Money and the Natural Rate of Interest: Structural Estimates for the U.K., the U.S., and the Euro Area*

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Abstract

In this paper, we look at the role of money in a general framework that encompasses three competing environments: the New Keynesian model with separable utility and static money demand; the nonseparable utility variant with habit formation; and the New Keynesian model modified to allow for adjustment costs for holding real balances. The last two models imply a *forward-looking* character of real money balances, that conveys on money an important role as a monetary policy indicator. We distinguish between these alternative specifications by conducting a structural econometric analysis for the U.S., the euro area, and the U.K. FIML estimates confirm the forward-looking character of money demand. Using these estimates we find that, in response to preferences and technology shocks, money incorporates useful information regarding future variations in the natural interest rate.

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1 Introduction

The growing use of sticky-price optimizing models, or a "New Keynesian" framework, in macroeconomics has simultaneously reaffirmed the relevance of monetary policy actions for the behavior of output and inflation, and downplayed the importance of monetary aggregates. The baseline version of the New Keynesian model, with household preferences separable across time and arguments, does generate a standard money demand function (the LM component of what McCallum and Nelson, 1999, call the "optimizing IS-LM specification"). But much work with New Keynesian models, exemplified by Rotemberg and Woodford (1997), uses the fact that the IS function, Phillips curve, and interest-rate policy rule contain no money term as grounds for not referring to money or the money demand function in the analysis at all. And insofar as money has an indicator role in this New Keynesian baseline, it is as a noisy indicator of current output (see e.g. Dotsey and Hornstein, 2003). The money stock then becomes one of many candidates as indicators of current economic activity—hardly a role that conveys great significance to money in macroeconomic analysis.

One modification to the New Keynesian model that restores an explicit role for money is to drop the assumption that household preferences are separable across consumption and real money balances. As shown in Andrés, López-Salido and Vallés (2003), Ireland (2004), Woodford (2003) and below, relaxing this assumption does introduce terms involving real balances into the model's IS and Phillips curve (or marginal cost) equations. But plausible calibrations do not seem to generate a sizable role for this channel (McCallum, 2000; Woodford, 2003), while econometric estimates so far provide even less empirical support (see Ireland, 2004, for the U.S., and Andrés, López-Salido and Vallés, 2003, for the euro area).

Nelson (2002) argues that neither the separable nor the non-separable preference specifications conveys on money the role stressed for it in the monetarist literature. That literature, as discussed in Artis (1993), Meltzer (2001), and references therein, rests on two propositions: first, that yields beside the short-term interest rate enter both the IS and the money demand functions; and secondly, that the money stock therefore provides information about determinants of aggregate demand beside short-term real interest rates. This perspective transforms the central issue from being whether money appears explicitly in the IS and Phillips curve equations, to whether money serves as a good proxy for movements in asset prices that do appear directly in the economy's IS equation, some of which may be difficult to observe directly. Nelson argues that a first step in capturing these ideas is to add to the New Keynesian model a forward-looking dimension to money demand, arising from portfolio adjustment costs. In this environment, it is not money's role as a static indicator of output, but instead the *interest-elastic* and *forward-looking* character of real money balances, that conveys on money an important role as an indicator.

The different perspectives on money suggested by the three model settings—the standard New Keynesian model with separable utility and static money demand; the non-separable utility variant; and the New Keynesian model modified to allow for dynamics in money demand—are brought out in Table 1. The departures from the baseline model both provide improved grounds for looking at money, but do so in different ways.

In this paper, we distinguish between the alternative views of the role of money in the transmission mechanism by conducting a structural econometric analysis of three economies: the U.S., the euro area, and the U.K. The dynamic stochastic general equilibrium model that we estimate delivers each model variant described in Table 1 as a special case. Using our estimated model, we are able to demonstrate the enhanced ability of money to capture the transmission mechanism of monetary policy when money demand has a forward-looking element. In particular, we show that the value of money as a proxy for variations in the natural interest rate and the real interest-rate gap is increased.

Relatively little of the study of money in the transmission mechanism has been in the context of optimizing models estimated by systems methods. The investigations of the role of money by Nelson (2002), Dotsey and Hornstein (2003), and Woodford (2003), for example, use calibrated models. The Bayesian maximum likelihood estimation of a DSGE model for the euro area by Smets and Wouters (2002) excludes money from the list of variables modeled.[1] A considerable amount of econometric work has been done on the role of money in the euro area, as discussed in Issing *et al* (2001) and the ECB (2003), but this work is typically either explicitly reduced form or has relied on postulated behavioral relationships that lack microfoundations (e.g. IS-LM systems without proper account for forward-looking behavior, or with

[1] Important recent papers for the U.S. that do include money among the variables of interest, though using a different estimations procedure from MLE, are Christiano, Eichenbaum, and Evans (2005) and Altig, Christiano, Eichenbaum, and Linde (2005).

lagged terms not traced explicitly to private sector optimization).

Work that does meet our joint criteria of using DSGE modelling, estimating by systems methods, and putting money in the likelihood, includes Ireland (2003, 2004) and Andrés, López-Salido and Vallés (2003).[2] Relative to these studies, the present paper makes three principal contributions. First, the model we estimate is sufficiently general to distinguish between all three model settings described in Table 1, not just the separable vs. nonseparable preference specifications. Secondly, we extend the analysis to the U.K. Thirdly, we use our estimates to carry out an analysis of the dynamic relationship between money and the natural rate, and the consequent usefulness of money to monetary policy.

Our model is laid out in Section 2. Section 3 includes some analytical results on the relation between money and the natural rate. Section 4 presents our empirical results. We find considerable support for the forward-looking money demand variant of the model, and in Section 5 we show how this specification improves the value of money as a proxy for the natural rate of interest. Section 6 concludes.

2 A Sticky-Price Model with Money

The model has many features commonly used in sticky-price versions of the New Keynesian model, but is closest to Andrés, López-Salido and Vallés (2003), Ireland (2004), and Nelson (2002). The economy consists of a representative household, a continuum of producing firms indexed by $j \in [0, 1]$ and a monetary authority. We abstract from capital accumulation. The model displays sufficient symmetry for our analysis to focus on the behavior of a representative goods-producing firm.

[2] Another example is Bergin (2003), but the model he estimates is not suited to the study of interest-rate policy rules.

2.1 Households

2.1.1 The Non-Separability Effect

The representative household of the economy maximizes the following expected stream of utility:

$$\max_{C_t,N_t,M_t,B_t} E_0 \sum_{t=0}^{\infty} \beta^t a_t \left[\Psi\left(\frac{C_t}{C_{t-1}^h}, \frac{M}{e_t P_t} \right) - \frac{N_t^{1+\varphi}}{1+\varphi} \right] \tag{1}$$

where C_t is the CES aggregator of the quantities of the different goods consumed:[3]

$$C_t = \left(\int_0^1 C_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

Here M_t/P_t and N_t represent real balances and hours, respectively; a_t is a preference shock, and e_t is a shock to the household's demand for real balances. The parameter $\beta \in (0,1)$ is a discount factor and $\varphi \geq 0$ represents the inverse of the Frisch labor supply elasticity.[4]

We allow for *non-separability* among consumption and real balances in preferences, as well as for habit formation in consumption. Intra-temporal non-separability makes it possible to test the relevance of an explicit money-balances term in the equations determining supply and demand decisions. This is the main influence of money emphasized in recent studies. The presence of habits has been emphasized by Fuhrer (2000) and Christiano, Eichenbaum, and Evans (2005), among others, as an important component of the monetary transmission mechanism that helps to account for the gradual response of output to monetary policy shocks. The dynamic interaction between nominal and real variables is further enriched by the presence of intertemporal non-separability that generates a battery of cross-equation restrictions. Finally, the marginal utility of consumption depends upon real balances but it is independent of labor supply decisions. In addition, the assumption of separability between a consumption/real balances basket and hours implies that aggregate demand relationships are invariant to the specification of the firm's problem (Driscoll, 2000).

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[3] $P_t = \left(\int_0^1 P_t(j)^{1-\varepsilon} \, dj \right)^{\frac{1}{1-\varepsilon}}$ is the aggregate price index that is consistent with the first-order conditions of the producing firms that face the differentiated demand, with $P_t(j)$ the price of good j.

[4] When $\varphi = 0$, preferences are linear in labor (Hansen, 1985) and the labor supply elasticity is infinite.

2.1.2 The Direct Effect

Recently, Nelson (2002) has elaborated on the idea that a key link between real balances and real aggregate demand occurs not via the non-separability channel, but through "direct effects" that are not well captured by short term real interest rates. In this framework, money is serving as an index for yields beside the short rate (in his application, the real long-term rate) that are relevant for aggregate demand (this builds on Meltzer, 2001, and the references therein). Nelson (2002) captures this idea by simply allowing for portfolio adjustment costs. Formally,

$$\max_{C_t,N_t,M_t,B_t} E_0 \sum_{t=0}^{\infty} \beta^t a_t \left[\Psi \left(\frac{C_t}{C_{t\text{-}1}^h}, \frac{M}{e_t P_t} \right) \text{-} \frac{N_t^{1+\varphi}}{1+\varphi} \right] - G(\bullet) \tag{2}$$

with

$$G(\bullet) = \frac{d}{2} \left\{ \exp \left[c \left\{ \frac{\frac{M_t}{P_t}}{\frac{M_{t\text{-}1}}{P_{t\text{-}1}}} \text{-}1 \right\} + \exp \left[\text{-}c \left\{ \frac{\frac{M_t}{P_t}}{\frac{M_{t\text{-}1}}{P_{t\text{-}1}}} \text{-}1 \right\} \right] - 2 \right] \right\} \tag{3}$$

and where $d > 0$, $c > 0$.[5]

The budget constraint each period is:

$$\frac{M_{t-1} + B_{t-1} + W_t N_t + T_t + D_t}{P_t} = C_t + \frac{B_t / r_t + M_t}{P_t} \tag{4}$$

Households enter period t with money holdings M_{t-1} and bonds B_{t-1}. At the beginning of the period, they receive lump-sum nominal transfers T_t, labor income $W_t N_t$, where W_t denotes the nominal wage, and a nominal dividend D_t from the firms. They use some of these funds to purchase new bonds at nominal cost B_t / r_t, where r_t denotes the gross nominal interest rate between t and $t+1$. The household carries M_t units of money into the period $t+1$.

