How Informative Are Central Bank Assessments of Macroeconomic Risks?

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Abstract

Surveying the forecasting practice of several central banks, we find that all these banks issue statements about risks to their macroeconomic forecasts. Often the balance of these risks is assessed as well. Upward [downward] risks to the forecast commonly imply that the outturn is expected to lie above [below] the central forecast. Investigating the inflation risk forecasts of the Bank of England and the Sveriges Riksbank, however, we do not find conclusive evidence for informativeness, that is, for a systematic connection between risk assessments and forecast errors. Thus, it seems questionable whether macroeconomic risk forecasts are meaningful.

Keywords: Forecast evaluation; risk forecasts; inflation forecasts

JEL-Classification: E37, C12, C53

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1 Introduction

> *"If Banks routinely report risk assessments, then those assessments should be systematically evaluated, just as the accuracy of Banks' inflation forecasts are evaluated. [...] If such an analysis finds no systematic connection between risk assessments and forecast errors, then the value of the risk assessments is called into question."*
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> — Eric Leeper (2003, p. 16)[1]

Nowadays, most major central banks publish point forecasts for macroeconomic variables playing an important role in the monetary policy decision process. Moreover, many central banks also give an assessment of the uncertainty surrounding these forecasts. In addition to these information, a large share of central banks issues statements about the probability of future outturns lying above or below the point forecasts, that is, about the asymmetry of the forecast density. In these statements, generally the term 'risk' appears. A single risk to the forecast is usually understood as a possible future event whose occurrence would lead to outturns differing markedly from the point forecasts. Statements about 'the overall risks', 'the balance of risks', or simply 'the risks' to a certain variable then apparently often refer to the entire set of single risks that have been identified and weighted by their perceived probabilities of materialization as well as their potential impact. These statements are supposed to contain information about the asymmetry of the forecast density. For example, a typical statement of this kind can read "Most participants viewed the risks to their projections for GDP growth as weighted to the downside".[2] We will further elaborate on the use of the term 'risk' by central banks below.

[1] In a footnote, Eric Leeper thanks Stefan Palmqvist for making this suggestion.

[2] See Board of Governors of the Federal Reserve System (2008, p. 41). Henceforth, we will refer to the Board of Governors of the Federal Reserve System simply as the Federal Reserve.

Determining the asymmetry of a forecast density, i.e. forecasting a phenomenon related to third moments is surely an extremely challenging task. This might be illustrated by the fact that many central banks assess their forecast uncertainty, i.e. a phenomenon related to second moments and therefore, in general, easier to assess, simply based on past forecast errors.[3] This is done due to the lack of models which can accomplish this task, as explained by Wallis (1989). However, if it is so difficult to forecast the uncertainty surrounding an institution's forecast appropriately, it is questionable whether risks can be forecast in a reasonable manner. Given that so many institutions engage in risk forecasting despite the difficulties to be encountered, it is important and interesting to find out how successful these risk forecasts are.

In this work, we study risk forecasts for inflation published by the Bank of England (henceforth BoE) and the Sveriges Riksbank (the central bank of Sweden, henceforth Riksbank). To the best of our knowledge, there have hardly been investigations of risk forecasts in the literature. For the BoE, which has the largest risk forecasting record, risk forecasts have at best been evaluated in the context of investigations of the entire forecast density. For instance, Wallis (2003) states that "the excessive concern with upside risk was not justified over the period considered." (p. 165). Mitchell and Hall (2005) could not reject the null hypothesis of equal density forecast accuracy for the BoE's asymmetric and the corresponding symmetric forecast densities. Both studies cited focus on one-year-ahead inflation forecasts.[4]

While these results already hint at the existence of problems with the risk

[3] See Deutsche Bundesbank (2010, pp. 34-36) for an overview.

[4] Wallis (2003) does not find major problems when studying the inflation nowcasts.

forecasts, it still remains to be analyzed if there is a systematic connection between the BoE's risk forecasts and its forecast errors. If there is a systematic connection, and if the point forecast is a mode forecast, then upside [downside] risks should on average be followed by outturns that are greater [less] than the point forecasts. Moreover, the size of the mode forecast error should on average correspond to the size of the (suitably defined) forecast risk. If this is the case, the risk forecasts can be considered optimal.[5] If there is no systematic connection, that is, if the risk forecasts do not help to predict the mode forecast errors, the risk forecasts are uninformative.

The analysis in this work is performed in the context of tests for forecast optimality. It turns out that there is considerable evidence against the optimality of risk forecasts at least for the BoE. For both central banks under study, we fail to find robust evidence in favour of an information content of risk forecasts. Put differently, it seems that there is no systematic connection between risk forecasts and mode forecast errors.

We present a survey of the current risk-forecasting practice at several central banks in Section 2. In Section 3, we analyze the risk forecasts of the BoE and the Riksbank with respect to optimality and information content. In Section 4, we briefly consider reasons for the apparent lack of information content. Section 5 concludes.

[5] To be more precise, they would at least be considered partially optimal in the sense of Diebold and Lopez (1996). The concept of optimality used will become clear in Section 3.1.

2 An Overview of Risk Forecasting

2.1 Definition of Risk

Although the term 'risk' is used in many forecasting-related publications of central banks, there is no unique definition of its meaning. In the New Palgrave Dictionary of Economics, Machina and Rothschild (2008) state that "A situation is said to involve *risk* if the randomness facing an economic agent presents itself in the form of exogenously specified or scientifically calculable objective probabilities, as with gambles based on a roulette wheel or a pair of dice." However, as mentioned above, the term 'risk' as used by central banks often seems to refer to important events with a rather uncertain probability of occurrence like a large change in oil prices or in exchange rates. A different interpretation of the term 'risk' as used by central banks is given by Kilian and Manganelli (2007) who link the risks to the preferences of central bankers. While this might be a valid interpretation with respect to several statements made by the Federal Reserve during a certain period, it is not adequate for the current risk forecasts of the Federal Reserve and many other central banks, among others those whose data are investigated in this study.

Many central banks devote a kind of stand-alone publication like a box, a chapter, or an article to their respective definitions of risk. An example-based but yet precise definition is given by the BoE in Britton et al. (1998, pp. 32-33).[6]

[6] It reads: "In deciding upon central assumptions and risks across key components of the forecast, it may become clear that the risks are unbalanced. A good example of this is the effect of 'windfall' gains to consumers from the conversion of several building societies to banks in 1997. Uncertainty about the pace at which the windfalls would be spent represented a risk to the forecast of consumer spending. The Bank's theoretical analysis suggested that only a small proportion of these gains would be spent in the first year, and correspondingly took this as a central view. In the Bank's judgment, the risks were much greater that actual expenditure would be in excess of the central forecast assumption than that it would be less. This was an upside risk to the forecast during most of 1997. In order to produce the fan chart, only one number is

According to the BoE, a risk is given by an uncertain and important event not taken into account in the central view, where the central view, i.e. the point forecast corresponds to the mode of the forecast density. In contrast to the definition of Machina and Rothschild (2008), the probability of the event is not exogenously specified or scientifically calculable, and in contrast to the interpretation of Kilian and Manganelli (2007), the risk is unrelated to preferences of the central bank. The balance of risks refers to the probabilities of the events mentioned producing values above or below the point forecast. So the balance of risks is directly related to the skewness of the forecast density which, in the case of the BoE, is measured as the difference between the mean and the mode of the forecast density. The fact that the mode (and not the mean) of the forecast density serves as the BoE's point forecast appears surprising, since aiming at the mode is associated with a rather implausible all-or-nothing loss function of the policy maker.[7] Interestingly, however, the mode also serves as the point forecast of many other central banks, as will be seen below.

Definitions of risk similar to the one used by the BoE can be found in the publications of other central banks. The Riksbank (1998, p. 36) writes: "[...] two aspects of the forecast distribution are assessed subjectively: whether the uncertainty in the forecast differs from the historical uncertainty [...], and whether the risk of forecasting errors is symmetric, upside or downside. In the absence of information to the contrary, the risk is assumed to be symmetric. [...] A

needed to summarise the degree of skewness (the balance of risks). Just as with the central view and the degree of uncertainty, there is more than one possible choice of parameter. The Bank's analysis focuses on the difference between the mean and the mode of the forecast distribution to be presented in the Report. This difference is of interest as a summary statistic of the balance of risks"

[7] See Wallis (1999) for a discussion.

skewed uncertainty (a difference between the upside and downside risks in the assessment of a particular variable, e.g. imports) affects the distribution of the inflation forecast by the amount of the variable's weight in the macro model. Skew is measured as the difference between the mean value and the most probable value (the mode of the distribution)." Again, the mode forecast serves as the point forecast. Further details concerning the forecasts of the Riksbank can be found in Blix and Sellin (1999). However, the Riksbank changed its forecasting procedure in 2007. Since then, it has only published symmetric forecast densities and has not mentioned the balance of risks, but just scenarios and risks. Its main forecast now seems to be a mean forecast: "The forecasts in the main scenario show the path which the Riksbank expects the economy to take and is a weighted consideration of various conceivable development paths (scenarios) and risks" (Riksbank, 2009 p. 22). The same approach is apparently used by the Norges Bank (the central bank of Norway).

Other central banks have followed the path set out by the BoE and the Riksbank, for example the Magyar Nemzeti Bank (the central bank of Hungary), which states that "The method that we follow in preparing fan charts broadly corresponds to that of the Bank of England, and the same holds true for the Swedish method." (Magyar Nemzeti Bank, 2004, p. 108).

The Federal Reserve (2008, p. 45) explains that the members of the Board of Governors and the presidents of the Federal Reserve Banks "provide judgments as to whether the risks to their projections are weighted to the upside, downside, or are broadly balanced. That is, participants judge whether each variable is more likely to be above or below their projections of the most likely outcome." So, in contrast to the approaches mentioned so far, the risk assessments are only qualitative (upside, downside, or broadly balanced risks), not quantitative. That is, no number is attached to the risk forecasts, but only the direction of the risk is given. The same applies to the risk forecasts of the European Central Bank (henceforth ECB). For example, the ECB states that "In the Governing Council's assessment, the risks to this improved economic outlook are slightly tilted to the downside" (ECB, 2010, p. 6).[8]

Several other central banks also link the overall forecast risks to the asymmetry of the forecast density, among them the Bank of Canada, the Banco Central de Chile, the Banco de España, the Bank of Japan, the Banco de Portugal, the Deutsche Bundesbank, and the International Monetary Fund (henceforth IMF).[9] Details concerning the corresponding references are given below.

We also found central banks which regularly report their assessments of individual risks, but which do not always mention the balance of these risks. For example, the Reserve Bank of Australia states that "Risks to these forecasts can be identified in both directions. A further deterioration in the outlook for global growth would be the main source of downside risk to the forecasts for domestic activity" (Reserve Bank of Australia, 2008, p. 68). Another example is the Swiss National Bank declaring that "The biggest risk for the global economy is the con-

[8]However, the ECB often also mentions "risks to price stability", where the term 'risks' rather appears to refer to the possibility that the ECB might not achieve its "aim of keeping inflation rates below, but close to, 2% over the medium term." (ECB, 2010, p. 6). In this case, the risks are apparently unrelated to the asymmetry of a forecast density. Note that the risks to price stability can also be asymmetric, as described in the statements "The information that has become available [...] has confirmed that [...] upside risks to price stability over the medium term prevail." (ECB, 2008, p. 5). Yet, this asymmetry supposedly just refers to the probability of observing inflation rates above 2% over the medium term being larger than 50%, and is therefore also unrelated to forecast densities.

[9]Of course, the IMF is not a central bank, but an intergovernmental organization. Nevertheless, we will consider the IMF here, because its interpretation of risk forecasts is identical to the one used by most central banks.

tinued increase in tension on financial markets [...]. At the same time, there are upside risks for the global economy [...]" (Swiss National Bank, 2010a, p. 40). In both cases, no overall assessment of risks follows. Yet, sometimes, such an assessment is made by these central banks.[10] The Reserve Bank of New Zealand behaves similarly to the two aforementioned central banks, with clear statements concerning the overall risks on some occasions and only mentioning individual risks on others. The same applies to the Bank of Israel.

In Table 1, we present an overview of the risk forecasting practice of several central banks.[11] Some caution is warranted with this overview because the approaches to risk forecasting might change over time like in the case of the Riksbank. Moreover, just because we have not been able to discover statements concerning the balance of risks, the definition of the point forecast or upward risks in some cases, it is not impossible that these exist.[12] All central banks shown regularly discuss risks to their forecasts. In Table 6 in Appendix A we show where these discussions can be found. The references on which Table 1 and several of the statements made above are based are also collected in Appendix A.

