas Economy equilibrium. Intuitively, financiers accumulate net worth sufficiently quickly to reach a state where the entire supply of risky investments can be bought with the financial intermediaries' capital.[30] In this state, the absence of leverage induces the financial intermediaries' capital to move one-for-one with stock prices, and financiers are no longer concerned about losing their net worth. The equilibrium dynamics of this case are illustrated in Figure 5. In contrast, under the restriction $\delta < \lambda - \rho$ financiers do not converge to the frictionless equilibrium. In this case, deposits are always strictly positive and the levered financiers are forever concerned about potential losses of net worth. The resulting equilibrium dynamics are illustrated in Figure 6.

In both cases, the *stochastic* steady state[31] is the point in Figure 7 where the drift of scaled net worth equals zero. In the first case, the stochastic steady state is the upper boundary of the state space: $\tilde{N}^{SS} = \frac{1}{\rho}$. The limiting distribution of scaled net worth is degenerate, with the entire probability mass concentrated at the stochastic steady state. In the second case, the stochastic steady state is in the interior of the state space; the stationary distribution of scaled net worth is reported in Figure 8. The distribution has a fat left tail, since negative shocks are amplified more than positive shocks. Therefore, while fundamental shocks are *iid* Gaussian, the banking economy suffers from endogenous financial disasters.

The autarky model shows that agents are concerned about both fundamental and financial risk. Furthermore, this concern creates lower demand for risky assets, particularly during bad times, and an endogenous amplification of shocks. These elements play a crucial role in the open economy that is analyzed in the next section.

[30] The balance of three effects regulates the asymptotic accumulation of aggregate net worth: financiers accumulate capital at the rate of time preference ρ, start-up capital allocated to new financiers increases aggregate net worth by δ, and net worth paid out by exiting financiers reduces aggregate net worth by λ.

[31] The stochastic steady state is defined as the point to which the state variable converges if shocks are possible but are not ever realized. This is in contrast to the most commonly analyzed *deterministic* steady state, which is defined as the point of convergence if the model features no shocks ($\sigma = 0$).

III Open Banking Economy: Single World Tree

To understand the role of the US in the global financial architecture, I introduce a simple model with two countries, Home and Foreign, which are symmetric other than the extent to which their respective financial systems are developed. This stylized model isolates the role of the asymmetry in the countries' financial sectors and describes the main result of this paper: the asymmetric risk sharing between the US and the RoW. The empirical *Facts 1-3* emerge from the implementation of this risk sharing.

The US, which acts as the key country in the global financial architecture, is characterized by the greater extent of its financial development and, in particular, the greater depth of its funding markets. This asymmetry is in the spirit of Kindleberger (1965), Caballero et al. (2008), and Mendoza et al. (2009), who were among the first to emphasize differences in financial development as a key driver of global imbalances.[32] Eichengreen (2011, pages 17-33) emphasizes how the development of funding markets for trade in New York in the 1920s was an important driver of the key country role switching from the UK to the US.

One can think of a general form of the credit constraint, where financiers have different abilities to divert assets or to walk away from their obligations. The less financiers are able to divert assets or to walk away from their obligations, the greater financial development is. This is meant to capture both the legal framework that is essential for the emergence of financial markets, and the broader institutional and regulatory design that affects the cost and efficiency of transactions in financial markets.

For simplicity I assume that Home financiers are unconstrained, while Foreign financiers face the limited-liability constraint described in the previous section.[33]

[32]The assumption is also supported by the literature on comparative financial and institutional development. Rajan and Zingales (1998), for example, motivate their empirical work, which assumes a frictionless financial market for the US, by noting that "capital markets in the United States are among the most advanced in the world".

[33]The choice of frictionless Home financial intermediation is one of convenience. It allows the model to be analyzed with a single state variable. More generally, one can think of Home intermediaries facing frictions, albeit lower than those faced by Foreign intermediaries. One-period and two-period versions of the model, which allow for frictions in both countries, yield similar qualitative results.

The global output of the sole good is generated by the process in equation (1); each country is endowed with half of the output. Almost the entire set-up of each of the two economies is identical to the autarky case, so I only describe the differences. I describe the model for the Foreign country, and only specify the corresponding Home country equations where necessary. Foreign variables are denoted by the superscript *.

Savers can only deposit funds with their domestic financial intermediaries; consequently, they solve a problem identical to equation (2). This restriction emphasizes the fact that private savings primarily enter the global financial system through domestic financial institutions.[34]

In addition to raising deposits domestically and investing in the risky asset, financiers can also lend and borrow in an international market for interbank[35] loans. These instantaneous interbank loans are promises to pay one unit of the consumption good. Both interbank loans and deposits are risk free in equilibrium, so I directly use this outcome to write their dynamics. The balance sheet of an individual financier is $Qs^* = n^* + d^* + b^*$, where b^* is the amount that the financier has borrowed in the interbank market.

In a technical simplification[36] from the autarky case, the exiting financiers have the option to reinvest their net worth with the incoming financiers. Since financiers maximize the value to their households of the intermediaries that they manage, they choose to reinvest the net worth whenever $\Omega^* > 1$ and to pay it out whenever $\Omega^* = 1$, where Ω^*, by analogy with the previous section, is the Foreign financier's marginal value of net worth. The representative financier problem is, therefore, equivalent to one for an intermediary not paying any net worth out to the household until a stopping time $t' \equiv inf\{t : \Omega^*(\tilde{N}^*(t)) = 1\}$. After that point is reached, exiting financiers pay their net

[34]While off-shore accounts certainly exist, they are a small phenomenon compared to on-shore savings.
[35]Literally, this market should be referred to as "inter-intermediary" rather than "interbank". In practice, various types of financial intermediaries participate in the interbank funding market, so it is commonly understood that it is not merely a market for banks.
[36]This assumption allows for the simplification of the equilibrium risk sharing between Home and Foreign without altering the economic implications of the model. In particular, it allows the equilibrium to be expressed as a function of a single state variable. See Appendix A for details.

worth to their households.[37] The representative financier's optimization problem is:

$$\max_{\{d^*(u),b^*(u),s^*(u)\}_{u=t}^{\infty}} \Lambda^*(t)V^*(t) = E_t\left[\int_{t'}^{\infty} \Lambda^*(u)e^{-\lambda(u-t')}\lambda n^*(u)du\right] \tag{11}$$
$$s.t. \quad dn^* = s^*(dQ + Ydt) - r_d^*\, d^*\, dt - r_b\, b^*\, dt$$
$$V^* \geq 0.$$

The Home financier's problem is symmetric, but without the last constraint. I assume that the start-up capital provided by households to new financiers is a function of the stochastic steady state[38] holdings of the risky asset in each country: $\bar{S}$ and $\bar{S}^*$, respectively. Consequently, new Home financiers receive $\delta\bar{S}Q$ and new Foreign financiers receive $\delta\bar{S}^*Q$. The aggregate net worth dynamics follow:

$$dN^* = r_d^* N^* dt + Q\{S^*[(\mu_Q - r_d^*)dt + \sigma_Q dz] + \delta\bar{S}^* dt\}$$
$$+B^*(r_d^* - r_b)dt.$$

An extra outflow of $\lambda N^* dt$ is detracted from the dynamics for all times after t'. The net transfers from financiers to their households are:

$$\Pi^* = -\delta\bar{S}^*Q.$$

An extra inflow of $\lambda N^* dt$ is added to the dynamics for all times after t'.

The Foreign trade balance is the difference between the Foreign share of world output and Foreign consumption. Foreign Net Foreign Assets (NFA) are the difference between the wealth owned in Home by Foreign residents and the wealth owned in Foreign by Home

[37] For this to be an equilibrium, the state where $\Omega^* = 1$ needs to be absorbing. As with the autarky case, this is guaranteed by the restriction $\delta = \lambda - \rho$, which is imposed in both this section and the next. See Appendices A and B for details.

[38] The assumption is meant to capture the fact that the household uses both the current value of assets and the long-run financial size of its country to judge how much start-up capital its new financiers need in order to operate. The specific functional form has been chosen to simplify the boundary analysis, and does not substantially affect the equilibrium.

residents.[39] Finally, the change in Foreign NFA is the Foreign Current Account (CA). Home definitions are symmetric. Thus, I have:

$$NX^* \equiv \frac{Y}{2} - C^*; \qquad NFA^* \equiv \left(S^* - \frac{1}{2}\right) Q - B^*. \tag{12}$$

The market clearing conditions are:

$$\begin{aligned} C + C^* &= Y; \qquad & S + S^* &= 1 \\ B &= -B^*; \qquad & N^* &= S^* Q - D^* - B^*. \end{aligned}$$

A Optimal Consumption and Investment

The Home country has no frictions; consequently, the Home marginal value of net worth is equal to one and the Home financiers' value function takes the form $V = n$. Foreign financiers instead value financial capital above one; as with the autarky case, their value function is $V^* = \Omega^*(\tilde{N}^*)n^*$. Since the Home and Foreign dynamic programming problems of both savers and financiers are extensions of those in the autarky case, they are reported in Appendix A. Here, I only include the corresponding Euler equations.

Lemma 3. *The optimality conditions for Home savers and financiers imply that they price assets according to:*

$$0 = \Lambda Y dt + E_t \left[d(\Lambda Q) \right] \tag{13}$$

$$0 = E_t \left[d(\Lambda D_a) \right] \tag{14}$$

$$0 = E_t \left[d(\Lambda B_a) \right]. \tag{15}$$

The optimality conditions for Foreign savers and financiers imply that they price assets

[39] Proportionally to their capital, all financial intermediaries within each country are identical. Equilibria with non-zero domestic interbank activity are possible, but are not materially different from the equilibrium where all interbank activity occurs across countries. Consequently, I focus on this last equilibrium and therefore include the interbank loans in the NFA position.

according to:

$$0 = \Lambda^*\Omega^* Y dt + E_t\left[d(\Lambda^*\Omega^* Q)\right] \tag{16}$$

$$0 = E_t\left[d(\Lambda^* D_a)\right] = E_t\left[d(\Lambda^*\Omega^* D_a)\right] \tag{17}$$

$$0 = E_t\left[d(\Lambda^* B_a)\right] = E_t\left[d(\Lambda^*\Omega^* B_a)\right], \tag{18}$$

where D_a is the deposit asset and B_a is the interbank asset.

Equations (13-15) show that the frictionless Home country only cares about consumption risk: the Home representative agent prices assets as though it had logarithmic preferences. By contrast, equations (16-18) show that the potentially constrained Foreign country also cares about financial risk, in addition to consumption risk. The Foreign representative agent discounts the stock more than an agent with logarithmic preferences if, as is the case in equilibrium, it has low returns when financial intermediaries have low capital. An immediate consequence of both deposits and interbank loans being risk free is that, to prevent arbitrage, their rates of return are equal: $r_b = r_d = r_d^*$.

B Equilibrium

B.1 Open Lucas Economy

Consider a Lucas open endowment economy (Lucas (1982)) with two symmetric countries, a single good generated by equation (1), and a representative agent with logarithmic preferences in each country, both of whom can trade claims to the tree and a risk-free bond. I refer to this economy in short as the Open Lucas Economy. If there are no frictions in the Foreign financial system, then the equilibrium of my model is equivalent to that of the Open Lucas Economy.[40]

Intuitively, the two countries are symmetric and the Foreign country is not affected by frictions, so that agents only care about consumption risk. Consequently, the international risk sharing and pricing equations reduce to the classic Lucas analysis. The equilibrium

[40]Details of the equilibrium are provided in Appendix A.

features of this economy are well known: symmetric equity portfolios, with each country owning half of the shares; no trading in the risk-free interbank market; equal Home and Foreign consumption state-by-state; and zero NFA, CA and NX. These results are a far cry from the stylized facts of the global financial system in *Facts 1-3*.

B.2 Open Banking Economy

The equilibrium of the open economy with frictions is affected by the wealth distribution, that is, the amount of capital inside the financial sector. However, only Foreign financial intermediaries are affected by frictions; consequently, the key variable is the fraction of the world's wealth that is held as capital by Foreign financial intermediaries.

Proposition 2. *The financier's and saver's optimization problems in the Home and Foreign countries can be written in terms of a single state variable: the aggregate Foreign financial sector net worth scaled by output* $\tilde{N}^*$*. Furthermore, the state variable is a strong Markov process with dynamics given by*

$$\begin{aligned}\frac{d\tilde{N}^*}{\tilde{N}^*} &= \left[(r_d - \lambda - \mu + \sigma^2) + \phi^*(\mu_Q - r_d - \sigma\sigma_Q) + \delta\frac{\bar{S}^* Q}{N^*}\right] dt + (\phi^*\sigma_Q - \sigma)dz \\ &\equiv \mu_{\tilde{N}^*} dt + \sigma_{\tilde{N}^*} dz,\end{aligned}$$

where $\phi^* \equiv \frac{S^*Q}{N^*}$*. The equilibrium is characterized by a system of two coupled second-order ODEs for the price-dividend ratio,* $\tilde{Q}(\tilde{N}^*)$*, and the marginal value of Foreign net worth,* $\Omega^*(\tilde{N}^*)$*:*

$$0 = \mu_Q - r_d - \sigma_C\sigma_Q \tag{19}$$

$$0 = \mu_{\Omega^*} - \sigma_{C^*}\sigma_{\Omega^*}, \tag{20}$$

where $\frac{dC}{C} = \mu_C dt + \sigma_C dz$ *and* $\frac{dC^*}{C^*} = \mu_C^* dt + \sigma_C^* dz$*.*

The system of ODEs has an intuitive interpretation; a formal analysis of the boundary conditions and the numerical solution method is included in Appendix B. The ODE (19) is a standard pricing equation: it shows that expected stock excess returns depend positively on the covariance between Home consumption and stock returns. The ODE (20) ensures that Foreign financiers and savers agree upon the price of risk-free deposits.[41]

The equilibrium allocation leads to an intuitive risk-sharing condition:

$$\frac{C^*}{C} = \frac{\Omega^*}{\xi}, \tag{21}$$

where ξ is a scaling constant that depends on the initial conditions and is akin to the relative weight of the Home country in a complete-market central-planner problem. The risk sharing is asymmetric: an increase in the marginal value of Foreign financial capital is associated with a relative increase in Foreign consumption over Home consumption. As Ω^* is counter-cyclical in equilibrium, this provides the foundations of the risk sharing that underpins the global financial architecture.

In bad times, the consumption of the more financially developed country falls more than that of the rest of the world.[42] This occurs because Home financial intermediaries are always able to achieve their desired investments in the risky asset by funding themselves in the deposit or interbank markets; this makes them less concerned than Foreign financial intermediaries about losses of capital. Consequently, the optimal risk sharing is for Home financial intermediaries to increase their investments in the risky asset by levering themselves in the international interbank market. Foreign financial intermediaries do exactly the opposite: they accumulate precautionary long positions in risk-free interbank deposits and decrease their investments in the risky asset. The portfolio implementation of the risk sharing condition therefore generates the asymmetric Net Foreign Asset portfolio of the Home and Foreign countries or, in actuality, of the US and the RoW (*Fact 1*).[43]

[41]In constrast to the autarky case, the restriction only imposes that the *direct* intra-temporal and inter-temporal effects of Ω^* on the risk-free rate offset each other. In both ODEs, there are also *indirect* effects of Ω^*, because consumption endogenously depends on Ω^*. See Appendix A for details.

[42]The assumption that the Home financial system is frictionless, while simplifying the analysis, inevitably produces a Home consumption path that is volatile. In a quantitative analysis, however, it is theoretically possible to extend the model to a case where Home also faces frictions, albeit lower than those faced by Foreign. In that case, the Home SDF would also feature an additional multiplicative term, like Ω, and this extra degree of freedom would allow the main results of the paper to be generated with a lower volatility of the Home consumption path.

[43]The equilibrium portfolio can be interpreted in the language of comparative advantage, as applied to trade in assets (Helpman and Razin (1978), Svensson (1988)). In autarky, Home's comparative advantage in financial markets results in higher Foreign than Home prices for "down state" Arrow securities. Once the two economies open for trade, Foreign will buy "down state" and sell "up state" Arrow securities from Home in order to achieve a safer portfolio overall.

The risk sharing condition has dynamic implications that emphasize the crucial role of the Home country during financial crises. Figures 9-10 show the equilibrium of the Open Banking Economy. Negative shocks cause capital losses in Foreign financial intermediaries and a fall in the stock market. As in the autarky case, a vicious cycle of fire sales sets in due to the systemic risk generated by the fact that all financial intermediaries hold the same risky asset. As Foreign financial intermediaries try to sell the risky asset, they further depress its price and, in turn, tighten their own credit constraints. Their increased concern for their net worth also heightens the Foreign financial intermediaries' desire to invest with Home financial intermediaries in the risk-free interbank market. In turn, Home financiers are willing to use the interbank funds that the Foreign financiers are providing to buy the stock that Foreign financiers are trying to sell. However, Home financiers require extra compensation for taking on this additional leveraged risk; this is achieved through a combination of an increase in the expected stock excess returns and a decrease in the interbank rate.

The global financial architecture is endogenously unstable. Since intermediaries are levered and invest in the same risky asset, negative shocks are amplified. The model endogenously generates a global flight to safety during crises, whereby Foreign financial intermediaries demand Home intermediaries' safe liabilities and Home's external portfolio loads more heavily on global risk. Part of the flight to safety occurs through a quantity adjustment of countries' portfolios, while the rest results from the price adjustment of assets. In the limit, as Foreign financial intermediaries lose all their net worth,[44] they only own Home financial intermediaries' safe liabilities.