2.2 Firm Behavior and Price Setting

The production function for firm j is

$$Y_t(j) = z_t N_t(j)^{1-\alpha} \tag{5}$$

[5] This functional form for portfolio adjustment costs used by Nelson is that of Christiano and Gust (1999), modified to apply to real balances and applied to a model without "limited participation" features.

where $Y_t(j)$ is output, $N_t(j)$ represents the number of work-hours hired from the household (i.e. $N_t = \int_0^1 N_t(j)\ dj$), z_t is a common technology shock and $(1-\alpha)$ parameterizes the technology. Letting $Y_t = \left(\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}}\ dj\right)^{\frac{\varepsilon}{\varepsilon-1}}$, the market-clearing condition implies $Y_t = C_t$.

The representative firm sells its output in a monopolistically competitive market and sets nominal prices on a staggered basis, as in Calvo (1983). Each firm has with probability $1-\theta$ an opportunity to reset its price in any given period, independently of the time elapsed since the last adjustment. Thus, each period a measure $1-\theta$ of producers reset their prices to maximize their stream of expected profits. Hence, θ^k will be the probability that the price set at time t will still hold at time $t+k$. Notice that, if there were no constraints on the adjustment of prices, the typical firm would set a price according to the rule $P_t(j) = (\frac{\varepsilon}{\varepsilon-1})MC_t(j)$, where $MC_t(j) = \frac{W_t}{\frac{\partial Y_t(j)}{\partial N_t(j)}}$ is the nominal marginal cost and $\frac{\varepsilon}{\varepsilon-1}$ is the steady-state price markup.

This framework implies that inflation is a purely forward-looking variable. Nevertheless, much recent research has pointed out the importance of allowing for a hybrid specification in which part of the inflation dynamics is explained by some backward-looking component in order to account for the inertia observed in inflation time series. Thus, following Christiano, Eichenbaum and Evans (2005), we allow for some degree of indexation. Those firms that do not set the optimal price at time t will adjust prices to lagged inflation: $P_{t+i}(j) = P_{t+i-1}(j)\left(\frac{P_{t+i-1}}{P_{t+i-2}}\right)^{\kappa} = P_{t+i-1}(j)\,(\pi_{t+i-1})^{\kappa}$, where κ is a parameter that indicates the degree of non-optimizers' price adjustment whose extreme values imply no indexation ($\kappa = 0$) or full indexation ($\kappa = 1$). The aggregate price level evolves as follows:

$$P_t = \left[\theta\left(P_{t\text{-}1}\left(\pi_{t\text{-}1}\right)^{\kappa}\right)^{(1\text{-}\varepsilon)} + (1\text{-}\theta)\left(P_t^f\right)^{(1\text{-}\varepsilon)}\right]^{\frac{1}{(1\text{-}\varepsilon)}} \tag{6}$$

2.3 Central Bank Reaction Function

We assume that the central bank sets the nominal interest rate following a general augmented Taylor-type interest rate rule. In particular, the nominal rate responds not only to the interest rate in the previous period and to deviations of output and inflation from their steady-state values, but also to nominal money growth:

$$\ln(r_t/r) = \rho_r \ln(r_{t-1}/r) + (1\text{-}\rho_r)\rho_\pi \ln(\pi_t/\pi) + (1-\rho_r)\rho_y \ln(y_t/y) + (1-\rho_r)\rho_\mu \ln(\mu_t/\mu) + \varepsilon_{r_t}$$

where the innovation ε_{r_t} is normally distributed with standard deviation σ_r; and $\mu_t = M_t/M_{t-1}$ is the rate of money growth. An interest-rate rule that depends on money growth (or the changes in real balances) might be rationalized, as in Svensson (1999), as part of an optimal reaction function when money-growth variability appears in the central bank's loss function. Alternatively, the response to money might be rationalized by money's usefulness in forecasting inflation.

2.4 Equilibrium

The symmetric equilibrium can be log-linearized to yield the following set of equations:[6]

$$\begin{aligned}\widehat{y}_t = & \frac{\phi_1}{\phi_1+\phi_2}\widehat{y}_{t-1} + \frac{\beta\phi_1+\phi_2}{\phi_1+\phi_2}E_t\widehat{y}_{t+1} - \frac{1}{\phi_1+\phi_2}\left[\widehat{r}_t - E_t\widehat{\pi}_{t+1}\right] - \frac{\beta\phi_1}{\phi_1+\phi_2}E_t\widehat{y}_{t+2} \\ & + \frac{\psi_2}{\psi_1}\frac{1}{(1\text{-}\beta h)}\left(\frac{1}{\phi_1+\phi_2}\right)\widehat{m}_t - \frac{\psi_2}{\psi_1}\frac{1}{(1\text{-}\beta h)}\left(\frac{1+\beta h}{\phi_1+\phi_2}\right)E_t\widehat{m}_{t+1} \\ & + \frac{\psi_2}{\psi_1}\frac{1}{(1\text{-}\beta h)}\left(\frac{\beta h}{\phi_1+\phi_2}\right)E_t\widehat{m}_{t+2} - \frac{\psi_2}{\psi_1}\left(\frac{1\text{-}\beta h\rho_e}{1\text{-}\beta h}\right)\left(\frac{1\text{-}\rho_e}{\phi_1+\phi_2}\right)\widehat{e}_t \\ & + \left(\frac{1\text{-}\beta h\rho_a}{1\text{-}\beta h}\right)\left(\frac{1\text{-}\rho_a}{\phi_1+\phi_2}\right)\widehat{a}_t \end{aligned} \tag{7}$$

$$\widehat{\pi}_t - \kappa\widehat{\pi}_{t\text{-}1} = \beta(E_t\{\widehat{\pi}_{t+1}\} - \kappa\widehat{\pi}_t) + \lambda\widehat{mc}_t \tag{8}$$

$$\begin{aligned}\widehat{mc}_t = & (\chi+\phi_2)\,\widehat{y}_t - \phi_1\widehat{y}_{t\text{-}1} - \beta\phi_1 E_t\widehat{y}_{t+1} - \frac{\psi_2}{\psi_1}\frac{1}{(1\text{-}\beta h)}\widehat{m}_t + \frac{\psi_2}{\psi_1}\frac{\beta h}{(1\text{-}\beta h)}E_t\widehat{m}_{t+1} \\ & + \frac{\psi_2}{\psi_1}\frac{(1\text{-}\beta h\rho_e)}{(1\text{-}\beta h)}\widehat{e}_t - \frac{\beta h(1\text{-}\rho_a)}{(1\text{-}\beta h)}\widehat{a}_t - (1+\chi)\,\widehat{z}_t \end{aligned} \tag{9}$$

$$\widehat{r}_t = \rho_r\widehat{r}_{t\text{-}1} + (1\text{-}\rho_r)\rho_y\widehat{y}_t + (1\text{-}\rho_r)\rho_\pi\widehat{\pi}_t + (1\text{-}\rho_r)\rho_\mu\,\widehat{\mu}_t + \varepsilon_{r_t} \tag{10}$$

$$\widehat{\mu}_t = \widehat{m}_t - \widehat{m}_{t\text{-}1} + \widehat{\pi}_t \tag{11}$$

[6] The symbol $\widehat{}$ denotes percentage deviations of a variable from its steady-state value.

$$\widehat{a}_t = \rho_a \widehat{a}_{t\text{-}1} + \varepsilon_{a_t} \tag{12}$$

$$\widehat{e}_t = \rho_e \widehat{e}_{t\text{-}1} + \varepsilon_{e_t} \tag{13}$$

$$\widehat{z}_t = \rho_z \widehat{z}_{t\text{-}1} + \varepsilon_{z_t} \tag{14}$$

where $\widehat{m}_t, \widehat{mc}_t$ represent (log-deviations of) real balances and real marginal costs, respectively; and the following relationships hold between structural parameters, the steady-state (upper-barred variables), and the composite parameters of equations (7)-(9),

$$\psi_1 = \left(\frac{-\Psi_1}{(\overline{Y})^{(1-h)}\Psi_{11}} \right) \qquad \phi_1 = \frac{(\psi_1^{-1}-1)h}{1-\beta h}$$

$$\psi_2 = \left(\frac{-\Psi_{12}}{(\overline{Y})^{(1-h)}\Psi_{11}} \right) \left(\frac{\overline{m}}{\overline{e}} \right) \qquad \phi_2 = \frac{\psi_1^{-1} + (\psi_1^{-1}\text{-}1)\beta h^2\text{-}\beta h}{1\text{-}\beta h}$$

$$\lambda = (1\text{-}\theta)(1\text{-}\beta\theta)(1\text{-}\omega)\xi \qquad \chi = \frac{\varphi+\alpha}{1\text{-}\alpha}$$

$$\xi = \frac{(1\text{-}\alpha)}{1+\alpha(\varepsilon\text{-}1)} \left[\theta + \omega(1\text{-}\theta(1\text{-}\beta)) \right]^{-1}$$

Equation (7) arises from the household's optimal intertemporal allocation of wealth. The case of non-separability across consumption and real balances makes the marginal utility of consumption a function of the amount of real balances optimally demanded by the households. The presence of habits makes the marginal utility of consumption also dependent on lags of output and further leads of money and output. Hence, in equilibrium, output will depend on current and expected real balances after accounting for the money demand shock. Notice that as $h \to 0$, expression (7) approaches the usual Euler equation for consumption under time-separable preferences. The real-balances term will disappear from the aggregate demand equation under the parameter restriction $\psi_2 = 0$, i.e. as long as the cross-derivative between consumption and real balances is zero in the utility function. As we discuss in the next section, however, a strong indicator role for money, not captured by standard money demand specifications, may prevail even if the restriction $\psi_2 = 0$ holds.