It should be noted that for many asymmetric distributions, the inequalities $E[Y] > mode(Y)$, $P(y > mode\,(Y)) > P(y < mode\,(Y))$ and $E[(Y - E[Y])^3] > 0$ imply each other.[13] This holds, for example, in case of the two-piece normal

[10] For example, the Swiss National Bank claims that "At present, the upside and downside risks are relatively balanced" (Swiss National Bank, 2010a, p. 26) for output growth.

[11] In Table 1, GDP denotes the growth rate of the real gross domestic product, CPI the inflation of the Consumer Price Index, PCD the inflation of the Private Consumption deflator, HICP the inflation of the Harmonised Index of Consumer Prices, PCE the growth rate of the price index for Personal Consumption Expenditures, and UND1X the inflation of the CPI excluding household mortgage interest expenditure and the direct effects of changes in indirect taxes and subsidies.

[12] We corresponded with members of several central banks in order to minimize the possibility that certain definitions slipped our attention.

[13] The same applies to the reversed inequalities $E[Y] < mode(Y)$, $P(y > mode\,(Y)) < P(y < mode\,(Y))$ and $E[(Y - E[Y])^3] < 0$. $E[Y]$ denotes the expectation of Y and $P\,(A)$ the probability

Central Bank	**Balance of Risks**	**Point Forecast**	**Meaning of** *"balance of risks tilted to the upside"*	**Figure related to forecast distribution**
Bank of Canada	quantitative; inflation	mode	$E[Y] >$ $\text{mode}(Y)$	asymmetric fan chart
Banco Central de Chile	quantitative; GDP, CPI, core CPI	mode	$P(y > \text{mode}(Y)) >$ $P(y < \text{mode}(Y))$	asymmetric fan charts
Bank of England	quantitative; GDP and CPI	mode	$P(y > \text{mode}(Y)) >$ $P(y < \text{mode}(Y))$	asymmetric fan chart
Banco de España	qualitative; GDP, PCD	mode	$P(y > \text{mode}(Y)) >$ $P(y < \text{mode}(Y))$	symmetric fan charts
Bank of Israel	rarely; inflation	no definition found	no definition found	symmetric fan chart
Bank of Japan	quantitative; GDP and CPI	forecast interval based on modes of individual forecast densities	positive skewness of (aggregated) forecast density	asymmetric histogram
Banco de Portugal	quantitative; GDP and components, HICP, exogenous variables	mode	$P(y > \text{mode}(Y)) >$ $P(y < \text{mode}(Y))$	asymmetric fan charts
Board of Governors of the Federal Reserve System	qualitative; GDP, PCE, unemployment	forecast interval based on modes of individual forecast densities	individual forecasts: $P(y > \text{mode}(Y)) >$ $P(y < \text{mode}(Y))$	histogram of individual point forecasts
Deutsche Bundesbank	qualitative; GDP and HICP	mode	positive skewness of forecast density	symmetric fan chart
European Central Bank	qualitative; GDP and HICP, global activity	none (forecast interval by staff)	no definiton found	none
International Monetary Fund	quantitative; world GDP, but also term spread, S&P500, oil price, inflation	mode	$P(y > \text{mode}(Y)) >$ $P(y < \text{mode}(Y))$	asymmetric fan chart
Magyar Nemzeti Bank	quantitative; GDP and CPI	mode	$P(y > \text{mode}(Y)) >$ $P(y < \text{mode}(Y))$	asymmetric fan chart
Norges Bank	no	mean (=mode)	-	symmetric fan chart
Reserve Bank of Australia	sometimes; growth and inflation	mode	$P(y > \text{mode}(Y)) >$ $P(y < \text{mode}(Y))$	none
Reserve Bank of New Zealand	sometimes; qualitative; activity, inflation, global growth	mode	no definition found	none
Sveriges Riksbank 2007-present	no	mean (=mode)	-	symmetric fan chart
Sveriges Riksbank 1999-2006	quantitative; CPI and UND1X	mode	$E[Y] >$ $\text{mode}(Y)$	asymmetric fan chart
Swiss National Bank	sometimes; qualitative; inflation	no definition found	no definition found	none

Table 1: Forecasts of central banks

distribution which is used by the BoE, the Magyar Nemzeti Bank, the IMF and the Riksbank (until 2006) for their asymmetric fan charts. We will elaborate on this distribution below. Some fan charts are almost always asymmetric (e.g. those of the Banco de Portugal), others only in special situations (e.g. those of the Bank of Canada).

To summarize, all central banks considered discuss risks to their forecasts, and many of them also assess the balance of these risks. All definitions of an asymmetric balance of risks relate to an asymmetric forecast density. Remarkably, the point forecast published corresponds to the mode of the forecast density for almost all central banks, although the mode is associated with a rather implausible all-or-nothing loss function of the policy maker.

It is perfectly possible that a central bank considers the balance of risks to be asymmetric and nevertheless publishes symmetric fan charts as in the case of the Deutsche Bundesbank, the Banco de España, or the Bank of Israel. This could be due to the fact that these central banks, as many others, assess the balance of risks in a qualitative manner only. The quantitative assessment is mostly published in the form of a fan chart. Only the Banco de Portugal and the Riksbank (1999-2006) release numbers measuring the risk of the forecast density in their main publications.[14] The Banco de Portugal shows the probabilities of an outturn below the central projection. The Riksbank (1999-2006) publishes the values of the mode and the mean of the forecast density.

In what follows, the term 'risk forecast' will refer to the balance-of-risk forecast, i.e. to a potentially asymmetric forecast density. It will thus not denote the

of event A.

[14] Numbers for the BoE's and the Magyar Nemzeti Bank's density forecasts can be downloaded from their respective websites.

assessment of certain individual risks without the evaluation of their overall effect on the forecast variable of interest.

2.2 Reasons for Risk Forecasting

The IMF (2008, pp. 42-43) offers four reasons for potential asymmetries of forecast densities. The first reason is given by non-linearities like capacity constraints or the zero-lower bound for interest rates. The second reason relates to the possibility that the numbers of the central forecast are already fixed when sudden large changes occur in important variables like oil prices or exchange rates. While, owing to time constraints, it might be infeasible to calculate a new central forecast and to adapt the often extensive explanatory notes coming with the central forecast, it might be possible to adapt the asymmetry of the forecast density to reflect the new information. Thirdly, forecasts are often based on "technical" assumptions, in many cases concerning exogenous variables. For example, exchange rates are commonly assumed to be constant for all forecast horizons. If such an assumption is used, maybe due to institutional reasons, but appears rather problematic owing to special circumstances, for example the forecaster's strong belief that the domestic currency is soon going to appreciate, the asymmetry of the forecast density can be employed to allow for this view. The fourth reason concerns the possibility of biased forecasts. If recent forecast errors appear to be biased, maybe owing to an undetected structural change, the forecaster might refrain from shifting the current central forecasts, but might be inclined to issue an asymmetric forecast density in order to account for the potential bias.

Another motivation, not mentioned by the IMF, could be given by the expectation of asymmetric shocks to a variable. For example, a forecaster could simply believe that, in a certain situation, positive shocks are more likely than negative shocks.[15]

In institutions where several decision makers have to agree on a single central forecast, as, for example, the board members of the BoE, there might exist an additional reason for issuing risk forecasts. If a minority of the decision makers does not agree with the central forecast decided upon by the majority, the view of the minority can be accommodated by the asymmetry of the forecast density.

Finally, risk forecasts could also be employed as a (subtle) communication device. For example, if the forecasts of an inflation-targeting central bank are made conditional on future interest rates as expected by market participants, it is rather unlikely that the inflation forecast deviates strongly from the target at the relevant policy horizon. If there was a strong deviation, this might cause market participants to believe that either the policy of the central bank has changed, or that the central bank's assessment of the economic conditions strongly differs from their own assessment. Both possibilities are not very attractive for a central bank that seeks to be transparent and predictable. In order to signal the possibility of an unexpected change in the policy rate, a central bank might therefore prefer to issue a risk statement. By doing so, the central bank's central forecast is in line with

[15] Actually, this reason for risk forecasts is very common in the publications of central banks. Two examples read: "Risks to this [inflation] outlook are fully confirmed to lie on the upside. These risks include the possibility of further rises in oil and agricultural prices, as well as of unanticipated increases in administered prices and indirect taxes" (ECB, 2007, p. 55) and "The balance of risks to the inflation outlook, relative to the central projection, lies on the upside, as the prospect of a faster exchange rate depreciation and the associated adjustment to the level of import prices is the dominant influence." (BoE, 2002, p. 49). It is unlikely that non-linear forecasting models are the motivations for assuming unbalanced risks in oil and agricultural prices or exchange rates. Moreover, apparently no technical assumption was made by the BoE with respect to exchange rates. Rather, the forecasters simply seem to anticipate an asymmetry of future shocks in both cases.

the forecasts of the market participants. At the same time, the central bank makes clear that these forecasts are subject to uncertainty, and that the materialization of certain risks considered to be likely by the central bank would require a policy response different from the one expected by the market. As the probabilities of these risks materializing will change over time, the market participants will adapt their interest rate expectations accordingly. However, Rasche and Thornton (2002) investigate the balance-of-risk statements of the Federal Reserve and find that these do not appear to be a crucial factor for the market expectations concerning the Fed Funds Rate.[16]

2.3 Methods for Risk Forecasting

Reading through the publications of central banks, it seems that risks to inflation or other aggregates are commonly identified via risks to variables that determine these aggregates. For example, an upward risk to inflation might be caused by an upward risk to oil prices, to the value added tax rate or by a risk of depreciation of the domestic currency. Thus, in order to correctly forecast the risks to inflation, one has to forecast the risks to these determinants. Actually, the process of risk forecasting might be thought of as a three-step process. In the first step, one has to identify those determinants which are subject to forecast risks. In the second step, one has to quantify these risks, and in the third step, their impact on the aggregate of interest has to be calculated.

All of these steps appear extremely demanding. The first step requires the identification of variables whose most likely future paths (represented by the mode

[16] Yet, the balance-of-risk statements supposedly had a different interpretation during that period. They were rather related to a loss-function as suggested by Kilian and Manganelli (2007) than to asymmetries of forecast densities.

forecast) differ from their expected future paths (represented by the mean forecast). This might be possible for fiscal variables like the value added tax rate, where one could imagine that a certain rate is likely, but that an alternative rate is discussed by the government at the time the forecast is made. For variables like oil prices and exchange rates, however, this task is very challenging. The subsequent quantification of the identified risks appears fairly difficult as well. Elekdag and Kannan (2009) propose methods to accomplish this task which can be applied to certain variables. However, their empirical performance is not evaluated. Apparently, most central banks rely on judgement for the identification as well as the quantification of risks. Yet, even if the risks to determinants are correctly identified and quantified, assessing their impact on the aggregate of interest is not trivial, as explained in Pinheiro and Esteves (2010).

3 Evaluation of Risk Forecasts

3.1 The Methods

The evaluation of risk forecasts can be based on tests for risk forecast optimality as discussed in Knüppel and Schultefrankenfeld (2011). In order to conduct such tests, a measure for the asymmetry of the forecast density has to be chosen. Knüppel and Schultefrankenfeld (2011) find that the Pearson mode skewness is strongly preferable to the standard third-moment based skewness, because the latter implies a very low power of the optimality tests. The Pearson mode skewness of a random variable Y is given by $(E[Y] - m)/\sigma$, where m is the mode of Y and σ its standard deviation.

Given a forecast horizon h, risk forecasts can be evaluated using the OLS regression equation

$$\frac{y_{t+h} - \hat{m}_{t+h|t}}{\hat{\sigma}_{t+h|t}} = \alpha + \beta \frac{\hat{\mu}_{t+h|t} - \hat{m}_{t+h|t}}{\hat{\sigma}_{t+h|t}} + \varepsilon_{t+h}, \quad t+h = 1, 2, ..., T, \tag{1}$$

where y_{t+h} is the realization of the variable of interest in period $t+h$, $\hat{m}_{t+h|t}$ is the mode forecast for period $t+h$ made in period t, $\hat{\mu}_{t+h|t}$ is the corresponding mean forecast, and $\hat{\sigma}_{t+h|t}$ is the corresponding forecast of the standard deviation. ε_{t+h} is a zero-mean error term, and α and β are the coefficients to be estimated. T denotes the sample size.

