Ultra Short Tenor Yield Curve

For Blockchain Trading and Settlement

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Abstract

Blockchain trading platform provides atomic settlement of transactions of digital assets. In such a short trading horizon and immediate settlement environment, it is crucial to have an ultra short tenor interest rate curve that is real time updated. Our paper is the first to study interest rate tenor that is shorter than one day. Many market-quoted rates are still accrued at the end of the trading day, typically with one day as the shortest tenor available. The shortest tenor is also one day in all previous studies of interest rate parity. This paper develops a practical money market model for the intraday equilibrium interest rates that has ultra short tenor that are re-estimated in real time, which can be vital for central banks' efforts in stabilizing the currencies during flash crashes. We show that on 15 January 2015 when the Swiss National Bank dropped the floor of CHF 1.2 per EUR, the implied CHF deposit rate should have been highly negative to discourage trading positions that aggravated the crash.

JEL-Classification: G01, G12, G14, G23, C01, C15, C41, C58

Keywords: Blockchain, Intraday Yield Curve, Flash Crash, Duration and Time Deformation

1 Introduction

Trading today occurs at high speed with traders opening and closing positions in rapid succession and it is estimated that more than 90% of currency positions are held for less than 24 hours, and typically only for minutes or a few hours (Golub et al. (2013)). If only the overnight positions attract interest rate charges, this means that in practice only a small portion of FX transactions actually triggers interest payments. In the event of a large intraday selling pressure of a particular currency, the central bank is helpless against this type of 'flash crash' (see description in Kirilenko et al. (2017)), which can potentially lead to permanent economic losses. Without the ultra short tenor rates, the central bank can only change the overnight 1-day rate or the base rate. Using the 1-day rate to manage ultra high frequency trading and to counteract flash crashes can cause disruptions to the financial system and the real economy. In another instance, in 2000 the Turkish Central Bank had to raise daily interest rates to 300% at the height of the crisis to prevent the currency from collapsing, driving many banks and corporations into bankruptcy and resulting in over 1 million people losing their jobs (Özatay (2002)). Today, Brexit and the instability of world political situations are the type of market conditions that germinate currency volatility, often in the ultra-high frequency space.

In conventional money market, using a six-month rate if reference period is three months is unacceptable. Similarly, if the trading and borrowing/lending period is a few hours or minutes, it is not appropriate to use the daily rate as reference rate. Current central bank interest rate tool is ineffective in delivering monetary policies or stabilizing financial markets in the high frequency space as long as the shortest end of the yield curve is one day. Our proposed ultra-short tenor yield curve allows central banks to quickly bring liquidity dislocations into equilibrium by (i) encouraging liquidity provision during market stress, (ii) discouraging traders from behaviors that amplify the market stress or dislocations by skewing the yield curve against the trend, (iii) has automatic-adjustment based on latest high frequency prices, and will re-adjust back during the re-estimation when the dislocation or stress is resolved.

Blockchain, a distributed ledger technology, allows for immediate settlement.[1] Chiu and Koeppl (2018) investigate optimal features for trading on the Blockchain and find, despite mining costs, gains in moving to Blockchain settlement; and Benos et al. (2017) discusses potential issues should distributed ledger technologies be adopted on a large scale. In a related paper, Malinova and Park (2017) uses a theoretical model to argue that a higher degree of transparency on the Blockchain would increase investors' welfare. An important effect of fast clearing and settlement is the reduction in costs, counterparty and liquidity risks (see Khapko and Zoican (2018) which discusses the implications of faster settlement). Traditional ways of clearing and settling trades based on batch-based serial processes that often result in multi-day settlement times, along with high costs and operational risks. Since blockchain allows for immediate settlement of transaction,[2] it will pave the way for the development of the ultra short tenor interest rate market.

The key concerns of policy makers are that systemic financial institutions are not undermined, market is resilient, and that flash events should ideally not happen nor produce systemic contagion across markets. In this context, Foreign Exchange Working Group (FXWG) developed the FX Global Codes to enhance coordinations in an orderly market. Among other things, the codes relate to market participants' obligation to avoid the disruptive consequences of their trading activities (see for example, the execution requirements during periods of poor liquidity); governance around algorithmic trades execution, and measures to boost resilience against the loss of data from public venues; and, in collaboration with several industry bodies, how market participants should determine the minimum (or maximum) point of pricing in a flash event.

Following the spirit of FXWG's code, this paper proposes an intraday model for the very short tenor interest rates based on the exchange rate dynamics, UIP and the condition of no-arbitrage. Our intraday (or hourly) updated yield curve is designed for trades settlement on blockchain where transactions are settled in milliseconds. Our findings show that the intraday yield curve update is vital especially during liq-

[1] See Peters and Panayi (2016) for a detailed description of how the technology works.

[2] Currently it needs about 10 minutes to update all the ledgers, but the technology is currently being developed such that settlement would be achieved in milliseconds (see for example McKinsey (2015)).

uidity blackouts and flash events. We argue that the yield curve used for settling millisecond transactions on blockchain should have a convincing and well considered adjustment for flash events. Our time-deformed model is well suited for capturing real time price discovery in ultra-high-frequency data and information flow.

The outline of the remaining paper is as follows: Section 2 describes how the blockchain functions in financial trading created the demand for ultra-short tenor yield curves in practice. Section 3 develops the methodology for estimating an ultra-short tenor yield curve based on the theory of no arbitrage and absence of abnormal returns in efficient markets. The estimation is implemented using UIP (Uncovered Interest Rate Parity) and a log-ACD (Autoregressive Conditional Duration) model for exchange rate returns. Section 4 examines the empirical application of the model using a case-study of the Swiss-Franc event on 15 January 2015, while Section 5 applies the model on an extreme price movement (EPM) identified using the methodology of Brogaard et al. (2018). Finally, Section 6 concludes.

2 The demand for ultra-short tenor yield curve

2.1 Blockchain trading and settlement

Removing the need of a central trusted party, blockchain and the distributed ledger technology have unleashed the wave of decentralised tradings. As blockchain ledger is immutable and updated in real time for all "node" members, even with only the support of a mobile phone application, buy and sell orders are matched resulting in atomic swap of assets. Trading can be executed as fast as the network connection between the traders permits, normally in the range of 10ms to 100ms. The limit orders system is price-spread-time dependent so that high-frequency traders will not be able to extract an unfair advantage from the pending limit orders as in the case of a standard price-time queuing system.

Blockchange based FX exchanges such as Lykke, HSBC, Santander One Pay etc. operating 24/7, and the currencies traded include USD, EUR, CHF, JPY, GBP, AUD, CAD and a whole range of crypto currencies. The maintenance of margin account and margin calls procedures are the same as the traditional leverage trading but with a much faster clock cycle. Interest rate payments are accounted for second by second, thus improving liquidity provision on the blockchain. Each broker has his own rollover/interbank swap rates connected to an ECN (Electronic Communication Network) with credit facility, e.g. IC Markets. Since blockchain trading reduces settlement period from a few days to a few minutes, it is served by a, yet developing, ultra-short tenor interest rate market. While this ultra short tenor interest rate market moves according to demand and supply in real time, it must satisfy the theory of no arbitrage and absence of abnormal returns in efficient markets. Since the equivalent intra day ultra short tenor forward contracts for interest and exchange rates do not yet exist, the only relevant parity exists empirically is the uncovered interest rate parity (UIP). The mechanism for linking the UIP with ultra-short tenor interest rates, explained in the Section 3, is the main contribution of this paper.

2.2 Uncovered Interest Rate Parity

The uncovered interest rate parity (UIP) relation postulates that the interest rate differential between two currencies should equal the expected exchange rate change as follow:

$$E_t(s_{t+1} - s_t) = (i_t - i_t^*), \tag{1}$$

where s_t is the log of the spot exchange rate (in terms of home currency price of a unit of foreign currency). If t represents a daily frequency, then i_t and i_t^* are, respectively, the one-day domestic and foreign continuously compounded interest rates,[3] and E_t is the conditional expectations operator.[4] In practice and without the expectation operator, eq. (1) becomes

$$r_{t+1} = s_{t+1} - s_t = \pi + (i_t - i_t^*) + \varepsilon_t, \tag{2}$$

where r_t is the daily (not annualised) exchange rate return at time t, π is a risk premium for the estimation period and ε_t is a zero-mean random error.

[3]Here, we see that the interest rate reference period is ex ante and not ex post.

[4]Over longer horizons, the differentials of inflation should be subtracted from the UIP, and the relationship is known as real exchange rate-real interest rate (RERI) (see for example Hoffmann and MacDonald (2009)). Since our focus is on high frequency intervals (daily or intraday), we do not consider the effects of inflation.

The UIP thus implies that a regression of exchange rate returns on the interest differential should give a slope coefficient of unity. This hypothesis has been consistently and decisively rejected by the data. Very often, the estimated slope coefficient is negative, meaning that the currency with the higher interest rate tends to appreciate. A carry trade (in which the investor borrows in the currency with the low interest rate and invests in the currency with a high interest rate) is profitable on average. Furthermore, Equation (2) suggests that $\pi > 0$ for a depreciating currency where $r_{t+1} > 0$ (i.e. $s_{t+1} > s_t$). The foreign currency becomes more expensive at time $t + 1$ than at time t, hence a risk premium is needed for holding the weaker currency for the amount not compensated by the interest rate differential.[5]

Based on pre-blockchain convention, investor received the interest rate differential only at the point when a position was rolled over from one day to the next with the rollover time determined by market convention. A position that was not held open overnight received no interest rate differential because intra-daily interest rates were often assumed to be zero. Today, transactions completed on blockchain will attract interest rate for the duration when the asset is held, which is a fraction of the daily rate (for example OIS-swap rate). However, the problem at hand is that the intraday rate for duration shorter than a day (say 10 minutes) may not be proportional to the 1-day rate, but is dictated by the supply and demand during the 10-minutes window at the time the currency exchange is executed, especially during exchange rate flash crashes. Hence, the demand for an ultra-short tenor interest rate curve is a direct outcome of the blockchain development. At the time of writing, 1-hour interest rates have become available for some currencies, but the data is too thin for research. Until 2019, the shortest tenor market interest rate quotes were for 1-day borrowing and lending. Our paper is the first to study interest rate tenor that is shorter than one day. In the next section, we present previous students that test UIP. The shortest tenor studied in the literature is the 1-day ON rate, which itself is updated in real time.