Aggregate demand also depends upon the present discounted value of current and future real short-term interest rates; so the sensitivity of output to interest-rate movements depends upon the coefficient ψ_1, which is inversely related to the households' degree of risk aversion.

The supply side of the model is characterized by two equations: first, a New Keynesian Phillips curve, (8), which allows for both expected and past inflation terms as well as real marginal costs to affect current inflation; and second, a relationship between real marginal costs, detrended output, real balances, and the technology shock, equation (9). Notice that, if we assume that all new prices (p_t^*) are set on a profit-maximising basis, i.e. $\omega = 0$, then inflation becomes a purely forward-looking variable. Moreover, the assumption of decreasing returns to labor implies that the link between output and inflation depends not only on the degree of nominal rigidities, but also the elasticity of output with respect to employment $(1-\alpha)$, and the labor supply elasticity (φ) through the coefficient χ. The non-separability in preferences across real balances and consumption implies a direct influence of the former variable on marginal costs and so on inflation. In the presence of habits, marginal cost also depends on leads and lags of output, money balances and the preference shock a_t. To close the model, we specify AR(1) processes for the aggregate demand shock (12), the money demand shock (13) and the technology shock (14), with innovations ε_{a_t}, ε_{e_t} and ε_{z_t} respectively, as well as a money demand equation, which we now discuss.

2.5 Money Demand

The model is completed with a specification of money demand behavior. The specification of portfolio adjustment costs determines the form of the money demand relationship. The model without adjustment costs implies that the money demand equation is as follows:

$$\begin{aligned}\widehat{m}_t = {} & \gamma_1 \widehat{y}_t - \gamma_2 \widehat{r}_t + \left[\gamma_2 (r-1)(h\phi_2 - \phi_1) - h\gamma_1\right] \widehat{y}_{t\text{-}1} \\ & - \left[\gamma_2 (r\text{-}1)\beta\phi_1\right] E_t \widehat{y}_{t+1} + \frac{\psi_2}{\psi_1} \frac{(r\text{-}1)\beta h \gamma_2}{(1\text{-}\beta h)} E_t \widehat{m}_{t+1} \\ & - \frac{(r\text{-}1)\beta h (1\text{-}\rho_a)}{(1\text{-}\beta h)} \gamma_2 \widehat{a}_t + \left[1 - (r-1)\gamma_2 \left(\frac{\psi_2}{\psi_1} \frac{\beta h \rho_e}{(1-\beta h)} + 1\right)\right] \widehat{e}_t \qquad (15)\end{aligned}$$

where $\gamma_1 = \left(r \frac{(\overline{Y})^{(1-h)}}{\overline{m}} \frac{\psi_2}{\psi_1} + (r-1)\frac{1}{\psi_1}\right) \gamma_2$ and $\gamma_2 = \frac{r}{(r-1)} \frac{\overline{e}}{\overline{m}} \left(\frac{\Psi_2}{(r-1)\overline{e}\Psi_{12} - r\Psi_{22}}\right)$.

Expressions (15), (10) and (11) describe the money market. Equation (15) is a generalized money demand equation, where the coefficients γ_1 and γ_2 are the long-run real-income and interest-rate response parameters. Again the presence of habits in the utility function generates a dynamic equation in which money demand depends also on future output and real balances as well as on the preference shock a_t. Equation (11) is an identity connecting nominal money growth, real balances, and inflation.

As noted above, allowing for non-separability across real balances and consumption gives real balances an explicit role in both the output and inflation equilibrium relationships. A reduced-form equation that has been proposed in the literature to look at the inflation-forecasting properties of monetary aggregates is the P^* model.[7] Finally, note that equation (15) can be solve forward in terms of m_t as a function of the present discounted value of future nominal interest rates (see also the next section). This underlies the so-called "direct effect," whereby money variations reflect determinants of aggregate demand other than the current short-term interest rate. To establish the role of money, we must separately identify such an effect from the "real balance effect" or, more precisely, "non-separability effect" related to the cross-derivative of the marginal utility of consumption and real balances. In order to do that, we need to consider a specification with portfolio adjustment costs.

To that end, if we consider the specification of preferences given by equations (2) and (3) we obtain an alternative money demand equation which allows us to identify both effects separately:

$$\begin{aligned}(1+\delta_0(1+\beta))\widehat{m}_t &= \gamma_1\widehat{y}_t\text{-}\gamma_2\widehat{r}_t+\left[\gamma_2(r\text{-}1)(h\phi_2\text{-}\phi_1)\text{-}h\gamma_1\right]\widehat{y}_{t-1}\text{-}\left[\gamma_2(r\text{-}1)\beta\phi_1\right]E_t\widehat{y}_{t+1} \\ &\quad +\delta_0\widehat{m}_{t\text{-}1}+\left[\frac{\psi_2}{\psi_1}\frac{(r\text{-}1)\beta h\gamma_2}{(1\text{-}\beta h)}+\delta_0\beta\right]E_t\widehat{m}_{t+1}\text{-}\frac{(r\text{-}1)\beta h(1\text{-}\rho_a)}{(1\text{-}\beta h)}\gamma_2\widehat{a}_t \\ &\quad +\left[1-(r-1)\gamma_2\left(\frac{\psi_2}{\psi_1}\frac{\beta h\rho_e}{(1\text{-}\beta h)}+1\right)\right]\widehat{e}_t \qquad (16)\end{aligned}$$

where $\delta_0 = \frac{dc^2}{\overline{m}}$. The two channels are captured trough the coefficients on past and expected future real balances. In particular, under no portfolio adjustment costs, i.e. $d \to 0$, then $\delta_0 \to 0$, the behavior of current real money balances does not depend on lagged real balances. In addition, even if there is no non-separability effect, i.e. $\psi_2 \to 0$, expected future real balances still matter for current values of that variable. Finally, note that it is not possible to separately identify the parameters d and c. We therefore normalize $c = 1$, allowing us to estimate the coefficient on adjustment cost d.

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[7] Svensson (2000) argues that the P^* model provides some basis for emphasizing the real balances gap (i.e. the difference between the current level of real balances and its long-run equilibrium level). The present setup provides a sound microfoundation for the presence of a sort of real balances gap, $\widehat{m}_t - \widehat{e}_t$, in inflation dynamics. Notwithstanding this, this model imposes cross-parameter restrictions that should be tested in order to assess the empirical relevance of this term; and in contrast to the P^* approach, the role of money specified here is integrated into a standard Phillips curve framework, where inflation depends on real marginal cost.

3 Money and the Natural Rate of Interest

In Wicksell's (1898) original outline of the link between price-level behavior and the spread between real and natural interest rates, he emphasized the connection of money creation with this spread. That is, to keep actual rates steady in the face of a real shock that raises the natural rate, the monetary authority must create additional money. In standard New Keynesian models, this connection is present, but because real money demand is a static function of current output and the policy instrument (the short-term nominal interest rate), all information about the natural rate contained in real money balances comes via the coefficients on these two variables. The remaining variation in real balances simply reflects money demand shocks that devalue the usefulness of money as an indicator.

When real money demand is forward-looking, however, the information in real balances about the natural rate is increased. If real money is registering weakness or strength that is hard to account for in the behavior of current income and the short-term nominal interest rate, that may be a signal of changes in current or expected future values of the natural real interest rate. We explore this property in our estimated model, but to provide intuition, in this section we briefly consider a version of the model with white noise IS and money demand shocks, and portfolio adjustment costs like those in equation (2), but no other source of nonseparability in utility. Then the money demand condition (16) may be written:

$$\widehat{m}_t = \mu_y \widehat{y}_t + \mu_r \widehat{r}_t + \mu_1 \widehat{m}_{t-1} + \beta\mu_1 \ E_t \widehat{m}_{t+1} + e'_t \tag{17}$$

where $\mu_1 \equiv \frac{\delta_0}{(1+\delta_0(1+\beta))}$, $\mu_y \equiv \frac{\gamma_1}{(1+\delta_0(1+\beta))}$, $\mu_r \equiv -\frac{\gamma_2}{(1+\delta_0(1+\beta))}$, and $e'_t = \frac{1}{(1+\delta_0(1+\beta))} e_t$. Notice that the long-run income elasticity and interest-rate semi-elasticity of money demand correspond to γ_1 and γ_2, respectively. The IS equation becomes:

$$\widehat{y}_t = E_t \widehat{y}_{t+1} - \sigma \widehat{rr}_t + \widehat{\nu}_t \tag{18}$$

where $\widehat{rr}_t = [\widehat{r}_t - E_t \widehat{\pi}_{t+1}]$, $\widehat{\nu}_t = (1 - \rho_a)\sigma \widehat{a}_t$ and $\sigma \equiv \frac{1}{\phi_2}$.