The term $\left(\hat{\mu}_{t+h|t} - \hat{m}_{t+h|t}\right)/\hat{\sigma}_{t+h|t}$ on the right-hand side is simply the risk forecast for period $t+h$ made in period t, where risk is measured by the Pearson mode skewness. On the left hand-side, $\left(y_{t+h} - \hat{m}_{t+h|t}\right)/\hat{\sigma}_{t+h|t}$ is the measure of realized risk. This measure is simply the scaled mode forecast error. If the risk forecasts are optimal, it should not be possible to reject the joint hypothesis $\alpha = 0, \beta = 1$. If the risk forecasts contain useful information, one should expect a rejection of the hypothesis $\beta = 0$.[17]

One would rather like to use $\left(y_{t+h} - m_{t+h}\right)/\sigma_{t+h}$ as the measure of realized risk, where m_{t+h} and σ_{t+h} are the true values of mode and standard deviation of the forecast variable in period $t+h$, but these quantities are, of course, unknown. However, as long as $\hat{m}_{t+h|t}$ and $\hat{\sigma}_{t+h|t}$ are unbiased forecasts of m_{t+h} and σ_{t+h}, no major complications arise. If $\hat{m}_{t+h|t}$ is biased, $\hat{\alpha}$, the estimate of α will be biased as

[17] One might think that testing for the presence of useful information should rather be based on the joint hypothesis $\alpha = 0, \beta = 0$. However, suppose that risk forecasts and realized risks are independent of each other, and that, on average, balanced risks are forecast. Suppose further that Y_{t+h} is always positively skewed. In this case, $\alpha > 0$ holds, although the risk forecasts contain no information.

well, but $\hat{\beta}$, the estimate of β, will continue to be consistent.[18] If $\hat{\sigma}_{t+h|t}$ is biased, also the estimate of β will be biased. Therefore, Knüppel and Schultefrankenfeld (2011) recommend to check for a potential bias of $\hat{\sigma}_{t+h|t}$, and to focus on the hypothesis $\beta = 1$ if there are doubts about the unbiasedness of $\hat{m}_{t+h|t}$. So if $\alpha = 0, \beta = 1$ is rejected while $\beta = 1$ is not, the risk forecasts might still be optimal. A complementing test of $\alpha = 0$ could be useful in this situation, because a rejection of $\alpha = 0$ would reinforce the supposition that the rejection of $\alpha = 0, \beta = 1$ could just as well be caused by biased mode forecasts.

Summing up, there are several null hypotheses of interest. Concerning risk forecast optimality, a suitable joint hypothesis is given by $\alpha = 0, \beta = 1$, resembling the hypothesis for mean forecast optimality used by Mincer and Zarnowitz (1969). The single hypothesis for risk forecast optimality $\beta = 1$ is robust with respect to a potential bias of the mode forecasts. In addition, the single hypothesis $\alpha = 0$ might also be of interest to complement the tests of $\alpha = 0, \beta = 1$ and $\beta = 1$. Finally, a test of the hypothesis $\beta = 0$ can be used to check for a systematic connection between forecast risks and realized risks, i.e. between forecast risks and the scaled mode forecast errors. In view of the excerpt from Leeper (2003) cited at the beginning, the latter hypothesis could actually be considered the most important one.

Although the tests have a much larger power if the Pearson mode skewness instead of the standard skewness[19] is used as the measure of asymmetry, their

[18] In what follows, a hat over a character always denotes the estimate or the forecast of the corresponding parameter. Whether the object is a forecast or an estimate will be clear from the context.

[19] Using standard skewness, the forecast risk would be given by $E\left[\left(Y_{t+h|t} - \hat{\mu}_{t+h|t}\right)^3\right] / \hat{\sigma}^3_{t+h|t}$ and the proxy for the realized risk by $\left(y_{t+h} - \hat{\mu}_{t+h|t}\right)^3 / \hat{\sigma}^3_{t+h|t}$. $E\left[\left(Y_{t+h|t} - \hat{\mu}_{t+h|t}\right)^3\right]$ would be

power can still be expected to be fairly low in empirical applications. This is due to several reasons. The available samples of macroeconomic risk forecasts are rather small. Moreover, the magnitude of the forecast risks is usually at best moderate. But the smaller these magnitudes are, the more difficult the inference about β becomes. In addition, the fact that m_{t+h} and σ_{t+h} are unknown also contributes to the low power, even if $\hat{m}_{t+h|t}$ and $\hat{\sigma}_{t+h|t}$ are unbiased forecasts.

As described above, many central banks do not publish quantitative, but only qualitative risk forecasts, giving assessments about the direction of the forecast risk. Therefore, it is important to evaluate direction-of-risk forecasts as well. The evaluation here is based on the categorical variables q_{t+h} and $\hat{q}_{t+h|t}$, determined by the quantitative risk forecasts and realizations, where q_{t+h} is related to the direction of the realized risk by

$$q_{t+h|t} = \begin{cases} 1 & \text{if } y_{t+h} > \hat{m}_{t+h|t} \\ 0 & \text{if } y_{t+h} < \hat{m}_{t+h|t} \\ na & \text{if } \hat{\mu}_{t+h|t} = \hat{m}_{t+h|t} \end{cases} \tag{2}$$

and $\hat{q}_{t+h|t}$ is related to the direction of the forecast risk by

$$\hat{q}_{t+h|t} = \begin{cases} 1 & \text{if } \hat{\mu}_{t+h|t} > \hat{m}_{t+h|t} \\ 0 & \text{if } \hat{\mu}_{t+h|t} < \hat{m}_{t+h|t} \\ na & \text{if } \hat{\mu}_{t+h|t} = \hat{m}_{t+h|t} \end{cases} \tag{3}$$

where na denotes a missing value. So if $q_{t+h|t} = \hat{q}_{t+h|t}$, the direction-of-risk forecast is succesful, and if $q_{t+h|t} \neq \hat{q}_{t+h|t}$, it is not.

In order to construct variables without missing values, we use the transforma-

calculated based on the forecast density of the random variable $Y_{t+h|t}$.

tions

$$\mathbf{q}_h^N = \mathbf{A}_h \mathbf{q}_h^T \tag{4}$$
$$\hat{\mathbf{q}}_h^N = \mathbf{A}_h \hat{\mathbf{q}}_h^T$$

where the vector $\mathbf{q}_h^T$ is given by $\mathbf{q}_h^T = \left(q_{1+h|1}, q_{2+h|2}, \ldots, q_{T+h|T}\right)'$, the vector $\hat{\mathbf{q}}_{t+h|t}$ is given by $\hat{\mathbf{q}}_{t+h|t} = \left(\hat{q}_{1+h|1}, \hat{q}_{2+h|2}, \ldots, \hat{q}_{T+h|T}\right)'$ and $\mathbf{A}_h$ is a known $(N \times T)$ selection matrix consisting of 1's and 0's, and with $N \leq T$. $\mathbf{A}_h$ is chosen such that the vectors $\mathbf{q}_h^N$ and $\hat{\mathbf{q}}_h^N$ do not contain missing values.[20]

Denoting the elements of $\mathbf{q}_h^N$ and $\hat{\mathbf{q}}_h^N$ by $q_{n+h|n}$ and $\hat{q}_{n+h|n}$, respectively, with $n = 1, 2, \ldots, N$, we test for the optimality of the direction-of-risk forecasts based on the regression

$$q_{n+h|n} = \alpha_q + \beta_q \hat{q}_{n+h|n} + \varepsilon_{t+h}. \tag{5}$$

Optimal risk forecasts do neither imply $\beta_q = 1$ nor $\alpha_q = 0$, as explained in Knüppel and Schultefrankenfeld (2011). However, a necessary condition for forecast optimality is given by the inequality $\beta_q > 0$. While a test of this inequality should not reject in case of optimal forecasts, the hypotheses $\beta_q < 0$ and $\beta_q = 0$ should be rejected. The latter hypothesis implies that the risk forecasts are not informative. Unfortunately, tests based on direction-of-risk forecasts, in general, suffer from even larger power problems than tests based on quantitative risk forecasts.[21] Yet, they might be more reliable in case of outliers or severe problems with the

[20] If there were no missing values, $\mathbf{A}_h$ would be a $(T \times T)$ identity matrix. If, for example, the risk forecast for the first forecast of horizon h was balanced, i.e. in the case $\hat{\mu}_{1+h|1} = \hat{m}_{1+h|1}$, N would equal $T-1$ and $\mathbf{A}_h$ would be given by $\mathbf{A}_h = [\mathbf{0}_N \;\; \mathbf{I}_N]$, where $\mathbf{0}_N$ denotes an $(N \times 1)$ vector of 0's and $\mathbf{I}_N$ denotes the $(N \times N)$ identity matrix.

[21] In principle, tests based on the binomial distribution could also be used, but the potential serial correlation of ε_{t+h} renders the regression-based approach presented here more appealing for our investigations.

forecasts of σ_{t+h}.

3.2 The Data

The risk forecast record of most central banks is rather limited. In order to be able to draw conclusions about the optimality of risk forecasts, we therefore attempt to focus mainly on quantitative risk forecasts. Moreover, these risk forecasts should imply at least moderate asymmetries, because, as stated above, with only small asymmetries, inference becomes too unreliable.

The first quantitative risk forecast that we are aware of was published by the BoE in its Inflation Report from February 1996 for inflation.[22] As stated above, the Riksbank issued quantitative risk assessments from 1999 to 2006. The Banco de Portugal started to give quantitative risk forecasts in December 2003, but these are annual forecasts, so that only very few observations are available. The IMF and the Bank of Japan only publish annual risk forecasts, too. The quantitative risk forecasting record of the Magyar Nemzeti Bank dates back to its Inflation Report from November 2002, and its forecasts have a quarterly frequency. However, the magnitudes of the asymmetries we backed out from its density forecast data are far too small for reliable inference about β in equation (1).[23] Finally, the Bank of Canada and the Banco de Chile do not publish data which allow a calculation of their quantitative forecast risks. Therefore, our subsequent analysis will be

[22] According to the numerical parameters that can be downloaded at http://www.bankofengland.co.uk/publications/inflationreport/irprobab.htm, the skewness of the forecast density then differed from zero for the first time.

[23] The Magyar Nemzeti Bank publishes certain quantiles of its forecast densities which are based on the two-piece normal distribution. From these information, the parameters of the distributions can be backed out in a similar way as described for the Riksbank in Appendix B.2. The spreadsheets containing the quantiles can be accessed via http://english.mnb.hu/Kiadvanyok/mnben_infrep_en. Further details are available upon request.

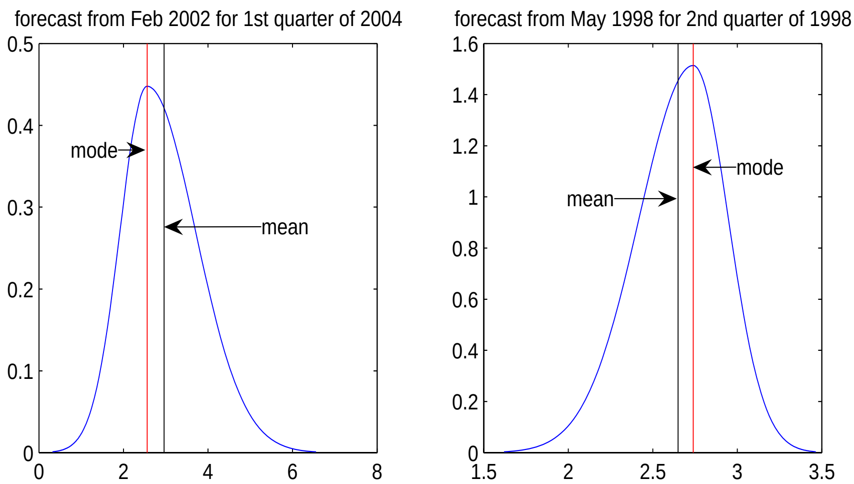

Figure 1: Two of the Bank of England's density forecasts for inflation. The forecast in the left panel implies an upward risk (mean > mode), the one in the right panel a downward risk (mean < mode).

restricted to the risk forecasts of the BoE and the Riksbank.

The BoE and the Riksbank use the two-piece normal distribution (henceforth tpn-distribution) as described, for example, in Wallis (2004, p. 66). The density of a tpn-distributed variable Y is given by

$$f(y) = \begin{cases} A \exp\left(-\frac{(y-m)^2}{2\sigma_1^2}\right) & \text{if } y \leq m \\ A \exp\left(-\frac{(y-m)^2}{2\sigma_2^2}\right) & \text{if } y \geq m, \end{cases}$$

with $A = \frac{2}{\sqrt{2\pi}(\sigma_1+\sigma_2)}$ and m denoting the mode of the distribution. The more σ_1 differs from σ_2, the more asymmetric the distribution becomes. If σ_1 and σ_2 are identical, a normal distribution is obtained. In Figure 1, two examples of tpn-distributions are displayed. These are inflation forecast densities of the BoE.