[5] Risk premium tends to vary slowly over time. Here, we assume a time-varying π estimated using a rolling window of 500 days.

2.3 Previous studies of interest rate parities

While the UIP regression is usually run over horizons from a month to a year, Lyons and Rose (1995) examine the relationship between interest differentials and exchange rates at high frequency. They considered pairs of currencies in the now-defunct European Monetary System (EMS), and found that currencies which were under attack but in fact stayed within the band actually appreciated intraday. Lyons and Rose argue that this intraday appreciation is a compensation for the risk of devaluation that might have occurred, but did not. Investors can be compensated for the risk of devaluation only by intraday appreciation, not by interest differentials, as there are effectively no interest rate differentials intraday at that time.

Chaboud and Wright (2005) examine UIP over extremely short horizons.[6] An intraday UIP regression, over a short period spans 17:00 New York time when interest rate is paid, yielded results in favor of the UIP hypothesis. The full overnight interest differential that accrues in such a window is offset by a jump in the exchange rate. Positive results are obtained for relatively large discrete interest payments accrue on positions held between Wednesday and Thursday, and especially on the multi-day interest rate differential days in the weekend.

Baglioni and Monticini (2010) show during liquidity crises, the intraday pattern of the overnight rate (ON) jumped by more than ten times (from 0.2 bp to 2.2 bp) in the reserve maintenance period from August 8th 2007. This is matched by an increase of the liquidity premium and the cost of collateral. The overnight interbank market actually operates round the clock where all loans must be repaid at the same time next day.[7] According to Baglioni and Monticini (2010), a bank short of liquidity say at 9 am has two alternatives: (i) borrow immediately in the interbank ON market, or

[6] Chaboud and Wright (2005) use bilateral Japanese yen, German mark/euro, Swiss franc and pound sterling 5-min average bid and ask spot exchange rates viz-a-viz the US dollar provided by Olsen and Associates over the 15-year period 1988-2002 and discarding weekends from 23:00 GMT on Friday to 22:55 GMT on Sunday.

[7] The difference between the rate charged on an overnight loan delivered at 9 a.m. and a loan with the same maturity delivered at 10 a.m. implicitly defines the price [difference] of an hourly loan. Baglioni and Monticini (2010) use tick-by-tick data for the e-MID interbank market, which was the most liquid market in the euro area for the exchange of interbank deposits at that time.

(ii) obtain intraday credit from the European Central Bank (ECB) and borrow later (say at 4 pm) in the ON market. During the period of high uncertainty, a risk averse bank might have a strict preference for borrowing early in the ON market, rather than borrowing later, in order to make sure that it has enough funds to achieve its end-of-day targeted liquidity position. This explains why a borrowing bank might be ready to pay an implicit interest rate higher than the cost of central bank daylight credit. This is then the "liquidity premium" on an ON loan delivered early in the day.

The second explanation for the jump in intraday interest rate is an increase in intraday credit from ECB due to a higher cost of collateral. Since ECB does not charge any fee on intraday credit, the only cost comes from the collateral requirement. A way to measure the cost of collateral is provided by the Euribor-Eurepo spread: this is the cost of borrowing eligible securities through a buy and sell back transaction, earning the Eurepo rate, and funding the deal by borrowing in the interbank market at the Euribor rate. The average three-month spread goes from 7.6 bp before the liquidity crisis to 51.6 bp during the crisis due to a higher credit risk perceived by market participants. This finding highlights that the ability of the central bank to curb the market price of intraday liquidity during a liquidity crisis is limited, despite the provision of free (collateralized) daylight overdrafts.

3 Estimating Ultra Short Tenor Yield Curve

Previous studies reviewed in section 2.3 above suggest that UIP works in short period when interest rate is charged for borrowing or compensated for lending. Here, we develop a methodology for estimating the the ultra short tenor yield curve based on UIP in a two-stage process. In the first stage, we model the irregularly spaced intraday exchange rate returns dynamics using the log-ACD (Autoregressive Conditional Duration) model with stochastic volatility to filter out the noise and volatility for tick-by-tick FX data in irregular time span. Since the real data typically does not cover a continuum of tenors, we simulate in the second stage sufficient data for constructing the required yield curve based on the principle of no arbitrage and the absence of abnormal return. We assume that the foreign rate is known or remain constant (within the hour).[8]

Let the ultra short term local and foreign interest rate be $i_{t_k}(\delta)$ and $i_{t_k}(\delta)^*$ respectively. This is the interest rate term structure that the trader faces at the time that the k^{th} trade is executed, i.e. at time t_k. δ is the fraction of a business day that the asset is held and restricted to be less than D, the entire business day. For $\delta > D$, the conventional interest rate term structure should be used. For day t with N transactions, the time-grid of transactions $t_k \in \{t_1, \ldots, t_N\}$ is irregularly spaced. Finally, the duration, d, is the time difference at the k^{th} transaction to the last executed transaction: $d_{t_k} = t_k - t_{k-1}$.

For convenience, we shorten the subscript t_k to k, and write the intraday interest rates at the time of the kth transaction as $i_k(\delta)$ and $i_k(\delta)^*$ respectively and write the duration at the k^{th} transaction as d_k. These intraday interest rates may or may not be observable. If they are unobservable, they can be derived from the intraday (logged) spot exchange rates, s_k, and the estimated risk premium, π.

3.1 Log ACD with stochastic volatility

To apply the UIP regression in Equation (2) to produce a yield curve for $i(d)$, for a continuum of tenors d, requires a projection of expected returns, $r(d)$. To fully exploit all the information contained in the tick-by-tick data, we follow Engle and Russell (1998) and Engle (2000) using transaction arrival times as stochastic events in the form of joint marked point processes. More recently, Feng et al. (2015) also estimated a time-deformed model for IBM stock returns with stochastic volatility using the method of

[8]While the assumption that the foreign interest rate is known or remain constant may sound contradictory, our focus here is to demonstrate that our methodology can be used to adjust short tenor interest rates for the local rate in real time and in response to flash cashes. If we allow all interest rates to be unknown, then we will need a more complex multi-variate setting to simultaneous estimate all parity relationships at the same time. This is beyond the scope of this paper and will be left for future research.

simulated moments.[9] Here, the FX returns are modeled as two simultaneous random processes- durations d_k and returns r_k, where the joint density is the product of the marginal density of duration and the conditional density of returns given duration:

$$f(d_k, r_k|\tilde{d}_{k-1}, \tilde{r}_{k-1}; \theta_k) = g(d_k|\tilde{d}_{k-1}, \tilde{r}_{k-1}; \theta_{1k})q(r_k|d_k, \tilde{d}_{k-1}, \tilde{r}_{k-1}; \theta_{2k}) \tag{3}$$

where $\tilde{x}_k = \{x_k, x_{k-1}, \ldots, x_1\}$ denotes the past of x and θs are parameters of the conditional densities.

To model the durations process, we use the log-ACD model of Bauwens and Giot (2000), which is the logarithmic version of the ACD model of Engle and Russell (1998). The duration between two quotes or transactions is expressed as

$$d_k = e^{\phi_k}\epsilon_k \tag{4}$$

where ϵ_k are IID and Weibull$(1, \gamma)$ distributed. ϕ_k is proportional to the logarithm of the conditional expectation of d_k, i.e. $\phi_k = \ln E(d_k|I_{k-1})$, and I_{k-1} denotes the information set available at t_{k-1} which contains at least $\tilde{d}_{k-1}$ and $\tilde{\phi}_{k-1}$. Furthermore, ϕ_k follows an autoregressive model

$$\phi_i = \omega_1 + \alpha_1 \ln \epsilon_{k-1} + \beta_1 \phi_{k-1} \tag{5}$$

which means it depends on its lagged past and the lagged "excess durations". The density of d_k is specified using the Weibull density

$$g(d_k) = \frac{\gamma}{d_k} m_k^{\gamma} e^{-m_k^{\gamma}} \tag{6}$$

where $m_k = \frac{d_k \Gamma(1+1/\gamma)}{e^{\phi_k}}$ and $\Gamma(\cdot)$ is the gamma function. The log likelihood function for observations $k = 1, \ldots, N$ can then be written as

$$\ln g(d_k) = \sum_{k=1}^{N} \ln \gamma - \ln d_k + \gamma \ln(d_k \Gamma(1 + 1/\gamma)) - \gamma \phi_k - \left(\frac{d_k \Gamma(1 + 1/\gamma)}{e^{\phi_k}}\right)^{\gamma} \tag{7}$$

where ϕ_k follows the process described in Equation (5).

[9] Feng et al. (2015)'s choice of MSM (Method of Simulated Moments) is motivated by the analytically intractable likelihood function. We decided against this estimation method after having difficulty with optimizing globally over all parameters when the sample includes an extremely large amount of data. The estimated model is very sensitive to the starting values used.