For future reference, we note also that equation (18) implies:

$$\widehat{y}^*_t = -\sigma \sum_{i=0}^{\infty} \widehat{rr}^*_{t+i} + \sigma \widehat{a}_t \tag{19}$$

and

$$\widehat{y}_t - \widehat{y}_t^{*} = -\sigma \sum_{i=0}^{\infty} (\widehat{rr}_{t+i} - \widehat{rr}^{*}_{t+i}) - \rho_a \sigma \widehat{a}_t \tag{20}$$

where $\widehat{rr}^{*}_t$ and $\widehat{y}^{*}_t$ are the natural levels of the short-term real interest rate and output, respectively.

3.1 Forward-Looking Money Demand Equation

Solving condition (17) using the methods described in Sargent (1987), we obtain:

$$(1\text{-}\psi L)\widehat{m}_t = \left(\frac{\psi}{\mu_1}\right) \sum_{i=0}^{\infty} (\beta\psi)^i E_t\{\mu_y \widehat{y}_{t+i} + \mu_r \widehat{r}_{t+i}\} + e'_{t+i} \tag{21}$$

where L is the lag operator, ψ is a stable root ($0 < \psi < 1$), and μ_1 is a function of δ_0, i.e a function of c in equation (3). Representation (21) establishes that real money demand is a function of its lagged value and the expected stream of output and nominal interest rates, as well as the white-noise money demand shock e'_t.

We now write this expression in a manner that separates the forward-looking terms from the current and lagged variables:

$$\widehat{m}_t = \psi \widehat{m}_{t-1} + b_0 \widehat{y}_t + c_0 \widehat{r}_t + \sum_{i=1}^{\infty} b_i E_t \widehat{y}_{t+i} + \sum_{i=1}^{\infty} c_i E_t \widehat{r}_{t+i} + e'_t \tag{22}$$

where $b_0 = \psi \frac{\gamma_1}{\delta_0}$, $c_0 = -\psi \frac{\gamma_2}{\delta_0}$, and b_i and c_i coefficients are defined in conformity with equation (21). Condition (22) can, in turn, be decomposed using equations (19), (20), and the Fisher relation $\widehat{rr}_t = [\widehat{r}_t\text{-}E_t \widehat{\pi}_{t+1}]$ as:

$$\begin{aligned} \widehat{m}_t \quad = \quad & \psi \widehat{m}_{t-1} + b_0 \widehat{y}_t + c_0 \widehat{r}_t + \sum_{i=1}^{\infty} b_i E_t \widehat{y}^{*}_{t+i} + \sum_{i=1}^{\infty} c_i E_t \widehat{rr}^{*}_{t+i} + \\ & \sum_{i=1}^{\infty} b_i E_t \{\widehat{y}_{t+i}\text{-}\widehat{y}^{*}_{t+i}\} + \sum_{i=1}^{\infty} c_i E_t \{\widehat{rr}_{t+i}\text{-}\widehat{rr}^{*}_{t+i}\} + \sum_{i=1}^{\infty} c_i E_t \widehat{\pi}_{t+1+i} + e'_t \end{aligned} \tag{23}$$

The above expression casts the forward-looking variables in terms of natural output levels, natural real interest rates, output gaps, real interest-rate gaps, and expected future inflation rates.

Further restrictions on this condition can be obtained by an explicit specification of price-setting behavior. We demonstrate here with two examples: one-period price setting and Calvo price setting.

3.2 Example 1: One-period-ahead price setting

Consider first the simple specification of price adjustment, used by Obstfeld and Rogoff (1996) and many others, where nominal prices must be set one period in advance but are then free to adjust. This specification implies that real variables are always expected to revert to their flexible-price (natural) values from next period onward: for $i > 0$, $E_t\widehat{y}_{t+i} = E_t\widehat{y}^*_{t+i}$ and $E_t\widehat{rr}_{t+i} = E_t\widehat{rr}^*_{t+i}$. The money demand expression may then be written as:

$$\widehat{m}_t=\psi\widehat{m}_{t\text{-}1}+b_0\widehat{y}_t+c_0\widehat{r}_t+\sum_{i=1}^{\infty}d_iE_t\widehat{rr}^*_{t+i}+\sum_{i=1}^{\infty}c_iE_t\widehat{\pi}_{t+1+i}+e'_t \tag{24}$$

where $d_i=c_i-\sigma(\sum_{i=1}^{\infty}b_i)$. Money demand thus contains valuable information beyond that recorded by its responses to current income and the current nominal rate: it varies in reaction to movements in expected future natural real rates, as well as expected future inflation.

3.3 Example 2: Calvo price setting

The basic version of Calvo price setting implies:

$$\widehat{\pi}_t=\beta E_t\widehat{\pi}_{t+1}+\alpha(\widehat{y}_t-\widehat{y}^*_t) \tag{25}$$

Solving this forward and substituting in equation (20), we have:

$$\widehat{\pi}_t=-\sigma\alpha\sum_{i=0}^{\infty}\beta^iE_t\{\sum_{j=0}^{\infty}(\widehat{rr}_{t+j}-\widehat{rr}^*_{t+j})\} \tag{26}$$

or

$$\widehat{\pi}_t=\text{-}\sigma\alpha\sum_{i=0}^{\infty}\phi_i(\widehat{rr}_{t+i}-\widehat{rr}^*_{t+i}) \tag{27}$$

where $\phi_i=-\sigma\alpha\sum_{j=0}^{i}\beta^j$. This expression implies that the money demand condition (23) may be written:

$$\widehat{m}_t=\psi\widehat{m}_{t\text{-}1}+b_0\widehat{y}_t+c_0\widehat{r}_t+\sum_{i=1}^{\infty}d_iE_t\widehat{rr}^*_{t+i}+\sum_{i=1}^{\infty}f_iE_t\{\widehat{rr}_{t+i}-\widehat{rr}^*_{t+i}\}+e'_t \tag{28}$$

where d_i is defined as above, and $f_i=-\sigma[\sum_{j=1}^{\infty}b_j]+c_i$, for $i=1$, and $f_i = -\sigma[\sum_{j=i}^{\infty}b_j]+c_i - c_{i-1}\alpha\sigma\sum_{j=0}^{i-2}\beta^j$ for $i>1$. Equation (28) reveals that *all* of the variation in real balances not arising from its "conventional" determinants (i.e. current real income, the current short interest rate, lagged balances and the money demand shock) is associated with movements in expected future real-rate gaps or expected future natural real interest rates.[8] We note that the relationship between real money balances and the natural rate is quite complex, not only because of the dynamics involved, but also because the natural rate enters with both negative and positive coefficients in the expression.

This perspective on the money demand relationship highlights three advantages of our estimation of our structural model by full-information methods. First, standard estimated money demand functions neglect forward-looking behavior. The resulting specification error overlooks the information about the natural rate in money demand, instead attributing the associated variation in real balances to money demand shocks, lagged adjustment, and responses to current income and the nominal interest rate. Our approach instead isolates the forward-looking component of money demand, and so offers the prospect of consistent estimation of the money demand parameters. Second, by specifying the shock processes and policy behavior explicitly, and so the implied path of the expectations terms that appear in agents' optimality conditions, we are able to extract natural-rate estimates from the other unobservable determinants of money demand. Third, other empirical estimates of natural rate and real-rate gap series using systems methods, whether with *ad hoc* models (e.g. Laubach and Williams, 2003) or DSGE models (e.g. Smets and Wouters, 2003), sacrifice information on the natural rate by not including real money balances in the set of variables modelled. Our systems estimates, by contrast, include money in the likelihood, and so exploit the valuable information in money suggested by equation (28).

4 Empirical Evidence

The maximum likelihood estimation follows Hansen and Sargent (1997) and recent applications can be found in Kim (2000) and Ireland (2003, 2004). The procedure involves expressing the stationary solution of the model state-space form and estimating the model's parameters using a recursive Kalman filter

[8] When we generalize (28) for the case of serially correlated IS shocks and habit formation in preferences, additional lagged variables and the current IS shock appear, but expected future values of the natural rate continue to appear prominently.

algorithm (see Ireland, 2003, for details).

4.1 Baseline Estimates

This section presents our parameter estimates for each economy. In line with the model, our estimated specifications exclude open-economy terms. For the U.S. and the euro area this abstraction is commonly made, reflecting the size of these economies. We also estimate the closed-economy specification for the U.K. This has the advantage of simplicity and preserves a symmetric specification across economies. On the aggregate demand side, the influence of open-economy elements can be thought of as entering implicitly by flattening the IS curve and so affecting the estimated interest elasticity of aggregate demand. On the supply side, since the price series we use in estimation is a consumer price index, exchange-rate or import-price terms should in principle appear in the Phillips curve. Both reduced-form and structural empirical evidence suggest, however, that excluding these terms does little damage in the modeling of U.K. consumer prices (Artis, 1993; Neiss and Nelson, 2003; U.K. Treasury, 2003, pp. 18–19). This is not to deny the importance of openness for inflation and output dynamics, but instead to argue that openness mainly matters through its effect on the behavior of domestic variables, rather than by introducing extra variables into the analysis.