Our analysis uses the BoE's inflation forecasts based on the assumption that the future official Bank Rate, i.e. the interest rate paid on commercial bank reserves follows a path implied by market interest rates. In line with Elder et al. (2005), for the purpose of forecast evaluations we consider this assumption more adequate than the assumption of a constant official Bank Rate.

The BoE's inflation forecasts employed in our analysis range from the first quarter of 1998 (henceforth denoted 1998q1) - the first time the aforementioned interest assumption was used - to 2010q2. Each of the BoE's quarterly projections covers the current and the subsequent 8 quarters. For some forecasts, mean and mode forecast coincide, so that the risk forecast equals zero, i.e. the risks to the inflation forecast are balanced. Until 2003, the BoE forecast the inflation of the All Items Retail Price Index (RPI) excluding mortgage interest payments (henceforth RPIX). Since 2004, it has forecast the inflation of the Consumer Price Index (henceforth CPI).[24] The BoE publishes several parameters of the forecast densities which allow a straightforward calculation of our risk measure, i.e. of the Pearson mode skewness. Details are given in Appendix B.1.

The BoE also publishes risk forecasts for GDP. We do not study these forecasts here, since the analysis of GDP risk forecasts would be more complicated due to the effects of data revisions. Such revisions play a substantial role for the assessment of the BoE's GDP forecasts, as noted by Elder et al. (2005).

Our forecast data from the Riksbank starts in December 1999, when the data used to produce the fan charts of the respective Inflation Report were made publicly available on the Riksbank's website for the first time.[25] The last asymmetric

[24] When outturns are compared with forecasts, this change has of course to be taken into account. For instance, an inflation forecast for the fourth quarter of 2004 has to be compared with CPI inflation data if the forecast was made in 2004. If the forecast was made before 2004, it must be compared with RPIX inflation data.

[25] See http://www.riksbank.se/templates/DocumentList.aspx?id=5031

fan chart appeared in October 2006. From 1999 to 2005, there were always 4 Inflation Reports per year, namely in February/March/April, May/June, October, and December. Like the Riksbank, we will refer to these as the Inflation Reports y:1, y:2, y:3, and y:4, respectively, where y stands for the year. The last (possibly) asymmetric fan chart appeared in the Inflation Report 2006:3. There was no Inflation Report 2006:4, and since 2007, the fan charts of the Monetary Policy Reports, which succeeded the Inflation Reports, are always symmetric. Therefore, our data sample covers the forecasts from the Inflation Reports 1999:4 to 2006:3.[26]

The Riksbank forecasts two monthly inflation measures, where we decide to focus on CPI inflation only. From the available 24 forecast horizons, we use every third monthly forecast, so that we are left with 8 forecast horizons, always with one observation per quarter.[27] The shortest forecast horizon is chosen such that it contains a forecast for the month of the publication of the Inflation Report, so that this forecast is actually a nowcast. Thus, we are considering the Riksbank's 0-, 3-, 6-, ..., and 21-month-ahead forecasts. In contrast to the BoE, the Riksbank only published the mode of the forecast density and the values of several quantiles. From these data and the corresponding statements in the Inflation Reports, we carefully back out the parameters which permit calculating the Pearson mode skewness of the forecast densities. Details are given in Appendix B.2.

A potential drawback of the data from the Sveriges Riksbank is given by the constant interest rate assumption underlying the forecasts in the Inflation Reports

[26] Unfortunately, the forecast data from the Inflation Report 2000:1 are not available, so that our sample contains only 27 instead of 28 forecasts.

[27] Using all monthly forecasts yields practically no additional insights, since the monthly risk forecasts are quantitatively very similar for adjacent forecast horizons, and also the forecast errors of adjacent horizons are often similar. Moreover, using every third monthly forecast only, the results can be compared more easily to those for the BoE.

1999:4 to 2005:2.[28] That is, it was assumed that the interest rates do not change during the forecasting period, but remain on the level they had attained at the time the forecast was produced. Starting with the Inflation Report 2005:3, the forecasts have been conditioned on interest rates expected by market participants.

In the case of a constant interest rate assumption, it might be difficult to assess the optimality of risk forecasts at least for larger horizons. For example, if the constant interest rate assumption leads to inflation forecasts that exceed the target at the relevant policy horizon, the policy maker is likely to raise the policy rate to dampen inflation. So the inflation forecast error mainly depends on the point forecast for inflation, and only to a very limited extent on the risk forecast. Only if the central inflation forecast implies that inflation will be close to target, the inflation forecast error could be well predicted by the risk forecast if the risk forecast is optimal.

For short horizons, testing for risk forecast optimality should be possible even with a constant interest rate assumption for two reasons. Firstly, a constant interest rate assumption is probably a good approximation to the behaviour of interest rates in the short run. Secondly, inflation responds to changes of the interest rate only with a certain delay. Therefore, it should be feasible to predict the inflation forecast errors for small horizons using the risk forecasts if the risk forecasts are optimal and the forecasts are conditioned on constant interest rates.[29]

The actual risk forecasts of the BoE and the Riksbank as measured by the

[28] See Sveriges Riksbank (2005, p. 57).

[29] One could think about "correcting" the point forecasts based on constant interest rates such that they resemble forecasts based on market interest rates. Such a correction, in most cases, would probably result in a shift of the point forecast towards the inflation target. However, without further knowledge about the historical interest rate expectations of market participants and the interest-elasticity of the inflation forecasts in Sweden, such a correction is infeasible.

Pearson mode skewness of the forecast densities are displayed in Appendix B.3 in Tables 7 and 8.

3.3 Bias of Volatility, Mode, Mean and Risk Forecasts

As mentioned above and explained in Knüppel and Schultefrankenfeld (2011), a biased forecast of the standard deviation $\hat{\sigma}_{t+h|t}$ leads to biased estimates of α and β in equation (1). If the true standard deviations σ_{t+h} are, on average, smaller than $\hat{\sigma}_{t+h|t}$, the estimates of α and β are biased towards zero. Conversely, if σ_{t+h} is larger than $\hat{\sigma}_{t+h|t}$, the estimates of α and β are biased away from zero. It is therefore important to test the unbiasedness of the volatility forecasts before evaluating the risk forecasts. We do so by testing the null hypothesis $\alpha^{\sigma^2} = 0$ in the equation

$$\left(y_{t+h} - \hat{\mu}_{t+h|t}\right)^2 - \hat{\sigma}^2_{t+h|t} = \alpha^{\sigma^2} + \varepsilon^{\sigma^2}_{t+h}$$

for $h = 0, 1, 2, \ldots, H$, where α^{σ^2} is a constant, $\varepsilon^{\sigma^2}_{t+h}$ is an error term and H is the largest forecast horizon. The case $h = 0$ corresponds to a nowcast. If the mean forecast $\hat{\mu}_{t+h|t}$ is biased, the term $\left(y_{t+h} - \hat{\mu}_{t+h|t}\right)^2$ is unlikely to be a good measure for the variance of y_{t+h}. Therefore, an additional test of the hypothesis $\alpha^{\mu} = 0$ in the equation

$$y_{t+h} - \hat{\mu}_{t+h|t} = \alpha^{\mu} + \varepsilon^{\mu}_{t+h}$$

is useful.

It would also be interesting to check for the unbiasedness of the mode forecasts $\hat{m}_{t+h|t}$, because biased mode forecasts lead to a biased estimate of α in equation (1). If, on average, the true modes m_{t+h} are smaller [larger] than $\hat{m}_{t+h|t}$, the estimate of α is biased downwards [upwards].

Unfortunately, to evaluate the bias of mode forecasts, one needs a rather restrictive assumption. Supposing that the risks are balanced on average implies $E\left[m_{t+h}/\sigma_{t+h}\right] = E\left[\mu_{t+h}/\sigma_{t+h}\right]$. Given this assumption, the bias of the (scaled) mode forecasts can be tested based on the equation

$$\frac{y_{t+h}}{\hat{\sigma}_{t+h|t}} - \frac{\hat{m}_{t+h|t}}{\hat{\sigma}_{t+h|t}} = \alpha^{m/\sigma} + \varepsilon_{t+h}^{m/\sigma}.$$

Finally, it might be interesting to know whether the central banks are mainly concerned with upward or downward risks to inflation. This question can be addressed by testing $\alpha^{(\mu-m)/\sigma} = 0$ in the equation

$$\frac{\hat{\mu}_{t+h|t} - \hat{m}_{t+h|t}}{\hat{\sigma}_{t+h|t}} = \alpha^{(\mu-m)/\sigma} + \varepsilon_{t+h}^{(\mu-m)/\sigma}.$$

If $\alpha^{(\mu-m)/\sigma}$ equals zero, the central bank assumes balanced risks on average.

Note that the error terms $\varepsilon_{t+h}^{\sigma^2}$, ε_{t+h}^{μ}, $\varepsilon_{t+h}^{m/\sigma}$ and $\varepsilon_{t+h}^{(\mu-m)/\sigma}$ can be serially correlated. We address this problem by using heteroskedasticity- and autocorrelation-consistent standard errors (henceforth HAC standard errors) as proposed by Newey and West (1987), and prewhitening as suggested by Andrews and Monahan (1992). The truncation lags are chosen based on the procedure of Andrews (1991). For all tests, also in what follows, we employ the conventional significance level of 5%.

The results in Table 2 show that all mean forecasts and most volatility forecasts are unbiased. The only exception is given by the nowcasts of the Riksbank where a significant overprediction of volatility occurs. This implies that the estimates of α and β in equation (1) can be expected to be biased towards zero for these nowcasts.

For the scaled mode forecasts, there is no significant bias for any forecast horizon. So if the risks are balanced on average, the mode forecasts are unlikely to cause a bias of the estimate of α in equation (1). Finally, the fact that $\hat{\alpha}^{(\mu-m)/\sigma}$ is always larger than zero suggests that both central banks are apparently slightly more concerned with upward risks to inflation. However, the estimates are small and not significantly different from zero. Thus, one cannot reject the hypothesis that, on average, the BoE and the Riksbank forecast balanced risks. Note that the latter result does not imply any kind of optimality, but is purely descriptive. For example, if the true densities of inflation are always positively skewed, $\alpha^{(\mu-m)/\sigma}$ would need to be positive in case of optimal risk forecasts.

3.4 Empirical Results

Before turning to the empirical optimality tests, it is instructive to take a brief look at the forecast risks and the associated realized risks used in this study. They are displayed in a scatter plot in Figure 2. The largest forecast risks equal about 0.5, the smallest about -0.3. The realized risks, i.e. the scaled mode forecast errors range from about -2.8 to about 4.7. For a large number of periods, the risks were actually forecast to be balanced.