The dynamic portfolio rebalancing of Home and Foreign is consistent with the empirical evidence in Curcuru, Dvorak, and Warnock (2010), who find that the RoW switches from equities to US safe assets precisely at times when the future performance

[44] The non-linear effects that occur in the autarky case when intermediaries' net worth is close to zero are no longer present. Their absence rests on the assumption that Home financiers are unconstrained and can therefore always help clear the market for the risky asset, provided that it offers an appropriate risk-return trade-off.

of these safe assets is poor compared to equities.[45]

In response to negative shocks, the static asymmetric Home and Foreign external portfolios and the dynamic effects combine to generate a wealth transfer from Home to Foreign (*Fact 3*). The wealth transfer supports the risk-sharing allocation by financing the relatively higher Foreign consumption in these states of the world. This is evident in Figure 9, where the value of the Home NFA portfolio falls in response to negative shocks and Home and Foreign consumption shares and trade balances move in opposite directions.[46]

On average, the Home country earns an expected compensation for the extra risk that it takes on in the global financial system. This stream of income finances higher Home consumption, and the Home country runs a trade deficit (*Fact 2*).[47]

The external adjustment of the US happens through both the traditional trade-balance channel and unrealized valuation effects on its NFA. Consistent with the empirical evidence of Gourinchas and Rey (2007), there are expected valuation effects on the NFA portfolio. These valuation effects are generated in my model by time-varying risk premia.

In the long run, Foreign financial intermediaries eventually accumulate sufficient capital to achieve their desired stock positions without raising deposits or borrowing or lending in the interbank market. The restriction $\delta = \lambda - \rho$ ensures that this upper state is absorbing. The stochastic steady state is one where Foreign financial intermediaries are no longer concerned about losses of net worth. The Home country runs an asymptotic trade deficit in the stochastic steady state. It does so not because it continues to earn higher risk compensation, but because the asymmetric risk sharing that occurred before reaching the steady state has allowed it to accumulate a positive NFA position.

[45] Curcuru et al. (2010) interpret the evidence in terms of bad timing of the purchase of US safe assets from RoW investors. In a consistent but alternative explanation, I interpret the empirical evidence in terms of risk compensation. RoW investors buy US safe assets during bad times because of their increased concern about fundamental and financial risk and their willingness to earn lower, or even negative, excess returns as compensation for the safety of the asset.

[46] In the figure, the trade balance is the difference between the country's consumption share and the red dotted line at 0.5. If a country's consumption share is more than 0.5, i.e. the country's share of the endowment, then it runs a trade deficit.

[47] In contrast to the Rueff (1971) interpretation of the US deficit as being "without tears", I emphasize that the US deficit is in fact financed by the "tears" of wealth transfers in bad states of the world.

In the data, the US NFA position is actually negative, but the US still runs a trade deficit. The model helps to rationalize this seemingly puzzling outcome: despite being a net debtor the US earns, on average, positive financial income since its assets, while lower, are riskier than its liabilities. This income finances the trade deficit. The model, however, cannot generate a long-run debtor position for the US because the stochastic steady state is one where risk taking is symmetric.[48] The stochastic steady state can be interpreted as a "very long run" outcome in which the RoW financial development and accumulation of capital make credit concerns irrelevant.

The model offers the view that some of the observed patterns in the data, including the global imbalances, are the outcome of equilibrium risk sharing. However, it stresses the substantial risks involved: the US benefits, on average, from positive financial income on its external portfolio only because it takes greater risks. The model also makes clear that the greater financial development of the US is not inconsistent with the 2008 crisis and its negative effects on the US banking system. In the model, it is precisely because US intermediaries are more efficient that they take more risk ex ante and, once a crisis hits, suffer the most severe losses.

While the motivational evidence for this paper is focused on the US, the same theoretical framework also sheds light on the role of the UK as the key country before the First World War. London's funding markets were then the deepest in the world; this was a key factor in determining Britain's financial dominance (Bagehot (1873)). My model is related to Kindleberger's (1965) hypothesis that the asymmetric external balance sheet of Britain, with respect to its colonies, was due to differences in "demand for liquidity" and did not necessarily represent a form of exploitation.[49]

My model also explains the global flight to safety toward the London funding markets, described by Bagehot (1873) for the financial crises of the nineteenth century. In contrast

[48] An extension of the paper could introduce mean reversion in the state variable, as was done in the closed economy, so that the US has a permanent advantage in financial intermediation. Logic suggests that this would allow the US, in extreme cases, to run both an asymptotic trade deficit and a negative NFA position.

[49] The similar claim of exploitation, or "exorbitant privilege", that was later directed at the US by the French Finance Minister Valéry Giscard d'Estaing, is often mentioned in connection with the stylized facts that concern my main analysis. I have shown how this can be demystified as the outcome of equilibrium risk sharing.

to the recent US history, however, Britain ran a sizable trade surplus at the time. In order to reconcile this with my framework recall that, though it is the focus of my model, I am not suggesting that financial development is the only determinant of the trade balance. Instead, my framework indicates that the key country runs either more of a trade deficit or less of a trade surplus than it would have otherwise done, if differences in the extent of financial development were not present. This allows other facts, such as Britain's industrial base, to also play a role in determining the overall trade balance.

The above shows how a simple asymmetry in the global financial system can explain the first three stylized facts (*Facts 1-3*) about the role of the US in the global financial architecture and provide meaningful foundations for its economic analysis. To analyze the missing stylized fact, the role of the US dollar as a reserve currency, I next extend the open economy to feature an exchange rate by introducing differentiated goods.

IV Open Banking Economy: Two Trees

I maintain the assumption from the previous section that the Home financial system is more developed than the Foreign one. In addition to applying this asymmetry to trade in assets, I also let financial frictions affect international trade in goods by introducing trade costs that are related to the state of the financial sector in each country. The model emphasizes that shifts in demand for Home and Foreign goods are important to understanding the dynamics of the exchange rate, particularly in times of global financial stress. As will become clear, both financial sector frictions and trade costs play an important role in determining these demand shifts.

There are two differentiated goods, one produced by Home and the other by Foreign. The output of the two goods is given by processes

$$\frac{dY(t)}{Y(t)} = \mu\ dt + \sigma\ d\vec{z}(t); \qquad \frac{dY^*(t)}{Y^*(t)} = \mu\ dt + \sigma^*\ d\vec{z}(t), \tag{22}$$

where $\sigma = [\sigma_z\ 0]$, $\sigma^* = [0\ \sigma_z^*]$, and $\vec{z}$ is a vector of two independent standard Brownian motions.

In both countries, agents have logarithmic preferences[50] over a basket of the two goods, with the Home and Foreign baskets given by, respectively:

$$C = C_H^{\alpha} C_F^{1-\alpha}; \qquad C^* = C_H^{*1-\alpha} C_F^{*\alpha}, \tag{23}$$

where $\alpha \in [\frac{1}{2}, 1)$ potentially allows for bias in each country's preferences toward its domestic good. I set a basket of the two goods, consisting of $\theta \in (0, 1)$ units of the Home good and $1 - \theta$ units of the Foreign good, as the numeraire. All prices are expressed in this common unit.

To model trade costs I assume, for simplicity, iceberg transport costs: if one unit of a good is shipped internationally, only $\frac{1}{\tau}$ units reach the destination, where $\tau \geq 1$. The most literal interpretation of the model, and the one that I follow, is that the relative variation in Home versus Foreign transport costs is due to the availability of credit. This can be directly modeled within my framework. A less literal interpretation is that Home and Foreign specialize in producing goods, the demand for which is affected differently during global crises.[51] Trade costs then need to be interpreted as reduced form demand shifts according to economic conditions.[52] As will become clear, both interpretations lead to a Home shift in demand toward its own good during bad times; therefore, both have similar equilibrium outcomes.

In keeping with the simplification that the Home country is unconstrained, I assume that there are no transport costs for Home exports. Foreign export transport costs are a function of the state of Foreign intermediaries. When intermediaries are well capitalized,

[50]In the models in Sections II and III, logarithmic preferences were mainly a matter of convenience. In the present section, logarithmic preferences permit one further simplification as agents have no desire to hedge their purchasing power risk (movements in the real exchange rate), thus allowing the model to be solved without introducing the ratio of the two trees as a state variable (see Coeurdacier and Gourinchas (2008), Pavlova and Rigobon (2007)). The downside of this simplification is that the equilibrium portfolios do not reflect this extra hedging demand, which would occur under general CRRA preferences. The central results of the paper, however, focus on how the portfolios are affected by the demand to hedge financial risk, which is not materially affected by the simplification to logarithmic preferences.

[51]The production specialization could actually be due to the development of the financial system, as in Antràs and Caballero (2009). This cannot be modeled directly here due to the assumption of exogenous endowments in the two countries.

[52]An alternative set-up is one where the coefficient of Home bias is not constant. A shift in Home demand in bad times toward its own good can be represented by an α that depends positively on Ω^*. The main implications of the model for the exchange rate and global portfolios carry over to this set-up. For a model of demand shocks that affect domestic bias see Pavlova and Rigobon (2010a).

Foreign exporters can easily access credit and trade costs are, therefore, low. By contrast, in periods of financial stress, Foreign exporters' access to credit dries up and trade costs increase correspondingly. This is modeled in reduced form by: $\tau = \Omega^{*\epsilon}$, where Ω^*, in line with the previous section, is the Foreign marginal value of net worth and $\epsilon \geq 0$.

A long-standing literature has highlighted the importance of trade costs for international finance (Samuelson (1954), Dumas (1992), Obstfeld and Rogoff (2001), Coeurdacier (2009)), while a fast-growing literature is analyzing the collapse in trade during the 2008 crisis. Chor and Manova (2011) find evidence that credit plays an important role in explaining the dynamics of exports during the 2008 crisis: countries that saw a more severe shutdown of their credit markets exported less to the US. Amiti and Weinstein (2011) and Paravisini, Rappoport, Schnabl, and Wolfenzon (2011) also find that credit conditions contribute to explaining the fall in exports for both Peru and Japan during the 2008 crisis. Another strand of the literature has emphasized the importance of both shifts in demand of tradables and global supply chains in explaining the 2008 collapse in trade (Eaton, Kortum, Neiman, and Romalis (2011), Levchenko, Lewis, and Tesar (2010)).

Since the focus of this paper is not on explaining the collapse in trade during a crisis, I want to clarify which elements of the empirical literature are relevant. I am interested in the *relative* variation in demand, according to the state of the economy, between the two countries for Home and Foreign goods. An overall symmetric increase in trade costs or a fall in world demand, while quantitatively interesting, are not the focus of this paper.

Standard static optimization of the consumption baskets gives the Home and Foreign demand for the two goods:[53]

$$C_H = \alpha \left(\frac{p}{P}\right)^{-1} C; \qquad C_F = (1-\alpha)\left(\frac{p^*\tau}{P}\right)^{-1} C \tag{24}$$

$$C_H^* = (1-\alpha)\left(\frac{p}{P^*}\right)^{-1} C^*; \qquad C_F^* = \alpha\left(\frac{p^*}{P^*}\right)^{-1} C^*, \tag{25}$$

where p and $p*$ are the prices of the Home and Foreign good, respectively, and P and P^* are the prices of one unit of the Home and Foreign consumption baskets, respectively.

[53]See Appendix A.

The terms of trade (ToT) are defined as the ratio of Foreign to Home goods prices, such that an increase in ToT represents a deterioration in the Home ToT. The real exchange rate ($\mathcal{E}$) is expressed as the Home price of Foreign currency and is given by the ratio of Foreign to Home price indices.[54] Thus, I have:

$$ToT \equiv \frac{p^*}{p}; \qquad \mathcal{E} \equiv \frac{P^*}{P} = (ToT)^{2\alpha-1} \ \tau^{\alpha-1}. \tag{26}$$

I denote the exchange rate dynamics by $\frac{d\mathcal{E}}{\mathcal{E}} = \mu_{\mathcal{E}} dt + \sigma_{\mathcal{E}} d\vec{z}$. Absent domestic bias ($\alpha = 0.5$), the exchange rate is only driven by movements in transport costs. In this case, an increase in transport costs generates a Home currency appreciation. In the presence of domestic bias ($\alpha > 0.5$) and barring changes in transport costs, the real exchange rate and the ToT are positively related.

Savers can only make deposits with domestic financial institutions. Deposits are instantaneous promises to pay one unit of the domestic consumption basket. Deposits are risk free for domestic agents because there is no default in equilibrium and deposits pay the consumption basket. The saver's problem is, therefore, identical to those in the previous sections and is reported in Appendix A.

Financiers in each country can raise domestic deposits, invest in any of the two stocks, and borrow or lend in an international interbank market. Interbank loans can be denominated in either Home or Foreign currency and are instantaneous promises to pay one unit of either the Home or Foreign consumption basket, respectively. The Foreign financier's balance sheet is $s^*_H \frac{Q}{\mathcal{E}} + s^*_F Q^* = n^* + d^* + b^*_H + b^*_F$, where s^*_H and s^*_F are the Foreign equity holdings of Home and Foreign stocks, Q and Q^* are the prices of the Home and Foreign stocks, both expressed in local currencies, and b^*_H and b^*_F are the amounts borrowed in the interbank market in Home and Foreign currency, both expressed in Foreign currency. As in the previous section, financiers who are selected to switch roles are allowed to reinvest their net worth with incoming financiers.

[54] An increase in the exchange rate equates to a Home currency depreciation.

The Foreign financier's optimization problem is:

$$\max_{\{d^*(u),b_H^*(u),b_F^*(u),s_H^*(u),s_F^*(u)\}_{u=t}^{\infty}} \Lambda^*(t)V^*(t) = E_t\left[\int_{t'}^{\infty} \Lambda^*(u)e^{-\lambda(u-t')}\lambda n^*(u)du\right] \quad (27)$$

$$\begin{aligned} s.t. \quad dn^* = &\, s_H^*\left(d\left(\frac{Q}{\mathcal{E}}\right) + \frac{pY}{P^*}dt\right) + s_F^*\left(dQ^* + \frac{p^*Y^*}{P^*}dt\right) + \\ & -r_d^*\, d^*\, dt - (r_b\, dt - d\mathcal{E})\, b_H^* - \; r_b^*\, b_F^*\, dt \\ V^* \geq 0, & \end{aligned}$$

where $\Lambda^* \equiv e^{-\rho t}\frac{1}{C^*}$ and $t' \equiv inf\{t : \Omega^*(\tilde{N}^*(t)) = 1\}$. The Home financier's problem is entirely similar, but without the last constraint.

As a generalization of the previous section, I assume that the start-up capital provided by households to new financiers is a function of the stochastic steady state holdings of the two stocks: $\{\bar{S}_H^*, \bar{S}_F^*\}$. Consequently, new Home financiers receive $\delta[\bar{S}_H Q + \bar{S}_F \mathcal{E} Q^*]$ and new Foreign financiers receive $\delta[\bar{S}_H^* \frac{Q}{\mathcal{E}} + \bar{S}_F^* Q^*]$.

The definitions of the Foreign trade balance and the NFA are generalizations of those in equation (12):

$$NX^* \equiv \frac{p^*}{P^*}Y^* - C^*; \qquad NFA^* \equiv S_H^*\frac{Q}{\mathcal{E}} - S_F Q^* - B_H^* - B_F^*.$$

The market clearing conditions are:

$$\begin{aligned} C_H + C_H^* &= Y; & \tau C_F + C_F^* &= Y^* \\ S_H + S_H^* &= 1; & S_F + S_F^* &= 1 \\ B_H &= -\mathcal{E}B_H^*; & B_F &= -\mathcal{E}B_F^* \\ N^* &= S_H^*\frac{Q}{\mathcal{E}} + S_F^* & Q^* - D^* &- B_H^* - B_F^*. \end{aligned}$$

A Optimal Consumption and Investment

In line with Section III.*A*, the Home financier's value function takes the form $V = n$ and that of the Foreign financier takes the form $V^* = \Omega^*(\tilde{N}^*)n^*$. The Home and Foreign dynamic programming problems of both savers and financiers and the corresponding Euler equations, which are extensions of those in Lemma 3, are reported in Appendix A. Here I want to emphasize the Home financier's Euler equation for the optimal trade-off between interbank loans denominated in Home and Foreign currency:

$$r_b^* - r_b + \mu_{\mathcal{E}} = -Cov_t\left(\frac{d\Lambda}{\Lambda}, \frac{d\mathcal{E}}{\mathcal{E}}\right) = \sigma_C\ \sigma_{\mathcal{E}}^T, \tag{28}$$

where the superscript ^{T} denotes the vector transpose. The Home currency safety premium, the compensation required to invest in Foreign currency by shorting Home currency, is determined by the covariance between Home consumption and the real exchange rate. If the Home currency appreciates ($\downarrow \mathcal{E}$) whenever Home consumption is low, then the Home currency has a positive safety premium. Intuitively, Home interbank loans are safer than their Foreign counterparts because they pay more in bad states of the world.

Since deposits and interbank loans are risk-free in their local currency, no arbitrage implies that $r_b = r_d$ and $r_b^* = r_d^*$.

B Equilibrium

B.1 Cole and Obstfeld Economy

In their classic analysis of the irrelevance of asset markets for international risk sharing, Cole and Obstfeld (1991) show that in an open economy with differentiated goods, agents with logarithmic preferences, and no trade costs, the central-planner's allocation can be achieved even without trade in asset markets.[55] I refer in brief to this economy as the Cole and Obstfeld Economy.

If there are no frictions, then the equilibrium of my model reduces to that of the Cole and Obstfeld Economy. Intuitively, if Foreign financiers face no frictions then: $\Omega^*(t) = 1$, so that the Euler equations and the demand equations for goods simplify to those in the frictionless world of Cole and Obstfeld.

[55] This occurs because the endogenous response of the ToT to supply shocks to the two goods is sufficient to implement the international wealth transfers that support the central planner's consumption allocation.