The log-linearized optimizing model that we estimate refers to deviations of variables from their steady-state values (or steady-state growth paths in the case of output and real money), rather than the levels of variables. For each economy studied, following Ireland (2003, 2004), we detrended output and real balances separately prior to estimation. Inflation and nominal interest rates also exhibit a (downward) trend over our sample; nevertheless, we continue to use the (demeaned) levels of these variables in estimation, on the grounds that the trends may be reduced or eliminated when these variables are cast as linear combinations (e.g. as a real interest rate).

In Tables 2 to 4 we present the results of the parameter estimates for the unrestricted (non-separable preferences) models of Section 2 for the U.K., U.S. and euro area ((7)-(14), and (16)). For comparability, we use narrow monetary aggregates in all three cases: M0 for the U.S. and the U.K. and M1 for the euro area. The sample periods used are also similar: 1979:3 to 2003:3 for the U.S., 1980:1 to 2004:4 for the euro area, and 1979:3 to 2004:4

for the U.K.[9]

The main result concerns the effect of money on output and inflation that may be captured by either ψ_2 and/or by δ_0. The null of $\psi_2 = 0$ cannot be rejected in all three countries. This implies separability of utility across consumption and real balances, so that a role for money does not arise from explicit terms involving money in either the IS equation or the Phillips curve. This result is consistent with those obtained by Ireland (2004) and Andrés, López-Salido and Vallés (2003). Money, however, seems to have a different kind of "direct effect," i.e. an important forward-looking element, as the strongly significant value of δ_0 obtained in all specifications indicates. Adjustment costs are thus important for the dynamics of real balances, in line with what Nelson (2002) finds for the U.K. and the U.S. As was discussed above, this forward-looking element of money demand confers on money considerable importance as an indicator of the determinants of aggregate demand.

Regarding the other parameters of interest, we find strong evidence of habit formation in all three economy. The value of h ranges from 0.90 to 0.975.[10] The reported h values for the U.S. and U.K. are fixed values; we obtained similar estimates in unrestricted estimation, but encountered convergence problems with allowing h to be estimated freely. The interest-rate elasticity of the IS function (ψ_1) is significantly positive, suggesting an intertemporal elasticity of substitution slightly below (but not significantly different from) one for the euro area and somewhat lower for the U.S. and the U.K.

Besides its appearance in the private sector behavioral equations, money is also related to the behaviour of output and prices through the money demand and the policy-rule equations. The interest-rate semi-elasticity of money demand is large and significant in the euro area model, and is fixed at a similar value for the U.S. after unrestricted estimation suggested values of that order. We obtain low estimated income elasticities of money demand. This may reflect the fact that we use detrended data, and therefore sacrifice

[9] Our data for the U.S. consist of series downloaded from the Federal Reserve Bank of St. Louis FRED database as well as an updated version of the Anderson-Rasche (2000) money base series, adjusted for temporary increases in the base during the millennium and September 2001. The U.K. data are from national sources, with the price index used being the RPIX, adjusted for tax-induced spikes in 1979:3 and 1990:2.

[10] Giannoni and Woodford (2005) also obtain values of h above 0.9 for the U.S., for a sample period similar to ours.

information from the levels of the data. In addition, Lucas (1988) argues that more plausible money demand estimates arise from fixing the income elasticity at unity. To explore the implications of this restriction for the hypotheses of interest to us, we have reestimated the model imposing a unit income elasticity (i.e., $\gamma_1 = 1$). These appear in column 2 in Tables 2 to 4. Although the rest of the parameters change somewhat, imposing this value does not affect the results regarding the role of money in the model: namely, the non-separabality in preferences can still be safely rejected; whereas the dynamic (forward-looking) component of money demand remains highly significant.

For the U.K., the estimated interest semi-elasticity of money demand is zero regardless of whether we impose an income elasticity of unity. This finding is not for lack of a strong reduced-form correlation between the U.K. nominal rate and U.K. real balances; on the contrary, in our sample the correlation between the nominal rate and the change in (detrended) real balances is -0.75. Rather, the system estimates prefer to attribute the bulk of the variation in real balances to the money demand shocks, absorbing much of the explanatory power usually accounted for by interest rates. In fact, the degree to which the exogenous shocks absorb variation in the data at the expense of endogenous dynamics is a generic problem with the U.K. estimates. It is likely that our assumptions (necessary for the systems estimation) of a constant-parameter U.K. monetary policy rule and a constant steady-state real interest rate are straining against the reality of major U.K. policy regime changes during the sample period. Notwithstanding these problems with estimating preference parameters, the key portfolio adjustment δ_0 parameter is significant in our U.K. estimates.

The estimates for the supply side of the economy reveal the importance of the forward-looking component of inflation and the low degree of indexation: κ is zero in the euro area. We found that this was also the case in the U.S., while κ is low and barely significant in the U.K. model. This general pattern for the three countries is one that it is not consistent with the estimated Phillips curves obtained by other methods (Fuhrer, 1997, Galí, Gertler, and López-Salido, 2001) in which an strong role for lagged inflation in the Phillips curve can be found. Nevertheless, our results are in line with recent microeconomic evidence (see e.g. the findings of the Inflation Persistence Network discussed in Angeloni *et al*, 2004). One way to rationalize the apparent lack of indexation is that our model implies a strong autoregressive pattern for the stochastic term in the Phillips curve (i.e., $\frac{\psi_2}{\psi_1}\frac{(1\text{-}\beta h\rho_e)}{(1\text{-}\beta h)}\widehat{e}_t - \frac{\beta h(1\text{-}\rho_a)}{(1\text{-}\beta h)}\widehat{a}_t - (1{+}\chi)\,\widehat{z}_t$). Additionally, the presence of habits in preferences changes the dynamic pattern of the marginal cost variable, which now depends on leads and lags of output.

An interesting difference between the three countries arises in the estimated slope parameter for the Phillips curve, λ. This parameter is strongly significant in all cases and the point estimates indicate that nominal inertia is stronger for the U.S. ($\lambda = 0.09$) than for the euro area (0.6) and the U.K. (0.4). In addition, from those estimates it is possible to back out the value of the elasticity of labor supply (i.e., from the parameter χ). The implied elasticities are similar for the euro area and the U.K., and somewhat lower for the U.S.

The estimated interest-rate rules also display many similarities across economies. There is significant interest-rate smoothing of similar magnitude (around 0.8), and the interest-rate response to output (ρ_y) is modest but significant in the U.S. and the euro area (around 0.15), while is zero in the U.K., possibly reflecting the problems with imposing a constant-parameter U.K. policy rule, noted above. The response of the nominal rate to the inflation rate is well above 1.0. Finally, money growth is present in all three policy rules, with a higher value in the U.K. This term may be approximating either genuine money targeting by the central bank during the sample, or a way of targeting future inflation, by responding to information beyond that contained in current π_t.

Summing up, all the models have reasonable point estimates for most structural parameters. Our estimates of money demand elasticities are less satisfactory, possibly reflecting the use of detrended data in estimation. Moreover, the main hypothesis of interest are not affected by the imposition of more conventional values for the money demand elasticities. The estimated values of the intertemporal elasticity of substitution in private spending appear reasonable. All three economies exhibit strong habit formation in preferences, while labor supply is highly elastic. The Phillips curve estimates suggest a very low degree of backward indexation, while both the U.K. and the euro area display less nominal stickiness (higher implied probabilities of price adjustment) than the U.S. The estimated policy rules indicate strong long-run responses to inflation and a high degree of interest-rate smoothing. The money demand shock and both real shocks display strong inertia.

5 Dynamics of Money and the Natural Rate

In this section we will examine the dynamics of money and the natural rate of interest in our estimated models. We consider each of the two shocks that drive the natural interest rate, i.e. the IS (preference) shock and the technology shock. As stressed in Section 3, when money demand is forward-looking, some variation in real balances, given current income and the nominal interest rate, will reflect portfolio responses to those real shocks (either aggregate demand or technology) and so, implicitly, variations in the natural rate of interest.[11] In this section we investigate such a relationship between the natural rate and the real money stock by examining key moments and impulse responses of the model. We thus aim to illustrate how The value of money is increased in our estimated models, relative to the New Keynesian baseline, by the specification of money demand dynamics for which we have found empirical support.

From equation (28) it can be seen that real money fluctuations are correlated with the natural rate, given the other determinants of money demand. Two factors therefore drive the response of the real balances to any real shock: first, the response of the natural rate to the shock, and second the policy response to the shock, as recorded in how actual rates in the next few quarters change relative to their natural value. In the next subsections we analyze how these two terms behave in response to each of the real shocks considered in our analysis, i.e. an IS shock and a technology shock.