If the risk forecasts are optimal, most points in Figure 2 should be located in the first and third quadrant if $\alpha \approx 0$. If the risk forecasts are not informative and $\alpha \approx 0$, the points should spread broadly evenly above and below the x-axis. The regression line in Figure 2 indicates that the latter condition is more likely to be fulfilled, since the line is not upward-sloping. However, the regression line is computed for the entire sample of risk forecasts, and there might well be differences,

h	0	1	2	3	4	5	6	7	8
					Bank of England				
T	50	49	48	47	46	45	44	43	42
coefficient estimates									
$\hat{\alpha}_h^{\mu}$	0.02	0.08	0.14	0.22	0.27	0.29	0.25	0.17	0.11
	(0.02)	(0.07)	(0.12)	(0.19)	(0.26)	(0.29)	(0.30)	(0.28)	(0.28)
$\hat{\alpha}_h^{\sigma^2}$	-0.05	0.00	0.13	0.31	0.38	0.42	0.24	0.04	0.01
	(0.03)	(0.06)	(0.11)	(0.22)	(0.32)	(0.45)	(0.38)	(0.34)	(0.39)
$\hat{\alpha}_h^{m/\sigma}$	0.09	0.24	0.32	0.43	0.51	0.51	0.45	0.34	0.27
	(0.10)	(0.15)	(0.23)	(0.28)	(0.37)	(0.38)	(0.38)	(0.41)	(0.45)
$\hat{\alpha}_h^{(\mu-m)/\sigma}$	0.04	0.04	0.04	0.04	0.04	0.03	0.04	0.04	0.03
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.03)	(0.04)	(0.05)
p-values									
$\alpha_h^{\mu}=0$	0.444	0.236	0.264	0.260	0.307	0.333	0.393	0.540	0.683
$\alpha_h^{\sigma^2}=0$	0.079	0.944	0.244	0.171	0.237	0.351	0.533	0.897	0.980
$\alpha_h^{m/\sigma}=0$	0.371	0.113	0.173	0.130	0.169	0.195	0.244	0.417	0.546
$\alpha_h^{(\mu-m)/\sigma}=0$	0.093	0.096	0.108	0.117	0.119	0.178	0.249	0.352	0.459
					Sveriges Riksbank				
T	27	27	27	27	27	27	27	27	
coefficient estimates									
$\hat{\alpha}_h^{\mu}$	-0.04	-0.01	-0.01	-0.08	-0.24	-0.37	-0.44	-0.41	
	(0.04)	(0.10)	(0.19)	(0.22)	(0.29)	(0.47)	(0.76)	(0.96)	
$\hat{\alpha}_h^{\sigma^2}$	-0.05	0.09	0.13	0.06	0.02	0.24	-0.02	0.28	
	(0.01)	(0.06)	(0.17)	(0.15)	(0.16)	(0.32)	(0.53)	(0.52)	
$\hat{\alpha}_h^{m/\sigma}$	-0.09	0.04	0.01	-0.11	-0.31	-0.42	-0.44	-0.36	
	(0.12)	(0.25)	(0.41)	(0.39)	(0.45)	(0.62)	(0.87)	(1.05)	
$\hat{\alpha}_h^{(\mu-m)/\sigma}$	0.01	0.03	0.03	0.03	0.03	0.03	0.03	0.03	
	(0.01)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	
p-values									
$\alpha_h^{\mu}=0$	0.323	0.894	0.968	0.710	0.424	0.444	0.566	0.670	
$\alpha_h^{\sigma^2}=0$	0.000	0.170	0.453	0.714	0.879	0.466	0.972	0.592	
$\alpha_h^{m/\sigma}=0$	0.458	0.884	0.984	0.771	0.499	0.500	0.618	0.735	
$\alpha_h^{(\mu-m)/\sigma}=0$	0.643	0.096	0.067	0.118	0.198	0.168	0.179	0.195	

Note: Figures in parentheses are HAC standard errors. T denotes the sample size.

Table 2: Regression-based tests for bias

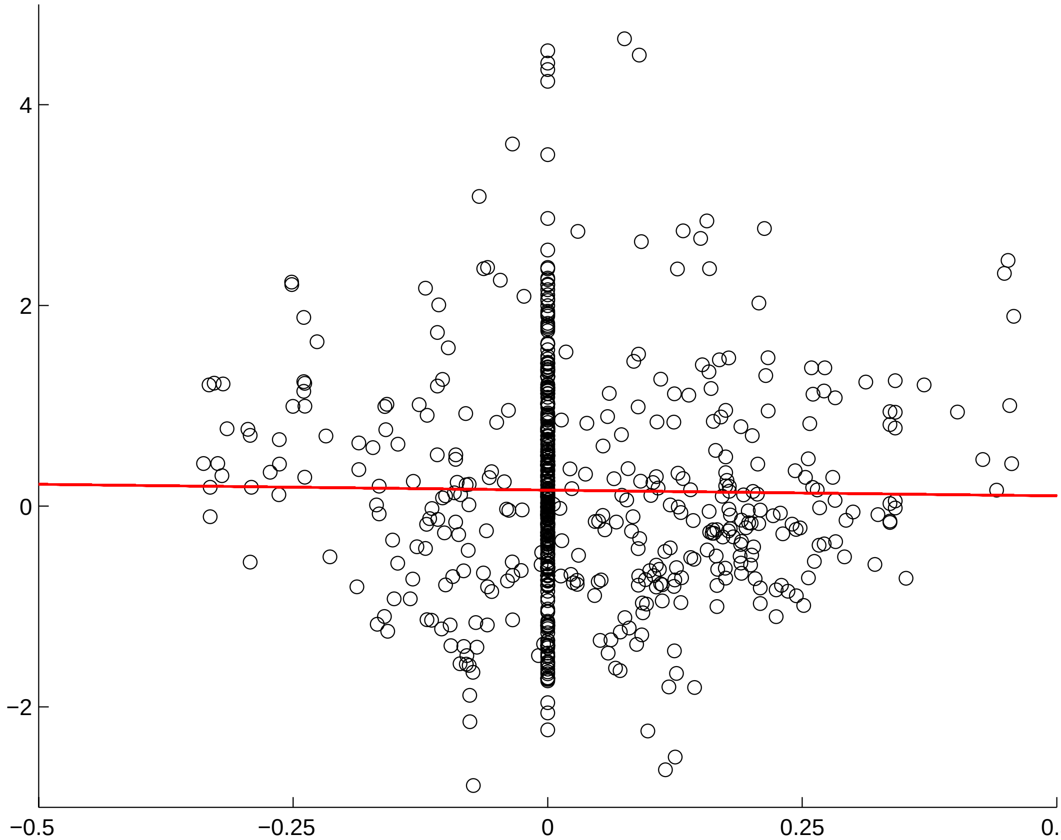

Figure 2: Forecast risks (on x-axis) and realized risks (on y-axis) of the BoE and the Riksbank, and a regression line.

for example, between distinct forecast horizons. If, for instance, risk forecasts are used to incorporate new information into the forecast in order to avoid last-minute changes to the central forecast, the risk forecasts would probably be informative at very short forecast horizons, but not necessarily at large horizons. In addition, there might also be differences between the BoE and the Riksbank with respect to their risk forecast performances. Moreover, it would remain to be analyzed whether the slope coefficient of the regression line in Figure 2 is significantly different from 1.

In the lights of the findings reported in Knüppel and Schultefrankenfeld (2011) and, additionally, potentially autocorrelated error terms, pronounced power problems of the tests can be expected especially for the Riksbank. In order to alleviate these problems, in what follows the risk forecasts are analyzed in a panel setup, where all forecast horizons are analyzed simultaneously, and where, in addition to the serial correlation, the cross-correlation of the error terms is taken into account. The cross-correlation is caused by the fact that, for example, if y_t is far from its unconditional mean, it is very likely that the realized risks, i.e. the scaled mode forecast errors associated with y_t are large for all but the short forecast horizons. For the panel estimation, we use the approach described in Greene (1997, p. 687). This approach is applicable also in the case of unbalanced panels which allows us to use all risk forecast data of the BoE. In a first step, the Prais-Winsten transformation is employed to remove autocorrelation. Then the transformed data are analyzed by means of the seemingly unrelated regressions model.[30]

In Table 3, the results of the tests for risk forecast optimality are shown for each forecast horizon of both central banks, respectively. Note that the Riksbank forecast unbalanced risks only in about half of its forecasts. The same holds for the BoE for nowcasts and forecasts up to one year ahead. For longer forecast horizons, the share of unbalanced risk forecasts equals about two-thirds. Interestingly, the estimates of β are negative for several forecast horizons in case of the BoE. For the Riksbank, a negative sign of $\hat{\beta}$ is only observed for $h = 1$. The standard errors

[30]Note that there is a minor error in the formula (15-57) by Greene (1997, p. 687). For the construction of the final covariance matrix of the errors, the covariance matrices of AR(1)-processes as defined in formula (13-10) are required.
When estimating the covariances of the errors, we make use of the entire unbalanced sample in case of the BoE. This approach is commonly attributed to Wilks (1932). Note that this approach does not necessarily produce a positive-semidefinite covariance matrix. Using only the balanced sample for the covariance estimation leaves the results virtually unchanged.

are often smaller for longer horizons than for shorter ones. This is due to the fact that the cross-correlations of the error terms are especially pronounced for longer forecast horizons.

In case of the BoE, the joint null hypothesis of risk forecast optimality $\alpha = 0, \beta = 1$ is rejected for all forecast horizons except $h = 0$ and $h = 8$, and $\hat{\beta}$ is significantly different from 1 for all horizons but $h = 8$. The hypothesis of no information content is rejected for the horizons $h = 4$, $h = 5$, and $h = 8$. However, $\hat{\beta}$ is negative for $h = 4$ and $h = 5$, so that these rejections do not appear very plausible. For the Riksbank, the estimation uncertainty turns out to be very large, so that only one hypothesis of interest can be rejected. For $h = 1$, $\hat{\beta}$ is strongly negative and differs significantly from 1.

So the results for the Riksbank basically show how difficult it can be to evaluate macroeconomic risk forecasts in a sample covering only 7 years of data.[31] The results for the BoE strongly indicate that its risk forecasts are not optimal, and that there is no systematic connection between realized risks and forecast risks. The only potential exception is the forecast horizon $h = 8$. However, it does not appear very convincing that the properties of the risk forecasts just for the horizon $h = 8$ should be so different from the neighbouring horizons $h = 7$, $h = 6$ etc. We will further elaborate on this issue below.

Since many central banks only publish direction-of-risk forecasts, it is interesting to investigate those as well. In addition, while Knüppel and Schultefrankenfeld

[31] For instance, if only the first observation is dropped from every equation, the estimate of β for $h = 7$ changes from 0.27 to -1.06 and becomes significantly different from 1. We also ran an estimation with the very restrictive assumption that α and β are identical, respectively, across all horizons. Even in this case, no hypothesis of interest can be rejected, and the estimates are not robust to minor variations of the sample size.

The estimates reported for the BoE are hardly affected by small changes of the sample size.

h	0	1	2	3	4	5	6	7	8
				Bank of England					
T	50	49	48	47	46	45	44	43	42
N	24	23	22	22	22	32	30	31	28
coefficient estimates									
$\hat{\alpha}$	0.09	0.15	0.19	0.25	0.33	0.32	0.24	0.04	-0.09
	(0.10)	(0.12)	(0.17)	(0.19)	(0.18)	(0.18)	(0.19)	(0.19)	(0.18)
$\hat{\beta}$	-0.45	0.05	0.13	-0.10	-0.52	-1.02	-0.30	0.08	0.71
	(0.61)	(0.39)	(0.26)	(0.24)	(0.21)	(0.29)	(0.26)	(0.19)	(0.21)
p-values									
$\alpha = 0, \beta = 1$	0.057	0.039	0.004	0.000	0.000	0.000	0.000	0.000	0.312
$\alpha = 0$	0.376	0.229	0.253	0.185	0.066	0.088	0.209	0.823	0.621
$\beta = 0$	0.465	0.905	0.619	0.666	0.017	0.001	0.267	0.680	0.002
$\beta = 1$	0.021	0.017	0.002	0.000	0.000	0.000	0.000	0.000	0.173
				Sveriges Riksbank					
T	27	27	27	27	27	27	27	27	
N	13	14	15	15	15	16	16	16	
coefficient estimates									
$\hat{\alpha}$	-0.09	0.14	-0.07	-0.21	-0.31	-0.40	-0.29	-0.18	
	(0.13)	(0.26)	(0.38)	(0.32)	(0.31)	(0.33)	(0.30)	(0.35)	
$\hat{\beta}$	1.08	-3.97	1.64	1.95	0.47	1.97	1.29	0.27	
	(1.68)	(2.27)	(2.02)	(1.37)	(1.05)	(1.09)	(0.80)	(1.13)	
p-values									
$\alpha = 0, \beta = 1$	0.803	0.094	0.944	0.681	0.497	0.377	0.604	0.676	
$\alpha = 0$	0.513	0.590	0.855	0.520	0.336	0.241	0.339	0.614	
$\beta = 0$	0.529	0.093	0.425	0.167	0.656	0.083	0.118	0.813	
$\beta = 1$	0.965	0.038	0.755	0.493	0.618	0.382	0.720	0.523	

Note: Figures in parentheses are HAC standard errors. T denotes the sample size. N denotes the number of forecasts with unbalanced risks.

Table 3: Tests for optimality of quantitative risk forecasts based on panel setup

(2011) find that quantitative risk forecasts are better suited for evaluation purposes because of higher testing power, this result is based on Monte Carlo studies with well-behaved realizations. In practice, as mentioned above, outliers could even lead to advantages of tests based on direction-of-risk forecasts in terms of power. Since we are not aware of approaches for analyzing categorical data with potentially autocorrelated residuals in a panel setup, in what follows we consider each forecast horizon separately.