Next, we model the ultra-high frequency returns using the UHF-GARCH model described in Engle (2000). Here we approximate the conditional mean of FX returns per square root of time with an ARMA(1,1) and include as in Engle (2000) observed durations as an additional regressor:

$$r_k/\sqrt{d_k} = \rho_2 r_{k-1}/\sqrt{d_{k-1}} + e_k + \phi_2 e_{k-1} + \kappa_2 d_k. \tag{8}$$

The conditional variance r_k is expressed as

$$h_k = d_k \sigma_k^2 \tag{9}$$

where σ_k^2 is the conditional volatility per unit of time which can be modelled as a GARCHX(1,1) process

$$\sigma_k^2 = \omega_2 + \alpha_2 e_{k-1}^2 + \beta_2 \sigma_{k-1}^2 + \gamma_2 d_k^{-1} \tag{10}$$

in which the conditional variance depends on the reciprocal of d_k and γ_2.[10]

Since the UHF-GARCH model can also be estimated using maximum likelihood, the overall time-deformed log ACD UHF-GARCH process can be estimated jointly using the log likelihood:

$$LL = \sum_{k=1}^{N} [\ln g(d_k|\tilde{d}_{k-1}, \tilde{r}_{k-1}; \theta_1) + \ln q(r_k|d_k, \tilde{d}_{k-1}, \tilde{r}_{k-1}; \theta_2)], \tag{11}$$

where $\theta_1 = \{\omega_1, \alpha_1, \beta_1, \gamma\}$ and $\theta_2 = \{\rho_2, \phi_2, \omega_2, \alpha_2, \beta_2, \gamma_2\}$

While our log-ACD model with stochastic volatility does not explicitly include a jump parameter or volatility regime, as we re-estimate the model very frequently after every 1000 trades, the model is applied in a dynamic sense to capture jump and volatility regime change through changing parameter estimates. In particular, equations (5), (8) and (10) capture the dynamics of duration, returns and volatility. If there is a jump in exchange rate, it will first appear as the residual term, e_k, in (8), and

[10]Easley and O'Hara (1992) argue that news arrival generates very frequent transactions with very short durations. In contrast, large duration should have lower volatility. Manganelli (2005) finds empirically that, for heavily traded stocks, volatility has a significant and negative impact on duration; low durations follow large volatilities. However, in an order-driven (instead of pricing-driven) market, a higher volatility should lead to a higher duration as traders are discouraged to trade immediately due to higher cost of liquidity consumption and higher benefit of liquidity provision.

affect the future returns via the autoregressive term, ϕ_2, in (8) and the future volatility via α_2 and β_2 in equation (10). When the crash effect ceases, ϕ_2 becomes insignificant, and $(\alpha_2 + \beta_2)$ return to their normal values.

3.2 Constructing the ultra short tenor yield curve

To construct the market implied ultra short tenor yield curve, we estimate the log-ACD UHF-GARCH model at time t_k using parameters estimated from the last 1000 quote observations to simulate the next 5000 time-deformed observations. We repeat the simulations 1000 times, i.e. we construct 1000 projected yield curves, and take the average.[11]

Using UIP, the ultra short tenor yield curve an investor faces at time $t = t_k$ to hold an asset till time $t = t_k + t_q$ is

$$\sum_{j=1}^{q} \widehat{r}_{k+j} = (i_{k+q} - i^*_{k+q}) + \left(\sum_{j=1}^{q} \widehat{\delta}_{k+j} \right) \pi + \varepsilon_k, \quad q = 1, \ldots, N \tag{12}$$

where $\widehat{\delta}_k = \frac{\widehat{d}_k}{D}$ is the duration d_k expressed as a fraction of a business day, D, both measured in the same time units (assuming that a business day is 24 hours), and N is the number of irregularly spaced quotes/observations. π is the daily exchange rate risk premium for a particular day, which arguably is a function of the volatility and can potentially be negative or zero depending on the relative strength of the two currencies. We assume π to be (slowly) time-varying over a particular interest rate regime and estimate it using Equation (2) using a rolling window of 500 days. We assume that it is 'constant' intraday, scaled by intraday duration when the asset is held.

The intraday interest rates, i_{k+q} and i^*_{k+q}, are the unscaled (i.e. not converted to daily or annual) local and foreign interest rates in basis points for the $\sum_{j=1}^{q} d_{k+j}$ duration at time t_k. If intraday interest rates are constant for day t, then the net interest

[11]The choice of using 1000 in-sample observations for estimations is arbitrary and constitutes for our dataset of the last 2-3 hours observations. Shortly after the crash on 15 Jan at 10a.m., the last 1000 observations consists of the last 1.5 hours of observations. One could also use for example all observations in the last hour, etc. Further fine-tuning for optimal in- and out-of-sample sizes could be made but is out-of-scope of this paper.

differential is simply $(i_{k+q} - i^*_{k+q}) = \sum_{j=1}^{q} \delta_{k+j}(i_t - i^*_t)$, where i_t and i^*_t are the local and foreign daily overnight rates respectively for day t. In the case where i^*_{k+q} is known, Equation (12) can be used to infer i_{k+q}, and *vice versa*.

Given $q = 1, \ldots, N$ estimated values of $\hat{i}_{k+q}$ and $\hat{d}_{k+q}$, we fit the Nelson-Siegel model to the time-deformed model-simulated yield curve to produce the fitted intraday yield curve for unscaled interest rate as follows:

$$\hat{i}_{k+q} = b_0 + b_1 \frac{[1 - exp(-d/\tau)]}{d/\tau} + b_2 \left(\frac{[1 - exp(-d/\tau)]}{d/\tau} - exp(-d/\tau) \right) \tag{13}$$

where b_0, b_1, b_2 and τ are the fitted parameters.

According to Nelson and Siegel (1987), b_0 is interpreted as the long run levels of interest rates (the loading is 1, it is a constant that does not decay), b_1 is the short-term component (it starts at 1, and decays monotonically and quickly to 0), b_2 is the medium-term component (it starts at 0, increases, then decays to zero), and τ is the decay factor: small values produce slow decay and can better fit the curve at long maturities, while large values produce fast decay and can better fit the curve at short maturities, τ also governs where b_2 achieves its maximum. In order to constrain $\tilde{i}_{k+q}$ to the overnight rate i_t when $\sum_{j=1}^{q} d_{k+j}$ equals one day, we do not estimate b_0 but simply use $b_0 = i_t$, the 1-day unscaled rate, i.e. the intraday yield curve should converge to the daily 'long run' yield, except during flash crashes when the ON rates may be stale.

4 Case Study: Swiss Franc Event, 15 January 2015

On 15 January 2015, SNB announced at 09:30 (UTC, local time +1 hour) the discontinuation of the minimum exchange rate of CHF 1.20 per euro. At the same time, it lowered the interest rate on sight deposit account balances by 0.5 percentage points to -0.75%, and the target range for the three-month Libor was to change from between -0.75% and 0.25% to between -1.25% and -0.25%. Prior to the SNB's intervention, the euro has depreciated considerably against the US dollar and this, in turn, has caused the Swiss franc to weaken against the US dollar. Hence, the SNB concluded that enforcing and maintaining the minimum exchange rate for the Swiss franc against the euro is no longer justified.

The SNB announcement caused a 41% rise in the Swiss franc in 20 minutes. It retracted over 60% of this move within a further 20 minutes. Automated trading algorithms withdrew from two-sided market-making and suspended streaming prices on public and bilateral platforms. Chicago Mercantile Exchange activated a trading halt in CHF currency futures which further amplified the flash crash. Later, the SNB stabilised markets by providing liquidity in a price range, giving market participants the confidence to re-enter the market. Market users initially reverted to more traditional transaction methods such as voice trading but resumed the use of all trading mechanisms once they had made the appropriate adjustments to their e-trading tools.

4.1 Data

We use intraday EURCHF quote data from OlsenData[12] for the period from 8th to 16th of January 2015. The data contains tick-by-tick bid and ask prices, and bid and ask volumes with time stamps in milliseconds. FX trading is 24 hours, hence we include the overnight period but exclude all weekend quotes (because of too few observations) which leaves us with seven days of data. For quotes within the same second, we keep only the last entry of the second (with the largest millisecond) as a representative observation.[13] To compute FX returns, we use the log returns of the midprice, i.e.

$$R_k = \ln(\frac{Ask_k + Bid_k}{2}) - \ln(\frac{Ask_{k-1} + Bid_{k-1}}{2})$$

where k denotes kth quote.

Table 1 shows the number of observations for EURCHF, each day before and after 'filtering' the data (from multiple quotes within the same second) as well as the descriptive statistics of the quote returns and durations.

[Table 1 about here.]

The descriptive statistics in Table 1 highlight the acute stressed market condition on 15th and 16th of January 2015. The mean return on these two days are, in absolute

[12] http://www.olsendata.com/

[13] Feng et al. (2015) also use this method to deal with multiple observations within a second. Another method is to use volume weighted averages within the same second (see for example, Engle and Russell (1998)).

term, about 3500-5000 times that of the previous day. The mean duration, on the other hand, is about halved. The number of filtered (unfiltered) quotes on January 15 is two times (four times) larger than that before the crash. These statistics show the dramatic change in market dynamics when the EURCHF floor was lifted.

Figure 1 presents the EURCHF exchange rate information on January 15, 2015. Figures 1a and 1b show vividly the sharp fall in EURCHF exchange rate in the first hour of trading and the huge volatility that follows. This is accompanied by a widening of the bid-ask spread in Figure 1c, which improved during the day but deteriorated until the Asian markets opened the next morning. Figure 1d presents the bid and ask prices from 9:24 to 10:24 hours. Interestingly, it shows the rate reaction took place a couple of minutes before the SNB announcement at 9:30 suggesting the possibility of insider trading. The rate recovered half of the lost ground by 10am. Figure 1e shows huge volume of trades and quotes concentrated around the announcement. Liquidity dried out when European and US markets closed, and before the Asian markets reopened the next morning. The shortage of liquidity is also reflected in the rise in trade duration around the same time as shown in Figure 1f.

[Figure 1 about here.]

The majority of the trades were executed very quickly within a few seconds. There is a significant rise in trading intensity and a drastic drop in duration during and shortly after the crash.