5.1 IS Shocks

In standard sticky-price models, one can conjecture that the reduced-form relationship between real balances and the IS shock is negative. This is based on the presumption that in response to a positive IS shock, both the natural rate and potential output shift up. If policymakers then partially

[11] The natural real rate of interest corresponds to the short-term real interest rate that would prevail when the Calvo probability approaches 1.0, i.e. when all prices are flexible (and all firms are forward-looking). The natural-rate process will be invariant to the monetary policy rule, but will be a (possibly dynamic) function of the two real shocks in the model. In the present application, obtaining a natural-rate series entails evaluating our model with parameters describing preferences and production at their estimated values, solving the model under flexible prices, and obtaining a Wold-style representation of the natural rate. The natural-rate estimates are then generated by a finite-order approximation of the Wold representation (see Neiss and Nelson, 2003, for details).

accommodate the shock, there will be increases in output, the nominal rate, and expected inflation, and the emergence of a negative real rate gap (i.e., actual rates below their natural levels). Under that scenario, the negative real balances-natural rate relation emerges. Money demand fundamentally depends upon the expected path of nominal rates, and provided expected future values of the natural rate move in the same direction as the nominal rate in response to the IS shock, a negative relationship between real balances and the natural rate will emerge in the data.

Figures 1(a), 2(a), and 3(a) report the responses to an IS shock in each economy (considering the case of a unitary income elasticity of money demand). The conjecture above is only confirmed by the impulse response of real balances and the natural rate in the U.S. and euro area economies. There an IS shock does drive up the natural rate, while real balances move down, so exhibiting an inverse relationship with the natural rate. The negative relationship is amplified by monetary policy, which raises the nominal interest rate in response to the (temporary) increase in real GDP.

But in the U.K. economy (Figure 3(a)), the natural rate actually *falls* slightly on impact in the wake of an IS shock, though it rises in subsequent periods. Why does the natural rate fall in the U.K.? This reaction reflects the high degree of habit formation in the model. Figure 4 plots the impact effect of the IS shock on the natural rate in our models as a function of the degree of habit formation (i.e., the parameter h), with all the other parameters of the model held at their estimated values. As can be seen, when the habits become very powerful (corresponding to h well above 0.90), households become so stubborn about maintaining their consumption at its previous level that they need to be induced by lower real interest rates to consume a higher quantity of output. [12]

Nevertheless an inverse relationship between the natural rate and real balances continues even in the U.K. case in the initial period. This, however, reflects a different channel from in the other two economies. In the U.K., our estimate of the interest sensitivity of money demand is zero, so IS shocks unambiguously raise real balances—only the expected real income stream matters for money demand. In the other two economies, interest rates matter for money demand, and real balances contract in anticipation of a sustained rise in the nominal rate. This difference also implies that the

[12] Notice that potential output always rises in response to the IS shock, regardless of the degree of habit formation. See also Amato and Laubach (2004) on this point.

negative relationship between the natural rate and real balances in the U.K. does not go beyond the initial period: the natural rate subsequently rises (consumers become accustomed to higher income, so habits no longer exert negative pressure on the natural rate), as does the nominal rate (a policy tightening in response to higher inflation), but real balances continue to adjust upwards in response to continued expectations of high output.

5.2 Technology Shocks

In Figures 5 to 7, we plot the response of key model variables to the technology shock for each of our three economies. In response to the shock, actual and potential output rise, and the natural rate of interest falls. The reduction in the natural rate is quantitatively similar in the U.S. and the U.K., but less pronounced in the euro zone. The natural rate will decline if the constraint on consumption implied by the level of potential output is relaxed more today than in the future. Here, given the increases in output, the entire path of consumption can be higher than previously, but because the productivity shock wears off over time, potential output is raised more in the immediate few quarters than in the later quarters, so the natural rate declines.

Real balances exhibit an inverse relationship with the natural rate, but tend to exhibit their peak response well after the natural rate has started returning to its natural level. For the U.S. and the euro area, this reflects differences between the response of the nominal rate and the natural rate of interest to the technology shock. The shock initially raises potential output relative to output because nominal rates respond positively to output, restraining the extent to which real aggregate demand can expand with the increased potential. This produces a reduction in inflation, which leads, via smoothing, to a protracted fall in nominal rates, and so a protracted rise in real balances. For the U.K., with an interest-rate dependence of real balances absent from the estimates, this pattern is absent so the peak in real balances comes sooner.

Figures 1 to 7 emphasize that the forward-looking character of the money demand enriches the relationship between real balances and the natural rate. To examine this further, we have computed some second moment statistics. Table 5 gives partial correlations between the real money stock and the natural interest rate for each of our estimated models. The partial correlations are the correlations between the two series holding constant the two determinants of money demand—current output and the nominal interest rate—that appear in the standard model without any forward-looking components of money demand.[13] The correlations are an outcome of the interaction of model structure, policy rule, and shock processes in the model. The natural rate depends on the IS and technology shocks, while the real money stock depends on these shocks plus two shocks that do not matter for the natural rate, i.e. the monetary policy and money demand shocks. Despite the noise created in the real money series, it nevertheless has a negative correlation with the natural rate in all three estimated models. It is important to emphasize that, in the absence of habit formation, these partial correlations *are identically zero* when money demand is described by the New Keynesian benchmark of Table 1, which has no forward-looking money demand (either from habit formation or from adjustment costs).[14] Non-zero correlations will reflect the increased value of money as an indicator of real shocks, imparted by the combination of portfolio adjustment costs and habit formation. These two features create forward-looking money demand dynamics that make the partial correlation negative for the three economies.

As the above discussion indicates, the impulse responses and model correlations are inherently a function of both the estimated policy rule and the structure of private sector behavior. The forward-looking character of money demand is part of the structure of the model, and would prevail across different policy rules. This structural feature should also be taken into account in an analysis of optimal policy in our model. How the forward-looking nature of money demand impacts the welfare analysis of an optimizing model such as ours is beyond the scope of this paper, but is an important area for future research.

6 Conclusions

In this paper, we have looked at the role of money in a general framework that encompasses three competing environments: the baseline New Keynesian model with separable utility and static money demand; non-separable utility between consumption and real balances, along with habit formation;

[13] The correlations are computed using expressions for the analytical moments from the models' VAR representations.

[14] Any variation in real balances in such a benchmark that is not recorded in the current nominal interest rate and output is uninteresting noise.

and the model modified to allow for adjustment costs for holding real balances. The last two models imply a forward-looking character of real money balances, that conveys on money an important role as a monetary policy indicator. We distinguish between these alternative views by conducting a structural econometric analysis for the U.S., the euro area, and the U.K. Our FIML estimates confirm the forward-looking character of money demand. A major source of this forward-looking behavior is the existence of portfolio adjustment costs.

We illustrated how the value of money is increased in our estimated models, relative to the New Keynesian baseline, by the specification of money demand dynamics for which we have found empirical support. We concentrated on the links between money and the natural rate, and demonstrated that money can have value as an indicator of future variations in the natural rate, even when inflation dynamics are viewed through a "neo-Wicksellian framework" of the type advocated by Woodford (2003).

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Table 1. Money's role in the transmission mechanism[(a)]

	New Keynesian model		
	Baseline	Non-separable utility	Forward-looking money demand
Is the money demand equation needed to obtain inflation and output paths?	NO	YES	NO
Do output and inflation have non-zero impulse responses to money demand shocks?	NO	NO	NO
Does money contain information about real shocks not present in scale variable?	NO	NO	YES

(*a*) In all cases, interest-rate rule assumed to have no response to money.

Table 2. Maximum likelihood estimates, Euro area, M1

Estimated Parameters	Unrestricted Estimates	Unit Elasticity
	(1)	(2)
β	0.9902	0.9902
	(0.0027)	(0.0026)
φ_1	0.9073	0.8992
	(0.0474)	(0.0504)
φ_2	0.0000	0.0047
	(0.0202)	(0.0201)
h	0.8907	0.9064
	(0.0277)	(0.0272)
δ_0	3.2920	2.9558
	(0.5775)	(0.2625)
γ_1	0.0527	1.0000
	(0.0226)	(–––)
γ_2	2.5323	3.1893
	(0.7861)	(0.8247)
κ	0.0000	0.0000
	(0.0007)	(0.1125)
χ	0.4677	0.7448
	(0.4040)	(0.4784)
λ	0.6148	0.3892
	(0.5297)	(0.0951)
ρ_r	0.7185	0.7451
	(0.0731)	(0.0389)
ρ_y	0.1818	0.1951
	(0.0448)	(0.0396)
ρ_π	1.8653	1.8550
	(0.1273)	(0.1383)
ρ_μ	0.1537	0.1653
	(0.0635)	(0.0676)
ρ_a	0.9859	0.9839
	(0.0168)	(0.0187)
ρ_e	0.9842	0.9852
	(0.0176)	(0.0162)
ρ_z	0.9796	0.9781
	(0.0163)	(0.0176)
σ_a	0.0772	0.0715
σ_e	0.0210	0.0202
σ_z	0.0041	0.0041
σ_r	0.0016	0.0015
Log-Likelihood	1704.19	1704.36