The results of the analysis of direction-of-risk forecasts are reported in Table 4. Recoding the data according to equations (2) and (3), and selecting only observations with corresponding non-zero risk forecasts according to (4), we are left with at most 32 observations per forecast horizon. The estimates for α_q are unrelated to forecast optimality and only reported for information.[32] Note that, according to Pesaran and Timmermann (2006), the problem of autocorrelation of the error terms in equations with categorical data like equation (5) can be addressed using HAC standard errors.[33]

The estimates for β_q are mostly negative in case of the BoE and mostly positive in case of the Riksbank. None of the hypotheses of interest can be rejected for the Riksbank. For the BoE forecasts, rejections of the hypothesis $\beta_q > 0$ occur for $h = 1$, $h = 3$ and $h = 6$, suggesting that the respective risk forecasts cannot be optimal. For $h = 3$ and $h = 6$, the hypothesis $\beta_q = 0$ is rejected as well. The latter result, however, does not seem very convincing, because it would imply an adverse information content of the risk forecasts at these horizons. The hypothesis

[32] However, the value $\hat{\alpha}_q = 1$ in case of $h = 3$ for the BoE forecasts implies that, whenever a downward risk was forecast, the realization was actually larger than the central forecast.

[33] Since Pesaran and Timmermann (2006) do not address the possibility of prewithening, we do not use it here.

$\beta_q < 0$ cannot be rejected for any forecast horizon of the BoE.

Interestingly, the best risk forecast performance in case of the BoE is again observed for $h = 8$. However, for this horizon, only 50% of the direction-of-risk forecasts were successful, so that the qualitative risk forecast performance is actually rather poor. This result might suggest that the good performance observed in the panel analysis based on quantitative risk forecasts is due to 'outliers', i.e. due to a few occasions where large risks were forecast and large realized risks were observed. Yet, it is rather the panel setup which leads to a significantly positive $\hat{\beta}$ for $h = 8$. A horizon-specific regression based on quantitative risks for $h = 8$ yields a negative estimate of β. Based on this estimate and its HAC standard error, one would reject the hypothesis $\beta = 1$, but not the hypothesis $\beta = 0$.[34] So on the whole, also for $h = 8$ there is no conclusive evidence for risk forecast optimality or against a lack of information content of the risk forecasts.[35]

In summary, considering quantitative and direction-of-risk forecasts, for the BoE there is evidence against the optimality of risk forecasts for almost all forecast horizons. In contrast to that, there is no conclusive evidence against the hypothesis that risk forecasts and realized risks are unrelated at all forecast horizons. For the Riksbank, only a single rejection of forecast optimality and no rejection of the risk forecasts' lack of information content occurs. The results for the Riksbank clarify that with only 7 years of data, an instructive evaluation of macroeconomic risk

[34] The horizon-specific estimate for β equals -0.46, its standard error being 0.62. Note that the point estimate of the panel analysis $\hat{\beta} = 0.71$ is inside of the 95% confidence interval around the horizon-specific estimate $\hat{\beta} = -0.46$.

[35] It should be mentioned that for no other forecast horizon the panel setup and the separate analysis of each horizon give such contradicting results. That is, for no other forecast horizon one of the hypothesis $\beta = 0$ or $\beta = 1$ is rejected in one setup, and the other hypothesis is rejected in the other setup. The same applies to the results for the Riksbank. In general, the standard errors are, of course, considerably larger when each horizon is analyzed separately, so that inference becomes rather difficult.

h	0	1	2	3	4	5	6	7	8
					Bank of England				
N	24	23	22	22	22	32	30	31	28
coefficient estimates									
$\hat{\alpha}_q$	0.57	0.86	0.86	1.00	0.71	0.58	0.60	0.58	0.42
	(0.15)	(0.14)	(0.11)	(0.00)	(0.17)	(0.16)	(0.11)	(0.14)	(0.16)
$\hat{\beta}_q$	-0.10	-0.42	-0.32	-0.60	-0.38	-0.18	-0.25	-0.11	0.02
	(0.18)	(0.20)	(0.19)	(0.10)	(0.24)	(0.19)	(0.12)	(0.15)	(0.20)
p-values									
$\beta_q = 0$	0.571	0.046	0.104	0.000	0.126	0.339	0.039	0.464	0.919
$\beta_q > 0$	0.286	0.023	0.052	0.000	0.063	0.169	0.020	0.232	0.541
$\beta_q < 0$	0.714	0.977	0.948	1.000	0.937	0.831	0.981	0.768	0.459
					Sveriges Riksbank				
N	13	14	15	15	15	16	16	16	
coefficient estimates									
$\hat{\alpha}_q$	0.33	0.60	0.50	0.33	0.33	0.17	0.17	0.33	
	(0.20)	(0.17)	(0.19)	(0.19)	(0.20)	(0.15)	(0.15)	(0.20)	
$\hat{\beta}_q$	0.10	-0.16	0.06	0.11	0.11	0.33	0.33	0.07	
	(0.34)	(0.22)	(0.26)	(0.23)	(0.18)	(0.25)	(0.25)	(0.26)	
p-values									
$\beta_q = 0$	0.787	0.498	0.832	0.632	0.549	0.201	0.201	0.804	
$\beta_q > 0$	0.606	0.249	0.584	0.684	0.726	0.900	0.900	0.598	
$\beta_q < 0$	0.394	0.751	0.416	0.316	0.274	0.100	0.100	0.402	

Note: Figures in parentheses are HAC standard errors. N denotes the sample size.

Table 4: Tests for optimality of direction-of-risk forecasts

forecasts can be extremely difficult.

Altogether, the results found above suggest that risk forecasts and realized risks lack a significant systematic connection. Given the number of central banks which engage in risk forecasting and, thus, given the resources which are devoted to it, the results found above might appear surprising. Yet, taking into account the often rather poor performance even of macroeconomic first moment forecasts, i.e. point forecasts, except for short forecast horizons, it does not seem too implausible that macroeconomic forecasts of third moments might be unsuccessful.

4 Possible Explanations for Poor Risk Forecasting Performance

4.1 Endogeneity of Outturns

If inflation actually reacts to a central bank's risk forecast, then it could of course happen that estimations of equations like (1) yield misleading results. Consider, for example, the case of upward risks to the inflation forecast. In this case, economic agents could anticipate a risk of rising interest rates. In response to this risk, economic activity could be dampened, leading to lower demand and, consequently, to lower inflation. Then, even if the upward risk to inflation materializes, inflation could still be lower than forecast. In this case, the risk forecasts could even seem to have an "adverse" information content, since β could be negative. The same could happen if the central bank actually set interest rates according to its risk forecasts, that is, if the central bank increases [decreases] interest rates when it forecasts upward [downward] risks.

As mentioned above, Rasche and Thornton (2002) found that, in case of the Federal Reserve, previous balance-of-risk statements were not a decisive determinant for the expectation of market participants with respect to future policy actions.[36] However, if a transmission channel from forecasts to realizations exists, then the analysis conducted above could easily come to wrong conclusions. Yet, the prevailing opinion in economics is that inflation can only be influenced with a lag by monetary policy makers. Actually, this is the reason why central banks are concerned with forecasting. They know that their current decisions will not affect the economy instantaneously, but only in the future.

Taking this fact into account, it should be clear that, if a transmission channel from forecasts to realizations exists, its importance should increase with the forecasting horizon. Therefore, we should be able to assess the importance of this channel by regressing the $\hat{\beta}$'s found for the different horizons on the respective h's. If the transmission channel is important and the risk forecasts contain the intended information, the coefficient associated with h should be negative, because for short horizons, the $\hat{\beta}$'s should correctly measure the information content of the risk forecasts and should therefore be relatively large. For larger horizons, the transmission channel would become more important, lowering the values of the $\hat{\beta}$'s.[37]

[36] However, remember that the balance-of-risk statements had a different interpretation during that period.

[37] Of course, there could also be another reason why the $\hat{\beta}$'s decline with the forecasting horizon. The difficulty of forecasting for longer horizons tends to lower the slope coefficients of longer horizons in standard Mincer-Zarnowitz (1969) regressions where one tests for the optimality of mean forecasts. However, if the $\hat{\beta}$'s do not decrease as h increases, this can be interpreted as unimportance of the transmission channel even if there is another reason why the $\hat{\beta}$'s should decrease.

	Bank of England	Sveriges Riksbank
H	9	8
coefficient estimates		
$\hat{a}$	-0.35	-0.25
	(0.31)	(1.29)
$\hat{b}$	0.05	0.24
	(0.06)	(0.31)
$p-$values		
$b = 0$	0.473	0.467
$b < 0$	0.236	0.233

Note: Figures in parentheses are standard errors.
H denotes the number of observations.

Table 5: Regression of $\hat{\beta}(h)$'s on a constant and h

Therefore, we estimate the equation

$$\hat{\beta}(h) = a + bh + \varepsilon_h$$

where ε_h is an error term and $\hat{\beta}(h)$ denotes the estimate of β for the forecast horizon h. We estimate this equation for the $\hat{\beta}$'s reported in Table 3. The results of these regressions are displayed in Table 5. None of the $\hat{b}$'s is negative, suggesting that the transmission channel from forecasts to realizations does not play a major role.

4.2 Reasons for Risk Forecasting Reconsidered

In Section 2.2, several reasons for risk forecasts were offered. These can now be reconsidered in the light of the risk forecasts' apparent lack of information content.

Non-linearities either played no role in the samples under study, or the forecasters' understanding of these non-linearities is too limited in order to lead to successful risk forecasts. For example, while the zero lower bound for the policy rate is a popular motivation for asymmetric forecast densities, one could ask whether the effects of quantitative easing might not counteract the zero-lower-bound problem in such a way that the forecast density is actually close to symmetric even if the policy rate is at a very low level.

If risk forecasts were used to incorporate last-minute information into the forecast densities without having to change the central forecast, one should have observed a higher risk forecast accuracy than found above especially at short horizons. So at least for the BoE and the Riksbank, without much doubt this reason for issuing risk forecasts does not matter.

If so-called technical assumptions are used in the forecasting process of the BoE and the Riksbank, and risk forecast are used as an attempt to improve upon the forecast accuracy implied by these assumptions, it seems that such an improvement is infeasible. Coming back to the example of the technical assumption of constant exchange rates, the difficulties to beat the random-walk forecast are actually well-known.[38]

An attempted bias correction by using risk forecasts has either not been carried out by the BoE and the Riksbank, or it was carried out but did not work as intended. However, it is not possible to assess the individual probability of these two explanations based on the results of the optimality tests.

If risk forecasts are employed to accommodate the view of a minority in a decision-making body, the results above suggest that the minority's view does not help to produce more accurate forecasts. Thus, according to the results found above, its view could just as well be ignored.

[38] See, for example, Meese and Rogoff (1983).

Finally, the likelihood of the possibility that risk forecasts are used as a communication device cannot be evaluated based on our risk forecast evaluation. But the basis for this possibility is the idea that the central bank tries to signal interest rate changes unexpected by the market without the central inflation forecast having to deviate too strongly from the inflation target. So, a possible future increase in interest rates could be signalled by forecasting an inflation rate that is only slightly above target and adding upward risks to this forecast. If risk forecasts are used in this way, they should be positively correlated with the deviations of the inflation forecasts from the target. However, neither for the BoE nor for the Riksbank we find evidence of positive correlations.[39]

The asymmetry of future shocks is apparently often assumed in the forecasts of central banks, but our results suggest that such assumptions might not be justified. Actually, the BoE gives some narrative support to this supposition. Referring to inflationary developments from 1997 to 1999, the BoE states that "In general, the modal inflation forecast has been closer to actual outturns than the mean projection. This is because the MPC judged the risks to the central projection to be on the upside, largely because of the risk that the sterling exchange rate might depreciate sharply. Up to 2000 Q2 this did not occur; indeed, the exchange rate tended to be higher than the central assumption." (Bank of England, 2000, pp. 63-64).

[39] Depending on the forecast horizon and the central bank under study, the correlation coefficients range from about -0.2 to 0.2. They are all insignificant.

5 Conclusion

Many central banks augment their point forecasts with assessments of the balance of risks. For virtually all central banks surveyed in this work, we actually find clear statements that the presence of unbalanced forecast risks corresponds to an asymmetry of forecast densities. Surprisingly, the point forecasts published almost always correspond to the modes of the forecast densities.

The risk forecasts are made and presented in a variety of ways. They can be quantitative as in case of the BoE and the Riksbank, or qualitative, hence only indicating the direction of the balance of risks, as, for example, in case of the Federal Reserve or the ECB. Risk forecasts can also be discussed without referring to the balance of risks, thereby just giving ideas how the central forecast would change if a certain risk materializes, as, for instance, in case of the Reserve Banks of Australia and New Zealand. Apart from the verbal presentation of the (balance of) risks, balance-of-risk forecasts are represented in potentially asymmetric fan charts, histograms or skewness-related numbers of the forecast densities.