4.2 Estimation Results

Since intraday prices and durations have seasonal patterns over a trading day, we need to adjust for these diurnal effects before estimating the econometric model. The intraday diurnal effects in duration is adjusted using a cubic spline as described in Engle and Russell (1998) and Bauwens and Giot (2000). The diurnal effect $\phi(\cdot)$ is estimated by taking the average duration conditional on the time of the day separated into 30-minute intervals. These average durations are used as mid-points in the respective intervals and a cubic spline smoothing is used to obtain the diurnal factor $\phi(\cdot)$. While there are differences in $\phi(\cdot)$ for each day of the week (see Bauwens and

Giot (2000)), we follow Feng et al. (2015) and Tse and Dong (2014) in disregarding this effect due to the limited number of days in our data.[14] Similarly, returns are adjusted for the diurnal effect in its volatility, where volatility per unit time, σ^2, is estimated using absolute values of returns divided by square root of duration ($r/\sqrt{d}$), and a smoothing spline is fitted over the average volatilities to obtain the diurnal factor. The absolute $r/\sqrt{d}$ is then divided by the estimated diurnal factor.

Our estimated diurnal factor for durations and volatility is shown in Figure 2, where the time of day is given in UTC which is one hour earlier than the Central Eastern Time. Durations are short when European and U.S. markets are open, but are longer during U.S. late afternoon trading period and when both markets are closed. For volatility, the diurnal factor is the highest during the European trading hours and tapers downwards in the late afternoon. A U-shape curve is observed during US market opening hours (14:00-22:00). Descriptive statistics of the durations and returns adjusted for diurnality are given in Table 2.

[Figure 2 about here.]

[Table 2 about here.]

Comparing the mean returns reported in Tables 1 and 2, the diurnal adjusted mean returns are much closer between non-crash days and crash days. The absolute values of mean returns on 15th and 16th January 2015 are now 28 and 42 times that before the crash, compare to 3500-5000 times before the adjustments. After the diurnal adjustment, the average durations are slightly less than half of that before the crash.

Using the diurnally adjusted returns and duration, we estimate the log-ACD-UHF-GARCH model before and after the SNB announcement, i.e. for the 9th and 13th January at 9 a.m., 10 a.m. and 11 a.m. (calm period) as well as for 15th January 9 a.m., 10 a.m., 11 a.m., 12 p.m., 1 pm, and 16th January 8 a.m., using the last 1000 observations at these specific times. The estimated parameters of the log-ACD UHF-GARCH model are given in Table 3. The estimated log-ACD parameters are mostly

[14]We also omit data from 15th and 16th of January when estimating the diurnal factor due to the flash event on 15th January 2015.

significant at 5% or better, with very large β_1 indicating strong persistence (see Equations (4) to (7) for the log-ACD specifications). The persistence drops sharply shortly after the crash at 11 a.m. on 15 January and the next day at 8 a.m.. Ljung Box statistics of the residuals are small and mostly insignificant, which indicates that the model has captured most of the autocorrelations in durations.

For the UHF-GARCH in Equations (8), (9) and (10), the estimated ARMA parameters in the conditional mean are mostly insignificant, except after the crash where returns take a downward trend with large negative AR (ρ_2) and negative MA (ϕ_2) parameters (as observed in Figure 1). Duration of quotes is added as an additional regressor to the conditional mean to capture the "bad news effect" of long durations following Diamond and Verrecchia (1987). The coefficients, κ_2, are mainly negative (as found in Engle (2000)) but they are mostly statistically insignificant.

In the variance equation, the α_2 and β_2 (ARCH and GARCH parameters) sum to a low number before the crash and tend to be insignificant. The coefficient, γ_2, on the reciprocal of duration is large and significantly positive for estimations before the crash. This was also observed in Engle (2000) and supports Easley and O'Hara (1992) hypothesis that long duration is interpreted as lack of news and decreases volatility. After the crash, however, the ARMA (ρ_2 and ϕ_2) and GARCH (α_2 and β_2) parameters become very large (with α_2 and β_2 summing to more than one) and statistically significant, while the coefficients on inverse durations (γ_2) becomes insignificant. The estimated high persistence is due to the extreme observations during the crash; the volatility persistence (β_2) over took the duration effect (γ_2)[15]. While the estimated UHF-GARCH process is nonstationary, Jensen and Rahbek (2004) show that inference of the process using maximum likelihood is consistent and parameters are asymptotically normal. The low Ljung-Box statistics in the residuals indicates that the UHF-GARCH model has captured much of the autocorrelation in intraday return.

[Table 3 about here.]

[15] We tried adding an indicator variable I_k to Equation (8) or (10) during the flash crash that takes a value of 1 for a structural break during or after a flash event and zero otherwise. This however did not alleviate the problem of explosive GARCH estimates.

Using only the estimated parameters that are significant, we simulate a horizon of 5000 time-deformed quote observations starting with the last observation in the sample. The simulations are repeated 1000 times and then the mean durations (d_k) and returns (r_k) are used. The diurnal factors are re-introduced into the mean simulated durations and returns. We then use the simulated returns, r_k, in the UIP equation (12) with π estimated using Equation (2) to extract the CHF rate, i, assuming that the EUR rate, i^*, remain stable during the day. To estimate the time-varying daily exchange rate risk premium π in Equation (2), we use rolling windows of previous 500 observations of daily EURCHF exchange rate, and EUR and CHF overnight daily interest rates[16] to obtain daily risk premium π. Figure 3 plots the time varying risk premium estimates (black line) and show EURCHF risk premiums have started to decrease since 2013. It drops sharply on 15 Jan 2015 and steadily increases thereafter. We remove the effect of the crash (using average of EURCHF exchange rate just before and just after the crash) to obtain a 'smoothed' time-varying risk premium (red dotted line).[17]

[Figure 3 about here.]

Both the EUR and CHF overnnight rates were negative during our sample period with the EUR rate being more negative than the CHF rate. Since pi (π) in Figure 3 is positive, it represents the reward for holding the EUR to compensate for the *peso effect* of a possible EUR devaluation. It is interesting to see in Figure 3 how this risk premium reached the minimum on the crash (re-valuation) date hinting a very efficient FX market that could anticipate the currency re-alignment. The premium remains positive after the crash and increases steadily after the crash suggesting that carry trade (with long position in EUR and short position in CHF) might still be viable due to the slow adjustment of the EURCHF FX rate and the two interest rates.

Assuming that the intraday EUR interest rate is fixed, the raw intraday CHF yield curves are extracted and plotted in Figure 4. The interest rate i_k (and i_{k+q}) is unscaled, i.e. it is the total amount of interest for d_k (and d_{k+q}) seconds. When i_k is negative, it

[16]OIS rates used are obtained from Bloomberg.

[17]Effects of the crash are removed from the risk premium estimates because this effect should be captured in the interest rates. In our framework here, interest rates of different tenors are the tools for combating the swings in exchange rate.

represents deposit rate only as lending rate would be zero bound for no arbitrage. We smooth the simulated estimated yield curves using the Nelson-Siegel model, where we constrain the first parameter b_0, which represents the level of the yield curve, to the daily CHF rate, except during the flash event when the ON rate might be stale. The estimated parameters are given in Table 4, where the parameters are mostly significant except for the crash period on 15 Jan at 10:00, 11:00 and 12:00. The smoothed curves are plotted in Figure 4. The Nelson-Siegel interest rate model was however unstable during the crash period on 15 Jan at 10:00, 11:00 and 12:00; the model failed to converge in these three cases. We also plot the fitted cubic splines in the same graph; cubic spline is more flexible than Nelson Siegel, but the resulting curve is also less intuitive.

[Table 4 about here.]

[Figure 4 about here.]

The CHF yield curves in Figure 4 tend to be downward sloping, and sometimes have a hump around the 10,000 secs mark, which is equivalent to about 3 hours. Since the interest rates are unscaled, it means for example in Figure 4(e) that an investor holding CHF for 5000 seconds (approximately 1 hr 20 mins) will have to pay an interest of about -2.0 bps in total for that 5000 secs. Holding CHF longer till 15000 seconds (approximately 4 hours 10 minutes) incurs an interest of about -2.2 bps for the time. In the very high frequency FX trading environment, (deposit) interest rate of very short tenor can fluctuate between positive and negative depending on the movements of the exchange rate. Hence, the high frequency short tenor CHF yield curve can become non-monotonic. Future research could investigate the one-sided quotes that will prevent a round trip arbitrage.

Shortly after the crash, the UHF-GARCH is nonstationary due to extreme observations and the graphs 4(h), (i) and (k) show very large volatility. We replot the graphs for these three periods for the first 500 seconds (approximately 8 minutes) in Figure 5 and also provide the graph at 9:00 just before the crash for comparison. At 9:00, the rates show no unusual activity. It is higher at the first few minutes, then drops to -1.92 bps and follows an upward trend there after. At 10:00, the rates in the first 300 seconds are only slightly negative. Thereafter it becomes extremely negative to -20 000 bps, which means to deposit CHF for 500 seconds (or 8.33 minutes) straight after the crash would cost 200%. The deposit rate has to be so negative at that point in time in order to deter further speculative attacks on the Euro (typically executed by shorting Euro and going long in CHF). At 11:00, the rates are stable at the first 300 seconds, and then displays a lot of volatility with unclear direction. At 12:00, the rates are stable only for the first 200 seconds, has several positive hump at around the 350 seconds and then dives down to -50 basis points for a 500 seconds deposit. Hence we can conclude that during the high turbulent period when SNB lifted the floor on EURCHF, the deposit rates for the very short horizon trading on blockchain have to be extremely negative, several hundred times more negative than the official -0.75% annualised rate, in order to stop the speculative trading and currency attack in these new platforms.

[Figure 5 about here.]

5 Robustness test using another 'flash' crash

We also apply our methodology on another crash period as a robustness test.[18] Here, we consider again EURCHF from 21st February to 11th July 2017 (121 days), a total of 2,924,840 observations after filtering out observations within the same second (as described in earlier section). We use the methodology of Brogaard et al. (2018) and identify 29 March 2017 as a day with the maximum number of EPMs.