As is well known, the Cole and Obstfeld equilibrium features: perfectly correlated Home and Foreign stock markets, symmetric aggregate stock market portfolio holdings,[56] zero holdings of risk-free bonds,[57] equal consumption state by state, zero NX, and indeterminate[58] NFA and CA. The exchange rate is either constant ($\alpha = 0.5$) or positively related to the ToT ($\alpha > 0.5$). These results are a far cry from the stylized facts in *Facts 1-4*.

B.2 Open Banking Economy: Two Trees

In line with Section III.*B*.2, the equilibrium is characterized by a single state variable, the aggregate scaled[59] net worth of Foreign financiers, and a system of three ODEs.

Proposition 3. *The equilibrium is characterized by a system of three coupled second-order ODEs for the Home price-dividend ratio,* $\tilde{Q}(\tilde{N}^*)$*, the Foreign price-dividend ratio,* $\tilde{Q}^*(\tilde{N}^*)$*, and the marginal value of Foreign net worth,* $\Omega^*(\tilde{N}^*)$*:*

$$0 = \mu_Q - r_b - \sigma_C \sigma_Q^T \tag{29}$$

$$0 = \mu_{Q^*} + \mu_{\mathcal{E}} + \sigma_{\mathcal{E}} \sigma_{Q^*}^T - r_b - \sigma_C (\sigma_{Q^*} + \sigma_{\mathcal{E}})^T \tag{30}$$

$$0 = \mu_{\Omega^*} - \sigma_{C^*} \sigma_{\Omega^*}^T. \tag{31}$$

The risk sharing allocations are a generalization of equation (21):

$$\frac{P^* C^*}{PC} = \frac{\Omega^*}{\xi} \tag{32}$$

[56]Individual stock market positions are indeterminate since the two stocks are perfectly correlated, but each country's holding of the aggregate stock market is determinate.
[57]In my setting there are zero holdings in the interbank market, which is the equivalent of the risk-free international bonds in Cole and Obstfeld (1991), but the deposit market is still active. However, note that without frictions the trading in the deposit market is merely a matter of internal accounting between savers and financiers in each country, without any real effects. In this sense the Cole and Obstfeld (1991) result on the irrelevance of international asset markets for risk sharing holds in my set-up when there are no frictions.
[58]The NFA indeterminacy is a consequence of the indeterminacy of the portfolio holdings of each stock. The CA is indeterminate because it is the change in NFA.
[59]Consistently with the previous sections, I normalized by the value of world output expressed in the appropriate currency. Consequently, Home variables are scaled by $\frac{pY+p^*Y^*}{P}$ and Foreign variables by $\frac{pY+p^*Y^*}{P^*}$.

$$C_H^* = \frac{(1-\alpha)\Omega^*}{\alpha\xi + (1-\alpha)\Omega^*} Y; \qquad C_H = \frac{\alpha\xi}{\alpha\xi + (1-\alpha)\Omega^*} Y \tag{33}$$

$$C_F^* = \frac{\alpha\Omega^*}{(1-\alpha)\xi + \alpha\Omega^*} Y^*; \qquad C_F = \frac{1}{\tau} \frac{(1-\alpha)\xi}{(1-\alpha)\xi + \alpha\Omega^*} Y^*. \tag{34}$$

Equation (32) is the risk sharing condition that underpins the global financial architecture. It states that the value of Foreign consumption increases relative to that of Home whenever the Foreign marginal value of net worth increases. Since in equilibrium Ω^* is counter-cyclical, this occurs in bad economic times.

The terms of trade and the exchange rate can also be understood in terms of movements in Ω^*. The risk sharing conditions above and the definitions in equation (26) imply that:

$$ToT = \frac{\xi(1-\alpha) + \alpha\Omega^*}{\alpha\xi + (1-\alpha)\Omega^*} \frac{Y}{Y^*}; \qquad \mathcal{E} = (ToT)^{2\alpha-1}\ \Omega^{*\epsilon(\alpha-1)}.$$

The ToT are determined by two effects. Movements in the ratio of the two trees affect the ToT by altering the relative supply of the two goods. If the Home good becomes relatively more scarce, then it also becomes relatively more expensive, and the Home ToT improve. This effect is present irrespective of domestic bias. In addition, if there is domestic bias ($\alpha > 0.5$), an increase in Ω^* weakens the Home ToT. This happens because an increase in Ω^*, according to equation (32), increases the relative consumption of Foreign residents. If the preferences of agents are biased toward the Foreign good ($\alpha > 0.5$), this induces a relative increase in the demand for the Foreign good. To clear the market, its price increases relative to the Home good. If these agents have no preference bias ($\alpha = 0.5$), then the ToT are unaffected.

The exchange rate is determined by the combination of three effects. The first two effects derive from the movement in the ToT analyzed above. If $\alpha = 0.5$, these two effects disappear because the Home and Foreign consumption baskets are identical and movements in the ToT have no effect on the exchange rate. The third effect is caused by variations in trade costs. An increase in Ω^* induces an increase in Foreign export costs. The higher costs increase the prices Home residents pay for the Foreign good (see equation (34)), thus relatively increasing the Home price index and causing the Home currency to appreciate.[60] The effect is absent in the limit $\alpha \uparrow 1$, because countries only consume their own good and never export their good.

The effects on the ToT and exchange rate can be understood in terms of the classic Keynes and Ohlin debate on the "transfer problem".[61] In my setting, an increase in Ω^* is associated with a wealth transfer from Home to Foreign. The reaction of the exchange rate depends on what Foreign (Home) residents do with the additional (reduced) wealth.[62]

Traditional international macroeconomic models predict that a transfer of wealth from the US to the RoW results in a US dollar depreciation. This prediction derives from the second channel described above: the increased relative demand for RoW goods due to the relatively higher wealth of RoW residents. If the US, in fulfilling its role as the key country in the global financial architecture, takes more risk in equilibrium than other countries, the wealth transfer occurs during bad economic times (*Fact 3*). Traditional models would then predict a US dollar depreciation in such times.[63] If this were the case, however, the US dollar would be a risky asset for RoW residents, since it would pay low in bad states of the world. The role of the US dollar as a reserve currency (*Fact 4*), therefore, is inconsistent with this traditional mechanism. This is the "reserve currency paradox": the key country's currency appreciates during a crisis despite the country suffering heavier wealth losses relative to other countries.

The model rationalizes this paradox by noting that in normal times, or even for mild negative shocks, the combination of the various effects produces an ambiguous exchange-rate response. However, for sufficiently large adverse shocks, such as global crises, the relative shift in demand toward the Home good, caused by an increase in τ, dominates and

[60] The trade cost does not affect the ToT, because in a Cobb-Douglas aggregator the unit elasticity of demand of the two goods implies that the wealth and substitution effects originating from trade costs exactly offset each other. In a more general CES aggregator, one could argue that an increase in Foreign trade costs reduces Home demand for Foreign goods and tends to improve the Home terms of trade.

[61] Following World War I, the Dawes committee imposed reparation payments from Germany to France. Keynes argued that, in addition to the primary burden of the wealth transfer, Germany would suffer a secondary burden due to the deterioration in its terms of trade (Keynes (1929)). Ohlin, on the contrary, argued that no secondary burden would occur as long as French people spent the transfer on German goods (Ohlin (1929)).

[62] The effect of trade costs on the "transfer problem" was analyzed by Samuelson (1952, 1954).

[63] This is reminiscent of "Triffin's dilemma". Triffin (1960) postulated that in running large trade deficits due to its effort to provide the world reserve currency, the US would suffer heavier losses, and potentially a run on its currency, during global crises.

the Home currency appreciates. This non-linearity allows for rich exchange rate dynamics. While it is consistent with the traditional view, with the exchange rate behaving much as predicted by traditional models in normal times, it extends this mechanism in order to make sense of the behavior of the exchange rate during extreme events.

Figures 11-12 isolate the role of the Home currency as a global safe asset by presenting the equilibrium of the model under no domestic bias ($\alpha = 0.5$). In this case, the exchange rate is entirely driven by movements in trade costs. Since the Home currency appreciates whenever intermediaries lose capital, it provides a hedge for the global financial system. Correspondingly, Figure 12 shows that the Home currency has a safety premium (*Fact 4*): global financial intermediaries are willing to earn negative expected excess returns as compensation for holding this safe currency.

Figure 11 shows how the equilibrium risk sharing allocation between Home and Foreign is implemented via the financial intermediaries' portfolios. First, note that for the case where $\alpha = 0.5$ the returns of the two stocks in the same currency are perfectly correlated, as in Cole and Obstfeld (1991), because changes in the ToT exactly offset the dividend shocks.[64] Therefore, I focus on intermediaries' holdings of the aggregate world stock market. The insight from Cole and Obstfeld (1991) on the relevance of asset markets for risk sharing has one further implication for my model (for the case $\alpha = 0.5$): despite the presence of two sources of fundamental risk, the two dimensional Brownian motion $\vec{z}$, two independent assets are sufficient to achieve the equilibrium risk sharing.[65] Figure 11 presents the case where intermediaries can trade the world stock market and lend or borrow in the Home-currency interbank market.

Foreign intermediaries invest in the risky asset, the stock market, and hold precautionary long positions in Home currency in the interbank market. Following a negative shock, Foreign intermediaries lose capital and their heightened concern for further

[64]See Appendix A for details. The drawback is that in this case, as in Cole and Obstfeld (1991), the NFA and CA are indeterminate. See Pavlova and Rigobon (2007, 2010a,b) for a discussion of how the result of Cole and Obstfeld (1991) affects a set-up with domestic bias and demand shocks.

[65]International markets' completeness requires three independent traded assets. Recall from Section IV.B.1 that in the Cole and Obstfeld Economy international asset markets are completely unnecessary to implement the risk sharing. Here the difference is that international asset markets are necessary to implement the asymmetric risk sharing, but an international asset market structure that falls short of the complete one is sufficient for the implementation.

losses leads to a fall in their investments in the risky asset and an increased demand for the Home currency. This global flight toward the Home currency leads to an increase in its safety premium through both an expected Home currency depreciation and a more pronounced fall in the Home interest rate than in the Foreign one. Even in the limit, as Foreign intermediaries lose all capital, their long positions in Home currency allow them to hedge the risk deriving from their long stock positions and, in contrast with Section III.*B*.2, Foreign intermediaries maintain a long position in the stock.[66]

I offer a view of the international role of the US dollar as a reserve currency based on risk. This contrasts with previous models of the key currency that had focused on its role as a vehicle currency, that is, a medium of exchange, in international transactions (Krugman (1980), Matsuyama, Kiyotaki, and Matsui (1993)).

The model provides a rationale for the "Global Saving Glut", the hypothesis formulated by Bernanke (2005) that the RoW demand for US safe assets lowered US interest rates and contributed to large US trade deficits. In the spirit of Caballero and Krishnamurthy (2009), the model stresses how Home safe liabilities are demanded by global financial intermediaries as a precautionary investment. More technically, the combination of the Home safe asset and the risky assets, the two stocks, allows Foreign intermediaries to replicate the Foreign risk-free asset, which might not be directly available to trade. Following negative shocks, the increased Foreign demand for the Home safe asset accentuates the fall in the Home risk-free rate. In contrast to the previous papers, however, I emphasize the importance of the exchange rate. Even the safest US assets, such as Treasuries and short term liabilities of the banking system, would not be particularly safe for RoW investors if the US dollar were to systematically depreciate during crises.[67]

A dimension along which the model could be extended is the heterogeneity of

[66]The equilibrium for the case that allows financial intermediaries to trade the stocks and the Foreign-currency denominated interbank deposits features identical allocations and asset prices. The portfolios, however, are different: Foreign intermediaries are long the stocks and lend in the interbank market in the Foreign currency. As the Foreign intermediaries lose capital they decrease their positions in the stock and increase their lending in the Foreign currency. In the limit, as Foreign intermediaries have no net worth, they own no stocks and only lend in the Foreign currency. These portfolios are not consistent with those observed in the data (*Fact 1*).

[67]Notable exceptions are countries with currencies that are (quasi-)pegged to the US dollar, such as China. The accumulation of precautionary reserves in US dollars, however, extends well beyond countries with pegged currencies.

financial intermediaries. Bernanke, Bertaut, DeMarco, and Kamin (2011) and Shin (2011) provide empirical evidence that there is heterogeneity in the portfolios of RoW financial intermediaries. For example, the evidence suggests that intermediaries based in Asia and other emerging markets have long positions in US safe assets, while some of the larger European banks are both funding themselves in the US and investing these funds in a wider spectrum of US assets, including bond and equities. In short, these European intermediaries are providing off-shore financial intermediation to the US, which Bernanke et al. (2011) and Shin (2011) suggest is related to lower capital requirements for European banks.[68]

Since the focus of my paper is on explaining the aggregate US international position, I have grouped the RoW intermediaries into one homogenous class. This simplification allows the model to sharpen its focus on aggregate flows, and leaves it to future research to also model the heterogeneity of the RoW intermediaries.

V Conclusion

A simple asymmetry in the global financial system, heterogeneity in financial development, can rationalize the economic role of the US in the global financial architecture. I have shown how the greater depth of financial development of the US leads to its role as the global risk taker with respect to both fundamental and financial risk. The four stylized facts that motivated my analysis emerge as consequences of the asymmetric risk taking that characterizes the global financial architecture.

The model not only provides a theoretical framework that jointly makes sense of these facts; its main contribution is to have done so by providing the underlying economic foundations through the explicit modeling of financial intermediation and its frictions. These foundations have highlighted the risks that affect the global financial system, as well as the costs and rewards for each country. The tractability of the foundations and the almost analytical solution method provide a base for future research.

[68]This is a form of regulatory or tax arbitrage. Similarly, one could argue that Switzerland and Singapore, countries with high financial development, also provide off-shore intermediation. The role of these financial centers in the global financial architecture seems to be mainly related to these regulatory and tax advantages.

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Appendix A: Proofs

Lemma 1. Given the conjecture that the saver's value function only depends on scaled deposits and scaled net transfers, $U(\tilde{D},\tilde{\Pi})$, the optimization problem is solved by the following Hamilton-Jacobi-Bellman (HJB) equation:

$$\begin{aligned}
0 &= \sup_{\tilde{C}} \left\{ log(\tilde{C})dt - \rho U(\tilde{D},\tilde{\Pi})dt + E_t\left[dU(\tilde{D},\tilde{\Pi})\right]\right\} \\
s.t. \quad d\tilde{D} &= \tilde{D}(r_d - \mu + \sigma^2)dt + (\tilde{\Pi} - \tilde{C})dt - \tilde{D}\sigma dz \\
d\tilde{\Pi} &= \{\tilde{N}\lambda(r_d - \lambda) + \tilde{Q}[\lambda(\mu_Q + \delta - r_d) - \delta\mu_Q - \sigma\sigma_Q(\lambda - \delta)] + \\
&\qquad + \tilde{\Pi}(\sigma^2 - \mu) + \delta\}dt + [\tilde{Q}\sigma_Q(\lambda - \delta) - \tilde{\Pi}\sigma]dz.
\end{aligned}$$

The first order condition (FOC) is: $\tilde{C}^{-1} = U'_{\tilde{D}}$, where the left hand side (LHS) is the first derivative of U with respect to the scaled deposits. The verification that the value function only depends on $\{\tilde{D},\tilde{\Pi}\}$ follows by substituting the FOC back into the HJB equation, and from the fact that:

$$\begin{aligned}
\tilde{N} &= \frac{\tilde{\Pi} + \delta\tilde{D}}{\lambda - \delta} \\
\tilde{Q} &= \frac{\tilde{\Pi} + \lambda\tilde{D}}{\lambda - \delta},
\end{aligned}$$

and from the fact that $\{\tilde{Q}, \mu_Q, \sigma_Q, r_d\}$ are going to only be functions of $\tilde{N}$ and can therefore be recovered by knowing $\{\tilde{D},\tilde{\Pi}\}$. The sufficiency of the HJB equation for the solution of the optimization problem follows standard steps from the Verification Theorem.[69] An explicit verification is omitted here and in the following proofs.

To establish the claim that $-r_d\, dt = E_t\left[\frac{d\Lambda}{\Lambda}\right]$, I employ the approach in Cox, Ingersoll, and Ross (1985). I take the difference between two expressions. The first expression is obtained by using the FOC above to write $\Lambda = e^{-\rho t}\frac{U'_{\tilde{D}}}{Y}$, and by applying Ito's lemma to this function. The second expression is obtained by taking the partial derivative of the HJB equation above with respect to $\tilde{D}$ and by then multiplying it by $\frac{e^{-\rho t}}{Y}$. Taking the difference between the two expression establishes, after tedious but standard algebra, the claim.

Lemma 2. Given the conjecture that the financier's value function depends on aggregate scaled net worth and the individual financier's net worth, $V(\tilde{N}, n)$, the optimization problem is solved by the following HJB equation:

$$\begin{aligned}
0 &= \sup_{s} \left\{ \lambda\Lambda_\lambda n dt + E_t\left[d(\Lambda_\lambda V(\tilde{N}, n))\right] + \chi(t)dt\; V(\tilde{N}, n)\right\} \\
s.t. \quad dn &= s(dQ + Ydt) - r_d\; d\; dt \\
d\tilde{N} &= \left[\tilde{N}(r_d - \lambda - \mu + \sigma^2) + \tilde{Q}(\mu_q - r_d + \delta - \sigma\sigma_Q)\right]dt + (\tilde{Q}\sigma_Q - \tilde{N}\sigma)dz,
\end{aligned}$$

[69] See Øksendal (2003, page 241).

where χ is the Lagrange multiplier. The FOC is:

$$\mu_Q - r_d = \sigma_C \sigma_Q - \sigma_\Omega \sigma_Q. \tag{35}$$

When substituting the FOC back into the HJB equation, I obtain a restriction that the function Ω has to satisfy for the conjecture of the value function to be valid:

$$0 = \lambda \frac{(1-\Omega)}{\Omega} + \mu_\Omega - \sigma_C \sigma_\Omega. \tag{36}$$

As long as $\{\tilde{Q}, \mu_Q, \sigma_Q, r_d\}$ only depend on $\tilde{N}$ in equilibrium, then the conjecture that Ω only depends on $\tilde{N}$ is verified.