Since the balance of risks is supposed to contain information about the third moments of the densities of the forecast variables, there should be a systematic connection between risk forecasts and realized risks, where the realized risks correspond to the scaled mode forecast errors. Investigating the risk forecasts for inflation of the BoE and the Riksbank, we fail to find such a connection. While the results for the Riksbank could also be caused by the small sample size, the results for the BoE indicate relatively clearly that the risk forecasts do not contain the intended information. Rather, it seems that risk forecasts and forecast errors are unrelated. This holds for quantitative forecasts as well as for direction-of-risk forecasts. We therefore conclude that it seems questionable whether macroeconomic risk forecasts are meaningful.

Yet, two caveats are to be indicated. Firstly, the success of risk forecasts might depend on the variable under study. If, for example, output growth is best described by a regime-switching process as proposed by Hamilton (1989), it should be possible to issue informative risk forecast for output growth. At least for short horizons, one would just have to forecast upward risks during recessions and downward risks during expansions. Secondly, if discrete random variables with few possible outcomes are an important determinant of the variable under study, risk forecasts could also perform well. As an example, one could think of a potential future change of the value-added tax rate and its effect on inflation. However, in such cases one might prefer to conduct scenario analyses as, for example, done by the Riksbank since 2007. This has the additional advantage of clarifying the comovements between variables in case that the risk materializes.

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A Appendix: Central Banks and Risk Forecasts

In what follows, the statements of central banks on which Section 2.1 including Table 1 is based are cited. Table 6 contains the positions in the publications of these central banks where their risk assessments can be found.

Bank of Canada:

"The staff projection is an outlook for the economy's most likely path."[40]

"While the underlying macroeconomic risks to the projection are roughly balanced, the Bank judges that, as a consequence of operating at the effective lower bound, the overall risks to its inflation projection are tilted slightly to the downside." (Bank of Canada, 2009, p. 27)

"Chart 23 and Chart 24 depict the 50 per cent and 90 per cent confidence bands for year-over-year core and total CPI inflation [...] In particular, they show the slight downward tilt to the confidence bands that results from monetary policy operating at the effective lower bound."[41] (Bank of Canada, 2009, p. 26)

Banco Central de Chile:

"Balance of risks: Evaluation of possible alternative scenarios to the baseline scenario used in projections (considered the most likely in the Monetary Policy Report) and their implications for future paths of output and inflation. The combined analysis of different sources of uncertainty to the baseline projection scenario are reflected in the balance of risks, which may be biased downward, upward or balanced, in terms of growth and inflation, as compared to the baseline scenario." (Banco Central de Chile, 2007, p. 32)

"When the balance of risks around the central or modal projection slopes upward (positive bias), values above the modal projection are considered more likely to occur than values below it." (Banco Central de Chile, 2001, p. 13)

Bank of England:

"The central projection of inflation is then interpreted as being the 'mode' of the statistical distribution - it is the single most likely outcome based on current knowledge and judgment" (Britton et al., 1998, p. 32)

"In order to produce the fan chart, only one number is needed to summarise the degree of skewness (the balance of risks). [...] The Bank's analysis focuses on the difference between the mean and the mode of the forecast distribution to be presented in the Report. This difference is of interest as a summary statistic of the balance of risks" (Britton et al., 1998, p. 32)

"If the MPC believed there was a higher probability that inflation would be above the mode than below, then the area under the curve would be skewed to the right" (Bank of England, 2002, p. 48)

[40] See http://www.bankofcanada.ca/en/monetary/monetary_decision3.html

[41] Note that this statement suggests that expected inflation is lower than the central projection of inflation. Moreover, it implies that the Bank of Canada produces quantitative risk forecasts.

Banco de España:

"[...] These factors [...] suggest that the risks surrounding the output growth projections are on the low side. This means that downward deviations from the growth path of the central scenario of this report are considered more likely than upward ones." (Banco de España, 2008, p. 4)

"[...] a central projection scenario is obtained [...]. This central scenario is considered the most likely." (Ortega et al., 2007, p. 19)

Bank of Israel:

"Nonetheless, within the horizon of a year or more, the upside risks of inflation balance out with its downside risks." (Bank of Israel, 2008, p. 26)

"The principal risk factor that could lead to the nonmaterialization of this forecast is the course of global developments." (Bank of Israel, 2010, p. 36)

Bank of Japan:

"Each Policy Board member submits his or her forecasts in the form of point estimates, the values to which he or she attaches the highest probability of realization [...] each Policy Board member also indicates, in the form of a probability distribution, the likelihood that upside or downside risks will materialize and cause divergence from the forecast value." (Bank of Japan, 2008, p. 9)

"The probability distribution for the rate of real GDP growth in fiscal 2008 is skewed to the left. This suggests that Policy Board members consider the downside risks to be greater than the upside risks." (Bank of Japan, 2008, p. 9)

Banco de Portugal:

"Risks on economic activity are on the downside, particularly in 2011." (Banco de Portugal, 2010, p. 10)

"According to the quantification of risks, the likelihood that GDP growth may fall below the present outlook stands at 54 per cent in 2010 and 63 per cent in 2011" (Banco de Portugal, 2010, p. 18)

"The baseline point forecasts are interpreted as the mode of the joint distribution."[42] (Pinheiro and Esteves, 2010, p. 1)

Board of Governors of the Federal Reserve System:

"Participants also provide judgments as to whether the risks to their projections are weighted to the upside, downside, or are broadly balanced. That is, participants judge whether each variable is more likely to be above or below their projections of the most likely outcome." (Board of Governors of the Federal Reserve System, 2008, p. 45)

"The projections now produced by FOMC participants are explicitly modal forecasts in that they represent participants' projections of the most likely outcome. Although participants provide qualitative assessments of whether the risks

[42] Note that this interpretation differs from those of most other central banks which consider the marginal modes.

around their projections are weighted to one side or the other, we do not have quantitative estimates of any skew." (Reifschneider and Tulip, 2007, p. 12)

"Most participants viewed the risks to their inflation projections as weighted to the upside. Recent sharp increases in energy and food prices and the passthrough of dollar depreciation into import prices could boost inflation in the near term by more than currently anticipated." (Board of Governors of the Federal Reserve System, 2008, p. 41)

Deutsche Bundesbank:

"It is generally assumed that uncertainties are distributed symmetrically around the most likely value, ie the baseline. Depending on the specific data situation and conditions, there may well be signs when the projections are produced that this will not be the case. Indeed, unlike in the historical patterns, there is often a skewed distribution. In this case, the terms upside or downside risks are used." (Deutsche Bundesbank, 2007, p. 27)

Eurpoean Central Bank:

"Risks to the outlook for price developments are slightly tilted to the upside. They relate, in particular, to the evolution of energy and non-oil commodity prices. Furthermore, increases in indirect taxation and administered prices may be greater than currently expected" (European Central Bank, 2010, p. 6)

"ECB/Eurosystem staff projections are presented in the form of ranges. The use of ranges acknowledges the inevitable uncertainty surrounding macroeconomic projections."[43]

International Monetary Fund:

"[...] the IMF staff has presented risks to the WEO projections using a fan chart [...] The methodology for constructing the fan chart is similar to that originally developed by the Bank of England. The central forecast is represented as the mode, or the most likely outcome [...] The skewness of the distribution, or the relative size of the two pieces of the normal distribution, represents the balance of risks to the central forecast." (International Monetary Fund, 2008, pp. 41-42)

"In the case of the balance of risk being tilted to the downside [...] the expected probability of outcomes being below the central forecast exceeds 50 percent" (International Monetary Fund, 2006, p. 25)

Magyar Nemzeti Bank:

"In order to plot the fan chart, the uncertainty distribution must be determined for each point in time on the projection horizon. [...] The mode of distribution is identical to the central projection. [...] In determining the skewness that indicates risk direction, the risk perception of the Economics Department prevails." (Magyar Nemzeti Bank, 2004, p. 107)

"The method that we follow in preparing fan charts broadly corresponds to

[43] See www.ecb.int/pub/pdf/other/newprocedureforprojections200912en.pdf

that of the Bank of England, and the same holds true for the Swedish method." (Magyar Nemzeti Bank, 2004, p. 108)

"On the whole, our inflation forecast is apparently jeopardised by significant upward risks, which means that actual price increases will more likely exceed our expectations, rather than fall short of them." (Magyar Nemzeti Bank, 2008, p. 46)

Norges Bank:

"There is also a risk that the global downturn will be deeper and more prolonged than expected. [...] Overall, the outlook and balance of risks suggest that the key policy rate should be gradually reduced further to a level of around 1% in the second half of 2009" (Norges Bank, 2009, p. 13)

Reserve Bank of Australia:

"[...] it would be a mistake to focus only on the point forecast; it makes much more sense to think of the central forecast as simply the modal point on the distribution of the possible outcomes, with a sequence of progressively less likely outcomes on either side. Nor is that distribution necessarily symmetric – it may be skewed one way or the other. [...] We do not use fan charts per se, but we do try to consider alternative scenarios to the central forecast. We attempt to use the results of that process to articulate some sense of the balance of risks – both on the inflation outlook and on growth prospects – in the published statements."[44]

"As always, there are risks in both directions around the forecasts, although overall, these risks are viewed as evenly balanced." (Reserve Bank of Australia, 2010, p. 57)

Reserve Bank of New Zealand:

"[...] the central forecast is only one of a large number of potential outcomes for the economy, albeit the outcome that the Bank considers to be the most likely." (Conway, 2000, p. 14)

"We continue to see the balance of risks to the central projection as being to the downside for activity and inflation." (Reserve Bank of New Zealand, 2009, p. 4)

Sveriges Riksbank, 2007-present:

"The forecasts in the main scenario show the path which the Riksbank expects the economy to take and is a weighted consideration of various conceivable development paths (scenarios) and risks. [*continued in footnote 11*] There are therefore no grounds to revise the main scenario afterwards in light of a certain specific risk. This approach was adopted previously in the Inflation Report" (Sveriges Riksbank, 2007, p. 22)

"The uncertainty bands for the forecasts for inflation and GDP growth are based on the Riksbank's historical forecast errors. [*continued in footnote 12*] This entails a change in the method used for designing the fan chart for inflation, which

[44] See http://www.rba.gov.au/speeches/2001/sp-ag-101001.html

has previously been calculated using a weighted average of underlying risks. With the new method, the uncertainty bands are symmetrical." (Sveriges Riksbank, 2007, p. 22)

Sveriges Riksbank 1999-2006:

"The distribution that is used as an approximation of the inflation forecast's distribution is known in statistical terminology as two-piece normal. [...] From Fig. B3 it will be seen that forecast inflation for the second quarter of 2000 carries a downside risk (the distribution in Fig. B3 is somewhat skewed to the left). The broken line is the inflation forecast in the main scenario (the mode)" (Sveriges Riksbank, 1998, pp. 36-37)

"Skew is measured as the difference between the mean value and the most probable value (the mode of the distribution)" (Sveriges Riksbank, 1998, p. 36)

"The overall assessment of different risks is that the probability of inflation being higher than in the main scenario is slightly greater than the probability of lower inflation." (Sveriges Riksbank, 2006, p. 38)

Swiss National Bank:

"[...] the higher inflation expectations and the possibility of second-round effects will keep inflation risks on the upside." (Swiss National Bank, 2008, p. 13)

"Uncertainty about the future outlook for the global economy remains high, however, and downside risks predominate." (Swiss National Bank, 2010b, p. 19)

B Appendix: The Data

B.1 Bank of England

Several parameters of the BoE's forecast densities can be downloaded directly from the BoE's website.[45] These include the mean μ, the mode m and an uncertainty measure ω. The standard deviations σ_1 and σ_2, however, have to be calculated in order to determine the Pearson mode skewness. Following Wallis (2004), it is helpful to define

$$s = \frac{\mu - m}{\omega}$$

from which the quantity γ can be determined as

$$\gamma = sign\,(s)\sqrt{1 - 4\left(\frac{\sqrt{1+\pi s^2}-1}{\pi s^2}\right)^2}.$$

Then σ_1 and σ_2 can be calculated as

$$\begin{aligned}\sigma_1 &= \frac{\omega}{\sqrt{1+\gamma}}\\ \sigma_2 &= \frac{\omega}{\sqrt{1-\gamma}}.\end{aligned}$$

[45] See http://www.bankofengland.co.uk/publications/inflationreport/irprobab.htm