Diurnal adjustments to durations and returns are made using diurnal factors estimated from the sample period prior to 29 March 2017 and the log-ACD UHF-GARCH model was estimated for the times 11:00, 14:00, 17:00, 20:00, 21:00, 22:00 and 23:00. The parameters indicate high volatility at 14:00 (when New York Stock Exchange opens) and from 17:00 to 23:00. During the flash crash at 22:00, we see that the ARMA parameters for the conditional mean become significant with a highly negative autoregressive parameter, which indicates downward pressure on the exchange rate. Volatility

[18] We thank a referee for this suggestion.

was also very high, indicated by the non-stationary GARCH parameters. These effects were no longer observed at 23:00, despite the volatility remaining high.

We estimate the simulated intraday yield curves at the same times using the last 1000 observations at each time considered using the time-varying exchange rate risk premium. Figure 6 plots the simulated yield curve (black solid line), a smoothing spline (red solid line) and a fitted Nielsen Siegel curve (dotted blue lines). We see that at 11:00, the intraday yield curve is upward sloping. The intraday CHF yield curves are then become downward sloping at 14:00, 17:00. 20:00 and 21:00. At 22:00 as the EURCHF rates plunged, the yield curve becomes highly negative except for the first few minutes. At 23:00, the yield curve returns to the gentle downward sloping curves observed at 17:00, 20:00 and 21:00. The highly negative curve observed at 22:00 arise from the explosive GARCH parameters which renders forecasting untenable at the longer end of the yield curve. Hence we plot the curves for the first eight minutes in Figure 7 for 22:00 (left) and 23:00 (right). Here we can observe the negative rates at 22:00 as compared to 23:00. This flash crash is naturally not as severe as the event on 15 January, but it does illustrate the mechanism that will disincentivize investors from panic buying of CHF and selling of EUR. The yield curve then re-adjusts at 23:00 as the liquidity dislocation resolves to the yield curve before the EPM.

[Figure 6 about here.]

[Figure 7 about here.]

6 Conclusion

In this paper, we propose an intraday model for deriving the implied equilibrium ultra short tenor CHF yield curve based on the exchange rate dynamics, uncovered interest rate parity and the condition of no-arbitrage. The necessity of an intraday ultra short tenor yield curve has been long overdue, with the rapid development of high frequency trading and the development of newer settlement platforms, for example the blockchain. Our ultra short tenor yield curve can be updated intraday (hour-by-hour, by a certain quote or trade volume or by other suitable criteria) and used, for example, for trades settlement on the blockchain where transactions are completed in milliseconds.

In order to capture all information in the ultra high frequency data, we adopt a time-deformed log-ACD UHF GARCH model to capture the dynamics of intraday durations and returns, and the real time price discovery across currency markets. Using the estimated model, we simulate time-deformed observations for a full range of ultra short tenor interest rates and construct a yield curve based on UIP. We then estimate a smoothed Nelson-Siegel yield curve using nonlinear least squares. We find that the log-ACD models intraday durations effectively, but the fit of the UHF-GARCH model for tick-by-tick quote returns are non-stationary during crash periods. We also find that the Nelson-Siegel curve to be unstable during flash crashes, and future work should consider developing other more robust models for such purpose.

Our findings show that during the crash triggered by the SNB announcement, the intraday yield curve dives sharply to -20000 basis points (for a 500 seconds deposit) in the first hour after the announcement. This very negative rates, discourages investors from the panic buying of CHF and selling EUR, and should automatically stabilize the currency in the short term before the effect of the daily interest rate adjustment kicks in at the end of the day. The next day after the crash, we notice large interest rates at the very short end of the intraday yield curve.

This paper is a first novel attempt at introducing an intraday ultra short tenor yield curve following the FXWG's code. It addresses the concerns that regulators have about the lack of incentives for liquidity provision during periods of market stress (Commission-Securities and Commission (2011), Kirilenko et al. (2017), Brogaard et al. (2018)) and at the same disincentivize investors from behavior that would aggravate flash crashes. We derive the intraday yield curves that would discourage ultra short term speculation which increases crash risks. During periods of shocks, the stabilizing mechanism of intraday interest rates become even more critical and lend central banks a useful tool in managing the stability of their currencies. The flash crash on January 15 saw the EURCHF fall by 40% in seconds. While FX trading venues could trigger circuit breakers to prevent extreme pricing, the execution system could not function properly because liquidity providers ceased to provide liquidity during the event. We strongly argue that had such ultra short tenor interest rates adjustments been implemented, many large swings in currency trades would have been prevented.

References

Baglioni, Angelo and Andrea Monticini (2010) "The intraday interest rate under a liquidity crisis: The case of August 2007," *Economics Letters*, Vol. 107, pp. 198–200.

Bauwens, L. and P. Giot (2000) "The logarithmic ACD model: An application to the bid-ask quote process of three NYSE stocks," *Annuals d' Économie et de Statistique*, Vol. 60, pp. 117–149.

Benos, Evangelos, Rodney Garratt, and Pedro Gurrola-Perez (2017) "The economics of distributed ledger technology for securities settlement," Working Paper 670, Bank of England Staff Working Paper.

Brogaard, Jonathan, Allen Carrion, Thibaut Moyaert, Ryan Riordan, Andriy Shkilko, and Konstantin Sokolov (2018) "High frequency trading and extreme price movements," *Journal of Financial Economics*, Vol. 128, pp. 253–265.

Chaboud, Alain P. and Jonathan H. Wright (2005) "Uncovered interest parity: it works, but not for long," *Journal of International Economics*, Vol. 66, pp. 349–362.

Chiu, Jonathan and Thorsten V. Koeppl (2018) "Blockchain-based Settlement for Asset Trading," working paper.

Commission-Securities, Commodity Futures Trading and Exchange Commission (2011) "Recommendations regarding regulatory responses to the market events of May 6, 2010,"Technical report, Commodity Futures Trading Commission-Securities and Exchange Commission.

Diamond, D.W. and R.E. Verrecchia (1987) "Constraints of Short-selling and Asset Price Adjustments to Private Information," *Journal of Financial Econometrics*, Vol. 18, pp. 277–311.

Easley, D. and M. O'Hara (1992) "Time and the process of security price adjustment," *Journal of Financial Economics*, Vol. 19, pp. 69–90.

Engle, Robert F. (2000) "The Econometrics of Ultra-High-Frequency Data," *Econometrica*, Vol. 68, pp. 1–22.

Engle, Robert F. and Jeffrey R. Russell (1998) "Autoregressive Conditional Duration: A New Model for Irregularly Spaced Transaction Data," *Econometrica*, Vol. 66, No. 5, pp. 1127–1162.

Feng, Dingan, Peter X.-K. Song, and Tony S. Wirjanto (2015) "Time-Deformation Modeling of Stock Returns Directed by Duration Processes," *Econometric Reviews*, Vol. 34, No. 4, pp. 480–511.

Golub, A., A. Dupuis, and R. Olsen (2013) "High Frequency Trading Strategies in FX Markets," in David Easley, Marcos López de Prado, and Maureen O'Hara eds. *High-Frequency Trading- New Realities for Traders, Markets and Regulators*: Risk Books.

Hoffmann, Mathias and Ronald MacDonald (2009) "Real exchange rates and real interest rate differentials: A present value interpretation," *European Economic Review*, Vol. 53, pp. 952–970.

Jensen, Soren Tolver and Anders Rahbek (2004) "Asymptotic Inference For Nonstationary Garch," *Econometric Theory*, Vol. 20, pp. 1203–1226.

Khapko, Mariana and Marius Zoican (2018) "Smart Settlement," working paper.

Kirilenko, A., A.S. Kyle, M. Samadi, and T. Tuzun (2017) "The flash crash: High frequency trading in an electronic market," *Journal of Finance*, Vol. 72, pp. 967–998.

Lyons, R.K. and A.K. Rose (1995) "Explaining forward exchange intraday bias," *Journal of Finance*, Vol. 50, pp. 1321–1329.

Malinova, Katya and Andreas Park (2017) "Market Design with Blockchain Technology," working paper.

Manganelli (2005) "Duration, Volume, and the Price impact of trades," *Journal of Financial Markets*, Vol. 8, pp. 377–399.

McKinsey (2015) "Beyond the Hype: Blockchains in Capital Markets," Working Papers on Corporate & Investment Banking 12, McKinsey&Company.

Özatay, Fatih (2002) "Turkey's 2000-2001 Financial Crisis and the Central Bank's Policy in the Aftermath of the Crisis," working paper, Bank of Albania 'In the second decade of transition' Conference.

Peters, Gareth W. and Efstathios Panayi (2016) "Understanding Modern Banking Ledgers through Blockchain Technologies: Future of Transaction Processing and Smart Contracts on the Internet of Money," in P Tasca, T. Aste, L. Pelizzon, and N. Perony eds. *Banking Beyond Banks and Money*: Springer.

Tse, Yiu-Kuen and Yingjie Dong (2014) "Intraday periodicity adjustments of transaction duration and their effects on high-frequency volatility estimation," *Journal of Empirical Finance*, Vol. 28, pp. 352–361.

List of Figures

Figure 1: Euro-Swiss Franc Exchange Rate

There are 93728 quotes from 08:14:57 15.01.2015 to 08:14:55 16.05.2015. The announcement to drop the minimum EURCHF rate at 1.2 was made by the Swiss National Bank at 9:30. The log quote returns, mid FX rates, bid-ask spreads, quote volumes and quote durations are plotted here, whereby Δt used are quote durations.