Using the saver's Euler equation in equation (4) and equation (36), algebraic manipulations yield the result in equation (6). The additional use of the financier's FOC yields the result in equation (5).

Proposition 1. The proofs of Lemma 1 and 2 state that to solve the saver's and financier's optimization problems one only needs to know the variable $\tilde{N}$, as long as $\{\tilde{Q}, \mu_Q, \sigma_Q, rd\}$ only themselves depend on that variable. The saver's Euler equation, equation (4), and the market clearing condition $C = Y$ together imply that the deposit rate is constant in equilibrium and is given by $r_d = \rho + \mu - \sigma^2$. Applying Ito's lemma to $\tilde{Q} = \frac{Q}{Y}$ and to the conjecture $\tilde{Q}(\tilde{N})$ and matching the corresponding drift and diffusion terms yields:

$$\begin{aligned}
\mu_Q(t) &= \frac{1 + \tilde{Q}[\mu + \tilde{Q}'(\delta - \mu - \rho)] + \tilde{N}\tilde{Q}'(\rho - \lambda)}{\tilde{Q}(1-\tilde{Q}')} + \frac{(\tilde{Q}-\tilde{N})\tilde{Q}'\sigma^2}{\tilde{Q}(1-\tilde{Q}')} + \frac{(\tilde{Q}-\tilde{N})^2\tilde{Q}''\sigma^2}{2\tilde{Q}(1-\tilde{Q}')^3} \\
\sigma_Q(t) &= \frac{\tilde{Q} - \tilde{N}\tilde{Q}'}{\tilde{Q}(1-\tilde{Q}')}\sigma.
\end{aligned}$$

Substituting these expressions into the financier's FOC (equation (35)) yields the ODE for $\tilde{Q}(\tilde{N})$, reported in implicit form in equation (9), thus verifying that $\tilde{N}$ is the only state variable. The proof that the state variable is a strong Markov process follows from its dynamics in equation (7), where the drift and diffusion terms only depend on $\tilde{N}$ itself.

Equation (36) is the ODE for Ω reported in equation (9). The ODEs in equations (8-9) are implicit and I report here their explicit expressions:

$$\tilde{Q}'' = \frac{2(-1+\tilde{Q}')\left\{-(-1+\tilde{Q}')[1+\tilde{Q}(\tilde{Q}'\delta - \rho) + \tilde{N}\tilde{Q}'(-\lambda+\rho)]\Omega + (-\tilde{N}+\tilde{Q}')(\tilde{Q}-\tilde{N}\tilde{Q}')\sigma^2\Omega'\right\}}{(\tilde{N}-\tilde{Q})^2\sigma^2\Omega} \tag{37}$$

$$\begin{aligned}
\Omega'' = {} & \frac{2(-1+\tilde{Q}')^2\lambda\Omega(-1+\Omega+\tilde{N}\Omega') + 2\Omega'\left\{-(-1+\tilde{Q}')\left[(-1+\tilde{Q}')(\tilde{Q}\delta+\tilde{N}\rho) + (-\tilde{N}+\tilde{Q})\sigma^2\right]\Omega\right\}}{(\tilde{N}-\tilde{Q})^2\sigma^2\Omega} + \\
& + \frac{2\Omega'(-\tilde{N}+\tilde{Q})(\tilde{Q}-\tilde{N}\tilde{Q}')\sigma^2\Omega'}{(\tilde{N}-\tilde{Q})^2\sigma^2\Omega},
\end{aligned} \tag{38}$$

where the superscript $''$ denotes the second derivative of a function.

Lucas Economy: Equilibrium Details

Assume that there are no frictions, so that the constraint $V(t) \geq 0$ is no longer present in the financier's optimization problem. Since financiers are unconstrained in raising deposits, $\Omega(\tilde{N}) = 1$ and $\tilde{Q}(\tilde{N}) = \frac{1}{\rho}$. These constant functions satisfy the ODEs in (37-38). The risk premium is constant and is given by $\mu_Q - r_d = \sigma^2$. Note that financiers can make arbitrary large losses on their investment strategy because they are raising risk-free deposits with a positive interest rate, and investing in a risky asset with a positive (and finite) risk premium. As a technical condition, to ensure that the financier's optimization problem is well defined, I rule out the "doubling portfolio strategy" by restricting the set of admissible investment strategies to those that are square integrable.[70]

To confirm that the underlying micro-foundations of the model are economically sensible, I analyze the dynamics of $\hat{N} \equiv \frac{N}{Q}$:

$$d\hat{N} = (\lambda - \rho)(\frac{\delta}{\lambda - \rho} - \hat{N})dt + \sigma(1 - \hat{N})dz.$$

Under the restriction $\delta < \lambda - \rho$, the above stochastic process is mean-reverting and lies in the interval $(-\infty, 1)$.[71] The stochastic steady state is $\hat{N}^{SS} = \frac{\delta}{\lambda - \rho}$. Note that deposits are always positive.

Under the restriction $\delta = \lambda - \rho$ the process, started at $\hat{N}(t = 0) < 1$, will eventually drift to the absorbing upper boundary[72] of 1. Consequently, the stochastic steady state is $\hat{N}^{SS} = 1$. In this scenario, financiers eventually accumulate enough capital to purchase all shares in the output tree without having to raise deposits.

Asset Pricing in the Banking Economy: Equilibrium Details

The financier's FOC yields:

$$\mu_Q - r_d = -Cov_t\left[\frac{d\Lambda}{\Lambda}, \frac{dQ}{Q}\right] - Cov_t\left[\frac{d\Omega}{\Omega}, \frac{dQ}{Q}\right] = Cov_t\left[\frac{dC}{C}, \frac{dQ}{Q}\right] - \frac{\Omega'}{\Omega}\tilde{N}\, Cov_t\left[\frac{d\tilde{N}}{\tilde{N}}, \frac{dQ}{Q}\right].$$

This implies that assets are priced according to a two factor asset pricing model, where the risk factors are consumption and the financial system's net worth:

$$\mu_Q(t) - r_d = \lambda_C \beta_C(t) + \lambda_{\tilde{N}}(t)\beta_{\tilde{N}}(t),$$

[70] See Duffie (2001, 6.c) for details.

[71] A precise proof of the boundary behavior is beyond the scope of this paper. I only note that, given that financiers' starting net worth is less than the price of the risky asset, in the limit of financiers accumulating sufficient net worth for deposits to shrink to zero (i.e. $\hat{N} \uparrow 1$) the diffusion term of $\hat{N}$ approaches zero and the drift term is negative. See Karlin and Taylor (1981) for a precise proof of the boundary behavior.

[72] This occurs because the drift of the process approaching the upper boundary is positive and decreases to zero in the limit, while the diffusion term converges to zero. See Karlin and Taylor (1981) for a rigorous description.

and where the prices of risk and betas are defined as:

$$\beta_C(t) \equiv \frac{Cov_t\left[\frac{dC}{C}, \frac{dQ}{Q}\right]}{Var_t\left[\frac{dC}{C}\right]}; \qquad \beta_{\tilde{N}}(t) \equiv \frac{Cov_t\left[\frac{d\tilde{N}}{\tilde{N}}, \frac{dQ}{Q}\right]}{Var_t\left[\frac{d\tilde{N}}{\tilde{N}}\right]}$$

$$\lambda_C \equiv Var_t\left[\frac{dC}{C}\right]; \qquad \lambda_{\tilde{N}}(t) \equiv -Var_t\left[\frac{d\tilde{N}}{\tilde{N}}\right]\frac{\Omega'}{\Omega}\tilde{N}.$$

The first term on the RHS of equation (39) is the CCAPM, where assets are risky if their returns covary positively with consumption. Compared to the CCAPM in the Lucas Economy, the volatility of asset prices varies endogenously and, consequently, the beta in the Banking Economy is time-varying. The second term implies that assets are riskier if they covary positively with the financial system's net worth. Both the market price and the beta of the financial risk factor are time-varying.

The risk-free deposit rate is constant and equal to the one in the Lucas Economy:

$$r_d = \rho + \mu - \sigma^2 - \underbrace{\lambda(1-\Omega) - \Omega'\mu_{\tilde{N}} - \frac{1}{2}\Omega''\sigma^2_{\tilde{N}} + \Omega'\sigma\sigma_{\tilde{N}}}_{=0 \text{ by ODE (9)}}.$$

The ODE (9) imposes that the increase in the risk-free rate that occurs because of the inter-temporal drift in the value of capital ($\mu_{\tilde{N}}$) and the role switching of financiers and savers ($\lambda(1-\Omega)$) is exactly offset by the precautionary motive to save that is induced by intra-temporal financial risk ($\sigma^2_{\tilde{N}}$) and the covariance between consumption and financial risk ($\sigma\sigma_{\tilde{N}}$). The result rests on two features of my set-up: firstly that savers are atomistic, and secondly that equilibrium consumption is exogenous. In the autarky model, there is no tension between a higher equity premium and a low and stable risk-free rate, thus accommodating the risk-free rate puzzle.

Even if dividends are a random walk, the model endogenously generates persistent effects of *iid* shocks and forecastable equity excess returns. This occurs because excess returns are a function of aggregate net worth, which in turn is persistent and pro-cyclical. For example, a negative shock results in a capital loss for financiers and increases the risk premium;[73] the only way to rebuild net worth is to earn the expected risk premium over time. Therefore, on impact, expected returns increase and then gradually decrease as financiers rebuild the stock of net worth.

Lemma 3. Since the Home country is unconstrained, the proofs for the autarky case make clear that its consumption and portfolio problems are identical to those of a representative agent with logarithmic utility. The Euler equations in (13-15) are standard for such an agent. Λ is the Home SDF. I focus here only on the optimization problems of Foreign agents.

Foreign savers solve a problem analogous to Lemma 1, so an entirely similar proof

[73] I refer here to the region of the equilibrium away from zero net worth. The sign of predictability, i.e. that a low price-dividend ratio predicts high excess stock returns, is also maintained in the region close to zero net worth. However, the relationship between net worth and the price-dividend ratio is inverted.

applies. Consider the problem of the representative financier in equation (11) for $t < t'$. Since the financier pays no net worth to the household for any $t < t'$, the discounted value of her intermediary needs to be a local martingale along the optimal path. The HJB equation is:

$$0 = \sup_{\{b^*(u),s^*(u)\}} E_t[d(\Lambda^* V^*)] + \chi(t)dt\ V^*,$$

where χ is the Lagrange multiplier. Conjecture that the value of the intermediary only depends on its capital and aggregate Foreign scaled net worth: $V(\tilde{N}^*, n^*) = \Omega^*(\tilde{N}^*)n^*$. The FOCs are:

$$\mu_Q - r_d^* = \sigma_{C^*}\sigma_Q - \sigma_{\Omega^*}\sigma_Q \tag{39}$$

$$r_b = r_d^*. \tag{40}$$

Substituting the FOCs in the HJB equation leads to a restriction that Ω^* has to satisfy:

$$0 = \mu_{\Omega^*} - \sigma_{C^*}\sigma_{\Omega^*}. \tag{41}$$

Now consider the problem of the financier for $t > t'$. I conjecture that in this case $\Omega^* = 1$ and the financier will pay out net worth when selected to switch roles. The HJB equation is:

$$0 = \sup_{\{b^*(u),s^*(u)\}} \left\{\lambda \Lambda_\lambda^* n^* dt + E_t[d(\Lambda_\lambda^* V^*)] + \chi(t)dt\ V^*\right\},$$

where $\Lambda^{*\lambda} = e^{-(\rho+\lambda)t}\frac{1}{C^*}$. The FOCs are analogous to those above for the case $t < t'$, except that $\sigma_{\Omega^*} = 0$. Plugging the FOCs back into the HJB equation verifies the guess that $\Omega^* = 1$. However, for this conjecture to be an equilibrium, the upper boundary of the state space needs to be absorbing. This restriction is verified in Proposition 2.

It remains to be verified that for $t < t'$ an individual financier will not want to deviate from the HJB problem described above for the representative financier. An individual financier faces the possibility that at some time t^A, where $t < t^A < t'$, she will switch jobs and the net worth of her intermediary will be reinvested with an incoming financier. Consider intermediary A with capital $n^{*A}(t)$ that is liquidated at time t^A, the capital of which is inherited by intermediary B. At time t^A, the value of intermediary B is a linear function of its net worth. The linearity allows me to only concentrate on the capital inherited by intermediary A and, without loss of generality, to ignore the start up capital injected in intermediary B by the household. It follows that $V^{*B}(t^A) = \Omega^*(\tilde{N}^*(t^A))n^{*A}(t^A)$. Using the definition of the value of the intermediary and the law of iterated expectations one has:

$$\begin{aligned} V^{*A}(t) &= E_t\left[\frac{\Lambda^*(t^A)}{\Lambda^*(t)}V^{*B}(t^A)\right] = E_t\left[\frac{\Lambda^*(t^A)}{\Lambda^*(t)}E_{t^A}\left[\int_{t'}^{\infty}\frac{\Lambda^*(s)}{\Lambda^*(t^A)}n^{*B}(s)\lambda e^{-\lambda(s-t')}ds\right]\right] \\ &= E_t\left[\int_{t'}^{\infty}\frac{\Lambda^*(s)}{\Lambda^*(t)}n^{*A}(s)\lambda e^{-\lambda(s-t')}ds\right]. \end{aligned}$$

Since the chosen timing of the liquidation t^A is arbitrary, this argument holds for a generic intermediary. This proves that the maximization problem for an individual intermediary is equivalent to the problem of the representative intermediary.

Using the Foreign saver's Euler equation and the restriction on the dynamics of Ω^* in equation (41) yields the financier's pricing equation for the deposit rate in equation (17). Using equation (40) gives the result in equation (18). Equation (39), equation (17), and equation (41) together yield equation (16). One concludes that the Foreign SDF is $\Lambda^*\Omega^*$.

Proposition 2. The pricing equations for the Open Banking Economy (13-18) and the fact that bankers can trade both the risk-less interbank rate and the stock together impose that:

$$\frac{d\Lambda^*\Omega^*}{\Lambda\Omega^*} = -r_b\ dt - \frac{\mu_Q - r_b}{\sigma_Q}dz$$
$$\frac{d\Lambda}{\Lambda} = -r_b\ dt - \frac{\mu_Q - r_b}{\sigma_Q}dz.$$

This in turn, yields:

$$\frac{C^*}{C} = \frac{\Omega^*}{\xi}, \qquad \text{where } \xi \text{ is a scaling constant to be determined.}$$

The verification that the equilibrium can be solved as a function of a single state variable, the scaled net worth of Foreign intermediaries, requires solving a system of equations. As for the autarky case, this is straightforward but algebra intensive. I provide here the steps of the substitutions that I follow, although the substitutions can clearly be made in different orders. To solve for the equilibrium I have normalized all variables for the size of the output tree, so that in the resulting system Y is no longer a state-variable. The equilibrium risk sharing condition in equation (21) shows that the ratio of the two countries' consumption is fully summarized by Ω^*. This relationship and the fact that the Home country is unconstrained together allow me to further reduce the number of state variables, since keeping track of Ω^* is sufficient to keep track of the ratio of net-wealth in the two countries $\frac{W^*}{W}$.

The conjecture that Ω^* only depends on $\tilde{N}^*$ remains to be verified. The steps are as follows. Use the risk sharing condition and goods market clearing to derive expressions for the drift and diffusion of consumption in each country. To compute the stock and international bond portfolio for each country use the standard derivation, as in frictionless open economies with complete markets à la Lucas. The Home country net wealth is $W(t) = SQ - B$ and the consumption optimality condition and budget constraint imply $W(t) = \frac{1}{\rho}C(t)$. Applying Ito's lemma to both sides of this last equality and requiring the equality of the resulting LHS drift and diffusion terms with those of the dynamic Home net wealth budget constraint yields two equations linear in two unknowns: the stock position S, and the international borrowing B. The market clearing condition for stock and international bond (interbank loans) markets yield S^* and B^*.