Central Bank	**Risk assessments published in**
Bank of Canada	Monetary Policy Report, chapter "Risks to the outlook"
Banco Central de Chile	Monetary Policy Report, chapter "Inflation scenarios", section "Risk scenarios"
Bank of England	Inflation Report, chapter "Prospects for inflation", section "Key judgements and risk"
Banco de España	Economic Bulletin, chapter "Spanish economic projections report", section "Risks to the projection" (chapter appears semi-annually)
Bank of Israel	Inflation Report, chapter "Update of the forecasts", section "Assessments regarding the development of inflation and the balance of its risks"
Bank of Japan	Outlook for Economic Activity and Prices, Figure "Risk balance charts" and section "Upside and downside risks"
Banco de Portugal	Economic Bulletin, chapter "Outlook for the Portuguese economy", section "Uncertainty and risks" (chapter appears semi-annually)
Board of Governors of Federal Reserve System	Monetary Policy Report to the Congress, chapter "Summary of economic projections", section "Uncertainty and risks"
Deutsche Bundesbank	Monthly Report, chapter "Outlook for the German economy", section "Risk assessment" (chapter appears semi-annually)
European Central Bank	Monthly Bulletin, "Editorial" and "The outlook for economic activity"
International Monetary Fund	World Economic Outlook, chapter "Global prospects and policies"
Magyar Nemzeti Bank	Quarterly Report on inflation, chapter "Inflation and real economy outlook", section "Inflation and growth risks"
Norges Bank	Monetary Policy Report, chapter "Monetary policy assessments and strategy", section "Uncertainty surrounding the projections (scenario analyses)"
Reserve Bank of Australia	Statement on Monetary Policy, chapter "Economic Outlook", section "Risks"
Reserve Bank of New Zealand	Monetary Policy Statement, chapters "Overview and key policy judgements" and "International developments and outlook"
Sveriges Riksbank 2007-present	Monetary Policy Report, chapter "Alternative scenarios and risks (scenario analyses)"
Sveriges Riksbank 1999-2006	Inflation Report, chapter "Inflation assessment", section "Risk assessment"
Swiss National Bank	Quarterly Bulletin, chapter "Monetary policy report", section "SNB inflation forecast"

Table 6: Publications of risk assessments by central banks.

The variance of the forecast density is given by

$$\sigma^2 = \left(1 - \frac{2}{\pi}\right) (\sigma_2 - \sigma_1)^2 + \sigma_1 \sigma_2,$$

from which the Pearson mode skewness $(\mu - m) / \sigma$ can be determined. Its values are displayed in Appendix B.3

B.2 Sveriges Riksbank

There are two inflation fan charts in each Inflation Report we study. One chart refers to the price index CPI, the other to the price index UND1X.[46] We focus on the CPI only. Forecast values are given for the current month and the next 24 months, and the inflation rate is calculated as the growth rate of the CPI with respect to the corresponding month of the previous year. The values which are published are the central forecast (mode forecast) and the quantiles belonging to the 50%, 75% and 90% confidence intervals. Unlike the intervals published by the BoE, these intervals are symmetric around the median, as clarified by Blix and Sellin (2000, footnote 7).

In contrast to the BoE, the Riskbank does not publish the means and standard deviations of the tpn-distributions that underlie the fan charts.[47] Therefore, we first have to back out the parameters of these distributions in order to calculate their means and standard deviations. The precision of the parameter estimates of course depends on the precision of the published forecast values. For Inflation Reports 1999:4 to 2004:4 they are very precise, having 14 decimal places. Then, for the Inflation Reports 2005:1 and 2005:2, the published values have 2 decimal places. Finally, the published values have only 1 decimal place for Inflation Reports 2005:3 to 2006:3. Surprisingly, the mode forecasts from Inflation Report 2004:4 only have 1 decimal place as well. So while backing out the exact parameter values is relatively easy until at least Inflation Report 2004:3, the estimates can be expected to be less precise for the following Inflation Reports due to the rounding of the forecast values.

Backing out the parameter values requires fitting a tpn-distribution to the modes and quantiles published. This fitting can either be done using a least squares criterion or a likelihood ratio criterion as shown by García and Manzanares (2007). For forecasts from Inflation Reports 1999:4 to 2004:4, the differences between the results obtained with both criterions are small in general. But especially when only 1 decimal place is used, the differences can become larger. In these cases, the

[46] The index UND1X is defined as the CPI excluding household mortgage interest expenditure and the direct effects of changes in indirect taxes and subsidies.

[47] That the Riksbank uses a tpn-distribution is stated in Sveriges Riksbank (1998, pp. 36-37).

likelihood ratio criterion gives more plausible results. For example, the skewness of the Riksbank's density forecasts published in a certain Inflation Report never changes sign from one forecast horizon to the next.[48] That is, the skewness might switch between zero and positive, or between zero and negative, but not between positive and negative. Looking at the Inflation Report 2005:4, however, with the least squares criterion, the skewness for the 5-month-ahead forecast is negative, while it is positive for 4 and 6 months ahead. With the likelihood ratio criterion, the skewness is positive for 4, 5 and 6 months ahead. Therefore, we use the likelihood ratio criterion to back out the parameters and to calculate the skewness of the inflation forecasts.

The skewness obtained in this way is cross-checked with the statements in the Inflation Reports. If an Inflation Report states that risks are balanced, the skewness of the corresponding forecast densities is set to zero, even if the estimated parameters indicate a (small) non-zero skewness. We do so because the parameter estimation results, as explained above, can be affected by imprecise data. The Pearson mode skewness for the selected forecast horizons are displayed in Appendix B.3

B.3 Pearson Mode Skewness of Forecast Densities

[48] This becomes clear when reading the chapters of the Inflation Reports discussing the balance of risks.

h	0	1	2	3	4	5	6	7	8
1998Q1	0.197	0.170	0.176	0.178	0.178	0.248	0.292	0.321	0.352
1998Q2	−0.338	−0.324	−0.320	−0.332	−0.332	−0.119	0.047	0.178	0.283
1998Q3	0.202	0.229	0.231	0.224	0.224	0.236	0.256	0.262	0.267
1998Q4	0.159	0.209	0.209	0.209	0.204	0.113	0.025	−0.034	−0.093
1999Q1	0.107	0.107	0.107	0.089	0.096	0.029	−0.026	−0.082	−0.118
1999Q2	0.166	0.166	0.189	0.166	0.182	0.195	0.197	0.199	0.207
1999Q3	0.230	0.272	0.267	0.265	0.256	0.206	0.143	0.103	0.073
1999Q4	−0.060	−0.108	−0.077	−0.080	−0.090	−0.132	−0.218	−0.272	−0.292
2000Q1	0.175	0.175	0.175	0.175	0.175	0.179	0.178	0.190	0.192
2000Q2	0.060	0.072	0.078	0.100	0.090	0.050	0.000	−0.040	−0.063
2000Q3	0	0	0	0	0	−0.034	−0.078	−0.114	−0.135
2000Q4	0.059	0.071	0.101	0.079	0.089	0.065	0.029	0.013	0
2001Q1	−0.120	−0.108	−0.103	−0.100	−0.090	−0.147	−0.214	−0.264	−0.291
2001Q2	−0.120	−0.108	−0.103	−0.080	−0.090	−0.116	−0.147	−0.172	−0.185
2001Q3	0	0	0	0	0	−0.050	−0.118	−0.159	−0.185
2001Q4	0.336	0.336	0.336	0.336	0.336	0.313	0.283	0.258	0.243
2002Q1	0.341	0.341	0.341	0.341	0.341	0.370	0.402	0.427	0.441
2002Q2	0	0	0	0	0	0.089	0.190	0.280	0.324
2002Q3	0	0	0	0	0	0.054	0.128	0.172	0.200
2002Q4	0	0	0	0	0	0.056	0.115	0.174	0.202
2003Q1	0	0	0	0	0	0.050	0.104	0.162	0.190
2003Q2	0.190	0.190	0.180	0.200	0.190	0.144	0.082	0.030	0.000
2003Q3	0	0	0	0	0	−0.038	−0.087	−0.128	−0.152
2003Q4	0	0	0	0	0	0	0	0	0
2004Q1	0	0	0	0	0	0	0	0	0
2004Q2	0	0	0	0	0	0	0	0	0
2004Q3	0	0	0	0	0	0	0	0	0
2004Q4	−0.240	−0.240	−0.226	−0.251	−0.240	−0.263	−0.292	−0.315	−0.328
2005Q1	−0.239	−0.239	−0.251	−0.250	−0.239	−0.264	−0.294	−0.319	−0.333
2005Q2	0	0	0	0	0	0	0	0	0
2005Q3	0	0	0	0	0	−0.047	−0.107	−0.158	−0.187
2005Q4	0	0	0	0	0	0	0	0	0
2006Q1	0	0	0	0	0	0	0	0	0
2006Q2	0	0	0	0	0	0	0	0	0
2006Q3	0	0	0	0	0	0	0	0	0
2006Q4	0	0	0	0	0	0	0	0	0
2007Q1	−0.165	−0.165	−0.151	−0.168	−0.160	−0.063	0.090	0.213	0.272
2007Q2	0	0	0	0	0	0.075	0.156	0.216	0.260
2007Q3	0	0	0	0	0	0.030	0.085	0.108	0.132
2007Q4	0	0	0	0	0	0	0	0	0
2008Q1	0	0	0	0	0	0	0	0	0
2008Q2	0.124	0.133	0.128	0.131	0.124	0.144	0.159	0.160	0.178
2008Q3	0.131	0.127	0.116	0.125	0.119	0.167	0.217	0.259	0.277
2008Q4	0	0	0	0	0	0	0	0	0
2009Q1	0	0	0	0	0	−0.059	−0.128	−0.179	−0.205
2009Q2	0.456	0.454	0.458	0.449	0.452	0.337	0.207	0.070	0
2009Q3	0	0	0	0	0	0	0	0	0
2009Q4	0	0	0	0	0	0	0	0	0
2010Q1	0.162	0.163	0.169	0.164	0.164	0.192	0.222	0.247	0.258
2010Q2	0.091	0.076	0.081	0.079	0.083	0.094	0.110	0.124	0.134

Note: The forecast horizon h is measured in quarters. A zero without decimal point refers to the situation of exactly balanced forecast risks.

Table 7: Pearson mode skewness of forecast densities of the BoE

h	0	3	6	9	12	15	18	21
Dec 1999	0.014	0.053	0.089	0.124	0.157	0.140	0.124	0.107
Jun 2000	0	0	0	0	0	0.018	0.038	0.059
Oct 2000	0.014	0.054	0.092	0.127	0.152	0.159	0.165	0.172
Dec 2000	0.023	0.089	0.150	0.207	0.261	0.272	0.282	0.293
Mar 2001	−0.000	−0.035	−0.067	−0.098	−0.126	−0.114	−0.101	−0.089
May 2001	0	0	0	0	0	0	0	0
Oct 2001	0	0	0	0	0	0	0	0
Dec 2001	0	0	0	0	0	0	0	0
Mar 2002	0.012	0.244	0.240	0.110	0.139	0.125	0.111	0.097
Jun 2002	0.252	0.244	0.080	0.111	0.140	0.126	0.112	0.098
Oct 2002	−0.006	−0.025	−0.043	−0.059	−0.070	−0.073	−0.076	−0.079
Dec 2002	−0.006	−0.023	−0.040	−0.055	−0.070	−0.073	−0.077	−0.080
Mar 2003	0	0	0	0	0	0	0	0
Jun 2003	−0.009	−0.035	−0.059	−0.082	−0.104	−0.095	−0.086	−0.077
Oct 2003	0	0	0	0	0	0	0	0
Dec 2003	0	0	0	0	0	0	0	0
Apr 2004	0.006	0.022	0.037	0.052	0.062	0.067	0.071	0.076
May 2004	0.000	0.024	0.046	0.067	0.087	0.093	0.093	0.092
Oct 2004	0	0	0	0	0	0	0	0
Dec 2004	−0.157	0	−0.004	−0.096	−0.167	−0.160	−0.132	−0.100
Mar 2005	0	0	0	0	0	0	0	0
Jun 2005	−0.000	−0.038	−0.057	−0.086	−0.109	−0.092	−0.077	−0.055
Oct 2005	0	0	0	0	0	0	0	0
Dec 2005	0	0.120	0.133	0.206	0.166	0.221	0.163	0.084
Feb 2006	0	0	0	0	0	0	0	0
Jun 2006	0	0	0	0	0	0	0	0
Oct 2006	0	0.120	0.201	0.300	0.253	0.214	0.169	0.159

Note: The forecast horizon h is measured in months. A zero without decimal point refers to the situation of exactly balanced forecast risks.

Table 8: Pearson mode skewness of forecast densities of the Riksbank