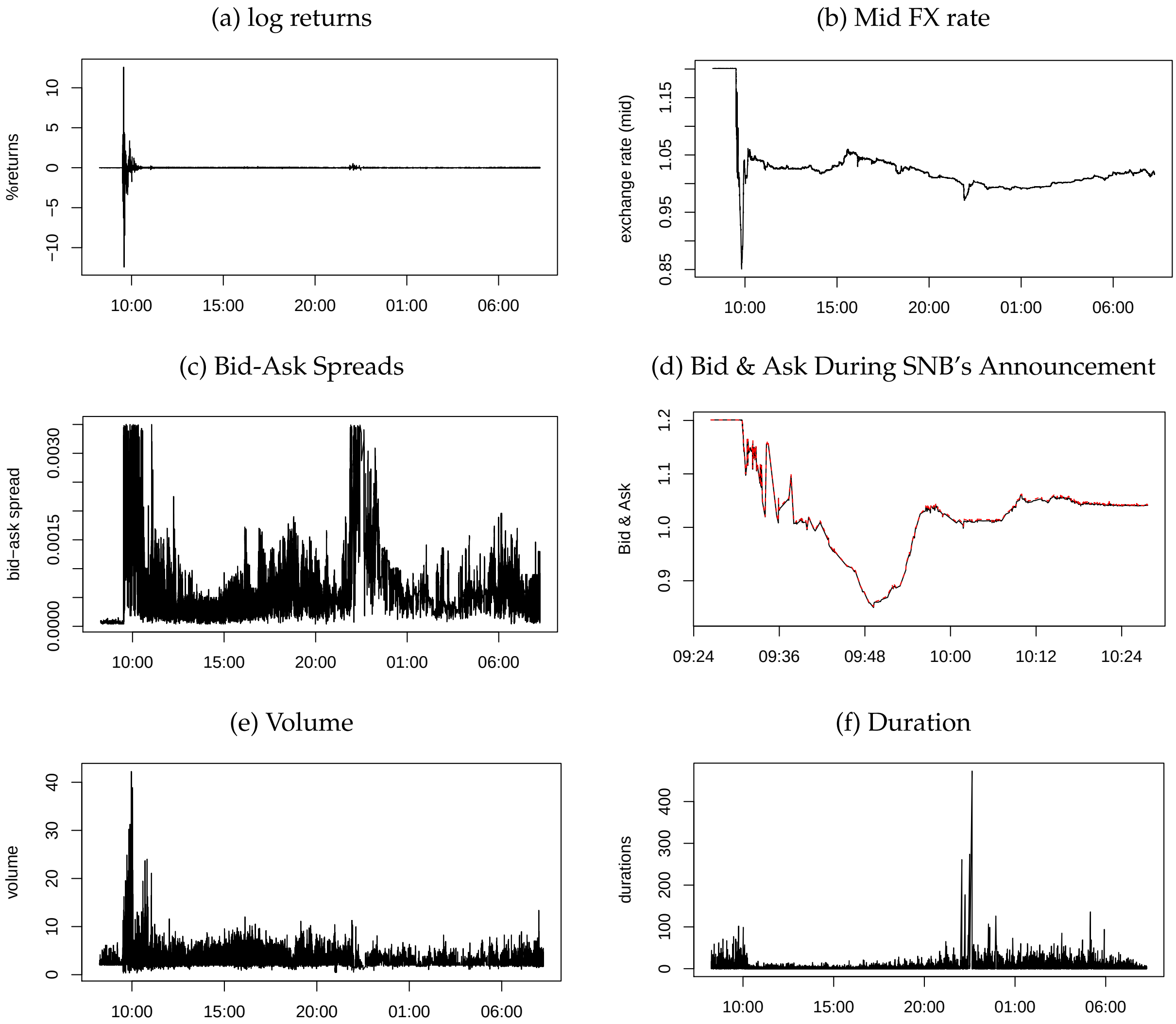


Figure 2: Diurnal effects in durations and volatility for the period from 8th to 14th January 2015: Black dotted lines show the average time-of-day effect of durations. Red solid lines show the smoothed diurnal factor using cubic splines. Time of day is given in UTC; add 1 hour for Central European Time.

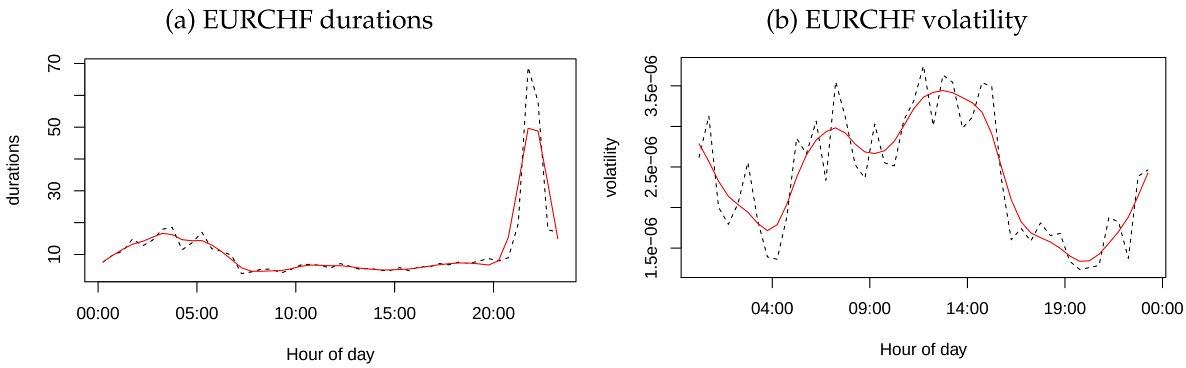


Figure 3: Exchange rate risk premium, pi, estimated using rolling windows of 500 observations. Red dotted lines are 'smoothed' estimates obtained by removing the effect of the crash on 15th January 2015.

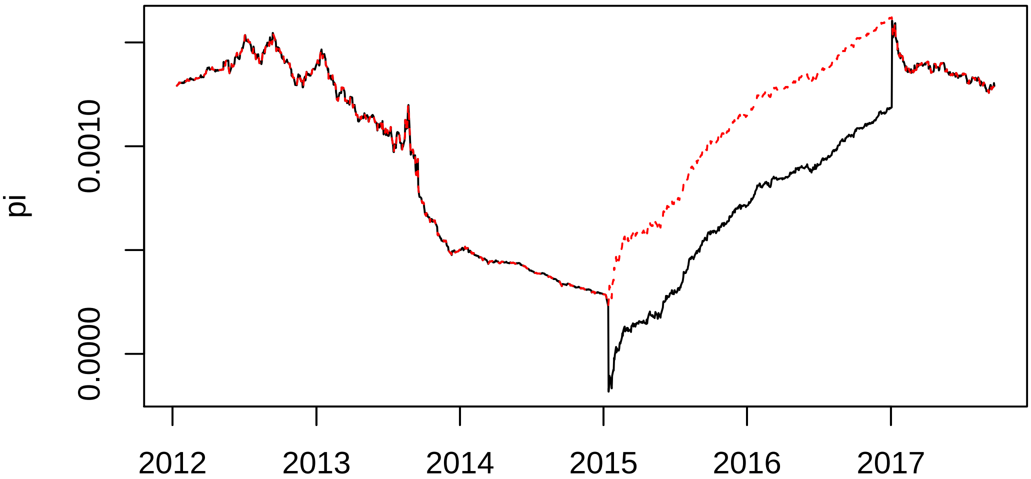
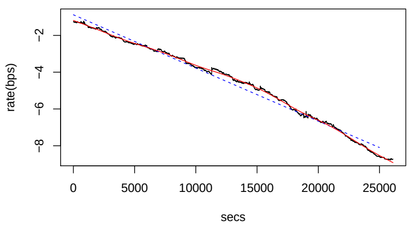
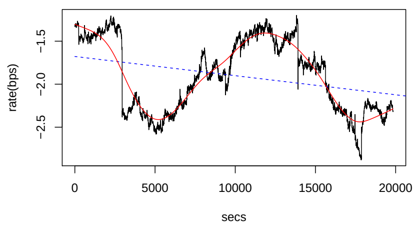


Figure 4: CHF Simulated Yield Curves: Simulated yield curves are plotted in black lines, with smoothed cubic splines given in red, and fitted Nelson Siegel curves in blue (dotted), for the respective days and times.