Table 3. Maximum likelihood estimates, U.S., M0

Estimated Parameters	Unrestricted Estimates	Unit Elasticity
	(2)	(2)
β	0.9902	0.9902
	(——)	(——)
φ_1	0.7305	0.5153
	(0.0029)	(0.0003)
φ_2	0.0000	0.0000
	(0.0003)	(0.0000)
h	0.9500	0.9500
	(——)	(——)
δ_0	8.3378	4.4166
	(0.0036)	(0.0014)
γ_1	0.0517	1.0000
	(0.0002)	(——)
γ_2	2.0000	2.0000
	(——)	(——)
κ	0.0000	0.0000
	(——)	(——)
χ	1.0223	1.3083
	(0.0063)	(0.0013)
λ	0.0925	0.0255
	(0.0010)	(0.0001)
ρ_r	0.7253	0.7874
	(0.0002)	(0.0001)
ρ_y	0.1437	0.1892
	(0.0001)	(0.0001)
ρ_π	1.8961	1.7659
	(0.0111)	(0.0016)
ρ_μ	0.1601	0.2209
	(0.0052)	(0.0005)
ρ_a	0.8347	0.7005
	(0.0013)	(0.0010)
ρ_e	0.8940	0.9423
	(0.0001)	(0.0004)
ρ_z	0.9489	0.9282
	(0.0003)	(0.0006)
σ_a	0.0132	0.0147
σ_e	0.0410	0.0276
σ_z	0.0079	0.0110
σ_r	0.0027	0.0026
Log-Likelihood	1561.83	1704.36

Table 4. Maximum likelihood estimates, U.K., M0

Estimated Parameters	Unrestricted Estimates	Unit Elasticity
	(3)	(2)
β	0.9911	0.9911
	(0.0022)	(0.0025)
φ_1	0.6383	0.7110
	(0.0872)	(0.1355)
φ_2	0.0000	0.0000
	(0.0200)	(0.0562)
h	0.9750	0.9750
	(—–)	(—–)
δ_0	7.9734	4.9709
	(0.1472)	(1.3139)
γ_1	0.0000	1.0000
	(0.0098)	(—–)
γ_2	0.0000	0.0000
	(0.3057)	(0.3459)
κ	0.3319	0.2965
	(0.2363)	(0.2073)
χ	0.3390	0.3178
	(0.1978)	(0.2974)
λ	0.4220	0.4601
	(0.3315)	(0.3022)
ρ_r	0.8400	0.8289
	(0.0430)	(0.0447)
ρ_y	0.0009	0.0000
	(0.0762)	(0.0919)
ρ_π	1.5128	1.4583
	(0.3024)	(0.2517)
ρ_μ	1.2646	1.2469
	(0.4808)	(0.3126)
ρ_a	0.8511	0.9182
	(0.0778)	(0.0613)
ρ_e	0.9754	0.9857
	(0.0297)	(0.0241)
ρ_z	0.9430	0.9409
	(0.0261)	(0.0589)
σ_a	0.0122	0.0142
σ_e	0.0294	0.0222
σ_z	0.0158	0.0162
σ_r	0.0023	0.0024
Log-Likelihood	1653.11	1658.7

Table 5. **Partial correlations of real money stock and natural rate, estimated models**

Holding current output and nominal interest rate constant

Euro area	−0.144
U.S.	−0.176
U.K.	−0.365

Figure 1. Responses to IS shock: Euro area

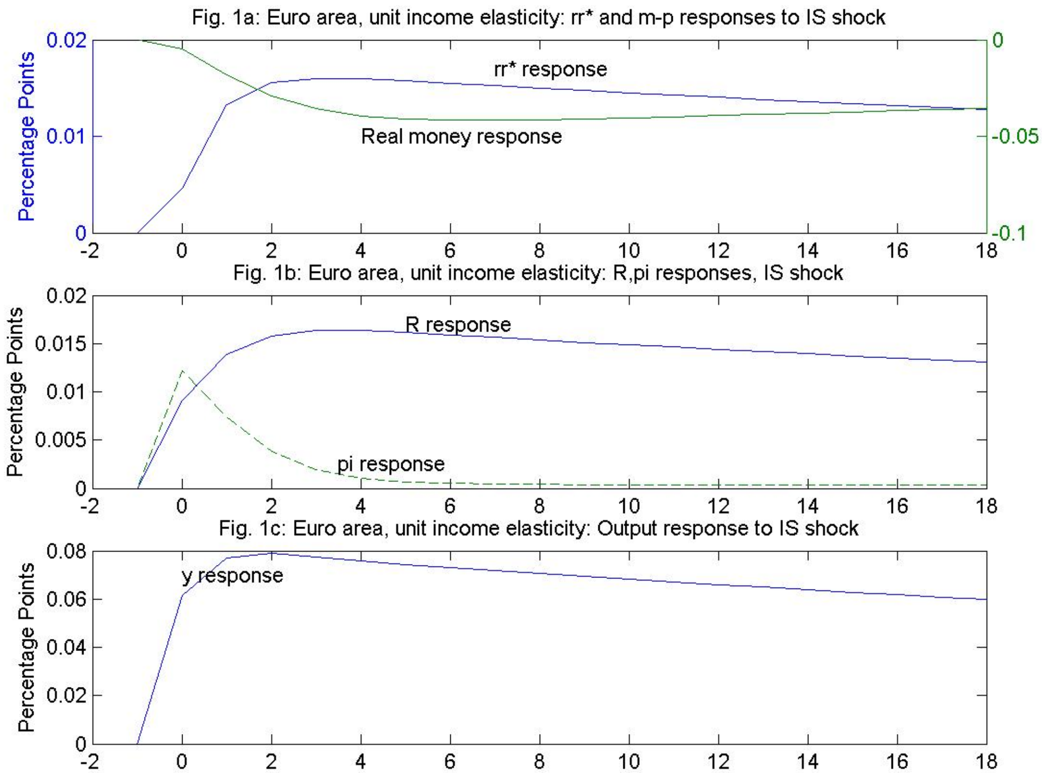

Figure 2. Responses to IS shock: U.S.

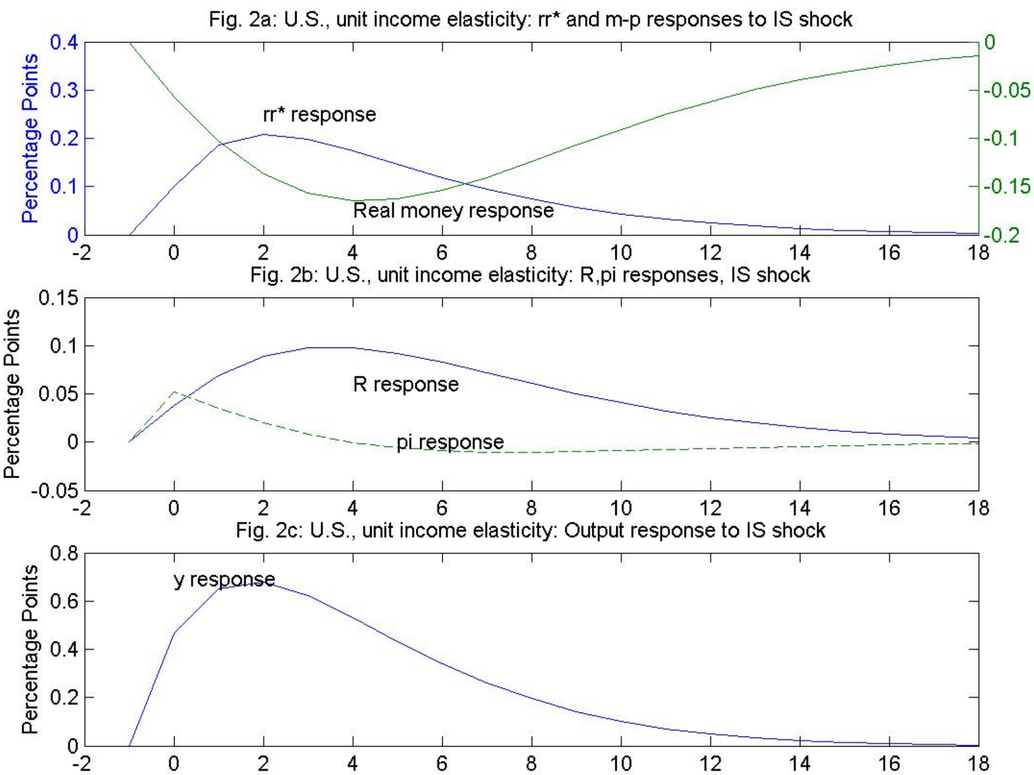

Figure 3. Responses to IS shock: U.K.