Use the Home saver pricing equation (14) to derive an expression for the risk-free rate. Finally, use the conjecture that $\{\tilde{Q}, \Omega^*\}$ only depend on $\tilde{N}^*$ to derive expressions for the drift and diffusion of these processes using similar steps to those in the proof of Proposition 1. These operations produce a system of equations in $\{\mu_Q, \sigma_Q, r_b, S, B, S^*, B^*\}$; its solution expresses these variables as functions of $\tilde{N}^*$ and the level and first two derivatives of the functions $\{\tilde{Q},\Omega^*\}$. Finally, substitute the variables in equations (39) and (41), the implicit ODEs reported in the main text, to obtain two coupled second order ODEs for $\{\tilde{Q},\Omega^*\}$, thus verifying the conjecture. I report here the extensive form of the ODEs:

$$\begin{aligned}\tilde{Q}'' = & -\frac{2(-1+\tilde{Q}'S^*)^2((1+\xi)(1+\tilde{N}\tilde{Q}'\rho)+\tilde{Q}(\tilde{Q}'\delta-(1+\xi)\rho))}{(\tilde{N}-\tilde{Q}S^*)^2(1+\xi)\sigma^2}+ \\ & -\frac{2(\tilde{Q}-\tilde{N}\tilde{Q}')(-1+\tilde{Q}'S^*)\Omega^{*\prime}}{(\tilde{N}-\tilde{Q}S^*)(\xi+\Omega^*)}-\frac{2(\tilde{Q}-\tilde{N}\tilde{Q}')\xi\Omega^{*\prime 2}}{\Omega(\xi+\Omega^*)^2}\end{aligned} \tag{42}$$

$$\begin{aligned}\Omega^{*\prime\prime} = & \frac{2(-1+\tilde{Q}'S^*)\left(\tilde{N}-\tilde{Q}S^*-\frac{(-1+\tilde{Q}'S^*)(\tilde{Q}\delta+\tilde{N}(1+\xi)\rho)}{(1+\xi)\sigma^2}\right)\Omega^{*\prime}}{(\tilde{N}-\tilde{Q}S^*)^2}+ \\ & +\frac{2(-\tilde{Q}S^*(\xi+\Omega^*)+\tilde{N}(\xi+\tilde{Q}'S^*\Omega^*))\Omega^{*\prime 2}}{(\tilde{N}-\tilde{Q}S^*)\Omega^*(\xi+\Omega^*)}+\frac{2\tilde{N}\xi\Omega^{*\prime 3}}{\Omega^*(\xi+\Omega^*)^2}.\end{aligned} \tag{43}$$

The scaling constant ξ is pinned down by requiring that the initial net wealth in each country equals the present value of future consumption. For the Home country, this implies the restriction $W(0) = \frac{1}{\rho}C(0)$. The starting conditions, $\{S(0) = 1/2, S^*(0) = 1/2, B(0) = 0, B^*(0) = 0, Y(0), N^*(0), D^*(0)\}$, are chosen so that countries are symmetric. Each country starts with half of the shares in the stock and no interbank loans. Within each country, the shares are held by its intermediaries, which have a starting balance sheet composed of $N(0)$ net worth and $D(0)$ deposits (where $1/2\ Q(0) = N(0) + D(0)$). Using the starting conditions and consumption rule for the Home country I have:

$$\frac{1}{2}\tilde{Q}(0) = \frac{\xi}{(\xi+\Omega^*(0))\rho}. \tag{44}$$

Given $\tilde{N}^*(0)$, the above equation pins down the value of ξ. As discussed in Appendix B, the solution for ξ is unique for all the numerical solutions of the model.

For the equilibrium to be well defined it remains to be verified that, having started the state variable such that $\tilde{N}^*(0) < \tilde{N}^{*SS} = \frac{1}{\rho(1+\xi)}$, the stochastic steady state (i.e. the upper boundary) is reached and is absorbing, and that V^* exists and is strictly positive for every $\tilde{N}^*(t)$ with $t < t'$. The imposed parameter restriction $\delta = \lambda - \rho$, as discussed in Appendix B, ensures that this is the case.

Open Lucas Economy: Equilibrium Details

Assume that there are no frictions in the Foreign financial sector, so that the constraint $V^*(t) \geq 0$ is no longer present in the Foreign financier's optimization problem. Since Foreign financiers are unconstrained in raising deposits, $\Omega^*(\tilde{N}^*) = 1$ and $\tilde{Q}(\tilde{N}^*) = \frac{1}{\rho}$. These constant functions satisfy the ODEs in equations (42-43). The risk sharing condition in equation (21) now simplifies to the statement that consumption in the two countries is equal in every state (the equality follows from $\xi = 1$ since the two countries are symmetric). The risk premium is constant and equal to $\mu_Q - r_d = \sigma^2$. The equilibrium allocation is supported by international portfolios, where each country's financiers own half of the stock and no interbank loans.

The stochastic steady steady state is $\tilde{N}^{*SS} = \frac{1}{2\rho}$, which is also the absorbing upper boundary of the state space.

Open Economy, Two Trees: Static Optimization for Consumption Baskets

Consider the problem for the Home country:

$$\begin{aligned} &\max_{C_H, C_F} C_H^{\alpha}\ C_F^{1-\alpha} \\ &s.t. \quad C_H\ p + C_F\ p\ \tau = C\ P, \end{aligned}$$

where CP, aggregate expenditure, is given. Substituting the budget constraint for C_F, and re-arranging the FOC for C_H yields the results in equations (24-25). The price indices for each country are derived by substituting equations (24-25) in the consumption basket, imposing $C = 1$, and rearranging to yield:

$$P = p^{\alpha}(p^*\tau)^{1-\alpha}\ \alpha^{-\alpha}(1-\alpha)^{\alpha-1}; \qquad P^* = p^{1-\alpha}p^{*\alpha}\ \alpha^{-\alpha}(1-\alpha)^{\alpha-1}.$$

Simple algebra then yields the expression for the exchange rate as a function of the terms of trade reported in equation (26).

Open Economy, Two Trees: The Home and Foreign Optimal Consumption and Investment Problems

As in the proof of Lemma 3, since Home agents do not face financial frictions their optimization problem is equivalent to that of a Home representative agent with logarithmic preferences. Since such an optimization problem is standard, I only report here the corresponding Euler equations:

$$0 = \Lambda\frac{pY}{P}dt + E_t\left[d(\Lambda Q)\right] \tag{45}$$

$$0 = \Lambda\frac{p^*Y^*}{P}dt + E_t\left[d(\Lambda\mathcal{E}Q^*)\right] \tag{46}$$

$$0 = E_t\left[d(\Lambda D_a)\right] \tag{47}$$

$$0 = E_t\left[d(\Lambda B_a)\right] \tag{48}$$

$$0 = E_t\left[d(\Lambda\mathcal{E}B_a^*)\right], \tag{49}$$

where $\Lambda \equiv e^{-\rho t}\frac{1}{C}$, D_a is the Home-currency deposit asset, B_a is the Home-currency interbank asset, and B_a^* is the Foreign-currency deposit asset with dynamics, respectively:

$$\frac{dD_a}{D_a} = r_d\ dt; \qquad \frac{dB_a}{B_a} = r_b\ dt; \qquad \frac{dB_a^*}{B_a^*} = r_b^*\ dt.$$

The no arbitrage condition implies: $r_d = r_b$. Equation (28) is derived by rearranging equations (48-49) and using the dynamics of the exchange rate.

The Foreign saver solves a problem identical to that in the previous sections and the corresponding Euler equation is: $0 = E_t\left[d(\Lambda^* D_a^*)\right]$.

The representative Foreign financier's optimization problem in equation (27) is solved analogously to the proof of Lemma 3, so I only describe here the differences. For $t < t'$

the HJB equation is:

$$0 = \sup_{\{b^*(u),b(u),s^*(u),s(u)\}} E_t[d(\Lambda^* V^*)] + \chi(t)dt\ V^*$$

where χ is the Lagrange multiplier. Conjecture that the value of the intermediary has the form: $V(\tilde{N}^*, n^*) = \Omega^*(\tilde{N}^*)n^*$. The FOCs are:

$$\mu_{Q^*} - r_d^* = \sigma_{C^*}\sigma_{Q^*}^T - \sigma_{\Omega^*}\sigma_{Q^*}^T \tag{50}$$

$$\mu_Q - \mu_{\mathcal{E}} + \sigma_{\mathcal{E}}\sigma_{\mathcal{E}}^T - \sigma_Q\sigma_{\mathcal{E}}^T - r_d^* = \sigma_{C^*}(\sigma_Q - \sigma_{\mathcal{E}})^T - \sigma_{\Omega^*}(\sigma_Q - \sigma_{\mathcal{E}})^T \tag{51}$$

$$(r_b^* - r_b + \mu_{\mathcal{E}}) = \sigma_{C^*}\sigma_{\mathcal{E}}^T - \sigma_{\Omega^*}\sigma_{\mathcal{E}}^T \tag{52}$$

$$r_b = r_d^*. \tag{53}$$

Substituting the FOCs in the HJB equation leads to a restriction that Ω^* has to satisfy:

$$0 = \mu_{\Omega^*} - \sigma_{C^*}\sigma_{\Omega^*}^T. \tag{54}$$

The problem for $t > t'$ follows the same logic as in the proof of Lemma 3 and requires $\Omega^* = 1$. Using the FOCs and the Foreign saver's Euler equation I obtain the Foreign representative financier's Euler equations:

$$0 = \Lambda^*\Omega^*\frac{pY}{P^*}dt + E_t\left[d(\Lambda^*\Omega^*\frac{Q}{\mathcal{E}})\right] \tag{55}$$

$$0 = \Lambda^*\Omega^*\frac{p^*Y^*}{P^*}dt + E_t\left[d(\Lambda^*\Omega^* Q^*)\right] \tag{56}$$

$$0 = E_t\left[d(\Lambda^*\Omega^* D_a^*)\right] \tag{57}$$

$$0 = E_t\left[d(\Lambda^*\Omega^*\frac{B_a}{\mathcal{E}})\right] \tag{58}$$

$$0 = E_t\left[d(\Lambda^*\Omega^* B_a^*)\right]. \tag{59}$$

The above Euler equations show that Λ and $\Lambda^*\Omega^*$ are the Home and Foreign SDFs respectively.

Proposition 3. The pricing equations (45-49,55-59) and the fact that bankers can trade at least three independent assets imply that $\Lambda = \Lambda^*\frac{\Omega^*}{\xi\mathcal{E}}$ and therefore:

$$\frac{P^*\ C^*}{P\ C} = \frac{\Omega^*}{\xi}, \qquad \text{where } \xi \text{ is a scaling constant to be determined.}$$

Substituting the demand functions for the consumption of each individual good in equations (24-25), and using the goods' market clearing conditions, $C_H + C_H^* = Y$ and $\tau C_F + C_F^* = Y^*$, yield the consumption allocations in equations (33-34).

The proof that the equilibrium can be solved as a function of a single state variable, the scaled net worth of Foreign intermediaries, follows steps similar to the proof of Proposition 2. The substitutions are algebra intensive but straightforward and are omitted in the interest of space. The ODEs, reported in implicit form in Proposition 3, are obtained by using: the Home Euler equations (45,48) to derive the Home financier's trade off between the Home stock and the Home interbank interest rate, which is the ODE in equation (29); the Home Euler equations (46,48) to derive the Home financier's trade off between the Foreign stock and the Home interbank rate, which is the ODE in equation (30); and the restriction on Ω^* in equation (54), which is the ODE in equation (31). The explicit form of the ODEs is omitted here because of the length of the expressions, but can be derived based on the information provided in this proof and is available on request.

The international asset market structure of the model includes, by design, redundant assets. Since the fundamental source of risk is the two-dimensional vector of Brownian motions $\vec{z}$, three assets with linearly independent returns are sufficient for a complete international asset market. For $\alpha > 0.5$ the two stocks are linearly independent and, therefore, the addition of either the Home or Foreign interbank asset is potentially sufficient to implement the equilibrium risk sharing. Various combinations are theoretically possible. The implementation that is of interest for this paper is the one where agents are not allowed to short-sell arbitrary large positions in the stocks and where the Foreign interbank market is shut-off. To derive the portfolio implementation of the equilibrium risk sharing recall that since the Home representative agent has logarithmic preferences one has: $W(t) = \frac{1}{\rho}C(t)$. Applying Ito's Lemma to both sides of this equation and using the Home dynamic budget constraint one has:

$$\left[Q\sigma_Q^T,\ Q^*\mathcal{E}(\sigma_e + \sigma_{Q^*})^T, -\sigma_{\mathcal{E}}^T\right][S_H,\ S_F,\ B_F]^T = \frac{C}{\rho}\sigma_C, \tag{60}$$

and B_H can be obtained as the residual term in the Home budget constraint. The portfolios are derived by solving this linear system of equations and by imposing restrictions on $\{S_H, S_F, B_F, B_H\}$.

The scaling constant ξ is pinned down in a fashion similar to the proof of Proposition 2. Recall that for the Home country one has $W(0) = \frac{1}{\rho}C(0)$. The starting conditions, $\{S_H(0) = 1, S_F^*(0) = 1, B_H(0) = 0, B_F(0) = 0, Y(0) = Y^*(0), N^*(0), D^*(0)\}$, are chosen so that countries are symmetric. Each country starts with all the shares in the domestic-tree stock and no interbank loans. Within each country, the shares are held by its intermediaries, which have a starting balance sheet composed of $N(0)$ net worth and $D(0)$ deposits (where $Q(0) = N(0) + D(0)$). Using the starting conditions and the consumption allocation for the Home country I have:

$$\tilde{Q}(0) = \frac{\xi}{(\alpha\xi + (1-\alpha)\Omega^*(0))\rho}. \tag{61}$$

Given $\tilde{N}^*(0)$, the above equation pins down the value of ξ. The solution for ξ is unique for all the numerical solutions of the model.

The stochastic steady state of this economy is $\tilde{N}^{*SS}\frac{1}{\rho(1+\xi)}$. Given the restriction $\delta = \lambda - \rho$, Appendix B verifies that this is the absorbing upper bound of the state space. The steady state stock positions $\{\bar{S}_H, \bar{S}_F\}$ are defined as the limits of the positions approaching the steady state.

Case $\alpha = 0.5$

The price-dividend ratios are given by

$$\begin{aligned}
\tilde{Q}(t) &= \frac{1}{\frac{p(t)Y(t)}{P(t)}} E_t\left[\int_t^\infty \frac{\Lambda(u)}{\Lambda(t)} \frac{p(u)}{P(u)} Y(u) du\right] = E_t\left[\int_t^\infty e^{-\rho(u-t)} \frac{\alpha\xi + (1-\alpha)\Omega^*(u)}{\alpha\xi + (1-\alpha)\Omega^*(t)} du\right] \\
\tilde{Q}^*(t) &= \frac{1}{\frac{\mathcal{E}(t)p^*(t)Y^*(t)}{P^*(t)}} E_t\left[\int_t^\infty \frac{\Lambda(u)}{\Lambda(t)} \frac{p^*(u)}{P(u)} Y^*(u) du\right] = E_t\left[\int_t^\infty e^{-\rho(u-t)} \frac{(1-\alpha)\xi + \alpha\Omega^*(u)}{(1-\alpha)\xi + \alpha\Omega^*(t)} du\right],
\end{aligned}$$

so that if $\alpha = 0.5$, one has $\tilde{Q} = \tilde{Q}^*$. It follows that $Q = Q^*\mathcal{E}\frac{p}{p^*}\frac{Y}{Y^*}$, and since in this case $ToT = \frac{Y}{Y^*}$, one concludes $Q = Q^*\mathcal{E}$. This establishes the claim in the main text that in the case $\alpha = 0.5$ stock returns, in the same currency, are perfectly correlated. The portfolio implementation of the equilibrium risk sharing can be derived using equations (60) and by imposing $B_F = 0$ and collapsing $\{S_H, S_F\}$ into a single world stock market position S. Equations (60), in this case, are a system of two equations in one unknown (S), but they admit a unique solution since the two equations are linearly dependent. This proves the claim in the main text that two assets are sufficient to implement the equilibrium allocation.

Cole and Obstfeld Economy: Equilibrium Details

Assume that there are no frictions in the Foreign financial sector, so that the constraint $V^*(t) \geq 0$ is no longer present in the Foreign financier's optimization problem. Since Foreign financiers are unconstrained in raising deposits, $\Omega^*(\tilde{N}^*) = 1$ and $\tilde{Q}(\tilde{N}^*) = \tilde{Q}^*(\tilde{N}^*) = \frac{1}{\rho}$. These constant functions satisfy the ODEs in equations (29-31). The risk sharing condition in equation (32) now simplifies to the statement that consumption in the two countries is equal in every state (the equality follows from $\xi = 1$ since the two countries are symmetric). The two stocks have perfectly correlated returns: $Q = Q^*\mathcal{E}$. The equilibrium allocation can be implemented with no trading in the stock and in the interbank market, and trading only in the deposit and goods markets.

The stochastic steady steady state is $\tilde{N}^{*SS} = \frac{1}{2\rho}$, which is also the absorbing upper boundary of the state space.

Appendix B: Numerical Solution Methods

The systems of ODEs in this paper are solved as boundary value problems (BVP) using the Matlab routine *bvp4c*.

Section II: Autarky

The system of coupled second order ODEs in equations (37-38) is to be solved over the interval $(0, \bar{\bar{\tilde{N}}})$, where $\bar{\bar{\tilde{N}}}$ is unknown. The ODEs are singular at both boundaries of the interval. To deal with the singularity, I use asymptotic approximations to derive the boundary conditions. The boundary conditions are:[74]

$$\tilde{Q}\left(\bar{\bar{\tilde{N}}}\right) = \bar{\bar{\tilde{N}}} \tag{62}$$

$$\tilde{Q}\left(\bar{\bar{\tilde{N}}}\right) = \frac{1}{\rho + \tilde{Q}'\left(\bar{\bar{\tilde{N}}}\right)(\lambda - \delta - \rho)} \tag{63}$$

$$\Omega\left(\bar{\bar{\tilde{N}}}\right) = \frac{\lambda + \Omega'\left(\bar{\bar{\tilde{N}}}\right)\tilde{Q}\left(\bar{\bar{\tilde{N}}}\right)(\delta + \rho - \lambda)}{\lambda} \tag{64}$$

$$\tilde{Q}(\epsilon) = a - \sqrt{\frac{a\sigma^2}{\delta}}\ \epsilon^{\frac{1}{2}} \tag{65}$$

$$\tilde{Q}'(\epsilon) = -\frac{1}{2}\sqrt{\frac{a\sigma^2}{\delta}}\ \epsilon^{-\frac{1}{2}} \tag{66}$$

$$\Omega(\epsilon) = 1 + \frac{e[1 - a(\rho - \sigma^2)]}{\lambda\sqrt{\frac{a\sigma^2}{\delta}}} + e\ \epsilon^{\frac{1}{2}} \tag{67}$$

$$\Omega'(\epsilon) = \frac{1}{2}\ e\ \epsilon^{\frac{1}{2}}, \tag{68}$$

where $\{a, e\}$ are unknown parameters, and ϵ is "small". The boundary condition in equation (62) is obtained by imposing that $\sigma_{\tilde{N}}(\bar{\bar{\tilde{N}}}) = 0$. The boundary conditions in equations (63-64) are obtained by imposing that $\lim_{\tilde{N}\to\bar{\bar{\tilde{N}}}} \tilde{Q}''(\tilde{Q} - \tilde{N}) = 0$ and $\lim_{\tilde{N}\to\bar{\bar{\tilde{N}}}} \Omega''(\tilde{Q} - \tilde{N}) = 0$. Intuitively, these conditions are requiring $\bar{\bar{\tilde{N}}}$ to be an upper bound for the state space and, since intermediaries are highly capitalized, the solutions to change "smoothly" approaching this upper bound.