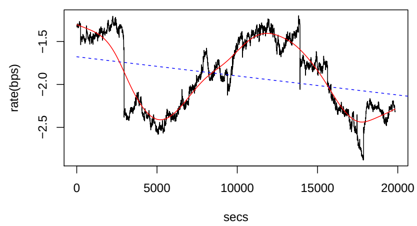
(a) Jan 09, 09:00



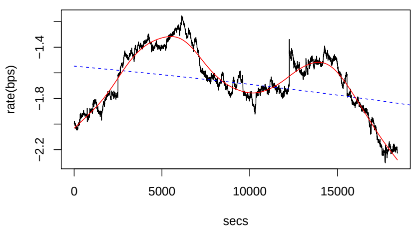
(b) Jan 09, 10:00



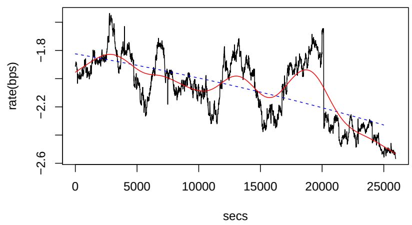
(c) Jan 09, 11:00



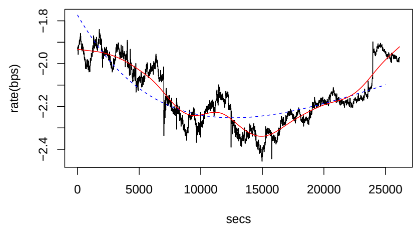
(d) Jan 13, 09:00



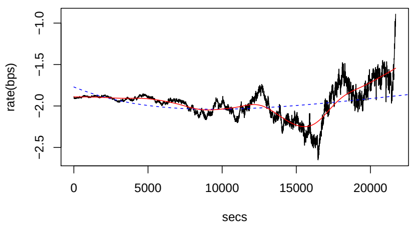
(e) Jan 13, 10:00



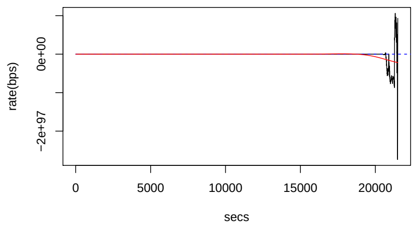
(f) Jan 13, 11:00



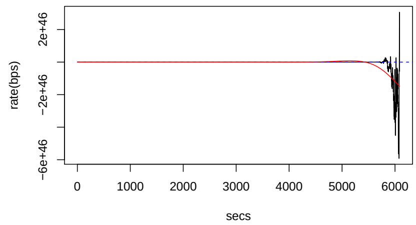
(g) Jan 15, 09:00



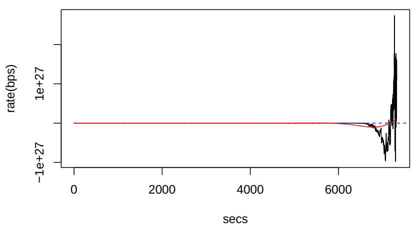
(h) Jan 15, 10:00



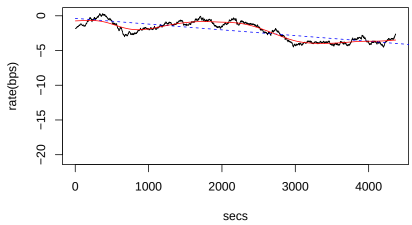
(i) Jan 15, 11:00



(j) Jan 15, 12:00



(k) Jan 15, 13:00



(l) Jan 16, 08:00

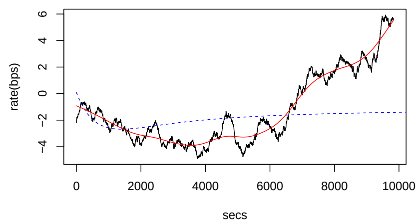
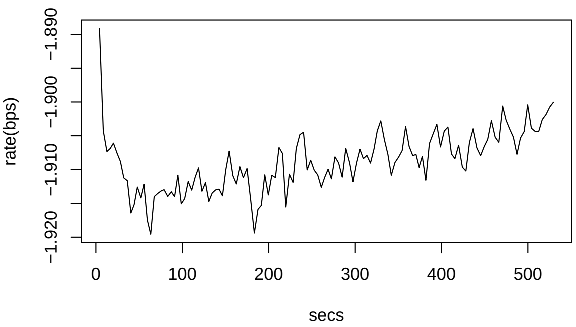
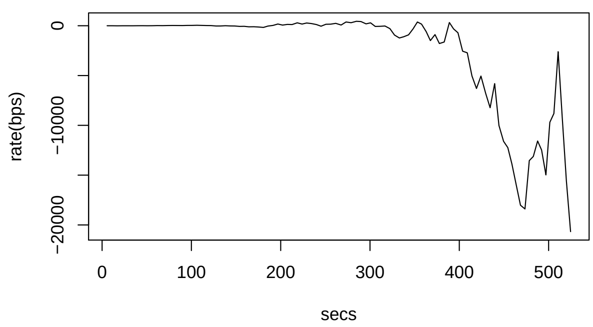


Figure 5: CHF simulated deposit rates just before and after SNB announcement on 15 January 2015 for tenors 500 seconds and shorter.

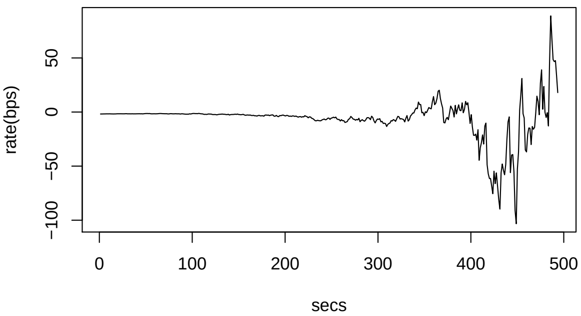
(a) Jan 15, 09:00



(b) Jan 15, 10:00



(c) Jan 15, 11:00



(d) Jan 15, 12:00

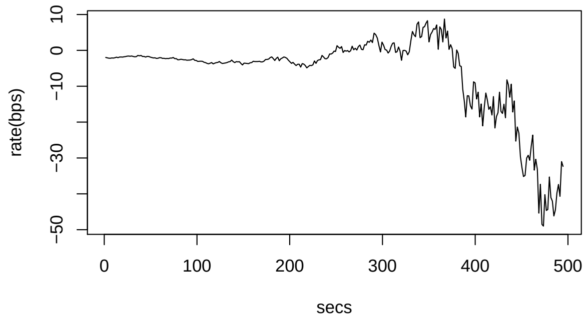


Figure 6: Estimated Intraday Yield Curves for 29 March 2017 at indicated times. Black solid lines indicated the simulated yield curves, red solid lines are fitted smoothing splines while blue dashed lines plot the fitted Nielsen Siegel curves.

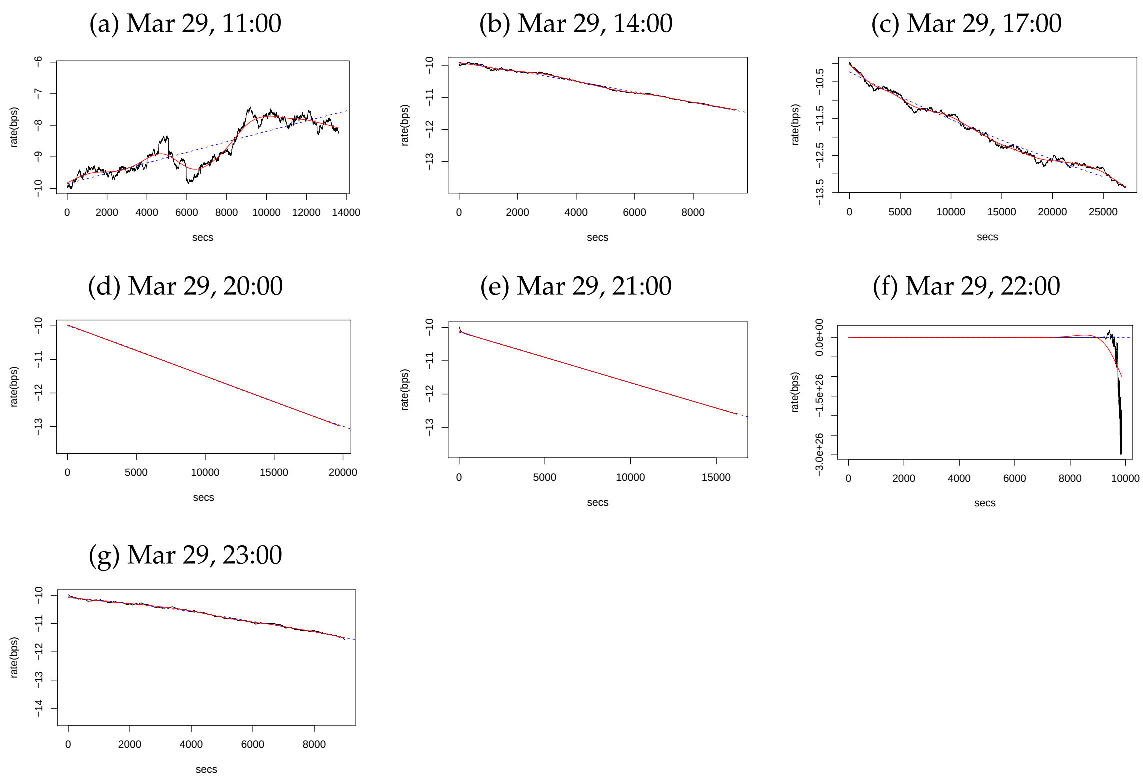
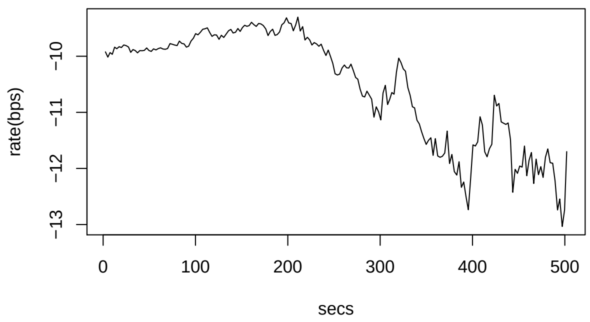
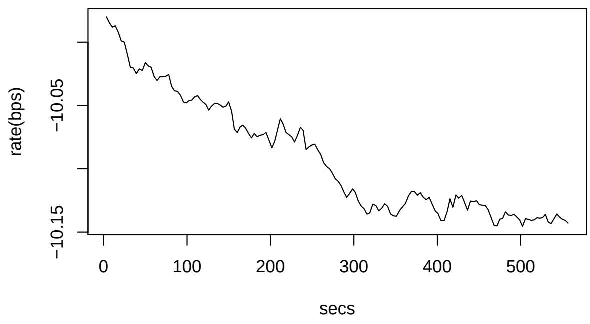


Figure 7: CHF simulated yield curves at the ultra short tenor on 29 March 2017 at 10 p.m (identified as period of EPM, Extreme Price Movements) and at 11 p.m.

(a) Mar 29, 10 p.m.



(b) Mar 29, 11 p.m.



List of Tables

Table 1: Descriptive statistics of returns and durations of intraday EURCHF FX data during the period from 8th to 16th of January 2015. N_{raw} is the total number of quotes without any filtering process. N_{filter} is the number of quotes after filtering out quotes within the same second. 15 January 2015 is the crash date when the Swiss National Bank removed the CHF/EUR floor.