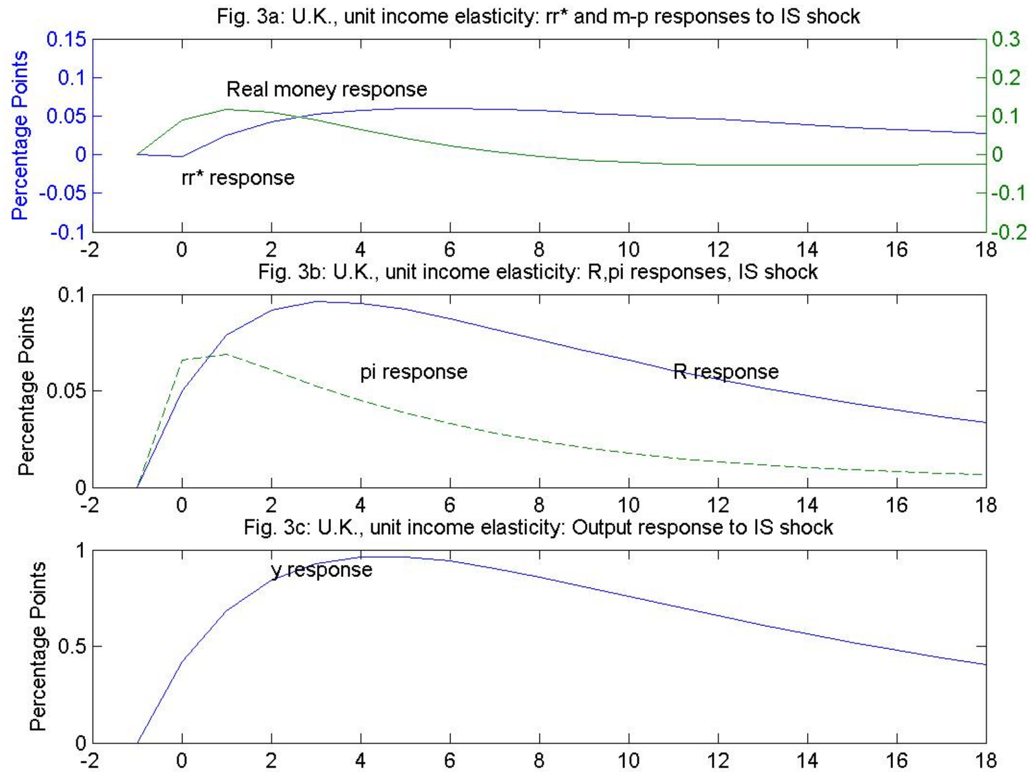

Figure 4. Habit formation and response of natural rate to IS shocks

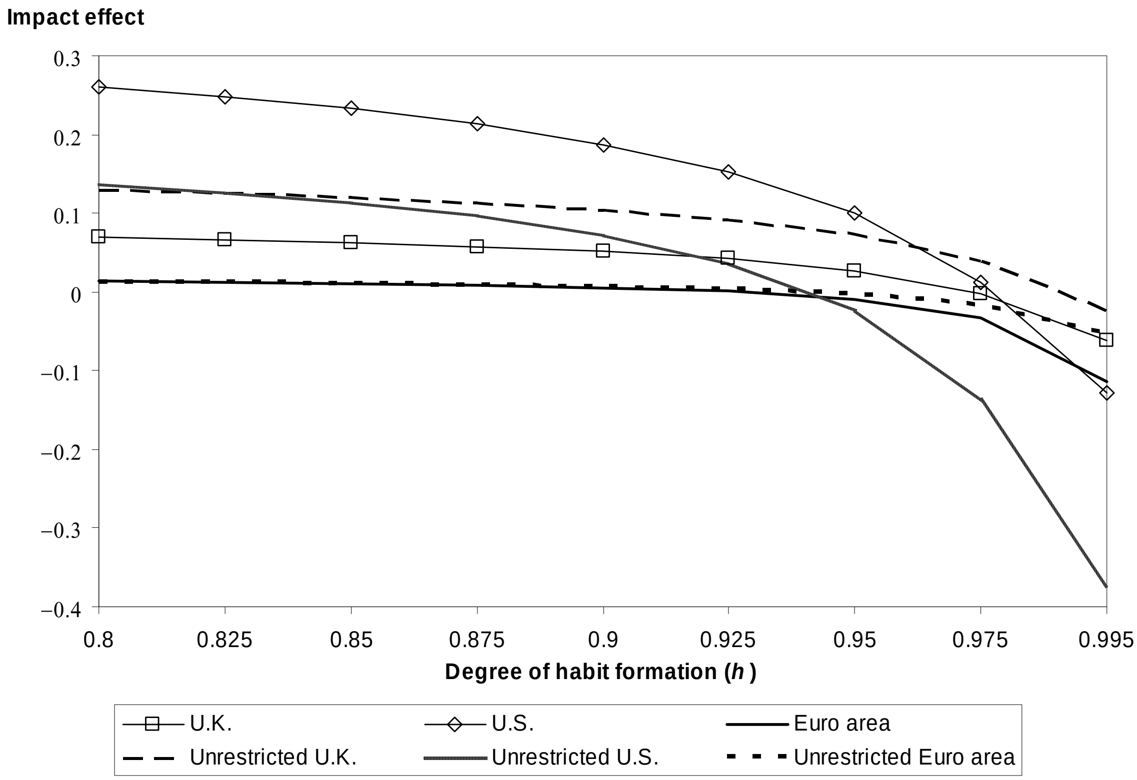

Figure 5. Responses to technology shock: Euro area

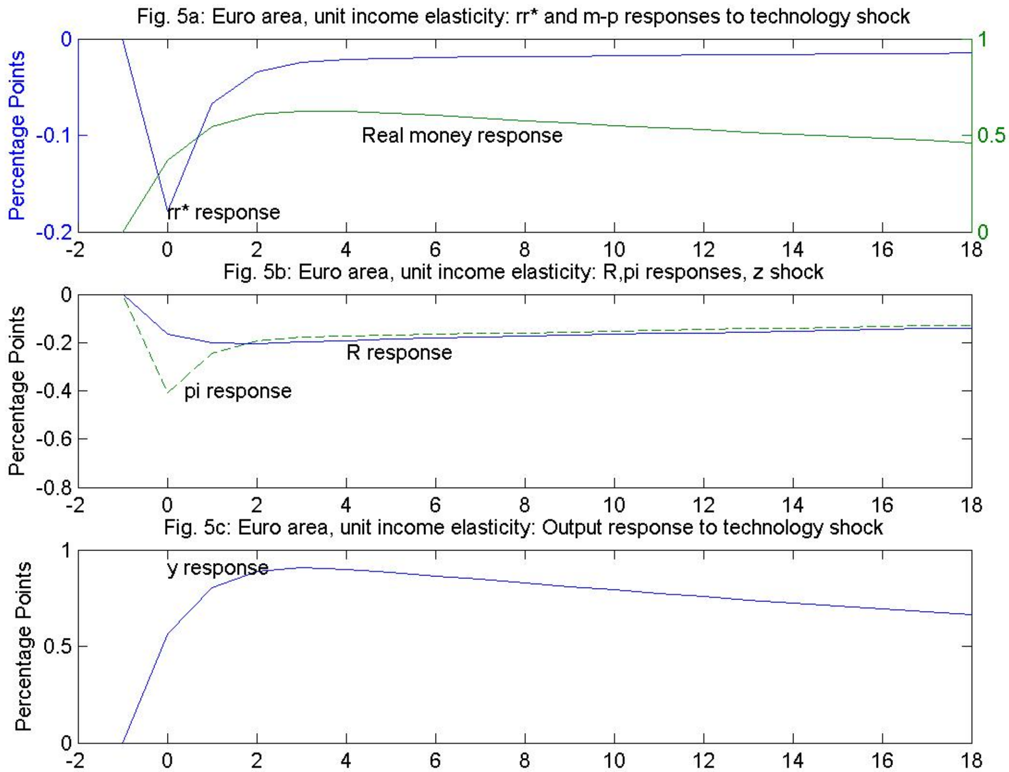

Figure 6. Responses to technology shock: U.S.

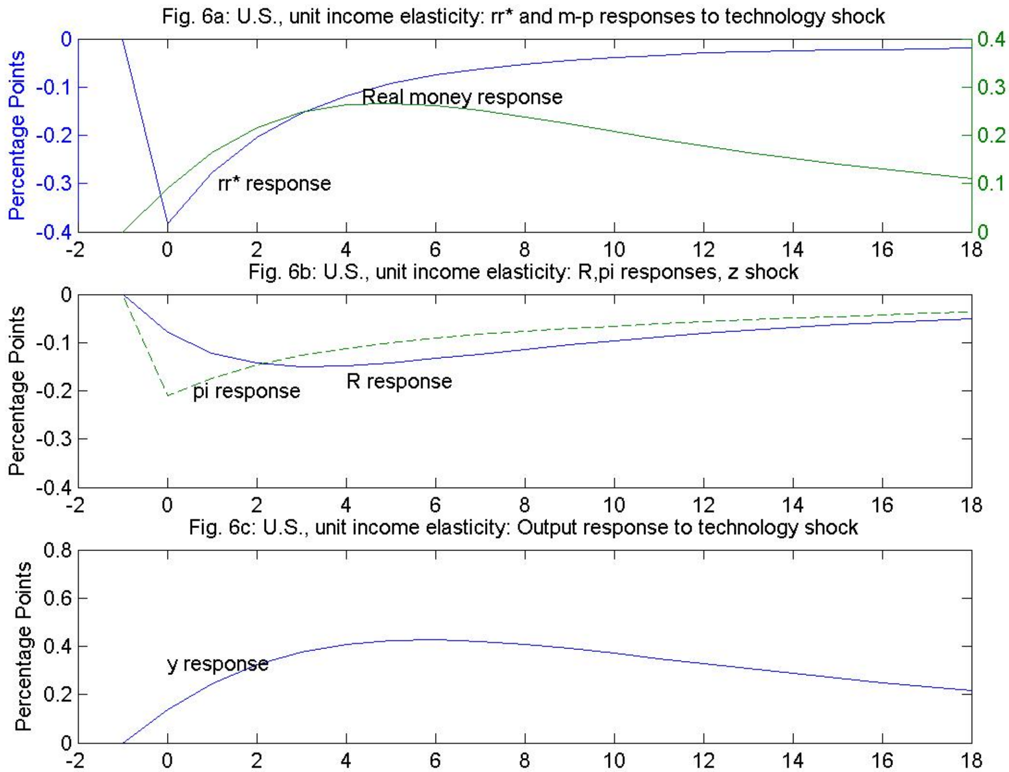

Figure 7. Responses to technology shock: U.K.