The boundary conditions in equations (65-68) are obtained by using Laurent asymptotic approximations of the ODEs[75] in the limit as $\tilde{N}$ approaches zero and by requiring zero to be a reflective boundary.

To adapt the problem to the the Matlab routine *bvp4c*, I re-write the system of ODEs

[74]Intuitively, seven boundary conditions are required to solve the system: four boundary conditions because it is a system of two second order ODEs, one boundary condition to pin down the unknown parameter $\bar{\bar{\tilde{N}}}$, and two boundary conditions to pin down the unknown parameters $\{a, e\}$ introduced by the asymptotic approximations of the ODEs at the lower boundary.

[75]I report here the first two terms of the approximations, which I found to be sufficient in practice for an accurate numerical solution. I have also experimented with including higher order terms.

by changing variables. Letting $x = \frac{\tilde{N}}{\bar{\tilde{N}}}$, I solve for the functions $\{\tilde{Q}(x), \Omega(x)\}$ on the interval$[\epsilon, 1-\epsilon]$.

Note that simpler boundary conditions can be used under the parameter restriction $\delta = \lambda - \rho$. In this case, $\bar{\tilde{N}} = \frac{1}{\rho}$ and the upper boundary conditions are $\tilde{Q}(\frac{1}{\rho}) = \frac{1}{\rho}$ and $\Omega(\frac{1}{\rho}) = 1$. Intuitively, in this case the upper bound of the state space is absorbing and coincides with the Lucas Economy equilibrium.

The upper boundary conditions impose that $\sigma_{\tilde{N}}(\bar{\tilde{N}}) = 0$; it remains to be verified that $\mu_{\tilde{N}}(\bar{\tilde{N}}) \leq 0$. An inspection of the dynamics of $\tilde{N}$ in equation (7) confirms that under the parameter restriction $\delta = \lambda - \rho$ one has $\mu_{\tilde{N}}(\bar{\tilde{N}}) = 0$, and under the restriction $\delta < \lambda - \rho$ one has $\mu_{\tilde{N}}(\bar{\tilde{N}}) < 0$.

Section III: Open Economy Single Tree

The system of coupled second order ODEs in equations (42-43) is to be solved over the interval $(0, \frac{1}{\rho(1+\xi)}]$. The ODEs are singular at both boundaries of the interval. To deal with the singularity, I use asymptotic approximations to derive the boundary conditions. The boundary conditions are:[76]

$$\tilde{Q}\left(\frac{1}{\rho(1+\xi)}\right) = \frac{1}{\rho} \tag{69}$$

$$\Omega^*\left(\frac{1}{\rho(1+\xi)}\right) = 1 \tag{70}$$

$$\frac{\tilde{Q}''(\epsilon)\ \epsilon}{\tilde{Q}'(\epsilon)} = -\frac{1}{2} \tag{71}$$

$$\frac{\Omega^{*''}(\epsilon)\ \epsilon}{\Omega^{*'}(\epsilon)} = -\frac{1}{2}, \tag{72}$$

where ϵ is "small". The boundary conditions in equations (69-70) are the equilibrium solutions for the Open Lucas Economy. Intuitively, the upper bound of the state space is absorbing and coincides with the Lucas Economy equilibrium. The boundary conditions in equations (71-72) are obtained by using Laurent asymptotic approximations of the ODEs in the limit as $\tilde{N}^*$ approaches zero and by requiring zero to be a reflective boundary.[77]

The upper boundary conditions impose that $\sigma_{\tilde{N}^*}(\frac{1}{\rho(1+\xi)}) = 0$; it remains to be verified that the upper bound of the state space is the absorbing stochastic steady state of the model. This is achieved by requiring that $\delta = \lambda - \rho$. Under this restriction, the numerical solution shows that $\mu_{\tilde{N}^*} > 0$ on the open interval $(0, \frac{1}{\rho(1+\xi)})$ and that $\mu_{\tilde{N}^*}(\frac{1}{\rho(1+\xi)}) = 0$. Intuitively the state variable, having started at $\tilde{N}^*(0) < \tilde{N}^{*SS}$, drifts toward the upper bound of the state space and remains there once it has been reached. Finally, the numerical solution shows that $\Omega^*(t) > 1\ \forall\ t < t'$, thus confirming that V^* exists and is non-zero.

[76] Intuitively, four boundary conditions are required to solve the system of two second order ODEs.

[77] In contrast with the autarky model, where the first two terms of the approximations are used as boundary conditions, it is sufficient for an accurate numerical solution to provide the numerical solver with information about the rate at which the solutions move approaching zero (i.e. the exponent of the series expansion, which I find to be equal to $\frac{1}{2}$).

In analogy with the autarky case, to deal with the singularities I solve the system on the interval $[\epsilon, \frac{1}{\rho(1+\xi)} - \epsilon]$.

For simplicity, instead of selecting a starting value $\tilde{N}^*(0)$, I guess a value for ξ, solve the ODE system, and then back out the implied value for $\tilde{N}^*(0)$ using equation (44). In all my numerical trials the implied value for $\tilde{N}^*(0)$ is unique.

Section IV: Open Economy Two Trees

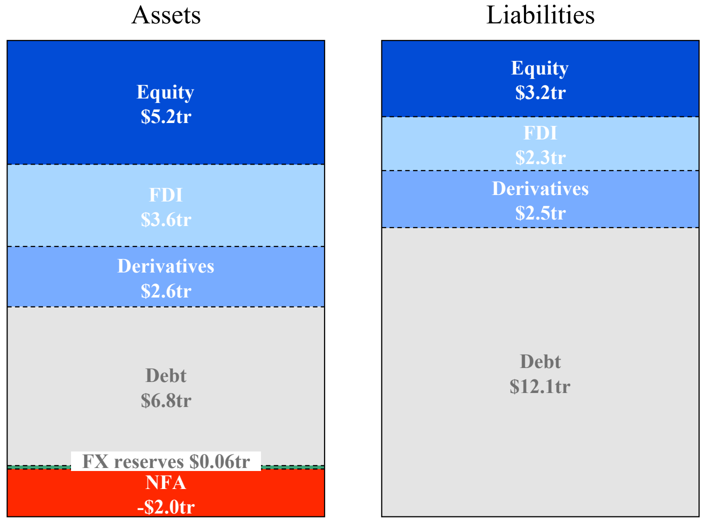
The system of coupled second order ODEs in equations (29-31) is to be solved over the interval $(0, \frac{1}{\rho(1+\xi)}]$. The ODEs are singular at both boundaries of the interval. To deal with the singularity, I use asymptotic approximations to derive the boundary conditions. The boundary conditions are:[78]

$$\begin{aligned}
\tilde{Q}\left(\frac{1}{\rho(1+\xi)}\right) &= \frac{1}{\rho} \\
\tilde{Q}^*\left(\frac{1}{\rho(1+\xi)}\right) &= \frac{1}{\rho} \\
\Omega^*\left(\frac{1}{\rho(1+\xi)}\right) &= 1 \\
\frac{\tilde{Q}''(\epsilon)\ \epsilon}{\tilde{Q}'(\epsilon)} &= -\frac{1}{2} \\
\frac{\tilde{Q}^{*\prime\prime}(\epsilon)\ \epsilon}{\tilde{Q}^{*\prime}(\epsilon)} &= -\frac{1}{2} \\
\frac{\Omega''(\epsilon)\ \epsilon}{\Omega'(\epsilon)} &= -\frac{1}{2},
\end{aligned}$$

where ϵ is "small". The intuition for the boundary conditions, the solution method, and the verification of the stochastic steady state are analogous to those for the Open Banking Economy with a single tree in the previous section.

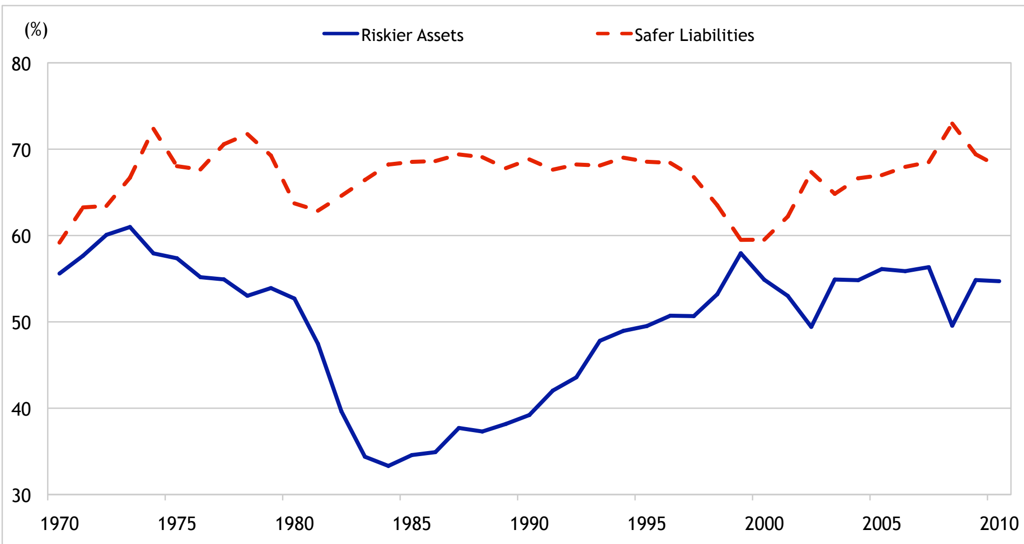
[78]Intuitively, six boundary conditions are required to solve the system of three second order ODEs.

Figure 1: US External Balance Sheet: 2007



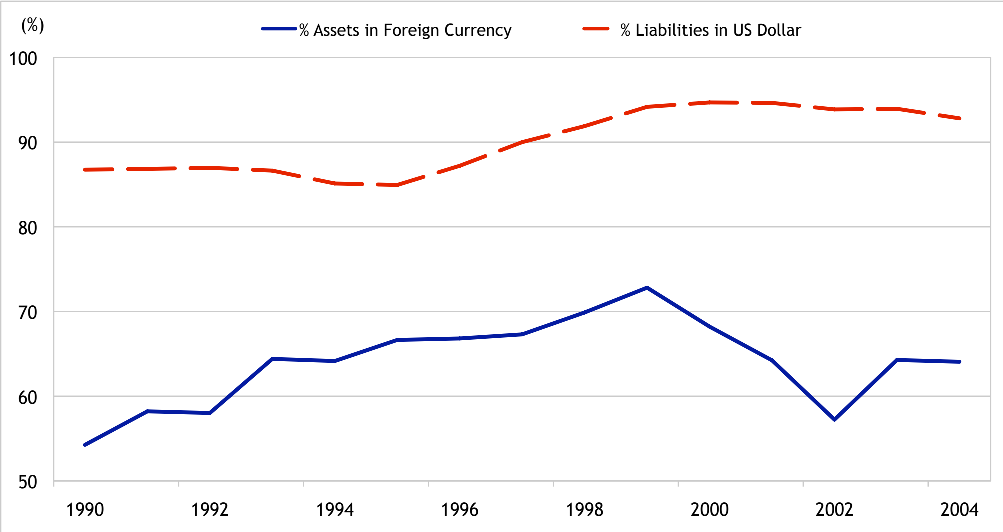
Source: Balance of Payment Statistics. US external balance sheet at year-end 2007. US external assets: US residents' holdings of assets abroad, by asset class. US external liabilities: RoW residents' holdings of assets in the US, by asset class. Debt assets and liabilities are (debt + other investments). The NFA position is reported in red and as a negative number on the asset side, to stress that it is the amount owed by the US to the RoW. See source for details on dataset construction.

Figure 2: Asset Class Composition of US External Assets and Liabilities



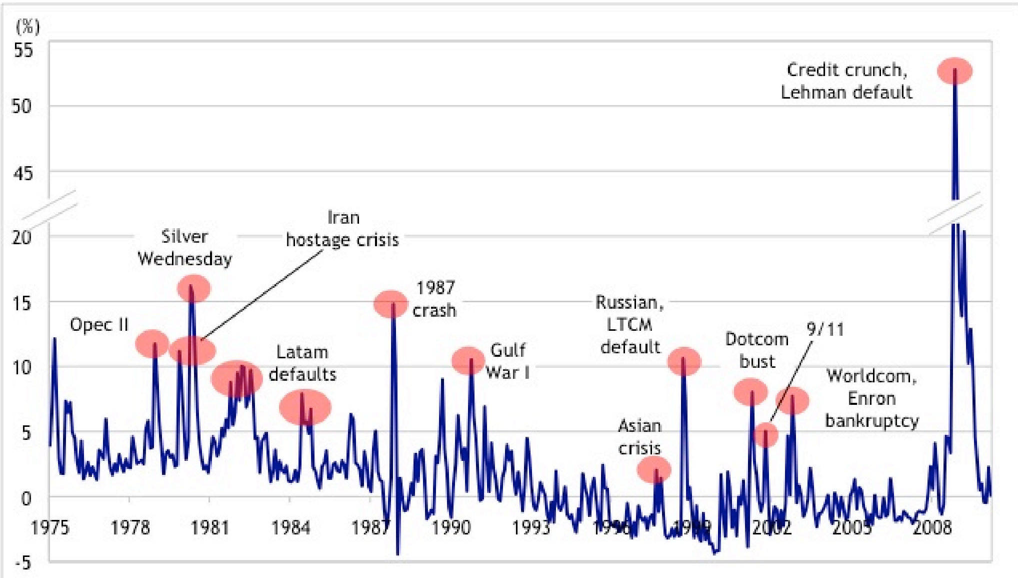
Source: Lane and Milesi-Ferretti (2007) and Balance of Payment Statistics. The data are annual 1970-2010. The percentages are computed as: (Equity+FDI)/(Total Assets-Derivatives) for assets and (Debt+Other Investments)/(Total Liabilities-Derivatives) for liabilities. Derivatives positions are excluded in order to avoid possible issues associated with the netting of contracts. In any case, data on derivatives held in the external portfolio are only available for the years 2005-10. See source for details on dataset construction.

Figure 3: Currency Composition of US External Assets and Liabilities



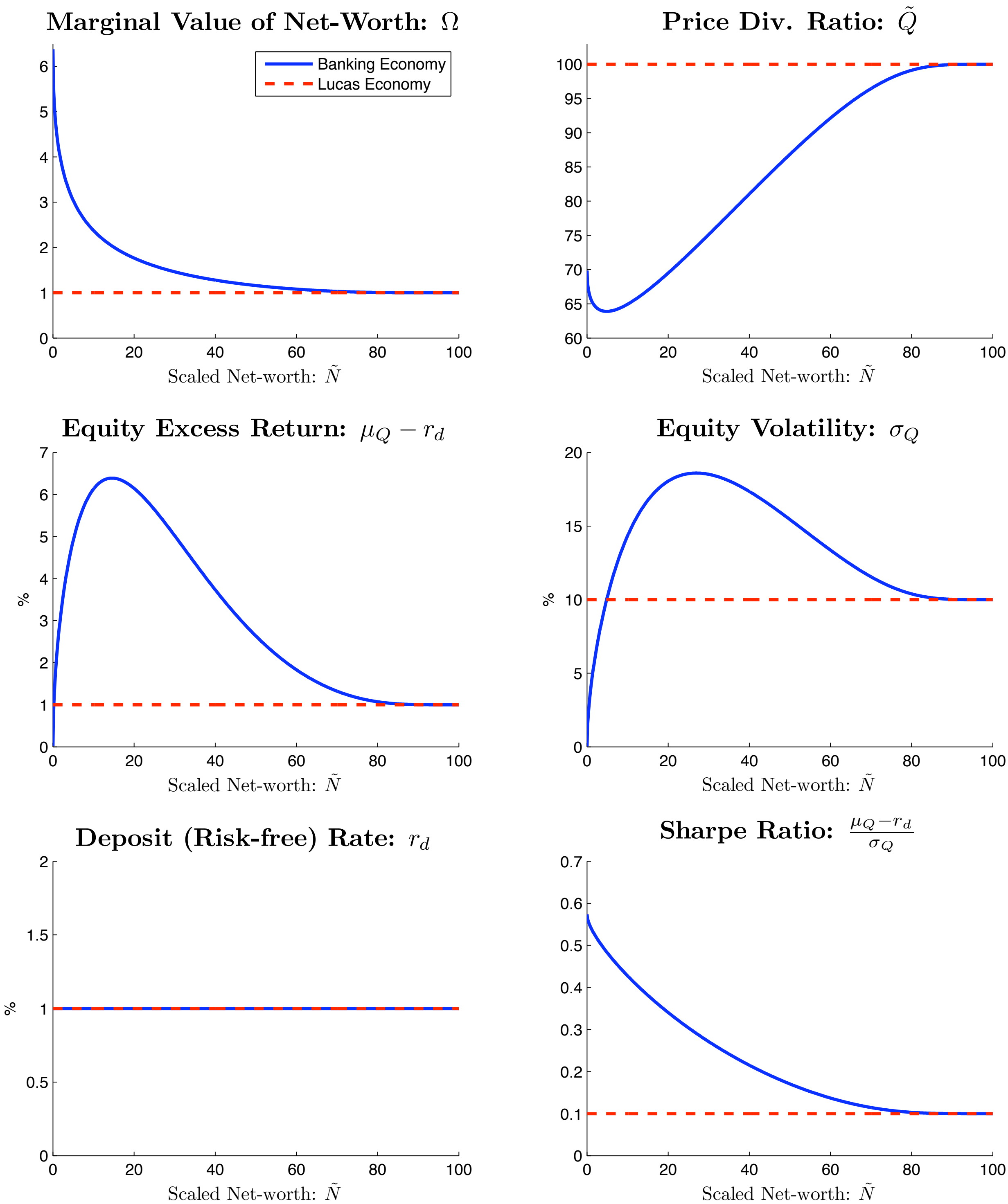
Source: Lane and Shambaugh (2010). The data are annual 1990-2004. Shares of total US external assets denominated in foreign currency and total US liabilities denominated in US dollars. See source for details on dataset construction.