Dates	Jan 8	Jan 9	Jan 12	Jan 13	Jan 14	Jan 15	Jan 16
	Number of EURCHF quotes						
N_{raw}	10887	17604	15045	12909	17390	78333	14752
N_{filter}	8900	13414	11790	10541	13523	30361	6962
	Returns ($\times 10^5$)						
mean	-.007	.0003	-.0071	-.0008	.0012	-6.228	4.141
std.dev	6.566	7.785	7.211	6.508	7.611	1517	111.6
median	0	0	0	0	0	0	0
min	-62.45	-62.45	-45.80	-45.80	-49.96	-123429	-1541
max	41.63	54.12	49.96	49.96	49.96	125843	1198
skew	-.1649	.0758	.1364	.1450	.0192	-6.060	-.4165
kurtosis	10.57	9.048	8.963	9.315	9.038	3579	27.04
LB(10)	344.4	1880	1455	1010	2011	3977	60.9
LB(25)	352.4	1909	1468	1057	2020	8913	155
	Durations (seconds)						
mean	6.633	5.904	7.326	8.195	6.389	2.845	3.927
std.dev	11.80	10.94	19.02	15.27	12.75	7.614	6.524
median	3	2	2	3	2	1	1
min	0	0	0	0	0	0	0
max	335	166	816	461	611	473	136
skew	6.715	4.900	17.25	9.061	15.44	18.58	5.338
kurtosis	104.3	37.55	546.0	175.3	548.9	765.7	53.96
LB(10)	630.2	2548	4410	2812	3304	20237	906.2
LB(25)	904	4552	7046	4339	4295	42984	1218

Table 2: Descriptive statistics of diurnally-adjusted returns and durations of EURCHF FX data during the period from 8th to 16th of January 2015.

Dates	Jan 8	Jan 9	Jan 12	Jan 13	Jan 14	Jan 15	Jan 16
	Adjusted returns						
mean	.0146	.0083	.0166	.0193	.0344	-.9852	1.446
std.dev	1.991	2.272	1.975	1.887	2.272	228.6	37.21
median	0	0	0	0	0	0	0
min	-31.64	-21.82	-15.02	-19.74	-28.12	-8317	-744.0
max	22.38	30.32	19.06	21.82	28.16	15221	436.5
skew	-.4004	.0850	.4561	.4215	.2354	25.76	-.4105
kurtosis	29.45	14.94	14.86	18.17	16.85	2092	46.19
LB(10)	184.3	1559	1177	650.9	1533	6242	40.73
LB(25)	198.7	1580	1185	701.2	1552	11057	109.8
	Adjusted durations (seconds)						
mean	.9609	.7843	.9457	1.058	.8246	.3672	.3578
std.dev	1.628	1.306	1.807	1.709	1.382	.8400	.5545
median	.3799	.3101	.3486	.4315	.3281	.1812	.1544
min	0	0	0	0	0	0	0
max	19.72	21.74	47.77	65.12	28.09	46.17	8.915
skew	4.119	4.438	7.248	8.028	5.086	15.53	4.654
kurtosis	25.99	32.43	109.6	206.9	48.08	498.6	37.17
LB(10)	672.9	1270	2067	978.4	2909	19481	726.2
LB(25)	1038	2060	3572	1797	4205	35828	936.7

Table 3: Estimations for the log-ACD UHF-GARCH model for 9th, 13th, 15th and 16th January 2015 for EURCHF using the last 1,000 observations at indicated times. Both returns and durations used have been adjusted for intraday diurnality. Standard errors for log-ACD are given in brackets. Robust standard errors are used for UHF-GARCH and also given in brackets. † indicates significance at 5% or better.

	log-ACD							UHF-GARCH									
	ω_1	α_1	β_1	γ	LL	$LB(10)$	$LB(25)$	ρ_2	ϕ_2	κ_2	ω_2	α_2	β_2	γ_2	LL	LB(10)	LB(25)
9 Jan, 9am	.0511†	.1024†	.8690†	.9211†	-951.02	3.702	11.94	-.0078	-.2531	-.0207†	.0884	.1375	.0000	.8903†	-1798.18	14.10	22.49
	(.0155)	(.0258)	(.0581)	(.0206)				(.1511)	(.1591)	(.0095)	(.0790)	(.1123)	(.0368)	(.1632)			
9 Jan, 10am	.0763†	.1418†	.9824†	.9320†	-859.64	14.89	36.44	.1358	-.4248†	-.0099	.0000	.1125	.0026	1.247†	-1952.41	11.00	20.02
	(.0149)	(.0263)	(.0195)	(.0208)				(.1185)	(.0830)	(.0081)	(.0000)	(.0793)	(.0276)	(.1457)			
9 Jan, 11am	.1130†	.2290†	.8810†	.8720†	-765.79	10.54	24.90	.0007	-.2947†	-.0076	.0000	.1659†	.0000	1.022†	-1952.42	11.36	23.85
	(.0235)	(.0316)	(.0400)	(.0192)				(.1304)	(.1282)	(.0055)	(.0000)	(.0611)	(.0767)	(.1622)			
13 Jan, 9am	.0381†	.0873†	.9796†	.9940	-750.10	4.580	22.02	.0352	-.3532	.0030	.1043	.1489	.0000	1.055†	-1944.55	18.92	42.92
	(.0104)	(.0227)	(.0180)	(.0222)				(.4551)	(.4235)	(.0155)	(.2573)	(.1136)	(.0684)	(.1976)			
13 Jan, 10am	.0697†	.1190†	.9572†	.9000†	-970.80	7.324	21.97	-.1982	-.0680	-.0093	.1622	.2741	.0000	.5191†	-1631.73	17.83	45.73
	(.0146)	(.0230)	(.0251)	(.0202)				(.1791)	(.2178)	(.0092)	(.1457)	(.1810)	(.0732)	(.1597)			
13 Jan, 11am	.0541†	.1055†	.8995†	.8986†	-911.87	4.233	23.91	.6596†	-.8017†	-.0044	.2153†	.5518†	.0000	.2487†	-1487	20.28	62.60
	(.0150)	(.0237)	(.0431)	(.0203)				(.1588)	(.1180)	(.0125)	(.0886)	(.2428)	(.0088)	(.0772)			
15 Jan, 9am	.0713†	.1319†	.9404†	.9236†	-950.85	8.836	15.33	.3501†	-.6877†	-.0009	.0036	.0065	.9931†	.0000	-1858.94	11.26	25.62
	(.0143)	(.0228)	(.0245)	(.0207)				(.1101)	(.0823)	(.0119)	(.0349)	(.0072)	(.0008)	(.0166)			
15 Jan, 10am	.0604†	.0937†	.9791†	.8483†	-987.95	10.43	30.37	-.1663	-.1462	.0151	.0000	.5952†	.6441†	.1764†	-3828.15	131.45	274.0
	(.0121)	(.0176)	(.0157)	(.0185)				(.1429)	(.1376)	(.0170)	(.0000)	(.1162)	(.0387)	(.0742)			
15 Jan, 11am	-.6500†	.3890†	.5700†	1.656†	861.65	6.995	23.82	.7270†	-.8793†	-.9537	18.43	.1614†	.8715†	.0000	-5086.88	47.77	71.60
	(.1566)	(.0454)	(.0951)	(.0311)				(.2383)	(.1663)	(3.870)	(87.55)	(.0480)	(.0430)	(7.492)			
15 Jan, 12pm	-.1900	.2100†	.8270†	1.473†	483.42	11.48	24.25	.4935†	-.6858†	-.1084	9.462	.0905†	.9220†	.0000	-5127	14.61	20.13
	(.1201)	(.0382)	(.0918)	(.0308)				(.1601)	(.1252)	(1.020)	(153.6)	(.0380)	(.0329)	(32.74)			
15 Jan, 1pm	-.0040	.0852†	.9855†	1.495†	504.81	9.290	24.00	.0206	-.1243	1.071	.0000	.0577	.0000	90.71†	-4464.11	3.109	20.40
	(.0210)	(.0198)	(.0153)	(.0305)				(.1541)	(.1499)	(1.618)	(.0015)	(.0504)	(.413)	(43.09)			
16 Jan, 8am	-.1610†	.2270†	.6880†	1.135†	-197.7	14.97	20.29	.9659†	-.9330†	.2412	18.01	.0660†	.9227†	.0000	-4882.29	7.170	35.08
	(.0821)	(.0400)	(.1093)	(.0242)				(.0245)	(.0384)	(.7643)	(28.98)	(.0235)	(.0278)	(7.664)			

Table 4: Nelson Siegel parameters estimated using nonlinear least squares. The long run factor, b_0, is constrained to the daily CHF O/N rate; it is not estimated except during the flash crash period. Standard errors are given in brackets. † indicates significance at 5% or better.

	b_0	b_1		b_2		τ	
9 Jan 09:00	-.4792	-.7451†	(.03022)	-7.600†	(.0741)	.0001†	(1.992e-6)
9 Jan 10:00	-.4792	-.6042†	(.0938)	-12.53†	(.1968)	3.428e-4	(5.488e-6)
9 Jan 11:00	-.4792	1.117†	(.0699)	-15.09†	(.1464)	4.265e-4	(4.000e-6)
13 Jan 09:00	-.7867	-1.120†	(.0133)	-3.732†	(.0404)	.0002†	(2.502e-6)
13 Jan 10:00	-.7867	-.5163†	(.0328)	-5.646†	(.0779)	.0004	(4.552e-6)
13 Jan 11:00	-.7867	-.4163†	(.0198)	-9.027†	(.0412)	7.086e-5†	(1.123e-6)
15 Jan 09:00	-.7686	-1.002†	(8.686e-3)	-2.410†	(.0293)	.0001†	(2.226e-6)
15 Jan 10:00	-.7686 (2.68e95)	.7686	(8.19e96)	.0100	(1.16e99)	.0100	(1.49e97)
15 Jan 11:00	6.921 (1.39e20)	.8492	(2.20e16)	.0766	(1.78e15)	-.0032	(2.95e12)
15 Jan 12:00	-.7686 (1.11e25)	.7686	(1.46e26)	.0100	(2.30e28)	.0100	(2.96e26)
15 Jan 13:00	-.7686	.4010†	(.0250)	-1.084e3	(1.232e-3)	1.530e-6	(1.745e-6)
16 Jan 08:00	-1.014	1.101†	(.2871)	-7.191†	(.5690)	.0016†	(.0001)