Figure 4: US Dollar Safety Premium



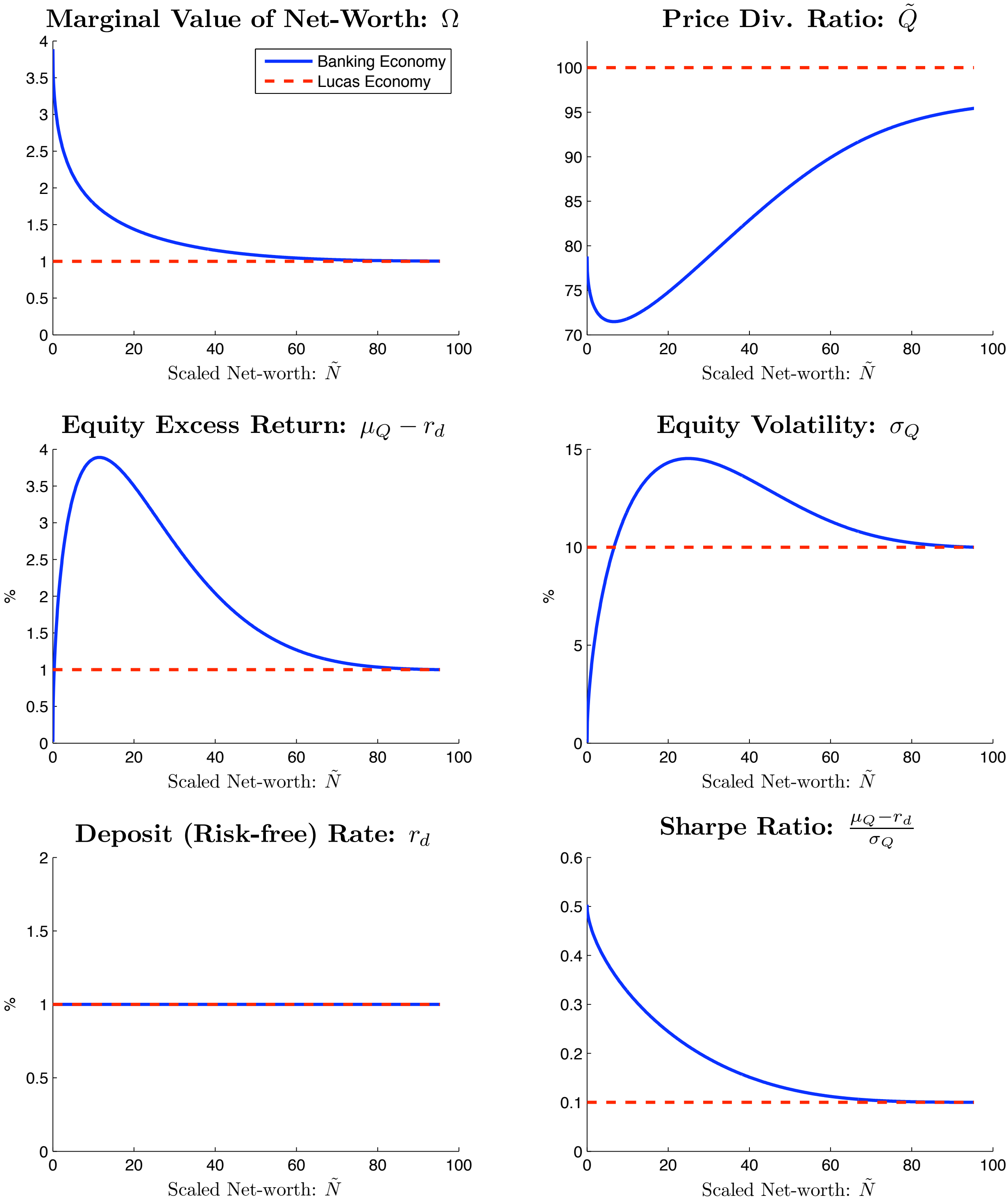
Source: Maggiori (2010). The data are monthly January 1975-March 2010. The estimated annualized compensation that an investor would require at each point in time to invest in a basket of foreign currency while shorting the US dollar. For example, the estimate of 53% for October 2008 is interpreted as investors requiring an annualized expected return of 53% to invest in foreign currency instead of the US dollar. The basket of foreign currency is weighted using the MSCI (All Country) World Index market capitalization. See source for details on dataset construction and estimation.

Figure 5: Autarky Equilibrium: Converges to Lucas Equilibrium



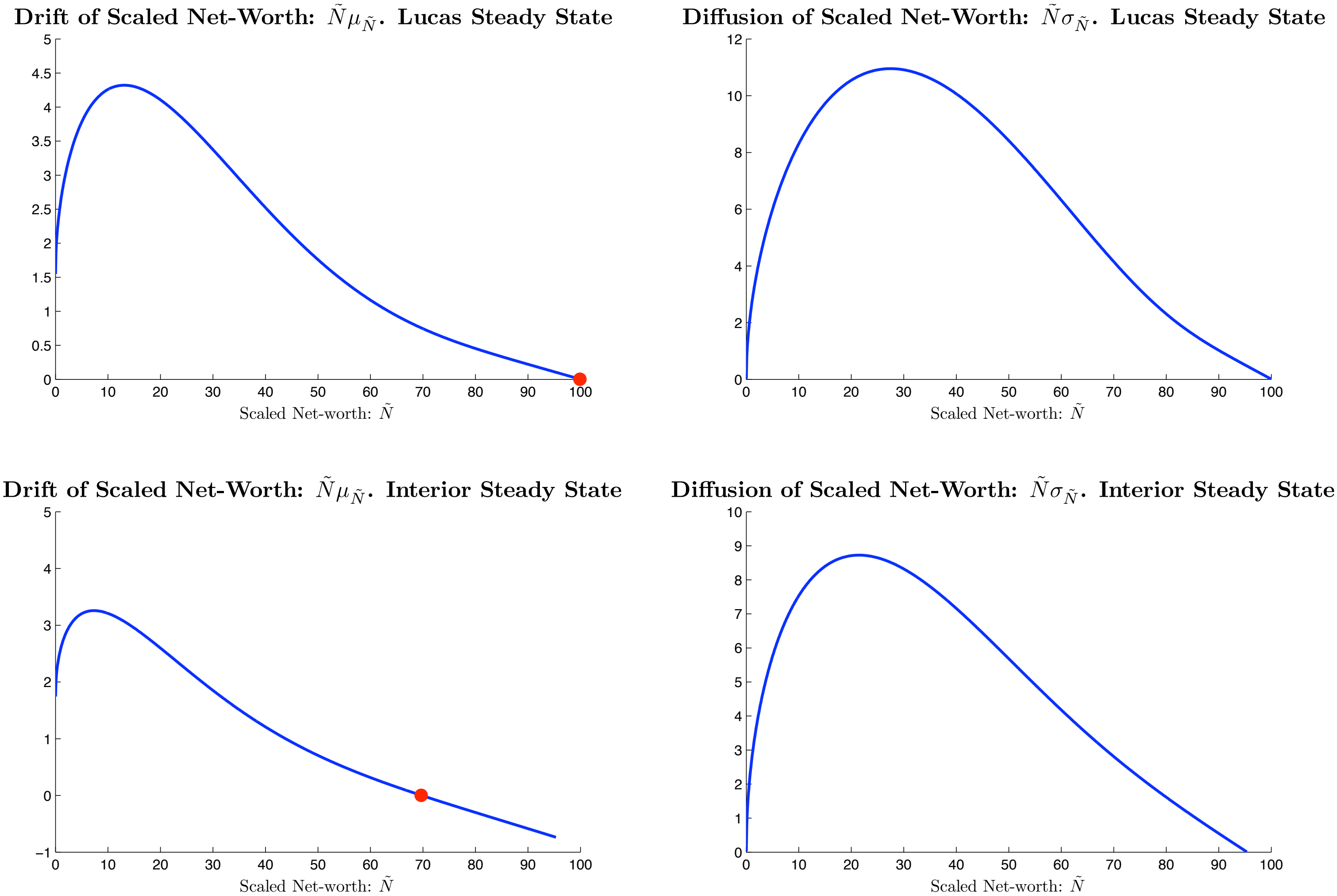
Numerical solution for the equilibrium in Section II for the case $\delta = \lambda - \rho$: the Banking Economy eventually converges to the Lucas Economy. Parameter values: $\rho = 0.01$, $\delta = 0.022$, $\mu = 0.01$, $\sigma = 0.1$. Note that the graphs plot the solution for the state space of the Banking Economy, the range of the state variable $\tilde{N}$. The Lucas Economy solution is plotted over the same state space for comparison purposes, but the state space of the Lucas Economy extends beyond the one of the Banking Economy. The state space of the Banking Economy is $(0, \frac{1}{\rho}]$, and the stochastic steady state is $\frac{1}{\rho}$.

Figure 6: Autarky Equilibrium: Interior Stochastic Steady State



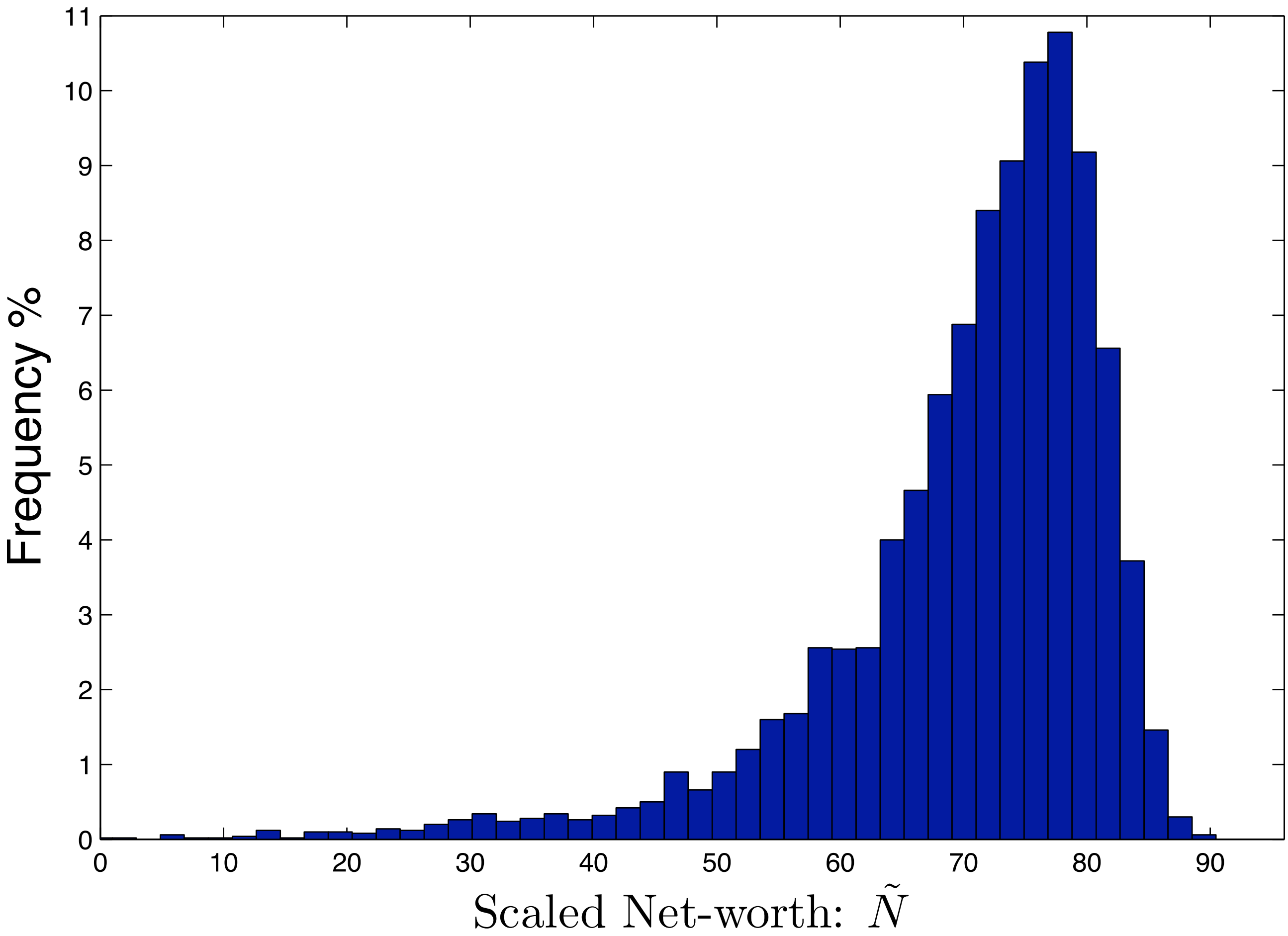
Numerical solution for the equilibrium in Section II for the case $\delta < \lambda - \rho$: the Banking Economy has an interior stochastic steady state. Parameter values: $\rho = 0.01$, $\delta = 0.022$, $\lambda = 0.0398$, $\mu = 0.01$, $\sigma = 0.1$. Note that the graphs plot the solution for the state space of the Banking Economy, the range of the state variable $\tilde{N}$. The Lucas Economy solution is plotted over the same state space for comparison purposes, but the state space of the Lucas Economy extends beyond the one of the Banking Economy. The state space of the Banking Economy is $(0, 95.43)$, and the stochastic steady state is 69.76.

Figure 7: Autarky Equilibrium: Stochastic Steady State



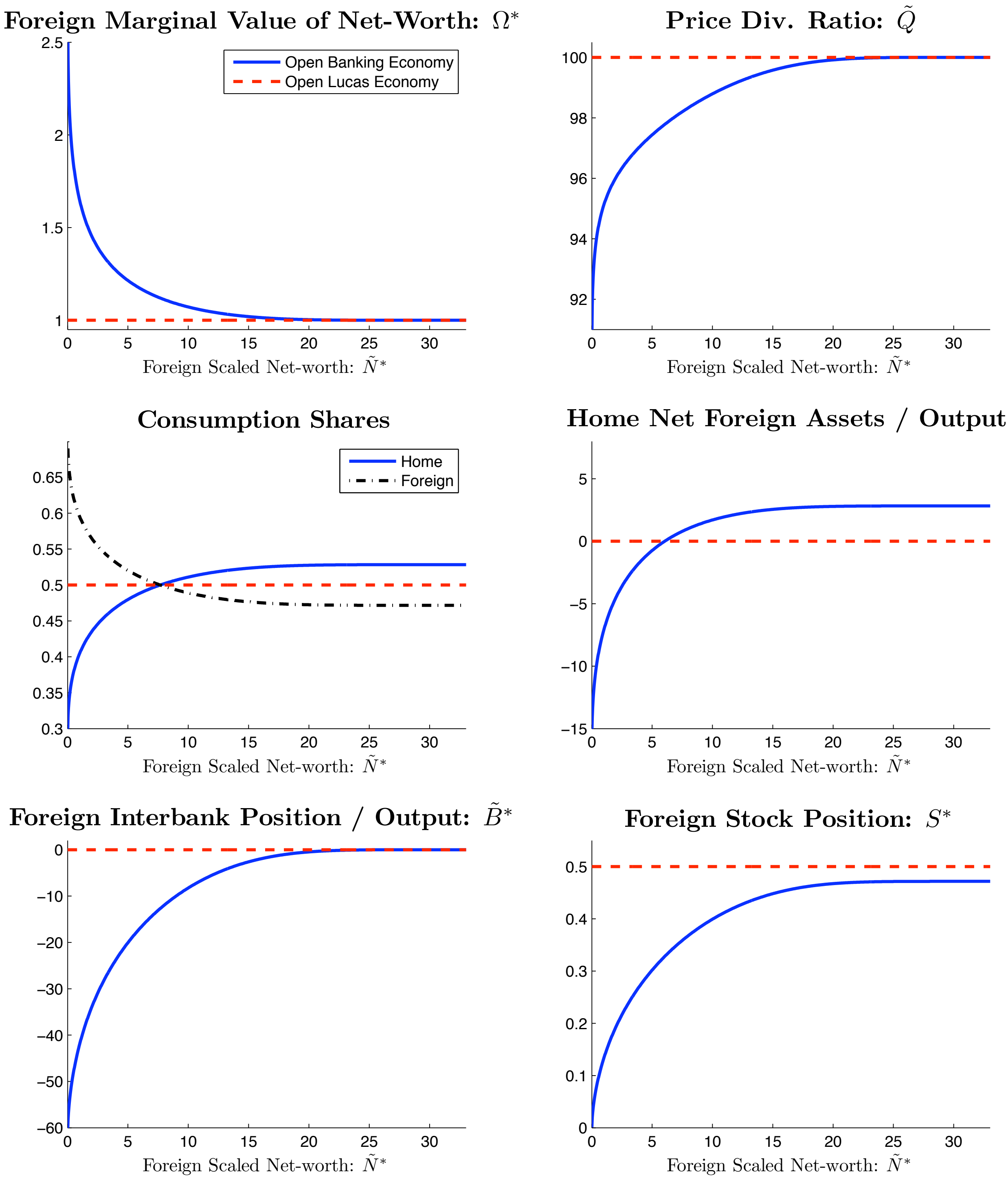
Numerical solution for the equilibrium in Section II for the case $\delta = \lambda - \rho$ (top two graphs) and $\delta < \lambda - \rho$ (bottom two graphs). Parameter values for the first case: $\rho = 0.01$, $\delta = 0.022$, $\lambda = 3.98$, $\mu = 0.01$, $\sigma = 0.1$. These are the drift and diffusion of the state variable, scaled net-worth $\tilde{N}$, for the equilibrium in Figure 5. Parameter values for the second case: $\rho = 0.01$, $\delta = 0.022$, $\lambda = 0.0398$, $\mu = 0.01$, $\sigma = 0.1$. These are the drift and diffusion of the state variable, scaled net-worth $\tilde{N}$, for the equilibrium in Figure 6. The red dot in the two graphs on the left corresponds to each case's stochastic steady state. The state space of the case $\delta = \lambda - \rho$ is $(0, \frac{1}{\rho}]$, and the stochastic steady state is $\frac{1}{\rho}$. The state space of the case $\delta < \lambda - \rho$ is $(0, 95.43)$, and the stochastic steady state is 69.76.

Figure 8: Autarky Equilibrium: Stationary Distribution



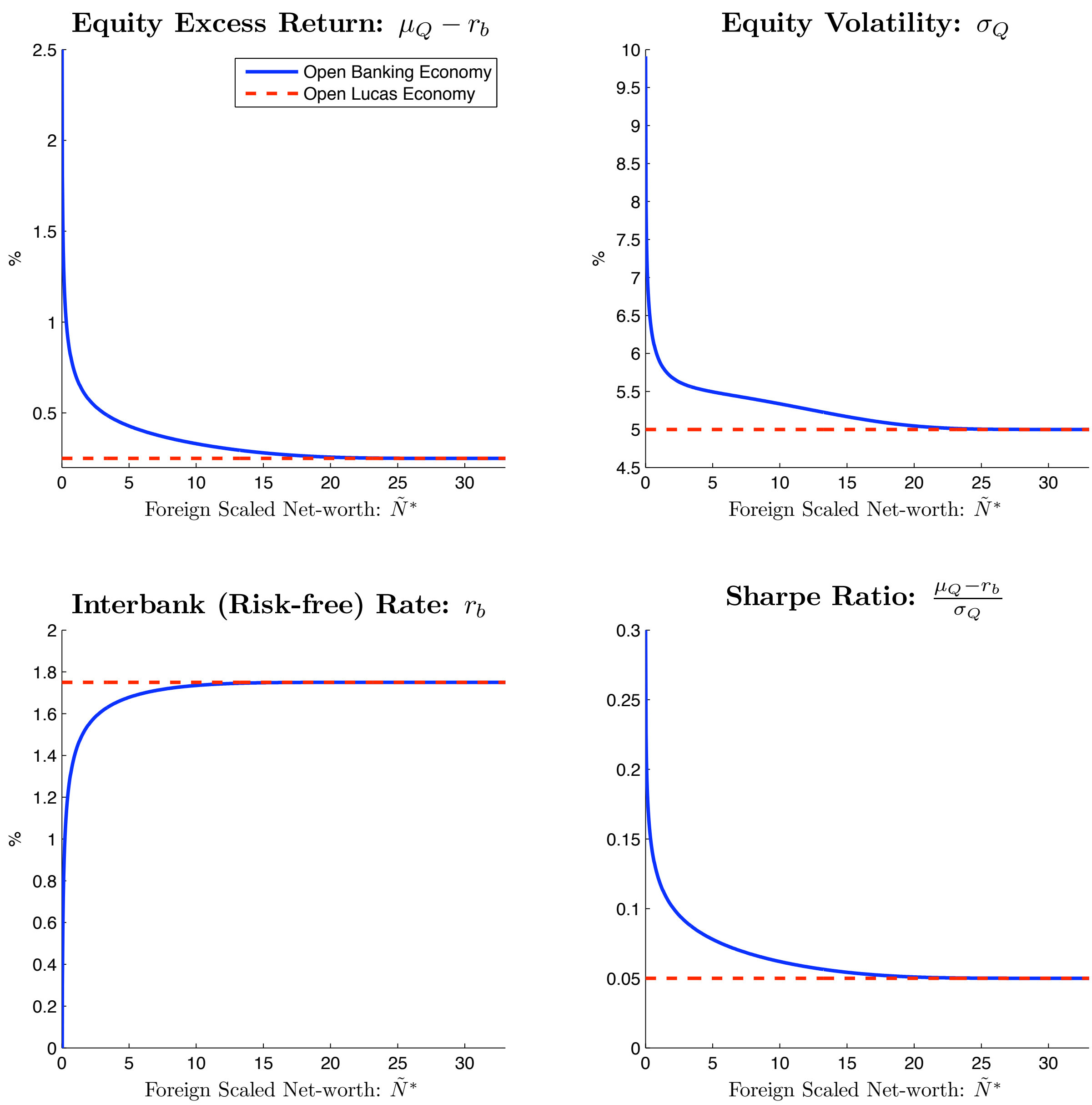
Plot of the limiting stationary distribution of the state variable, scaled net-worth $\tilde{N}$, for the equilibrium in Section II for the case $\delta < \lambda - \rho$. Parameter values: $\rho = 0.01$, $\delta = 0.022$, $\lambda = 0.0398$, $\mu = 0.01$, $\sigma = 0.1$. This is the stationary distribution for the equilibrium in Figure 6. The state space is $(0, 95.43)$, and the stochastic steady state is 69.76. The approximation to the stationary distribution is obtained by simulating 5,000 paths for 100 years at daily frequency (36,500 periods) for the process $\tilde{N}$.

Figure 9: Open Economy Equilibrium, Single Tree: Allocations



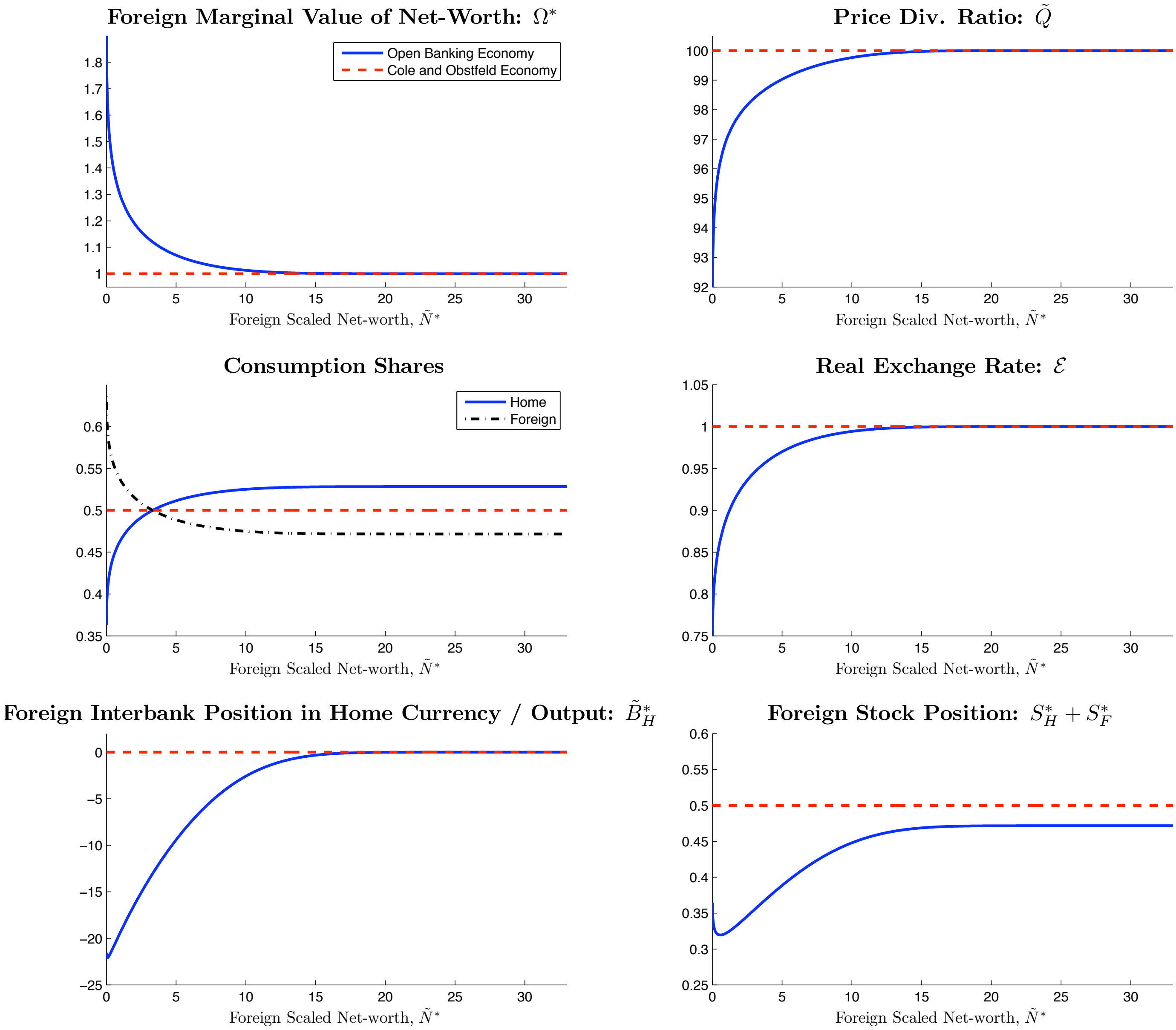
Numerical solution for the equilibrium in Section III. Parameter values: $\rho = 0.01$, $\delta = 0.004$, $\lambda = 0.014$, $\mu = 0.01$, $\sigma = 0.05$. *The starting scaled net-worth is* $\tilde{N}^*(0) = 5.2$*, which results in* $\xi = 1.12$. *Note that the graphs plot the solution for the state space of the Open Banking Economy, the range of the state variable* $\tilde{N}^*$. *The Open Lucas Economy solution is plotted over the same state space for comparison purposes, but the state space of the Open Lucas Economy extends beyond the one of the Open Banking Economy. The state space of the Open Banking Economy is* $(0, \frac{1}{\rho(1+\xi)}]$*; in the figures above it has been cut on the right to allow for better visualization. The stochastic steady state is* $\frac{1}{\rho(1+\xi)}$.

Figure 10: Open Economy Equilibrium, Single Tree: Asset Prices



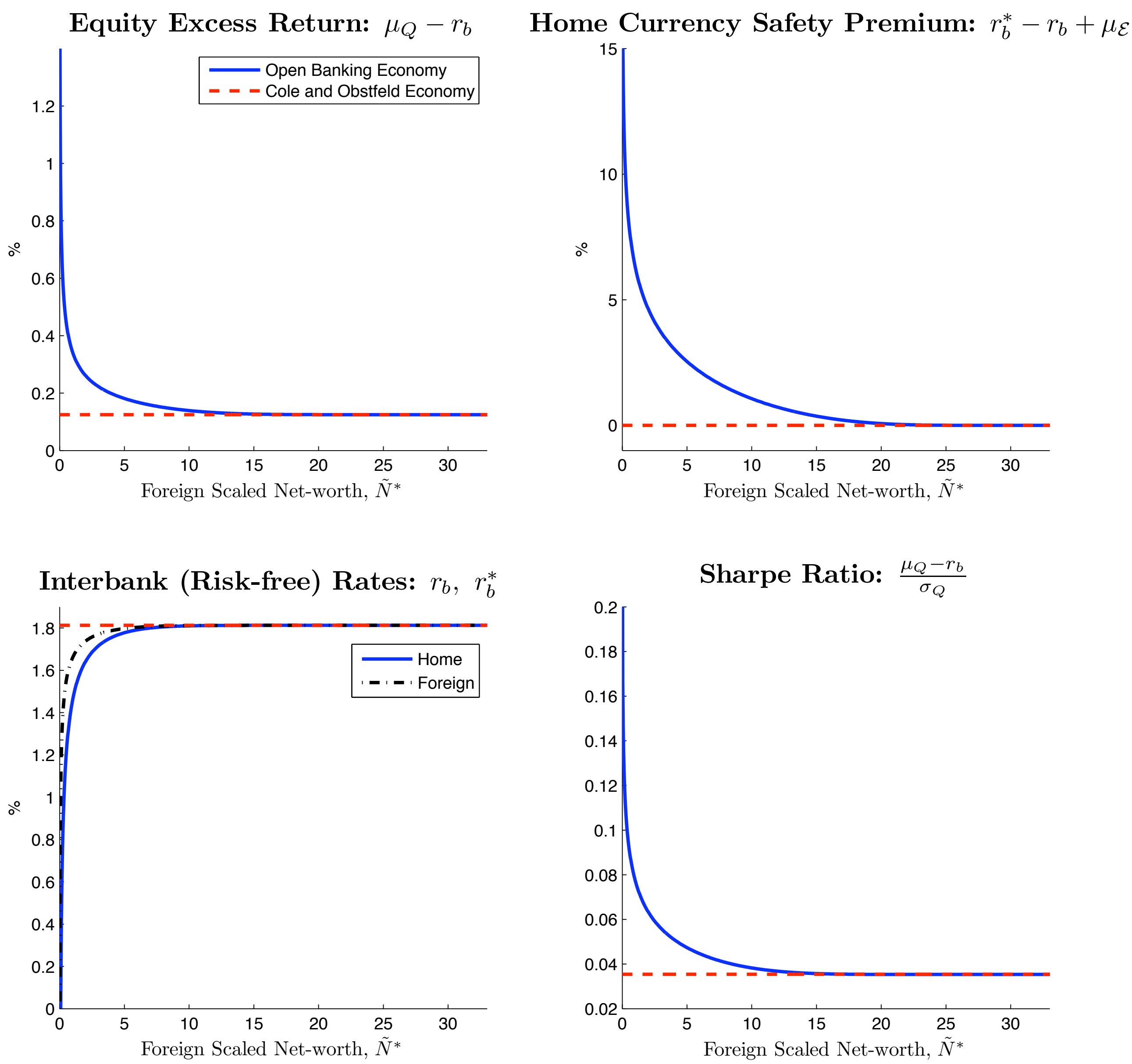
Numerical solution for the equilibrium in Section III. Parameter values: $\rho = 0.01$, $\delta = 0.004$, $\lambda = 0.014$, $\mu = 0.01$, $\sigma = 0.05$. *The starting scaled net-worth is* $\tilde{N}^*(0) = 5.2$, *which results in* $\xi = 1.12$. *Note that the graphs plot the solution for the state space of the Open Banking Economy, the range of the state variable* $\tilde{N}^*$. *The Open Lucas Economy solution is plotted over the same state space for comparison purposes, but the state space of the Open Lucas Economy extends beyond the one of the Open Banking Economy. The state space of the Open Banking Economy is* $(0, \frac{1}{\rho(1+\xi)}]$; *in the figures above it has been cut on the right to allow for better visualization. The stochastic steady state is* $\frac{1}{\rho(1+\xi)}$.

Figure 11: Open Economy Equilibrium, Two Trees, No Domestic Bias: Allocations



Numerical solution for the equilibrium in Section IV. Parameter values: $\rho = 0.01,\ \delta = 0.004,\ \lambda = 0.014,\ \mu = 0.01,\ \sigma_z = \sigma_{z^*} = 0.05,\ \alpha = 0.5$*. The starting scaled net-worth is* $\tilde{N}^*(0) = 3.5$*, which results in* $\xi = 1.12$*. Note that the graphs plot the solution for the state space of the Open Banking Economy with two trees, the range of the state variable* $\tilde{N}^*$*. The Cole and Obstfeld Economy solution is plotted over the same state space for comparison purposes, but the state space of the Cole and Obstfeld Economy extends beyond the one of the Open Banking Economy. The state space of the Open Banking Economy is* $(0, \frac{1}{\rho(1+\xi)}]$*; in the figures above it has been cut on the right to allow for better visualization. The stochastic steady state is* $\frac{1}{\rho(1+\xi)}$*.*

Figure 12: Open Economy Equilibrium, Two Trees, No Domestic Bias: Asset Prices



Numerical solution for the equilibrium in Section IV. Parameter values: $\rho = 0.01,\ \delta = 0.004,\ \lambda = 0.014,\ \mu = 0.01,\ \sigma_z = \sigma_{z^*} = 0.05,\ \alpha = 0.5$. *The starting scaled net-worth is* $\tilde{N}^*(0) = 3.5$, *which results in* $\xi = 1.12$. *Note that the graphs plot the solution for the state space of the Open Banking Economy with two trees, the range of the state variable* $\tilde{N}^*$. *The Cole and Obstfeld Economy solution is plotted over the same state space for comparison purposes, but the state space of the Cole and Obstfeld Economy extends beyond the one of the Open Banking Economy. The state space of the Open Banking Economy is* $(0, \frac{1}{\rho(1+\xi)}]$; *in the figures above it has been cut on the right to allow for better visualization. The stochastic steady state is* $\frac{1}{\rho(1+\xi)}$.